Chapter 0

The Many-Body Problem for Everybody

0.0 What the many-body problem is about

The many-body problem has attracted attention ever since the philosophers of old speculated over the question of how many angels could dance on the head of a pin. In the angel problem, as in all many-body problems, there are two essential ingredients. First of all, there have to be many bodies present—many angels, many electrons, many atoms, many molecules, many people, etc. Secondly, for there to be a problem, these bodies have to interact with each other. To see why this is so, suppose the bodies did not interact. Then each body would act independently of all the others, so that we could simply investigate the behaviour of each body separately. In other words, without interaction, instead of having one many-body problem, we would have many one-body problems. Thus, interactions are essential, and in fact the many-body problem may be defined as *the study of the effects of interaction between bodies on the behaviour of a many-body system.*

(It might be noted here, for the benefit of those interested in exact solutions, that there is an alternative formulation of the many-body problem, i.e., how many bodies are required before we have a problem? G. E. Brown points out that this can be answered by a look at history. In eighteenth-century Newtonian mechanics, the three-body problem was insoluble. With the birth of general relativity around 1910 and quantum electrodynamics in 1930, the two- and one-body problems became insoluble. And within modern quantum field theory, the problem of zero bodies (vacuum) is insoluble. So, if we are out after exact solutions, no bodies at all is already too many!)

The importance of the many-body problem derives from the fact that almost any real physical system one can think of is composed of a set of interacting particles. For example, nucleons in a nucleus interact by nuclear forces, electrons in an atom or metal interact by Coulomb forces, etc. Some examples are shown schematically in Fig. 0.1. Furthermore, it turns out that in the calculation of physical properties of such systems—for example, the energy levels of the atom, or magnetic susceptibility of the metal—interactions between particles play a very important role.

It should be clear from the variety of systems in Fig. 0.1 that the many-body problem is *not* a branch of solid state, or nuclear, or atomic physics, etc. It deals rather with *general* methods applicable to *all* many-body systems.

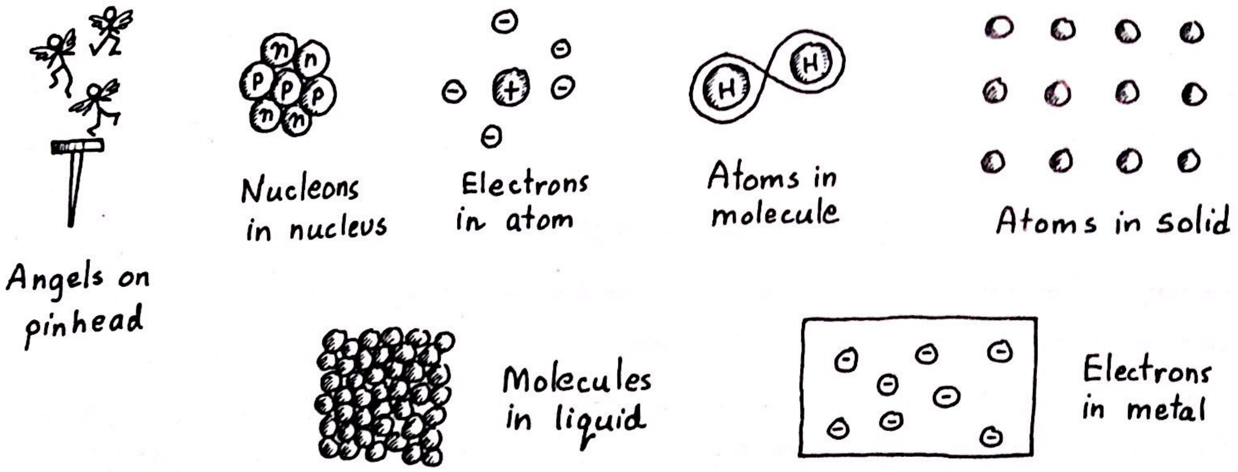

Fig. 0.1 *Some Many-body Systems*

The many-body problem is an extraordinarily difficult one because of the incredibly intricate motions of the particles in an interacting system. In Fig. 0.2 we contrast the simple behaviour of non-interacting particles with the complicated behaviour of interacting ones. Because of the complexity of the many-body problem, not much progress was made with it for a long time. In fact one of the preferred methods for solving the problem was simply to ignore it, i.e., pretend there were no interactions present. (Surprisingly enough, in some cases this 'method' produced good results anyway, and one of the great mysteries was how this could be possible!)

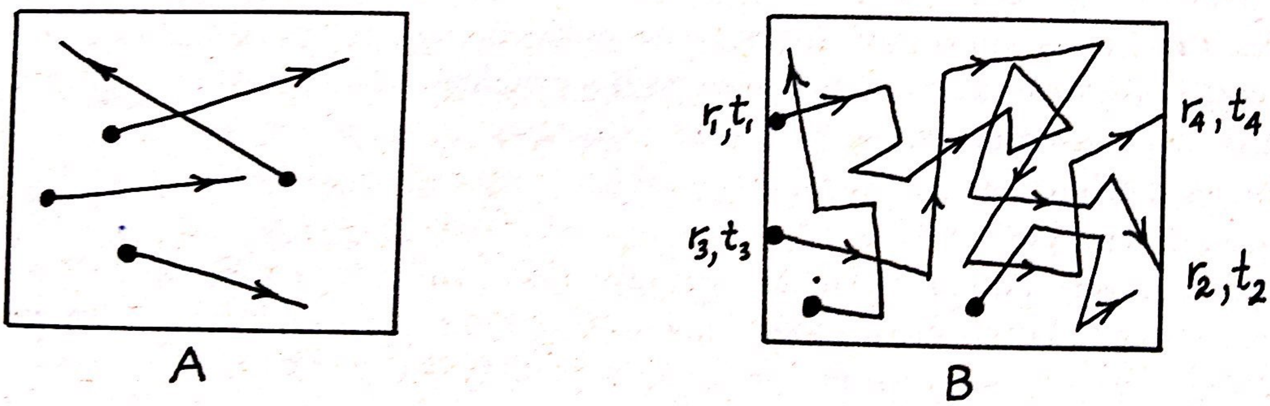

Fig. 0.2 A. *Non-interacting Particles*
B. *Interacting Particles*

Another of the early approaches to the problem, and one which is still used extensively today is the *canonical transformation* technique, described in appendix $\mathscr{A}$. This involves transforming the basic equations of the many-body system to a new set of coordinates in which the interaction term becomes small. Although considerable success has been achieved with this technique, it is not as systematic as one would like, and this sometimes makes it difficult to apply. It was this lack of a systematic method which kept the many-body field in its cradle well up into the 1950s.

The situation changed radically in 1956–7. In a series of pioneering papers, it was shown that the methods of *quantum field theory*, already famous for its success in elementary particle physics, provided a powerful, unified way of attacking the many-body problem. The new key opened many doors, and in rapid succession the idea was applied to nuclei, electrons in metals, ferromagnets, atoms, superconductors, plasmas, molecules—virtually everything in sight.

From that time on, much of the most exciting and fundamental research into the nature of matter has been based on the quantum field theory method. One of the things emerging from this research is a new simple picture of matter in which systems of interacting real particles are described in terms of approximately non-interacting fictitious bodies called 'quasi particles' and 'collective excitations'. Another thing is new results for calculated physical quantities which are in excellent agreement with experiment—for example, energy levels of light atoms, binding energy of nuclear matter, Fermi energy and effective electron mass in a variety of metals.

In this introductory chapter, we will give a physical picture of quasi particles and collective excitations. Then in the next chapter we show qualitatively how to describe quasi particles and calculate their properties by means of the quantum field theoretical technique known as the method of *Feynman diagrams*.