

Microfluidica en el límite de Stokes

Resistencia hidráulica y equivalente eléctrico
Resistencia y movilidad de partículas
Efectos Inercia

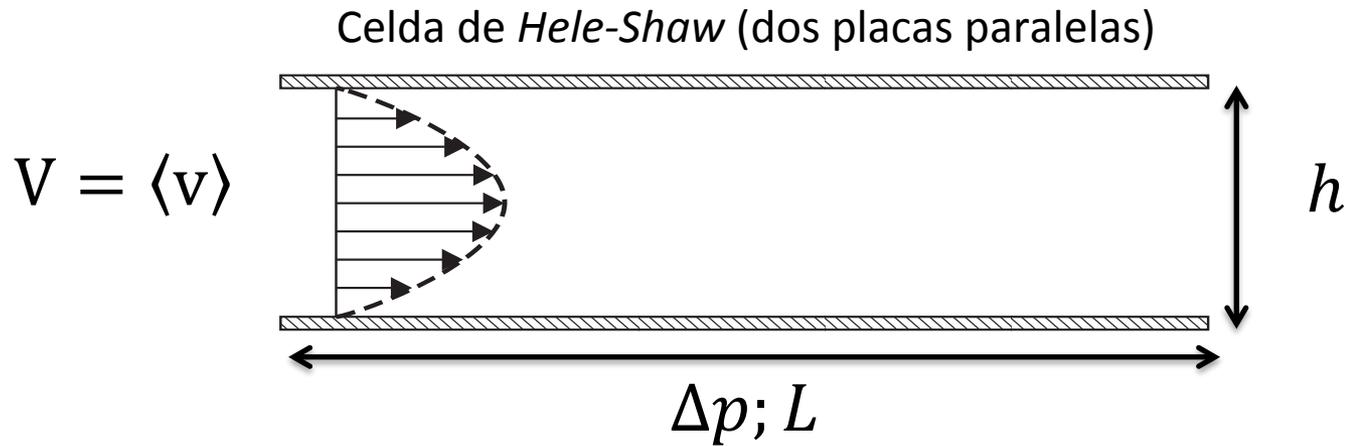


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departamento de física
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Microfluidica

Resistencia hidráulica de canales



Linealidad

$$V \propto \frac{\Delta p}{L}$$

Ley de escala
Análisis dimensional

$$V \propto \frac{h^2}{\mu} \frac{\Delta p}{L}$$

Resultado Analítico

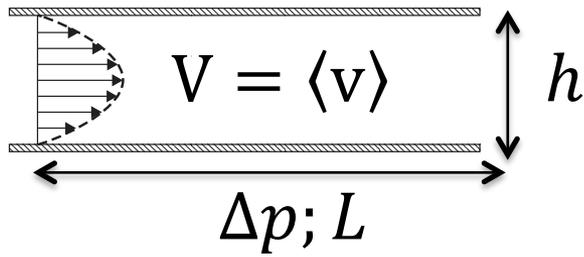
$$V = \frac{1}{\mu} \left(\frac{h^2}{12} \right) \frac{\Delta p}{L}$$

General

$$V = \frac{k_H}{\mu} \frac{\Delta p}{L}$$

Microfluidica

Resistencia hidráulica de canales



$$V = \frac{1}{\mu} \left(\frac{h^2}{12} \right) \frac{\Delta p}{L}$$

En general $V = \frac{k_H \Delta p}{\mu L}$

k_H Permeabilidad
Propiedad solo de la geometría !!

Sección transversal $S \rightarrow$ Caudal $Q = S V$

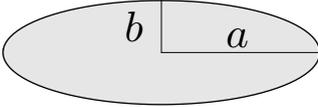
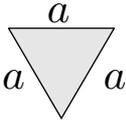
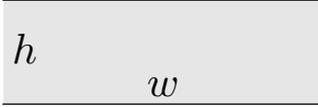
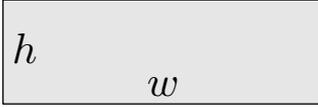
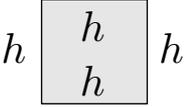
$$\Delta p = R_H Q$$

$$R_H = \mu \frac{12 L}{h^2 S}$$

R_H Resistencia hidrodinámica

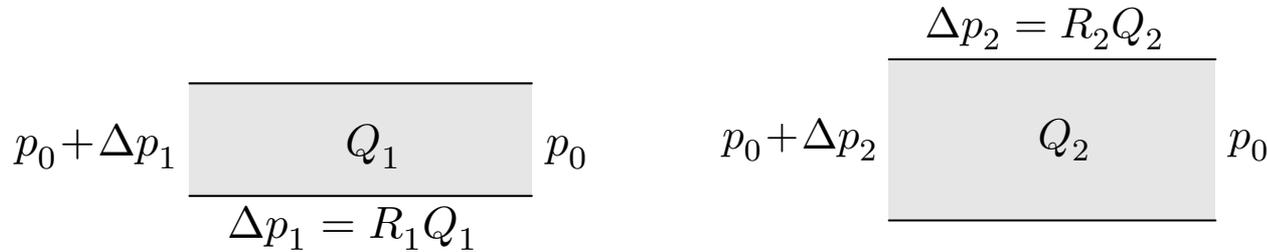
Microfluidica

Resistencia hidráulica

shape		R_{hyd} expression
circle		$\frac{8}{\pi} \eta L \frac{1}{a^4}$
ellipse		$\frac{4}{\pi} \eta L \frac{1 + (b/a)^2}{(b/a)^3} \frac{1}{a^4}$
triangle		$\frac{320}{\sqrt{3}} \eta L \frac{1}{a^4}$
two plates		$12 \eta L \frac{1}{h^3 w}$
rectangle		$\frac{12 \eta L}{1 - 0.63(h/w)} \frac{1}{h^3 w}$
square		$\frac{12 \eta L}{1 - 0.917 \times 0.63} \frac{1}{h^4}$

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Resistencias en serie y en paralelo



En series

(que quiere decir?):

Mismo

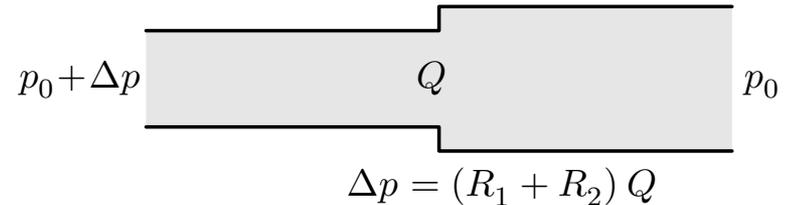
Caudal Q

$$\Delta p_1 = R_1 Q$$

$$\Delta p_2 = R_2 Q$$

$$\Delta p = \Delta p_1 + \Delta p_2$$

$$R = R_1 + R_2$$



En paralelo

(que quiere decir?):

Mismo

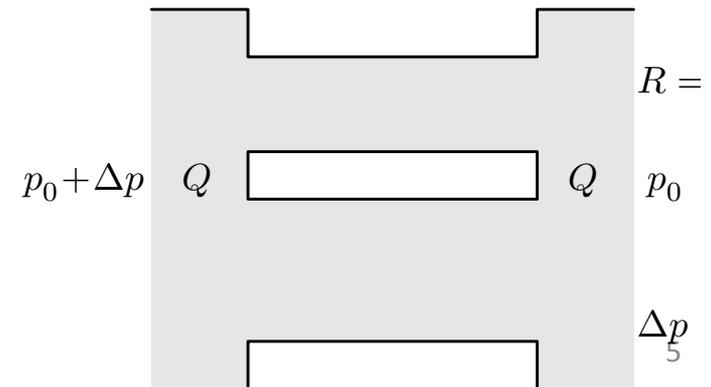
Δp

$$\Delta p = R_1 Q_1$$

$$\Delta p = R_2 Q_2$$

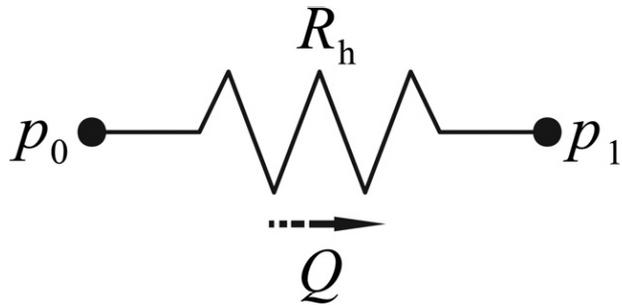
$$Q = Q_1 + Q_2 = \frac{\Delta p}{R_1} + \frac{\Delta p}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \Delta p$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

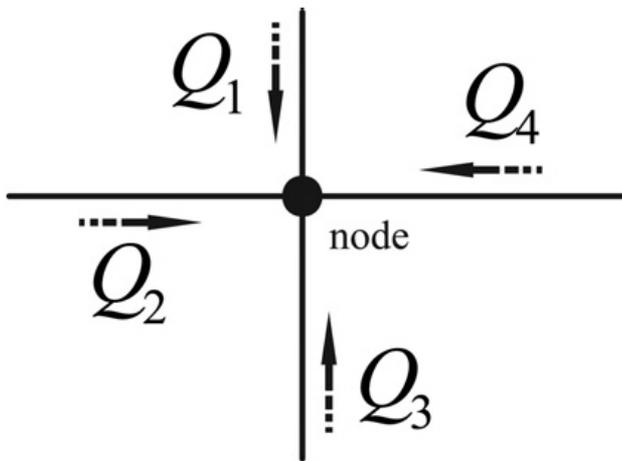


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Analogía Eléctrica



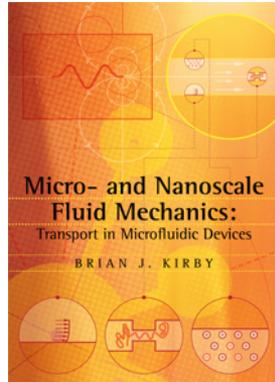
Circuito Eléctrico	Análogo en microfluidos
Potencial; Voltaje ΔV	Presión Δp
Corriente I	Caudal Q
Densidad de corriente i	Velocidad v
Resistencia R	Resistencia R_H
Capacitancia C	Deformación o compresibilidad



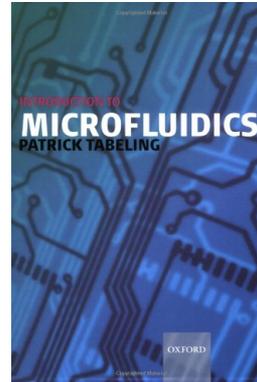
$$\sum_i Q_i = 0$$

Leyes de Kirchhoff

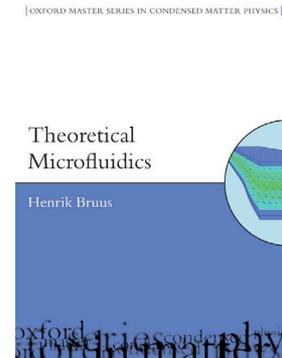
Microfluidica



Capítulo 3



Capítulo 2

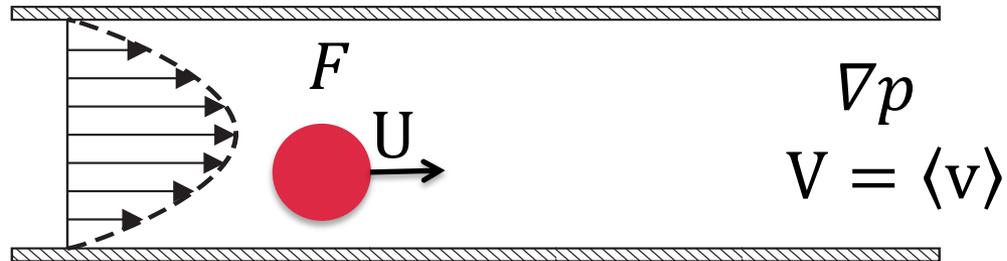


Capítulo 4

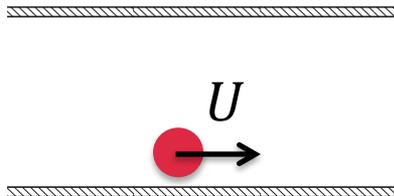
Problema: Crear una bomba de caudal constante usando diferencia de presiones (diferencia de altura) y un tubo capilar (tamaño arbitrario). Pensar en el problema análogo en circuitos eléctricos.

Microfluidica

Movimiento de partículas

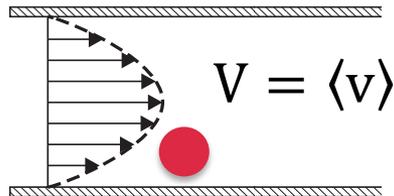


Separarlo en problemas simples !
Cada problema es lineal !
Podemos definir Resistencias y Movilidades

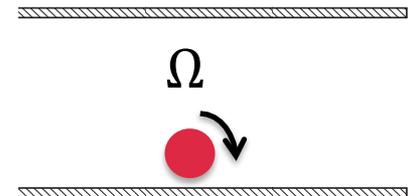


$$F = R_{FU} U$$

$$U = M_{FU} F$$



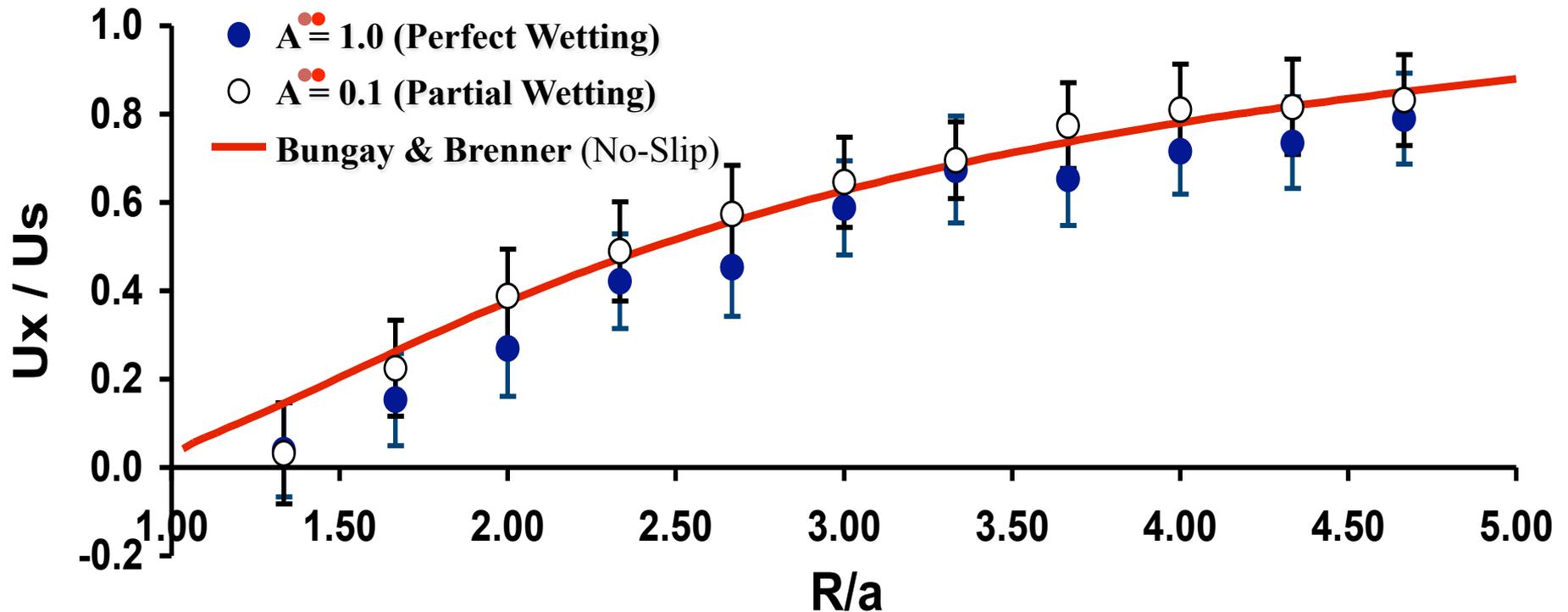
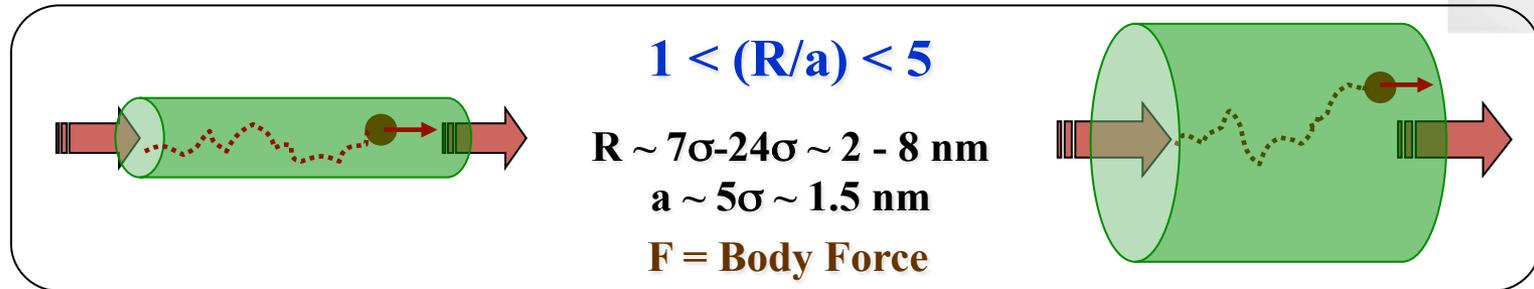
$$F = R_{FV} V$$



$$F = R_{F\Omega} \Omega$$

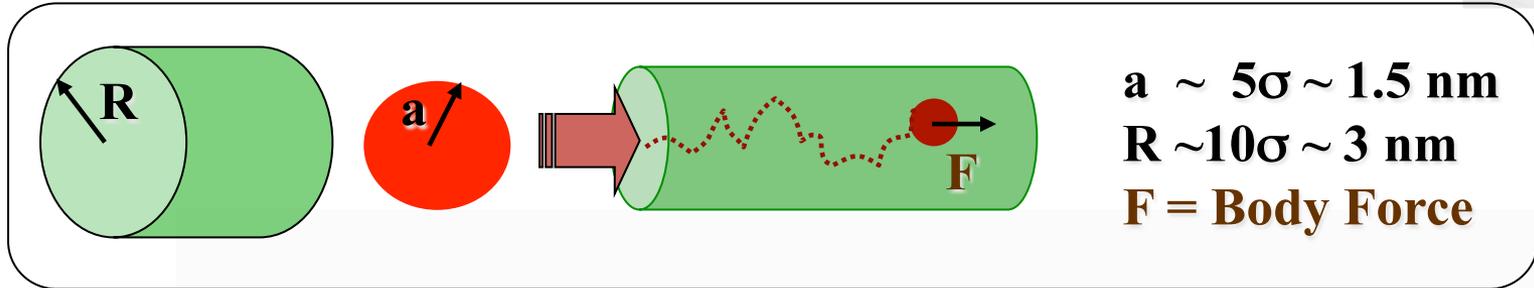
Nanofluidos

Simulaciones usando Dinámica Molecular



Microfluidica

Adsorción / *Stick-slip*



$A^* = 1.0$
Complete
Wetting



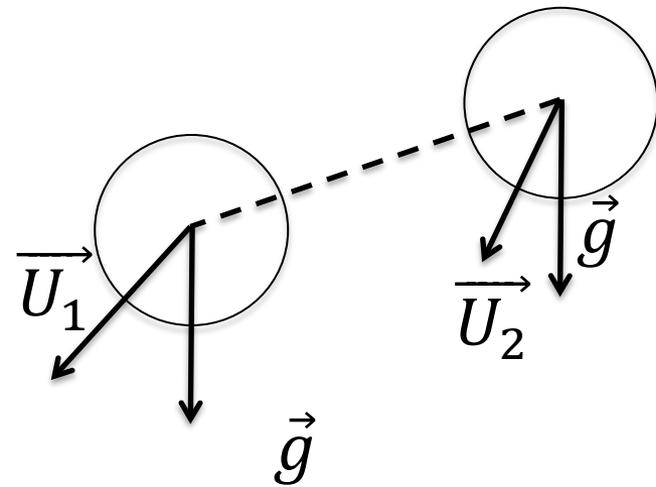
$A^* = 0.3$
Partial
Wetting



Microfluidica

Movimiento de partículas

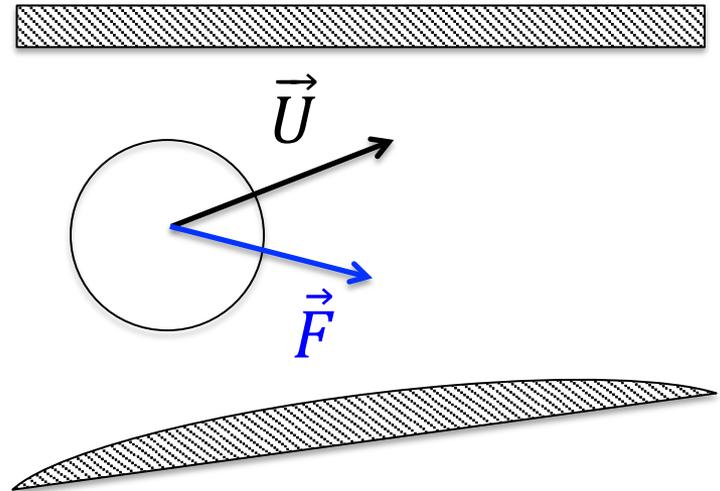
Como es el movimiento de dos esferas idénticas?



$$\vec{U}_1 = \vec{U}_2 !$$

Cambia en algo si hay una pared?

En general;
Geometría arbitraria



$$\vec{U} = \boxed{\mathcal{M}} \vec{F}$$

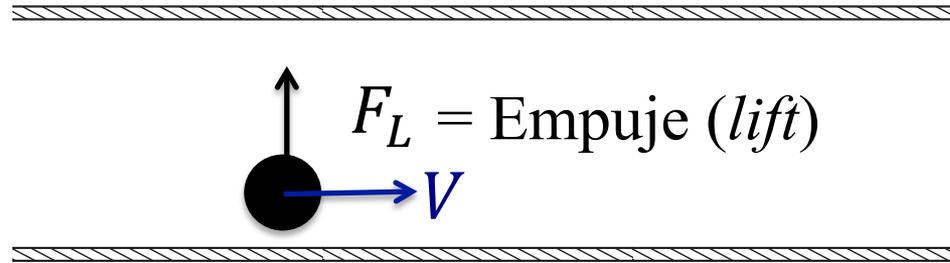
↑
↑
↑

vector ? vector

Tensor

Microfluidica

Cuando son importantes los efectos inerciales?



Estimar la velocidad W perpendicular a la pared.

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) = -\nabla p + \mu \nabla^2 \vec{v}$$

Para estimar las fuerzas inerciales en la dirección perpendicular, solo tenemos un término: la derivada convectiva !

$$F_L \propto \rho \frac{V^2}{a} v_a \propto \rho a^2 V^2$$

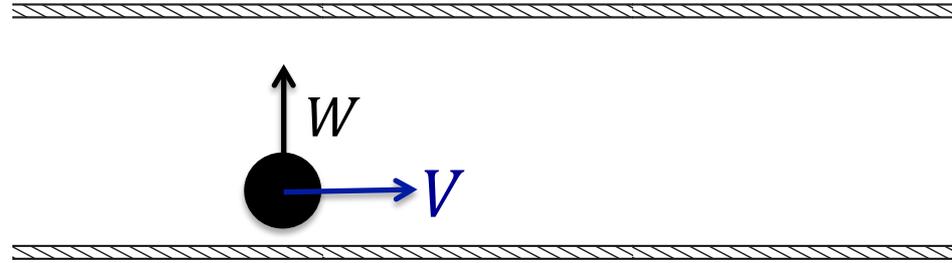
Que término es importante para balancear esta fuerza?

(Ayuda: la esfera se mueve con velocidad W perpendicular a la pared.)

$$F_W = 6\pi\mu a W$$

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Cuando son importantes los efectos inerciales?

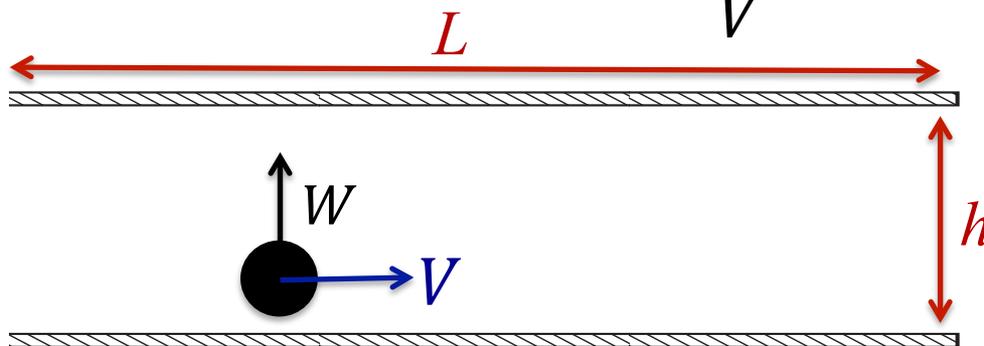


$$F_W = F_L \Rightarrow \rho a^2 V^2 = \mu a W \Rightarrow W = \frac{\rho a V^2}{\mu} = \text{Re } V$$

$$\text{Re} = \frac{\rho a V}{\mu} \quad \text{Re}_p \text{ Número de Reynolds de la partícula}$$

$$\text{Re} = \frac{W}{V} \ll 1$$

Es suficiente?

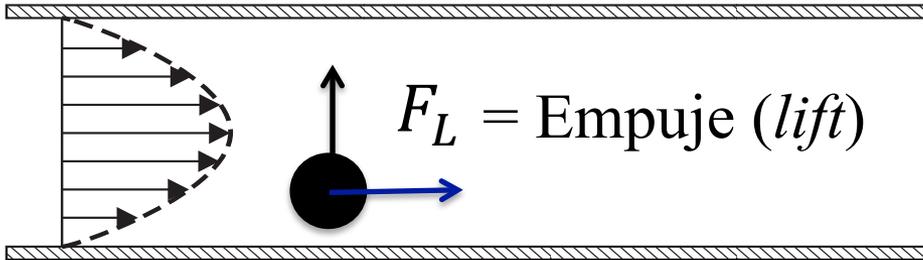


Que pasa si

$L \gg h$?

Migración Inercial

Migración Inercial en flujo de Poiseuille



G. SEGRÉ, A. SILBERBERG,
“Radial Particle Displacements
in Poiseuille Flow of Suspensions”.
Nature 189, 209-210 (21 January 1961)

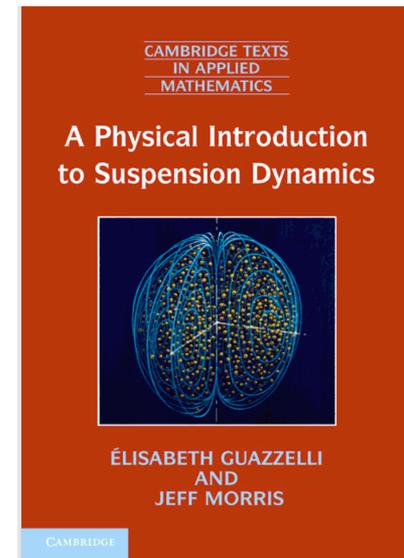
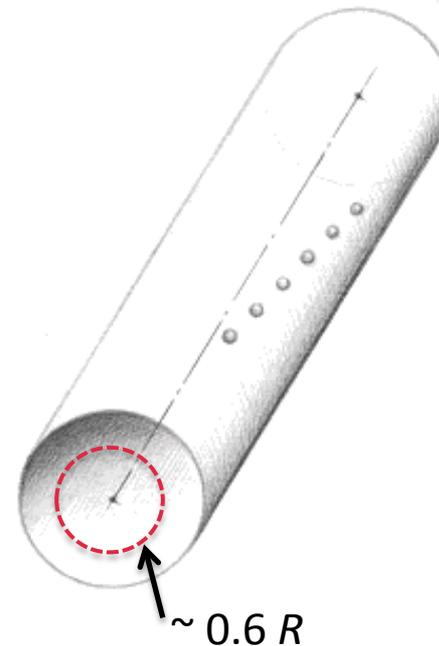
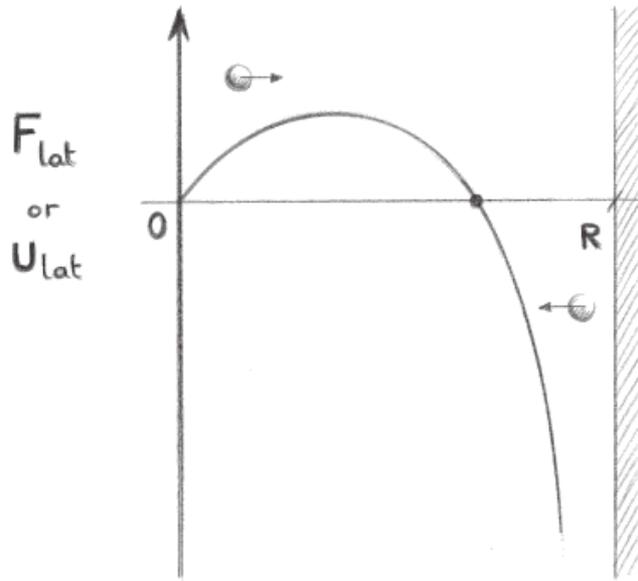


Figure 8.7 The form of the particle lateral force (or velocity) in Poiseuille pipe flow at finite Re .



Inertial microfluidics

Dino Di Carlo*

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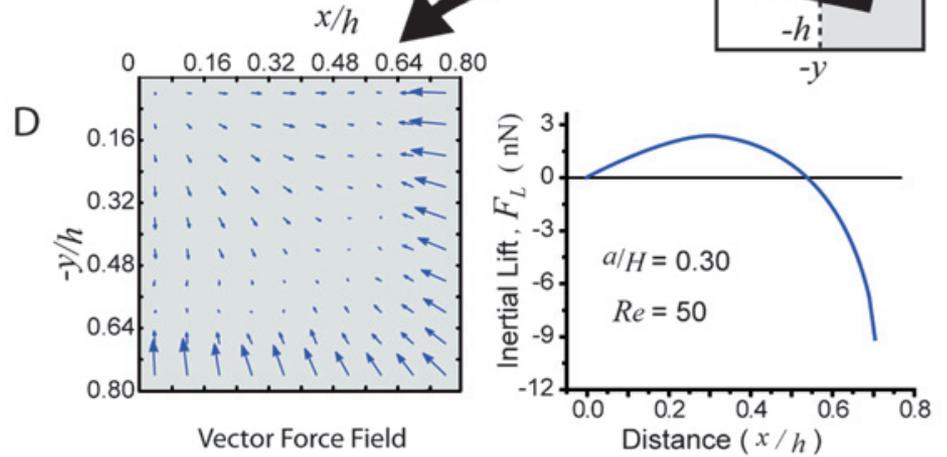
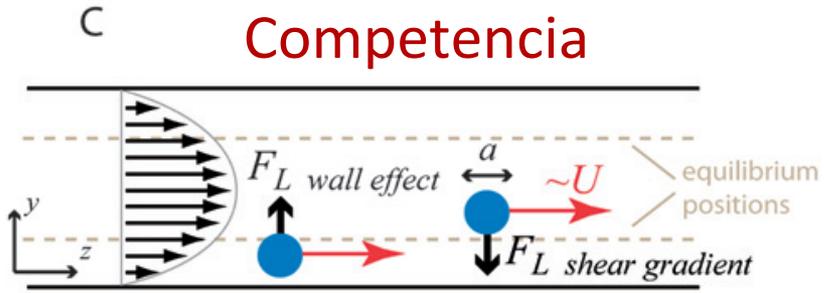
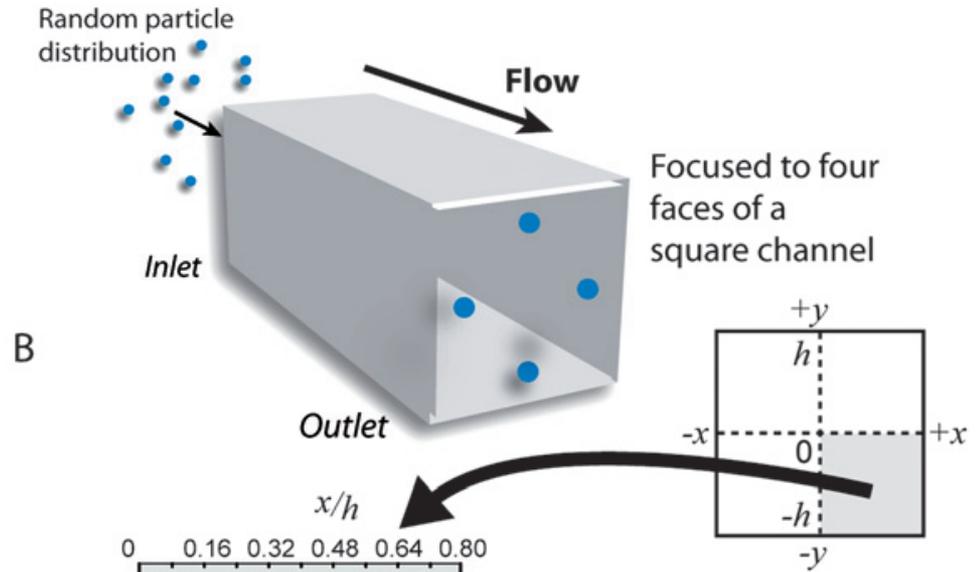
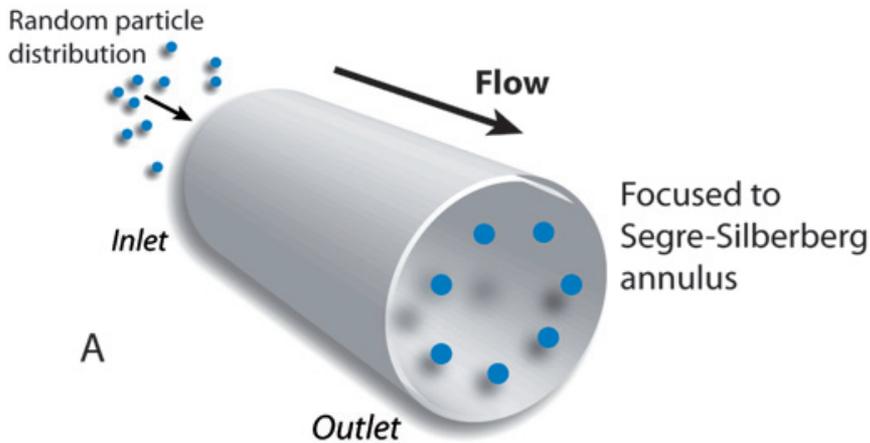
Despite the common wisdom that inertia does not contribute to microfluidic phenomena, recent work has shown a variety of useful effects that depend on fluid inertia for applications in enhanced mixing, particle separation, and bioparticle focusing. Due to the robust, fault-tolerant physical effects employed and high rates of operation, inertial microfluidic systems are poised to have a critical impact on high-throughput separation applications in environmental cleanup and physiological fluids processing, as well as bioparticle focusing applications in clinical diagnostics. In this review I will discuss the recent accelerated progress in developing prototype inertial microfluidic systems for a variety of applications and attempt to clarify the fundamental fluid dynamic effects that are being exploited. Finally, since this a nascent area of research, I will suggest some future promising directions exploiting fluid inertia on the microscale.

Microfluidica

Migración Inercial

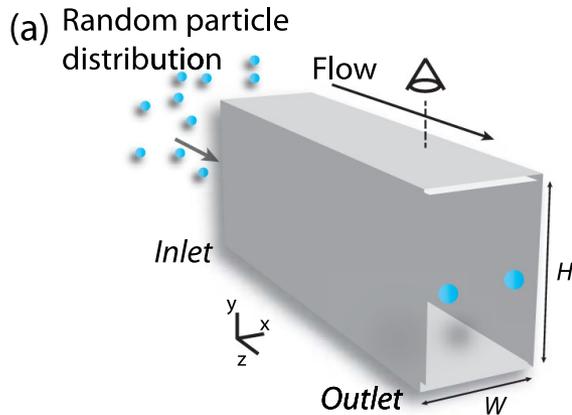
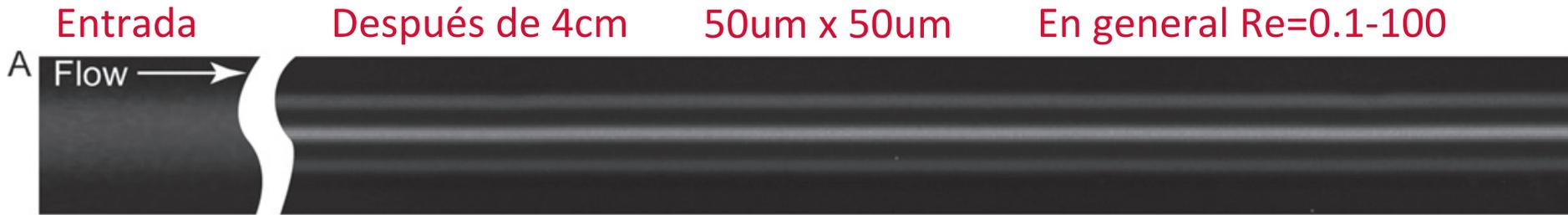


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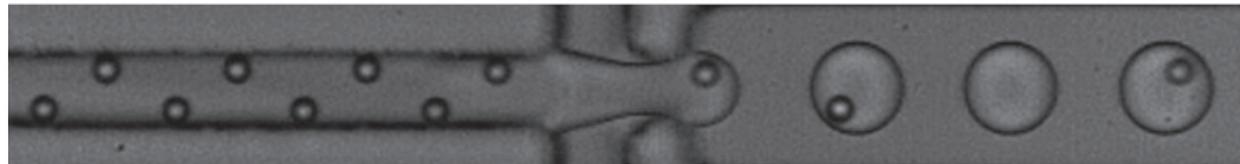


Microfluidica

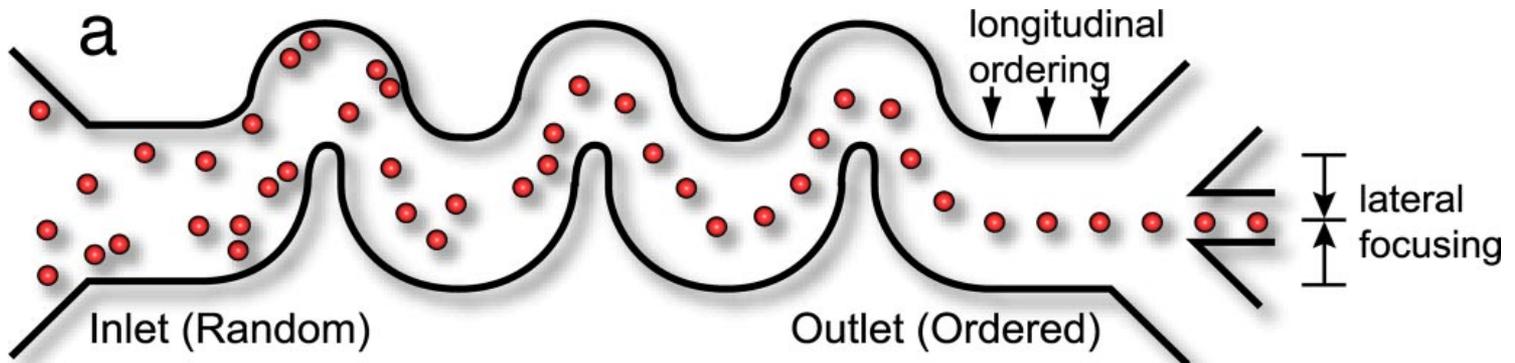
Centrado/Enfoque de partículas



50um x 30um → dos posiciones de equilibrio

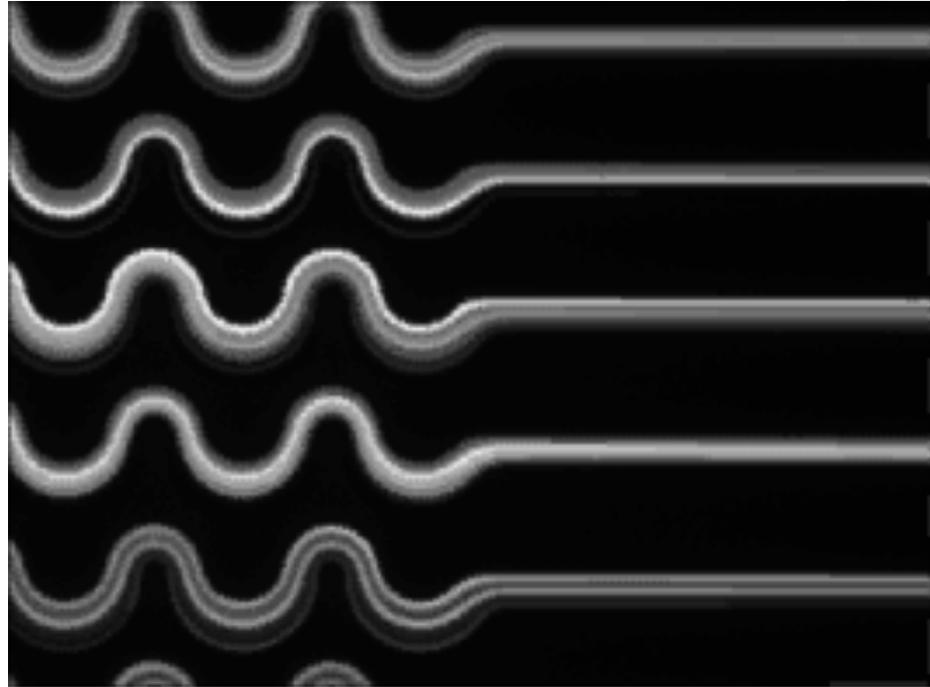


También se ordenan transversal y longitudinalmente!!

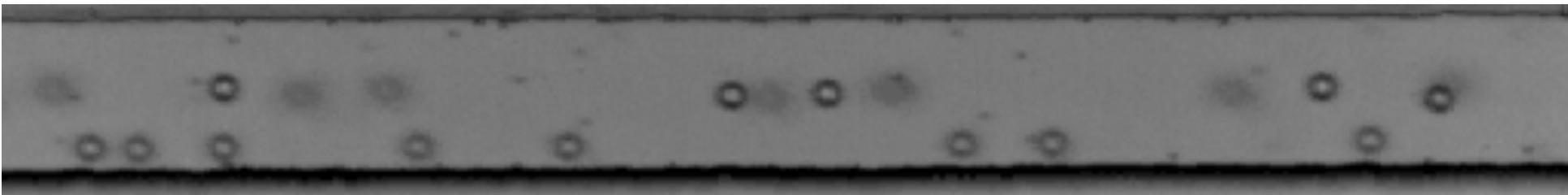


Microfluidica en el límite de Stokes

El número de Reynolds
aumenta de 0.3 a 12,
alrededor de $Re=1.5$
la suspensión de partículas
se focaliza y permanece
estable

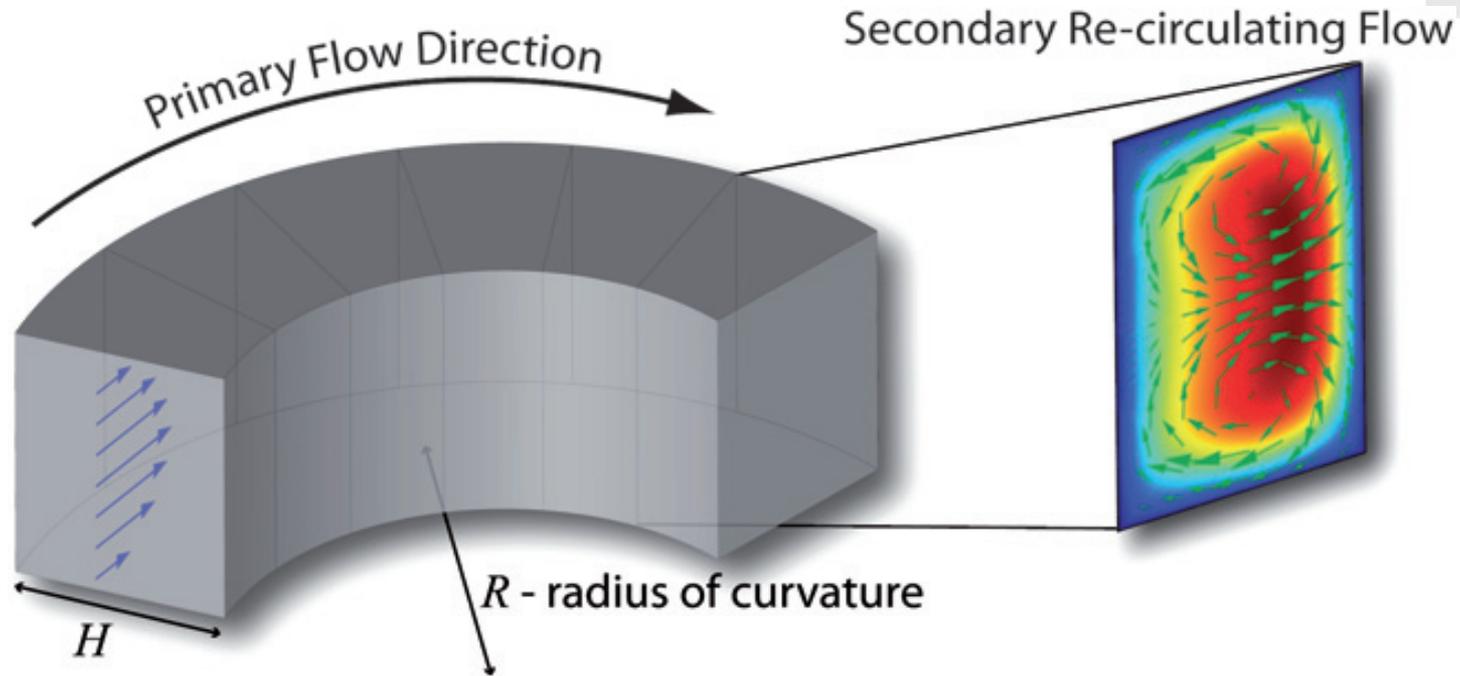


$Re=120$; Canal $50\mu\text{m} \times 50\mu\text{m}$; Estimar la velocidad



Microfluidica

Flujo Secundario de Dean



Para que el fluido *doble* hace falta una fuerza centripeta v^2/R

La mayor velocidad es en el centro del tubo \rightarrow mayores fuerzas inerciales

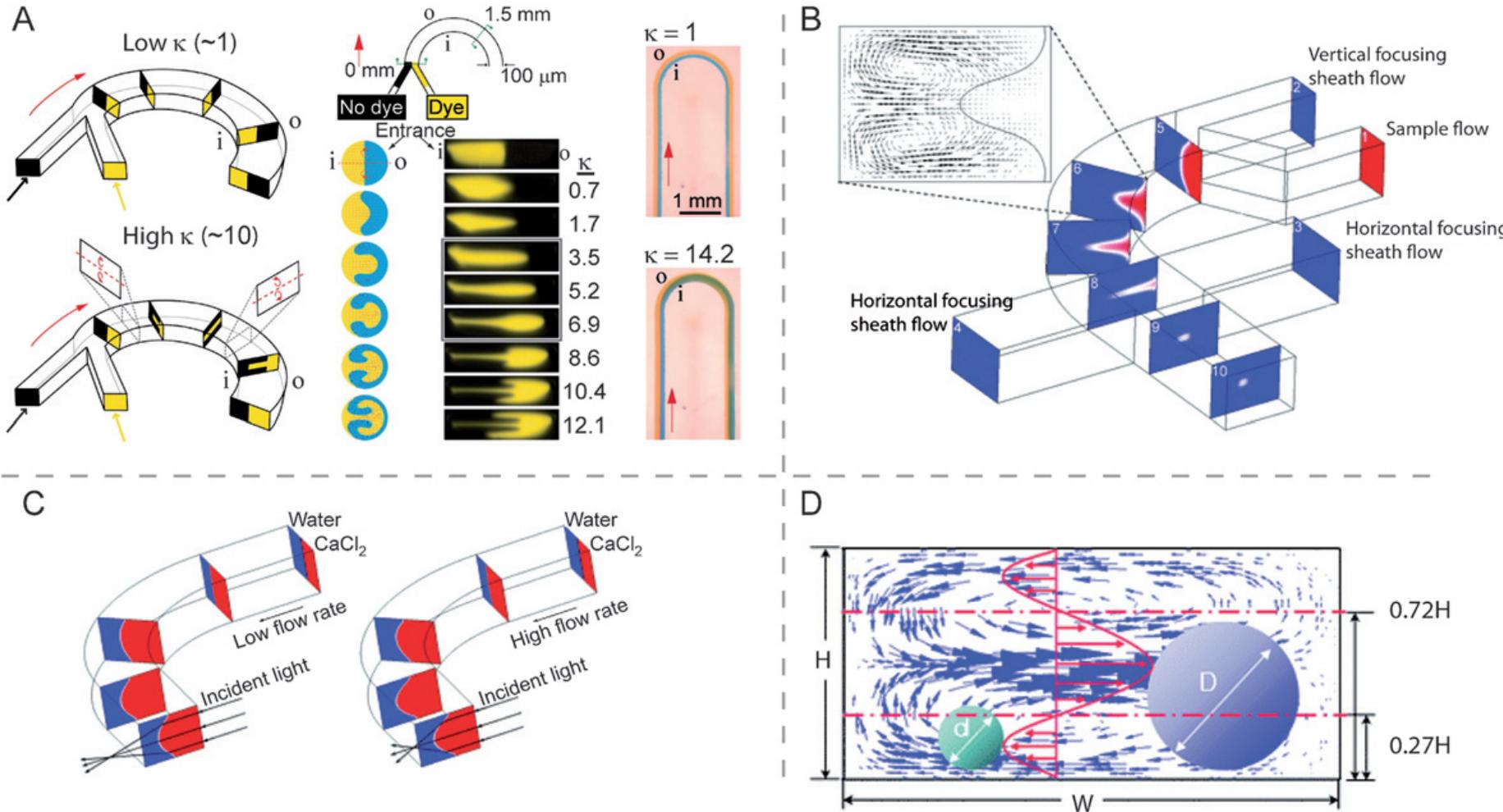
Inestabilidad equivalente a la de Rayleigh-Bernard y Taylor-Couette

Mayor velocidad \leftrightarrow Mayor densidad

Disipación: Conductividad de calor \leftrightarrow Viscosidad

Microfluidica

Aprovechando la inestabilidad de Taylor-Dean Mezclado; separación; manipulación de fluidos



Microfluidica en el límite de Stokes



Mezclado

En el régimen de Stokes ($Re=0$)
También es un problema!