ZEEMAN EFFECT

Fabry-Perot Spectroscopy: Observation of the Anomalous Zeeman Effect in Mercury 5461Å Line

Objective
This experiment employs a Fabry-Perot interferometer used as a high resolution spectrograph to observe the anomalous Zeeman splitting of the 5461Å line of the mercury spectrum. An electromagnet is provided to allow measurement of the effect at several different magnetic field values.

It should be noted that the 5461Å line of mercury will exhibit the anomalous, not the normal, Zeeman effect and will be split into nine, as opposed to three, components. A polarizer is provided to allow the isolation of either three $\Delta m = 0$ lines or six $\Delta m = \pm 1$ lines.

A linear correlation between the magnetic field strength and the width of the splitting should be found. An experimental value of $\mu_0/\hbar c$ should also be determined.

For a more detailed consideration of high-resolution spectroscopy using Fabry-Perot interferometer, refer to Chapter 7 in Melissinos I.

Experimental Procedure
All optical components are placed on a single rail, which should ease the alignment procedure. Figure 1 shows a diagram of the apparatus.

1. Using the power supply provided, obtain a calibration curve of the electromagnet. Do not exceed a current of more than 5 amperes or a potential difference of much above 70 volts, whichever comes first.

2. Magnetic field strength can be obtained by using a Hall probe. Make sure to first calibrate the probe using a reference magnet. The power supply has a more precise built-in voltmeter than an ammeter; therefore, obtain the magnetic field strength at different voltages.

3. Position the discharge lamp in the center of the magnet gap. The lamp is excited by a 15 kV neon sign transformer. Use caution in adjusting the lamp while it is on, since the resulting voltage can give a very unpleasant, although not lethal, shock.

Figure 1: Suggested setup for the Zeeman effect lab
4. Place the light shield in front of the magnet. On the rail, place \( L_1 \) and the Fabry-Perot interferometer. \( L_1 \) has a focal length of 209 mm. Place the lens around this distance away from the discharge tube. Also place the green filter in between the lens and the interferometer. Turn on the lamp and look into the interferometer. A set of concentric rings should be visible. Adjust the orientation of the interferometer to center the pattern.

5. In order to obtain correct data, the interferometer may need to be aligned so that its plates are perfectly parallel. This can be accomplished by using the three screws on the interferometers. **Do not attempt this alignment without first consulting with an instructor!** While making adjustments to the interferometer, it will not be necessary to turn any of the three screws more than one-quarter turn in either direction. As you turn the screw, the image will contract or expand. First, adjust the two screws on the bottom of the interferometer so that the ring pattern neither expands nor contracts with any horizontal movement of the eye. A slight distortion can be expected near the edge of the plates, but otherwise the pattern should remain constant. Now, adjust the third screw so that the pattern remains constant with any vertical movement of the eye. Check the horizontal adjustment again and readjust if necessary. Continue the adjustment until the ring pattern is constant with any movement of the eye.

6. The interferometer is now correctly aligned. A sharp, symmetrical pattern of concentric rings should be visible. This is the most critical adjustment of the experiment. The quality of data depends mostly on the sharpness of the ring pattern.

7. Remove the green filter from the rail. Place \( L_2 \) behind the interferometer. \( L_2 \) also has a focal length of 209 mm. Place the microscope lens approximately one focal length away from \( L_2 \); place the green filter in between. Look into the microscope lens. A magnified image of the ring pattern should be visible. Adjust the microscope lens and/or \( L_2 \) to get the image in focus. This image needs to be properly focused; the CCD camera does not see the image any better than a naked eye can.

8. Once the image is clear in the microscope lens, place the polarizer.

9. Place the CCD camera behind the microscope lens.

10. Power up the computer and run kSA software for CCD image acquisition. Using single image acquisition mode, take several images of the ring pattern adjusting the following variables until a satisfactory image is obtained.

- CCD camera focal length and aperture.
- \( L_1 \), the interferometer and \( L_2 \) placements and apertures.
- Image exposure time.
- Optical railing alignment.

Suggested starting setup (approximate distances, in cm, on rail from the lamp):

<table>
<thead>
<tr>
<th>( L_1 )</th>
<th>FP</th>
<th>( L_2 )</th>
<th>Filter</th>
<th>Polarizer</th>
<th>Lens</th>
<th>CCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>41</td>
<td>58</td>
<td>65</td>
<td>79</td>
<td>83</td>
<td>97</td>
</tr>
</tbody>
</table>

- Polarizer 90 degrees
- CCD focal length: Infinity (for no B field), 4 m (with applied B field)
- CCD aperture: 4
11. A successful image should look like those in Fig. 7.27 of Melissinos I (Fig. 6.17 of Melissinos II).

12. The image needs to satisfy the following requirements:
   - First ring must be visible in full. This is needed to obtain the value of the center of the concentric rings.
   - At least five sets of concentric rings must be visible.
   - The experimental setup tends to produce images that are more clearly focused along a horizontal axis. Align the optical components so that the center of the image roughly corresponds to this line of improved focus.

13. Once a satisfactory image is obtained with no magnetic field, power up the power supply. As you turn up the magnet voltage, you will notice that rings split. Rotate the polarizer so only 3 splitting $\Delta m = 0$ lines are visible, which should occur with the polarizer arm placed horizontal to the table. The image tends to get somewhat brighter as the magnetic field is turned up. Adjust the aperture and exposure times accordingly.

14. Obtain images at least three different magnetic field strengths. Make sure the splitting is clearly visible for analysis.

**Data Analysis**

1. Plot the magnet calibration data (magnetic field strength vs. applied voltage). Confirm that there is a linear relationship between the applied voltage and the magnetic field strength.

2. Using the kSA software Line Profile tool, determine the center of the concentric rings. This can be accomplished by analyzing the first ring which should be visible in full. Make sure to take measurements along the diameter of the ring. Repeat measurements may be necessary.

3. Correct analysis of the data requires the determination of the relative radius of each ring. On the Line Profile tool, extend the analysis line horizontally from the center. Using the Data Cursor, read the column numbers of all visible and identifiable rings. It is useful to shrink the analysis line to isolate each set of rings. Make sure the analysis line remains horizontal along the center of the rings.

4. Repeat steps 2 and 3 for all images.

5. Determining the fractional order at the center:
   a) Just as the first ring is labeled as first order, an order is assigned to the center of the rings and this value is called the fractional order at the center. It is fractional since the interference order at the center is in general not an integer. (Hence there is no bright interference pattern at the center.)
   b) The fractional order at the center is used to compare the overall shifting of the ring pattern. By comparing the fractional order at different magnetic field strengths, the effect of a magnetic field application to the ring pattern can be determined.
   c) The fractional order at the center is determined as the intercept on the ring order axis on the plot of the ring order vs. relative radii squared.
d) Determine the radius of each ring by using the column value of each ring as determined in step 3 and of the center as determined in step 2. Tabulate the radius squared vs. the ring order.
e) Plot the ring order vs. radius squared. Fit a linear line and determine the intercept on the ring order axis. This is fractional order at the center.
f) Repeat the procedure for all other images. For images with an applied magnetic field, use only the central component (‘b’).

6. Determining the Zeeman splitting:
a) The Zeeman splitting data will be analyzed using square array reduction method as described in Melissinos I.
b) For images with the applied magnetic field, set up the square array as Table 7.4 described on page 319 of Melissinos I.
c) After tallying the data in the array, determine \(<\Delta>, <\delta_{ab}>\) and \(<\delta_{bc}>\). Remember to take every other column of \(\Delta\) to determine \(<\Delta>\). If there are any \(\Delta, \delta_{ab}\) and/or \(\delta_{bc}\) with significant deviation from other values, they can be discarded.
d) The wave number separation, \(\Delta v\), is calculated as:
\[
\Delta v_{ab} = \frac{<\delta_{ab}>}{2t <\Delta>}
\]
where \(t\) is the Fabry-Perot interferometer spacing. (Remember that the Fabry-Perot interferometer has a spacer of width 0.28 ± 0.02 cm).
e) Plot the wave number separation against applied magnetic field values. To produce a chart similar to Melissinos I, it may be necessary to label ‘bc splitting’ value as negative. Fit a linear curve with the intercept at 0 to determine \(\Delta v/B\) values for both ‘ab’ and ‘bc’ splitting. Take the average of ‘ab’ and ‘bc’ splitting to get the central line.

7. Determining the experimental value of \(\mu_0/\hbar c\):
   a) Once \(\Delta v/B\) values for both ‘ab’ and ‘bc’ splitting are determined, take the average. This is the experimental \(\Delta v/B\) value.
   b) Since \(\Delta v = \mu_0 B / 2\hbar c\) (Equation 6.2, page 326 in Melissinos I), the experimental value of \(\mu_0/\hbar c\) can be determined. [Don’t forget (0,0) is a point!]
   c) Compare the experimental value with the accepted theoretical value of:
   \[
   \mu_0/\hbar c = 4.669 \times 10^{-5} \text{ cm}^{-1} / \text{gauss}
   \]

For a more detailed explanation and procedure for data analysis, refer to Chapter 7, Section 6 in Melissinos I.

THE YELLOW DOUBLET LINES OF MERCURY (if a 3-week experiment)

To observe the mercury yellow lines remove the green interference filter from the optical path and insert the yellow interference filter mounted in the heavy brass cylinder into the output hole of the interferometer. Note that there will now be two sets of interference rings present since the filter passes both of the yellow lines. With luck the two sets of rings may be conveniently spaced so that their Zeeman patterns do not overlap at moderate fields. If not, you will have to change the spacing of the mirrors slightly.

Measure the structure and splitting of both components of the mercury yellow doublet. This will require special care because of possible overlapping of the two interference patterns at high field values. Note the slight difference in the Zeeman patterns of the 5769.6 Å and 5461 Å lines. How do they differ and why?
Additional Questions

1. Present a classical reason why the triple splitting of the spectrum (i.e., the normal Zeeman effect) occurs. Why would the three split components be polarized the way they are? Why would the central component be missing if viewed along the pole of the magnet? (Hint: Try to imagine a randomly orbiting, photon-emitting electron under a magnetic field.)

2. Did you confirm that the central component of concentric rings remain stable with the application of magnetic fields?

3. Did you observe a linear relationship between the splitting strength and the applied magnetic field value?

4. Did you obtain an acceptable experimental value for $\mu_0/hc$?

5. Why is the lamp put at the focal plane of the first lens?

6. Construct energy level diagrams of the mercury transitions and identify the observed Zeeman component frequencies with the various allowed transitions between the magnetic substates.
APPENDIX A: THEORY OF THE FABRY-PEROT INTERFEROMETER

A Fabry-Perot interferometer consists of two precisely parallel glass plates with optically flat and highly reflective surfaces facing one another, as illustrated in Figure 6. To use it as a spectrometer one must have, in addition, a lens to focus parallel rays to a point in its image plane and a magnifying eyepiece for examining the intensity pattern of light in the focal plane, i.e. a telescope.

FIG. 6: Geometrical optics of the Fabry-Perot interferometer. Only one of many multiply reflected paths is shown.

Consider a ray of light of wavelength $\lambda$ emitted by an excited mercury atom at $S$ and making an angle $\theta$ with the axis, is incident on the Fabry-Perot from the left at the point $A$. It will be partially transmitted at each of the two Fabry-Perot mirror surfaces, and will arrive at $P$ after passing through the telescope lens. The portion of the ray reflected at $B$ will be reflected again at $C$ and partially transmitted at $E$. It will enter the telescope lens parallel to the original ray and will be focussed to the same point $P$ after having traversed an additional distance $2D\cos\theta$. If this additional distance is an integer number of wavelengths, i.e.,

$$2D\cos\theta = m\lambda \quad (7)$$

then the two rays (and all the additional multiply reflected rays) will interfere constructively when brought to the focus at $P$. Constructive interference among all the multiply reflected rays passing through the interferometer at an angle $\theta$ to the axis will produce a circle of interference maxima in the focal plane, i.e., a bright ring of the $m$th order of interference. If the separation of the plates is increased, then the angular radius of the $m$th order ring will expand so that the decrease in $\cos\theta$ compensates for the increase in $D$. 