The nature of particle–wave complementarity

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(Received 15 April 1996; accepted 1 July 1997)

To resolve the challenges (paradoxes) attending the genesis of quantum theory, Niels Bohr in 1927 proposed a generalization of the conceptual framework of classical physics, termed complementarity, which posited pairs of classical concepts that could be manifested only in mutually exclusive experimental arrangements with both needed to embrace the full range of physical experience. In this context particle–wave complementarity has encountered two major obstacles; (1) No formulation of it has commanded a general consensus; and (2) empirical evidence abounds that contradicts the claim of mutual exclusivity of particle and wave properties of quantum objects in a single experiment. These problems are rooted in the unrestricted scope of this claim. It is shown that a restricted version of particle–wave complementarity, here called ''*which value–interference*'' complementarity, is consistent with observations, is naturally contained in the mathematical formalism of quantum theory and, in fact, holds in any representation of quantum theory defined by the eigenvectors of a complete set of commuting observables, such as the configuration space position (kinematic) variables or the complementary linear momentum (dynamic) variables. Thus, the *which value–interference* form of particle–wave complementarity exists within kinematic–dynamic complementarity, a ''layered'' complementarity, so to speak. It is hoped that a consistent and valid exposition of particle–wave complementarity will be helpful to those who find the framework of complementarity useful and appropriate for some purpose.

I. INTRODUCTION

The increasingly persuasive evidence for wave–particle duality for matter and radiation and the challenges attending a deterministic space–time description of atomic processes prompted Bohr[1] in 1927 to propose the framework of complementarity and Heisenberg to expound upon the uncertainty (indeterminacy) relations. Heisenberg[5–10] has provided several accounts of his involvement with Bohr in these developments and has rather extensively discussed Bohr's work. Bohr's account[11] of these collaborations with Heisenberg is brief. Several authors, including Jammer,[12] Petersen,[13] Rosenfeld,[14] MacKinnon,[16] Mehra,[17] Miller,[18] and Pais[19] have explored these developments in historical context.

Folse[20] has emphasized that Bohr did not promote complementarity as a principle of physics but as a point of view or conceptual framework within which the quantum world could be rationally analyzed and understood. Thus it has not been easy to find a clear, succinct statement or definition of complementarity in Bohr's writings. What probably comes closest to such a definitive statement appears in the Introduction to Bohr's first book of essays on the quantum description of nature, where he writes:[21]

> ... The fundamental postulate of the indivisibility of the quantum of action ... forces us to adopt a new mode of description designated as *complementary* in the sense that any given application of classical concepts precludes the simultaneous

use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomenon.

In his celebrated treatise on quantum theory Pauli echoes with somewhat greater specificity this Bohrian statement about complementarity:[22]

> If ... the use of a classical concept excludes that of *another*, we call both concepts (e.g., position and momentum co-ordinates of a particle) *complementary* (to each other), following Bohr. We might call modern quantum theory as ''The Theory of Complementarity'' (in analogy with the terminology ''Theory of Relativity'').

Note that Pauli's version omits any reference to the need for both complementary concepts for a complete ''elucidation of the phenomenon.'' This example of canonically conjugate noncommuting variables as complementary concepts was frequently used by Bohr, though cast in somewhat more general language such as the complementarity of space–time description and claims of causality[23] or complementarity of space–time coordination and momentum–energy conservation.[24] In this paper I shall refer to this example of complementarity as kinematic–dynamic complementarity, following Murdoch.[25] This example of complementarity has a close kinship to the quantitatively expressed Heisenberg uncertainty (indeterminacy) relations.

A second example of complementarity in physics emphasized by Bohr[26] is the complementarity of the particle picture and wave picture of phenomena. The purpose of this paper is to explore the nature of this wave–particle complementarity, especially its relation to kinematic–dynamic complementarity. In Sec. II, the way in which Bohr and some of his followers viewed these issues will be reviewed and in Sec. III these issues will be clarified by appeal to the formal structure of quantum theory. Some implications of the analysis will be drawn in Sec. IV, where limits on the scope of wave–particle complementarity will be explained.

II. REVIEW OF THE WAVE–PARTICLE, KINEMATIC–DYNAMIC RELATIONSHIP

On the issue of whether (1) the wave picture is associated with the kinematic space–time description and the particle picture with the dynamic momentum–energy description or (2) *vice versa*, i.e., a wave–dynamic, particle–kinematic correlation, or (3) some other relationship, it is natural to examine the views of Bohr and his followers, who were the prime movers of the complementarity doctrine. Unfortunately, no consensus will emerge from this examination.

Bohr himself was not very expansive on the issue at hand. However, he did write:[27]

> As regards light, its propagation in space and time is adequately expressed by the electromagnetic theory. Especially, the interference phenomena *in vacuo* and the optical properties of material media are completely governed by the wave theory superposition principle. Nevertheless, the conservation of energy and momentum during the interaction between radiation and matter as evident in the photo-electric and Compton effect, finds its adequate expression just in the light quantum idea put forward by Einstein.

This statement suggests that Bohr is advocating position (1) above, i.e., the wave–kinematic (space–time) and particle–dynamic (momentum–energy) correlations. In any event, L. Rosenfeld, one of Bohr's most eloquent spokesmen and closest collaborators, co-author with him of the Biblical articles on measurability and complementarity of electromagnetic field quantities, adhered to the correlation stated in (1). He wrote:[28]

> ... Bohr wanted ... to understand the logical nature of the mutual exclusion of the two agents opposed in the wave–particle dualism. From this point of view the indeterminacy relations appear in a new light. The conjugate variables for which they hold refer, respectively, to the mutually exclusive classical picture centred (sic) on the idealized concepts of particle and wave; to the former are attached the momentum and energy variables, to the latter the coordinates of space and time.

In this passage it is not completely clear whether Rosenfeld was stating his own or Bohr's views on the point. He may well be stating the position of both himself and Bohr since they very probably agreed on the matter. Scheibe[29] concurs with the correlation stated by Rosenfeld. Given the context of his exposition, he apparently means to express Bohr's view, which, as with Rosenfeld, probably agrees with his own. In a major study of quantum paradoxes, Selleri[30] also adopts the view that the wave picture and particle picture correlate with the kinematic and dynamic variables, respectively.

However, option (2) above, namely, the association of the kinematic space–time description with the particle picture (particle trajectories in space–time) and the momentum–energy variables with the wave picture (via the de Broglie–Einstein relations $p=h/\lambda$ and $E=h\nu$, where λ and ν are wavelength and frequency, respectively, of the wave) also has distinguished advocates. A. Pais, who reported[31] that he discussed complementarity with Bohr for ''countless hours,'' quoted a statement by Bohr:[32]

> The very nature of the quantum theory ... forces us to regard the space–time coordination [meaning: particle behavior] and the claim of causality [meaning: wave behavior], the union of which characterizes the classical theories, as complementary but exclusive features of the description ... .

The inserts in brackets were made by Pais as an expression presumably of Bohr's position as well as perhaps his own of particle–kinematic and wave–dynamic correlation. This correlation, option (2), is also espoused by Bohm,[33] Scully *et al.*,[34] and Englert.[35] This claim of Bohr's position is obviously at variance with the claims of the previous paragraph.

Then there are those who claim that neither option (1) nor option (2) stated at the beginning of this section reflects Bohr's position. Born reports:[36]

> ... I have used the expression ''complementarity'' to describe the two aspects of rays of light or matter-particles and waves. But this is not what Bohr had in mind, as I have learned from numerous discussions with him. He proposes to use this expression for two physical situations produced by experimental arrangements, which

> have the same material object but are intended for the determination of different ''conjugate'' properties restricted by the uncertainty relation. Such arrangements are mutually exclusive and complementary in this sense, that they define together all observable features of the object. The conceptions ''particles'' and ''waves'' have no such complementary character, as in many cases *both are needed* (WGH emphasis) for a proper prediction of observations (the waves giving the probabilities of finding particles). One can speak here about the ''dual'' aspect of matter.

As the last sentence indicates, Born does not mean to dispense with wave–particle duality but to argue that Bohr's idea of complementarity is not applicable to these phenomena. Grünbaum[37] concurs that:

> the terms ''wave-like'' and ''particle-like'' should *not* be used as surrogates for *bona fide* complementary parameter values [the conjugate variables of the Heisenberg principle].

Murdoch agrees. After an extended discussion of complementarity, he concludes.[38]

> I do not believe ... that there is an invariable, systematic association of one or other of the two models with either position or momentum measurement operations: wave–particle complementarity and kinematic–dynamic complementarity are logically independent notions.

On further analysis, Murdoch comes to doubt that the thesis of wave–particle complementarity can be sustained.[39]

Folse,[20] as the title suggests, orients his entire book around the concept of complementarity. He deals extensively with the issue of the correlation between wave–particle and kinematic–dynamic complementarity.[40] He does not propose to jettison wave–particle complementarity completely but does warn that[41]

> ... we must be careful not to attempt any one-to-one correlation of one type of picture [wave–particle] with one or the other descriptive modes [kinematic–dynamic].

Faye[42] assigns to Bohr the position taken by Rosenfeld[28] on the correlation of the complementarities under discussion and then argues against Bohr's position on the grounds that[43] [in concurrence with Pais[31]]

> ... sometimes the wave picture can be ... correlated with the measurement of momentum and energy, while spatio-temporal measurement is connected with the particle picture

and concludes[44] with Murdoch

> ... that wave–particle complementarity and kinematic–dynamic complementarity are logically distinct notions.

Writing together, Faye and Folse further assert:[45]

> ... the connection between wave–particle dualism and the complementarity of kinematic and dynamic properties remains a problematic issue both for the analysis of Bohr's philosophy and generally for the interpretation of quantum mechanics.

The diversity of views documented in this section on the relation among some of the most foundational concepts of quantum mechanics such as wave–particle dualism, complementarity, and the indeterminacy relations is an intriguing but very troubling aspect of the 70 year old effort to achieve some coherent consensus on these relationships. That such an unseemly condition exists at the conceptual roots of one of the most stunningly successful theories in physics is unacceptable. Fortunately, a simple and straightforward remedy exists for reconciling these diverse positions, as Sec. III demonstrates.

III. WAVE–PARTICLE COMPLEMENTARITY WITHIN KINEMATIC–DYNAMIC COMPLEMENTARITY

A clue to the remedy just mentioned is found in the work of Scully *et al.*,[35] who analyze the complementarity attending the designation of the path taken by a quantum entity (in their case an excited Rydberg atom) through double slits. They show that such a path designation (no matter how delicate), which they call ''Welcher-Weg'' or ''which path'' information, is sufficient to destroy the interference pattern that would otherwise occur at the detection screen where the two paths meet. Complementarity is manifested in this situation by the appearance of interference phenomena (wave-like) if there is no specification of which path (slit) the quantum entity traversed; but, if the path is designated (particle-like), the interference disappears.

Moreover, this *which path–interference* complementarity occurs with the double-slit arrangement rigidly in place, defining spatial coordination of the phenomenon in the sense of Bohr,[46] i.e., within the kinematic mode of description. But in quantum theory, the kinematic–dynamic complementary modes of description stand on a similar footing, so that a *definite value–interference* complementarity should exist for the dynamic mode of description, the change in terminology from ''which path'' to ''definite value'' being occasioned by the fact that the ''which path'' designation for space–time kinematic description is not too appropriate for the dynamic mode. Henceforth, we shall refer to this form of complementarity as *definite value–interference* or *which value–interference* complementarity. This form of complementarity seems to have meaning only within a representation of quantum mechanics defined by the eigenvectors of a complete set of commuting physical observables, such as the complementary kinematic or dynamic variables.

All of this is consistent with and contained within the standard formalism of quantum theory, as will now be set forth.[47] In this formalism the state of an object is represented by a unit vector $|\psi_s\rangle$ in the Hilbert space H_s of the object system. This vector may be written as a superposition of the eigenvectors $|q\rangle$ of a complete set of commuting observables of the object, i.e.,

$$|\psi_s\rangle = \sum_q |q\rangle\langle q|\psi_s\rangle, \tag{1}$$

the sum being an integral for any set of the eigenvalues that are continuous. The probability of a particular eigenvalue o_i of some physical quantity O_s of the object system is

$$\text{Prob of } o_i = \operatorname{Tr} \Pi_{o_i} \rho_s = |\langle o_i|\psi_s\rangle|^2, \tag{2}$$

where Πo_i is the projection operator $|o_i\rangle\langle o_i|$ on the eigenstate $|o_i\rangle, \rho_s$ is the density operator $|\psi_s\rangle\langle\psi_s|$ for the state $|\psi_s\rangle$, and Tr means the trace of the product operator $\Pi o_i \rho_s$. The insertion of the expansion Eq. (1) for $|\psi_s\rangle$ into Eq. (2) leads to

$$\text{Prob of } o_i = \sum_{q,q'} \langle o_i|q\rangle\langle o_i|q'\rangle^*\langle q|\psi_s\rangle\langle q'|\psi_s\rangle^*, \tag{3}$$

in which appear cross terms with $q \neq q'$ corresponding to interference characteristic of wave behavior associated with superposition of amplitudes as in Eq. (1).

If each $|q\rangle$ state is specifically and distinctively identified, which can be indicated by labeling each $|q\rangle$ with a vector $|L_q\rangle$ in the Hilbert space H_L of the labeling or identifying system, i.e., each $|q\rangle \Rightarrow |q\rangle \otimes |L_q\rangle$, then

$$|\psi_s\rangle \rightarrow |\psi_{sL}\rangle = \sum_q |q\rangle \otimes |L_q\rangle\langle q|\psi_s\rangle, \tag{4}$$

where $|\psi_{sL}\rangle$ is an entangled state in the product Hilbert space $|H_s\rangle \otimes |H_L\rangle$ of the object and labeling system. Then, the probability of the eigenvalue o_i for the physical quantity O_s of the object system is

$$\text{Prob of } o_i = \text{Tr } \Pi o_i \rho_{sL}, \tag{5}$$

where $\rho_{sL} = |\psi_{sL}\rangle\langle\psi_{sL}|$.

The insertion of $|\psi_{sL}\rangle$, Eq. (4), into Eq. (5) and the calculation of the trace result in

$$\text{Prob of } o_i = \sum_{qq'} \langle q|\psi_s\rangle\langle o_i|q\rangle \times \langle o_i|q'\rangle^*\langle q'|\psi_s\rangle^*\langle L_q|L_{q'}\rangle. \tag{6}$$

By the requirement that the states $|q\rangle$ be distinctively labeled, $\langle L_q|L_q'\rangle = \delta q,\ q'$, and

$$\text{Prob of } o_i = \sum_q |\langle o_i|q\rangle|^2\langle q|\psi_s\rangle|^2, \tag{7}$$

and the interference terms have disappeared, characteristic of a situation in which the object system S has definite values q, each with probability$=|\langle q|\psi_s\rangle|^2$, a particle-like picture complementary to the wave-like interference among the different nonlabeled $|q\rangle$ states in Eq. (3).

What has been set forth here in the q representation can be done equally well in any other representation defined by the eigenvectors of a complete set of commuting observables, so that, if the $|q\rangle$ states are associated with the position variables in configuration space, a set of results analogous to Eqs. (1)–(7) holds, for example, for the conjugate momentum states $|p\rangle$. However, the terminology particle–wave complementarity or *which path–interference* complementarity for the situation represented by Eq. (7) and Eq. (3), respectively, while appropriate for the position space representation, should generally be replaced by the terminology ''definite value–interference'' or ''which value–interference'' complementarity, for any basis other than that defined by the configuration space position variables.

According to this analysis, we see that there is a *definite value–interference* or *which value–interference* complementarity within any representation (or basis set of eigenvectors) in quantum theory, within which the basis vectors may be distinguished or labeled by vectors in the Hilbert space of the labeling system. Thus there should be, in principle, a *definite value–interference* complementarity within each of the complementary kinematic–dynamic or space–time momentum–energy modes of descriptions (I am ignoring the well-known problem that time is not really a conjugate variable), a *layered* complementarity, so to speak.

Most of the relevant concrete examples involve *which path–interference* complementarity in the kinematic space–time mode of description such as two-path arrangements like Young's double-slit experiment.[48] Scully and co-workers[34,35] have discussed extensively one such example and have provided references to several others. It would be desirable to have empirical examples of *which value–interference* complementarity for the dynamic momentum–energy states, but unfortunately this author has not been able to identify any.

Section IV examines some consequences of this layered complementarity, i.e., of *which value–interference* complementarity (a surrogate for particle–wave complementarity) within kinematic–dynamic complementarity.

IV. CONSEQUENCES OF LAYERED COMPLEMENTARITY

We can see that neither the assignment to Bohr by Rosenfeld[28] (and adopted by others[29,30]) of a wave–kinematic and particle–dynamic complementarity correlation nor the assignment to Bohr by Pais[32] (also supported by others[33–35]) of a particle–kinematic (trajectories in space–time) and wave–dynamic $[p=h/\lambda,\ E=h\nu]$ correlation completely captures the range of possibilities that seem to occur both in nature and in the quantum formalism. For example, whereas Rosenfeld allows for wave interference in space–time and for objects with specified values of momentum and energy, his position does not embrace objects with which path characteristics in space–time. While Pais's position allows for the latter, it does not embrace the former (wave interference in space).

Layered complementarity is not restricted in these ways. In the kinematic (space–time) mode of description, waves can interfere in space–time (*contra* Pais) but labeling the paths destroys interference and provides which path designation (*contra* Rosenfeld). Momentum and energy are needed to complete the description.

Thus as Murdoch,[38] Folse,[41] Faye,[43] and Faye and Folse[45] argue, there is no simple one-to-one correlation of the wave–particle picture with the kinematic–dynamic descriptive modes, but layered complementarity with *definite value–interference* complementarity within both kinematic and dynamic modes seems to do justice to the range of possibilities.

Furthermore, as discussed in Sec. II, Born[36] and others[37–39] stand ready to jettison particle–wave complementarity altogether, partly on grounds that in some situations *both are needed* for an explanation of what happens. The latter is true but it is possible, perhaps even desirable, to retain both this truth *and* particle–wave complementarity in the form of *which value–interference* complementarity discussed here. This will now be explained.

There are, in fact, several aspects of the explanation. First, there are several examples of phenomena that disagree with the general tenet of wave–particle complementarity, i.e., the general prohibition of simultaneous wave–particle behavior, but that are compatible with the more restricted *which value–interference* complementarity discussed here. Here are some of them:

(1) Nothing in the analysis presented here precludes the simultaneous kinematic manifestation of a ''which path'' designation along with some kind of wave behavior other than interference, such as occurs in the experiment described by Ghose *et al.*,[49] and carried out by Mizobuchi and Ohtake.[50] In this experiment a beam of light of frequency ν directly shines at normal incidence on one side of a 45° prism with an index of refraction sufficiently high that the light undergoes total internal reflection at the hypotenuse of the prism. A nonpropagating exponentially decaying *wave* amplitude extends out of the prism at the hypotenuse into space, so that, if the hypotenuse of a second 45° prism is brought into close proximity to the first prism, transmission of light across the gap separating the two prisms will occur with an intensity that depends on the square of the wave amplitude crossing the gap, which in turn depends on the ratio of the gap distance and the wavelength of light. The light that is not transmitted is reflected. For a weak light beam carrying only one photon, the photon may be transmitted or reflected with relative probabilities that depend on the wave amplitude crossing the gap. Detectors placed in the transmission and reflection paths count the photons in anticoincidence, i.e., a photon transmitted across the gap is not reflected and *vice versa*, consistent with the concept of ''photon.'' Thus this experiment demonstrates simultaneously the transmission and reflection of a *wave* at the gap and *particle* ''which path'' anticoincidence detection in the transmission or reflection channel.

The results can be interpreted, as Ghose *et al.*[49–51] do, as contradictory to the orthodox tenet of mutual exclusivity (complementarity) of wave and particle behavior,[52] but does not contradict the more narrow *which path–interference* complementarity in the kinematic (space–time) description advocated here, since interference is not a feature of this experiment.

(2) Particle-like objects with specific mass, charge, and energy tunnel through potential barriers as in α decay of nuclei or field emission of electrons from metals. In each case the Schrödinger wave function of the particle penetrates through the barrier to free space and Born's doctrine[36] of ''The wave giving the probability of finding the particle'' provides the quantitative description for particle leakage through the barrier. Contrary to orthodoxy, coexisting wave and particle behavior seems necessary to comprehend this phenomenon, but there is no inconsistency with *which value–interference* complementarity since interference plays no role in this process.

(3) The lifetime of spontaneous photon emission during transition between energy states of an atom is modified if the atom moves near the face of a wave-reflecting surface. According to Selleri.[53]

> ... The lifetime phenomenon can be attributed to stimulation or inhibition of the emission *of the light quantum* (WGH emphasis), the stimulating field being the radiation field of the atom capable of emitting the quantum itself ... it appears virtually impossible to give any rational explanation [of the phenomena] in terms of a purely undulatory (or purely corpuscular) description of radiation.

Again the requirement of wave–particle mutual exclusivity is not met here, but no violation of *which value–interference* complementarity occurs, since there is no designation of ''which value'' kinematic quantities to destroy the interference of the emitted and reflected wave that influences the lifetime of the state.

(4) The Planck formula for the spectrum of blackbody radiation contains terms reflecting the presence of both waves (frequency v) and quanta ($E=hv$). This is perhaps most clearly demonstrated by Einstein's expressions for the mean square energy and momentum fluctuation of radiation at frequency v from a blackbody at temperature T.[54] These expressions contain two terms, one based explicitly on the wave characteristics of radiation and the other on a classical gas of molecules or point-like quanta of energy hv (particle-like),[55] a result inconsistent with the standard doctrine but consistent with the more narrow *which value–interference* complementarity since neither wave interference nor ''which value'' designation seems relevant in this situation.

Another advantage of the point of view espoused here about *which value–interference* complementarity and the ''both are needed'' doctrine (i.e., the concomitant appearance of both wave and particle properties) of Born comes from the realization that substantial interference can occur among states with only a slight departure from distinctive ''which value'' identification, as Wooters and Zurek show.[56] Moreover, the extent of wave and particle properties can be quantified[57] for this *which value–interference* form of particle–wave complementarity analogous to the quantifying by the Heisenberg indeterminacy relations of kinematic–dynamic complementarity, an opportunity that does not exist for the apparently incorrect orthodox claim of general particle–wave complementarity.

V. SUMMARY

It is clear from the empirical evidence discussed in Sec. IV that the orthodox position on particle–wave complementarity that generally prohibits simultaneous wave and particle behavior in a single experiment is too broad in scope[58] and violates a number of observations. It is thus not too surprising that a diversity of views as to the nature of particle–wave complementarity came to exist. It is further not surprising that the more restricted *which value–interference* form of particle–wave complementarity agrees with the empirical evidence, since it is rooted in the formal structure of quantum theory which has an unparalleled record of success in accounting for the behavior of the microworld. Further credibility would accrue to this more limited concept of particle–wave complementarity if some clear, cogent examples of *which value–interference* complementarity could be identified outside the *which path–interference* examples that exist within the kinematic (space–time) mode of description, such as examples involving dynamic variables.

Finally, since whatever truth about nature embraced by the concept of complementarity seems to reside within the formal structure of quantum theory, the concept seems dispensable for physics,[59] and many physicists have largely ignored it in their work, finding interest in it only as a historical relic in guiding Bohr and some of his colleagues in the monumental achievement of the early development of quantum theory. However, the concept still has currency in some quarters, including with authors of physics textbooks. Thus it should be used and explained correctly. This paper seeks to provide a valid elucidation of complementarity for those who find some relevance in, and need for, it.

ACKNOWLEDGMENTS

Helpful communications are gratefully recognized about various aspects of this work with Horace Crater of the University of Tennessee Space Institute; Joe Ferguson, Mississippi State University; R. E. Mickens, Clark-Atlanta University; Marlan Scully, Texas A&M University; H. C. von Baeyer, College of William and Mary; and D. F. Walls and S. M. Tan of the University of Auckland, none of whom bears a burden for its shortcomings.

DEDICATION

It is an honor to dedicate this work to several of my late mentors: (1) an older brother, Dr. Alvis M. Holladay (1915–1996), who pioneered the path to postgraduate education for me and several other close relatives; (2) Guy Forman (1906–1993), my first physics teacher who encouraged my entering physics as a career; (3) Sherwood K. Haynes (1910–1990) director of my Master's thesis who first introduced me to the wonders of quantum physics; (4) Ingram Bloch (1920–1995), whose polymathic talents inspired in me an abiding interest in theoretical physics; and (5) Robert T. Lagemann (1912–1994) who faithfully sustained my early professorial career in physics at Vanderbilt.

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[32]Reference 19, p. 315. The statement is from Bohr, Ref. 2, pp. 54–55.

[33]D. Bohm, *Quantum Theory* (Prentice-Hall, New York, 1951), pp. 132 and 609.

[34]M. Scully, B.-G. Englert, and H. Walther, ''Quantum Optical Tests of Complementarity,'' Nature **351**, 111–116 (9 May 1991).

[35]B.-G. Englert, ''Complementarity,'' in *Foundations of Quantum Mechanics*, edited by T. D. Black, M. M. Nieto, H. S. Piloff, M. O. Scully, and R. M. Sinclair (World Scientific, Singapore, 1992), pp. 181–192.

[36]M. Born, *The Restless Universe* (Dover, New York, 1951), p. 283.

[37]A. Grünbaum, ''Determinism in the light of recent physics. Complementarity in quantum physics and its philosophical generalization,'' J. Philos. Soc. **LIV**, 713–727 (1957), fn. 7, p. 717.

[38]D. Murdoch, in Ref. 25, p. 67.

[39]Reference 25, p. 79.

[40]H. J. Folse, in Ref. 20, pp. 108–141, 154–160.

[41]Reference 20, p. 121.

[42]J. Faye, *Niels Bohr, His Heritage and Legacy: An Anti-realist View of Quantum Mechanics* (Kluwer, Dordrecht, 1991), p. 144.

[43]See Ref. 42.

[44]See Ref. 42.

[45]*Niels Bohr and Contemporary Philosophy*, edited by J. Faye and H. J. Folse (Kluwer, Dordrecht, 1994), from p. xvi of The Introduction.

[46]N. Bohr, ''Discussion with Einstein on epistemological problems in atomic physics,'' in *Albert Einstein: Philosopher-Scientist*, edited by P. A. Schilpp (Library of Living Philosophers, Evanston, 1949), pp. 201–241. Reprinted in N. Bohr, *Atomic Physics and Human Knowledge* (Wiley, New York, 1958), pp. 32–66 and in Ref. 4, pp. 9–49. Essential excerpts of this article appear in French and Kennedy, Ref. 16, pp. 121–140.

[47]The subsequent discussion leans heavily on the work of: (a) J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton U.P., Princeton, 1955), especially, Chap. VI. Translation by R. T. Beyer of The German edition (Springer-Verlag, Berlin, 1932). (b) F. W. London and E. Bauer, Chap. II-1 ''The theory of observation in quantum mechanics,'' in

Ref. 4, pp. 217–259. (c) J. M. Jauch, ''The Problem of Measurement in Quantum Mechanics,'' Helv. Phys. Acta **37**, 293–316 (1964). (d) P. A. Moldauer, ''Is there a quantum measurement problem?,'' Phys. Rev. D **5**, 1028–1032 (1972). (e) H. Krips, *The Metaphysics of Quantum Theory* (Clarendon, Oxford, 1987), especially Chap. 5. These works focus on the theory of measurement which involves irreversibility with an increase in entropy. The discussion here is not about measurement, only about identifying or labeling states, a procedure that may be reversed with the possibility of restoring interference among the previously labeled states, a circumstance of special interest to Scully *et al.* (Ref. 34).

[48]The earliest experiments known to me of destroying interference by labeling were by Arago and Fresnel in 1816 where a beam of light hitting a birefringent crystal produces a so-called ordinary and extraordinary ray with orthogonal states of polarization. When recombined these two rays do not produce an interference pattern. This work is reviewed in R. W. Wood, *Physical Optics* (Macmillian, New York, 1923), pp. 148–151. A laser version of the classical Arago–Fresnel experiments is described in J. L. Ferguson, ''A simple bright demonstration of the interferences of polarized light,'' Am. J. Phys. **52**, 1141–1142 (1984), where references to a number of related works can be found.

[49]P. Ghose, D. Home, and G. S. Agarwal, ''An experiment to throw more light on light,'' Phys. Lett. A **153**, 403–406 (1991).

[50]Y. Mizobuchi and Y. Ohtake, ''An experiment to throw more light on light,'' Phys. Lett. A **168**, 1–5 (1992).

[51]P. Ghose, D. Home, and G. S. Agarwal, ''An experiment to throw more light on light: Implications,'' Phys. Lett. A **168**, 95–99 (1992). See also P. Ghose and D. Home, ''The Two-Prism Experiment and Wave-Particle Duality of Light,'' Found. Phys. **26** (7), 943–953 (1996).

[52]Bohr's tenet of general mutual exclusivity of wave and particle pictures has become widely accepted. One of the most highly respected and widely used current textbooks in introductory physics, D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics with Modern Physics* (Wiley, New York, 1993) 4th ed., has on p. 1171 the following statement:

> The wave and particle aspects of a quantum entity are both necessary for a complete description. However, both aspects cannot be revealed simultaneously in a single experiment. Which aspect is revealed is determined by the nature of the experiment being done.

It is the second sentence being disputed here. While it appears to hold for *which value-interference* properties in both the kinematic and dynamic modes of description, it does not universally hold for all manifestations of particle and wave properties as the discussion in Sec. IV demonstrates. Moreover, note that, while which value designation precludes interference among the states so designated, the existence of interference precludes only anterior, not posterior, designation. In the two slit experiments, designation of which slit traversal obliterates the interference pattern, but with no such designation, the waves from the two paths interfere, which is revealed by the fringe pattern of particle-like deposition of quanta at localized regions of the detection screen. This experiment, which is more fully described in A. P. French and E. Taylor, *An Introduction to Quantum Physics* (Norton, New York, 1978), in Sec. 2.10, ''The coexistence of wave and particle properties,'' pp. 87–93, clearly violates the orthodox prohibition contained in Halliday, Resnick, and Walker (above) that, both [particle and wave] aspects cannot be revealed simultaneously in a single experiment.

[53]In Ref. 30, p. 133. This reference provides more details about this experiment.

[54]An illuminating discussion of these formulas appears in A. Pais, *Subtle is the Lord* (Clarendon, Oxford, 1982), pp. 402–405.

[55]The essence of this description is from N. C. Combourieu and H. Rauch, ''The wave particle dualism in 1992: A Summary,'' Found. Phys. **22** (12) 1403–1424 (1992).

[56]W. K. Wooters and W. H. Zurek, ''Complementarity in the double-slit experiment. Quantum nonseparability and a quantitative statement of Bohr's principle,'' Phys. Rev. D **19**, 473–484 (1979). See also L. S. Bartell, ''Complementarity in the double-slit experiment on simple realizable systems for observing intermediate particle-wave behavior,'' *ibid.* **21**, 1698–1699 (1980). Both papers are reprinted in Ref. 4, pp. 443–454 and 455–456, respectively.

[57]D. M. Greenberger and A. Yasin, ''Simultaneous wave and particleknowledge in a neutron interprometer,'' Phys. Lett. A **128**, 391–394 (1988); G. Jaeger, A. Shimony, and L. Vaidman, ''Two interferometric complementarities,'' Phys. Rev. A **51**, 54–67 (1995); B.-G. Englert, ''Fringe visibility and which-way information: An inequality,'' Phys. Rev. Lett. **77**, 2154–2157 (1996); J. L. Cereceda, ''An apparent paradox at the heart of quantum mechanics,'' Am. J. Phys. **64**, 459–466 (1996). A readable account of the quantification of particle–wave properties appears in H. C. von Baeyer, ''The Quantum Eraser,'' Sciences **37** (1), 12–14 (1997).

[58]In this connection Cushing wrote:

> In Bohrian complementarity we have another example of his general philosophical prohibition overstepping the necessary logical implications of the mathematical formalism of quantum mechanics (i.e., in categorically forbidding certain possibilities).

In J. T. Cushing, *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony* (University of Chicago Press, Chicago, 1994), Chap. 3, ''Standard quantum theory,'' p. 124.

[59]It is interesting to note that such well-known, classic treatises on quantum theory as those by Weyl, Dirac, von Neumann, and Landau and Lifshitz do not contain the word ''complementarity.'' On the other hand Pauli (Ref. 22) gives it center stage.

SCHOLARS AND CHARLETANS

Unfortunately the opinion is already to prevalent that a profound knowledge of any branch of science is not necessary to a good teacher of that branch but rather detrimental. It is evident that what ever may be a persons capacity for communicating knowledge he cannot teach more than he knows. The man of profound acquirement it is true may not possess a happy faculty of imparting knowledge and he may err in attempting to give too much but it will be found on the other hand that the sucesful popular teacher in general is little more than a charletan who does not attempt to give his pupuls precise ideas but substitutes crude and partial hypotheses for the true generalizations of science.

Joseph Henry, letter to Benjamin Silliman, Jr. (1846), in *The Papers of Joseph Henry*, edited by Marc Rothenberg (Smithsonian Institution Press, Washington, 1992), Vol. 6, p. 477.