

Physical aspects of the greenhouse effect and global warming

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According to the simplest model of the earth's radiative balance, global warming will occur with certainty as humankind increases its production and consumption of nonsolar energy. This prediction is revisited, using a broader model that allows the greenhouse effect to be considered. The new model predicts a global warming of $\Delta T_E = (114 \text{ K})\epsilon$, where ϵ is the rate of surface energy release in units of the average incident solar radiation, 342 W m^{-2} , and ΔT_E is the average temperature rise at the earth's surface. Present values of these quantities, excluding geothermal sources, are $\epsilon = 0.69 \times 10^{-4}$ and $\Delta T_E = 7.9 \text{ mK}$. The model assigns a small number of optical parameters to the atmosphere and surface and qualifies the simple warming prediction: It is rigorous only if parameters other than ϵ are unchanged. The model is not complex and should serve as an aid to an elementary understanding of global warming. © 1999 American Association of Physics Teachers.

I. INTRODUCTION

The simplest model for the earth's radiation balance¹ portrays the planet as a spherical blackbody or "black ball" receiving radiation from the sun, which is also considered as a blackbody with an effective temperature of 5800 K. In the steady state, the model black ball has a temperature of 279 K, which is remarkably close to 288 K, the average temperature at the earth's surface.^{2,3} This temperature is currently measured globally by satellite sensing of the microwave radiation from certain transitions in the oxygen molecule⁴ and by more conventional weather balloon and surface sensors.⁵

Photographs of the earth show that it is hardly black and has considerable reflectivity. Nonetheless, the better-than-order-of-magnitude success of the black-ball model encouraged Rose^{6,7} to use it to point out that as humans produce and consume energy that is not of current solar origin, the temperature of the earth rises. Rose warned that the increase would become significant if this human activity were to increase by a factor of 10 and possibly dangerous if it were to increase by another factor of 10.

It is well established that the greenhouse effect⁸⁻¹¹ is responsible for keeping the temperature near the the earth's surface higher than its effective radiative temperature. The greenhouse effect is due to the atmosphere behaving differently with respect to incoming (mainly visible-ultraviolet) and outgoing (mainly infrared) radiation. The surface and lower troposphere exchange energy with the upper atmosphere, which radiates at a lower temperature.¹⁰ With its single temperature parameter the black-ball model is necessarily silent about the greenhouse effect. We therefore ask whether, and how, Rose's argument should be modified.

Current discussions on long-term changes in the surface temperature of the earth deal with the possible effect of human activities. These discussions rarely include Rose's energy effect itself, because it is quite small. Our model is able to address the broader human effect as well as the Rose effect through its reflectivity, emissivity, and absorptivity parameters. External factors also affect the surface temperature: The solar radiation intensity varies¹² and changes in the solar wind may affect the atmosphere's optical parameters.¹³ A general climatological approach to global warming is well beyond the scope of this paper, but the reader can find points of entry into the literature in texts,^{2,14,15} reviews,^{10,16,17} and in a Resource Letter.¹⁸ The story of the development of a modern interest in global warming has been told by Weart.¹⁹

In Sec. II the model on which the original Rose calculation is based is described and expanded into a two-layer, two-temperature model. In Sec. III surface energy release is introduced and its consequences evaluated. The sensitivity of the average surface temperature to each of the model parameters is calculated. In Sec. IV, we compare the temperature changes caused by human energy production with global temperature variations of similar orders of magnitude and discuss some of the issues raised by the new model. Five brief problems (with answers) are found at the end of the paper.

II. THE ELEMENTARY MODEL AND THE GREENHOUSE EFFECT

A. Introduction: The black-ball model and an extended model

The solar flux density incident on a plane normal to the propagation direction at the earth²⁰ is 1368 W m^{-2} and is called the solar constant. Expressed as an average over the surface and over time, it is reduced by a factor of 4 to an effective solar constant of $S_0 = 342 \text{ W m}^{-2}$. The factor of 4 may be understood as follows: Energy is captured from solar flux in accordance with the cross section of the earth (πR_E^2 , where R_E is the earth's radius) and in steady state is radiated from a total area $4\pi R_E^2$. Therefore the ratio of the outgoing flux density to the full solar constant is 1/4. We will refer to S_0 as the solar constant in keeping with common usage in steady-state modeling. The flows are sketched in Fig. 1(a).

The temperature T_E required of a blackbody to radiate with flux density S_E is obtained from the Stefan-Boltzmann law,

$$S_E = \sigma T_E^4, \quad (1)$$

where $\sigma = 5.6705 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$. We take $S_E = S_0$ and $T_E = (S_E/\sigma)^{1/4} = 279 \text{ K}$. As sketched in Fig. 1(b), when a realistic reflectivity of 35% is introduced, the ball needs to radiate only 222 W m^{-2} . Including this factor not only reduces the predicted surface temperature to $[(0.65 \times 342)/\sigma]^{1/4} = (222/\sigma)^{1/4} = 250 \text{ K}$, but also raises the question of the validity of using the blackbody at all in the model. We will return to this question after a look at Rose's application of the simple model.

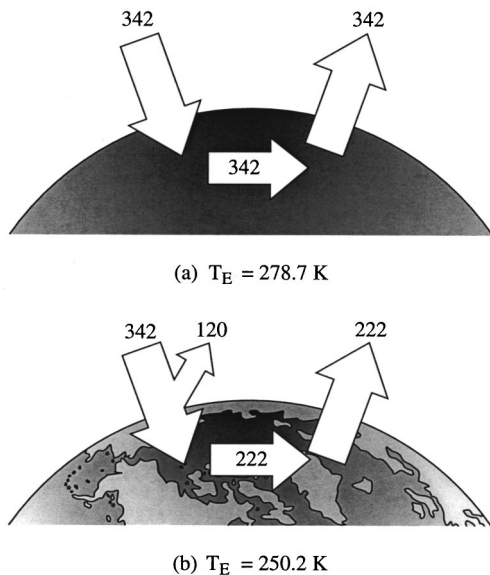


Fig. 1. (a) Energy flow in the simplest model of the earth in radiative steady state. Absorptivities and emissivities are taken to be unity and the incoming solar flux density is 342 W m^{-2} . Radiative dissipation of this flux density necessitates a surface temperature of 279 K . In this and similar figures, flux densities are assumed to be global and are given in terms of their time- and geometrically averaged values in W m^{-2} . Horizontal arrows represent the redistribution of the energy at the surface. (b) The same as (a) except for the assumption of a 35% reflectivity. The surface temperature adjusts to 250 K .

Human activity produces additional power output from fossil fuels and nuclear sources that must be dissipated in the steady state. Because neither convection nor conduction is available to the isolated earth, its rate of radiation and therefore its temperature must increase:

$$S_E + \Delta S = \sigma(T_E + \Delta T_E)^4, \quad (2)$$

where ΔS is the additional time- and surface averaged flux density and ΔT_E is the associated temperature rise. Energy derived from sources associated with contemporary solar energy such as wind, water power, and photovoltaic cells are excluded from ΔS . With $\epsilon' \equiv \Delta S/S_E$, we write

$$\Delta T_E = T_E \left[(1 + \epsilon')^{1/4} - 1 \right] \xrightarrow{\epsilon' \ll 1} (T_E/4)\epsilon'. \quad (3)$$

As we will see below, ϵ' is currently of order 10^{-4} , so the approximate form is adequate in practice.

Using Eq. (3), Rose⁶ discussed the temperature rise that would occur if humans were to increase their energy consumption. He pointed out that increasing ϵ' to 0.1 or 1 would be disastrous ($\Delta T_E = 7$ and 55 K , respectively) and settled on a value of $\epsilon' = 0.01$ as the “absolute upper limit,” for which $\Delta T_E \approx 1 \text{ K}$. He chose this limit because⁶ “experts may disagree on whether such a temperature rise would have major [deleterious] effects . . .”

Rose dismissed the greenhouse effect as a mere wake-up call: “It is a lucky accident that it alerts us to seriously consider a change in world energy supply” (Ref. 7, p. 159). His concerns seem well worth revisiting in the context of a model that accounts for reflectivity and does not automatically preclude a greenhouse effect.

B. A two-temperature, two-layer model

We seek an understanding at the simplest meaningful level and consider two layers whose temperatures are different.

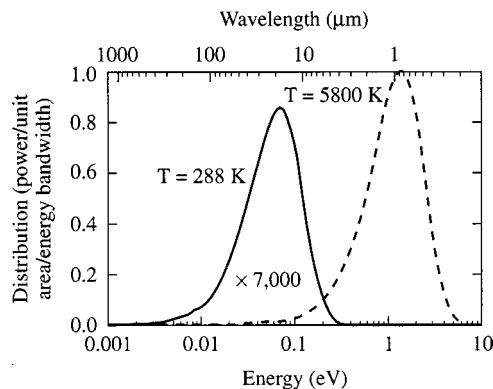


Fig. 2. The blackbody spectrum of the earth and the sun, assumed to be at 288 and 5800 K , respectively, are seen to occupy nearly disjoint regions of the spectrum. On this plot, which expresses the distributions in terms of flux density per energy bandwidth, the sun’s distribution, scaled to unity, peaks at 1.39 eV (889 nm). The 288 K curve has been multiplied by 7000 , but is otherwise scaled correctly. Choosing 0.40 eV ($3.1 \mu\text{m}$) as a boundary, we find that 2% of the 5800 K energy lies in the infrared and 0.008% of the 288-K energy lies in the visible-ultraviolet.

Such a two-layer model, which in its present form dates back to Arrhenius,⁹ makes use of the fact that the radiation involved can be discussed in terms of two distinct spectral regions, the near-infrared-visible-ultraviolet (“UV”) and the infrared (IR). The importance of the distinction is that important atmospheric constituents differ greatly in their optical properties in the two regions. In particular, IR radiation encounters heavy absorption by water vapor and carbon dioxide while UV radiation does not.²¹ There is a fairly natural split between these regions at a wavelength of about $3 \mu\text{m}$ (Fig. 2). Unless precise values of parameters are required, the dividing line between the two regions is inconsequential and can be placed anywhere in the largely transparent red or infrared region of the spectrum.

The two-layer model does not appear in many physics texts despite its simplicity and importance.²² The general principle is brought out in Fig. 3(a), the two-layer generalization of Fig 1(a). We will set up equations that determine the steady-state temperatures of the two layers. The earth’s surface, the lower layer, will be assigned a temperature T_E and an average UV reflectivity r_S . As before, it will emit in the IR with a flux density $S_E = \sigma T_E^4$ because it is assigned 100% absorptivity in that region. The atmosphere, or upper layer, is more detailed. If it were a perfect blackbody radiator at a temperature T_A , its surface emission flux density would be $S_A = \sigma T_A^4$. We assign the layer an IR absorptivity g , which by Kirchhoff’s law dictates an identical value for its emissivity, so that radiation occurs with a flux density gS_A . The upper layer is also given two UV properties that can be varied, namely, an average UV reflectivity r_A and the fraction f of nonreflected incident UV that is absorbed. Completing the list of six variable parameters in the model are S_0 , discussed above, and S_{NR} , a nonradiative flux density estimated to have a rate of 100 W m^{-2} (Ref. 10) that takes energy from the lower layer to the upper layer through convection and latent heat of evaporation. This convective flux density enters as the parameter $h = S_{NR}/S_0$.

The number of parameters in the model is kept small by assuming that all IR incident at the surface is absorbed, that no IR is reflected by either the surface or the atmosphere, and

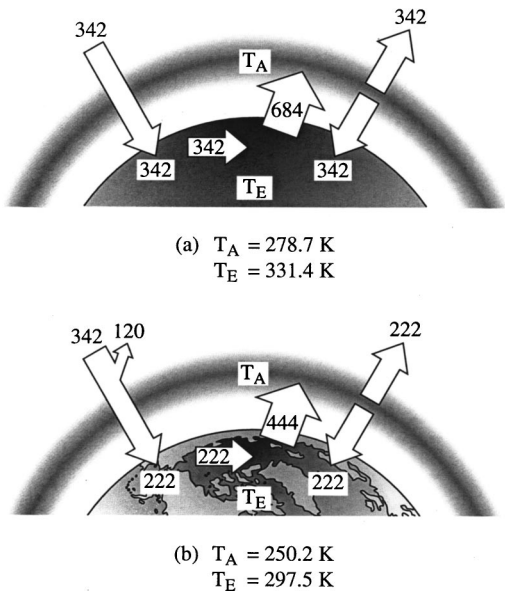


Fig. 3. (a) Energy flow in a two-layer model of the earth in radiative steady state. For this “maximal greenhouse” model, all IR absorptivities and emissivities are taken to be unity and the UV absorptivity and reflectivities are zero. The upper layer radiates from both its inner and outer surfaces, causing the need for a higher dissipation at the lower layer. The new flux density of 684 (see Fig. 1 caption for units) necessitates a surface temperature of 331 K. The upper layer takes on the same temperature as the single layer of Fig. 1(a). (b) The same as (a) except for the assumption of 35% reflectivity of the upper layer. The surface temperature adjusts to 298 K. The upper layer assumes the same temperature as the single layer of Fig. 1(b).

that the UV reflectivity r_A is the same at the top and bottom of the upper layer. Our convention allows the absorption as well as reflection parameters to range simply from 0 to 1. In summary, five material parameters have been introduced: the upper-layer absorptivities f and g , the UV reflectivities r_A and r_S , and one parameter h for the net upward nonradiative flow.

Our strategy is to compute T_A and T_E by finding S_A and S_E , the two unknowns that appear linearly in two balance equations. Setting up the equations is straightforward but is complicated by the fact that surface UV reflectivity implies a second encounter with the surface, a third with the upper level, and so on, all of which generate an infinite multiple-reflection sum. The upper-layer balance equation is

$$[f + f(1 - r_A)k_m r_S(1 - f)](1 - r_A)S_0 + gS_E + S_{NR} = 2gS_A, \quad (4a)$$

where the terms on the left are inputs and the one on the right is the output. The first term on the left is the UV input, which is proportional to the initially received nonreflected flux density $(1 - r_A)S_0$. The factor in square brackets contains the primary UV absorptivity f and a term representing the absorption of the multiply reflected secondary UV (see Appendix A). The multiple-reflection parameter k_m is given by $k_m \equiv 1/(1 - r_A r_S) \geq 1$. The term gS_E is the absorbed fraction of the surface’s IR emission and S_{NR} is the nonradiative flow from the surface. A factor of 2 appears on the right because the upper layer has two radiating surfaces, one facing the lower layer and one facing outward. Finally, g appears on the right in its role as IR emissivity.

The UV flux density first arriving at the surface is $S_1 = (1 - f)(1 - r_A)S_0$. Of this flux, $(1 - r_S)S_1$ is absorbed and $r_S S_1$ is reflected. As shown in Appendix A, the total flux density absorbed by the surface is increased by multiple reflections to $k_m(1 - r_S)S_1$. The resulting lower-layer equation is

$$k_m(1 - r_S)(1 - f)(1 - r_A)S_0 + gS_A = S_E + S_{NR}. \quad (4b)$$

In Eq. (4b) the terms on the left are inputs, namely the UV that is absorbed by the surface in the first and subsequent passes and IR received from the upper layer. On the right is the output, the sum of the IR and net nonradiative flux densities leaving the lower layer.

For simplicity, we have assumed that the total areas of all layer surfaces are equal. Taking the earth’s radius to be 6.4 Mm and a typical layer altitude of ~ 10 km, the error produced by this assumption is approximately $20/6400 = 0.31\%$ in emissions and $(20/6400)/4 = 0.08\%$ in the temperatures.

Equations (4a) and (4b) are readily solved if they are re-arranged into matrix form,

$$\begin{pmatrix} 2g & -g \\ -g & 1 \end{pmatrix} \begin{pmatrix} S_A \\ S_E \end{pmatrix} = \begin{pmatrix} AS_0 + S_{NR} \\ BS_0 - S_{NR} \end{pmatrix} = \begin{pmatrix} A + h \\ B - h \end{pmatrix} S_0, \quad (5)$$

where $A = f(1 - r_A) + fk_m r_S(1 - f)(1 - r_A)^2$ and $B = k_m(1 - r_S)(1 - f)(1 - r_A)$. The result is

$$\begin{pmatrix} S_A \\ S_E \end{pmatrix} = \begin{pmatrix} \sigma T_A^4 \\ \sigma T_E^4 \end{pmatrix} = \begin{pmatrix} 2g & -g \\ -g & 1 \end{pmatrix}^{-1} \begin{pmatrix} A + h \\ B - h \end{pmatrix} S_0 = \frac{S_0}{g(2 - g)} \begin{pmatrix} A + h + g(B - h) \\ g(A + h) + 2g(B - h) \end{pmatrix}. \quad (6)$$

We will discuss some interesting special cases of this solution and set up a standard parameter set for discussion of Rose’s ideas and related topics.

It is first useful to examine the ideal two-layer radiative model,²² Fig. 3(a), defined in terms of our parameters by $f = 0$, $g = 1$, $r_A = r_S = 0$, and $h = 0$. The upper layer transmits all UV and absorbs all IR. Under these conditions Eqs. (4a) and (4b) reduce to

$$S_E = 2S_A, \quad (7a)$$

$$S_0 + S_A = S_E. \quad (7b)$$

The solutions of these equations are $S_A = S_0 = 342 \text{ W m}^{-2}$ and $S_E = 2S_0 = 684 \text{ W m}^{-2}$, yielding $T_E = 331 \text{ K}$ and $T_A = 279 \text{ K}$, as shown in Fig. 3(a). For these “perfect greenhouse” conditions, the surface is very warm indeed because it must dissipate twice as much energy as does the surface in the single-layer model. If we again account for reflectivity by setting $r_A = 0.35$, effectively reducing S_0 in Eqs. (7a) and (7b) to $0.65S_0$, we obtain $S_A = 222 \text{ W m}^{-2}$ and $S_E = 444 \text{ W m}^{-2}$, implying $T_E = 298 \text{ K}$ and $T_A = 250 \text{ K}$, as shown in Fig. 3(b). This calculation puts the surface almost as close to the “true” temperature as it was in the original black-ball model; the near-miss suggests adjusting the model parameters to produce the observed value $T_E = 288 \text{ K}$.

C. The case of zero reflectivity

Before attempting to fine-tune the model, we consider the case in which the surfaces do not reflect UV and in which the

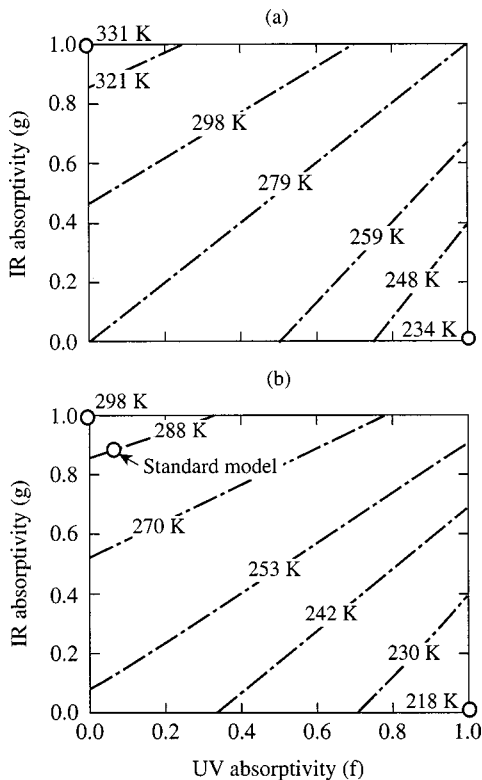


Fig. 4. Temperatures T_E of the lower layer (surface) as determined by the atmospheric absorptivity parameters f and g . In (a), the reflectivities are zero. In (b), the reflectivities are those of the standard model (upper layer 25.5%, lower layer 16%). A very narrow range of values of f and g exists for which the surface may attain a temperature of 288 K.

nonradiative term is absent. The results are instructive and show how an outer layer can act in some cases as a thermal shield instead of an amplifier.

Figure 4(a) shows a selected set of computed T_E isotherms as the parameters f and g range from zero to unity. At the upper left, where $(f, g) = (0, 1)$, the “maximal greenhouse” case is found. As we proceed directly downward on the diagram, the outgoing IR begins to penetrate the atmosphere. Because more energy has escaped, less is reradiated to the surface and the surface temperature drops. The lower left corner $(f, g) = (0, 0)$ represents an “IR-thin” atmosphere that reproduces the black-ball result at the surface. The atmosphere’s temperature T_A , not shown, has also dropped, but not to zero; less IR is being absorbed but the rate of emission is correspondingly smaller by Kirchhoff’s law. Using Eq. (5), we find that the limiting value of T_A for $f=0$ is $279/2^{1/4} = 234$ K. An IR-thin and UV-transparent atmosphere essentially represents removal of the second layer. The point $(f, g) = (0, 0)$ is, in fact, pathological because the limiting value of T_A depends strongly on the value of f (see Problem 3).

For the case $f=g$, we move up the diagonal of Fig. 4(a) and find that the surface temperature is constant. The surface loses UV input and makes up the deficit exactly from the IR reradiation (see Problem 1). At $(f, g) = (1, 1)$ we find what one might call “maximal nuclear winter,” a completely opaque atmosphere that is evoked to various degrees in discussions of the aftermath of a nuclear war. The temperature of the atmosphere is again $T_A = 279$ K, matching the tem-

perature of the surface, which is in thermal equilibrium with the atmosphere (now the surface’s only source of energy). While an average temperature of 279 K does not seem too wintry, it is characteristic of the deepest ice-age climates (~ 9 K below the current average; see Ref. 15, p. 396). Even if we could stand the cold, life as we know it would be impossible inside this extended black ball because “high-quality” UV radiation, the very top of the food chain, is prevented from reaching the surface.

Moving down the right side of Fig. 4(a), at $(f, g) = (1, 0)$ the lowest surface temperatures and the highest atmospheric temperatures to be found anywhere on the diagram are reached. The surface, still seeing only the weakly emitting atmosphere, has dropped to $(279)/2^{1/4} = 234.3$ K. The atmosphere’s temperature formally increases without limit because, despite its weak emissivity, it must dispose of virtually all the absorbed UV radiation. When $g = 10^{-4}$, for example, the temperature required to accomplish this emission is $T_A = 10(279)/2^{1/4} = 2343$ K. In reality, at these higher temperatures visible-UV emission from the atmosphere would become important, halting the model-induced divergence and warming the surface. The model atmosphere in this case mimics the existing upper atmosphere, which is physically very thin and yet absorbs UV quite well, reaching temperatures of about 1000 K in the thermosphere and exosphere (Ref. 15, p. 63).

Figure 4(b) shows the effect of including UV reflectivities. The partial symmetry of Fig. 4(a) is gone, and because less energy is being received by the whole system, the temperature at every point is lower. Variations in the temperature across the diagram are qualitatively the same but are less severe, and they may be understood generally on the basis of the same physical arguments that were used with Fig. 4(a).

D. Choice of parameters for a realistic atmosphere

An implementation of the two-layer model will be regarded as reasonable if it (1) predicts a surface temperature of 288 K, (2) predicts the atmosphere’s temperature of 245 K to within a few degrees,^{23,24} (3) predicts an albedo, defined as the fraction of the total incident UV returned to space by reflection, in the region of 30%–35% (see Ref. 10, p. 198), and (4) involves parameter values that are compatible with known properties of the surface and atmosphere. Except as noted, the following values of the parameters have been selected on the basis of these criteria and have not been subjected to optimization or independent estimates.

When reflectivity is absent, a wide range of absorptivities is available for the target surface temperature of 288 K. To narrow the parameter space, we maintain $f=0$, $g=1$, and $r_A=0$, and introduce a surface reflectivity $r_S=0.150$, which is within the range estimate quoted by Sellers¹⁴ and is equal to the average quoted by Henderson-Sellers and Robinson (Ref. 15, p. 47). This choice produces an albedo of 15% and surface temperature of 319 K, neither of which is acceptable. As we increase the atmospheric reflectivity r_A from its initial value of zero, it is found that by the time the albedo has climbed to 37%, the surface temperature has dropped only to 295 K. The reasonableness criterion not having been met, adjustments in f , g , and r_A must be made. Through a rapidly converging process of trial and error, our most successful set has been found to be $f=0.080$, $g=0.890$, $r_A=0.255$, and $r_S=0.160$. Along with $h=0$ they are adopted as standard

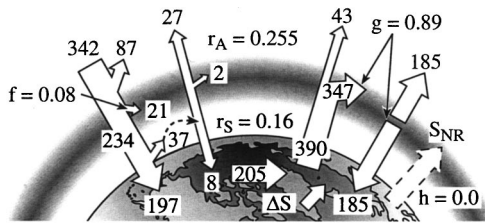


Fig. 5. Energy flow in a modified two-layer model of the the earth in steady state. The parameters f , g , h , r_A , r_S , and ϵ_C are given standard-model values to produce a surface temperature $T_E=288$ K, an atmosphere temperature $T_A=246$ K, and an overall albedo of 33.3% (see the text, Sec. II D). The dashed arrow represents nonradiative heat flow and the small arrow “ ΔS ” represents terrestrial energy inputs such as geothermal and that produced by civilization’s activities. See the caption of Fig. 1 for units.

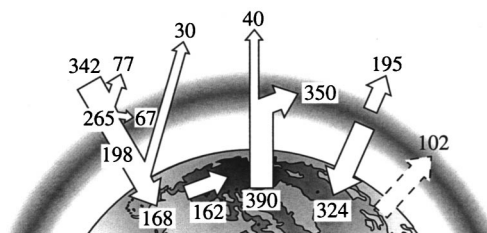


Fig. 6. Characterization of the results of more complex atmospheric energy flow calculations in terms of the two-layer picture. The numbers are adapted from a report of Kiehl and Trenberth (Ref. 10) and are discussed in the text, Sec. II D. See the caption of Fig. 1 for units.

model parameters, or in brief “the standard model.” The predicted albedo is 33%, the atmosphere is at 245 K, and the surface is at 288 K. We claim no uniqueness for this parameter set, but experience shows that any simple alteration pushes the results away from “reasonable.” The flux densities involved with the standard parameter set are shown in Fig. 5 and Table I.

An adaptation of the results of climatological estimates of the components of the earth’s energy budget¹⁰ is given in Fig. 6. The agreement with the corresponding components in Fig. 5 is not bad, considering the simplicity of the two-layer picture and its associated physics. Quantitative balancing of the flux densities can be seen most easily in the individual figures, while Table I draws attention to discrepancies between the simple treatment and the climatological treatment. Most striking is the fact that the climatologically modeled atmosphere radiates more IR to the surface than it does to outer space, a fact that probably can be traced to the heterogeneity of the temperature distribution of the atmosphere and its internal radiative transfer processes that are absent in our model. Another difference between the two treatments is the much larger amount of UV absorbed in the real atmosphere. This second disagreement is qualitatively consistent with the first: The additional downward IR needs a source, part of which is the additional UV absorbed and another part of which is the nonradiative flow S_{NR} . The latter is not in-

cluded in our standard parameter set ($h=0$). Issues associated with nonradiative flow are discussed in Sec. IV C.

III. EFFECTS OF ENERGY SOURCES AT THE SURFACE

A. Formalism

Nonsolar energy sources at the surface originate primarily in the activity of civilization and in geological phenomena. These sources can be treated in terms of fluxes averaged over the surface, called, respectively, ΔS_C and ΔS_G , and characterized in the usual way by parameters $\epsilon_C=\Delta S_C/S_0$ and $\epsilon_G=\Delta S_G/S_0$. They are readily included in the two-layer formalism as an additional source term for the lower layer:

$$\begin{pmatrix} 2g & -g \\ -g & 1 \end{pmatrix} \begin{pmatrix} S_A \\ S_E \end{pmatrix} = \begin{pmatrix} AS_0 + S_{NR} \\ BS_0 - S_{NR} + \Delta S \end{pmatrix} = \begin{pmatrix} A+h \\ B-h+\epsilon \end{pmatrix} S_0, \quad (5')$$

where $\Delta S=\Delta S_C+\Delta S_G$ and $\epsilon=\epsilon_C+\epsilon_G$. All other quantities are defined as in the original Eq. (5). The solutions are as easily modified,

$$\begin{pmatrix} S_A \\ S_E \end{pmatrix} = \begin{pmatrix} \sigma T_A^4 \\ \sigma T_E^4 \end{pmatrix} = \begin{pmatrix} 2g & -g \\ -g & 1 \end{pmatrix}^{-1} \begin{pmatrix} A+h \\ B-h+\epsilon \end{pmatrix} S_0 \\ = \frac{S_0}{g(2-g)} \begin{pmatrix} A+h+g(B-h+\epsilon) \\ g(A+h)+2g(B-h+\epsilon) \end{pmatrix}. \quad (6')$$

Table I. Comparison of standard two-layer model results with those of estimates of the annual global mean energy budget (Kiehl and Trenberth, Ref. 10).

Descriptions	Energy budget (W m ⁻²)	Two-layer model (W m ⁻²)	Two-layer model expression
Solar input	342	342	S_0
UV reflected by atmosphere	77	97	$r_A S_0$
UV reflected by surface (net)	30	22	$(1-f)^2 k_m r_S (1-r_A) S_0$
UV absorbed by atmosphere	67	19	$[f+f k_m r_S (1-f)(1-r_A)](1-r_A) S_0 = A S_0$
UV absorbed by surface	168	204	$k_m (1-r_S)(1-f)(1-r_A) S_0 = B S_0$
IR emitted by surface	390	388	S_E
IR absorbed by atmosphere	350	349	$g S_E$
IR emitted by atmosphere (up)	195	184	$g S_A$
IR emitted by atmosphere (down)	324	184	$g S_A$
IR to space (total)	235	223	$g S_A + (1-g) S_E$
Thermals, evapo-transpiration	102	0 ^a	$h S_0$

^aSee the text. The parameters in the standard model must be modified to handle this nonradiative flux.

Table II. Temperatures, albedo, and fluxes resulting from the standard model, three attempts to include an adequate nonradiative flow within the confines of the two-layer model, and the maximal warming case. Parameters are defined in the text, and the calculations are discussed in Sec. IV. Subscripts on the partial derivatives indicate evaluation with all parameters held constant and equal to the values shown.

Quantity	Units	Standard model	Calculation 1 (seeks T_E, S_{NR})	Calculation 2 (seeks UVabs, S_{NR})	Calculation 3 (seeks compromise)	Maximal warming
S_0	W m^{-2}	342	342	342	342	342
f	1	0.080	0.011	0.224	0.090	0
g	1	0.890	0.990	1.00	0.965	1
h	1	0	0.297	0.298	0.117	0
r_A	1	0.255	0.253	0.225	0.270	0
r_S	1	0.160	0.04	0.234	0.100	0
T_E	K	288	288	269	288	331
T_A	K	246	257	254	252	279
albedo	%	33.3	27.5	31.5	31.5	0
S_{NR}	W m^{-2}	0	102	102	40	0
UV \rightarrow atm	W m^{-2}	23	2.9	68	24	0
UV \rightarrow sfc	W m^{-2}	205	145	166	210	342
$(\partial T_E / \partial \epsilon_C)_0$	K	114	126	154	122	82.8
$(\partial T_E / \partial f)_0$	K	-27.4	-42.6	-30.6	-35.1	-41.4
$(\partial T_E / \partial g)_0$	K	64.9	71.2	67.4	69.6	82.9
$(\partial T_E / \partial r_A)_0$	K	-85.8	-95.2	-100.2	-92.9	-82.9
$(\partial T_E / \partial r_S)_0$	K	-60.6	-70.3	-71.1	-59.6	-82.9
$(\partial T_E / \partial S_0)_0$	mK/W m^{-2}	211	210	197	211	242

The formalism tends to obscure the interesting fact that additional flux density originating at the surface is also subject to the greenhouse effect and is “reflected to the surface” as infrared radiation by the atmosphere. To appreciate this fact we directly modify the simplified two-layer model, Eqs. (7a) and (7b):

$$S_E = 2S_A, \quad (7a')$$

$$S_0 + S_A + \Delta S = S_E. \quad (7b')$$

The solutions of Eqs. (7a') and (7b') are $\sigma T_A^4 = S_A = S_0 + \Delta S$ and $\sigma T_E^4 = S_E = 2S_0 + 2\Delta S$. The first solution illustrates the radiative disposition of the excess flux density outside the atmosphere and the second illustrates the factor of 2 enhancement of the surface flux density contribution to the surface temperature.

In applications of global temperature models, response to small changes of parameters is usually of greatest interest. We represent these changes by partial derivatives with a subscript “0,” which means that all parameters except the varying parameter are held constant, and all parameters are evaluated at their standard model values (or other values as noted). An important example of these derivatives is $(\partial T_E / \partial \epsilon)_0$, whose analytical form is readily obtained from Eq. (6'):

$$\left(\frac{\partial T_E}{\partial \epsilon}\right)_0 = \frac{T_E S_0}{4 S_E} \frac{2g}{g(2-g)} = \frac{T_E}{4} \frac{2}{A + 2B - h + 2\epsilon}. \quad (8)$$

A complete set of analytical expressions for the partial derivatives will not be presented because they are readily computed numerically in typical applications of the theory. Table II contains some important examples of the surface temperature derivatives.

The maximal-greenhouse value is obtained in Eq. (8) by setting $g = 1$ and $f = r_A = r_S = h = \epsilon = 0$, in which case $A = 0$,

$B = 1$, and $(\partial T_E / \partial \epsilon)_0 = T_E / 4$, identical in form to the result for the black-ball model! See Eq. (3) and the discussion following it. The reason is that *both* T_E and ΔS are enhanced by the greenhouse effect, as mentioned above. Rose’s application of the black-ball model is not unreasonable for determining variations in surface temperature despite using a surface flux that is wholly inadequate for computing the surface temperature itself on that model.

The coincidence just described is only an approximate one for general values of the parameters, in which case Eq. (8) must be used. Of greater importance is the nature of the derivative. Rose’s calculations imply a total derivative, there being no other model parameters. Processes causing an increase in ϵ may also cause atmospheric parameters to change. Examples are changes in absorptivities caused by atmospheric pollutants, whether introduced geologically or by power plants. In general,

$$\frac{dT_E}{d\epsilon} = \left(\frac{\partial T_E}{\partial \epsilon}\right)_0 + \left(\frac{\partial T_E}{\partial f}\right)_0 \frac{df}{d\epsilon} + \dots \quad (9)$$

In principle there are sources for which the terms for f , g , h , and r_A in Eq. (9) are negligible. We define *clean* energy production as that which creates or increases ΔS without changes in any model atmospheric parameters. The surface reflectivity r_S is specifically excluded from the definition, partly because it is more easily controlled and estimated than the atmospheric parameters and partly because atmospheric pollution is the principal consequence of power plants that are not clean in the popular sense of the word.

One example of a clean source is the nonvolcanic geothermal flux²⁵ whose magnitude is $\Delta S_G = 82 \text{ mW m}^{-2}$. It is constant, not human-controllable, and will not be considered further beyond noting that according to our model ΔS_G is

Table III. World energy production (based on data in Ref. 28). The lower half of the table shows the production obtained by excluding contributions of solar, geothermal, and wind sources. ΔT_E is based on the standard model.

	Power production TW	Globally-averaged flux density mW m^{-2}	$\epsilon_C \times 10^4$	ΔT_E mK
<i>Total</i>				
1970	7.86	15.4	0.45	5.1
1980	9.57	18.7	0.55	6.2
1990	11.62	22.7	0.66	7.6
2000	13.11	25.6	0.75	8.5
<i>Modified total</i>				
1970	7.47	14.6	0.43	4.9
1980	8.95	17.5	0.51	5.8
1990	10.77	21.1	0.62	7.0
2000	12.05	23.6	0.69	7.9

responsible for 27 mK of the current average earth temperature [use Table II: $(0.082/342) \times 114 = 0.027$].

B. Human surface input

We now consider ΔS_C , the contribution by civilization to ΔS . Rose used 8 TW, the total 1970 world production listed by Häfele.²⁶ We take data for the period 1980–1996 from the U.S. Energy Information Administration as quoted in the 1998 Statistical Abstract of the U.S.²⁷ Shown in the upper half of Table III is the total power production for 1970,²⁸ 1990, and 2000 (extrapolated). The lower half of the table shows similar numbers but with hydroelectric, photovoltaic, wind, and geothermal sources excluded. These sources can be traced to energy already flowing into the surface and therefore are not formally considered *produced* by civilization.

For the year 2000 our standard model predicts that the warming artificially produced by human activity will be about 8 mK. Häfele made a 30-year projection to 2000; had his scenario become reality, we would have reached a production of 70 TW with a concomitant temperature increase of 42 mK. Rose did not make specific projections, but argued more generally that it would be very unwise to countenance large increases in ϵ_C caused by increased use of nonsolar sources. The flavor of his argument is seen in Table IV, which is a slightly revised version of his own similar tables.²⁹ Rose was concerned with the prediction that an eventual factor of 100 increase in ϵ_C would bring a one or two degree rise in the average temperature. There is no headlong rush toward this factor of 100, but any sudden availability of large amounts of inexpensive energy would make these calculations well worth our attention.

Apparently Rose did not apply any correction for solar-related sources and he rounded up the temperature predictions to the nearest order of magnitude, which slightly biased the argument. To reach the “absolute upper limit” of $\Delta T_E = 1$ K, ϵ_C must rise to 0.0088 on the standard model, 210 times its 1970 value. Our year 2000 values of ϵ_C are such that a factor of 130 is still required to attain the 1-K limit.

All of the foregoing numbers must be viewed in the context of the idea of *clean* nonsolar energy. We have no *a priori* idea how any major production increase will affect the atmospheric parameters f , g , h , and r_A . Many forms of

Table IV. An updated Rose calculation of surface temperature increases as a function of ϵ_C , clean surface energy input in units of S_0 . The compromise model is the one called Calculation 3 in Table II and the text. Temperature changes are given in K.

ϵ_C	ΔT_E , standard model (K)	ΔT_E , compromise model (K)	Comment
1	77.0	80.2	“Life not as we know it”
0.1	10.8	11.3	Major flooding
0.01	1.13	1.20	“Absolute upper limit”
0.001	0.114	0.120	
0.0001	0.011	0.012	
0.688×10^{-4}	0.0079	0.0077	World level, 2000

pollution will increase g and therefore T_E , but if r_A were to increase because of pollutants or cloud cover, one of the feedback phenomena that has concerned climatological researchers from Arrhenius to the present, T_E would have reason to decrease.

One or two simple connections can be made with current practice. A well-known set of predictions examined by the International Panel on Climate Change is that the increase in concentration of greenhouse gases is producing a surface temperature increase of 300 ± 100 mK per decade.³⁰ Let us apply the sensitivity parameters of Table II to see how our standard model would make such a prediction. If all of the 300-mK increase could be associated with the infrared parameter g , then a change $\Delta g = 0.30/64.9 = 0.0046$ per decade would be required, just over 0.5% of the current model value of g . If this increase and proportional increases in f and r_A were to occur simultaneously, the predicted net change would be $\Delta T_E = 175$ mK. Adjusting all the increments by a constant factor so that $\Delta T_E = 300$ mK, we have $\Delta g = 0.0079$, $\Delta f = 7.1 \times 10^{-4}$, and $\Delta r_A = 2.3 \times 10^{-3}$, each having increased by 0.85%. The infrared absorptivity has the dominant influence in this linear-behavior scenario and the UV properties are not negligible.

IV. SPECIAL TOPICS

A. Anthropomorphic and other millikelvin-level effects

We have seen that the surface energy release by civilization induces millikelvin-magnitude changes in the global mean temperature. How might such anthropomorphic effects be identified? An analysis of satellite data appears to show that the average earth temperature, based on a 10-year data set, has a 7-day periodic variation with an amplitude of 10 mK and a peak on Wednesdays.³¹ The report shows no error bars and has otherwise been discounted,³² but we mention it because it represents one kind of observation that would clearly signal the product of human activity. The connection between atmospheric CO_2 content and the earth’s average temperature is very complex, and it is difficult to establish a causal link.^{23,33}

There do exist periodic surface temperature effects that rise above the noise in the millikelvin range. At full moon, the earth’s temperature is 20 mK higher than at new moon,³⁴ according to a study that does include an error analysis and for which at least part of the amplitude can be explained.³⁵ That part is attributed to the barycenter effect:

The earth is closer to the sun at full moon than it is at new moon, which causes a variation $(-2\Delta R_{ES}/R_{ES})S_0$ in the solar constant. The temperature variation due to this effect is considered in Problem 4. Variations caused by the IR and UV radiation from the moon are about an order of magnitude smaller.

B. Placement of solar-cell farms

Rose contended that the increased use of solar energy would alleviate concern about ε_C rising to values dangerous to life as we know it. In its operation, solar collection is likely to be a very clean source in any sense of the word. The effect on surface temperature is not altogether negligible, however. If solar collectors are not to affect the radiation balance, they must be positioned in such a way that their absorption and reflectivity match that of the original surface. It follows that highly efficient collectors on a desert, where dry sand produces a reflectivity of 37% (Ref. 15, p. 47), would add to global warming because of the reduced reflectivity of the area. This effect may be made quantitative, and we use it as an example of the application of our model.

Suppose that we wish to use solar cells to avoid the heat output of additional conventional sources in Rose's critical case of ε_C increasing by a factor of 100 (causing a surface temperature rise of 0.72 K on the standard model). The specific goal would be to produce all of the additional power by solar cells,

$$\Delta P = (\varepsilon_C^{\text{after}} - \varepsilon_C^{\text{before}}) S_0 A_E = 99 \times 0.680 \times 10^{-4} S_0 A_E, \quad (10)$$

where $A_E = 4\pi R_E^2$ is the earth's surface area. We make the simplifying assumption that the efficiency of the cells is the same as that of the conventional energy source. That is, if (say) 30% of the energy processed becomes useful work and 70% is due to heat loss during that production, the numbers apply equally to the cells and the conventional source. The same cannot be assumed of the duty cycles of the two producers. Solar cells are at least a factor of 3 at a disadvantage (8 h of useful sunlight is an optimistic estimate) and must produce at least three times the power.

If an area A_{SC} of reflectivity r'_S is covered with solar cells of reflectivity r_{SC} , the change in the average earth reflectivity will be

$$\Delta r_S = (A_{SC}/A_E)(r_{SC} - r'_S). \quad (11)$$

The minimal required area A_{SC} is obtained by making the power absorbed by the cells equal to three times the required power, Eq. (10):

$$S_2 A_{SC} (1 - r_{SC}) = 3 \Delta P = (3) 99 \times 0.680 \times 10^{-4} S_0 A_E, \quad (12)$$

where S_2 is the net incoming UV flux density at the surface. Equation (11) assumes that all nonreflected energy is absorbed, which is consistent with our surface model. With $S_2 = 205 \text{ W m}^{-2}$, we find

$$A_{SC} = 0.0337 A_E / (1 - r_{SC}), \quad (13)$$

about 3.5% of the earth's total area! The change of reflectivity, Eq. (11), would then be

$$\Delta r_S = 0.0337 (r_{SC} - r'_S) / (1 - r_{SC}), \quad (14)$$

which, for $r_{SC} = 0.1$ and $r'_S = 0.37$ (sand) becomes -0.0101 . By the standard model (Table II), $\Delta T_E = +0.61 \text{ K}$. Choosing deserts for placement of the cells would therefore defeat over 80% of the benefit of switching to solar. Again we remind the reader about the clean energy assumption. No account has been taken of the risks that would accrue from fossil fuels and nuclear sources in making this comparison. They are assumed to be as clean as the solar source.

C. Nonradiative transfer considerations

We address two issues relating to the robustness of our radiative transfer model if corrections due to the existence of nonradiative transfer are made. Can the model accommodate this process quantitatively and maintain reasonable radiative flux densities? Can the model even make definitive surface-driven rising-temperature predictions if the nonradiative pathway exists?

The two-layer model is radiative in concept and design. However, an appreciable net upgoing nonradiative flux density S_{NR} of about 100 W m^{-2} is known to exist in the form of thermal currents and evapo-transpiration.¹⁰ This flux density is nearly 1/3 of the solar constant and therefore deserves some attention. We have included it as an independent parameter that essentially short-circuits the upward radiative flow (see Fig. 5).

Three series of model calculations were done to test the influence of nonradiative terms on our model. In the first, the goal was to match the surface temperature, albedo, and nonradiative flux density as closely as possible, regardless of the effect on f , g , r_A , and r_S . In the second, the goal was to model as many of the fluxes of the climatological picture as possible (Fig. 6). In the third, we sought a compromise in which the surface temperature was matched and a reasonable portion of the nonradiative flux density could be reproduced. In none of the three cases was any formal optimization algorithm used, because the model's few parameters have little hope of arriving at a unique and satisfactory fit to so much data.

Table II shows the results of the calculations. In calculation 1, where T_E and S_{NR} are matched, the albedo is rather low because a very small surface reflectivity is required. The UV atmospheric absorption must also be greatly reduced. Both of these contribute to a large amount of UV absorbed at the surface, which appears necessary to feed the nonradiative flows while maintaining the surface temperature (compare calculation 2 in this respect).

In calculation 2, the price paid for matching UV absorption to the climatological results and retaining the nonradiative flow is a very low surface temperature. This appears to be a direct result of the "short circuit" effect on the IR surface emission. The atmosphere already has a lot of energy from direct absorption, and receives more from S_{NR} , so much of the normal upward IR flux density is not needed.

Calculation 3 results in a compromise parameter set in which a fairly good fraction (41%) of the observed S_{NR} is accounted for without sacrificing albedo, surface reflectivity, or surface temperature. Again the UV atmospheric absorption is low, but higher than in the standard model of Table I (shown also in Table II). If a nonzero nonradiative flow is considered absolutely essential to the viability of a particular application of the two-layer model, then calculation 3's parameters can well serve as an alternative "standard model."

The prediction that increased surface production inevitably increases the surface temperature was very convincing in the black-ball model, and it is convincing in our treatment as long as the new energy is clean. However, consider a source such that all of its surface release $\Delta S = \varepsilon S_0$ somehow leaves the surface by the S_{NR} pathway. It might be thought that no increase in S_E and therefore no increase in T_E would be necessary. This hypothetical situation does not materialize, because the dissipation of ΔS by the atmosphere entails additional radiation to the surface as well as outward. This new IR radiation, being indistinguishable from the rest of the downward IR radiation, lacks a mechanism to reaggregate into the upward nonradiative stream. It must return to the upper layer by an increase in S_E , which then requires some increase in T_E . This thermodynamic reasoning can be verified within the model by showing that the total derivative $dT_E/d\varepsilon$ is positive when $h = h_0 + \varepsilon$ (see Problem 5). Here h_0 is the value of h before the new source was introduced.

V. SUMMARY

We have reaffirmed Rose's quantitative estimates of surface-energy-driven temperature increases. Our model shows clearly that these estimates are valid when certain global environmental parameters are unchanged by the energy production, and it provides a relatively simple way of assessing the effect of these parameters. Difficulties of modeling aside, Rose's message is preserved: When significantly larger amounts of energy are to be consumed, it will not be sufficient to consider only the chemical and physical changes of the atmosphere and surface introduced by the energy production. The effect of the *quantity* of energy consumption on the earth's average temperature must be assessed as well.

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APPENDIX A: MULTIPLE REFLECTIONS AND THE FLUX DENSITY EQUATIONS

After reflection at the outer surface and absorption within it, the incoming solar flux density is reduced to $S_1 \equiv (1-f)(1-r_A)S_0$. Upon the first upward reflection at the surface, $r_S S_1$ returns toward the upper layer and $(1-r_S)S_1$ enters the surface layer. The upgoing part encounters the atmosphere, causing $r_A r_S S_1$ to reflect toward the surface and $(1-r_A)r_S S_1$ to reenter the atmosphere. A second upward reflection occurs, then a second downward reflection, and so forth; the following assertions may be appreciated by setting m or n equal to 1 and then 2.

At the m th surface reflection, $r_A^{m-1} r_S^m S_1$ goes upward and $(1-r_S)r_A^{m-1} r_S^{m-1} S_1$ enters the surface. At the n th downward reflection, $r_A^n r_S^n S_1$ goes down and $(1-r_A)r_A^{n-1} r_S^n S_1$ reenters the upper layer. We compute the densities of flux entering the layers by summing each of the above terms over all positive integers m and n :

into surface:

$$\begin{aligned} & \sum_{m=1}^{\infty} (1-r_S)(r_A r_S)^{m-1} S_1 \\ &= \frac{1}{1-r_A r_S} (1-r_S) S_1 = k_m (1-r_S) S_1, \end{aligned} \quad (A1)$$

into atmosphere:

$$\begin{aligned} & \sum_{n=1}^{\infty} (1-r_A) r_S (r_A r_S)^{n-1} S_1 \\ &= \frac{1}{1-r_A r_S} (1-r_A) r_S S_1 = k_m (1-r_A) r_S S_1. \end{aligned} \quad (A2)$$

Here $k_m \equiv (1-r_A r_S)^{-1}$ acts effectively as an enhancement factor for the energy deposited upon first reflection at each surface.

All nonreflected UV reaching the surface is absorbed by it. Therefore Eq. (A1) is ready for Eqs. (5) and (5'). We substitute the value of S_1 and obtain for the lower layer

$$\begin{aligned} \text{UV input to surface} &= k_m (1-r_S) S_1 \\ &= k_m (1-r_S) (1-f) (1-r_A) S_0 \\ &\equiv B S_0. \end{aligned} \quad (A3)$$

For the upper layer, this substitution does not finish the job because we have calculated only the *reentering* surface-reflected flux. Of this flux, a fraction f is absorbed. Finally, the first-pass absorbed amount $f(1-r_A)S_0$ must be added. The result is

$$\begin{aligned} \text{UV input to atmosphere} &= [f(1-r_A) + f k_m r_S (1-f) (1-r_A)^2] S_0 \equiv A S_0. \end{aligned} \quad (A4)$$

It is suggested that a flow diagram be constructed in connection with this standard multiple-reflection calculation.

APPENDIX B: DERIVATIONS AND DISCUSSIONS AS EXERCISES (WITH SOLUTIONS)

1. (a) Consider the two-layer greenhouse model with no UV reflection and no nonradiative flux density for the case $f = 1/2$ and $g = 1/2$. Complete a flow diagram similar to Figs. 2, 4, or 6, preferably without solving any equations. Take $S_0 = 342 \text{ W m}^{-2}$. Read part (b) of the question for a hint. (b) Using the kinetic equations, Eqs. (4a) and (4b), prove that the surface temperature is equal to that of the black-ball model for all values of f when $g = f$.

Answer: (a) Because $f = 1/2$, the incoming flux density of 342 is equally divided between absorption in the atmosphere and at the surface. Similarly, because $g = 1/2$, the flux leaving the surface will be divided equally between absorption in the atmosphere and a return to space. At this point one sees that the atmosphere must radiate a total of $171 + S_E/2$ equally toward space and back toward Earth. The diagram if carefully sketched will suggest that $S_E = 342$ is the solution; alternatively, the solution may be obtained through the hint or through a simple balance equation involving the one unknown, S_E .

(b) For the case described, Eqs. (4a) and (4b) become

$$\begin{aligned} fS_0 + fS_E &= 2fS_A, \\ (1-f)S_0 + fS_A &= S_E, \end{aligned} \quad (\text{B1})$$

whose solution is $S_A = S_E = S_0$; alternatively, a substitution of the parameters in Eq. (6) can be made. In either case, one is forced to deal with the meanings of all the symbols. Completion of part (b) involves associating $S_0 = 342 \text{ W m}^{-2}$ with the black-ball temperature 279 K, preferably commenting that the surface emissivity in the IR is assumed unity in the model.

2. (a) Complete the multiple-reflection diagram described in Appendix A and verify all the expressions in the text for the m th upward and n th downward reflections. (b) Find an expression for the albedo, defined as the fraction of the total incident UV returned to space by reflection, in terms of the model parameters. Check your result by computing the albedo for any of the models in Table II that involve nonzero reflectivities.

Answer: (b) By our definition

$$\text{albedo} = \frac{\text{all UV leaving the system}}{S_0}, \quad (\text{B2})$$

and there are two contributions to the numerator. The first is the amount $r_A S_0$ initially reflected from the atmosphere and the second is based on the amount entering the atmosphere from all surface reflections, Eq. (A2). Only a fraction $1-f$ of the latter amount escapes and contributes to the albedo. Therefore

$$\text{albedo} = r_A + (1-f)^2(1-r_A)^2 k_m r_S. \quad (\text{B3})$$

The second term on the right in (B3) may be thought of as ‘‘an enhanced surface reflectivity $k_m r_S$ that is modified because the radiation it represents must twice avoid reflection and absorption by the atmosphere.’’ On the standard model, the two terms contribute as follows: $0.255 + 0.078 = 0.333$.

3. (a) Return to exercise 1(b) and use the kinetic equations to determine S_A and T_A for the case $f=g$. Your result for $f=g=0$ will differ from that quoted in Sec. II C. (b) Set $g=0.001$ and calculate T_A for $f=0, 0.001, \text{ and } 0.002$. Why are there such large changes in T_A ?

Answer: (a) When $r_A = r_S = h = \varepsilon_C = 0$, the input factors A and B are f and $1-f$, respectively. Therefore, from Eq. (6),

$$S_A = \frac{S_0}{g(2-g)} [f + g(1-f)]. \quad (\text{B4})$$

When f is set equal to g , one immediately has $S_A = S_0(T_A = 279 \text{ K})$, solving this part of the question. However, the function (B4) is discontinuous at $f=g=0$, accounting for the different result when, as in the text, the origin is approached along the line $f=0$. (b) The results of the numerical calculations are shown in the accompanying table:

f	g	T_A (K)	T_E (K)
0	0	N/A ^a	278.7
0	0.001	234.4	278.7
0.001	0.001	278.7	278.7
0.002	0.001	308.4	278.6

^aMultiple-valued, depending on direction of approach.

When g is small but nonzero, the large variations in T_A as f varies over a small range are easily seen in the mathematics. From a physical viewpoint, the case $f=g=\varepsilon$, with ε however small, corresponds to a situation very similar to that of $f=g=1/2$: Although most of the radiation is passing through the atmosphere in both directions with no effect, the amounts absorbed from the UV and the from the outgoing IR are identical. Despite the small amount of energy to be radiated, the temperature of the atmosphere must remain at the relatively high black-ball level because of its low emissivity. When f falls to zero while g remains at ε , the atmosphere loses half of its input and its temperature drops to a level that will maintain its own balance. When f increases to 0.002, the atmosphere must now deal with about twice as much UV. Since its emissivity is still small, its temperature must rise to maintain the steady state. In the present example, where $g=\varepsilon$ and $f=2\varepsilon$, it is easily shown that the enhancement over the value for the case $f=g=\varepsilon$ is a factor of $(3/2)^{1/4} = 1.107$.

4. (a) Calculate the difference between Earth’s surface temperature in full-moon and new-moon conditions, taking into account only the variation of the solar constant due to the motion of the Earth-moon system (the barycenter effect). (b) Find the temperature of the moon’s illuminated side, assuming that no heat flows to its dark side (a principal difference from Earth’s case) and that it acts as a ‘‘black ball’’ with 7% reflectivity. (c) Compare the amount of IR radiation reaching Earth from the full moon with the amplitude of the variations in solar radiation associated with the barycenter effect.

Answer: (a) Here one must first find the change ΔR_{ES} in the Earth–Sun distance between full-moon and new-moon conditions. Ignoring variations in the distance of the barycenter (Earth-moon center of gravity) to the sun, the result is twice the distance from Earth to the barycenter, which can be computed as $2R_{EM}(m_M/m_E) \approx 9320 \text{ km}$ (the masses are those of the moon and Earth, respectively). Because of the inverse-square dependence of solar intensity at Earth, we have $\Delta S_0 = -2S_0 \Delta R_{ES}/R_{ES} \approx (1.25 \times 10^{-4})(342 \text{ W m}^{-2}) = 42.6 \text{ mW m}^{-2}$. From Table II, the resulting variation in T_E is 8.9 mK.

(b) If the moon is a black ball and may radiate from only half of its surface, a straightforward modification of black-ball theory shows that its temperature is $2^{1/4}$ times 278.7 K, or 331.4 K. Since only 93% of the radiation is absorbed by the moon, a flux reduction occurs. A further small correction (a factor of 0.995) results from the moon’s greater distance from the sun under full-moon conditions. The final result is $T_M = 325.2 \text{ K}$.

(c) If Earth sees the full moon as a 325-K blackbody, then the ‘‘lunar constant’’ may be calculated by simple analogy to the model calculation of the solar constant (Ref. 1, pp. 111, 115):

$$S_{0M} = S_0 \left(\frac{T_M}{T_S} \right)^4 \left(\frac{R_M}{R_{EM}} \right)^2 \left(\frac{R_S}{R_{ES}} \right)^{-2}. \quad (\text{B5})$$

The various quantities have obvious meanings in terms of temperatures, radii, and distances. Their values are $T_S = 5800 \text{ K}$, $R_{EM} = 0.384 \text{ Gm}$, $R_{ES} = 149.6 \text{ Gm}$, $R_M = 1.74 \text{ Mm}$, and $R_S = 0.696 \text{ Gm}$. The result is $S_{0M} = 3.19 \text{ mW m}^{-2}$, producing (again according to Table II) a contribution to T_E of 0.67 mK, an order of magnitude less

than the contribution of the barycenter effect. Incidentally, the 7% of the moon's intercepted light that remains as UV will contribute at a level smaller by yet another order of magnitude.

5. By computing $dT_E/d\varepsilon$ in the case $h=h_0+\varepsilon$, show that global warming resulting from an increase in "clean" production of nonsolar energy cannot be defeated by channeling that energy into the nonradiative channel (see the discussion at the end of Sec. IV B).

Answer. Only two parameters are varying. Therefore

$$\frac{dT_E}{d\varepsilon} = \left(\frac{\partial T_E}{\partial \varepsilon}\right)_{h=h_0+\varepsilon} + \left(\frac{\partial T_E}{\partial h}\right)_{h=h_0+\varepsilon} \frac{dh}{d\varepsilon}. \quad (\text{B6})$$

The first term is obtained from Eq. (8),

$$\left(\frac{\partial T_E}{\partial \varepsilon}\right)_{h=h_0+\varepsilon} = \frac{T_E}{4} \frac{2}{A+2B-h_0+\varepsilon}. \quad (\text{B7})$$

From Eq. (6'),

$$\begin{aligned} \frac{\partial T_E}{\partial h} &= \frac{T_E}{4S_E} \frac{\partial S_E}{\partial h} = -\frac{T_E}{4S_E} \frac{S_0}{(2-g)} \\ &= -\frac{T_E}{4} \frac{1}{A+2B-h+2\varepsilon}. \end{aligned} \quad (\text{B8})$$

Since $dh/d\varepsilon=1$,

$$\left(\frac{\partial T_E}{\partial h}\right)_{h=h_0+\varepsilon} \frac{dh}{d\varepsilon} = \frac{T_E}{4} \frac{1}{A+2B-h_0+\varepsilon} \quad (\text{B9})$$

and finally

$$\frac{dT_E}{d\varepsilon} = \frac{T_E}{4} \frac{1}{A+2B-h_0+\varepsilon}. \quad (\text{B10})$$

Recall that $(A+B)S_0$ is the total UV flux density absorbed by the two layers. The quantity $A+2B-h_0$ is therefore highly unlikely to be negative in any physically reasonable application of the model. (B10) is therefore a positive quantity, *QED*.

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¹See, for example, C. Kittel and H. Kroemer, *Thermal Physics* (Freeman, San Francisco, 1980), 2nd ed., Exercise 4.5, p. 111.

²D. L. Hartmann, *Global Physical Climatology* (Academic, San Diego, 1994), p. 2.

³We adopt as our observed "surface temperature" the time- and geographically averaged lower-atmosphere value measured by satellite. This temperature is consistent with the use of elementary models that deal with a single global surface temperature, interpreted as a similarly averaged value.

⁴R. W. Spencer and J. R. Christy, "Precise monitoring of global temperature trends from satellites," *Science* **247**, 1558–1562 (1990); J. R. Christy, R. W. Spencer, and W. D. Braswell, "How accurate are satellite thermometers?" *Nature (London)* **389**, 342 (1997).

⁵P. D. Jones and K. R. Briffa, "Global surface air temperature variations during the 20th century: I. Spatial, temporal, and seasonal details," *The Holocene* **2**, 165–179 (1992).

⁶A. Rose, "A global view of solar energy in rational units," *Phys. Status Solidi A* **56**, 11–26 (1979); "Solar energy: A global view," *CHEMTECH* **11**, 566–571 (1981); **11**, 694–697 (1981). These papers contain almost identical material.

⁷A. Rose, "World energy supply and greenhouse effect," *Mol. Cryst. Liq. Cryst.* **175**, 159–168 (1989).

⁸J. B. J. Fourier, *Mém. de l'Ac. R. d. Sci. de l'Inst. de France*, t. vii, (1827).

⁹S. Arrhenius, "On the influence of carbonic acid in the air upon the tem-

perature of the ground," *Philos. Mag.* **41**, 237–276 (1896).

¹⁰J. T. Kiehl and K. E. Trenberth, "Earth's annual global mean energy budget," *Bull. Am. Meteorol. Soc.* **78**, 197–208 (1997).

¹¹The greenhouse effect was apparently first recognized by Fourier (Ref. 8), whose terminology has persisted despite the fact that the atmosphere does not behave in the manner of hothouse glass. The role of "greenhouse gases" were first treated in detail by Arrhenius (Ref. 9) who summarizes the relevant 19th-century literature. The greenhouse effect is an implicit factor in every modern model involving radiative and other means of energy transport in the atmosphere. Kiehl and Trenberth (Ref. 10) provide a brief, readable, yet comprehensive review.

¹²H. van Loon and K. Labitzke, "The 10–12-year atmospheric oscillation," *Meteorol. Z.* **3**, 259–266 (1994); J. Lean and D. Rind, "Climate forcing by changing solar radiation," *J. Climate* **11**, 3069–3094 (1998).

¹³B. A. Tinsley and K. A. Beard, "Links between variations in solar activity, atmospheric conductivity, and clouds: An informal workshop," *Bull. Am. Meteorol. Soc.* **78**, 685–687 (1997).

¹⁴W. D. Sellers, *Physical Climatology* (University of Chicago Press, Chicago, 1965); M. I. Budyko, *The Earth's Climate: Past and Future* (Academic, New York, 1982).

¹⁵A. Henderson-Sellers and P. J. Robinson, *Contemporary Climatology* (Addison-Wesley-Longman, Edinburgh Gate, UK, 1986).

¹⁶R. E. Dickinson, "Impact of human activities on climate—a framework," in *Sustainable Development of the Biosphere*, edited by W. C. Clark and R. E. Munn (University Press, Cambridge, 1986), Chap. 9, pp. 252–289.

¹⁷S. H. Schneider, "Introduction to climate modeling," in *Climate System Modeling*, edited by K. E. Trenberth (Cambridge U.P., Cambridge, UK, 1992), pp. 3–26.

¹⁸J. W. Firor, "Resource letter: GW-1: Global warming," *Am. J. Phys.* **62**, 490–495 (1994).

¹⁹S. R. Weart, "The discovery of the risk of global warming," *Phys. Today* **50** (1), 34–40 (1997).

²⁰R. C. Willson, "Total solar irradiance trend during solar cycles 21 and 22," *Science* **277**, 1963–1965 (1997).

²¹Texts such as Refs. 2, 14, and 15 may be consulted for the details of the atmospheric absorption spectra. In our model, effective overall absorptivities are assigned to the atmosphere as single parameters.

²²The simplest two-layer model is discussed clearly by C. Kittel and H. Kroemer, Ref. 1, 4th printing, pp. 115–116.

²³R. E. Dickinson, "Modeling climate changes due to carbon dioxide increases," in *Carbon Dioxide Review 1982*, edited by W. C. Clark (Clarendon, Oxford, and Oxford U. P., New York, 1982), pp. 103–142.

²⁴Because the temperature and density of the upper atmosphere vary greatly with altitude, an effective radiation temperature or "skin" temperature, based on the overall IR emission, is usually assigned. The value 255 K is typically quoted (Refs. 17 and 23). In our elementary model, IR originates both at the surface and in the atmosphere, and the latter therefore has a somewhat lower temperature (246 K). The number to be compared with the typical 255 K is the temperature of an ideal body emitting the total IR flux, in our standard model 228 W m^{-2} , and therefore our $T_A^{\text{skin}}=251 \text{ K}$.

²⁵C. M. R. Fowler, *The Solid Earth: An Introduction to Global Geophysics* (Cambridge U.P., Cambridge, UK, 1990), p. 234. Average geothermal flux densities are continents and shelves, 57 mW m^{-2} ; oceans and basins, 99 mW m^{-2} ; surface average, 82 mW m^{-2} .

²⁶W. Häfele, "A systems approach to energy," *Am. Sci.* **62**, 438–447 (1974).

²⁷*U.S. Bureau of the Census, Statistical Abstract of the United States 1998* (USGPO, Washington, DC, 1998), 118th ed. World primary energy production appears in Table 962, p. 595.

²⁸Data for 1970 are extrapolated from the more detailed 1980–1996 data in order to effectively incorporate recent data revisions and methods of defining energy categories. Our total 1970 production agrees with Häfele's (Ref. 26) for 1970.

²⁹Rose adopted the "sun" as his unit of flux density, informally choosing one sun to be 200 W m^{-2} , the approximate flux density of incoming UV at the surface. Because this quantity varies with model parameters, we prefer to retain our own sun units with $\varepsilon_C=\Delta S_C/S_0$, slightly smaller than two-thirds of Rose's parameter, which he did not name.

³⁰J. T. Houghton, L. G. Meira Filho, J. Bruce, H. Lee, B. A. Callander, E. Haites, N. Harris, and K. Maskell, editors, *Climate Change 1994. Radiative Forcing of Climate Change and an Evaluation of the IPCC IS92 Emission Scenarios* (Cambridge U.P., New York, 1995).

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³²D. E. Lenschow, "Next step for weekday warming," *Nature* (London) **369**, 193 (1994).

³³D. H. Douglass, *Bull. Am. Phys. Soc.* **44**, 1422 (1999).

³⁴R. C. Balling, Jr. and R. S. Cerveny, "Influence of lunar phase on

daily global temperatures," *Science* **267**, 1481–1483 (1995); **269**, 1285 (1995).

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MEDICAL AUTHORITY

When the Japanese pitcher Hideki Irabu joined the Yankees last season, he...quickly became known for...taping tiny magnets on his arms before games.... This season, Darryl Strawberry, Paul O'Neill, Derek Jeter and Scott Brosius are following Mr. Irabu's lead and wearing U-shaped magnetic bracelets, which are touted by their Japanese manufacturer, Tsujimoto, to increase circulation, ease pain, help balance and promote energy.

"From what I've heard about it, it's going to make me healthy and I'm going to live to be 150," Mr. O'Neill said.

But Dr. Carlos Vallbona, a medical authority on magnet therapy, said that any benefit the Yankees receive from the bracelets is mind over magnets. "For the magnetic field to work, magnets have to be applied directly to the area of distress," he said, adding, however, that the jewelry did work nicely with the team's 1996 World Series rings.

Joe Brescia, *New York Times*, August 23, 1998.