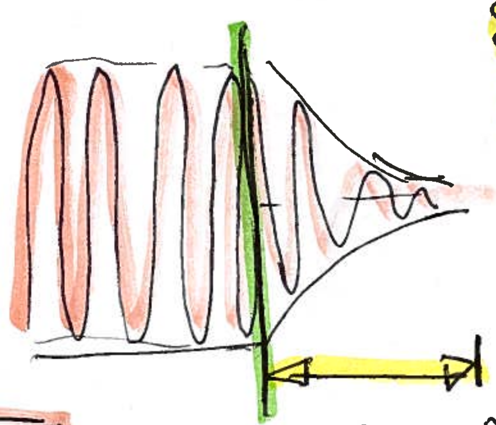
ANOMALOUS SKIN EFFECT

MICROWAVE FREQ.
LOW T's



LOCAL RELATION

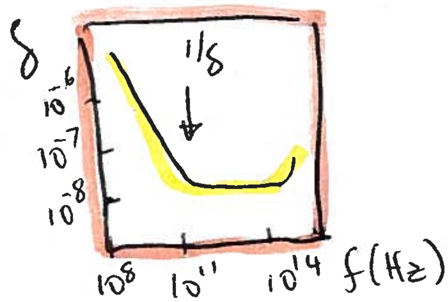
$$J = \sigma E$$

OK

PROVIDED

$$\ell \ll \delta$$

mean-free path



$$\delta = \frac{\lambda_0}{2\pi\kappa}$$

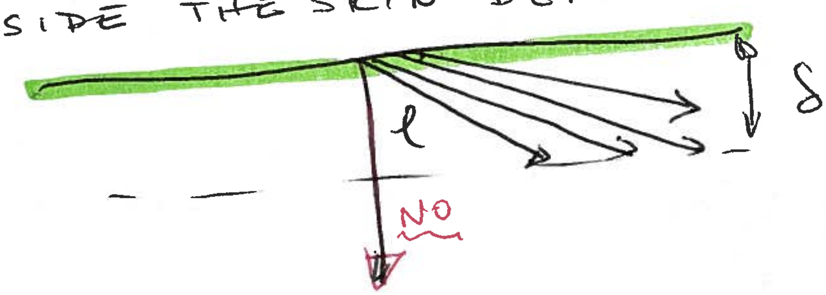
skin depth

IF $\ell \gtrsim \delta \Rightarrow J(q,\omega) = \sigma(q,\omega)\, E(q,\omega)$

SPATIAL DISPERSION

[ORDINARY THEORY ~~DOESN'T~~ WORK BECAUSE
FIELD VARIES RAPIDLY BETWEEN COLLISIONS

THE ONLY CARRIERS PARTICIPATING IN THE ABSORPTION ARE THOSE INSIDE δ w/ PATH INSIDE THE SKIN DEPTH



SFCE ROUGHNESS MATTERS

$\sigma \sim \frac{\delta}{\ell}\sigma_0$ BUT THEN $\delta = \frac{c}{\sqrt{2\pi\sigma\omega}}$

$\Rightarrow \sigma \sim \frac{1}{\omega^{1/3}}\left(\frac{c\,\sigma_0}{\ell}\right)^{2/3}$ (COMPARE WITH $\sigma \sim \gamma^{-1}$)

SURFACE PLASMONS

CONSIDER P-POLARIZED LIGHT (TM)

$$\vec{E} = \left(E_x(z), 0, E_z(z)\right) e^{i(kx-\omega t)}$$

MAXWELL'S EQS.

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \qquad \vec{\nabla} \wedge \vec{H} = \frac{1}{c}\epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\frac{i\omega}{c}\vec{H} = \hat{y}\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right]$$

$$\begin{cases} -\frac{i\omega\epsilon}{c}E_x = -\frac{\partial H_y}{\partial z} \\ -\frac{i\omega\epsilon}{c}E_z = \frac{\partial H_y}{\partial x} \end{cases}$$

$$\begin{cases} \frac{\epsilon\omega^2}{c^2}E_x + \frac{\partial^2 E_x}{\partial z^2} = \frac{\partial^2 E_z}{\partial x \partial z} \\ \frac{\epsilon\omega^2}{c^2}E_z + \frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 E_x}{\partial x \partial z} \end{cases}$$

$$\begin{cases} \dfrac{\epsilon\omega^2}{c^2}\mathcal{E}_x + \dfrac{\partial^2 \mathcal{E}_x}{\partial z^2} = ik\dfrac{\partial \mathcal{E}_z}{\partial z} \\[2em] \mathcal{E}_z = \dfrac{ik}{\left(\dfrac{\epsilon\omega^2}{c^2} - k^2\right)} \dfrac{\partial \mathcal{E}_x}{\partial z} \end{cases}$$

$$\Downarrow$$

$$\frac{\epsilon\omega^2}{c^2}\mathcal{E}_x + \left[1 + \frac{k^2}{\left(\frac{\epsilon\omega^2}{c^2} - k^2\right)}\right]\frac{\partial^2 \mathcal{E}_x}{\partial z^2} = 0$$

$$\Downarrow$$

$$\mathcal{E}_x + \frac{1}{\left(\frac{\epsilon\omega^2}{c^2} - k^2\right)}\frac{\partial^2 \mathcal{E}_x}{\partial z^2} = 0$$

EVANESCENT WAVES !

$$\begin{cases} z > 0 \qquad \mathcal{E}_x = E_0^{(+)} e^{-\alpha z} \qquad \alpha^2 = k^2 - \dfrac{\omega^2}{c^2} \\[2em] z < 0 \qquad \mathcal{E}_x = E^{(-)} e^{\beta z} \qquad \beta^2 = k^2 - \dfrac{\epsilon\omega^2}{c^2} \end{cases}$$

$$\begin{cases} z>0 & E_z = \dfrac{ik}{\underbrace{\left(\frac{\omega^2}{c^2}-k^2\right)}_{-\alpha^2}}(-\alpha)\,E^{(+)}\,e^{-\alpha z} \\[2em] z<0 & E_z = \dfrac{ik}{\underbrace{\left(\frac{\epsilon\omega^2}{c^2}-k^2\right)}_{-\beta^2}}(+\beta)\,E^{(-)}\,e^{\beta z} \end{cases}$$

BOUNDARY CONDITIONS

E_x & ϵE_z CONTINUOUS

$\Downarrow$

$$E^{(+)} = E^{(-)} \quad \& \quad \frac{\cancel{ik}\,(-\alpha)}{\frac{\omega^2}{c^2}-k^2} = \frac{\cancel{ik}\,\epsilon\beta}{\frac{\epsilon\omega^2}{c^2}-k^2}$$

$$\frac{-\alpha}{-\alpha^2} = \frac{\epsilon\beta}{-\beta^2} \Rightarrow \frac{1}{\alpha} = -\frac{\epsilon}{\beta}$$

SOLUTION ONLY EXISTS IF

$\epsilon < 0$!

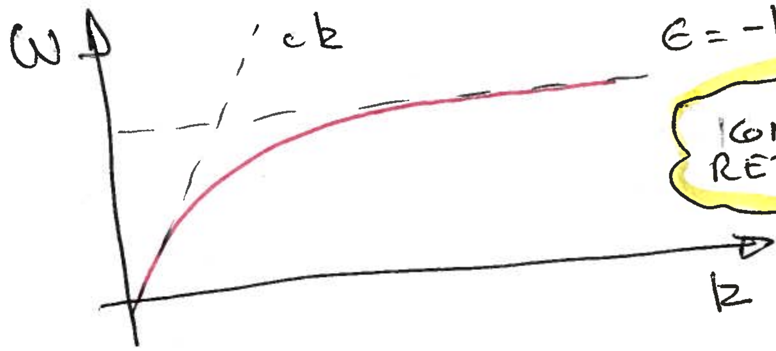
$$\frac{\sqrt{k^2+|\epsilon|\omega^2/c^2}}{|\epsilon|} = \sqrt{k^2-\omega^2/c^2}$$

$$|k^2| = \frac{|\epsilon|\left[|\epsilon|+1\right]\omega^2/c^2}{|\epsilon|^2-1}$$

$$|k^2| = \frac{|\epsilon|}{|\epsilon|-1}\,\omega^2/c^2$$

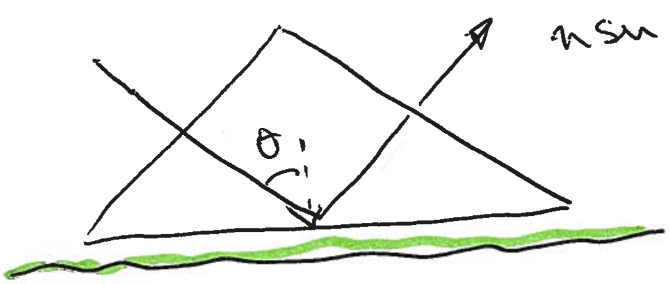
FOR $|\epsilon| = \dfrac{\omega_p^2}{\omega^2} - 1$

$\epsilon = -1 \quad \omega = \omega_p/\sqrt{2}$



IGNORING RETARDATION $\quad \epsilon = -1$

$n \sin\theta > 1$



METAL

EXTREMELY SENSITIVE SFCE. PROBE