Mathematical Modelling of Social Dynamics

A physicist perspective

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Outline:

1. Physics, Complexity, and Social Sciences: Sociophysics?
2. Axelrod´s model for cultural dissemination
3. Schelling´s model of segregation
4. Game Theory. Social and strategic interactions

*T. C. Schelling, J. Math. Soc. **1**, 143-186 (1971)*

Micromotives and Macrobehavior, Norton, N.Y. 1978

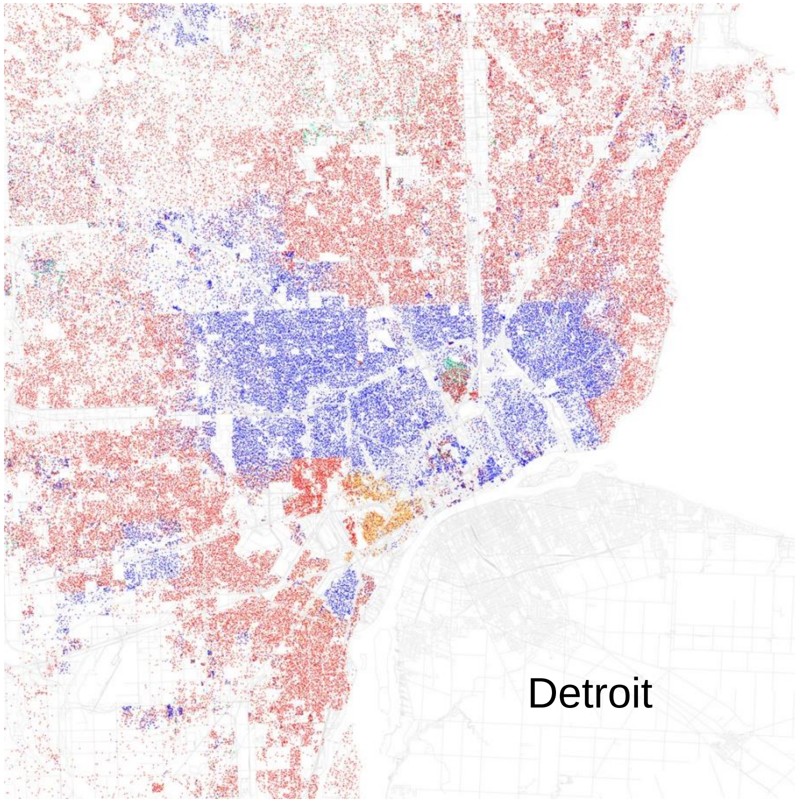
Segregation

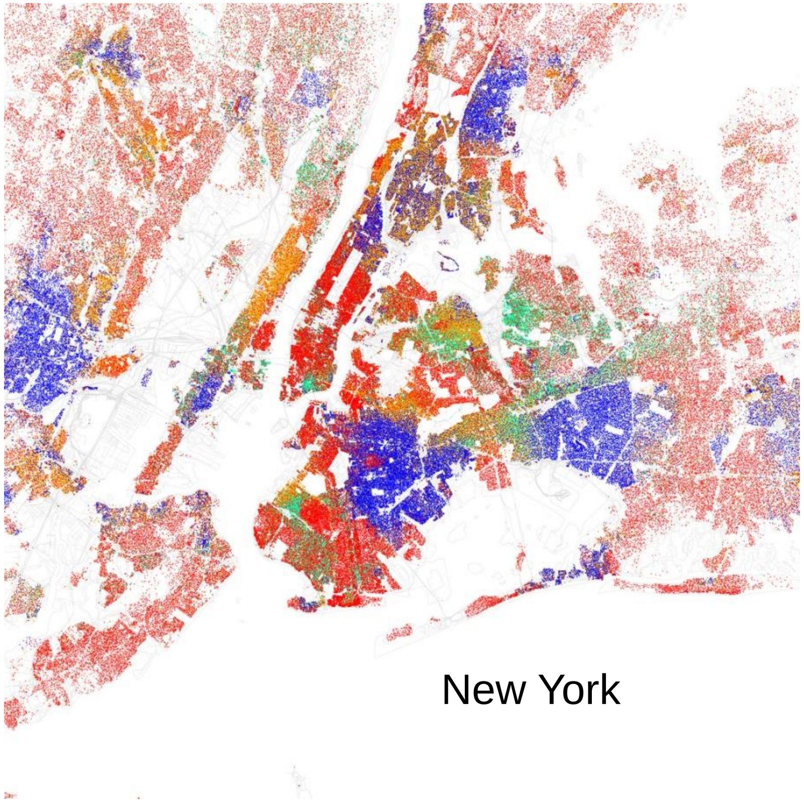
Residential areas in the US are predominantly inhabited by black or by white people. Why?

How are ghettos formed?

Is this a consequence of racial prejudice and individual intolerance?

Data from Census 2010





White
Asian
Black
Hispanic
Other

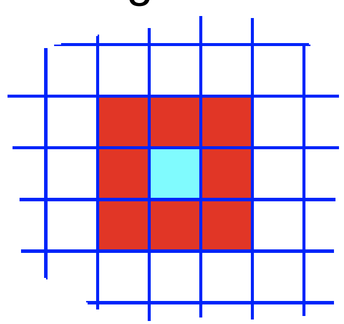
Flickr *Eric Fisher*

Two types of agents: black/ white, rich/poor, boy/girl, etc.

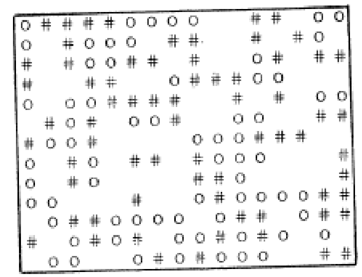
Agents on a lattice, with a density ρ_0 of **empty** sites

Tolerant agents: if at least a fraction *f=1/2* of neighbors is of the same group, the agent is happy and does not move,
otherwise it moves to a free and satisfactory neighboring site

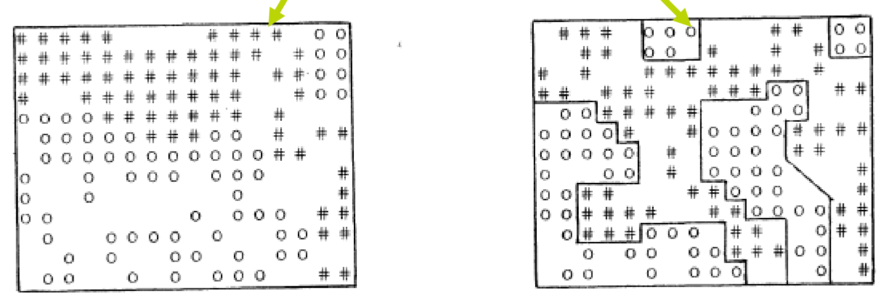
Moore's neighborhood



Random initial condition

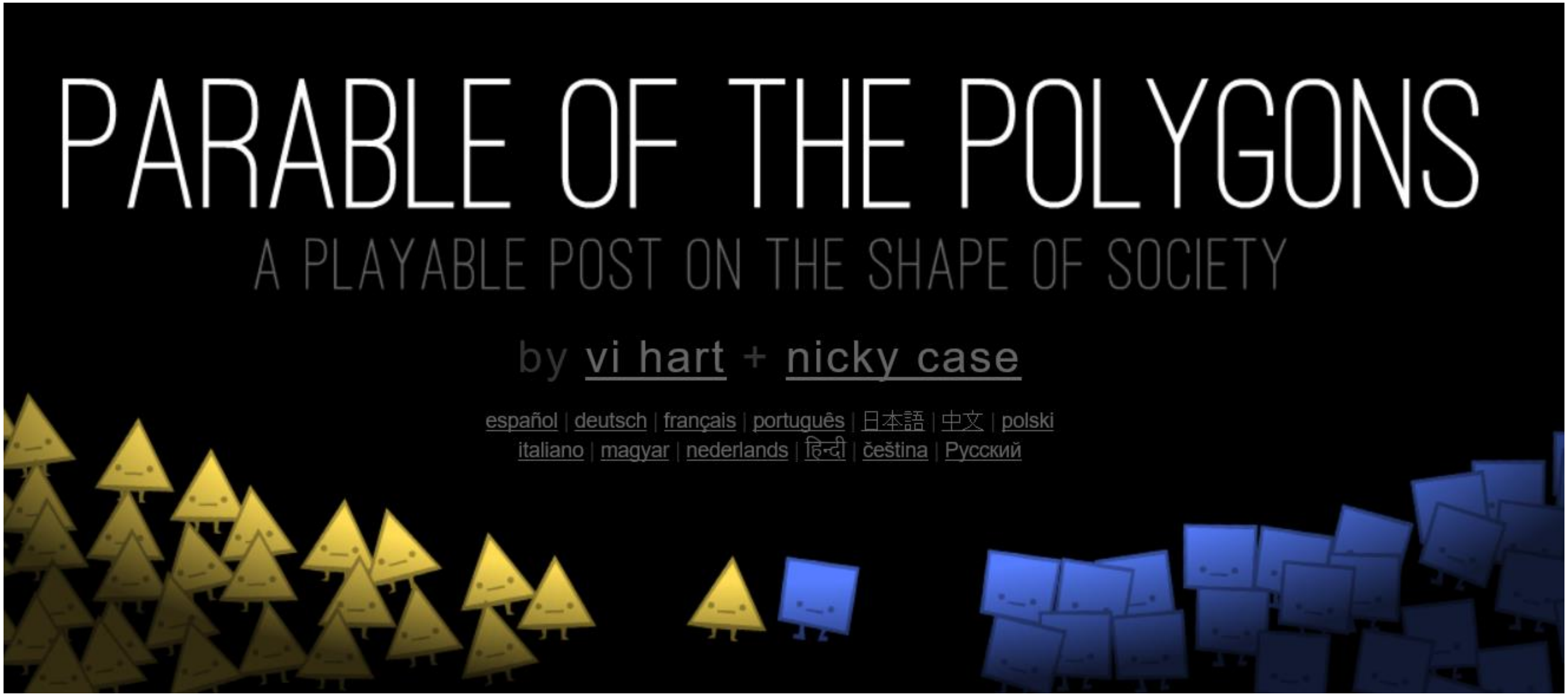


2 possible final segregated configurations



T. C. Schelling, J. Math. Soc. ***1****, 143 (1971)*

This is a story of how harmless choices can make a harmful world.



http://ncase.me/polygons/

Importance of Schelling´s model

Robust result: Segregation occurs even when individuals have a very mild preference for neighbors of their own type: System evolves from an initially well-mixed population to one that is highly segregated.

It is wrong to infer from the highly segregated society that is produced, that the agents in that society must be extremely prejudiced.

-It was one of the first models of a complex system to show emergent behavior due to interactions among agents.

-Simplicity: Status of Ising model in social context

-The 'hand-made simulations' done by T. Schelling by moving pawns on a chessboard can be considered as the first agent-based simulations ever done in social science

-General question of mechanisms of clustering (related to KI models) and group formation (social, coevolution,…)

-Concept of individual threshold (Granovetter)

Long range vs. short range (neighborhood) motion:
Original model: short range implying segregation of empty sites
Realistic model of spatial segregation: long range

Constrained: motion of unhappy only if satisfying site is found (original)
Solid-like (flows stop) Frozen small clusters
Unconstrained: Several versions not requiring increase of happiness.
Liquid-like Large clusters and full segregation can occur

Short range unconstrained motion: (unhappy move locally with no constraint)

Two parameters:
- -Density or number of vacancies
- -Threshold of similarity for happiness

L. Gauvin et al. Eur. Phys. J. B 70, 293 (2009)

Noisy Constrained model (long range):

At each time step an individual chosen at random is swapped with a randomly chosen vacancy which has a satisfying neighborhood.

Constrained: happy after moving, but happy and unhappy can move.

Noise effect: Motion of happy. Global utility can decrease allowing for full segregation

ρ Density of vacancies

T similarity threshold

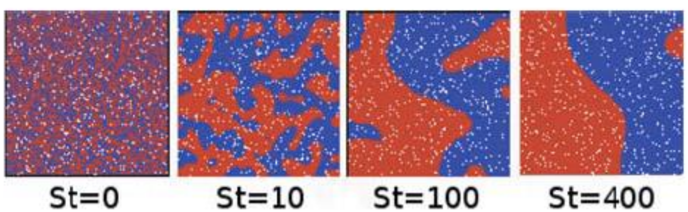


Fig. 1. Evolution of the configuration for a vacancy concentration $\rho = 5\%$ and a tolerance $T = 0.5$ with a network size $L = 100$. St stands for the number of time steps. The red and blue pixels correspond to the two types of agents, the white pixels to the vacancies. The system evolves from a random configuration – where the vacancies and the two types of agents are intimately mixed – to a completely segregated configuration. After just 10 steps there exist two percolating clusters, one of each color, which are very convoluted, fractal-like.

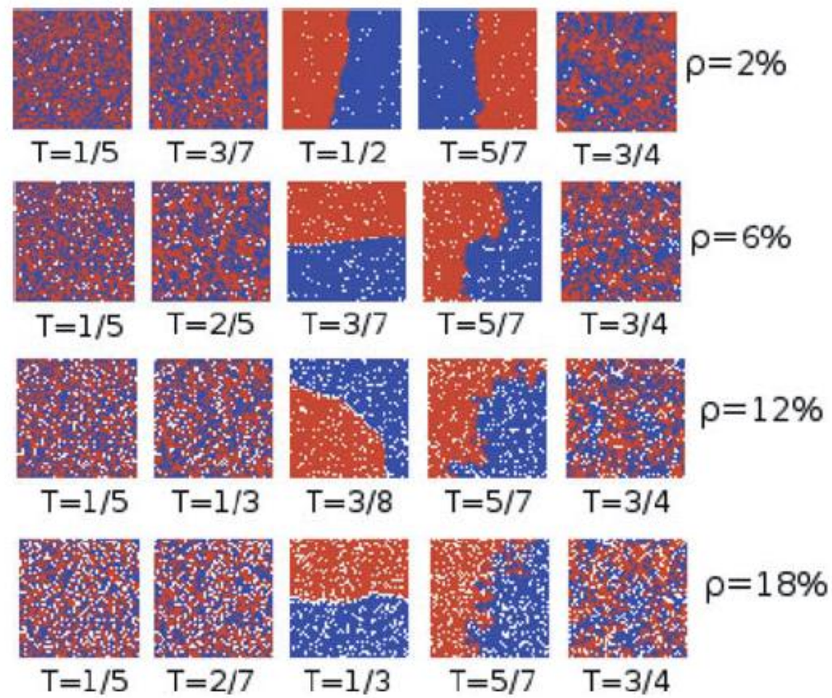


Fig. 2. Configurations obtained at large times for selected values of ρ and T.

Final eq configurations

L. Gauvin et al. Eur. Phys. J. B 70, 293 (2009)

Noisy Constrained model (long range):

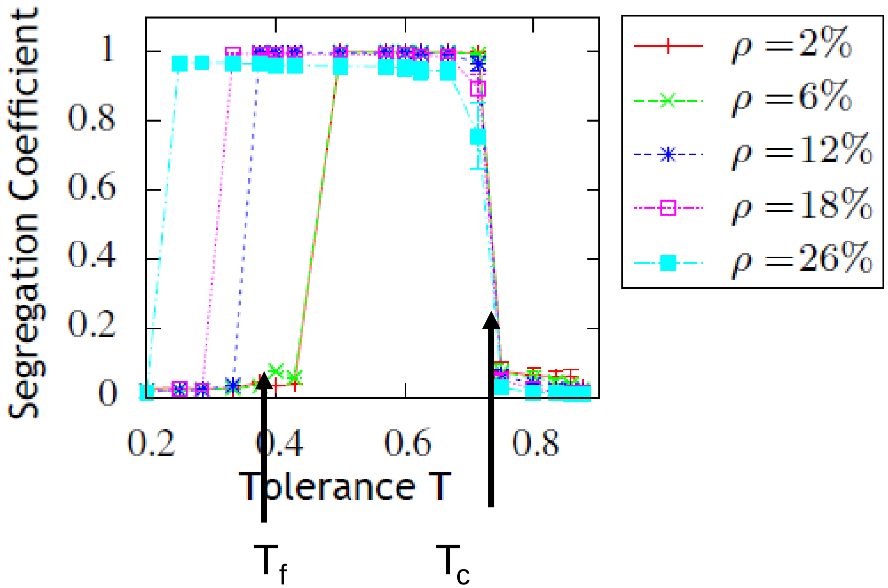
Order parameter: Segregation coeff. <s>

$S = \sum_{\{c\}} n_c \, p_c$ Size of clusters

$p_c = \frac{n_c}{N_{tot}}$ Weight of cluster c with n_c agents

$N_{tot} = L^2(1-\rho)$ Number of agents

$$s = \frac{2}{L^2(1-\rho)} S = \frac{2}{(L^2(1-\rho))^2} \sum_{\{c\}} {n_c}^2$$



Two sharp transitions for each ρ:

T_f: From frozen mixed to segregated

T_c: From segregated to mixed

L. Gauvin et al. Eur. Phys. J. B 70, 293 (2009)

Noisy Constrained model (long range):

Phase diagram:

Four different phases

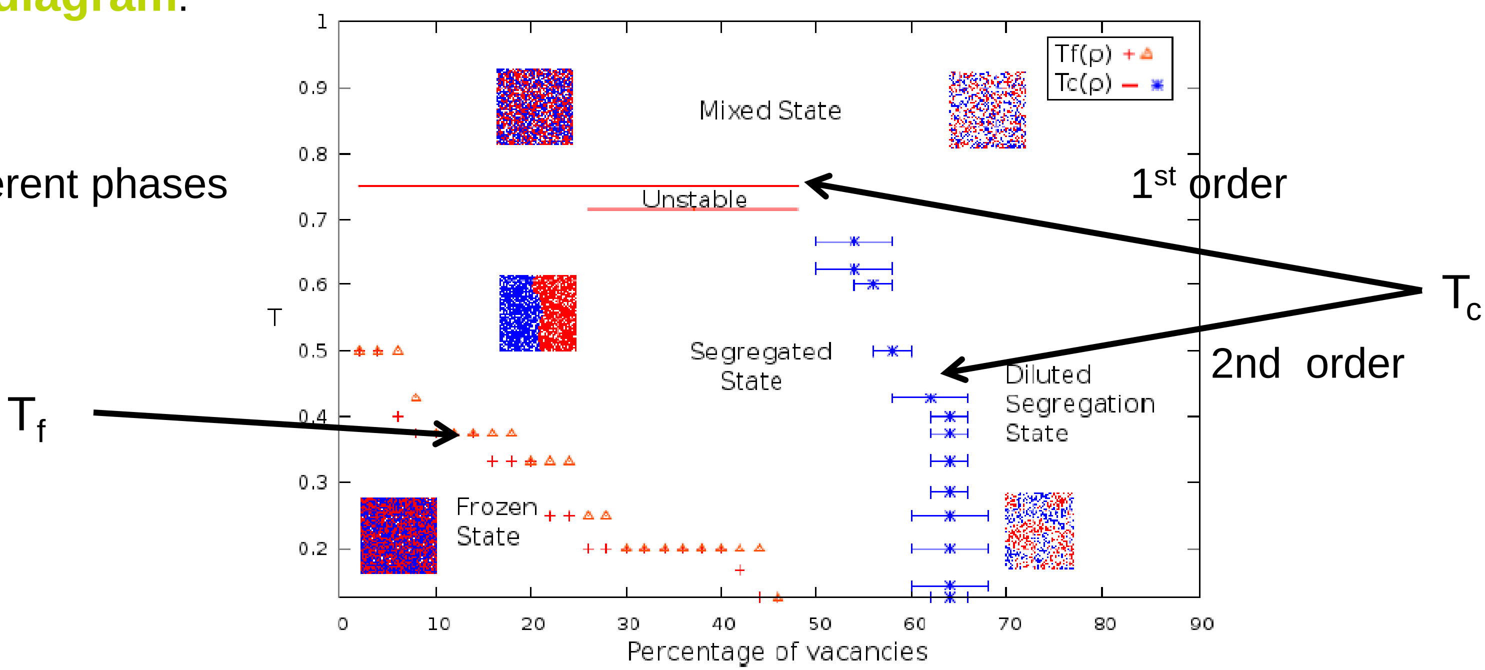


Figure 10: Phase diagram of the studied Schelling model. The blue crosses correspond to the continuous transition between the segregated and dilute segregated states. The red triangles and pluses are the upper and lower limits of the transition between the frozen and segregated states. The red lines separate the segregated state from the mixed one. Note that the tolerance T only takes discrete values.

Noisy Constrained Model

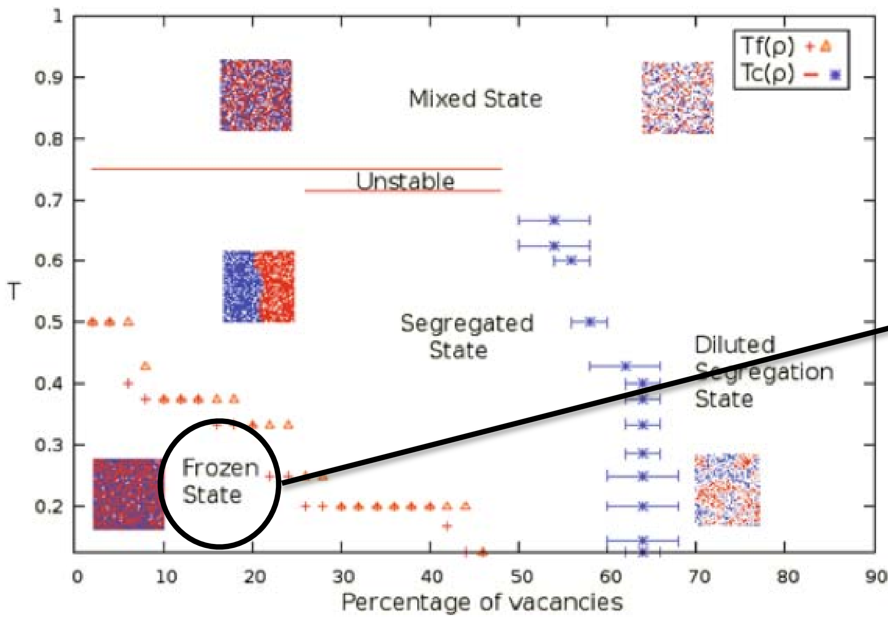
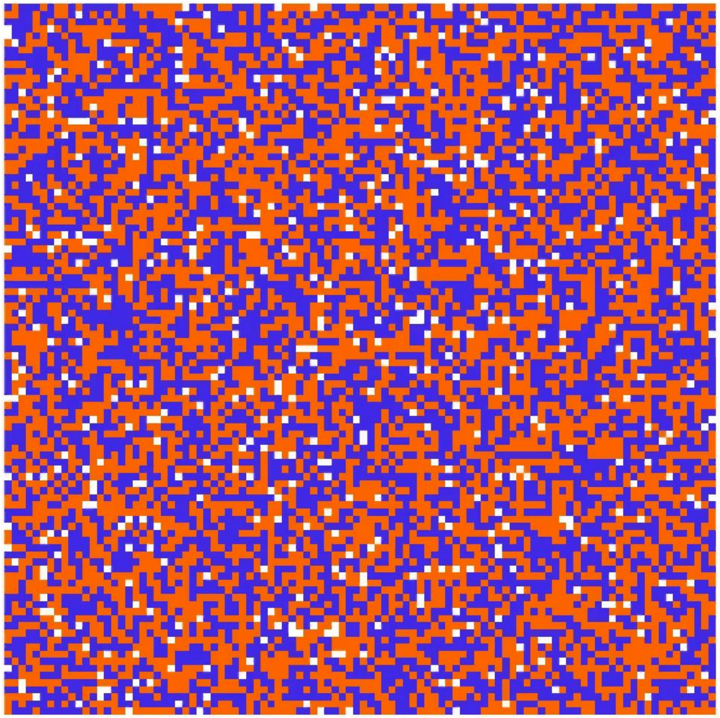
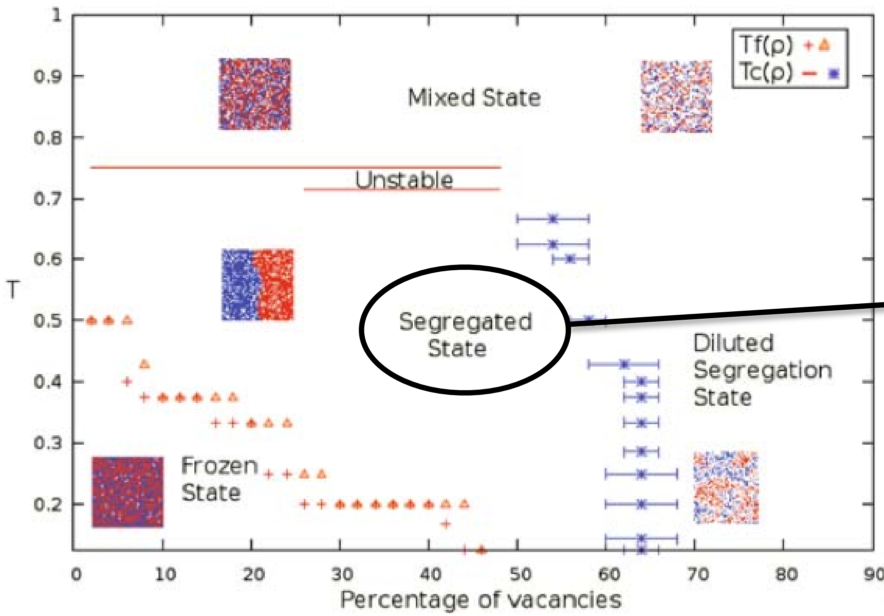


Fig. 10. Phase diagram of the studied Schelling model. The blue crosses correspond to the continuous transition between the segregated and dilute segregated states. The red triangles and pulses are the upper and lower limits of the transition between the frozen and segregated states. The red lines separate the segregated state from the mixed one. Note that the tolerance T only takes discrete values.



$$L = 100,\ T = 0.125,$$
$$\rho_0 = 0.05,\ time = 10^3 ts$$

Noisy Constrained Model



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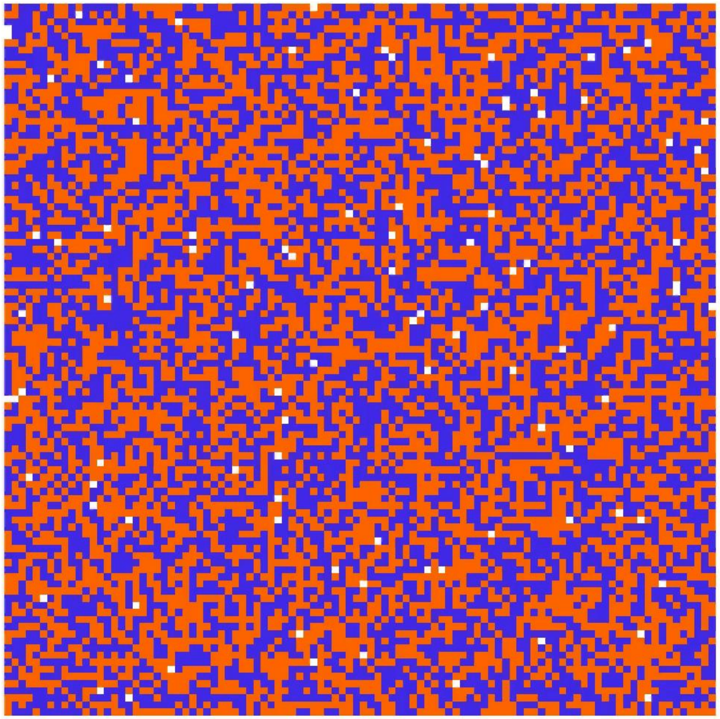
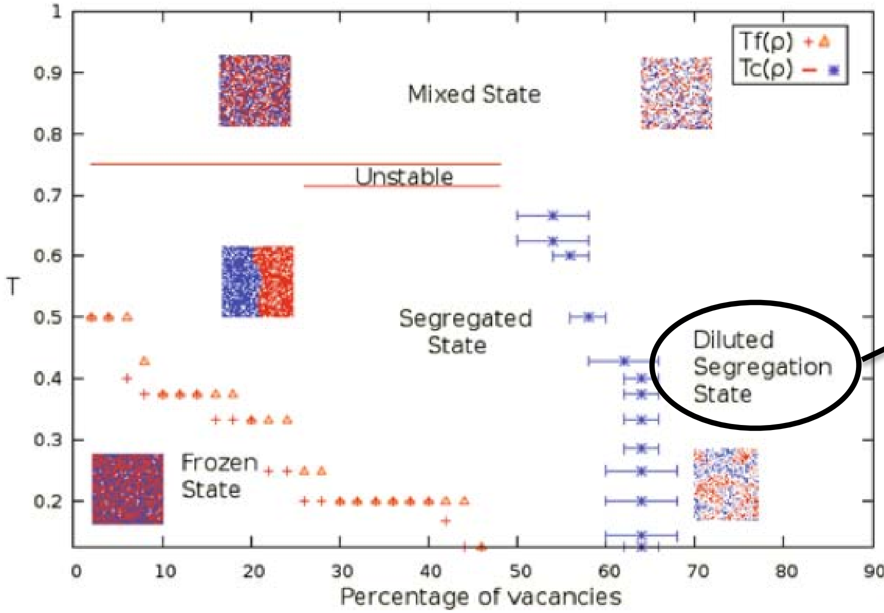


$$L = 100,\ T = 0.625,$$
$$\rho_0 = 0.01,\ time = 10^7 ts$$

Noisy Constrained Model



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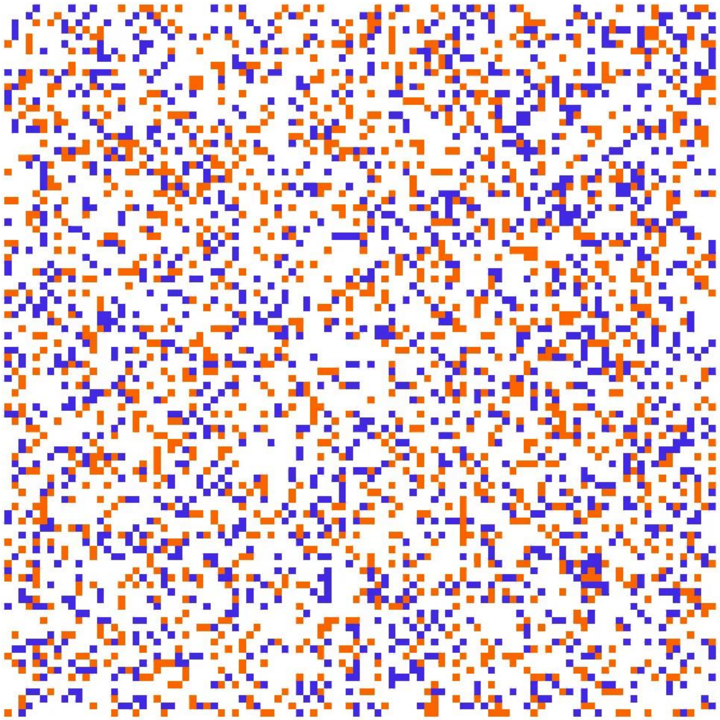
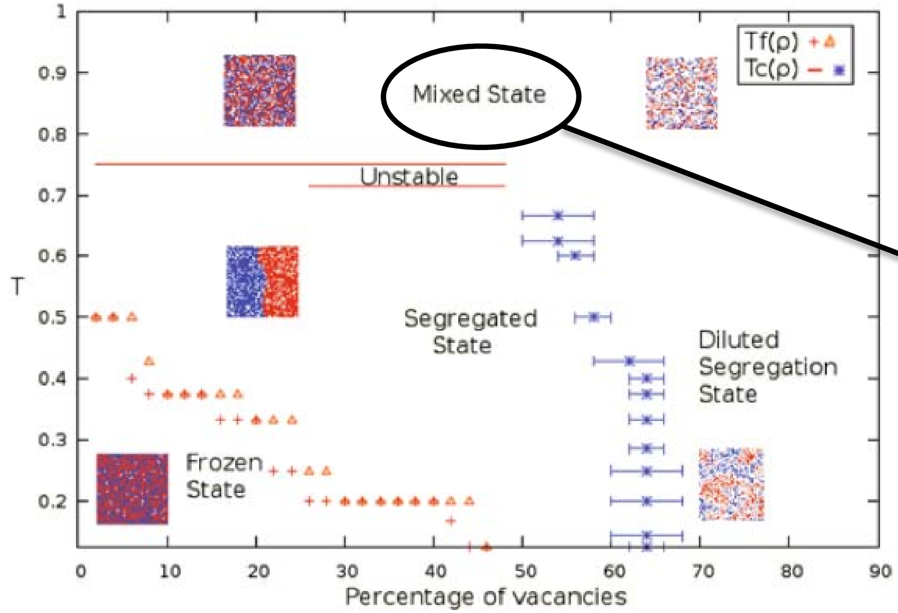


$$L = 100,\ T = 0.375,$$
$$\rho_0 = 0.7,\ time = 10^5 ts$$

Noisy Constrained Model



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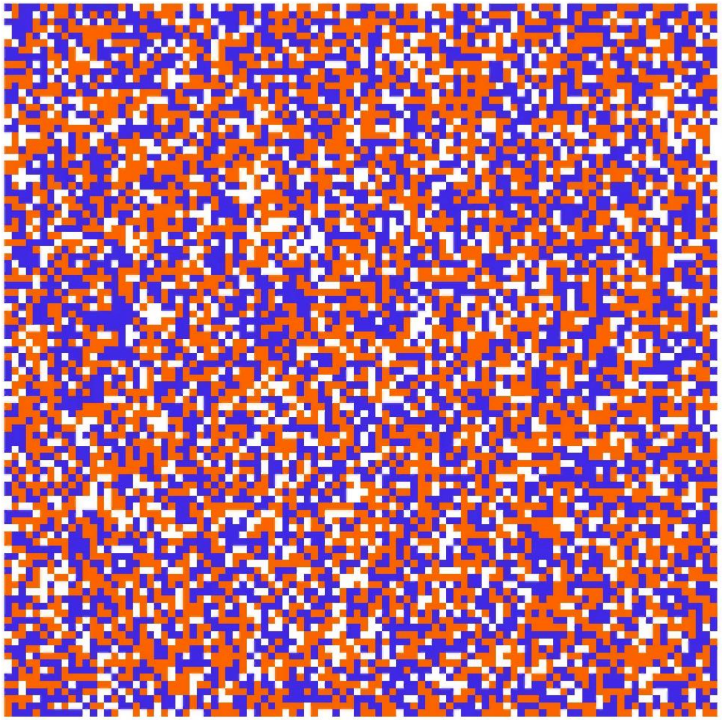


$$L = 100,\ T = 0.875,$$
$$\rho_0 = 0.2,\ time = 10^5 ts$$

L. Gauvin et al. Eur. Phys. J. B 70, 293 (2009)

Constrained model has as a Lyapunov potential: the Blume-Emery-Griffitth spin 1 hamiltonian

$$E_S = -\sum_{\langle i,j\rangle} c_i c_j - K \sum_{\langle i,j\rangle} c_i^2 c_j^2, \qquad K = 2T - 1$$

c_i=1,0,-1

<i,j>= nn, nnn

Constrained model (only unhappy move to happy locations):

Zero temperature dynamics of BEG with conservation laws

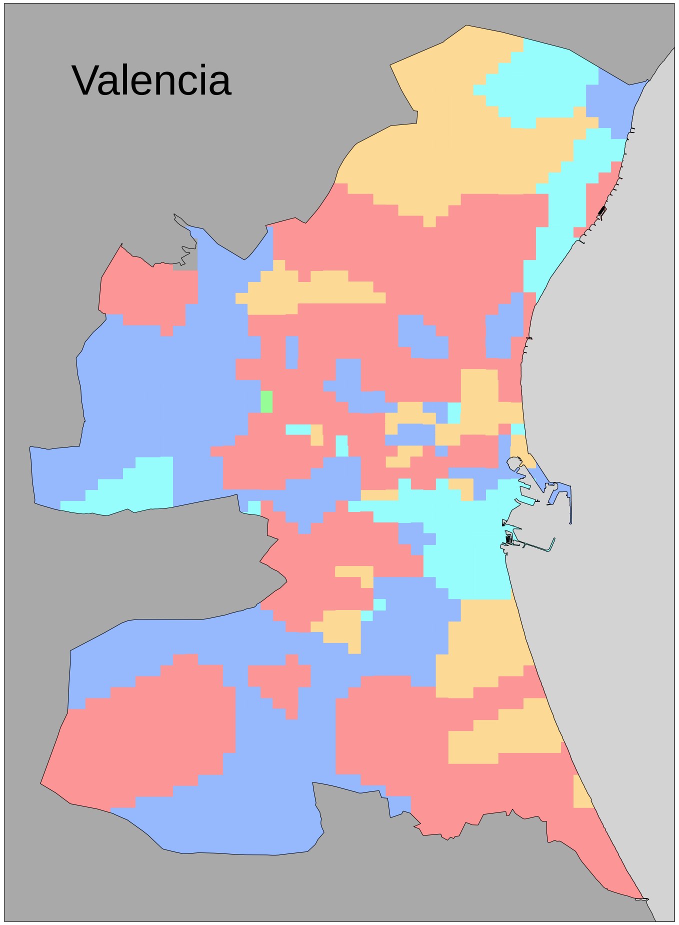
Schelling: Short range, constrained, T=1/2

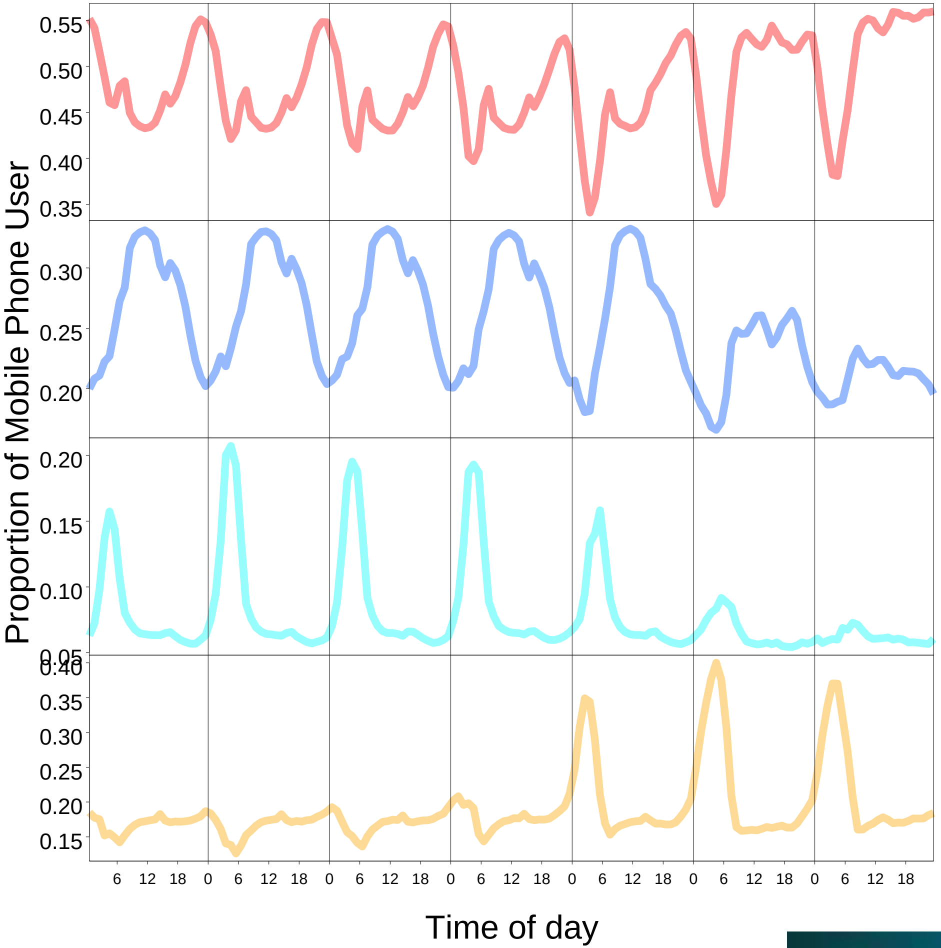
Zero temperature dynamics of Kinetic Ising model
with exchange dynamics and vacancies

Noisy constrained model: motion of happy agents plays the role of T and dynamics is no longer a simple minimization, but something like finite T sampling.

M. Lenormand et al., Royal Society Open Science 2, 150449 (2015)

Landuse from the functional network of the city





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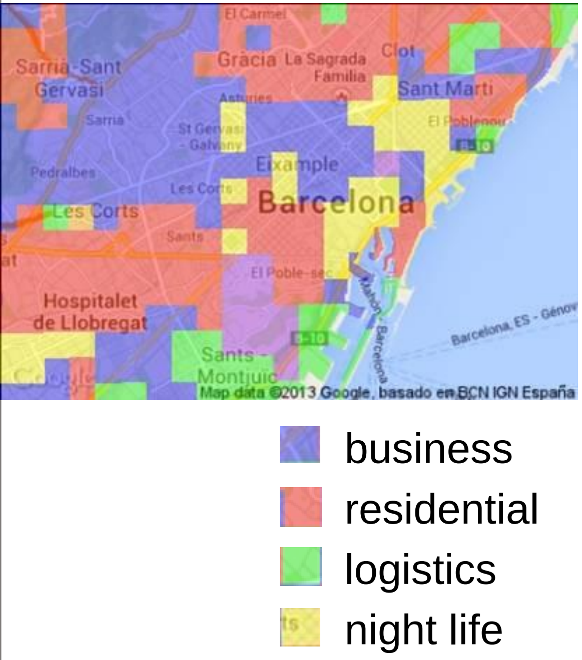
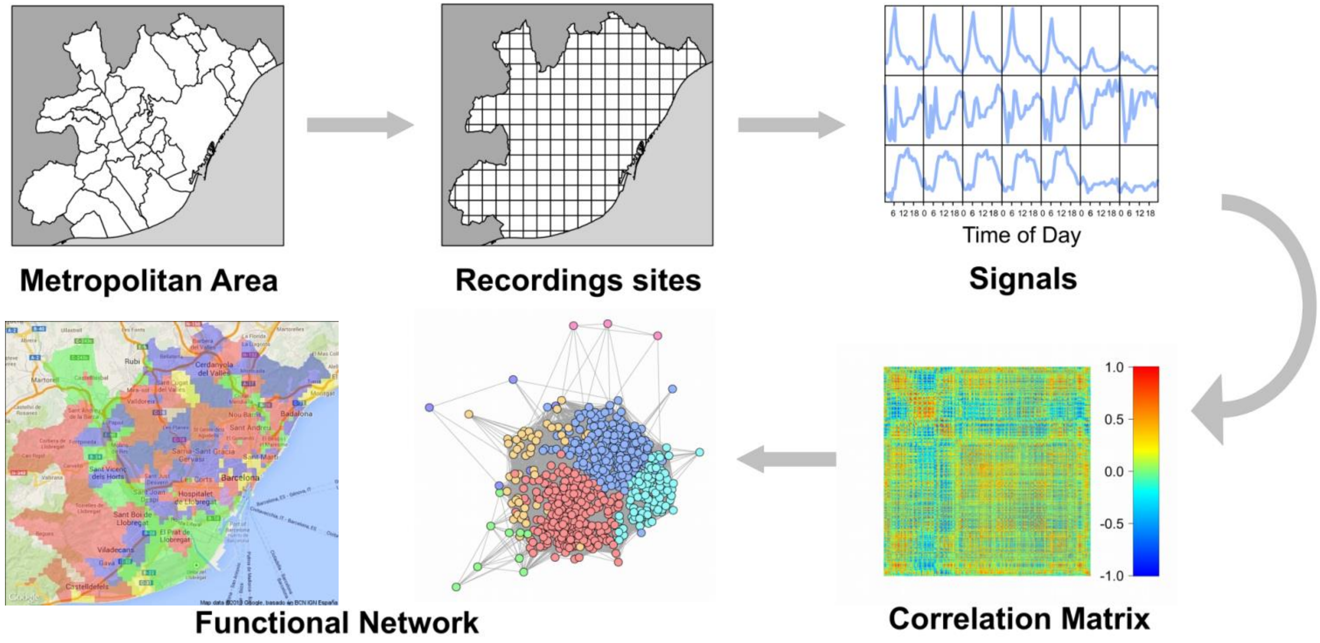
Landuse from the functional network of the city

- Network approach to determine land-uses from mobile phone data
- Automatic detection of 4 main land use whose relative proportions are very close from one city to another





M. Lenormand et al., Royal Society Open Science 2, 150449 (2015)

Segregation model *a la* Schelling

Satisfaction S_i of a cell based on the fraction of land use type among its neighbors

p_t^i is the fraction of neighbours of *i* of type *t*.

$$\text{if } t_i = L, \quad S_i = \delta_{p_L^i,1},$$

$$\text{if } t_i = N, \quad S_i = p_N^i \delta_{p_L^i,0},$$

$$\text{if } t_i = R, B, \quad S_i = \begin{cases} \delta_{p_L^i,0} & \text{with probability } \gamma, \\ p_{R,B}^i \delta_{p_L^i,0} & \text{with probability } 1-\gamma, \end{cases}$$

Logisitcs gives repulsive forces: logistic cells S_i=1 only if they are surrounded by cells of the same type, and that cells of other types have zero satisfaction if surrounded by any logistic one. Residence and business also attract each other with the only adjustable parameter

Global satisfaction:

$$S = \sum_i S_i$$

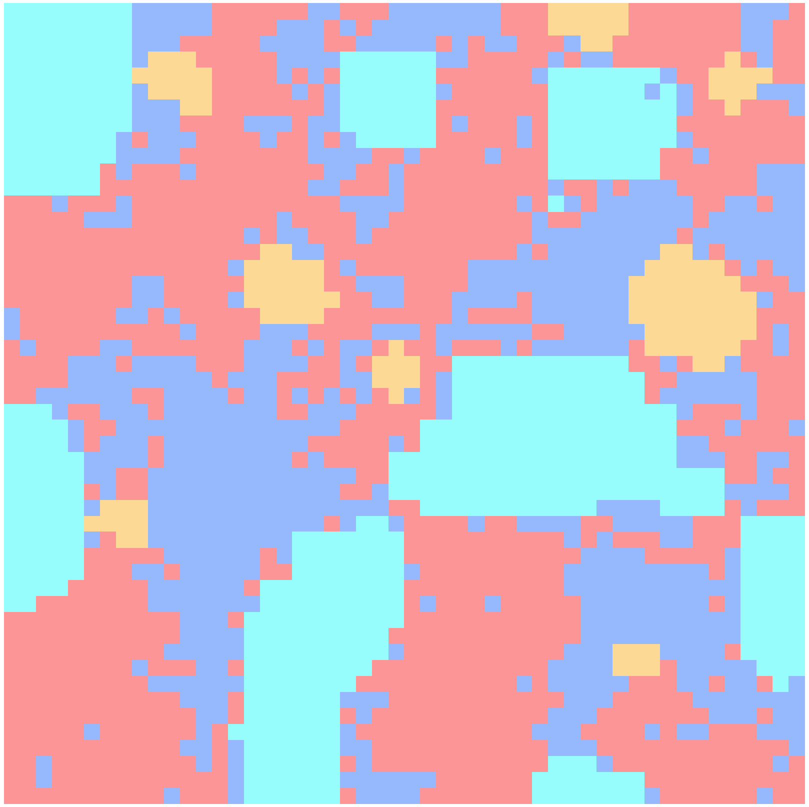
Dynamics:

The model is updated by choosing random pairs of cells and interchanging their land use if the exchange increases *S*. This process is repeated until the satisfaction reaches a stationary state.

M. Lenormand et al., Royal Society Open Science 2, 150449 (2015)

Segregation model *a la* Schelling

t = 300,000



M. Lenormand et al., Royal Society Open Science 2, 150449 (2015)

Segregation model *a la* Schelling

Entropy index at different scales. City divided in cells of size DxD

f_i : fraction of area of use α in cell i in city subdivison at scale D

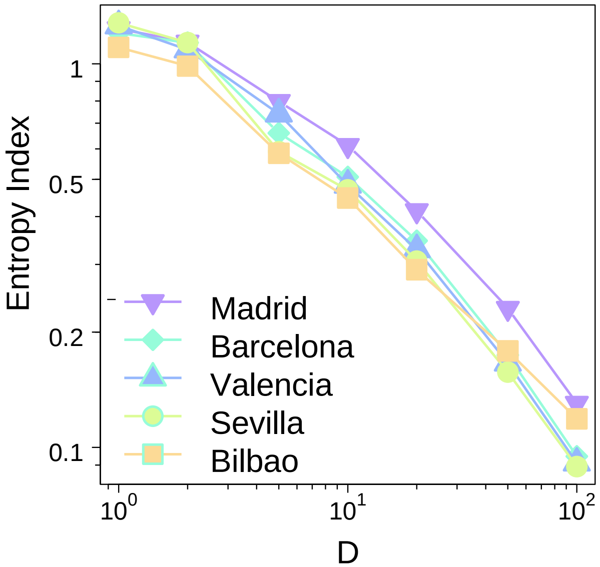
$$E_i = -\sum_{\alpha} f_i^{\alpha} \ln(f_i^{\alpha})$$

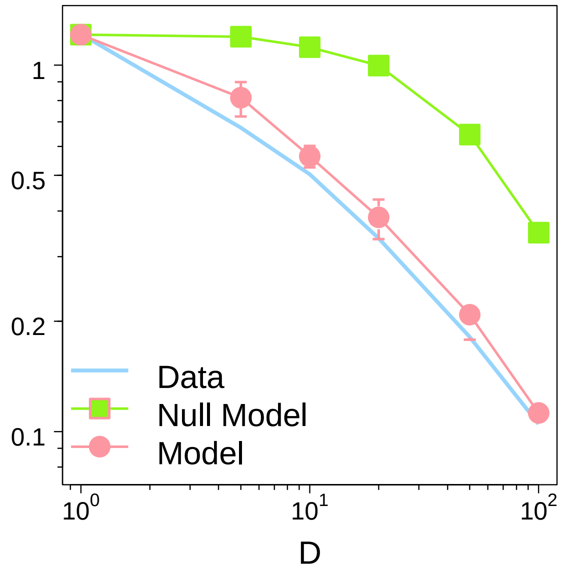
α: L,R,B,N

E(D): average of

E_i over cells i

at scale D





Outline:

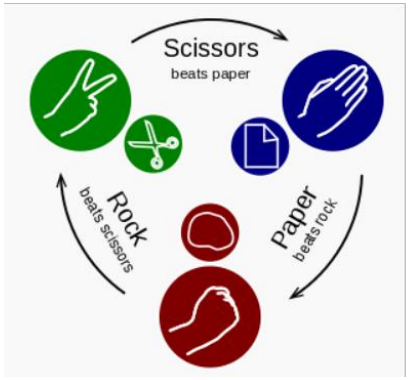
1. Physics, Complexity, and Social Sciences: Sociophysics?
2. Axelrod´s model for cultural dissemination
3. Schelling´s model of segregation
4. Game Theory. Social and strategic interactions

- Rational agents that play strategies.
- **Normal form** representation: Pay-off matrix
- Pay off-matrix: Gain for each agent for chosen strategies.

Nonsymmetric matrix

Example: Rock - Paper - Scissors

Game rules:



		Player B		
		stone	*paper*	*scissors*
Player A	*stone*	$(0,\ 0)$	$(-1,\ 1)$	$(1,\ -1)$
	paper	$(1,-1)$	$(0,0)$	$(-1,1)$
	scissors	$(-1,1)$	$(1,-1)$	$(0,0)$

Usually, only the payoffs received by player A from player B

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}.$$

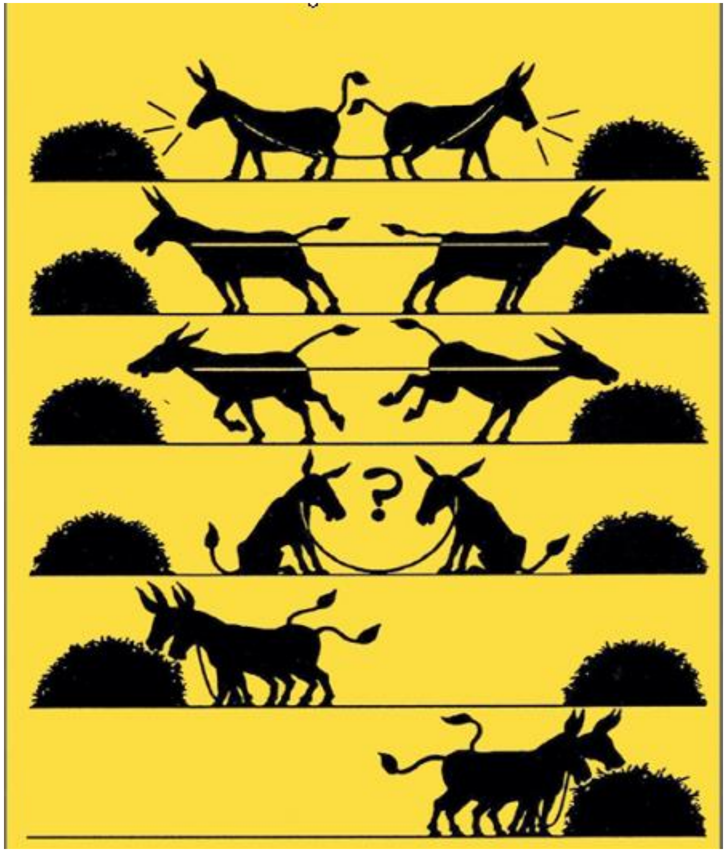
Zero-sum game: Gain of player A equals the loss of player B, and vice versa

Minimax Theorem:

There exist optimal mixed strategies X_s and Y_s such that

- **Each player minimizes the maximum payoff possible for the other**
- **Zero-sum: she also minimizes her own maximum loss**
  (i.e. maximize her minimum payoff).

Buridan´s ass



Coordination games (CG)

Individuals choose between two strategies. Their pay off is larger when they choose the same strategy than the other player (coordination)

The fully symmetrical coordination game (doorway game, driving game) is described by the payoff matrix

	L	*R*
L	1,1	0,0
R	0,0	1,1

Nash equilibrium:

- A set of strategies (one per player) from which no player benefits by changing unilaterally

- A set of strategies such that each one of them is a ***best response*** (highest payoff) to the joint strategies of the rest

Ask what each player would do, *taking into account* the decision-making of the others: Each player is told the strategies of the others. Suppose then that each player asks himself or herself: "Knowing the strategies of the other players, and treating the strategies of the other players as set in stone, can I benefit by changing my strategy?" If any player would answer "Yes", then that set of strategies is not a Nash equilibrium. But if every player prefers not to switch (or is indifferent between switching and not) then the set of strategies is a Nash equilibrium.

→ The largest pay-off is not necessarily achieved at the Nash equilibrium.
A unique Nash equilibrium does not exist for every game

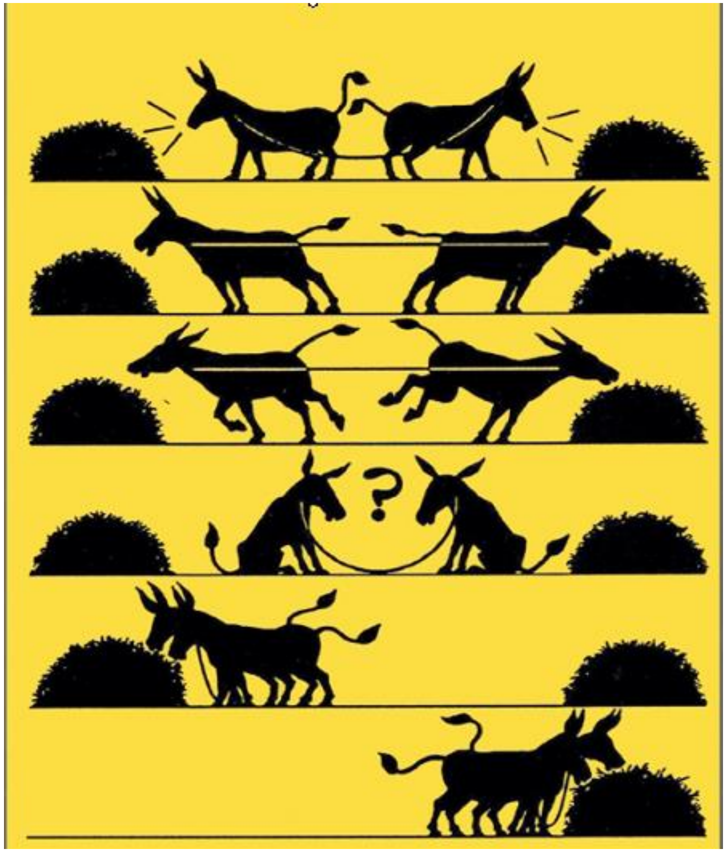
Nash´s Theorem

“An equilibrium point is an n-tuple such that each player’s ***mixed strategy*** maximizes his payoff if the strategies of the others are held fixed. Thus each player’s strategy is optimal against those of the others.”

“Every finite game, with N players, with a finite number of strategies per player, has at least one Nash equilibrium, possibly in mixed strategies” [PNAS 36, 48 (1950)]

→ In zero-sum games, the minimax solution is the same as the Nash equilibrium

Buridan´s ass



Coordination games (CG)

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	L	R
L	1,1	0,0
R	0,0	1,1

→ Two degenerate Nash Equilibria in pure strategies: Coordination in ++ (LL) or in - - (RR)

→ Third Nash Equilibium: Mixed strategies 50%-50%

Pay-off (Pareto Dominance) and Risk Dominance

Pay-off matrix (*b*>0)

	L	R
L	$1,1$	$0,-b$
R	$-b,0$	$2,2$

RR equilibrium: pay-off dominant

LL also equilibrium: Although each player is awarded less than optimal payoff, neither player has incentive to change strategy due to a reduction in the immediate payoff

Expected pay-off for player 1 (or 2) playing L: $\langle \Pi_L \rangle = ½\ 1 + ½\ 0$

Expected pay-off for player 1 (or 2) playing R: $\langle \Pi_R \rangle = ½\ 2 + ½\ (-b)$

$\langle \Pi_L \rangle > \langle \Pi_R \rangle \Rightarrow b>1$

- *b>1,* **LL risk dominant** equilibrium
- *b*<1, **RR pay-off dominant** equilibrium

Stag-Hunt Game

Two players may choose to hunt a stag ***(strategy: Hunt)*** or a rabbit ***(strategy: Gather)*** , the former providing more meat (5 utility units) than the latter (4 utility units). The caveat is that the stag must be cooperatively hunted, so if one player attempts to hunt the stag, while the other hunts the rabbit, he will fail in hunting (0 utility units), whereas if they both hunt the rabbit they will split the payload (2, 2).

If one hunter trusts that the other will hunt the stag, he should hunt the stag; however if he suspects that the other will hunt the rabbit, he should hunt the rabbit

	Hunt	Gather
Hunt	5, 5	0, 4
Gather	4, 0	2, 2

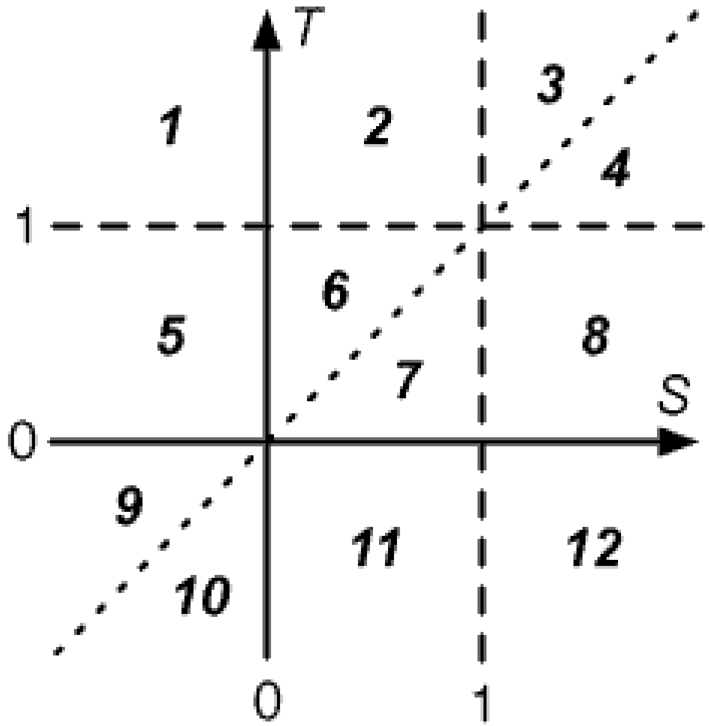
HH: Payoff dominant equilibrium

GG: Risk dominant equilibrium

2x2 game classification

	C	X
C	1,1	S,T
X	T,S	0,0

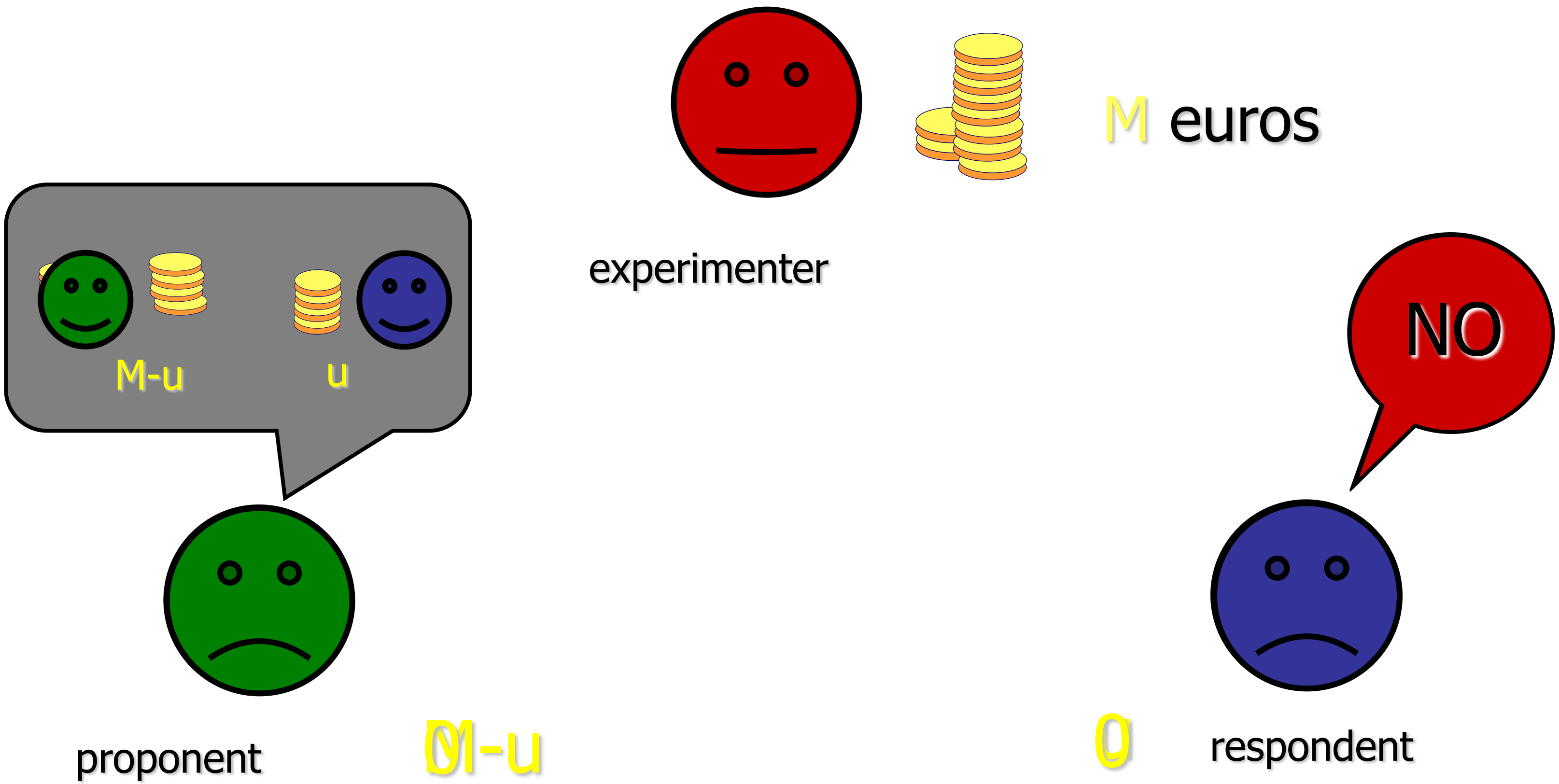
T > 1 : temptation to defect
S < 0 : risk in cooperation



1 Prisoner's dilemma
2 Chicken/Hawk-Dove/Snowdrift
3 Leader
4 Battle of the sexes
5 Stag hunt
6 Harmony
12 Deadlock

(Güth, Schmittberger & Schwarze, 1982)

Ultimatum



Subgame perfect Nash equlibrium is (offer the minimum, accept)

- To what extent the mechanisms of human interaction are well captured in game theory?
  - Choice of strategies: EXPERIMENTS
- Dynamics? Approach to Nash equilibrium?
- Iterated games
- N players and Evolutionary Game Theory:
  - Evolutionary Stable Strategy
- N players: Random pairing vs spatial effects. Networks
- **Competition of strategic (the self) and socially motivated decisions**

D. Vilone et al. Sci. Rep. 2, 686 (2012)

Social imitation **Strategic behavior**

Fast and Slow Thinking

VOTER MODEL **COORDINATION GAME**

Agents located at the nodes of a ER Random Network or a Barabasi-Albert Scale Free Network

Voter Model (VM)

- Social Imitation:

Agents copy the strategy of a randomly selected neighbor in the network.

→ **Asymptotic state of dynamical disorder (no global coordination)**

Coordination game dynamics of N interacting agents (CG)

-Agents play the coordination game with all their neighbors and aggregate a pay-off.

-STRATEGIC dynamical rule of UNCONDITIONAL IMITATION (UI): at the end of each round, individuals imitate the strategy of their neighbour with largest pay off

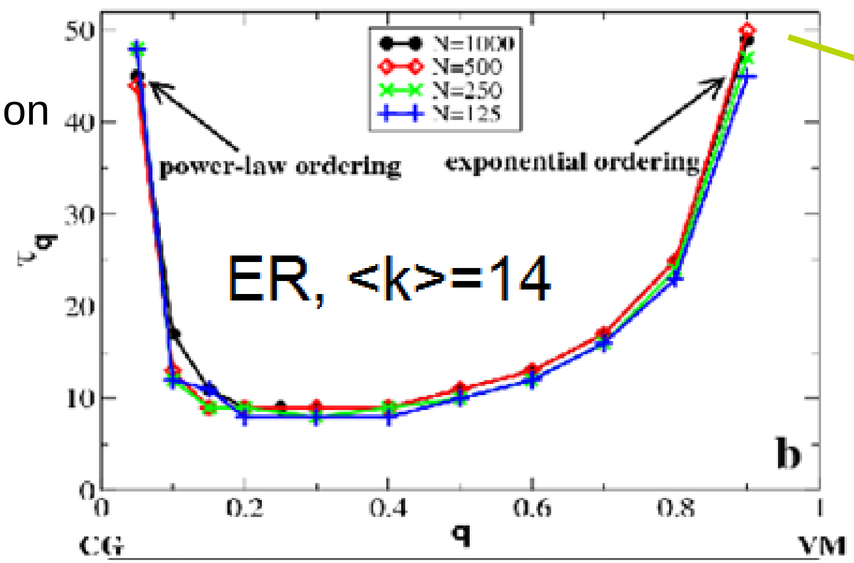
→ **Asymptotic state of frozen disorder (no global coordination)**

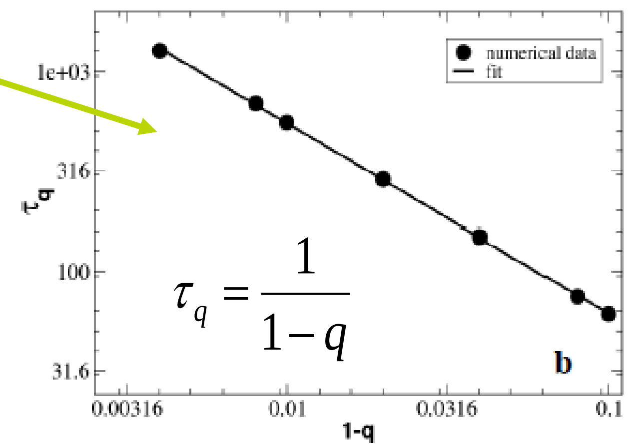
D. Vilone et al. Sci. Rep. 2, 686 (2012)

With probability q and 1-q updating follows VM or CG rules

- No coordination for q=0,1. **BUT global absorbing state of coordination for any** $q \neq 0,1$
- Crossover regime from slow ordering (q<q*) to fast ordering (q>q*)

Time to coordination



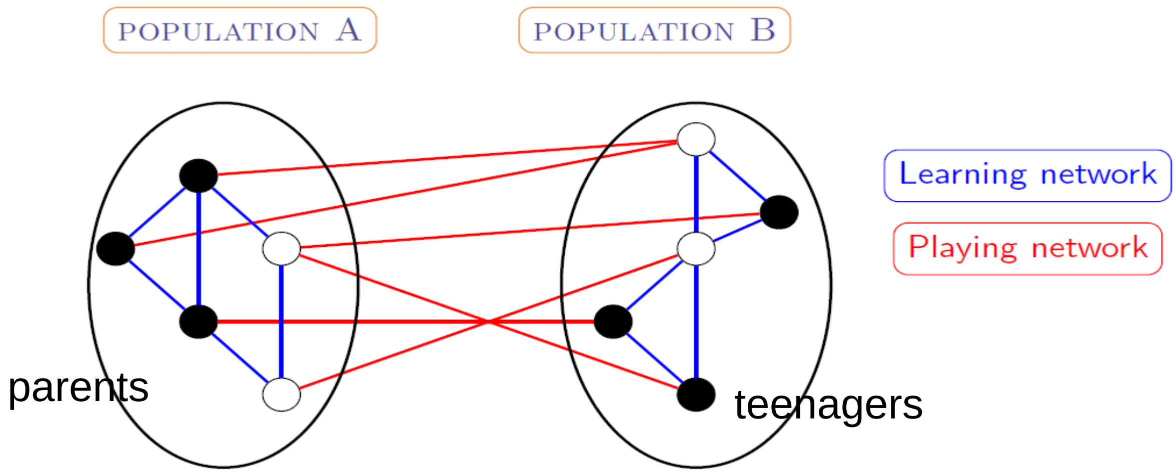


Pure strategic or social imitation leaves the system uncoordinated, but any amount of mixing of them allows to reach total consensus.

- **Optimum mixture for q=q* with a smallest time to reach consensus τ_q**

Social Learning and Coordination

H. Lugo et al. Sci. Rep. ***55,*** *776 (2015)*



Interlayer: General Coordination Game

Intralayer: Learning network → Strategy update

$T \in [0, 1]$

T<0.5 Herding

T>0.5 Skeptical

Strategy Satisfaction by pay-off π →

Social Satisfaction by *d* neighbors with opposite strategy →

S	P1	P2	U
$\pi_i = n_i$ $d_i < T$	$\pi_i = n_i$ $d_i > T$	$\pi_i < n_i$ $d_i < T$	$\pi_i < n_i$ $d_i > T$

Satisfied: No strategy change

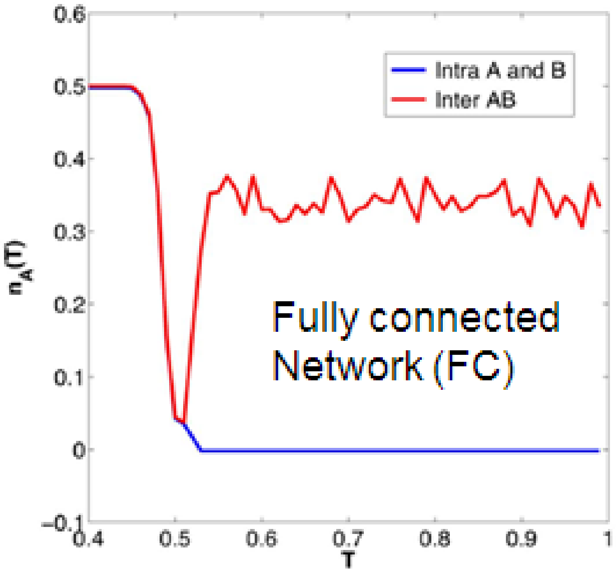
Unsatisfied: Change of strategy

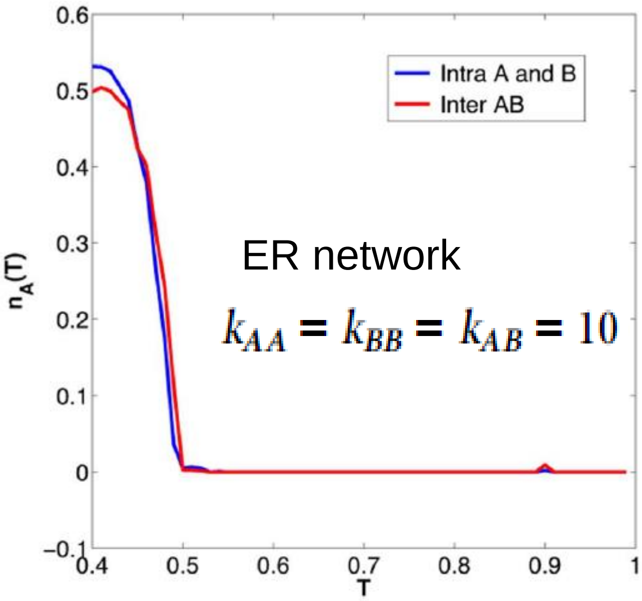
Partially satisfied: Imitates strategy of her best performing neighbor in the learning network with payoff larger than her payoff.

*H. Lugo et al. Sci Rep **55,** 776 (2015)*

Links connecting nodes in different states

b=1.1





-No coordination for herding populations T<0.5

-Bistability: Coordination and Interlayer anti-Cooordination in FC networks

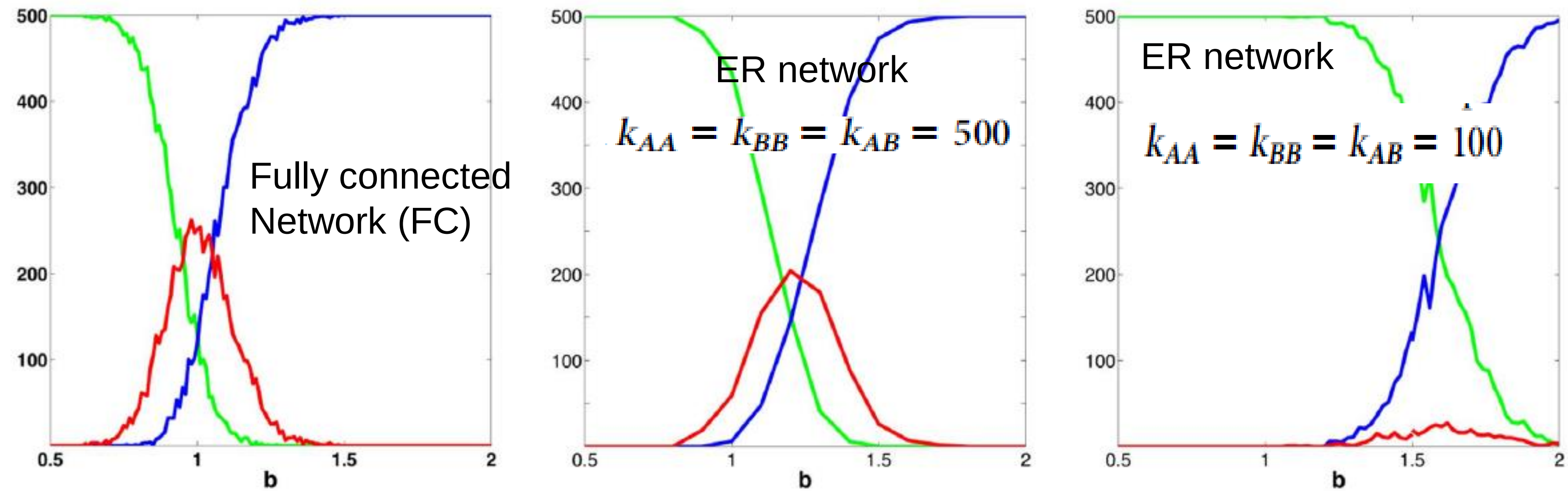
-Full coordination by local interactions

Skepticism about the wisdom of crowd and the local connectivity are driving forces for full coordination

*H. Lugo et al. Sci Rep **55,** 776 (2015)*

Number of realizations with L or R interlayer coordination, or anticoordinated

L, Risk dominant eq. **R, Pay-off dominant eq.** T=0.7



Skepticism together with local interactions shift risk coordination to b>1

Full coordination is possible for b>1 in the socially efficient Pareto-dominant strategy in spite of being the riskier one.

Data Analysis and Mathematical Modelling of Social Science

- **Science: Prediction vs. UNDERSTANDING vs Managing or Controlling**

  Data analysis vs modeling

  Data driven modeling vs Question driven data gathering

- **Identification of mechanisms**
- **Modeling: proving wrong statements, understanding counterintuitive observations**
- **Cause-Effect relations instead of correlations**
- **Choice of variables: Relevant and irrelevant variables**

Data Analysis and Mathematical Modelling of Social Science

CLASH OF TITANS

INDUCTIVISM
F. Bacon

Data driven
Big Data
Inference

DEDUCTIVISM
Carl Popper

Model driven

Complexity science is the discipline in which both these approaches merge as one.

Peter Sloot: Visions for Complexity (2016)