Soluciones de Algunos Ejercicios: Guía 1

Ejercicio 1:

$$
\begin{aligned}
dx &= \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz \\
dz &= \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \\
dz &= \left(\frac{\partial z}{\partial x}\right)_y \left[\left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz\right] + \left(\frac{\partial z}{\partial y}\right)_x dy \\
dz &= \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz + \left[\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x\right] dy
\end{aligned}
$$

Ejercicio 1a

$$dz = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz + \left[\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x\right] dy$$

Esta ecuación es general, y no depende del camino: se debe cumplir para **cualquier camino**. Elegimos dos caminos particulares: 1) $dy = 0$

$$\begin{aligned} dz &= \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz \\ \Rightarrow & \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y = 1 \end{aligned}$$

$$\left(\frac{\partial z}{\partial x}\right)_y = \frac{1}{\left(\frac{\partial x}{\partial z}\right)_y}$$

Ejercicio 1b

$$dz = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz + \left[\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x\right] dy$$

Camino 2) $dz = 0$

$$0 = \left[\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x\right] dy$$

$$\Rightarrow \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

Ejercicio 1c

$$
\begin{aligned}
dx &= \left(\frac{\partial x}{\partial y}\right)_w dy + \left(\frac{\partial x}{\partial w}\right)_y dw \\
dy &= \left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw \\
dx &= \left(\frac{\partial x}{\partial y}\right)_w \left[\left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw\right] + \left(\frac{\partial x}{\partial w}\right)_y dw \\
dx &= \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w dz + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw
\end{aligned}
$$

Ejercicio 1d

$$dx = \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w dz + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

Camino $dz = 0$

$$dx_z = \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw_z$$

Dividimos por dy_z:

$$\left(\frac{\partial x}{\partial y}\right)_z = \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] \left(\frac{\partial w}{\partial y}\right)_z$$

$$= \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z \left(\frac{\partial w}{\partial y}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial y}\right)_z$$

Ejercicio 1d

$$\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z \left(\frac{\partial w}{\partial y}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial y}\right)_z$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial x}{\partial y}\right)_w + \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial y}\right)_z$$

Ejercicio 3

$V(l)$	p (atm)
5.1	5
7.2	4
9.9	3
15.0	2
30.0	1

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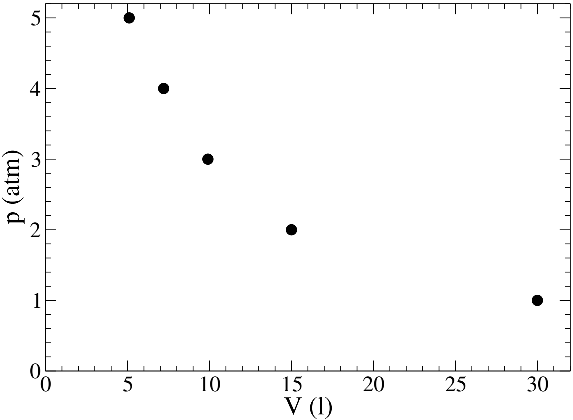


Ejercicio 9

Cambios de variable:

- $u \rightarrow E$
- $p \rightarrow F$
- $v \rightarrow x$

$$\begin{aligned} du &= \left(\frac{\partial u}{\partial v}\right)_T dv + \left(\frac{\partial u}{\partial T}\right)_v dT \\ dE &= \left(\frac{\partial E}{\partial x}\right)_T dx + \left(\frac{\partial E}{\partial T}\right)_x dT \end{aligned}$$

$$\begin{aligned} E &= \frac{1}{2} k\, x^2 + c\, T \\ dE &= k\, x\, dx + c\, dT \end{aligned}$$

Ejercicio 9a

$$\begin{aligned} dE &= \left(\frac{\partial E}{\partial x}\right)_T dx + \left(\frac{\partial E}{\partial T}\right)_x dT \\ dE &= k\,x\,dx + c\,dT \\ &\Rightarrow \left(\frac{\partial E}{\partial T}\right)_x = c \end{aligned}$$

$$\begin{aligned} c_v &= \left(\frac{\partial u}{\partial T}\right)_v \\ c_x &= \left(\frac{\partial E}{\partial T}\right)_x = c \end{aligned}$$

Ejercicio 9b

$$\begin{aligned} c_p &= c_v + \left[p + \left(\frac{\partial u}{\partial v}\right)_T\right]\left(\frac{\partial v}{\partial T}\right)_p \\ c_F &= c_x + \left[F + \left(\frac{\partial E}{\partial x}\right)_T\right]\left(\frac{\partial x}{\partial T}\right)_F \end{aligned}$$

$$\begin{aligned} dE &= \left(\frac{\partial E}{\partial x}\right)_T dx + \left(\frac{\partial E}{\partial T}\right)_x dT \\ dE &= k\,x\,dx + c\,dT \\ &\Rightarrow \left(\frac{\partial E}{\partial x}\right)_T = k\,x \end{aligned}$$

Ejercicio 9b

$$
\begin{aligned}
c_F &= c_x + \left[F + \left(\frac{\partial E}{\partial x}\right)_T \right] \left(\frac{\partial x}{\partial T}\right)_F \\
&= c + \left[-k\,x + b\,\mu\,T + k\,x \right] \left(\frac{\partial x}{\partial T}\right)_F \\
&= c + b\,\mu\,T\,\left(\frac{\partial x}{\partial T}\right)_F
\end{aligned}
$$

Ejercicio 9b

$$
\begin{aligned}
F &= -k\,x + b\,\mu\,T \\
dF &= -k\,dx + b\,\mu\,dT\,.
\end{aligned}
$$

Para F=cte, $dF = 0$:

$$
\begin{aligned}
k\,dx_F &= b\,\mu\,dT_F \\
\left(\frac{\partial x}{\partial T}\right)_F &= \frac{b\,\mu}{k}
\end{aligned}
$$

Recapitulando:

$$
\begin{aligned}
c_F &= c + b\,\mu\,T\left(\frac{\partial x}{\partial T}\right)_F = \\
&= c + \frac{b^2\,\mu^2\,T}{k}
\end{aligned}
$$

Chequear unidades!!

Ejercicio 9c (adiabáticas)

$$dq = du + p\,dv$$

En $1d$:

$$\begin{aligned} dq &= dE + F\,dx \\ &= \left(\frac{\partial E}{\partial T}\right)_x dT + \left(\frac{\partial E}{\partial x}\right)_T dx + F\,dx = \\ &= c\,dT + [k\,x + F]\,dx = \\ &= c\,dT + b\,\mu\,T\,dx \end{aligned}$$

Ejercicio 9c (adiabáticas)

$$dq \quad = \quad c\,dT + b\,\mu\,T\,dx$$

Adiabática $dq = 0$:

$$\frac{dT}{T} = -\frac{b\,\mu}{c}\,dx$$

Integramos:

$$\ln T = -\frac{b\,\mu}{c}\,x + K$$

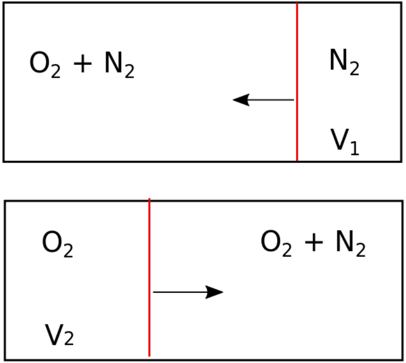
por lo cual, la ecuación de las adiabáticas es

$$T\,e^{\frac{b\,\mu}{c}\,x} = \text{const}$$

Ejercicio 13

$$
\begin{aligned}
W_a &= -n_2\, R\, T \ln \frac{V_f}{V_i} = \\
&= -n_2\, R\, T\, \ln \frac{V - V_1}{V} \\
W_b &= -n_1\, R\, T \ln \frac{V_f}{V_i} = \\
&= -n_2\, R\, T\, \ln \frac{V - V_2}{V}
\end{aligned}
$$

(a) $W = W_a + W_b = W_b + W_a$.

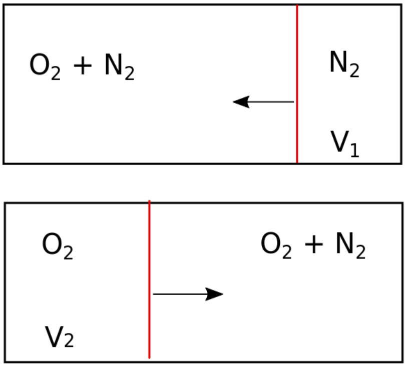


Ejercicio 13b

Completamente separado:

$V_1 + V_2 = V$

$$
\begin{aligned}
W &= W_a + W_b = \\
&= n_2\, R\, T \ln \frac{V}{V - V_1} + \\
&\quad + n_1\, R\, T \ln \frac{V}{V - V_2} = \\
&= n_2\, R\, T \ln \frac{V}{V - V_1} + \\
&\quad + n_1\, R\, T \ln \frac{V}{V_1}
\end{aligned}
$$

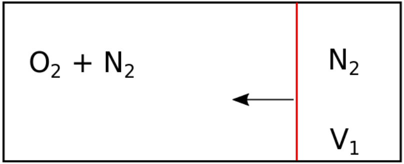


Ejercicio 13b

Completamente separado:

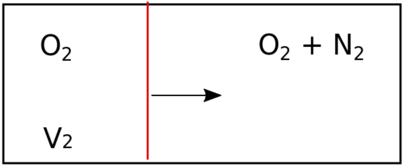
$V_1 + V_2 = V$

$$W = n_2\,R\,T\ln\frac{V}{V-V_1} + \\ + n_1\,R\,T\ln\frac{V}{V_1}$$



Buscamos el extremo:

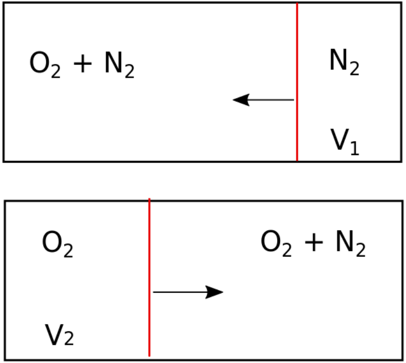
$$\frac{dW}{dN_1} = n_2\,R\,T\,\frac{1}{V-V_1} + \\ - n_1\,R\,T\,\frac{1}{V_1}$$



Ejercicio 13b

Extremo:

$$
\begin{aligned}
\frac{dW}{dN_1} &= n_2\,R\,T\,\frac{1}{V-V_1} + \\
&\quad - n_1\,R\,T\,\frac{1}{V_1} = \\
&= n_2\,R\,T\,\frac{1}{V_2} + \\
&\quad - n_1\,R\,T\,\frac{1}{V_1} \\
\frac{n_2}{V_2} &= \frac{n_1}{V_1} \\
\frac{V_1}{V_2} &= \frac{n_1}{n_2} \;\Rightarrow \frac{p_1}{p_2} = 1
\end{aligned}
$$



Ejercicio 13c

$$W = n_2\, R\, T \ln \frac{V}{V_2} + n_1\, R\, T \ln \frac{V}{V_1}$$

Datos:

- $m_2 = 1$ kg $\Rightarrow n_2 = 31.25$ mol
- $n1 = 4\, n_2 =$ 125 mol
- $V_2 = \frac{n_2\, R\, T}{p} = 0.769\, \mathrm{m}^3$
- $V_1 = 4\ V_2 = 3.08\, \mathrm{m}^3$

Reemplazando: $W = 195$ kJ. (Chequear!!)