

Spherical aberration in spatial and temporal transforming lenses of femtosecond laser pulses

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Abstract

We study the influence of third order spherical aberration on the Group Velocity Dispersion (GVD) and on the Propagation Time Delay (PTD) of a plane pulse that focalizes by a thin lens.

Applications in refractive/diffractive PTD compensation systems and in gaussian temporal-shaped pulses are analyzed.

1. Introduction

It is well known that the temporal and spatial profile of a femtosecond laser pulse is strongly affected by the dispersion properties of the optical media.

The pioneering work of Z. Bor¹⁻³ showed, in the framework of geometrical optics, that two kind of effects occur in propagation of optical laser pulses through lenses: PTD (propagation time delay) and GVD (group velocity dispersion).

Travelling in dispersive medium causes a temporal delay of the pulse front, which is defined as the surface that coincides with the peak of the pulse, with respect to the phase front. This effect (PTD) is responsible for a first contribution to pulse broadening in time. The dependence of the group velocity on wavelength (GVD), in dispersive optical systems,

causes a second contribution to the broadening of the pulse.

Later, Kempe et al⁴ investigated the transmission of ultrashort light pulses by singlet lens and achromatic lens doublet taking into account diffraction effects. In terms of Fourier optics, he derived the basic equation for the transformation taking into account the dispersion in first and second order. Also, Kempe showed that the spatial size of a femtosecond pulse in a focal plane is considerably larger compared with that of a focused monochromatic wave. In two subsequent papers, Kempe and Rudolph⁵⁻⁶ presented a theoretical and experimental study of the interplay of spherical and chromatic aberration of focusing femtosecond light pulses by single lenses.

Reflective optics was proposed as a solution to these kind of pulse front distortion because the light does not travel through any dispersive media but the problem with this approach is experimental (path of the beam propagation and coating for high power).

Another solution is to use achromatic doublets since the delay between the phase front and the pulse front is constant over the lens cross-section¹. Also, the group-velocity dispersion is uniform across the beam so it can be compensated with grating or prism compressors. As the propagation time is still dependent on wavelength, the construction of achromats could be very complicated, depending on the working frequency range. All-glass achromats fabrication involve exotic glasses, large apertures and extreme curvatures of the refractive surfaces.

To avoid these problems and to achieve PTD compensation, the combination of lens and zone plate was studied⁷ from a geometrical point of view. It was shown that partial compensation is possible only if the two elements are separated at a certain distance. Later, based in the paraxial approximation of the diffraction theory, it was demonstrated⁸ that complete compensation is possible, even when the distance between the diffractive and refractive elements is zero.

However the former bibliography does not take into account the frequency dependence of the spherical aberration of the lens. In this paper we describe how the third order spherical aberration of a thin lens influences the GVD and PTD effects of a plane pulse that focalizes

in the gaussian plane. We applied these results to the cases of a refractive/diffractive lens system and to a gaussian temporal profile.

2. Amplitude field behind a thin lens of an incident plane pulse

A. The transforming lens

Let a monochromatic plane wave with amplitude $A(\omega)$ be incident to a thin lens whose generalized pupil function is $P(x_1, y_1)$. Fig. 1 shows the coordinate system, notation, and lens parameters.

In the framework of Fourier optics, the amplitude field at distance z behind a lens is⁹:

$$U(x_2, y_2, z, \omega) \propto \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_1 dy_1 P(x_1, y_1) A(\omega) t(x_1, y_1) \exp\left[\frac{j k_a}{2z} [(x_2 - x_1)^2 + (y_2 - y_1)^2]\right] \quad (1)$$

where $t(x_1, y_1)$ is the phase transformation factor that describes the operation of the lens:

$$t(x_1, y_1) = \exp[j k_l d] \times \exp\left[-j(k_l - k_a) \frac{x_1^2 + y_1^2}{2} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right] \quad (2)$$

with k_l and k_a the wave-numbers vectors inside the lens and in air, respectively. R_1 and R_2 are the radii of curvature of the lens surfaces and d is the thickness of the lens along the optical axis.

B. Spherical aberration

Considering axial incidence on the lens with object in infinity, we can restrict our analysis of primary aberrations to the case of spherical aberration in the gaussian image plane (i.e.; z corresponds to the paraxial focus of the lens). This situation can be described with the following generalized pupil function:

$$P(x_1, y_1) = \exp[jk_a \phi(x_1, y_1)], \text{if } x_1^2 + y_1^2 \leq r_1^2 \quad (3)$$

$$0, \text{else} \quad (4)$$

with:

$\phi(x_1, y_1)$: spherical aberration function.

$$\phi(x_1, y_1) = -\frac{1}{4}B(x_1^2 + y_1^2)^2 \quad (5)$$

Geometrical theory of aberrations¹⁰ shows that the coefficient B can be expressed as:

$$B = \frac{1}{2}\beta + \frac{n(\omega)^2}{8(n(\omega) - 1)^2}\wp^3 - \frac{n(\omega)}{2(n(\omega) + 2)}\aleph^2\wp + \frac{1}{2n(\omega)(n(\omega) + 2)}\wp\left[\frac{n(\omega) + 2}{2(n(\omega) - 1)}\sigma + 2(n(\omega) + 1)\aleph\right]^2 \quad (6)$$

with:

$n(\omega)$: frequency-dependent refractive index of the lens material

\wp : power of the lens

$$\wp = \frac{1}{f} = (n(\omega) - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\sigma = (n(\omega) - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

\aleph : Abbe invariant of the lens

$$\aleph = -\frac{\wp}{2}$$

β : deformation coefficient of the lens

$$\beta = (n(\omega) - 1)\left(\frac{C_1}{R_1^3} - \frac{C_2}{R_2^3}\right)$$

C_i : deformation coefficients of the lens surfaces

C. The incident pulse

To specify the frequency dependence of the wave numbers, we expand them about the center frequency ω_0 of the incident pulse up to second order in $\Delta\omega = \omega - \omega_0$:

$$k_l = \frac{\omega}{c}n(\omega) = k_0n_0[1 + a_1\Delta\omega + a_2(\Delta\omega)^2] \quad (7)$$

$$k_l - k_a = \frac{\omega}{c}[n(\omega) - 1] = k_0(n_0 - 1)[1 + b_1\Delta\omega + b_2(\Delta\omega)^2] \quad (8)$$

with:

$$a_1 = \frac{1}{\omega_0} + \frac{1}{n_0}D$$

$$a_2 = \frac{1}{\omega_0 n_0}D + \frac{1}{2n_0}D'$$

$$D \equiv \left(\frac{dn}{d\omega}\right)_{\omega=\omega_0}$$

$$D' \equiv \left(\frac{d^2n}{d\omega^2}\right)_{\omega=\omega_0}$$

$$k_a = \frac{\omega}{c}$$

c : speed of light in vacuo.

$$k_a = k_0\left(1 + \frac{\Delta\omega}{\omega_0}\right) \quad (9)$$

$$k_0 \equiv k_a(\omega_0)$$

$$n_0 \equiv n(\omega_0)$$

$b_i \rightarrow a_i$, changing n_0 by $(n_0 - 1)$

We also need to specify the frequency dependence of the aberration function:

$$B = B_0 + B_1\Delta\omega + B_2(\Delta\omega)^2 \quad (10)$$

The expressions that relate B_0 , B_1 , and B_2 with the lens parameters n_0 , R_1 , R_2 , C_1 , C_2 , D , D' , and the focal length of the thin lens for the central frequency, $1/f_0 \equiv (n_0 - 1)(1/R_1 - 1/R_2)$, can be seen in Appendix.

D. Pulse evolution

We have used polar coordinates both in the pupil and in the image gaussian plane because of the axial simmetry:

$$x_1^2 + y_1^2 = r_1^2 = (a \cdot r)^2 \quad (11)$$

$$x_2^2 + y_2^2 = r_2^2 \quad (12)$$

$$v \equiv \frac{ak_0 r_2}{f_0} \quad (13)$$

Then, replacing Eqs. (9-12) in (3-4), the generalized pupil function in polar coordinates will be:

$$\begin{aligned} P(r) = & \exp[-jk_0 \frac{a^4 B_0}{4} r^4] \times \exp[-jk_0 (\frac{a^4}{4} [B_1 + B_0/\omega_0]) r^4 \Delta\omega] \\ & \times \exp[-jk_0 (\frac{a^4}{4} [B_2 + B_1/\omega_0]) r^4 (\Delta\omega)^2] \times \text{circ}(r) \end{aligned} \quad (14)$$

The first factor represents the ordinary monochromatic contribution to the spherical aberration due to the center frequency ω_0 . The other factors will influence the PTD and the GVD phenomena respectively.

In these coordinates and considering the former definitions, we can integrate Eq. (1) over the azimuthal coordinate to obtain:

$$\begin{aligned} U(v, \Delta\omega) \propto & A(\Delta\omega) \exp[-jk_0 n_0 d \Delta\omega (a_1 + a_2 \Delta\omega)] \exp[-j \frac{v^2}{4N} (1 + \frac{\Delta\omega}{\omega_0})] \\ & \times \int_0^1 r dr J_0 \{rv [1 + \frac{\Delta\omega}{\omega_0}]\} \\ & \times \exp[-j \frac{k_0}{2f_0} (ar)^2 \Delta\omega (b_1 - \frac{1}{\omega_0} + b_2 \Delta\omega)] \\ & \times \exp[-jk_0 \frac{B_0}{4} (ar)^4] \\ & \times \exp[-jk_0 \frac{1}{4} (B_1 + B_0/\omega_0) (ar)^4 \Delta\omega] \\ & \times \exp[-jk_0 \frac{1}{4} (B_2 + B_1/\omega_0) (ar)^4 (\Delta\omega)^2] \end{aligned} \quad (15)$$

where J_0 denotes the Bessel function of the first kind of order zero and $N \equiv a^2 k_0 / 2f_0$ is the Fresnel Number. In general the relation $N \gg v^2/4$ is valid, and we can neglect the corresponding phase factor in Eq. (15).

If we perform an inverse Fourier transform the pulse field will be:

$$\begin{aligned} U(v, t) \propto & \int_{-\infty}^{\infty} d(\Delta\omega) A(\Delta\omega) \int_0^1 r dr J_0 \{rv [1 + \frac{\Delta\omega}{\omega_0}]\} \\ & \times \exp[-jk_0 \frac{B_0}{4} (ar)^4] \end{aligned}$$

$$\begin{aligned}
& \times \exp[-j(\Delta\omega)^2\{\delta' - \delta r^2 - \delta'' r^4\}] \\
& \times \exp[-j(\Delta\omega)\{t - \tau' + \tau r^2 + \tau'' r^4\}]
\end{aligned} \tag{16}$$

with:

$$\delta \equiv \frac{a^2 k_0}{2f_0} b_2 \tag{17}$$

$$\delta' \equiv k_0 n_0 d a_2 \tag{18}$$

$$\delta'' \equiv \frac{a^4 k_0}{4} \left(B_2 + \frac{B_1}{\omega_0} \right) \tag{19}$$

$$\tau \equiv \frac{a^2 k_0}{2f_0(n_0 - 1)} D \tag{20}$$

$$\tau' \equiv k_0 n_0 d a_1 \tag{21}$$

$$\tau'' \equiv \frac{a^4 k_0}{4} \left(B_1 + \frac{B_0}{\omega_0} \right) \tag{22}$$

The third and fourth exponential factors represent the GVD and PTD effects, respectively, with the corrections owing to the contribution of the spherical aberration in the first and second order of the expansion.

Not only the spherical aberration distorts the radius-dependent chirp due to the focusing properties of the lens, $(\Delta\omega)^2 \delta r^2$, but the radius-dependent delay in the focal plane (time delay between the pulse front and the phase front or PTD), τr^2 , as well.

We can conclude, for example, that the design of a system capable of compensating PTD should take into account the remanent PTD due to spherical aberration ($\tau'' r^4$).

E. Relative weight of the spherical aberration

We can estimate the contribution of the spherical aberration to GVD and PTD, by considering the ratio of their corresponding phase terms:

$$W_{GVD} \equiv \frac{\delta'' r^4}{\delta r^2} \Big|_{MAX} = f_0 a^2 \frac{(B_2 + B_1/\omega_0)}{(3/2)(D/\omega_0)(n_0 - 1)} \tag{23}$$

$$W_{PTD} \equiv \frac{\tau'' r^4}{\tau r^2} \Big|_{MAX} = f_0 a^2 \frac{(B_1 + B_0/\omega_0)}{2D/(n_0 - 1)} \tag{24}$$

As the coefficients B_0 , B_1 , and B_2 are complicated functions of the lens parameters (see Appendix I) we reduce their degrees of freedom choosing the lens glass type. In this case

we consider fused silica so $n(\lambda)$, and its derivatives, turn out to be known functions¹¹. As we fixed the central wavelength of the incident pulse radiation at $620nm$, we were able to determine univocally the values of $n(0)$, D and D' .

Considering usual ultrafast microscopy parameters, we analyzed the case of focal length $f_0 = 19.0mm$ and half aperture $a = 6.3mm$. Hence we can express R_2 as a function of R_1 and choose it as the first degree of freedom. Allowing the deformation parameters of the lens surfaces to be equal ($C_1 = C_2 \equiv C$) we obtained the second degree of freedom.

In this way, W_{GVD} and W_{PTD} are functions of only two variables: R_1 and C .

Another useful relation is the aberration expressed in terms of the wavelength as a function of R_1 and C . Considering that the first exponential factor in Eq. (14) represents the monochromatic contribution of the spherical aberration we can define:

$$N_{\lambda_0} \equiv \frac{a^4}{4\lambda_0} B_0 \quad (25)$$

as a measure of the amount of spherical aberration present in the lens.

Fig. 2, 3 and 4 shows maps of N_{λ_0} , W_{GVD} and W_{PTD} as functions of R_1 and C . These figures describe the global influence of the spherical aberration over the main features of a pulse transforming lens.

3. Applications

I. The refractive/diffractive lens system

Once we know the effect of spherical aberration over pulse transformation, we will apply the former results to the case of PTD compensation (assuming as usual that GVD effects are negligible) using a diffractive lens.

Due to aberrations, the use of a refractive/diffractive lens system could not be fully effective to compensate PTD effects (i.e.; spherical aberration could be an important remanent contribution to PTD).

To place a diffractive lens right after the refractive lens imply to insert the following transmission function in Eq. (1):

$$F(r) = \cos\left\{\left(\frac{k(\omega)}{2r_0}\right)r_1^2\right\}, \quad (26)$$

being r_0 the primary focus. In our notation, this function can be expressed as:

$$F(r) = \frac{1}{2}\left\{\exp\left[j\left(\frac{k_0 a^2}{2r_0}\right)(1 + \Delta\omega/\omega_0)r^2\right] + \exp\left[-j\left(\frac{k_0 a^2}{2r_0}\right)(1 + \Delta\omega/\omega_0)r^2\right]\right\} \quad (27)$$

Eq. (27) shows that the diffractive lens will influence the PTD effect by its exponential factors in $\Delta\omega$.

Applying the criteria of reference⁸ to the amplitude field of the composed system (i.e.; Eq. (1) with $F(r)$), we found the conditions of PTD compensation (and hence to compensate first order chromatic aberration of the refractive lens by the opposite dispersion in the diffractive lens). In other words, we found the value of z_c of z and the relationship between the focii of both lenses that make exponents up to $\Delta\omega$ equal zero:

$$\frac{1}{r_0} = \frac{1}{2} \frac{D\omega_0}{f_0(n_0 - 1)} + \frac{a^2}{4} B_1 r^2 \quad (28)$$

$$\frac{1}{z_c} = -\frac{1}{f_1} \pm \frac{1}{r_0} - \frac{a^2}{2} B_0 r^2 \quad (29)$$

We can see that the former Eqs. are spatial-aberration dependent through coefficients B_0 and B_1 . According to the magnitude of a , the deviations from the ideal situation could be important.

II. Gaussian pulses

For a temporal gaussian-shaped plane pulse that is transformed by a singlet, we find the temporal and spatial distribution of the field.

In this case, the following functions are Fourier pairs:

$$a(t) = \exp[-(t/T)^2] \longleftrightarrow A(\Delta\omega) = \exp[-T^2(\Delta\omega)^2] \quad (30)$$

where T is the temporal width of the pulse.

We will take into account pulses as short as $100fs$, so we can assume that $(\Delta\omega/\omega_0) \ll 1$ in the argument of J_0 with considerable accuracy for most cases of experimental relevance⁴.

Solving the integral of Eq. (16) on $\Delta\omega$ we find that:

$$\begin{aligned}
U(v, t) \propto & \int_0^1 r dr J_0(rv) \exp[-jk_0 \frac{B_0}{4}(ar)^4] \\
& \times \left\{ \frac{1 + j[\delta' - \delta r^2 - \delta'' r^4]/T^2}{1 + ([\delta' - \delta r^2 - \delta'' r^4]/T^2)^2} \right\}^{\frac{1}{2}} \\
& \times \exp\left[-\frac{[t - \tau' + \tau r^2 + \tau'' r^4]^2}{T^2(1 + ([\delta' - \delta r^2 - \delta'' r^4]/T^2)^2)}\right] \\
& \cdot (1 + j([\delta' - \delta r^2 - \delta'' r^4]/T^2))
\end{aligned} \tag{31}$$

Assuming that GVD does not affect the intensity distribution essentially (i.e.; aberrations do not contribute to increase second order effects relative to first order effects) we can consider only the features that lead to the generalized PTD:

$$\begin{aligned}
U(v, t) \propto & \int_0^1 r dr J_0(rv) \exp[-jk_0 \frac{B_0}{4}(ar)^4] \\
& \times \exp\left[-\left(\frac{t - \tau'}{T} + \frac{\tau r^2}{T} + \frac{\tau'' r^4}{T}\right)^2\right]
\end{aligned} \tag{32}$$

We depicted the normalized intensity distribution $I(v, t) \propto |U(v, t)|^2$ as a function of $(t - \tau')/T$ for a pulse temporal width of $30fs$. Fig. 5 shows the case free from spherical aberration (in agree with reference⁴) and Fig. 6 corresponds to a λ_0 -spherical aberration ($N_{\lambda_0} \sim 1$). It is possible to see how the spherical aberration modifies the temporal scale by moving the peak of the intensity distribution.

4. Conclusions

In the general frame of deviations from ideal plane pulse transformation by lens systems, we have studied the contribution of the spherical aberration to the GVD and PTD effects of a pulse that is focalized in the gaussian plane by a thin lens. Even though, in the case of a thin singlet, spherical aberration can be diminished by defocusing, the deviations from the ideal situation on those effects are still present.

Hence, to describe those deviations from ideality, we chose the radius of curvature and the surface deformation coefficient of the lens as degrees of freedom. This fact allowed us to quantify the additional chirp and time delay due to spherical aberration and its dependence on those parameters, as Fig. 2, 3, and 4 showed.

We applied the former results to two special situations which deviations from ideality were expected.

First, considering PTD compensation by refractive/diffractive systems, we calculated the corrections due to spherical aberration in the foci relation and in the focalization condition.

Second, for gaussian pulses in temporal profile, we showed that even a λ_0 -spherical aberration is capable to modify the space and temporal structure of the intensity distribution.

Additionally, as a tool for calculation we presented a compact form of the expansion of the spherical aberration up to second order in the frequency shift.

Finally, we considered that these studies can be generalized to more complex lens systems commonly used in ultrashort pulse phenomena.

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Appendix A

Eq. 6 shows the spherical coefficient B as a function of $n(\omega)$ and the lens parameters. The task is to expand each of its terms up to second order in $\Delta\omega$, to perform all the operations,

and to arrange the final expression by the corresponding order. In other words, to calculate the explicit expressions of B_0 , B_1 , and B_2 .

We present a compact form of the final result for numerical applications.

Defining:

$$K_0 \equiv \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (\text{A1})$$

$$K_1 \equiv \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (\text{A2})$$

$$K_2 \equiv \left(\frac{C_1}{R_1^3} - \frac{C_2}{R_2^3} \right) \quad (\text{A3})$$

We can express B_0 as:

$$B_0 = \sum_{i=1}^6 B_{0i} \quad (\text{A4})$$

with:

$$B_{01} = \frac{1}{8}(n_0 - 1)n_0^2 K_0^3 \quad (\text{A5})$$

$$B_{02} = -\frac{1}{8} \frac{(n_0 - 1)^3 n_0}{(n_0 + 2)} K_0^3 \quad (\text{A6})$$

$$B_{03} = \frac{1}{2}(n_0 - 1)K_2^3 \quad (\text{A7})$$

$$B_{04} = \frac{1}{8} \frac{(n_0 - 1)(n_0 + 2)^2}{n_0} K_0 K_1^2 \quad (\text{A8})$$

$$B_{05} = -\frac{1}{2} \frac{(n_0 + 1)(n_0 - 1)^2}{n_0} K_1 K_0^2 \quad (\text{A9})$$

$$B_{06} = \frac{1}{2} \frac{(n_0 - 1)^3 (n_0 + 1)^2}{n_0 (n_0 + 2)} K_0^3 \quad (\text{A10})$$

As we will see later, it is convenient to associate the above six quantities to the components of a vector B_{0i} . Assuming the double index summation convention, we can write:

$$B_0 = B_{0i} U^i, \quad (\text{A11})$$

if we define U^i as the unit vector (i.e.; the six components equal 1).

The expansion up to second order makes us to define the following quantities:

$$g_{10} \equiv \frac{1}{n_0} D \quad (\text{A12})$$

$$g_{20} \equiv \frac{1}{2n_0} \frac{D}{\omega_0} \quad (\text{A13})$$

$$g_{11} \equiv \frac{1}{(n_0 - 1)} D \quad (\text{A14})$$

$$g_{21} \equiv \frac{1}{2(n_0 - 1)} \frac{D}{\omega_0} \quad (\text{A15})$$

$$g_{12} \equiv \frac{1}{(n_0 + 1)} D \quad (\text{A16})$$

$$g_{22} \equiv \frac{1}{2(n_0 + 1)} \frac{D}{\omega_0} \quad (\text{A17})$$

$$g_{13} \equiv \frac{1}{(n_0 + 2)} D \quad (\text{A18})$$

$$g_{23} \equiv \frac{1}{2(n_0 + 2)} \frac{D}{\omega_0} \quad (\text{A19})$$

If we construct two new vectors G_1^i and G_2^i with the following components:

$$G_1^1 \equiv g_{11} + 2g_{10} \quad (\text{A20})$$

$$G_1^2 \equiv 3g_{11} + g_{10} - g_{13} \quad (\text{A21})$$

$$G_1^3 \equiv g_{11} \quad (\text{A22})$$

$$G_1^4 \equiv g_{11} + g_{13} - g_{10} \quad (\text{A23})$$

$$G_1^5 \equiv g_{12} + 2g_{11} - g_{10} \quad (\text{A24})$$

$$G_1^6 \equiv 3g_{11} + 2g_{12} - g_{10} - g_{13} \quad (\text{A25})$$

and:

$$G_2^1 \equiv g_{21} + g_{10}^2 + 2g_{20} + 2g_{11}g_{10} \quad (\text{A26})$$

$$G_2^2 \equiv g_{11}^2 + g_{21} + g_{20} + g_{13}^2 - g_{23} - g_{10}g_{13} + 3g_{11}g_{10} - 3g_{11}g_{13} \quad (\text{A27})$$

$$G_2^3 \equiv g_{21} \quad (\text{A28})$$

$$G_2^4 \equiv g_{21} + g_{23} + g_{11}g_{13} + g_{10}^2 - g_{20} + g_{11}g_{10} - g_{13}g_{10} \quad (\text{A29})$$

$$G_2^5 \equiv g_{22} + g_{11}^2 + 2g_{12}g_{11} + g_{10}^2 - g_{20} - g_{12}g_{10} - 2g_{11}g_{10} \quad (\text{A30})$$

$$G_2^6 \equiv 3g_{11}^2 + 3g_{21} + g_{12}^2 + 2g_{22} + 6g_{11}g_{12} + g_{10}^2 - g_{20} + g_{13}^2 - g_{23} + g_{10}g_{13} \\ - [3g_{11} + 2g_{12}][g_{10} + g_{13}], \quad (\text{A31})$$

it is possible to demonstrate that:

$$B_1 = B_{0i}G_1^i \tag{A32}$$

$$B_2 = B_{0i}G_2^i \tag{A33}$$

In brief, Eq. (A4), (A32), and (A33) describes the coefficients B_0 , B_1 , and B_2 as a function of the lens parameters.

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Figure captions

Fig. 1: (a) Coordinate system and (b) lens parameters.

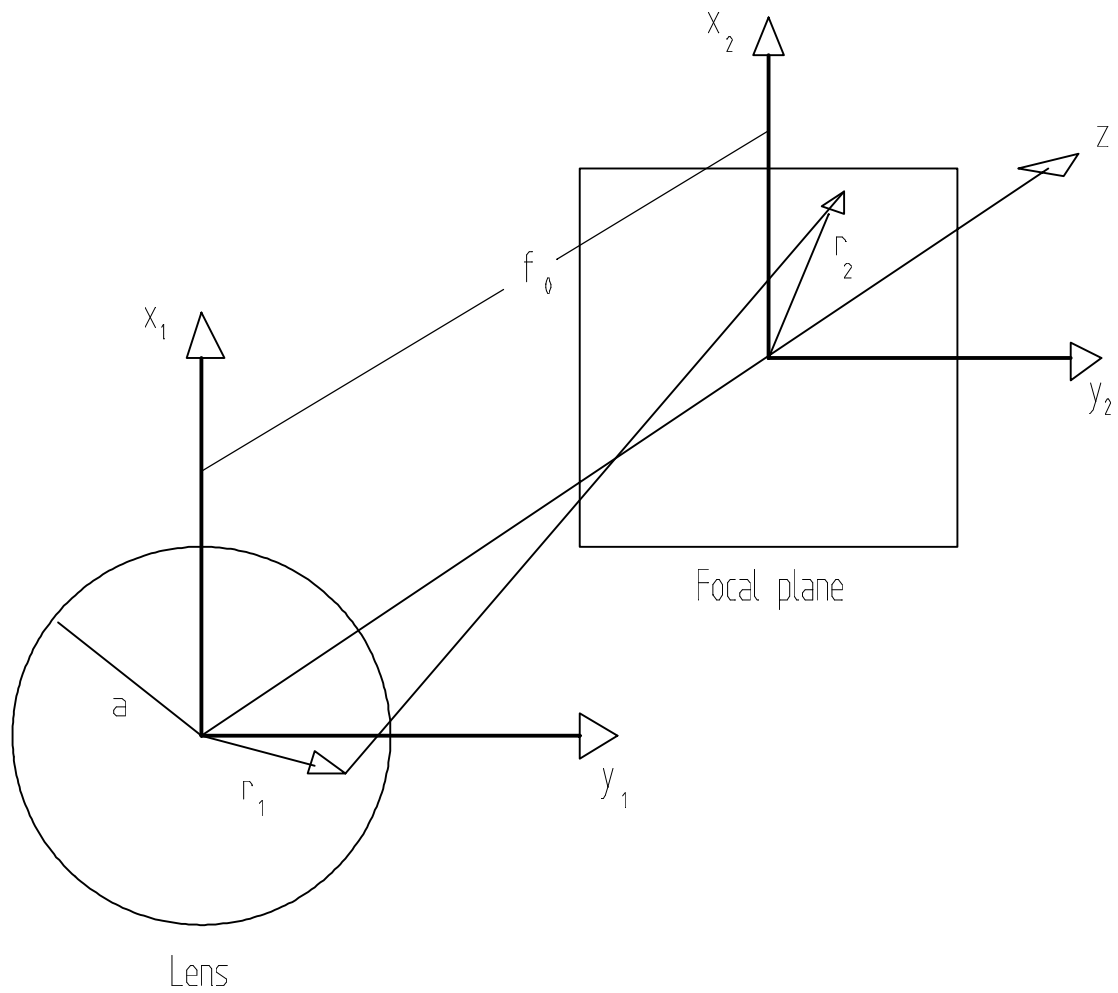
Fig. 2: Spherical aberration as a function of the radius of curvature R_1 and the deformation coefficient of one surface of the lens C . The arrow points in the direction of increasing C which takes equidistant values from -0.74 to -0.44 for each curve.

Fig. 3: Relative weight between the lineal chirp and the spherical aberration GVD-contribution as a function of the radius of curvature R_1 and the deformation coefficient of one surface of the lens C . The arrow points in the direction of increasing C which takes equidistant values from -0.74 to -0.44 for each curve.

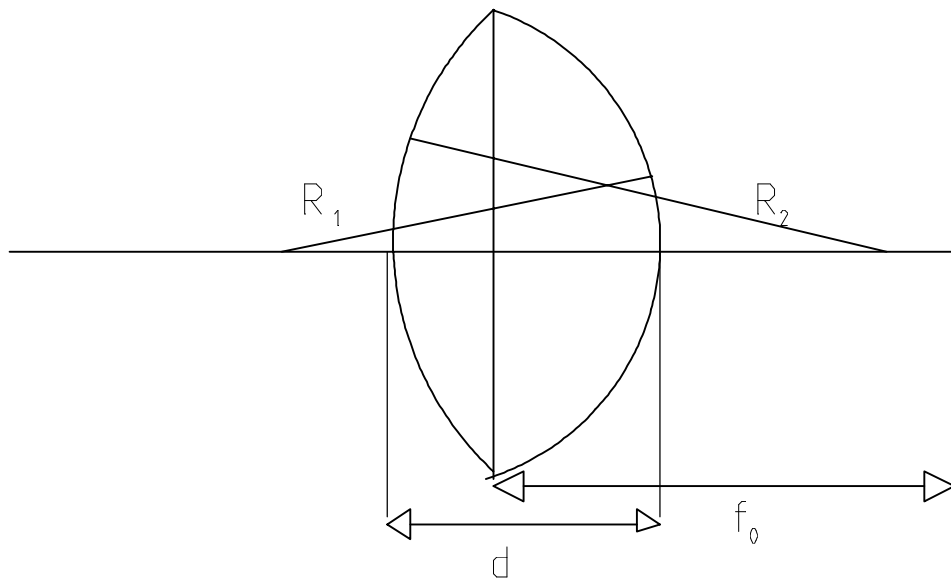
Fig. 4: Relative weight between the time delay between and spherical aberration PTD-contribution as a function of the radius of curvature R_1 and the deformation coefficient of one surface of the lens C . The arrow points in the direction of increasing C which takes equidistant values from -0.74 to -0.44 for each curve.

Fig. 5: Intensity distribution $I(v,t)$ in the focal plane. Free from aberrations.

Fig. 6: Intensity distribution $I(v,t)$ in the focal plane. Spherical aberration of λ_0 .



(a)



(b)

Fig # 1. (G. Mattei & M. Gil, "Spherical aberration in spatial...")

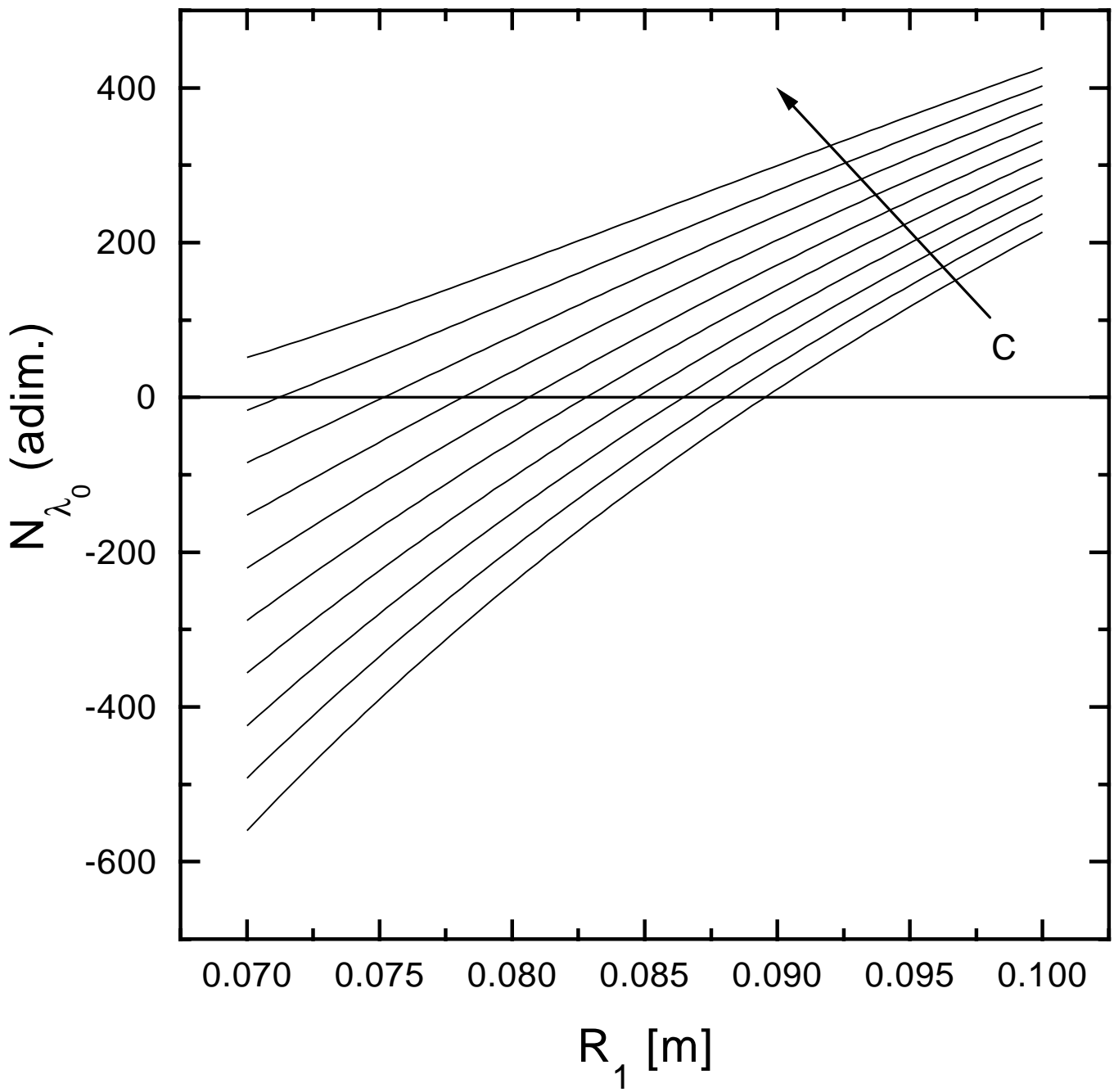


Fig. # 2 (G. Mattei & M. Gil, "Spherical aberration in spatial...")

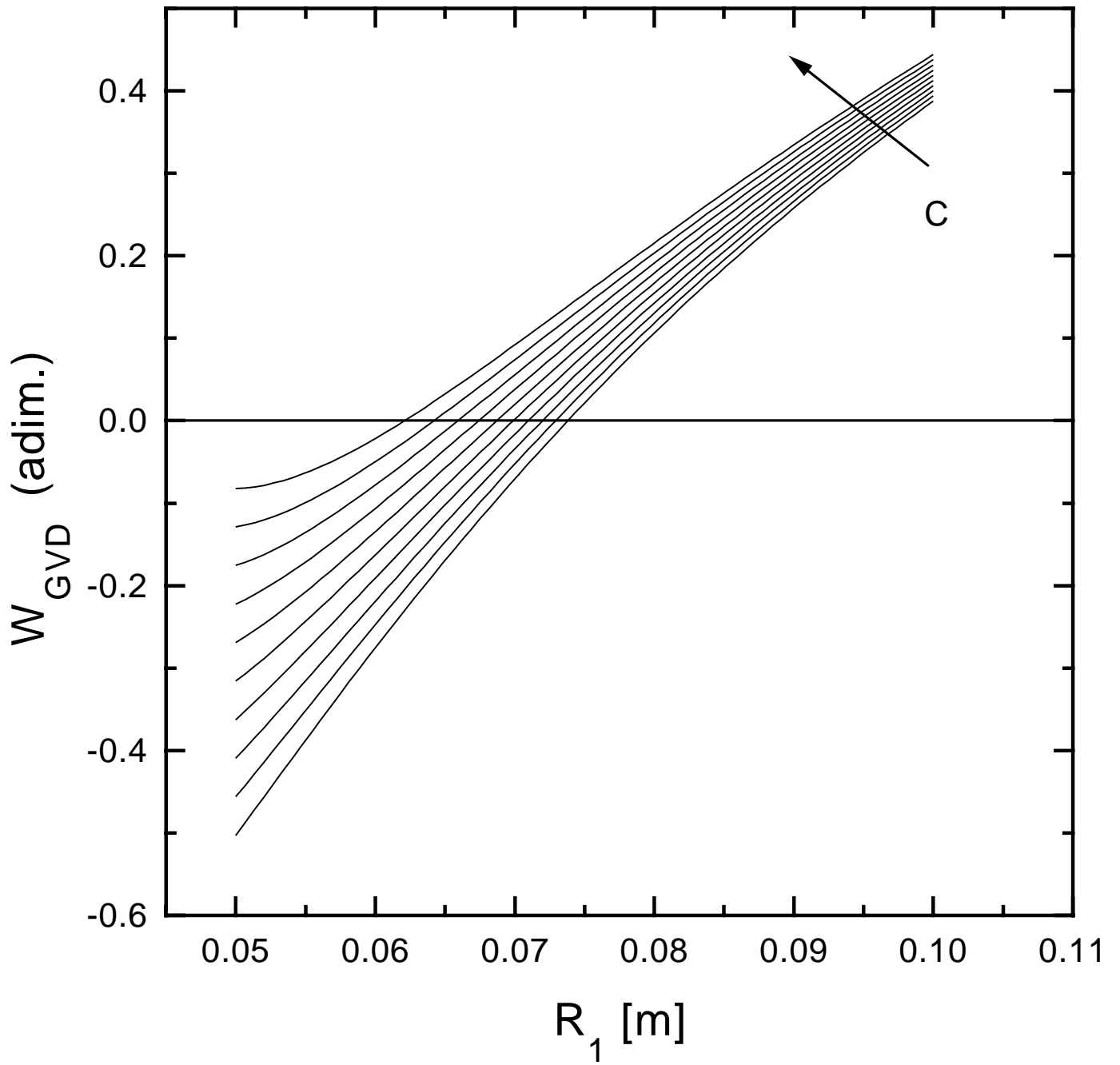


Fig # 3 (G. Mattei & M. Gil, "Spherical aberration in spatial...")

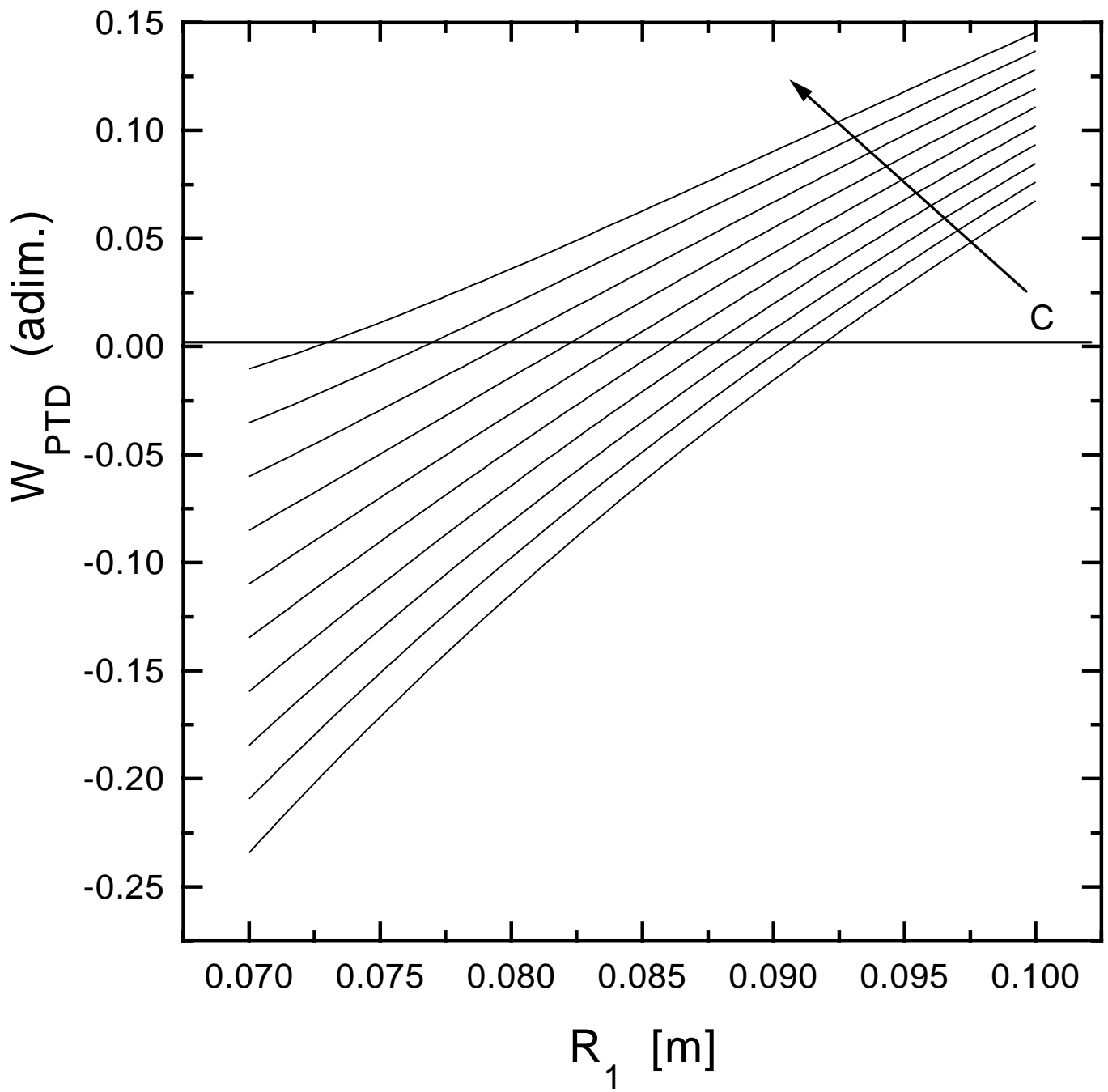


Fig. # 4 (G. Mattei & M. Gil, "Spherical aberration in spatial...")

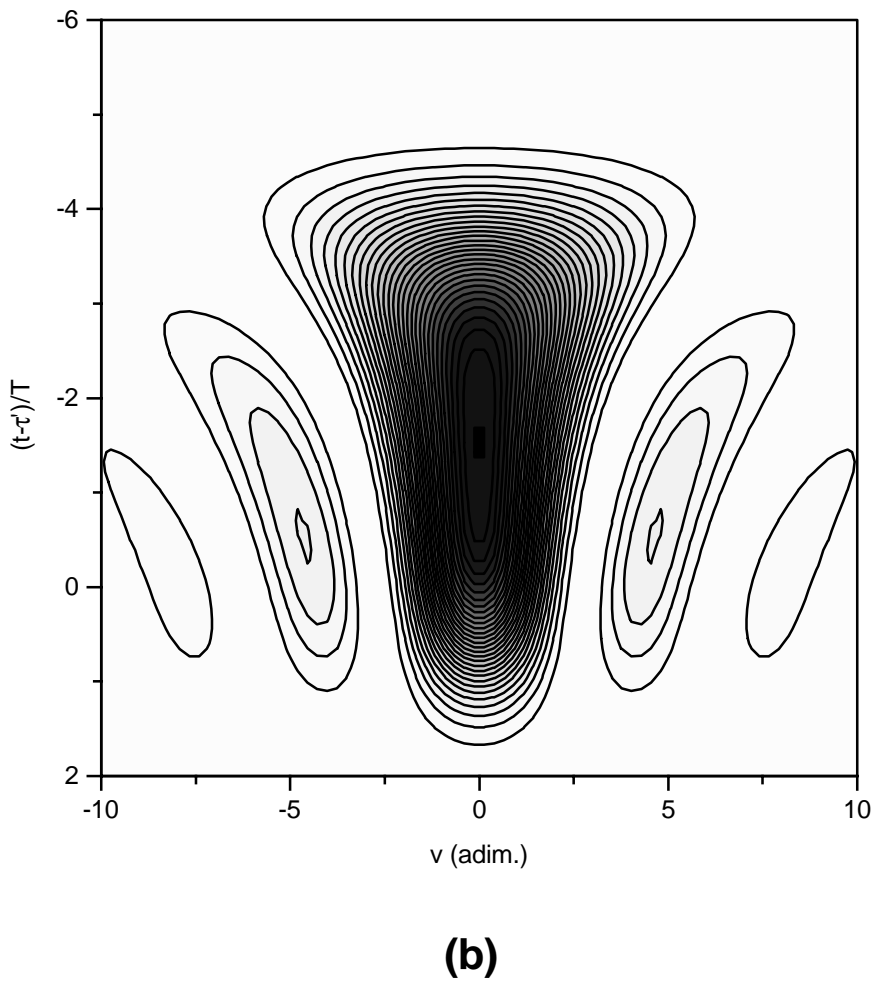
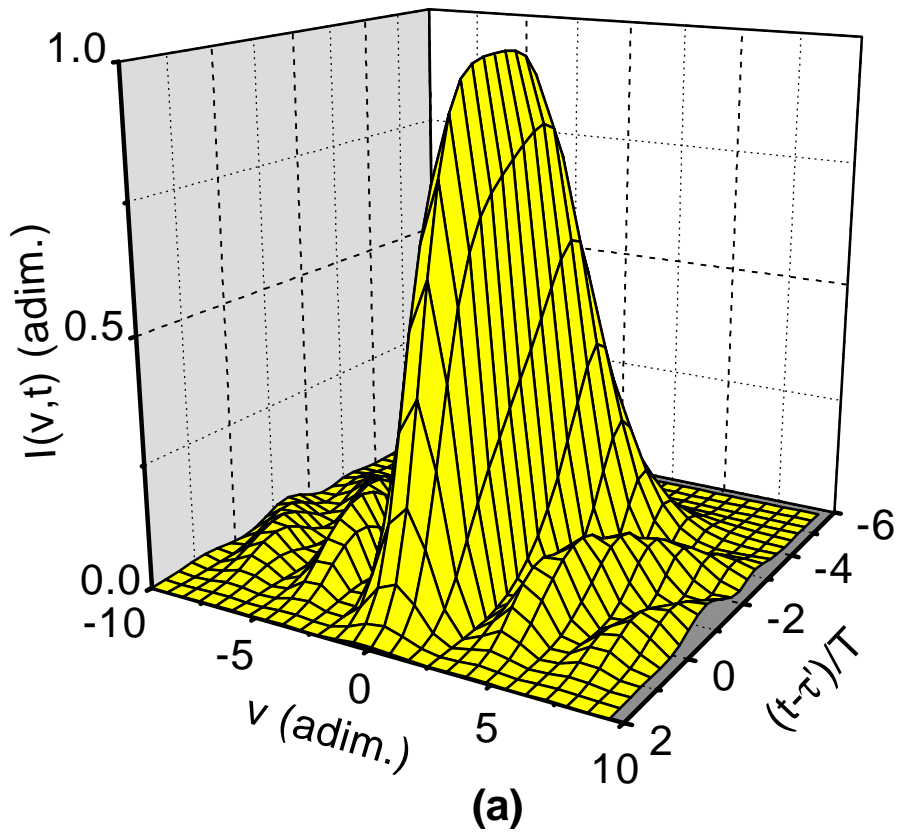


Fig. # 5, G. Mattei and M. Gil, "Spherical aberration spatial..."

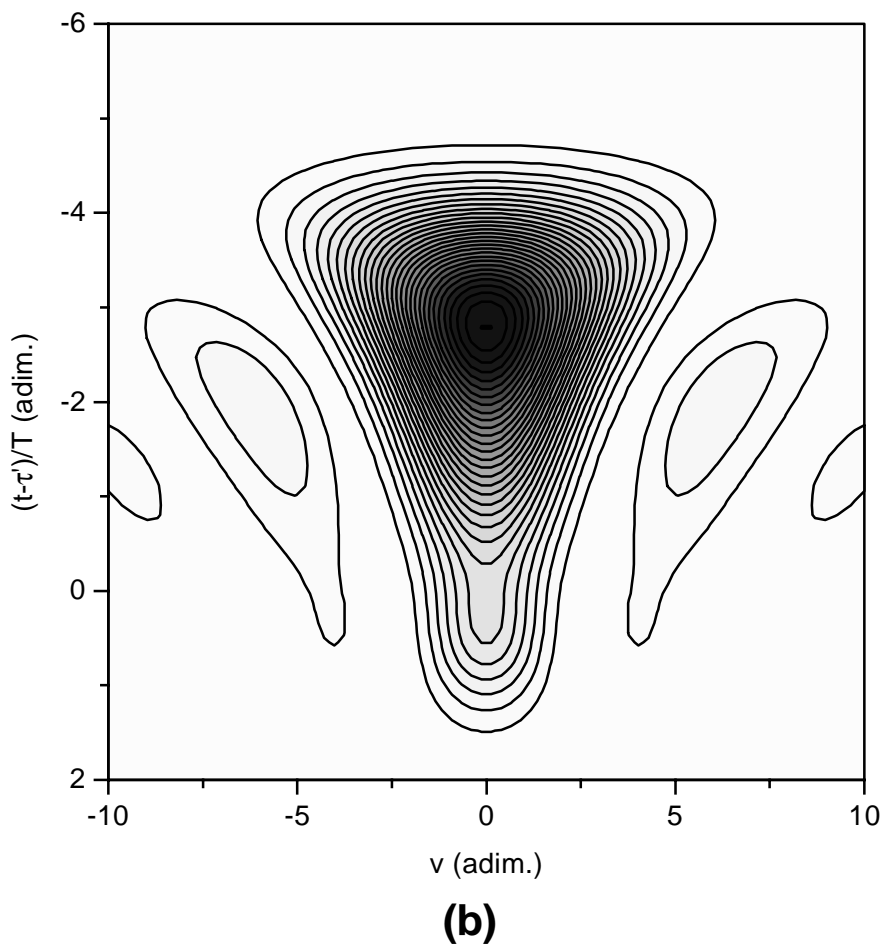
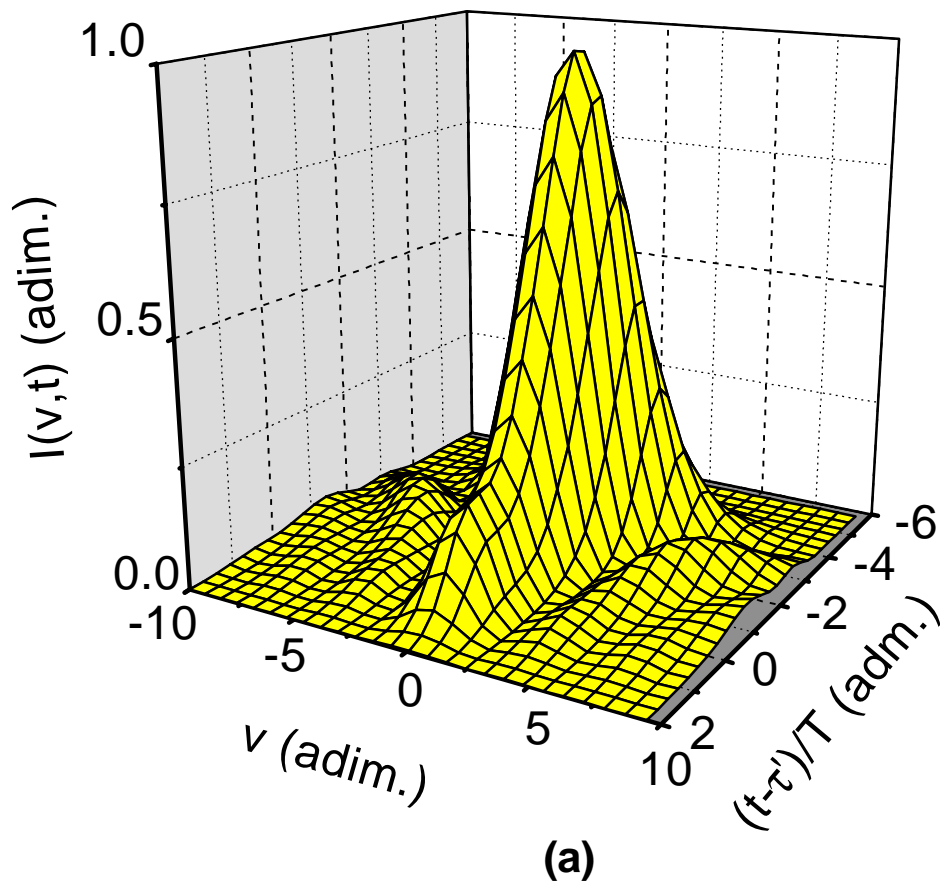


Fig. # 6, G. Mattei and M. Gil, "Spherical aberration of spatial..."