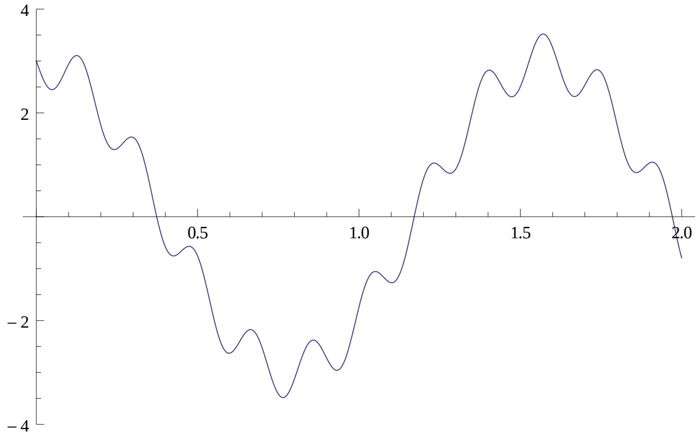
Paquetes de Onda

Ondas

In[1]:=

```
A[x_] := x^2 / 125 - Sin[35 * x] / 2 + 3 * Cos[4 * x]
Plot[ A[x], {x, 0, 2},
 PlotRange → {-4, 4}
]
```

Out[2]=

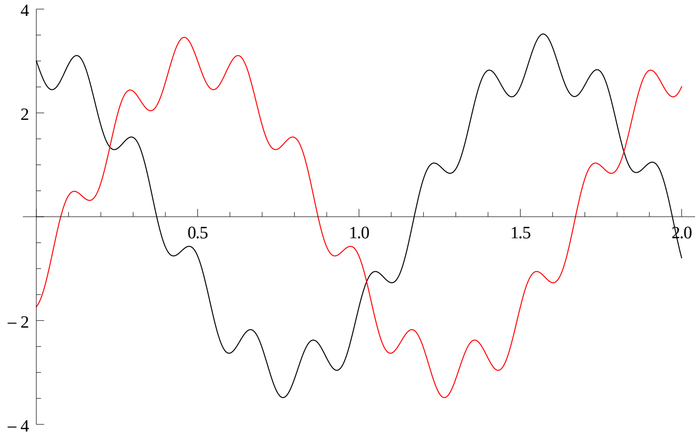


In[3]:=

```
v = 0.5
t = 1;
Plot[ {A[x], A[x - v * t]}, {x, 0, 2},
 PlotRange → {-4, 4},
 PlotStyle → {RGBColor[0, 0, 0], RGBColor[1, 0, 0] }
]
```

Out[3]= 0.5

Out[5]=



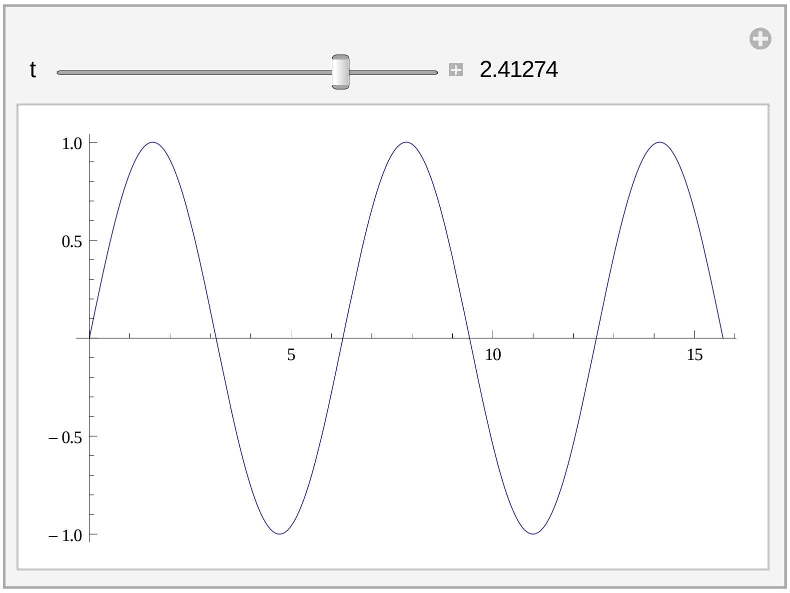
Dibujo de Ondas Armonicas

In[6]:=

```
v = 2;
T = 2 * π / v;
Onda[x_, t_] := Sin[x - v * t];
```

In[9]:=
```
Manipulate[Plot[{Sin[x], Onda[x, t]}, {x, 0, 5 π}],
 {{t, 0, "t"}, 0, T, Appearance → "Labeled"}]
```

Out[9]=



Ejercicios:

1. **Hallar las velocidades de avance de la onda (grupo o fase?)**
2. **Que pasa si en lugar de Sin[x] se pone Cos[x]?**

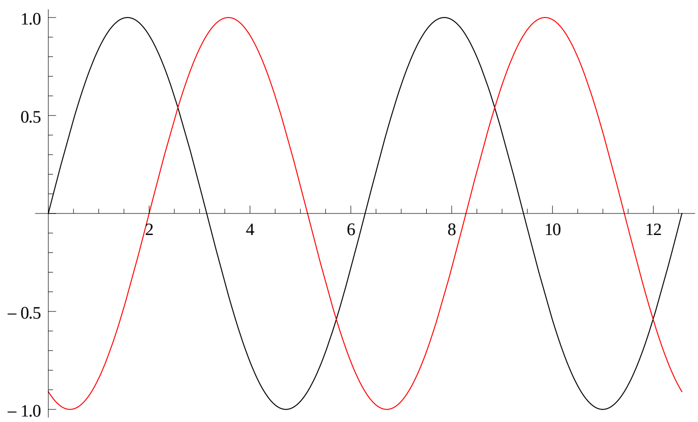
Construccion de paquetes de ondas

In[10]:=
```
Clear[Onda];
Onda[k_, ω_, x_, t_] := Sin[k * x - ω * t];
```

In[12]:=
```
Δk = k / 10 ;
Δω = ω / 10;
```

In[14]:=
```
k = 1;
ω = 2;
Plot[ {Onda[k, ω, x, 0], Onda[k, ω, x, 1] }, {x, 0, 4 π},
 PlotStyle → {RGBColor[0, 0, 0], RGBColor[1, 0, 0] }
]
```

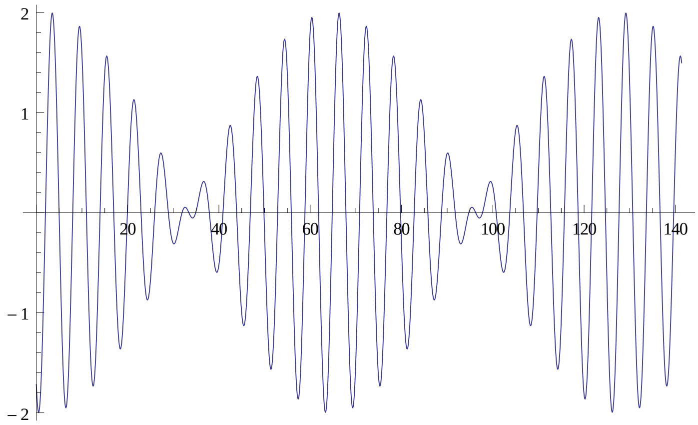
Out[16]=



In[17]:=
```
Δk = k / 10 ;
Δω = ω / 10;
```

In[19]:=
```
Plot[ Onda[k, ω, x, 1] + Onda[k + Δk, ω + Δω, x, 1] , {x, 0, 45 π} ]
```

Out[19]=

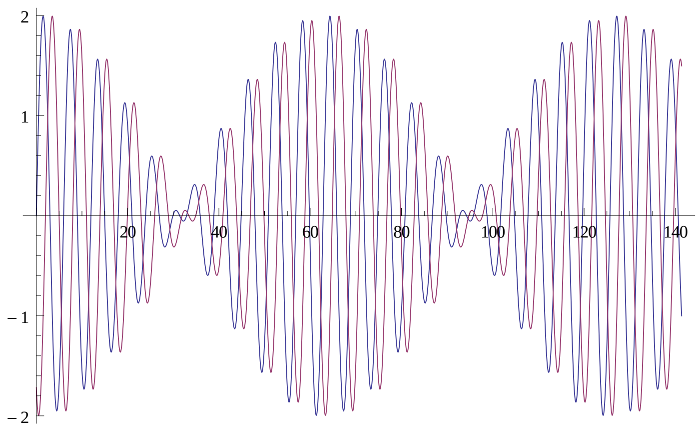


Ejercicios

1. **Hallar las velocidades de avance de la onda (grupo o fase?)**
2. **Construir un paquete de ondas sumando muchas ondas**
3. **Avanza el paquete? A que velocidad?**

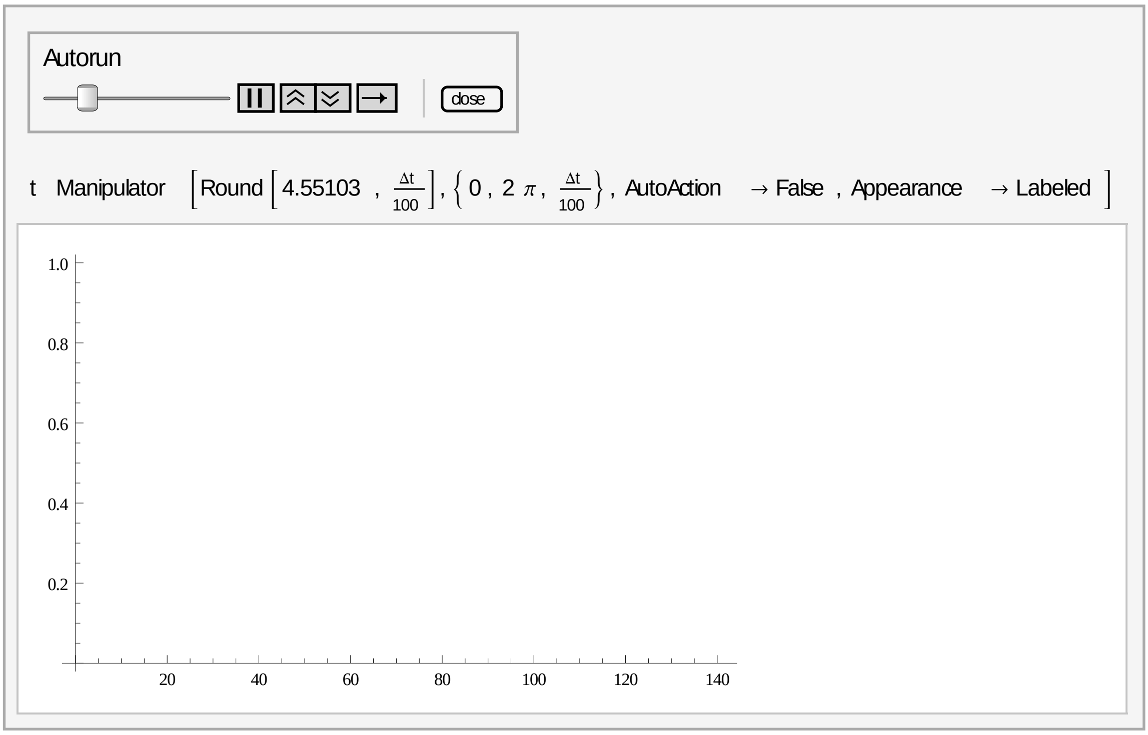
In[20]:=
```
Ondasum2[x_, t_] := Onda[k, ω, x, t] + Onda[k + Δk, ω + Δω, x, t]
Plot[ {Ondasum2[x, 0], Ondasum2[x, 1] }, {x, 0, 45 π} ]
```

Out[21]=



In[22]:=
```
T = 4 * π * Δk / Δω;
Manipulate[Plot[ {Ondasum2[x, 0], Ondasum2[x, t]} ,
  {x, 0, 45 π} ], {{t, 0, "t"}, 0, T, Δt / 100, Appearance → "Labeled"}]
```

Out[23]=



Paquete de Ondas

In[25]:=
```
CearAll[k, ω, x, t, wavepacket];
```

In[26]:=
```
nondas = 10;
k = 1;
ω = 2;
Δk = k / nondas ;
Δω = ω / nondas;
```

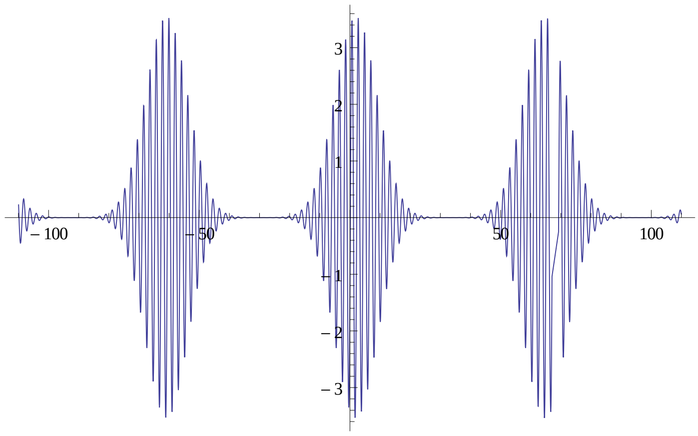
In[31]:=
```
δk = 0.2;
```

In[32]:=
```
Onda[k_, ω_, x_, t_] := Exp[I (k * x - ω * t)];
```

In[33]:=
```
wavepacket[k_, ω_, x_, t_] :=
 Sum[Exp[- (i Δk / δk) ^ 2] Onda[k + i Δk, ω + i Δω, x, t], {i, -1 nondas, 1 nondas, 1}]
```

In[34]:=
```
Plot[Re[wavepacket[3, 2, x, 1]], {x, -35 π, 35 π}, PlotRange → All]
```

Out[34]=



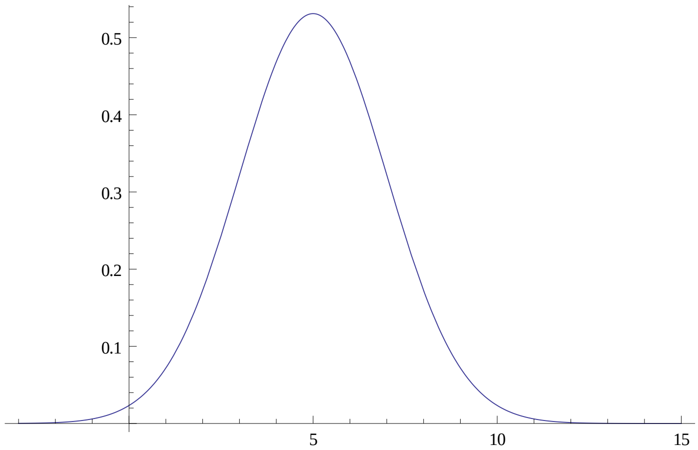
Paquete de Ondas Gaussiano

In[39]:=
$$G[x\_] := \frac{1}{\sqrt{\sigma \sqrt{\pi}}} e^{-\frac{(x-a)^2}{2 \sigma^2}}$$

```
σ = 2;
a = 5;

Plot[ G[x] , {x, -3, 15}]
```

Out[42]=



Ejercicios

1. Comprobar que G^2 esta normalizado
3. Que significan σ y a

Transformacion Fourier

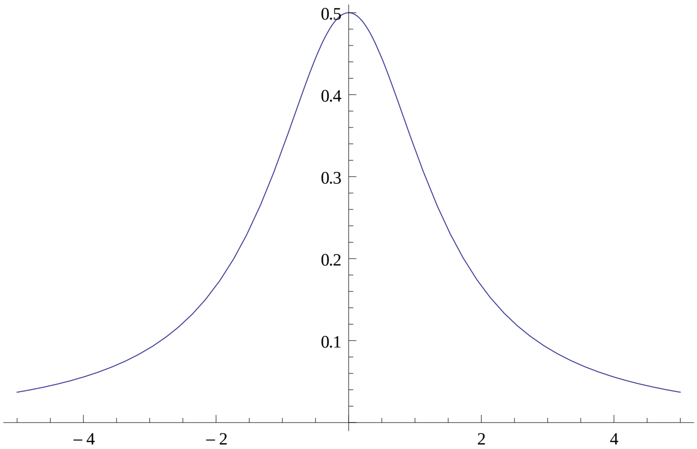
In[43]:=
```
Clear[x, k, Yx, Fk];
Yx[x_] = 1/(x^2 + 2);
```

In[45]:=
```
Yk[k_] = 1/(Sqrt[2 π]) Integrate[ Yx[x] * Exp[I k x], {x, -Infinity , Infinity}]
```

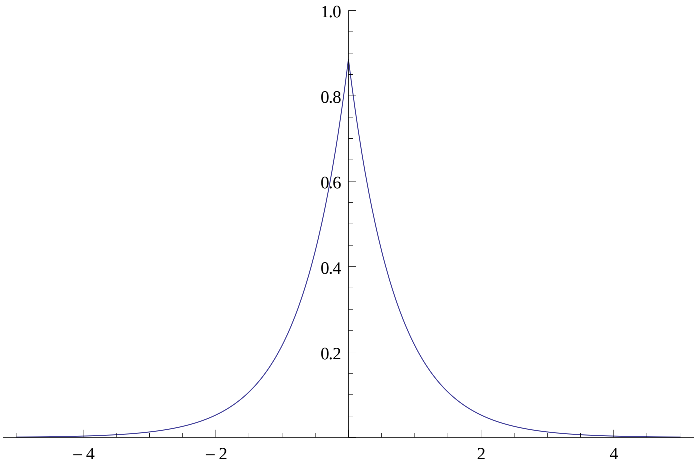
Out[45]= $\text{ConditionalExpression}\left[\frac{1}{2} e^{-\sqrt{2}\ \text{Abs}[k]} \sqrt{\pi}, k \in \text{Reals}\right]$

In[46]:=
```
Plot[ Yx[x] , {x, -5, 5}]
Plot[ Yk[k] , {k, -5, 5}, PlotRange → {0, 1}]
```

Out[46]=



Out[47]=

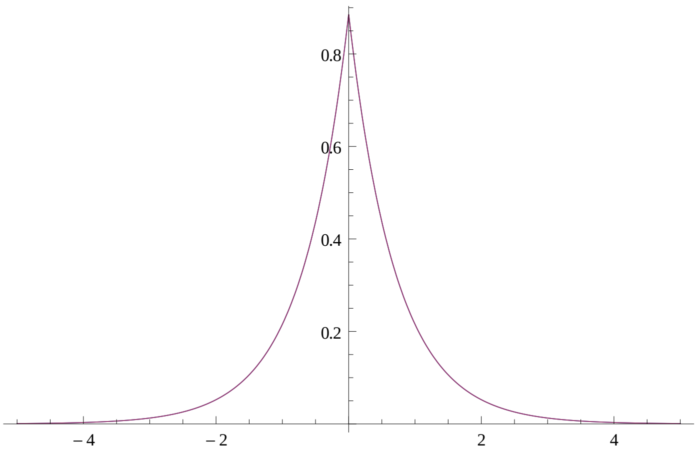


In[48]:=
```
Yfk[k_] = FourierTransform[Yx[x], x, k]
```

Out[48]= $\frac{1}{2} e^{-\sqrt{2}\,\text{Abs}[k]} \sqrt{\pi}$

In[49]:=
```
Plot[ {Yk[k], Yfk[k] }, {k, -5, 5}]
```

Out[49]=



In[50]:=
```
Clear[x, k, σ, a, Gk];
```

$$G[x\_] := \frac{1}{\sqrt{\sigma \sqrt{\pi}}} e^{-\frac{(x-a)^2}{2 \sigma^2}}$$

In[52]:=
```
Gk[k_] = FourierTransform[G[x], x, k]
```

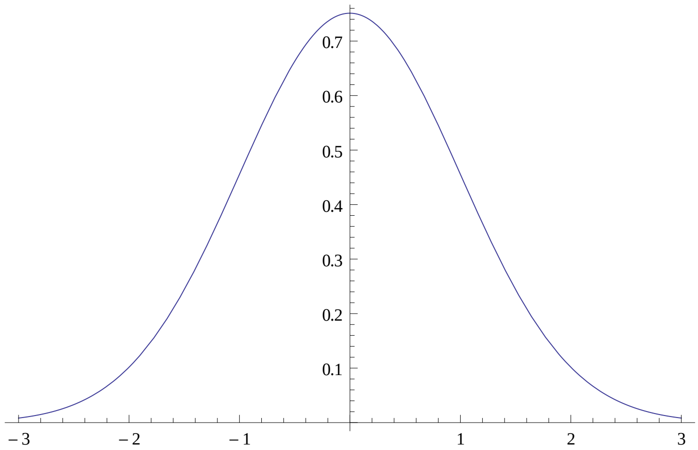
Out[52]=

$$\frac{e^{i a k-\frac{k^2 \sigma^2}{2}}}{\pi^{1/4} \sqrt{\frac{1}{\sigma^2}} \sqrt{\sigma}}$$

In[53]:=
```
Clear[x, k, σ, a, Gk];
Gk[k_] := FourierTransform[G[x], x, k]
σ = 1 ;
a = 0 ;
```

In[57]:=
```
Plot[ G[x] , {x, -3, 3}]
```

Out[57]=

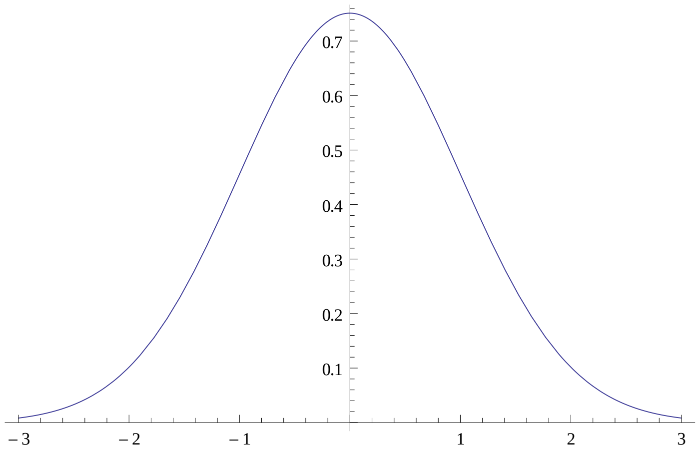


In[58]:=
```
Gk[1.5]
```

Out[58]= 0.243855 + 0. i

In[59]:=
```
Plot[ Re[Gk[k]] , {k, -3, 3}]
```

Out[59]=



Ejercicios

1. Comprobar que Gk^2 esta normalizado
3. Como varian los anchos y posiciones de la transformada?

Valores Medios

In[60]:=
```
σ = 1 ;
a = 2;
```

In[62]:= ClearAll[σ, a]

In[63]:= amed :=
 Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * x * G[x], {x, -Infinity, Infinity}]]

In[64]:= a2med := Assuming[{σ > 0, Element[σ, Reals]},
  Integrate[G[x] * x ^ 2 * G[x], {x, -Infinity, Infinity}]]

In[65]:= amed

Out[65]= a

In[66]:= a2med

Out[66]= $a^2 + \frac{\sigma^2}{2}$

In[67]:= σx := a2med - (amed) ^ 2

In[68]:= σx

Out[68]= $\frac{\sigma^2}{2}$

In[69]:= P[x_] := -I h * G'[x]

In[70]:= P[2]

Out[70]= $\frac{i\,(2-a)\,e^{-\frac{(2-a)^2}{2\sigma^2}}\,h}{\pi^{1/4}\,\sigma^{5/2}}$

In[71]:= pmed := Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * P[x], {x, -Infinity, Infinity}]]

In[72]:= pmed

Out[72]= 0

In[73]:= P2[x_] := -I h * P'[x]

In[74]:= p2med :=
 Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * P2[x], {x, -Infinity, Infinity}]]

In[75]:= p2med

Out[75]= $\frac{h^2}{2\,\sigma^2}$

In[76]:= σp := p2med - (pmed) ^ 2

In[77]:= σp

Out[77]= $\frac{h^2}{2\,\sigma^2}$

In[78]:= σx σp

Out[78]= $\frac{h^2}{4}$