Theory and experiment of two-photon 'direct' interference for type-II spontaneous parametric down-conversion with an ultrafast pulse laser pump

2002 J. Opt. B: Quantum Semiclass. Opt. 4 260

(http://iopscience.iop.org/1464-4266/4/5/306)

Institute of Physics Publishing | Journal of Optics B: Quantum and Semiclassical Optics

J. Opt. B: Quantum Semiclass. Opt. **4** (2002) 260–264 | PII: S1464-4266(02)34998-X

Theory and experiment of two-photon 'direct' interference for type-II spontaneous parametric down-conversion with an ultrafast pulse laser pump

S L Lu[1], B Liu, B Q Sun, D S Jiang and Z Y Xu

National Laboratory for Superlattices and Microstructures, Institute of Semiconductors, Chinese Academy of Sciences, PO Box 912, Beijing 100083, People's Republic of China

E-mail: lsljhx@red.semi.ac.cn(S L Lu)

Received 11 March 2002, in final form 21 May 2002
Published 11 July 2002
Online at stacks.iop.org/JOptB/4/260

Abstract

Fourth-order spatial interference of entangled photon pairs generated in the process of spontaneous parametric down-conversion pumped by a femtosecond pulse laser has been performed for the first time. In theory, it takes into account the transverse correlation between the two photons and is used to calculate the dependence of the visibility of the interference pattern obtained in Young's double-slit experiment. In this experiment, a short focal length lens and two narrow band interference filters were adopted to eliminate the effects of the broadband pump laser and improve the visibility of the interference pattern under the condition of nearly collinear light and degenerate phase matching.

Keywords: Spontaneous parametric down-conversion, femtosecond pulse laser, entangled two-photon state

1. Introduction

During the past few years, investigations of the correlated properties of entangled two-photon states have attracted much attention in quantum optics and information fields, such as quantum imaging, quantum cryptography and quantum teleportation [1–3]. Experimentally, spontaneous parametric down-conversion (SPDC) is an effective way of producing entangled photon pairs [4–8]. At the simplest level, almost all those experiments were performed by using a continuous-wave (cw) laser as the pump source. While the emission times of the two photons are well known with respect to one another, the absolute time of emission is completely random. The phenomenon depends on the fact that the process is pumped with a cw source, which has an essentially infinite coherence length [9]. In order to control the emission times of two photons to within a short period, one method is to use a femtosecond ultrafast pulse laser as a source to pump the nonlinear crystal to produce entangled photon pairs. It has been shown that this is crucial for performing important experiments in quantum teleportation [10], entangled swapping [11] and the production of three-photon entangled (GHz) states [12].

Meanwhile, spatial (transverse) correlated properties of entangled photon pairs have been extensively investigated, and almost all the research has concentrated on the case using a cw pump source [13–17]. But when the pump source is an ultrafast pulse laser, besides the coherence length of the pulse, consideration of multi-frequency components in the pump source is also needed. In contrast with cw-pumped down-conversion, the two photons produced in pulsed type-II down-conversion have different spectra. The existence of a broad range of k vectors in the pump results in the situation that, for a given single photon direction, there is a cone distribution of its conjugate photon. However, Young's double-slit arrangement provides an extension of point–point transverse correlation using an interference pattern. Therefore this makes it difficult to observe a perfect interference pattern under the condition of a pulse-pumped source.

[1] Author to whom any correspondence should be addressed.

In this paper, we investigate the dependence of the interference visibility of Young's interference pattern for entangled photon pairs generated in the process of SPDC pumped by a pulse laser. A direct interference pattern with high visibility of nearly 1 is also obtained.

2. Theory of two-photon direct interference and the derivation of interference visibility

The SPDC process occurs under the condition of phase matching when a laser pumps the nonlinear crystal. In this process, photons from a pump laser are converted into a pair of photons, where one is called the signal photon and the other is called the idler photon. Although the pumping laser photons may be highly coherent, neither the signal nor idler down-converted photons carry this coherence. The temporal and spatial correlations between the signal and idler photons are established at the photon source by the energy and phase matching constraints.

The process of down-conversion can be investigated in the interacting presentation and the evolution of the state vector can be expressed as

$$|\psi(t)\rangle = \exp\left[\frac{1}{\mathrm{i}\hbar}\int_{t_0}^{t}\mathrm{d}t'\hat{H}_I(t')\right]|\psi_0\rangle \qquad (1)$$

where $|\psi_0\rangle$ is the state vector at time t_0 and $\hat{H}_I(t)$ is the interacting Hamiltonian.

The Hamiltonian is depicted as

$$H_I(t) = \int_V \mathrm{d}^3r\,\chi^{(2)}E_p^{(+)}(r,t)E_o^{(-)}(r,t)E_e^{(-)}(r,t) + \mathrm{h.c.} \qquad (2)$$

where V is the volume of the crystal covered by the classical pump beam $E_p^{(+)}(r,t)$ and $\chi^{(2)}$ is an electric susceptibility tensor which describes the nonlinear coefficient of the crystal. The positive frequency part of

$$E_j(r,t) = \int \mathrm{d}\omega_j A(\omega_j)\hat{a}_j(\omega_j)\mathrm{e}^{\mathrm{i}[k_j(\omega_j-\omega_j t)]}, \qquad j = p, e, o$$

where $A(\omega_j) = \mathrm{i}\sqrt{\hbar\omega_j/2\varepsilon_0 n^2(\omega_j)}$ is a slowly varying function of frequency and can be taken outside the integral; $\hat{a}_j(\omega_j)$ is an annihilation operator and $E_j^{(-)}(r,t) = [E_j^{(+)}(r,t)]^{+}$.

The two-photon state can be written as [9]

$$|\Psi\rangle = \frac{2\pi A}{\mathrm{i}\hbar}\int_V \mathrm{d}^3r\,\chi^{(2)}\int \mathrm{d}\omega_o\int \mathrm{d}\omega_e\,\alpha(\omega_o+\omega_e)\hat{a}_e^{(+)}(\omega_e) \times \hat{a}_o^{(+)}(\omega_o)\mathrm{e}^{-\mathrm{i}\{[\vec{k}_o(\omega_o)+\vec{k}_e(\omega_e)-\vec{k}_p(\omega_0+\omega_e)]\cdot\vec{r}\}}|\psi_0\rangle + \mathrm{h.c.} \qquad (3)$$

where ω_o and ω_e are the circular frequencies of the down-converted o photon and e photon, respectively, $\alpha(\omega_o+\omega_e)$ represents the spectral envelope function of pump lasers and $\hat{\alpha}^{(+)}(\omega_o)$ and $\hat{\alpha}^{(+)}(\omega_e)$ are the creation operators of an o photon and an e photon, respectively.

The difference between 'direct' and 'ghost' interference lies in the position of the scanning detector in relation to Young's double slits. In the case of 'direct' interference, the coincidence interference fringe was obtained by scanning the signal detector placed behind Young's double slits. However, in the case of 'ghost' interference, the interference fringe was obtained by scanning the detector placed in the idler beam, although the slits were placed in the signal beam. The initial experiments were performed by Ribeiro *et al* [18] and Shih *et al* [19], respectively.

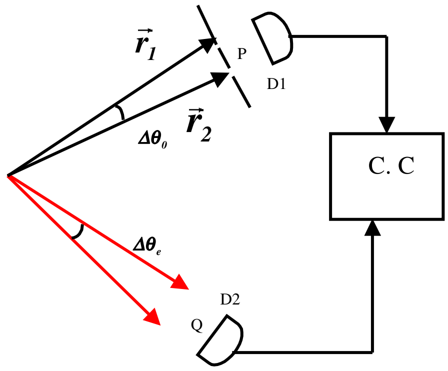

Figure 1. A simple schematic diagram of 'direct' interference is shown, where D1 is used to detect o photons that pass through the double slits and D2 is used to detect e photons.
(This figure is in colour only in the electronic version)

The illuminating schematic diagram of 'direct' interference is illustrated in figure 1. The type-II down-converted photon pairs were sent to a polarization beam splitter (PBS) to reflect the signal photons (o-ray) through Young's double slits into the detector D1 at the position P and to transmit the idler photons (e-ray) into the detector D2 at position Q.

Thus the electric field vector at the position P can be expressed as

$$\hat{E}_o^+(P,t) = \hat{E}_o^+(\vec{r}_1,t) + \hat{E}_o^+(\vec{r}_2,t-\tau_o) = \sum_{\omega_o}\hat{a}_o(\omega)\mathrm{e}^{-\mathrm{i}\omega_o t}\mathrm{e}^{\mathrm{i}\vec{k}_o(\omega_o)\cdot\vec{r}_1} + \sum_{\omega_o}\hat{a}_o(\omega)\mathrm{e}^{-\mathrm{i}\omega_o(t-\tau_o)}\mathrm{e}^{\mathrm{i}\vec{k}_o(\omega_o)\cdot\vec{r}_2} \qquad (4)$$

where $\hat{a}_o(\omega)$ is the annihilation operator of the o-ray at frequency ω, ω_o is the circular frequency and $\vec{k}_o$ is the wavevector of the o-ray. $\vec{r}_1$, $\vec{r}_2$ are the shifts from O to the double slits, respectively, as shown in figure 1.

The electric field vector at Q can be written as

$$\hat{E}_e^+(Q,t) = \sum_{\omega_e}\hat{a}_e(\omega)\mathrm{e}^{-\mathrm{i}\omega_e t}\mathrm{e}^{\mathrm{i}\vec{k}_e(\omega_e)\cdot\vec{r}_e}. \qquad (5)$$

It is similar to equation (8), where $\hat{a}_e(\omega)$ is the annihilation operator of the e-ray at frequency ω, ω_e is the circular frequency and $\vec{ke}$ is the wavevector of the e-ray. $\vec{r}_e$ is the displacement from the point O to Q.

The coincidence rate between the two photons being detected by the two detectors is proportional to the fourth-order coherence function. The fourth-order coherence function can be written as

$$\Gamma_\tau^{(2,2)} = \langle 0|\hat{E}_o^-(P,t+\tau)\hat{E}_e^-(Q,t)\hat{E}_e^+(Q,t)\hat{E}_o^+(P,t+\tau)|\Psi\rangle = |\langle 0|\hat{E}_e^+(Q,t)\hat{E}_o^+(P,t+\tau)|\psi\rangle|^2 \qquad (6)$$

where $\hat{E}_o^+(P,t)$ and $\hat{E}_e^+(Q,t)$ are electric field vectors at positions P and Q, respectively. τ is the relative delay between the o and e photons, which is determined by the crystal length.

Substituting equations (3), (4) and (5) into (6), we have

$$\Gamma_\tau^{(2,2)} \propto |\langle 0|\hat{E}_e^+(Q,t)\hat{E}_o^+(P,t+\tau)|\psi\rangle|^2$$
$$= \left| \begin{array}{c} \int \mathrm{d}\omega_e \int \mathrm{d}\omega_o \int \mathrm{d}\omega_e\, \alpha(\omega_o+\omega_e)\mathrm{e}^{-\mathrm{i}(\omega_o+\omega_e)t}\Phi(\omega_o,\omega_e,\omega_o+\omega_e) \\ [\mathrm{e}^{-\mathrm{i}[\vec{k}_e(\omega_e)\cdot\vec{r}_e+\vec{k}_o(\omega_o)\cdot\vec{r}_1]} + \mathrm{e}^{\mathrm{i}\omega_o\tau_o}\mathrm{e}^{\mathrm{i}[\vec{k}_e(\omega_e)\cdot\vec{r}_e+\vec{k}_o(\omega_o)\cdot\vec{r}_2]}]\mathrm{e}^{-\mathrm{i}\omega\tau} \end{array} \right|^2 \tag{7}$$

where

$$\Phi(\omega_o,\omega_e,\omega_o+\omega_e) = \int_v \mathrm{d}^3\vec{r}\, \mathrm{e}^{-\mathrm{i}[\vec{k}_o(\omega_o)+\vec{k}_e(\omega_e)-\vec{k}_p(\omega_o+\omega_e)]\cdot\vec{r}}.$$

Suppose that the quantum efficiencies of the two detectors are β_o and β_e, respectively, and the acceptance time of the coincidence counting circuit is T_R. Then the coincidence rate of the two photons can be written as

$$R_{oe} \propto \beta_o\beta_e \int_{-T_R/2}^{T_R/2} \mathrm{d}\tau$$
$$\times \left| \begin{array}{c} \int \mathrm{d}\omega_o \int \mathrm{d}\omega_e\, \alpha(\omega_o+\omega_e)\mathrm{e}^{-\mathrm{i}(\omega_o+\omega_e)t}\Phi(\omega_o,\omega_e,\omega_o+\omega_e) \\ [\mathrm{e}^{-\mathrm{i}[\vec{k}_e(\omega_e)\cdot\vec{r}_e+\vec{k}_o(\omega_o)\cdot\vec{r}_1]} + \mathrm{e}^{\mathrm{i}\omega_o\tau_o}\mathrm{e}^{\mathrm{i}[\vec{k}_e(\omega_e)\cdot\vec{r}_e+\vec{k}_o(\omega_o)\cdot\vec{r}_2]}]\mathrm{e}^{-\mathrm{i}\omega\tau} \end{array} \right|^2. \tag{8}$$

We can simplify the function (8) as

$$R_{oe} \propto \beta_o\beta_e|\alpha(\omega_o+\omega_e)|^2|\Phi(\omega_o,\omega_e,\omega_o+\omega_e)|^2$$
$$\times (1+\mu\cos(\omega_{o0}\tau_o)) \tag{9}$$

where μ is the interference visibility of the coincidence interference fringe and is given by

$$\mu = \frac{1}{2\Delta\omega}\int_{\omega_{o0}-\Delta\omega}^{\omega_{o0}+\Delta\omega} \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}\vec{k}_{o0}\cdot(\vec{r}_1-\vec{r}_2)}$$
$$\approx \frac{1}{2\Delta\omega}\int_{-\Delta\omega}^{+\Delta\omega} \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}k_{o0}d\cos[\vec{k}_{o0}\cdot(\vec{r}_1-\vec{r}_2)]} \tag{10}$$

$$d\cos\lfloor\vec{k}_{o0}\cdot(\vec{r}_1-\vec{r}_2)\rfloor \approx d\sin\Delta\theta_o \approx k_{o0}d\Delta\theta_o \tag{11}$$

where $\Delta\omega$ is the difference between the circular frequency and the central circular frequency which is induced by the broadband pulse laser, $\Delta\omega = \omega_o - \omega_{o0}$; d is the distance between the double slits and $\vec{k}_{o0}$ is the wavevector of the central wavelength. Within a small range of $\Delta\omega$, the separation angle $\Delta\theta_o$ between the wavevector $\vec{k}_o$ and the central wavevector $\vec{k}_{o0}$ is proportional to $\Delta\omega$, and this conclusion is also adopted for the e-ray. Therefore we have

$$\Delta\theta_o = m_o\Delta\omega_o, \qquad \Delta\theta_e = m_e\Delta\omega_e$$

where m_o and m_e are proportional coefficients.

Then the final expression for the interference visibility can be obtained as

$$\mu \propto \frac{1}{2\Delta\omega}\int_{-\Delta\omega}^{\Delta\omega} \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}k_{o0}dm_o\omega}$$
$$= \frac{1}{2\Delta\omega}\frac{1}{k_{o0}dm_o}(\mathrm{e}^{\mathrm{i}k_{o0}d\kappa_o\Delta\omega} - \mathrm{e}^{-\mathrm{i}k_{o0}d\kappa_o\Delta\omega})$$
$$= \frac{1}{\Delta\omega}\frac{1}{k_{o0}dm_o}\sin(k_{o0}d\Delta\theta_o) = \frac{\sin k_{o0}d\Delta\theta_o)}{k_{o0}d\Delta\theta_o}. \tag{12}$$

It can be confirmed that the interference visibility is mainly determined by $\Delta\theta_o$ under the condition that $k_{o0}d$ is a constant. It will be close to 1 if the value of $\Delta\theta_o$ is infinitesimal.

For the case of a broadband femtosecond pulse laser pump, we have to consider the effect induced by the bandwidth of the laser. Under the condition of phase matching, the conservation of momentum can be written as

$$k_o\cos\theta_o + k_e\cos\theta_e = k_p \tag{13}$$
$$k_o\sin\theta_o - k_e\sin\theta_e = 0. \tag{14}$$

For the sake of the existence of the uncertain quality of the bandwidth, $\Delta\vec{k}_p$, we differentiate the above equations, which can be derived into the formulae

$$\Delta k_o\cos\theta_o - k_o\sin\theta_o\Delta\theta_o + \Delta k_e\cos\theta_e - k_e\sin\theta_e\Delta\theta_e = \Delta k_p \tag{15}$$

$$\Delta k_o\sin\theta_o + k_o\cos\theta_o\Delta\theta_o - \Delta k_e\sin\theta_e - k_e\cos\theta_e\Delta\theta_e = 0. \tag{16}$$

Then the final relation between $\Delta\theta_o$ and $\Delta\theta_e$ is

$$\Delta\theta_o = \frac{k_e\cos(\theta_e+\theta_o)}{k_o}\Delta\theta_e + \frac{\sin(\theta_e+\theta_o)}{k_o}\Delta k_e - \frac{\sin\theta_e}{k_o}\Delta k_p \tag{17}$$

where the first, second and third terms express the uncertain relation of the output direction, the spectrum of the e-ray and the pulse laser source, respectively. It demonstrates that, for the case of the pulse laser pump, the uncertain conditions (emitting direction) of the o-ray are not only determined by the uncertain properties of idler photons, but are also determined by the spectrum distribution of the pump laser. So, just as depicted in the introduction, it will degrade the visibility of the interference pattern. In order to improve the visibility, we use a short focal length lens and two interference filters to eliminate the effect of broad bandwidth.

3. Experimental set-up

The schematic experimental set-up is illustrated in figure 2, which is similar to the 'direct' experimental set-up proposed by Ribeiro *et al* [18] except that a short focal length lens, whose focal length is only 60 mm, is placed in front of the stationary detector D2. An argon-ion laser with 780 nm wavelength serves as a pump for the mode-locked Ti–sapphire laser whose light is doubled in a second-harmonic generator. It results in a train of 80 fs pulses with an 80 MHz repetition rate at a central wavelength of 390 nm. A beta-barium borate (BBO) crystal with 3 mm length is cut at a degenerate type-II phase matching angle to produce a pair of orthogonally polarized signal (o-ray of the BBO) and idler (e-ray of the BBO) photons ($\lambda_o = \lambda_e = 780$ nm, where λ_o, λ_e are the wavelengths of the o-ray and e-ray photons, respectively). The pair of photons emerge from the crystal nearly collinear. The residual pump light is then separated from the down-converted photons by a dispersion prism and the o-ray and e-ray are sent in different directions by a PBS. The o-ray passes through Young's double slits (slit width $a = 0.15$ mm and slit distance $d = 0.3$ mm) and is detected by the detector D1, which can scan transversely, while the e-ray is detected by the stationary detector D2, which is exactly located at the focal point of the lens L. In our experiment single-photon counting modules (SPCM-14) from EG&G with 60% quantum efficiency at 780 nm were used. In front of the two detectors are placed two interference filters with bandwidths of 4 nm FWHM centred at the degenerate wavelength 780 nm. The output pulses of each detector are sent to a coincidence counting circuit with a 4 ns acceptance time window for the two-photon coincidence measurement.

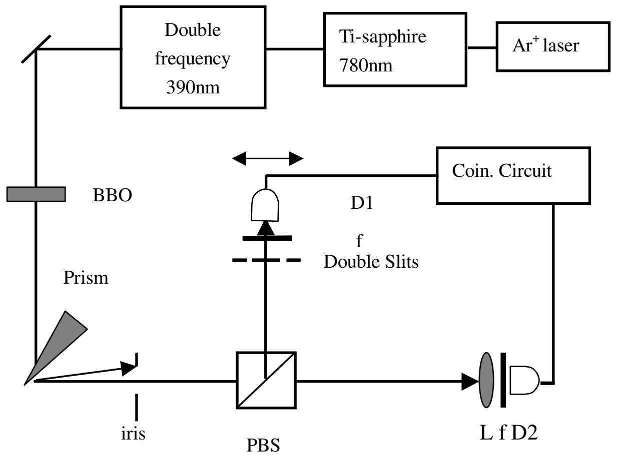

Figure 2. Layout of the experiment, where the detector D1 can scan transversely while the detector D2, which is exactly located at the focal point of the short focal length lens L, remains stationary. Young's double slits are placed in front of the detector D1.

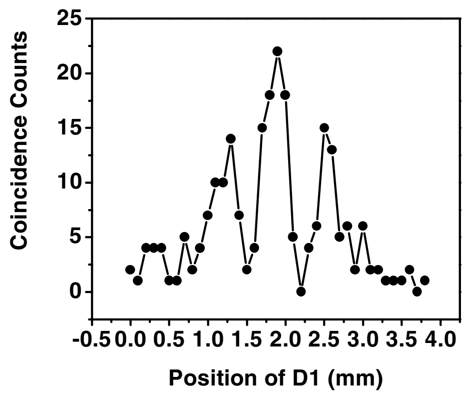

Figure 3. Typical observed interference pattern: the dependence of the coincidence on the position of D1. Every measurement is taken for 120 s.

4. Experimental results and discussion

By recording the coincidence counts as a function of the position of the scanning detector, the typical interference pattern without any correction is observed, as shown in figure 3. As can be seen from this figure, the interference visibility is close to 1, which demonstrates that we have effectively improved interference visibility by using a short focal length lens and two interference filters under the condition of nearly collinear light and degenerate phase matching.

In fact, the uncertain relation between the wavevectors of the *o*-ray and *e*-ray is also much larger even in the case of a cw-pumped laser. Thus the *o-ray* and *e-ray* both express statistical properties that are similar to noise. So the interference pattern would not be observed on the screen if only a single beam was employed to perform the single-photon Young's double-slit interference. We can limit it by positioning a small hole on the path of the idler, e.g. just as shown in [15], which is performed under the cw condition to control the visibility of the interference pattern and one detector acts as a source for its conjugate. This conclusion is also adapted for direct interference [16]. The larger the collection hole was, the worse the visibility was.

However, for the pulse laser pump, in order to control the value of $\Delta\theta_o$, we must decrease the influence of the third term on the emitting direction of the *o*-ray. This cannot be obtained by limiting the collection hole before the detector. In the experiment, we use a short focal length lens to improve the visibility of the interference pattern. Because of focusing and the small effective acceptance diameter (only 0.2 mm), in addition, the focal length of the lens is only 60 mm. These conditions can ensure only those photons whose wavevectors are parallel or nearly parallel can be detected by detector D2, which determines that $\Delta\theta_e$ and θ_e are both minimum (under the conditions of collinear and degenerate phase matching). Therefore $\Delta\theta_o$ is also minimum through coincidence measurement. The second term of equation (5) is limited by a narrow band filter. Under this condition, we have obtained a perfect interference pattern with high visibility using coincidence measurement.

In [16], the finite dimensions of the light source and detectors were also taken into account and shown to be responsible for the partial coherence that affects the visibility of the fringes. In addition, in the literature, in order to simplify the case, the pump, signal and idler fields are quasimonochromatic. For the case where the pump beam diameter is much larger than the dimensions of the idler detector and the slit width, the visibility of the interference pattern did not always equal 1 unless the diameter of the detector can be regarded as a point source. In our experiment, the dimension of the pump beam incident on the nonlinear crystal is about 2 mm, i.e. much larger than the slit width (0.15 mm) and the diameter of the idler detector (0.2 mm). That is to say, we can hardly obtain a high visibility interference pattern by using the traditional experimental conditions. However, the interference visibility can be improved using our experimental set-up.

5. Conclusions

In conclusion, we have investigated direct Young's interference of entangled photon pairs generated in the process of SPDC pumped by a femtosecond pulse laser in theory and derived the dependence of the interference visibility by considering the broad bandwidth of the pulse laser. The interference visibility depends on the magnitude of $\Delta\theta_o$. In our experiment, a short focal length lens and two interference filters were adopted to improve the interference visibility under the condition of nearly collinear light and degenerate phase matching to obtain the perfect interference pattern.

Acknowledgment

This research was supported by the innovative foundation of the Chinese Academy of Sciences.

References

[1] Pittman T B, Shih Y H, Strekalov D V and Sergienko A V 1995 *Phys. Rev.* A **52** R3429
[2] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[3] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W 1993 *Phys. Rev. Lett.* **70** 1895
[4] Souto Riberiro P H, Padua S, Machado da Silva J C and Barbosa G A 1995 *Phys. Rev.* A **51** 1631

[5] Strealov D V, Sergienko A V, Klyshko D N and Shih Y H 1995 *Phys. Rev. Lett.* **74** 3600
[6] Joobeur A, Salech B E A and Teich M C 1994 *Phys. Rev.* A **50** 3349
[7] Joobeur A, Salech B E A, Larchuk T S and Teich M C 1994 *Phys. Rev.* A **53** 4360
[8] Pittman T B, Strealov D V, Klyshko D N, Rubin M H, Sergienko A V and Shih Y H 1996 *Phys. Rev.* A **53** 2804
[9] Grice W P and Walmsley I A 1997 *Phys. Rev.* A **56** 1627
[10] Bouwmeester D, Pan J-W, Mattle K, Eibl M, Weinfurter G and Zeilinger A 1997 *Nature* **390** 575
[11] Pan J-W, Bouwmeester D, Weinfurter H and Zeilinger A 1998 *Phys. Rev. Lett.* **80** 3891
[12] Greenberger D M, Horne M and Zeilinger A 1989 *Bell's Theorem, Quantum Theory and Conceptions of the Universe* ed M Kafatos (Dordrecht: Kluwer)
[13] Rubin M H 1996 *Phys. Rev.* A **54** 5349
[14] Barbosa G A 1996 *Phys. Rev.* A **54** 4473
[15] Souto Ribeiro P H and Barbosa G A 1996 *Phys. Rev.* A **54** 3489
[16] Souto Ribeiro P H 1997 *Phys. Rev.* A **56** 4111
[17] Fonseca E J S, Souto Ribeiro P H, Padua S and Monken C H 1999 *Phys. Rev.* A **60** 1530
[18] Souto Ribeiro P H, Padua S, Machado da Silva J C and Barbosa G A 1994 *Phys. Rev.* A **49** 4176
[19] Shih Y H, Sergienko A V, Pittman T B, Strekalov D V and Klyshko D N 1995 *Ann. NY Acad. Sci.* **755** 121
Strekalov D V, Sergienko A V, Klyshko D N and Shih Y H 1995 *Phys. Rev. Lett.* **74** 3600