

An introduction to global warming

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(Received 9 April 1999; accepted 28 June 99)

The physics of climate and of climate changes associated with increasing concentrations of greenhouse gases in the atmosphere are briefly presented. Construction of a “toy model” of the climate is discussed. Possibilities for reducing carbon dioxide emissions are indicated. Degrees of uncertainty characterizing predictions of climate responses to anthropogenic greenhouse gas emissions are presented. © 1999 American Association of Physics Teachers.

I. INTRODUCTION

Global warming primarily results from introducing large quantities of greenhouse gases into the atmosphere. Only some aspects of global warming are understood quantitatively according to physical principles, the climate system being very complex. We will present some physical principles and briefly discuss some complexities. We also discuss the potential for reducing carbon dioxide emissions. Several textbooks^{1,2,3} and reports^{4(a),4(b),4(c)} issued by the Intergovernmental Panel on Climate Change (IPCC) present excellent discussions of global warming as viewed from mainstream science.⁵

Carbon dioxide, methane, and nitrous oxide are naturally occurring greenhouse gases, but their concentrations are rapidly increasing due to human activity.⁴ The main anthropogenic greenhouse gases are shown in Table I. Major greenhouse gases not shown are water and ozone. Water vapor is the most important greenhouse gas because of its high concentration. Its concentration depends on temperature and not directly upon mankind's activities. Ozone is difficult to quantify and is omitted from Table I for that reason.

Atmospheric CO₂ has increased 26% since the industrial revolution, largely due to fossil fuel combustion and deforestation.⁶ The annual increase in CO₂ can be estimated from emissions as follows. The mass of air above each square meter is P/g , where g is gravitational acceleration and standard atmospheric pressure at sea level is $P = 101$ kPa. The mass of the atmosphere is approximately

$$M = 4\pi r^2 P/g = 5.25 \times 10^{18} \text{ kg}, \quad (1)$$

where the earth's radius is $r = 6.37 \times 10^6$ m. The average molecular weight of air is ~ 29 g mole⁻¹, so the atmosphere contains $\sim 1.8 \times 10^{20}$ moles of air. The mass of carbon from fossil fuel combustion emitted per year globally is 6.0×10^{12} kg,⁷ or 5×10^{14} moles. Thus the ratio of CO₂ in the atmosphere is estimated to increase annually by $5 \times 10^{14}/1.8 \times 10^{20} = 2.8$ ppmv. The observed rate of increase is only about 55% as great, primarily because of carbon sinks in the oceans and biosphere. The latter effects are difficult to estimate accurately and are subjects of current research.

The increase in N₂O is most likely due to agricultural practices. Methane concentration has doubled, primarily due to rice cultivation, cattle rearing, natural gas releases in oil fields, and leaks from pipelines. The chlorofluorocarbons (CFC) are made exclusively by humans. They were devel-

oped by General Motors researchers in the 1930's seeking nonflammable, nontoxic replacements for the refrigerants sulfur dioxide and ammonia.

The concentrations of natural greenhouse gases have fluctuated. A 160 000 year history at the Antarctic is shown for CO₂ in Fig. 1. These concentrations were measured by testing small samples of air found in ice cores. The temperatures shown in Fig. 1 are primarily determined from isotope ratios, such as the ratio ¹⁸O/¹⁶O found in H₂O. When the oceans are warmer, relatively more of the heavier isotopically-substituted molecule vaporizes and then precipitates as snow. The two important aspects of Fig. 1 are that the temperature and CO₂ concentration are correlated; and the CO₂ concentration is now at its highest level in the last 160 000 years.

From Fig. 1 it is not clear whether the CO₂ rise is a response to the temperature rise, or *vice versa*. However, one can imagine plausible scenarios. For example, volcanic activity may cause a CO₂ concentration increase, which can cause a temperature rise. Other causes of climate change (such as the earth's orbital variations) are much slower than the temperature increase observed during the last 100 years.

II. SUNLIGHT AND EARTHLIGHT

The solar energy absorbed by the earth must be balanced by energy leaving the earth, or the temperature would change substantially. For example, an excess of 1% of absorbed over emitted flux for one year would be equivalent to a heating of about 7 °C of the entire atmosphere.⁹ The balance is achieved by the outflow of “earthlight,” the infrared light emitted into space.

Both sunlight and earthlight can be approximated as blackbody radiation, whose spectrum (see Fig. 2) is defined by the Planck radiation law:

$$\frac{dF}{d\lambda} = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/k_B\lambda T) - 1}, \quad (2)$$

where F is the radiative flux (power per unit area) emitted by the surface of the blackbody, λ is the wavelength, T is the absolute temperature, and the other symbols have their usual meanings. The Stefan–Boltzmann equation is obtained by integrating Eq. (2) over wavelength:

$$F = \sigma T^4, \quad (3)$$

where σ is the Stefan–Boltzmann constant (5.67×10^8 W m⁻² K⁻⁴). The solar flux B incident on the earth depends

Table I. Key anthropogenic greenhouse gases. The chlorofluorocarbons are represented by CFC.

Name	Carbon dioxide	Methane	Nitrous oxide	CFC-11	CFC-12
Chemical formula	CO ₂	CH ₄	N ₂ O	CFCl ₃	CF ₂ Cl ₂
Atmospheric mixing unit ^a	ppmv	ppbv	ppbv	pptv	pptv
Pre-industrial	278	700	275	0	0
Present day (1992)	356	1714	311	268	503
% change per year	0.45	1.1	0.47

^appmv: parts per million by volume; ppbv: parts per billion by volume; pptv: parts per trillion by volume. Source: Ref. 4(c).

on the sun's surface temperature ($T_0 \sim 5800$ K), the solar radius, and the radius of the earth's orbit. The incident solar flux absorbed by the earth is

$$(1 - \alpha) \pi r^2 B = 1.2 \times 10^{17} \text{ W}, \quad (4)$$

where the albedo $\alpha = 0.3$ is the fraction directly reflected. For comparison, the global energy use by people is about 1.4×10^{13} W, one part in 10 000 of the solar influx. Energy use per unit area in U.S. metropolitan areas is roughly 2% of the solar influx. The absorbed flux averaged over the year and surface of the earth is

$$S = (1 - \alpha) B / 4. \quad (5)$$

The net energy flux can be written

$$N = S - F. \quad (6)$$

Here F is the average earthlight flux. In balance, $N = 0$ and the average flux of earthlight just beyond the atmosphere is $F = (1 - \alpha) B / 4 = 235 \text{ W m}^{-2}$.

III. CLIMATE AND THE GREENHOUSE EFFECT

The basic function of a greenhouse is twofold: glass is transparent in the visible and black in the infrared. Thus it admits sunlight, which is essential to the plants. The heat which would escape is largely in the form of infrared light. The glass absorbs infrared light, is warmed by it, and emits in the infrared. Only about half of the emission is directed out of the structure. Thus the net effect of the glass is to resist loss of infrared radiation from the interior. The interior becomes warmer, so the rate of emission increases inside the greenhouse until the heat loss matches the gain from absorp-

tion of sunlight. The glass also prevents convective loss, which would otherwise be an important mechanism for heat transport.

The characteristics of the earth and atmosphere are similar to a greenhouse. Convective and conductive heat losses are prevented by the vacuum surrounding the earth, so radiation is the means of energy gain and loss. The net emission of infrared light depends on the temperature, and on the transparency of the atmosphere to the infrared. Greenhouse gases decrease the transparency, forcing the earth's surface to become warmer until the energy emitted by the climate system balances the solar energy absorbed.

The solar spectrum reaching the earth's surface is similar to a blackbody spectrum at the sun's surface temperature, corrected for atmospheric absorption (see Fig. 2). The best known absorption is in the ultraviolet by stratospheric ozone. In the visible, the atmosphere is relatively transparent on a clear day.

The spectrum of earthlight from the earth's surface is also shown in Fig. 2. This spectrum is approximated by radiation from a blackbody at 288 K (15 °C); there is essentially no overlap with the solar spectrum. Absorption by the atmosphere is strong. Key absorption bands arise from exciting the bending vibrations of H₂O, CO₂, and O₃ (Ref. 9, p. 107).

The absorption from ground level to the top of the atmosphere is almost complete for wavelengths greater than 14 microns. There is however a "window" for most of the range from 8 to 13 microns. This window will become slightly less transparent as the CO₂ concentration is increased (see Fig. 3). This darkening is the main greenhouse effect of anthropogenic CO₂. Other greenhouse gases, especially chlorofluorocarbons, are potent agents of the green-

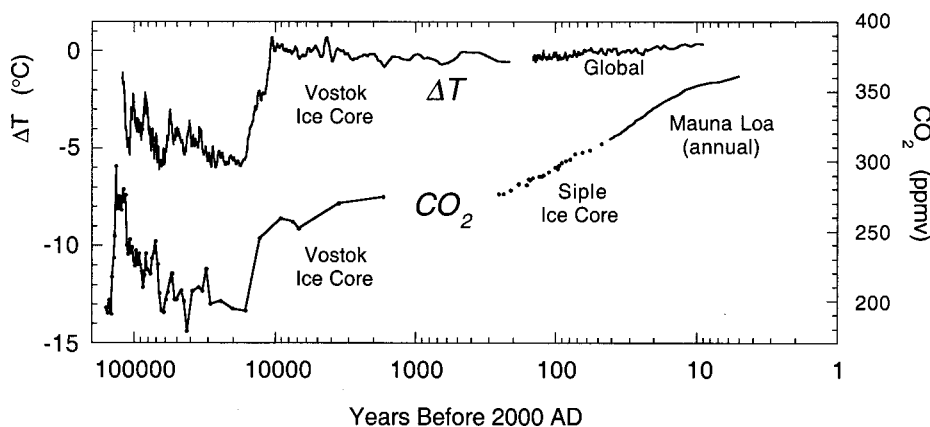


Fig. 1. Carbon dioxide mixing ratio and temperature variations for the last 160 000 years. In ice cores, the isotopic composition of the ice is a surrogate for temperature, and gas bubbles preserve samples of ancient air laden with CO₂. The global temperature change is based on historical meteorological records (Ref. 8).

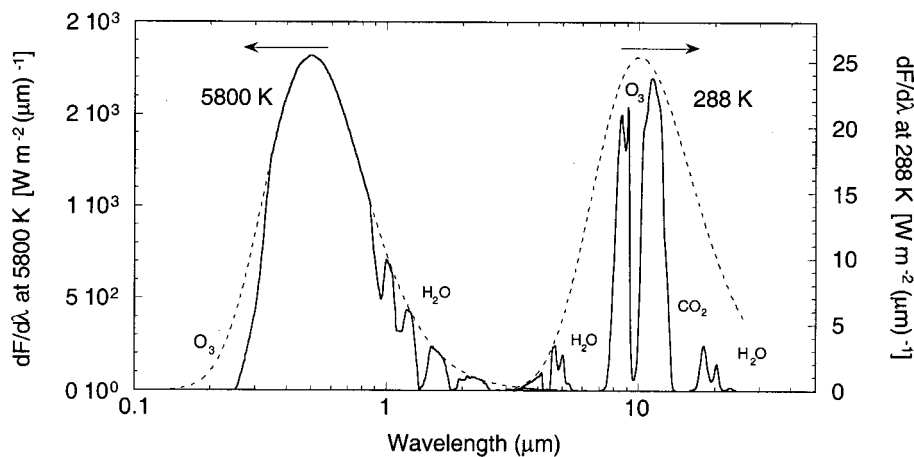


Fig. 2. The spectral radiative flux incident on the earth and escaping from the atmosphere, approximated as blackbody emission. Absorptions due to major natural greenhouse gases are shown. (Data from Ref. 10.)

house effect because they absorb in the atmospheric window and have long atmospheric lifetimes, but their concentrations are much lower than that of CO_2 .

A. A simple model of energy fluxes

A simple model of energy flows is shown in Fig. 4. Radiation fluxes are averaged annually and over the earth's surface. The incident radiation outside the atmosphere is

$$Q = \pi r^2 B / (4 \pi r^2) = B/4 = 342 \text{ W m}^{-2}. \quad (7)$$

Some 107 W/m^2 is reflected by clouds, and by water, snow and ice on the surface, corresponding to an albedo of about 0.3. The net flux absorbed is $\sim 235 \text{ W/m}^2$.

Most of the earth's output of infrared light is from the lower atmosphere; relatively little light emitted by the surface escapes directly into space. The observed average global surface temperature is $T_s = 288 \text{ K}$. If the earth's surface is approximated as a blackbody, the flux emitted by the surface is

$$F_s = \sigma T_s^4 = 390 \text{ W m}^{-2}. \quad (8)$$

If there were no absorbing atmosphere, the balance between the incoming and outgoing fluxes would be achieved with $F_s = 235 \text{ W m}^2$ at an average surface temperature of $\sim 254 \text{ K}$. The $\sim 34 \text{ K}$ difference between the observed 288 K and that just calculated is today's greenhouse effect. It is the direct result of atmospheric absorption. How much greater will the surface temperature become as the fraction absorbed by the atmosphere is increased by the increasing greenhouse gas concentration? The analysis is given in the following.

B. Radiative forcing and global warming potentials

The technical term "radiative forcing" is the net change in the flow of radiative energy due to specific changes, such as atmospheric composition, when other properties of the system, like temperature distributions, are held constant. It is the net radiative flux downward, averaged over a year and over the earth's surface at the tropopause (the boundary at $\sim 12 \text{ km}$ altitude between the lower atmosphere and the stratosphere). The tropopause is a convenient boundary, because the lower atmosphere is closely coupled with the surface.

Let a_i be the radiative forcing due to a unit increase in the mass of gas i in the atmosphere. It depends on the strengths and wavelengths of its infrared absorptions. The impact of a

gas over a time horizon H is its "global warming potential" (GWP).^{4(c)} The GWP of a gas is defined as the time integrated radiative forcing from a 1 kg increase of gas i relative to that of 1 kg of CO_2 :

$$\text{GWP}(i, H) = \frac{\int_0^H a_i C_i(t) dt}{\int_0^H a_0 C_0(t) dt}, \quad (9)$$

where $C_i(t)$ and $C_0(t)$ are concentrations of the i th greenhouse gas and CO_2 , respectively, which perturb the gases already present. If the gas concentrations decay exponentially with lifetimes τ_i and τ_0 , respectively, then

$$\text{GWP}(i, H) = \frac{a_i [1 - \exp(-H/\tau_i)]}{a_0 [1 - \exp(-H/\tau_0)]}. \quad (10)$$

Some lifetimes and global warming potentials are shown in Table II. There are two complications which we do not discuss: The decay of CO_2 in the atmosphere is not represented by a single exponential; and the impact of methane is largely indirect, through its effect on ozone concentration.

C. Aerosols and albedo

Radiative forcing due to greenhouse gases is relatively well understood. However, another important cause of forcing is anthropogenic aerosols. Aerosols are dust particles

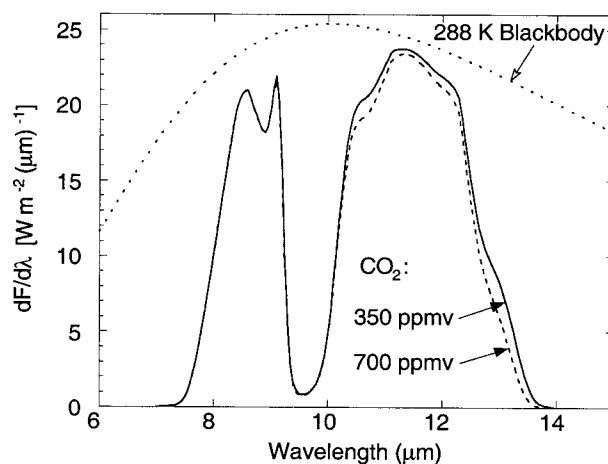


Fig. 3. Spectral radiative flux emitted from the earth. The range 8 to 13 microns is the atmospheric window, where most of the energy flux escapes. (Data from Ref. 10.)

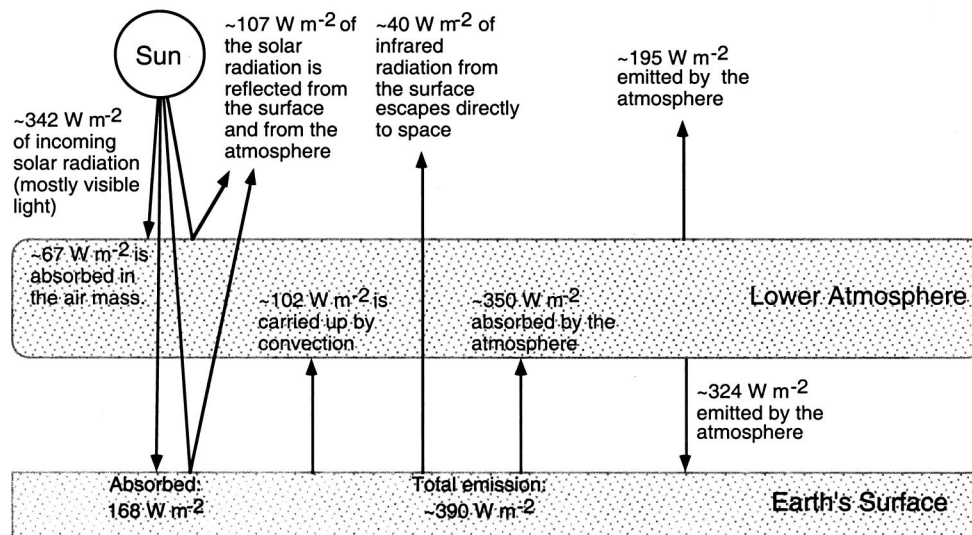


Fig. 4. Earth energy budget. (Adapted from Ref. 11.)

suspended in the air, or tiny droplets that incorporate, for example, sulfuric acid. When high-sulfur fuels are burned, the resulting sulfur dioxide gas (SO_2), is oxidized in the atmosphere to produce sulfuric acid and sulfates. These substances nucleate the formation of aerosols and cloud droplets. Thus, SO_2 affects the degree of haziness and cloudiness. Clouds and haze reflect sunlight (producing increased albedo) and help to absorb infrared light. Figure 5 shows the estimated direct effect from the increased albedo due to aerosols. The indirect effects of clouds nucleated by the aerosols are more difficult to estimate and remain one of the most uncertain aspects of climate change.

D. The climate system

Climate is characterized by the statistical properties of the weather over a period of time, including averages of local variables such as temperature, winds, humidity, pressure, and precipitation. One problem in trying to detect significant climate change is that considerable time must elapse while sufficient data are obtained.

The climate system is driven by incoming sunlight and outgoing earthlight. Ocean currents, general atmospheric circulation and storms are all ultimately powered by sunlight and the geographical imbalances between input and output energy fluxes. Heating is strongest in the tropics and cooling is dominant near the winter pole. The latitudinal heating gradient drives atmospheric and oceanic circulation which transfer heat from the equator towards the poles. Figure 6 shows the geographical energy imbalances. The top panel shows the higher reflectivity near the poles. There is a net influx of energy between 30°N and 30°S and a net out-flux at higher latitudes.

The climate system is conveniently divided into five parts: atmosphere, oceans, cryosphere, biosphere, and geosphere, each of which will be briefly discussed.

The atmosphere. The earth's surface absorbs the bulk of the incoming solar radiation. Direct heating of the surface and evaporation of water cause heat transfer between the surface and the atmosphere in the form of infrared radiation and as sensible heat (energy related to temperature changes when there is no phase change) and latent heat (energy re-

lated to phase change at constant temperature). The atmosphere transports this heat by means of weather systems, on a time scale of days.

Some other important atmospheric processes are condensation, scattering of light by aerosols, and responses to the earth's topography. Condensation causes cloud formation, with major effects. Aerosols also have effects; for example, scattering of light by stratospheric aerosols increases the albedo. The location of mountains and land-sea distributions is important to evaporation and precipitation, and thus to regional climates.

The oceans. Over half of the absorbed solar radiation is absorbed by the oceans and redistributed by ocean currents. The ocean's horizontal and vertical structure depends on the winds, the topography of the ocean floor and distribution of the continents. The currents are driven by winds, salinity variations, and temperature gradients. Carbon dioxide dissolves in and evaporates from the surface of the ocean, depending on local atmospheric and seawater concentrations. Dissolved CO_2 is used in photosynthesis by plankton, which later die and settle into deep water, sequestering carbon for periods longer than 100 years.

The cryosphere. The cryosphere consists of seasonal snow cover, sea ice, ice sheets (Greenland and Antarctica), mountain glaciers, and permafrost. Among its important roles are the high albedo of snow and ice, and the freshwater reservoirs in ice sheets. Melting or movement of ice sheets would affect sea level, but it is thought that the time scales for these processes are at least millennia, except for speculation about instability of the West Antarctic Ice Sheet.¹⁴

The biosphere. The combined land and ocean biosphere

Table II. Global warming potentials. Sources: Refs. 4(b) (Table 5.2) and 12 (Table 7-2).

Gas	Lifetime (years)	GWP ($H=20$ yrs)	GWP ($H=100$ yrs)
CO_2	50–200	1	1
CFC-11	50	5000	4000
CFC-12	102	7900	8500
Methane	14.5	62 ± 20	24.5 ± 7.5

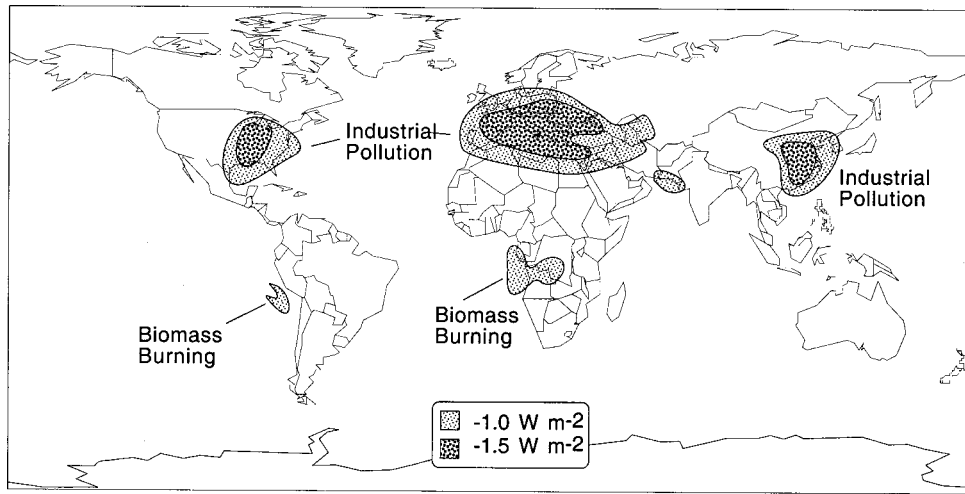


Fig. 5. Estimated radiative forcing due to sulfate aerosols produced by the combustion of sulfur-bearing fuels and by biomass burning. [Adapted from Fig. 4.3 in Ref. 4(b).]

controls the main natural greenhouse gases and their interchanges with the atmosphere, oceans and land. The biosphere contains much of the carbon involved in short time scale interchanges, for example, in plankton, soil and crops.

The geosphere. The geosphere consists of the continents. The continents are permanent for the purposes of this discussion, but fresh water storage in the soil and run-off have time scales from days to months.

E. Feedback mechanisms and the greenhouse effect

Feedback is the influence of system responses on the operation of greenhouse gas forcing. It is subsidiary to the main process, either tending to reinforce the main process (positive feedback) or tending to resist the main process (negative feedback). The major feedback mechanisms include the following.

Water vapor feedback. Due to greenhouse warming, more water evaporates and more water vapor is present in the atmosphere. But water is a greenhouse gas, so one result of greenhouse gas emissions is to increase the concentration of another greenhouse gas, a positive feedback.

Snow/ice albedo feedback. A warmer climate results in less snow and ice cover and therefore the albedo is reduced, a positive feedback. However, because atmospheric circulation and polar atmospheric stability may also be affected, the situation is difficult to analyze.

Cloud feedback. Clouds cause warming by closing the window to the escape of earthlight, a positive feedback; but they also increase the albedo by efficiently reflecting solar radiation, a negative feedback. The net result averaged over the earth's surface is calculated to be a cooling, a negative feedback. However, if radiative forcing increases substantially, the analysis is more complicated. Although warming increases humidity, higher temperatures may reduce cloud cover. Similarly, warming may increase the water content of clouds, increasing the efficiency of solar reflection, but also increasing the efficiency of infrared absorption.

F. Feedback and the climate sensitivity factor (Ref. 15)

The simplest radiative model is the radiative energy balance model described by Eq. (7). When radiative balance is achieved, $N=0$. Consider a perturbation:

$$\Delta N = \Delta S - \Delta F. \quad (11)$$

Assume the balance is reestablished by changing the surface temperature ΔT_s . The temperature change produces a change in the net radiative flux opposite to the perturbation:

$$-\Delta N = \Delta T_s \left\{ \frac{\Delta F}{\Delta T_s} - \frac{\Delta S}{\Delta T_s} \right\} = G, \quad (12)$$

where G is called the direct radiative forcing. (For doubled CO_2 , $G=4 \text{ W/m}^2$.) The climate sensitivity factor is defined as its reciprocal:

$$\lambda = \Delta T_s / G. \quad (13)$$

Because F and S depend on other climate processes, λ depends on the details of the climate system response. For example, the term involving ΔS can be written as a Taylor

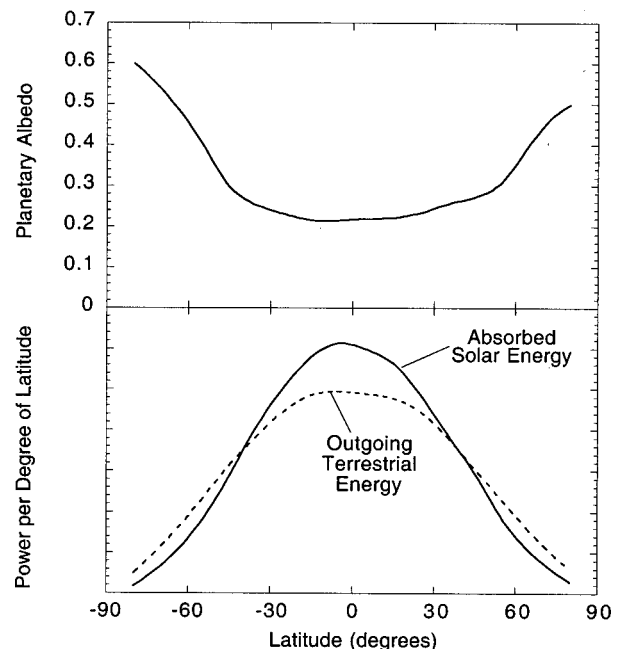


Fig. 6. Planetary albedo and energy flux (annual average). (Adapted from Ref. 13.)

series which depends on the solar flux $Q = B/4$, the albedo α , the average cloud fraction f_c , etc.,

$$\frac{\Delta S}{\Delta T_s} = -Q \left\{ \frac{\partial \alpha}{\partial T_s} + \frac{\partial \alpha}{\partial f_c} \frac{\Delta f_c}{\Delta T_s} + \dots \right\}. \quad (14)$$

Assume the outgoing flux depends only on the surface temperature according to the Stefan–Boltzmann law. This assumption leads to the no-feedback climate sensitivity factor:

$$\lambda_{\text{nf}} = 1/(\delta F/\delta T_s) = T_s/(4F) = 0.3 \text{ K m}^2/\text{W}. \quad (15)$$

Observational data¹⁶ give the following approximate relationship between F and T_s :

$$F = 1.55 T_s - 212 \text{ W m}^2 \quad (16)$$

and

$$\lambda_{\text{obs}} = 0.6 \text{ K m}^2/\text{W}, \quad (17)$$

which is twice as large as the no-feedback case. This value can be used with the estimated⁴ $\sim 4 \text{ W/m}^2$ radiative forcing associated with doubling the CO_2 mixing ratio (from 350 ppmv to 700 ppmv) to obtain the temperature change associated with the doubling: $T_{2x} (4 \text{ W/m}^2 \times \lambda_{\text{obs}} = 2.4 \text{ K})$. This value is close to the estimate preferred by the IPCC, although they have assigned a large range of uncertainty (1.5 K to 4.5 K).⁴

Feedback mechanisms also add complexity. Consider snow/ice albedo feedback. A simple parameterization considering only the surface temperature shows the essential factors. One such parameterization is based on observations:¹⁵

$$\text{Bare soil: } \alpha = 0.3 \quad T_s > 270 \text{ K}, \quad (18a)$$

$$\text{Mixed: } \alpha = 0.3 - 0.01(T_s - 270) \quad 230 \leq T_s \leq 270 \text{ K}, \quad (18b)$$

$$\text{Bare ice/snow: } \alpha = 0.7 \quad T_s < 230 \text{ K}. \quad (18c)$$

Figure 7 shows F and S , where the solar input is given by $S = (1 - \alpha)B/4$. One sees that there are three states of the system which satisfy the radiative balance condition $N = 0$. Multiple equilibria are a feature of many climate models. In Fig. 7 the highest temperature intersection corresponds to the present climate. The lowest ($T_s = 202 \text{ K}$) corresponds to complete ice coverage. The intermediate solution is unstable. It is not surprising that there should be a stable low-temperature state, because as the albedo increases, the surface temperature declines, further increasing the albedo. In general, we would expect the earth's climate to exhibit a variety of complicated behaviors in response to forcing.

G. General circulation models and projections of climate change

Projections of climate change are of limited accuracy because the feedback mechanisms are complex and not fully understood, and because anthropogenic activities cannot be predicted with great confidence. One approach that avoids the need to fully understand the climate system is the Paleo-Analog method, which uses ancient climates as analogs for future climate. Although this method is intuitively attractive, there are many uncertainties attached to the reconstruction of past climates and to accounting for influences on past climates.

The second approach is to simulate the climate system using general circulation models which are based on funda-

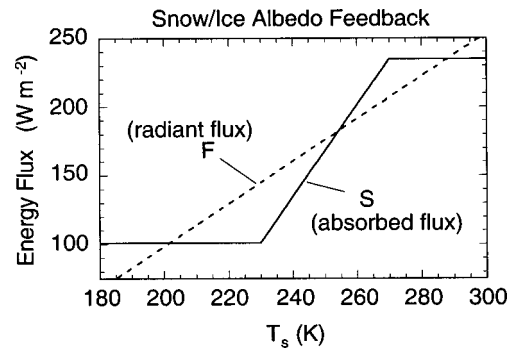


Fig. 7. Snow/ice albedo feedback. (Adapted from Ref. 15.)

mental physics principles, estimates for the more poorly understood climate processes, and calculated radiative forcings. General circulation models use a grid to divide the climate system both geographically and vertically into cells; the larger the number of cells, the more accurate the calculation. These models vary greatly in their complexity, depending on the number of cells, on whether ocean-atmosphere couplings are explicitly modeled, and on whether the time dependence is analyzed instead of assuming equilibrium. The more complex models are expensive in computer time. A practical strategy is to run the more complex models a few times to calibrate the simpler models, which can then be run many times to explore dependencies.

The predictions of today's general circulation models must be validated. We can hope that a model will make reasonable predictions only if it adequately describes the present climate. Validation is an iterative process. If the model does not work well, the semi-empirical relationships and parameters are adjusted until it works well enough. The danger lies in the possibility that the parameters and semi-empirical relationships, which enable the model to describe the present climate, are compensating for errors, which prevent the model from accurately describing responses to radiative forcing.

The good news in the face of these difficulties is that climate has many features. Observations and validation tests include the following properties, which are functions of season, altitude, and geographical region: temperature, precipitation, pressure, cloud cover, humidity, snow and ice cover, and winds. Moreover, there are several special observables associated with global warming. These include much warmer polar winters, glacial retreat, borehole temperature trends, historical correlation between CO_2 concentration and temperature, cooling of the stratosphere, drier soils in northern mid-latitude summers, sea level rise, and decreased ocean circulation. Such observables can be compared with developments in recent history. Most models have been adjusted so that they work reasonably well for most properties. The most troublesome aspects are related to clouds. In any event, model calculations must be regarded with caution because future climates may not correspond to the test cases, so the reliability of model predictions is not easily evaluated.

IV. A "TOY" CLIMATE MODEL

To gain insight into the climate system and learn about some problems facing modelers, we construct a "toy" climate model. Such models cannot represent the climate system accurately, but they are fun to construct. By adjusting

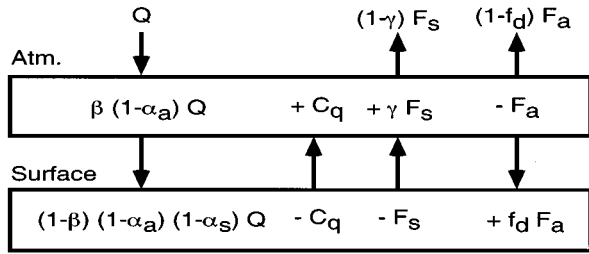


Fig. 8. Schematic of two-level radiative equilibrium model showing the terms corresponding to Eq. (20).

the parameters and the assumed relationships, toy models can even produce semiquantitative results. This “validation process” is analogous to the process used in constructing state-of-the-art general circulation models.

Consider a simple system of two layers, the atmosphere and the surface (see Fig. 8). If there were no atmosphere, one differential equation would suffice to describe the surface temperature. (A typical problem is to assess the fraction of black and white paint stripes needed on a satellite with an internal heat source in order to maintain its temperature within a desired range.) In more accurate models, the atmosphere is divided into many layers at each geographical location. The number of layers is increased until the solution “converges,” that is, no longer changes significantly as more layers are added. In addition to radiative transfer, general circulation models include equations for conservation of mass, momentum, and energy.

In our simple two-layer model, “short wave” solar energy (Q) is incident on the system and a fraction (α_a) is reflected by the atmosphere, a fraction (β) is absorbed by the atmosphere, and the rest is transmitted to the surface, which reflects a fraction (α_s) and absorbs the remainder.

The surface emits infrared light according to the Stefan–Boltzmann law, $F_s = \sigma T_s^4$. A fraction (γ) of this “long wave” radiation is absorbed by the atmosphere and the rest is lost to space. This absorption by the atmosphere leads to heating and to emission of long wave radiation. We assume the emission rate depends on atmospheric temperature according to the Stefan–Boltzmann law ($F_a = 2\sigma T_a^4$, the factor of 2 arises because the emission is directed both upward and downward).

Finally, heat and water vapor are transported by turbulent convection, giving rise to an energy flux C_q (as sensible and latent heat) from the surface to the atmosphere.

With the aid of Fig. 8, we can write differential equations for the rate of change of energy in the two layers:

$$\text{Atmosphere: } \frac{dE_a}{dt} = \beta(1 - \alpha_a)Q + C_q + \gamma\sigma T_s^4 - 2(1 - f_d)\sigma T_a^4. \quad (19a)$$

$$\text{Surface: } \frac{dE_s}{dt} = \beta(1 - \alpha_a)(1 - \alpha_s)Q - C_q - \sigma T_s^4 + 2f_d\sigma T_a^4. \quad (19b)$$

The parameter f_d describes the fraction of long wave emission from the atmosphere which is directed back to the surface.

Table III. Toy climate model parameters.

Parameter	Symbol	Value	Source
“Clear air” albedo	α_{a0}	0.05	Estimated
Cloud albedo	α_c	0.40	Estimated
Snow-ice albedo	α_i	0.70	Estimated
Soil albedo	α_r	0.10	Estimated
Relative humidity	R_{H_2O}	0.8	Assumed
H ₂ O scale height	H_{H_2O}	2 km	Assumed
Fraction emitted downward	f_d	0.69	Adjusted
Incident sun	Q	342 W m ⁻²	Ref. 11
Heat flux at T_{s0}	H_0	102 W m ⁻²	Ref. 11
Reference surface temp.	T_{s0}	288.0 K	Ref. 11
Reference atoms. temp.	T_{a0}	253.8 K	

Assuming radiative equilibrium, the left-hand-sides of the equations are set equal to zero and the two simultaneous algebraic equations are solved for T_s by iteration or other methods. Of course, we also need to obtain reasonable values of the parameters (Table III).

The *planetary albedo* is the average reflectance of short-wave radiation. Most measurements and models give an albedo of ~ 0.3 .¹¹ The albedo of clouds depends on their altitude, liquid water content, droplet size, and temperature, etc. This complexity causes clouds to present the most difficult problems in climate modeling. For simplicity, we ignore these distinctions and assume all clouds have the same albedo α_c . Even “clear air” has a “clear air albedo” α_{a0} , due to scattering by the molecular constituents and aerosols. These different albedos must be combined; we assume the relationship for average atmospheric albedo:

$$\alpha_a = f_c \alpha_c + (1 - f_c) \alpha_{a0}, \quad (20)$$

where f_c is the fraction of cloud cover. For the present atmosphere, we assume $f_c = 0.62$.¹¹ The albedo of the surface is assumed to be $\alpha_r = 0.1$. The energy budget of Kiehl and Trenberth,¹¹ enables estimation of albedo parameters for the present atmosphere within the context of this model (Table III).

Of the long-wave energy radiated by the atmosphere, only the fraction f_d reaches the earth’s surface. For a single layer atmosphere, one would expect $f_d = 1/2$, but the actual atmosphere is more complex. The “tuned” value $f_d = 0.69$ (Table III), which was adjusted in the model to obtain $T_s = 288$ K, is in reasonable agreement with the estimate shown in Fig. 4.

In our model, we assume that clouds are perfect absorbers of long wave radiation, because liquid and solid water are good absorbers in the infrared. In addition, a fraction γ of the long wave radiation from the surface is absorbed by the cloud-free atmosphere, which is how greenhouse gases produce their effect. For monochromatic infrared light in clear air, this fraction can, in principle, be calculated using the Beer–Lambert law. Techniques exist for accurately solving the actual radiative transfer problem, but none is practical for a toy climate model. We use an approximate method based on “equivalent widths,”¹⁰ and obtain atmospheric transmittances for variable amounts of CO₂ and H₂O, and a fixed amount of stratospheric O₃. [The three terms in the square bracket of Eq. (21) are associated with these three gases.] We fit the results empirically to obtain

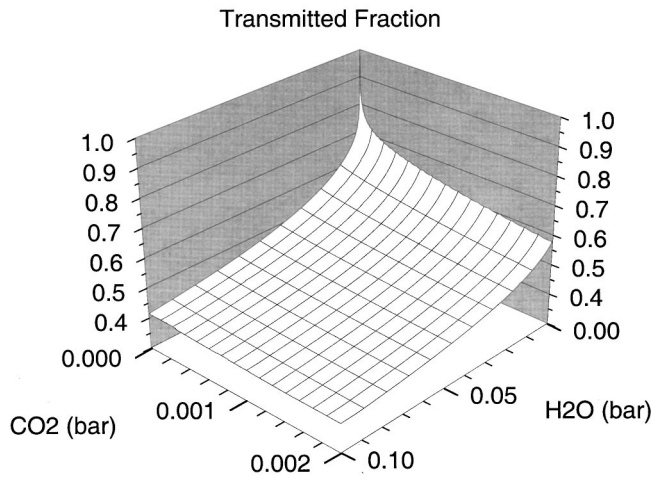


Fig. 9. Surface plot of the fraction of 288 K blackbody radiation transmitted by the atmosphere for various partial pressures of CO₂ and H₂O (at the surface).

$$\gamma \approx \{1 - \exp[0.082 - (2.38P_{\text{H}_2\text{O}}H_{\text{H}_2\text{O}}R_H + 40.3f_{\text{CO}_2})^{0.294}]\}, \quad (21)$$

where f_{CO_2} is the mixing ratio of carbon dioxide (for example, $f_{\text{CO}_2} = 3.5 \times 10^{-4}$ corresponds to 350 ppmv), R_H is the relative humidity (assumed to be 80%), and $P_{\text{H}_2\text{O}}$ is the saturation vapor pressure of water (in bars):

$$P_{\text{H}_2\text{O}} = 1.76 \times 10^6 \exp(-5318/T_s) \text{ bars}. \quad (22)$$

The parameter $H_{\text{H}_2\text{O}}$ is the scale height in km of water vapor (assumed to be 2 km). ‘‘Scale heights’’ are often used to describe the partial pressures of gases in the atmosphere.²

$$p_i(z) = p_i(0) \exp(-z/H_i), \quad (23)$$

where H_i is the scale height, p_i is the partial pressure, and z is the altitude. Usually H_i depends only weakly on altitude.

Equation (21) shows that the fraction of long wave radiation absorbed by the clear atmosphere is strongly non-linear in the partial pressures of CO₂ and H₂O (Fig. 9). The absorbance is only about 50%, even for large amounts of CO₂.

Perhaps the most important climate feedback mechanism is associated with water vapor. We assume that $P_{\text{H}_2\text{O}}$ increases with T_s according to the saturation vapor pressure, while the relative humidity and scale height are constant, even when global warming takes place. These assumptions could be the subject of further investigation. General circulation models include specific mechanisms for evaporation of surface water, condensation of clouds, and precipitation, etc.

Snow/ice albedo feedback is incorporated into the toy model using the empirical form for surface albedo:

$$\alpha_s = \alpha_i + (\alpha_r - \alpha_i) \frac{1}{2} \left[1 - \tanh \left[- \left(\frac{250 - T_s}{20} \right) \right] \right], \quad (24)$$

where α_i is the snow/ice albedo and α_r is the albedo for bare soil. The numerical values (Table III) imitate Eq. (18).

Additional feedback mechanisms, for the fraction of cloud cover f_c and convective heat flux C_q are described empirically:

$$f_c = 0.5 \{1 - \tanh[-0.245 + (1 - P_{\text{H}_2\text{O}}P_{\text{H}_2\text{O}}^0)^s]\} \quad (25)$$

$$C_q = C_{q_0} (T_s/T_{s_0})^t, \quad (26)$$

where $T_{s_0} = 288$ K is the surface reference temperature, $P_{\text{H}_2\text{O}}^0$ is the saturation vapor pressure of water at T_{s_0} , and C_{q_0} is the convective heat flux (102 W m^{-2}) at T_{s_0} . For T_{s_0} , Eq. (25) gives $f_c = 0.62$, in good agreement with observations.¹¹ Choices for cloud albedo feedback and heat flux feedback are made by selecting parameters s and t . In the model these are free parameters. Positive, negative, and zero feedback are selected by choosing a parameter greater than, less than, or equal to zero. One way to set s and t is to ‘‘tune’’ the model to give a desired climate sensitivity factor. For example, to achieve $T_{2x} = 2.5$ K, we arbitrarily chose $t = 1.00$ and then varied s until that sensitivity factor was achieved ($s = 0.92$). Because the model includes some arbitrary assumptions, these parameters have little numerical significance. However, one can explore the feedback mechanisms by changing the parameter magnitudes and signs.

We have implemented the model as a spreadsheet (Microsoft Excel). (A copy of the spreadsheet can be obtained from web site: <http://www.physics.lsa.umich.edu/phys419> or from the authors.) Numerical solutions are obtained by combining Eqs. (19) at equilibrium and using the secant iterative root-finding method to find T_s . The iterations are started by choosing T_{start} , an estimate for the surface temperature. By choosing various values, one finds the equilibrium solutions. For example, when $f_{\text{CO}_2} = 350$ ppmv and the parameters s and t are set to 0.92 and 1.00, respectively, three real solutions are found at $T_s = 288$ K, 241 K, and 229 K for $T_{\text{start}} = 300$ K, 250 K, and 200 K, respectively. The lowest temperature solution corresponds to the frozen earth. The middle solution is unstable. Stability is assessed by adding a small positive perturbation to the solution. If an increase in net energy input results, the perturbation is amplified, signaling instability. As f_{CO_2} is increased from 350 ppmv to around 500 ppmv, the two lower temperature solutions converge and above ~ 500 ppmv of CO₂ there is only one real solution. This is an example of a ‘‘pitchfork bifurcation.’’¹⁷

Advanced students can construct their own models. Or students can modify or enhance the current model. For example, additional feedback mechanisms can be incorporated, and different empirical approximations can be adopted and tested. What would happen to the results if the H₂O scale height and/or relative humidity varied as a function of T_s ? All students can benefit from exploring the behavior of the current model when the feedback parameters are changed. For example, how does the pitchfork bifurcation change when the feedback parameters are altered, retaining the climate sensitivity factor, but changing the sense of the feedback?

Given a suitable climate model, we can estimate climate changes as functions of greenhouse gas concentrations. The next task is to estimate anthropogenic greenhouse gas emissions.

V. PREDICTIONS OF ANTHROPOGENIC EMISSIONS

We do not know the future, but most CO₂ emissions are associated with large amounts of capital equipment: energy-intensive mills, buildings and their equipment, and vehicles. These facilities cannot change rapidly, thus placing limits on

the rate at which CO₂ emissions can vary. If an activity increases at a constant relative rate v , the growth is exponential, with doubling time t_2 :

$$t_2 = \ln 2/v. \quad (27)$$

For a growth rate of 3% per year, typical of well-functioning economies, the doubling time is 23 years. So at least a quarter of a century is likely to be involved in a doubling of energy use, or in a reduction to one half. For example, it would probably take at least a quarter century for the U.S. to change from coal-based steam turbine electrical generation to natural-gas based “combined-cycle” generation (that is, from high carbon intensity to low carbon intensity). Similarly, if we were to adopt new automotive technology, it would take at least a quarter century to change most automobiles to those emitting, say, one-half or one-third as much.

Prognosticators cannot accurately predict the effects of economic downturns, or of unusually rapid growth that may occur, or of political decisions that may be made to reduce carbon emissions. The way to handle these uncertainties is to create a family of alternative scenarios (within reasonable constraints).⁴ One important scenario involves vigorous efforts to reduce carbon emissions.

The potential for reducing carbon emissions. The world energy system is dominated by fossil fuels. The most important anthropogenic greenhouse gas is carbon dioxide from the burning of fossil fuels, accounting for more than half of the forcing. There are at least four approaches to reducing greenhouse gas emissions associated with fossil energy use: (1) improve the efficiency of processes; (2) transfer some carbon from the atmosphere to the biosphere, for example, by growing trees; (3) substitute fuels with lower or no carbon content; and (4) separate and sequester the CO₂ from fossil fuels. Improving efficiency represents a very large opportunity. It is the best and least expensive of these approaches, but is too large a topic to be discussed here. Moreover, important as efficiency is, it has limits. At this time technology is far from those limits in most activities, but societies would be hard-pressed to achieve, through efficiency alone, all the reduction some scientists say would be necessary. We next briefly discuss the other options.

Improved forest management and growing more trees would reduce CO₂ concentrations, and in might be beneficial in other ways.¹⁶ The capacity is not very large, up to 7% of annual carbon emissions in the U.S. might be sequestered annually. Accurate assessment is difficult because much carbon storage occurs underground in root and soil systems.

More natural gas and less coal and oil are being used for reasons of cost and convenience, and this use also helps to slow the increase in carbon emissions. If global warming is as serious a threat as it seems, it will be necessary in the long term to switch to energy forms not based on carbon emissions. Nuclear fission, biomass, wind, and photovoltaics have been discussed in this connection. The costs and timing of expanding nuclear power or of adopting energy forms like wind and photovoltaics raise big questions; but instead we consider a less well known alternative: using fossil fuels without carbon emissions.

The emissions from fuel combustion. From the atomic constituents of a fuel and its heating value, we can readily calculate the emission of CO₂ per unit of heat release in normal combustion. The relative numbers of atoms of hydrogen, carbon or oxygen in a fuel are given by the empirical formulas

Table IV. Relative numbers of carbon and hydrogen atoms in various fuels.

Fuel	Empirical formula	Mass fraction of carbon ^a	Heating value ^b (MJ/kg)	Carbon intensity (kg/GJ)
Methane	CH ₄	0.75	55.5	13.5
Typical natural gas	CH _{3.8}	0.76	50	15
Gasoline	CH _{1.9}	0.86	47.3	18.2
Light diesel fuel	CH _{1.8}	0.87	44.8	19.4
Propane	C ₃ H ₈	0.82	50.4	16.3
Typical US crude oil ^c	CH _{1.65}	0.86	42.2	20.4
Typical utility coal	CH _{0.8}	0.58	23.8	24
Methanol	CH ₄ O	0.375	22.7	16.5
Ethanol	C ₂ H ₆ O	0.52	29.7	17.5

^aIn the cases of crude oil and utility coal, corrected for impurities.

^bReference 18. The value given is the higher heating value (see text).

^cReference 19, Table 4.

shown in Table IV. From the empirical formula and atomic weights, the mass fraction of carbon (F_C) is obtained. The mass of carbon (M_C) released when a fuel is burned to produce energy Q is

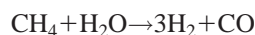
$$M_C = \frac{F_C Q}{H_h}, \quad (28)$$

where H_h is the “higher heating value” of the fuel. The higher heating value is the heat released when one kilogram of fuel is burned in air to produce gaseous CO₂ and liquid water at ambient temperature. (The “lower heating value” is similar, except that the water is assumed to remain in the vapor state and thus the latent heat of condensation has not been released.) Rather than focusing on the emission of carbon in the CO₂, many analyses state the emissions as mass of CO₂. Because the molecular weight of CO₂ is 44 g mole⁻¹, the mass of CO₂ is $M_{CO_2} = (44/12)M_C$.

Using this and information on fuel consumption, students could investigate potential reductions in carbon emissions from switching fuels. Or they could determine how much a tax on carbon emissions would increase the effective price of a fuel.

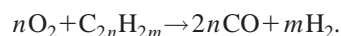
Exploiting fossil fuels without greenhouse gases emissions. One proposal is that fossil fuels be burned in oxygen and the CO₂ captured (instead of being released up the stack) and sequestered (see below). This process would probably be least expensive at large power plants. It may instead be more practical to separate the CO₂ at large chemical facilities, creating hydrogen fuel along with electricity, followed by burial of the CO₂.^{20,21}

Technology for converting carbonaceous fuels to hydrogen. The first step is to produce synthesis gas or “syn gas,” a mixture of hydrogen and carbon monoxide. If one starts with methane, a good method is *steam reforming*:

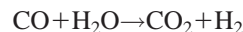


at moderately high temperature and pressure over a catalyst. This process is widely used in the petroleum refining industry to make hydrogen, which is used to upgrade the quality of petroleum products by increasing the H/C ratio and removing sulfur. Hydrogen is also used in the chemical industry as the precursor to production of ammonia and methanol.²²

If one starts with heavy fuel oil or coal, syn gas is made by the *partial oxidation* of the fuel:



This well-established process in the petrochemical industry involves high pressure and temperature; no catalyst is needed, but oxygen is needed, not air. Following either of the syn gas processes, the *water gas shift reaction* is used to convert the CO. At relatively low temperature over a catalyst:



which is followed by separation of the CO₂ and hydrogen.

Thus, well-established industrial processes make hydrogen from hydrocarbon fuels, leading to separate streams of hydrogen and carbon dioxide. However, the processes are fairly costly because of the pressures and heat exchangers involved. There are potential new technologies which could bring those costs down.^{20(a)} Expressing the cost increase as a charge per tonne (metric ton) of avoided C emissions, the cost might be about \$50/tonne, which would roughly double fuel costs. However, because the final use of hydrogen in fuel cells should be more energy-efficient than use of the fuels from which it is made, the overall efficiency is favorable, and the overall cost may not be high.^{20(b)}

Sequestration. If pumping CO₂ into deep ocean waters is feasible and the CO₂ would escape to the atmosphere only slowly with little damage, then the storage capacity is in principle enormous. The costs of moving the CO₂ large distances and compressing it would probably be large. One obvious disadvantage is that much energy use is far from deep ocean.

Another potentially large site for sequestration is underground saline aquifers. They are deeper than fresh water aquifers and are believed to have many voids. It is not known how much CO₂ could be stored securely in this manner, but some speculate hundreds of years worth.^{23,24}

Much less costly but more limited than either of those possibilities is storage in connection with enhanced oil recovery and extraction of coal-bed methane. CO₂ has been extensively pumped down wells in oil fields to enhance oil production from adjoining wells. (So far, however, the CO₂ has been extracted from geologic deposits rather than being a product of fossil fuel combustion.) Such pumping has been a major activity in periods of high oil prices; the cost is paid for by the extra oil.

Methane is attracted to the porous surface of coals and is the source of explosions in many coal mines. It has been observed that CO₂ is more strongly attracted to coal than methane and will displace methane when injected into a coal bed. Coal beds are estimated to offer a major source of natural gas, perhaps doubling natural gas resources.²⁵ So it might be economically and environmentally advantageous to use CO₂ in this way. The potential for global storage of CO₂ in this manner may be a decade or a few decades of fossil fuel combustion, comparable to the sequestration potential in trees.²¹

VI. CLIMATE CERTAINTIES AND UNCERTAINTIES

We now summarize what is known, what is fairly certain, and what is still very uncertain about climate change in the near future.²⁶ Controversy swirls around the problem, but

some aspects are no longer the subject of serious debate in the scientific community, although it is always possible to find an “expert” who disagrees.

Virtually certain facts not dependent on the accuracy of climate models:

- (1) Greenhouse gases are increasing as a result of human activity.
- (2) The increasing concentration of greenhouse gases produces an increase in radiative forcing.
- (3) Anthropogenic greenhouse gases remain in the atmosphere from decades to centuries.
- (4) Over the past century, global surface temperatures have increased by 0.5 °C (±0.2 °C).
- (5) Increases in CO₂ have produced about 1 °C stratospheric cooling. The cooling is caused by increased infrared-emitting CO₂ in the stratosphere, where the emissions are not intercepted by an absorbing atmosphere.
- (6) Natural climate variability makes it difficult to detect long-term trends.
- (7) Key uncertainties about the effects of water in the system, especially clouds, are likely to remain a modeling problem for a decade or more.

Very probable projections. These projections are likely to be true (~90% probability) within stated ranges:

- (1) The increase in global average surface temperature over the past century is consistent with model projections. No viable alternative explanations for the warming have been identified.
- (2) If atmospheric CO₂ is doubled compared with pre-industrial levels (which is likely to occur in the next century), equilibrium global warming is 1.5 °C to 4.5 °C.
- (3) Because the effect of CO₂ saturates (see Fig. 3), the forcing does not increase in proportion to CO₂ concentration.
- (4) By 2100 AD, sea level will rise by 50 cm (±20 cm). The pace is slow because the main mechanism, simple warming of the water, is slow.
- (5) Global mean precipitation will increase by 2% (±0.5%) for each 1 °C of warming, due to increased evaporation and enhancement of the hydrologic cycle.
- (6) There will be substantial changes at high latitudes in the Northern Hemisphere. Temperature increases will be far above the global average, sea ice will be reduced, and precipitation increased.

Probable projections. These projections have a greater than even chance of being correct:

- (1) In northern mid-latitudes, soil moisture will be reduced in summer.
- (2) Around Antarctica the oceans are resistant to warming and little change is expected for a century or more.
- (3) Formation of deep ocean currents at high latitudes of the North Atlantic will be attenuated. Thus the Gulf Stream which carries warm water into the North Atlantic and the associated deep ocean return current may be threatened by increased fresh water in the Arctic, which would reduce salinity and thus the descent of water which initiates the return current.²⁷
- (4) Once formed, tropical storms will become more *intense* on average.
- (5) The range of natural temperature fluctuations will remain about the same.

Unsubstantiated projections. These projections are plausible, but are unsupported at present by climate science:

- (1) The *number* of tropical storms, hurricanes, and typhoons will increase.
- (2) The winds in mid-latitude cyclones (low pressure areas) will become more intense.

There is no evidence for either projection.

VII. CONCLUSIONS

We have outlined some of the basic physical principles involved in the global warming problem. The climate system is extremely complex and physicists have an important role to play in dissecting the feedback mechanisms and in making predictions for the future. We humans possibly face a huge dilemma. Do we pay the cost now to minimize global warming, or do we wait until later and then possibly pay the much higher cost of dealing with a greatly changed climate? A rational decision making process requires more accurate projections of future climate change.

Much work needs to be done to improve the predictive models, and much of it will be done by physicists. State-of-the-art general circulation models are extremely elaborate affairs, but they have recognized deficiencies. Many feedback mechanisms have been identified, but many quantitative uncertainties remain. The uncertainties are especially true for mechanisms related to clouds and aerosols. Much work remains to be done in making critical observations of the climate system and in formulating rigorous physical descriptions which can be incorporated into the models. Therefore, work exists to suit any taste: experimental, theoretical, and computational. The climate change problem is not only extremely important for the future of the human race, but it is a fascinating scientific problem.

¹T. E. Graedel and P. J. Crutzen, *Atmospheric Change. An Earth System Perspective* (W. H. Freeman and Company, New York, 1993).

²S. H. Seinfeld and S. N. Pandis, *Atmospheric Chemistry and Physics. From Air Pollution to Climate Change* (Wiley, New York, 1998).

³R. P. Turco, *Earth Under Siege. From Air Pollution to Global Change* (Oxford U.P., Oxford, 1997).

⁴(a) *Climate Change: The IPCC Scientific Assessment*, edited by J. T. Houghton, G. J. Jenkins, and J. J. Ephraums (Cambridge U.P., Cambridge, 1990); (b) *Climate Change 1994: Radiative Forcing of Climate Change and An Evaluation of the IPCC IS92 Emission Scenarios*, edited by J. T. Houghton, L. G. Meira Filho, J. Bruce, H. Lee, B. A. Callander, E. Haites, N. Harris, and K. Maskell (Cambridge U.P., Cambridge, 1995); (c) *Climate Change 1995: The Science of Climate Change*, edited by J. T. Houghton, L. G. Meira Filho, B. A. Callander, N. Harris, A. Kattenberg, and K. Maskell (Cambridge U.P., Cambridge, 1996).

⁵This article is excerpted from material on climate change in a senior-level course, Energy Engineering, offered at the University of Michigan.

⁶The 40-year history of CO₂ concentration measured at Mauna Loa shows an annual oscillation of about 8 ppmv associated with seasonal storage and release of carbon in the biosphere superimposed on the long-term rise.

⁷*Annual Energy Review 1997* (Energy Information Administration, U.S. Dept. of Energy, 1998).

⁸References for Fig. 1: J. M. Barnola *et al.*, "Vostok ice core provides 160000-year record of atmospheric CO₂," *Nature* **329**, 408–414 (1987); J. Jouzel *et al.*, "Climatic interpretation of the recently extended Vostok ice records," *Clim. Dyn.* **12**, 513–21 (1996), A. Neftel *et al.*, *Historical CO₂ Record from the Siple Station Ice Core* (Physics Institute, University of Bern, CH-3012 Bern, Switzerland, 1994); C. D. Keeling; and T. P. Whorf, *Atmospheric CO₂ concentrations from Mauna Loa, Hawaii* (Scripps Institution of Oceanography, 1996); K. R. Briffa and P. D. Jones, "Global surface air temperature variations over the twentieth century: Part 2, Implications for large-scale high-frequency paleoclimatic studies," *Holocene* **3**, 82–93 (1993).

⁹J. P. Piexoto and A. H. Oort, *Physics of Climate* (American Institute of Physics, New York, 1992), p. 119.

¹⁰J. T. Houghton, *The Physics of Atmospheres*, 2nd ed. (Cambridge U.P., Cambridge, 1986).

¹¹J. T. Kiehl and K. E. Trenberth, "Earth's annual global mean energy budget," *Bull. Am. Meteorol. Soc.* **78**, 197–208 (1997).

¹²D. L. Albritton and R. T. Watson, editors, *Scientific Assessment of Ozone Depletion: 1991 Global Ozone Research and Monitoring Project-WMO Report No. 25* (World Meteorological Organization, 1991).

¹³T. H. Von der Haar and V. E. Suomi, "Measurement of the earth's radiation budget from satellites during a five year period. Part I. Extended time and space means," *J. Atmos. Sci.* **28**, 305–314 (1971).

¹⁴R. Binschadler, "Future of the West Antarctic Ice Sheet," *Science* **282**, 428–428 (1998).

¹⁵J. T. Kiehl, "Atmospheric General Circulation Modeling," in *Climate System Modeling*, edited by K. E. Trenberth (Cambridge U.P., Cambridge, 1992).

¹⁶Office of Technology Assessment, U.S. Congress, *Changing by Degrees*, 1991, Chap. 7.

¹⁷R. C. Johnson, "Unicycles and bifurcations," *Am. J. Phys.* **66**, 589–92 (1998).

¹⁸J. B. Heywood, *Internal Combustion Engine Fundamentals* (McGraw-Hill, New York, 1987), p. 915, Appendix A and Appendix D.

¹⁹W. L. Nelson, *Petroleum Refinery Engineering*, 3rd ed. (McGraw-Hill, New York, 1949).

²⁰(a) R. W. Williams (private communication); (b) R. W. Williams, Fuel decarbonization for fuel cell applications and sequestration of the separated CO₂, in *Eco-restructuring: Implications for Sustainable Development*, edited by R. U. Ayres (UN University Press, Tokyo, 1998), pp. 180–222. "A technological strategy for making fossil fuels environment- and climate-friendly," *World Energy Council Journal*, 59–67, July 1998.

²¹E. A. Parson and D. W. Keith, "Fossil fuels without CO₂ emissions," *Science* **282**, 1053–1054 (1998).

²²L. F. Hatch and S. Mater, *From Hydrocarbons to Petrochemicals* (Gulf Publishing, Houston, 1981).

²³C. Hendricks, Carbon Dioxide Removal from Coal-Fired Power Plants, Ph.D. thesis, Department of Science, Technology and Society, Utrecht University, Utrecht, The Netherlands, 1994.

²⁴S. Holloway (British Geological Survey), ed., *The Underground Storage of Carbon Dioxide*, Report prepared for the Joule II Programme (DG XII) of the Commission of the European Communities, Contract No. JOU2 CT92-0031, Brussels, February 1996.

²⁵D. Rice, B. Law, and J. Clayton, "Coal-Bed Gas-an Undeveloped Resource," in *The Future of Energy Gases*, U.S. Geological Survey Professional Paper, No. 1570 (U.S. Government Printing Office, Washington, DC), pp. 389–404.

²⁶Unless another reference is given, the points raised here were discussed by J. D. Mahlman, "Uncertainties in projections of human-caused climate warming," *Science* **278**, 1416–1417 (1997).

²⁷W. S. Broecker, "Thermohaline circulation, the achilles heel of our climate system: Will man-made CO₂ upset the current balance?," *Science* **278**, 1582–1588 (1997).