

When order matters: Last-come first-served effect in sequential arithmetic operations

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Cognitive psychologists have relied on dual-task interference experiments to understand the low-capacity and serial nature of conscious mental operations. Two widely studied paradigms, the Attentional Blink (AB) and the Psychological Refractory Period (PRP) have demonstrated a first-come first-served policy; processing a stimulus either impedes conscious access (AB) or postpones treatment (PRP) of a concurrent stimulus. Here we explored the transition from dual-task paradigms to multi-step human cognition. We studied the relative weight of individual addends in a sequential arithmetic task, where number notation (symbolic/non-symbolic) and presentation speed were independently manipulated. For slow presentation and symbolic notation, the decision relied almost equally on all addends, whereas for fast or non-symbolic notation, the decision relied almost exclusively on the last item reflecting a last-come first-served policy. We suggest that streams of stimuli may be chunked in events in which the last stimuli may override previous items from sensory buffers.

Keywords: Sequential operations; arithmetic; decision making; multi-step cognition.

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1. Introduction

Despite the massively parallel organization of the human brain (Felleman & Van Essen, 1991), the most distinctively human higher brain functions are strikingly serial (Calvin, 1987), as suggested by our limitations to attend to several objects at the same time, understand multiple spoken conversations, or respond simultaneously to different stimuli.

In order to determine which mental operations contribute to serial processing, cognitive psychologists have relied on dual-task interference experiments. Two experimental paradigms have been particularly useful in highlighting the processing limits of the human brain: the Psychological Refractory Period (PRP) and the Attentional Blink (AB). In PRP experiments, two stimuli requiring speeded responses are presented in close succession (Pashler, 1994). It is observed that as the delay between the two stimuli decreases, the response time (RT) to the second-task (T2) increases in proportion, while the RT to the first-task (T1) remains unaffected (Pashler, 1994). In the AB, the second stimulus is masked, and the first target impedes conscious access of the second within a window of about 500 ms (Raymond *et al.*, 1992).

In both the PRP and the AB, interference has been explained in terms of a passive queuing of T2 during certain processing stages of T1, establishing a processing bottleneck which operates on a first-come first-serve basis (Chun & Potter, 1995; Sigman & Dehaene, 2005; Kamienskowski & Sigman, 2008; Zylberberg *et al.*, 2009, 2010; Kamienskowski *et al.*, 2011). Neuronal correlates of delayed processing and refractory periods after task-relevant stimuli have been found through EEG, MEG and time-resolved fMRI, and even in the fine-grained analysis of the pupillary response (Sigman & Dehaene, 2008; Clearwater *et al.*, 2009; Marti, Sigman & Dehaene, 2011; Zylberberg, Oliva & Sigman, 2012). However, robust departures from strict seriality have been repeatedly found. In the AB, the blink vanishes when participants report all of the items in a sequence (Nieuwenstein, 2006), and is strongly reduced when the targets can be perceived as part of the same object (Raymond, 2003) or goal (Ferlazzo *et al.*, 2007). In the PRP, RT1 can be affected by the compatibility with the second response (Hommel, 1998; Logan & Schulkind, 2000), and the decision process for T2 can proceed largely in parallel with T1 (Zylberberg *et al.*, 2012). In a task that required chaining two arithmetic operations, the second operation starts before the completion of the first task (Sackur & Dehaene, 2009), questioning the validity of extrapolating simple models derived from dual-task paradigms to multi-step human cognition.

Here we study how successive arithmetic operations are scheduled on a task where the same operation has to be executed repeatedly on sequential stimuli. We explore the contribution (the relative weight) of each operation of the sequence to the decision. Task parameters were varied from values for which the choice was made according to rule-based and precise calculation, characteristic of conscious thought (Anderson & Lebiere, 1998) to very fast or noisy presentations for which the

low-capacity of working memory becomes determinant (Luck & Vogel, 1997). We show that, under the last circumstances, the bulk of the decision relies exclusively on the last item of the decision reflecting a last-come first-served policy.

2. Methods

2.1. *Experimental design*

Each trial started with a fixation point presented at the center of a computer screen for 500 ms, followed by four displays presented successively with a time delay given by the Stimulus Onset Asynchrony (SOA) (Fig. 1(a)). Each display contained two numbers, presented right and left of the fixation point (participants sat at approximately 1 m away from the monitor; letters were centered at 2.2° in the horizontal axis from the fixation point; numbers were presented in Arial font size 60). Numbers were presented either as Arabic Digits (symbolic condition) or a set of dots (non-symbolic condition). The left and right numbers of the display $i \{i = 1, 2, 3, 4\}$ are referred respectively as x_i and y_i . We refer to each $z_i = y_i - x_i$ as an addend, and the accumulated sum in each trial as: $d = \sum_i z_i$. Participants were asked to report whether the numbers in the right summed more or less than those on the left (i.e., the sign of d) (Fig. 1(a)). Responses were made with the left ($d < 0$) and right ($d > 0$) arrow keys of a computer keyboard, and a blue (red) circle indicated whether the response was correct (incorrect) after each trial.

Sequences were generated randomly with the following constraints: numbers ranged from four to nine, the absolute value of d ranged between one and three and the same number could not occupy the same side on successive displays. The SOA for each trial was selected pseudo-randomly from a list of values [100, 125, 150, 200, 250, 300, 350, 400, 450, 500, 600, 700] ms. The numbers were removed from the screen after 150 ms or when the next display was presented. The experiment was performed by four participants (three males, 27–30 years old). Sessions were divided in eight blocks of 36 trials, and participants performed two sessions per day for 10 days.

Control experiments differed from the main experiment in the following manner:

- Experiment 2 (Masking of the Last Display). In 50% of the trials the last display was followed by a mask, composed of two randomly selected letters, presented at the same position and font as the numbers (Fig. 3(a)). The time delay between the onset of the last addend and the onset of the mask was 300 ms for the shortest SOA and 350 ms for all other SOA values. We only studied the symbolic condition and short SOA values (SOA = [100, 150, 250, 350] ms).
- Experiment 3 (Sign of the First Addend). In this experiment participants had to respond based only on the first addend, i.e., all displays after the first had to be treated as distractors. We only studied the symbolic condition and a smaller set of SOA values (SOA = [100, 150, 250, 350, 450, 550, 700] ms).

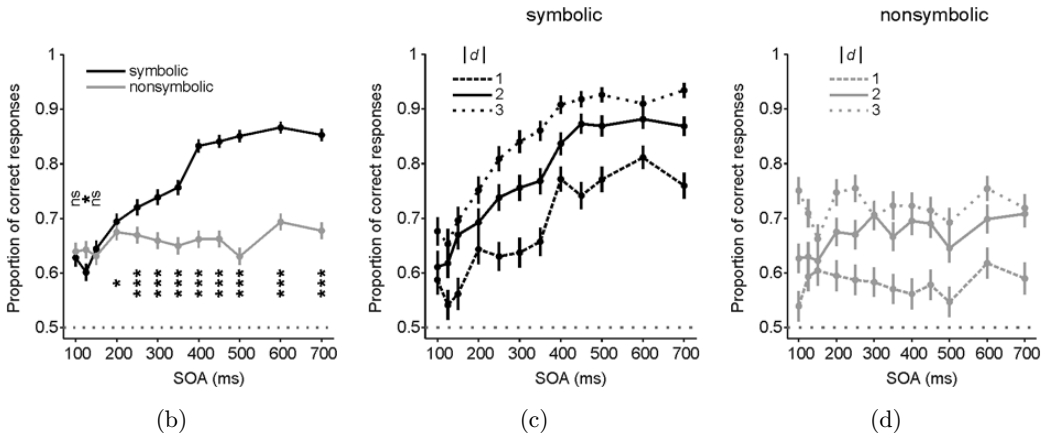
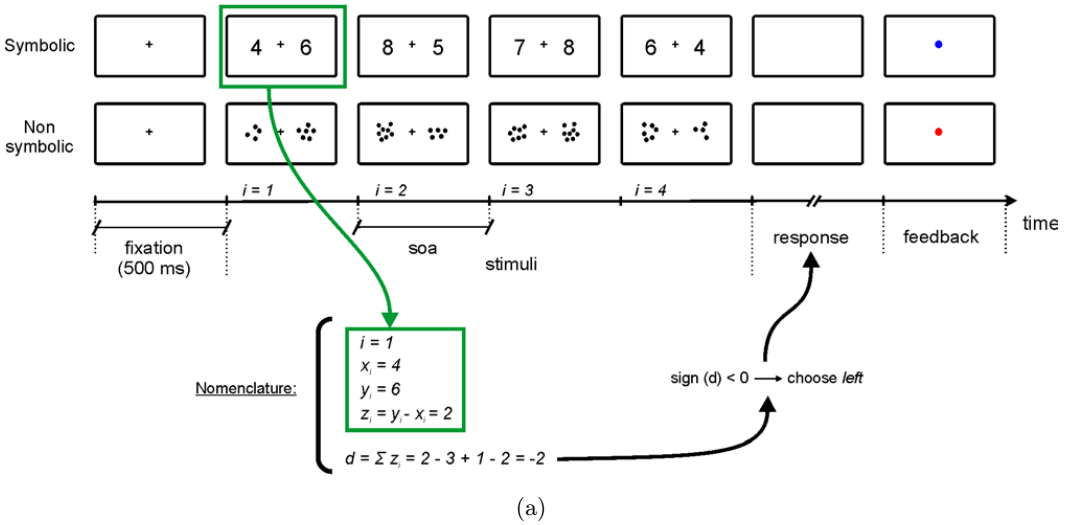


Fig. 1. Effects of notation, speed and distance on sequential arithmetic operations. (a) Experimental Design for the main experiment. (b) Effects of the SOA on average performance, for symbolic (black) and non-symbolic (grey) conditions. Significance levels were calculated with a paired *t*-test (DF = 39); ns: $p > 0.05$, *: $p < 0.05$, ***: $p < 10^{-4}$. (c) Task performance as a function of SOA for the three possible absolute values of d_i for trials where the numbers were presented as Arabic Digits. (d) Same as (c), for the non-symbolic condition. Error bars indicate the standard error of the mean.

- Experiment 4 (Cueing of the First Display). An auditory cue (pure tone of 660 Hz plus 15% of white noise) lasting 150 ms was presented 300 ms before the first display. Only the symbolic condition was explored, and SOA values were $SOA = [100, 150, 250, 350, 450, 550, 700]$ ms. The last display was masked as in experiment 2.
- Experiment 5 (Task Temporal Context). In this experiment all numeric quantities were presented in symbolic notation, exploring only large SOA values ($[550, 700, 1000, 1250]$ ms).

Five participants (3 males, age 22–30) performed experiments 2–5. Two of them also participated in the main experiment described above. Before conducting experiments 2–5, participants performed six sessions (420 trials each) of the main experiment (only for the symbolic condition, with a smaller set of SOA values (SOA = [100, 150, 250, 350, 450, 550, 700] ms)). Following these six sessions, subjects performed experiments 2 to 5 in random order. Each session contained 8, 7, 7, and 4 blocks of 60 trials each, for experiments 2 to 5, respectively. All participants were college students or graduates, with normal or corrected-to-normal vision.

2.2. Data analysis

Behavioral responses were analyzed with a logistic regression model (Hosmer & Lemeshow, 2000). The response (dependent) variable was a binary variable indicating if the decision made on each trial was correct or incorrect. The explanatory (independent) variables were:

- SOA.
- Addend in each position; a separate variable was included for each SOA.
- Binary variable which took a value of 1 if the largest addend had the same sign as the total sum and a value of 0 otherwise; this variable can detect the strategy of focusing only on the largest addend of the sequence.
- Binary variable which took a value of 1 if one or more displays in the trial contained addends with value zero ($z_i = 0$ for any i , which potentially could be filtered out), and a value of 0 otherwise.
- Session number.
- Participant, as a categorical variable.
- Side of the required response, as a categorical variable (to account for side biases).

The main aim of this experiment was to understand the contribution of each addend to the decision as a function of SOA. The goodness-of-fit of the model was assessed with a Hosmer–Lemeshow Test (Hosmer & Lemeshow, 2000).

3. Results

3.1. Effects of SOA, distance and notation on accuracy in multi-step arithmetic

We explored the effect of three factors that were independently manipulated: Notation (symbolic or non-symbolic number presentation), SOA, and Distance (the total sum d (see methods) which took values between -3 and 3 excluding 0).

We first studied the effect of these factors (and their interactions) on task accuracy (Fig. 1(b)–1(d); Table 1, ANOVA on performance with SOA, Notation and Distance as main independent variable). We found that accuracy was higher for increasing SOA values, for digits than for dots, and for large numeric distances. The effect

Table 1. Results of an ANOVA on performance with SOA, NOTATION and ABSOLUTE DISTANCE as main independent variables, and SUBJECT * SESSION (4 subjects * 10 sessions each, resulting in *d.f.* = 39) as random variable.

| Factor | d.f. | F | P |
|---------------------|------|--------|--------------|
| SOA | 11 | 22.12 | $< 10^{-16}$ |
| NOTATION | 1 | 143.95 | $< 10^{-13}$ |
| DISTANCE | 2 | 219.76 | $< 10^{-16}$ |
| SOA * NOTATION | 11 | 18.65 | $< 10^{-16}$ |
| SOA * DISTANCE | 22 | 0.96 | 0.52 |
| DISTANCE * NOTATION | 2 | 0.21 | 0.81 |

of Distance was additive, not showing an interaction with the other factors (Fig. 1(c), 1(d), Table 1). We only observed a significant interaction between Notation and SOA (Fig. 1(b)). In the symbolic condition, performance increased with SOA from chance levels at the shortest SOA of 100 ms to above 80% accuracy (Fig. 1(b)). However, accuracy reached a plateau at an SOA of 400 ms (a pos-hoc Tuckey-test showed no significant difference ($p > 0.05$) for values in the range 400–700 ms) at a performance which was quite accurate but yet far from perfect. Below we present an experiment specifically designed to understand the origins of this plateau. In marked contrast, performance in the non-symbolic condition did not vary with SOA: a 7-folds increase in inter-stimulus interval made no significant difference on performance (Fig. 1(b), 1(d)).

3.2. Critical determinants of the decision

The less-than-perfect performance displayed by the participants even at the largest SOA values indicate that subjects relied on partial cues in the formation of the decision, accessing only a subset of the relevant information. In this section, we explore — through a logistic regression model — which variables were determinant for the formation of a decision. In formal and perfect calculation, changing the order of the addends is irrelevant for the final result since addition is commutative. The main objective of the regression analysis was to determine whether addends had equal or unequal weights in the final, approximate decision.

The logistic regression model assigns to each trial a continuous value of correct probability. Figure 2(a) and 2(b) show — for each SOA — the coefficients of the regression (β_i) which reflect the weight of each addend z_i on the categorical decision. For the non-symbolic task (Fig. 2(b)) the weights of the addends were independent of SOA, as we previously showed for overall performance. The contribution of each addend in the sequence showed a clear *last-come first-served* effect. The last addend virtually determined the decision and the previous addends had progressively decreasing weight. In the symbolic task (Fig. 2(a)), the coefficients of the regression varied with SOA. For short SOA values, as for the non-symbolic task, the decision

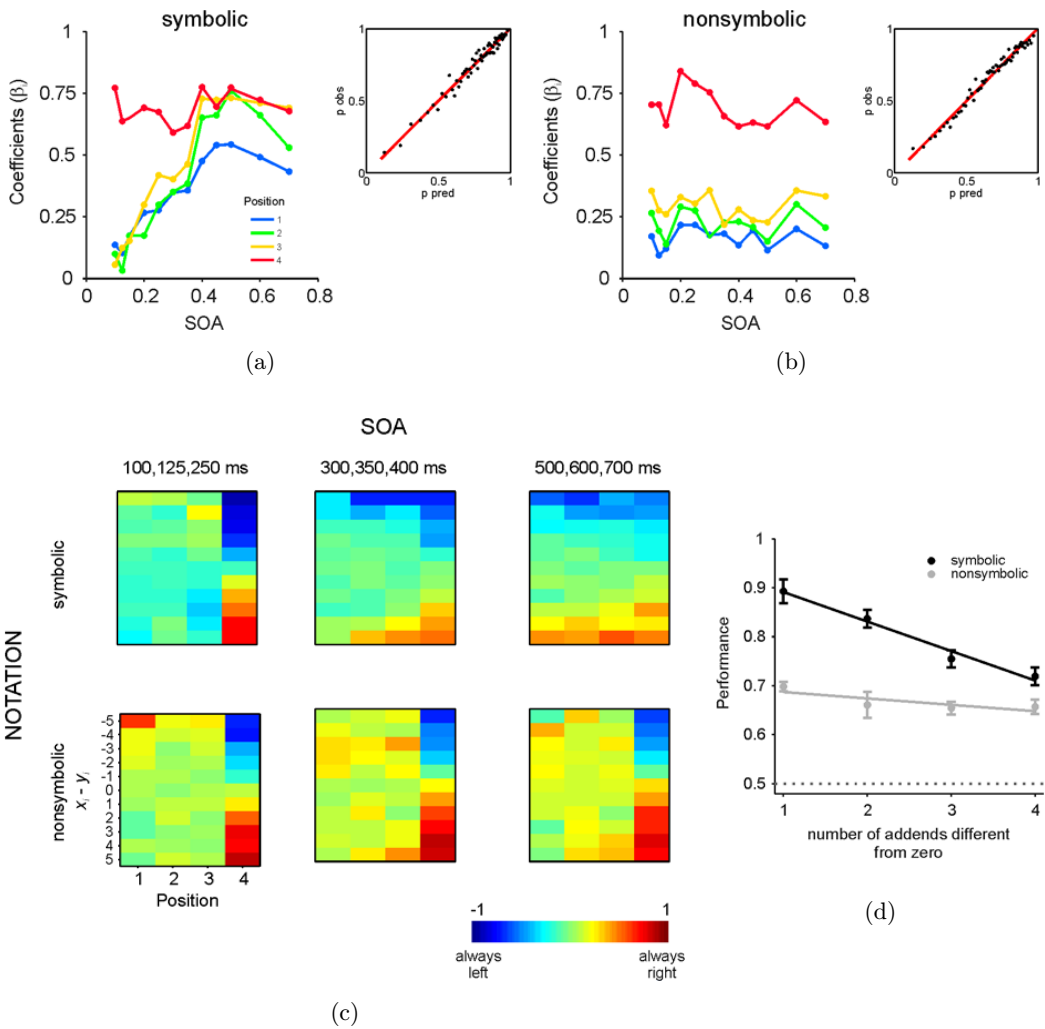


Fig. 2. Choice contribution of individual addends. (a)–(b) Contribution of each addend (β_i) to the decision, as a function of SOA. Coefficients β_i were obtained through a logistic regression model. Insets: Comparison of correct probability predicted by the logistic model with the one observed experimentally. (c) Average distribution of responses (normalized to the range $(-1, 1)$) for every combination of addend value (z_i), position, notation, and SOA category. By convention, $+1(-1)$ stands for a rightwards (leftwards) response. (d) Task performance as a function of the number of addends with values different from zero. Error bars indicate the standard error of the mean.

relies almost exclusively on the last addend. As the SOA increases, the previous addends have progressive weight in the decision process. For SOA values larger than 400 ms, the coefficients of the last three addends were identical and only the first addend had a comparatively lower weight in the decision. To assess the overall goodness of the model, we first sorted trials according to the correct probability assigned by the model and parsed this distribution in 120 percentile groups. We estimated for each group, the mean probability of correct response assigned by the model (Figs. 2(a), 2(b), insets, horizontal axis) and the proportion of correct

responses measured in the experiment (Figs. 2(a), 2(b), insets, vertical axis). As observed in the figure, the model fitted the experimental data very accurately (the correlation between the measured and predicted performance for these groups was $R^2 > 0.97$ for both symbolic and non-symbolic conditions). To formalize the goodness of this fit, we conducted a Hosmer–Lemeshow test, which tests the hypothesis that the distributions of predicted and measured performance are different. We performed independent tests for symbolic and non-symbolic notations with 120 groups each. The χ^2 statistics from the Hosmer–Lemeshow test showed that there was no significant difference between measured and predicted distributions for symbolic ($\chi^2 = 97.40, p = 0.92$) and non-symbolic ($\chi^2 = 120.2, p = 0.43$) conditions.

Another manner of assessing the relative weight of each addend in the formation of the decision is to calculate the probability (relative frequency) of responding right (1) or left (−1) as a function of the value of the addend in each position. This analysis led to essentially the same results than the logistic regression analysis (Fig. 2(c)). It is also evident that even very high values ($z_i = x_i - y_i \sim \pm 5$) of the first addends ($i < 4$) were ignored in the non-symbolic condition and in the symbolic condition for short SOA values (bottom row and first column of the upper row, Fig. 2(c)).

To further understand the determinants of task performance, we studied the effects of learning and side biases on accuracy. Learning effects were evaluated with logistic regression with *performance* as dependent variable and *session number* and *participant* as independent variables. We started with a “full” model which included all sessions from one to twenty, and tested the regression coefficient for *session number* calculating *p*-values based on the standard error of the coefficients (Hosmer & Lemeshow, 2000). Then, we evaluated models with progressively less sessions, iteratively removing the trials belonging to the earliest session of the previous iteration and recalculating the regression. We found that *session number* made a significant contribution ($p < 0.05$) to performance, only when the first four sessions were included in the regression, and made no significant contribution for the remaining sessions neither in the symbolic nor the non-symbolic conditions. Furthermore, the side of the required response had a significant effect on task performance, which was different for digits and dots. For the symbolic condition, performance was significantly higher when the required response was on the left side ($p < 5.10^{-8}$, *t* statistic for the *side* coefficient of the logistic regression model, see Sec. 2), while performance for the non-symbolic condition was higher when the required response was on the right ($p < 5.10^{-8}$). The cause of this asymmetry, which was consistently observed at every SOA, could be investigated in future studies; here, we focused on the relative contribution of individual addends in the sequence to the decision as a function of notation and SOA.

3.3. *Perceptual masking cannot explain the last-come first-served effect*

The previous analysis showed that the first addends of the sequence were virtually ignored. This is reminiscent of perceptual backward masking, a common

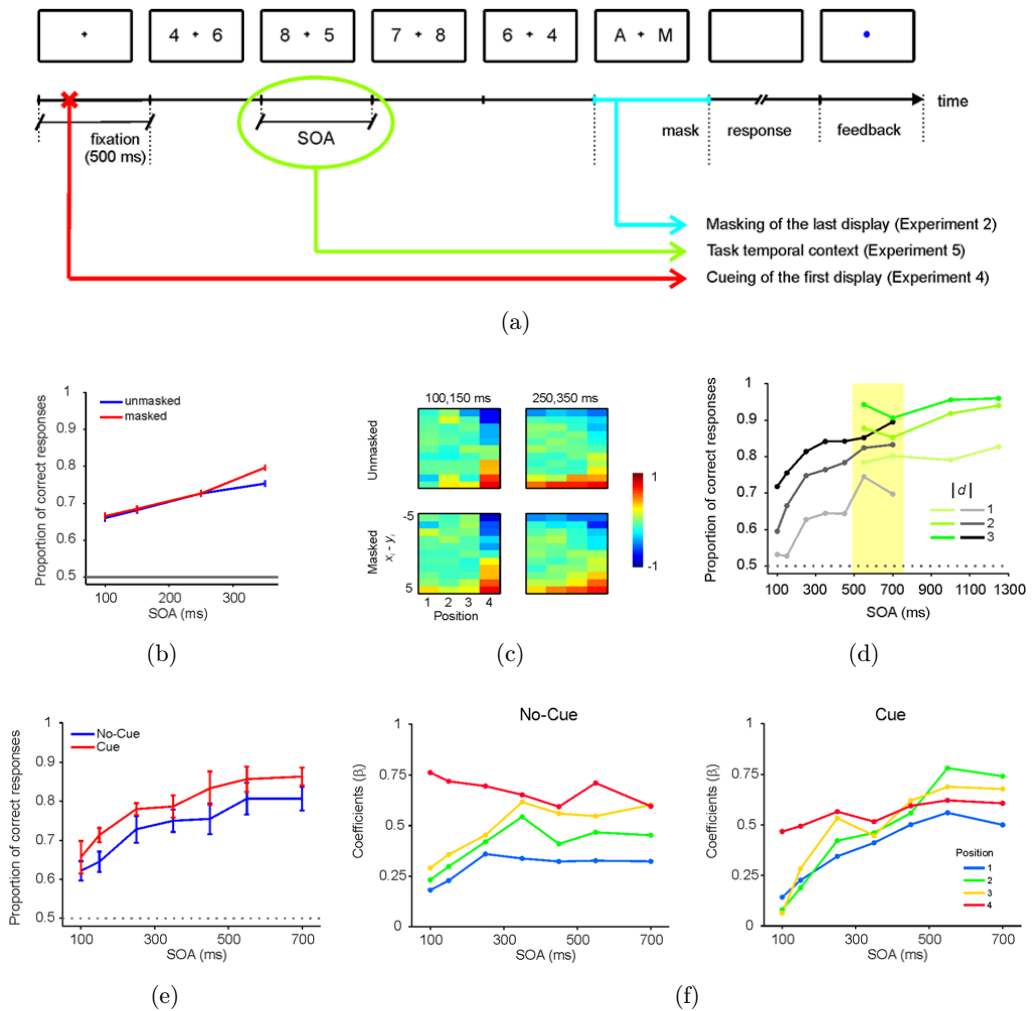


Fig. 3. Sensitivity to perceptual masking, attentional cueing and temporal context. (a) Experimental Design for experiments 2, 4 and 5. (b) Experiment 2: Performance as function of SOA and mask condition. (c) Probability of responding right (1) or left (-1) as a function of addend value (z_i), position, mask condition and SOA group. (D) Experiment 5: Performance as a function of distance and SOA, for two experiments differing only in the range of possible SOA values. The yellow box highlights the SOA values present in both experiments. (E) Experiment 4: Performance as a function of SOA for Cue and No-Cue conditions. The presence of a cue enhances performance without interacting with SOA. (f) Contribution of each addend (β_i) to the decision — estimated by a linear regression analysis — for both Cue (right) and No-Cue (left) conditions and as a function of SOA.

manipulation in psychophysical experiments where a brief mask presented after a stimulus renders it invisible. Even though the temporal scale involved in this experiment (from 100 to 700 ms) is larger than in perceptual masking, control experiments were conducted to ensure that visual masking did not account for the over-weighting of the last addend (Fig. 3(a)).

In experiment 2, the last addend was followed by a mask. Performance after masking the fourth addend was not significantly different from the no-mask condition

(71.98% and 70.62% correct, ANOVA with factors of mask and SOA, $p > 0.29$) (Fig. 3(b)). Also, the correspondence between the value of each addend and the final decision were virtually identical to the unmasked condition (Fig. 3(c)).

In experiment 3 participants were simply asked to respond which of the two digits of the first display was larger. If subsequent displays masked previous ones, then participants should be unable to perform this task at high levels of accuracy. We found performance to be very accurate, with a mean average performance of 96.05%, and high performance levels even at the shortest SOA (85.67% at SOA = 100 ms and 93.33% at SOA = 150 ms). Thus, from the previous experiments, we conclude that perceptual masking did not have a major impact in the overweight of the last addend.

Our results contrast with those of dual-task experiments like the PRP and the AB where later but not earlier stimuli are affected by interference. To investigate whether we could force participants to a first-come first-serve policy, we run a variant of the task where an auditory cue was presented 300 ms before the onset of the first array (Experiment 4, see Fig. 3(a)) — a time interval for which auditory cues can facilitate visual processing (Schmitt, Postma & De Haan, 2000). Cueing resulted in a significant increase in performance (ANOVA with Cue/No-Cue and SOA as factor and subjects as random effect; Cue/No-Cue effect: $p < 10^{-4}$, $F = 307.69$; Fig. 3(e)). This effect was found to be additive over all the SOA range (Cue/No-Cue \times SOA interaction: $p = 0.98$, $F = 0.18$; Fig. 3(e)). Analysis of the contribution of each addend to the decision showed that the cue effectively enhanced the response contribution of the first addend (Hosmer–Lemeshow with 12 groups for the logistic regression model in the cued (no-cued) condition: $\chi^2 = 10.15$ (3.94), $p = 0.43$ (0.95)). Still, the main effect was not reversed (Fig. 3(f)): the last addend was still the most determinant element in the decision.

3.4. *Accessing and filtering partial elements of the task*

We showed that presenting a pair of letters after the last addend had no effect on the overweighting of later addends. Here we explore whether interference is reduced if the two numbers in the display are equal. We reasoned that if subjects could filter out algebraic equality — which would only be expected for the symbolic condition — then sequences in which some addends would be zero (when the two presented numbers were identical, i.e., $x_i = y_i$ and thus $z_i = 0$) should have better performance and the contribution of the previous addend immediately before the one with zero sum should be increased.

We thus analyzed accuracy and weights of the addends from the main experiment according to the number of non-zero addends. We found that performance was a decreasing function of the number of addends different from 0 ($\sum_i z_i \neq 0$). This filtering effect of algebraic equality was observed only in the symbolic condition, an effect which may result from the difficulty of evaluating numeric equality in the non-symbolic notation (Fig. 2(d)).

Performance was unchanged, even for short SOA values, when $z_4 = 0$ (performance for $\text{SOA} < 450$ ms when $z_4 = 0$: $(67.0 \pm 0.1)\%$ and when $z_4 \neq 0$, $(67.9 \pm 0.1)\%$; paired t -test: $t = -0.98$, $\text{DF} = 3$, $p = 0.40$). This was surprising because we had previously shown that for short SOA, the decision relied mainly on the last addend. How can performance be maintained when the last addend does not contribute useful information to the decision? According to the last-come last-served hypothesis one could expect that when $z_4 = 0$, the decision relies in the previous addend (z_3). We examined this hypothesis, measuring the coefficient of the regression for $z_3(\beta_3)$ depending on whether z_4 was equal or not equal to zero. We found that β_3 was significantly higher when $z_4 = 0$ ($\beta_3(z_4 = 0) = (0.31 \pm 0.07)$); the difference in regression coefficients between the two conditions is $([\beta_3(z_4 = 0) - \beta_3(z_4 \neq 0)]) = (0.11 \pm 0.05)$, z -value = 2.13 , $p < 0.05$; Hosmer–Lemeshow Test (120 groups): $\chi^2 = 118.20$, $p = 0.48$). Furthermore, if interference results mainly from a backwards effect then β_3 should be independent of whether the previous addend (z_2) was zero or non-zero. We also verified this prediction $([\beta_3(z_2 = 0) - \beta_3(z_2 \neq 0)]) = (-0.04 \pm 0.05)$, z -value = -0.72 , $p = 0.47$). Thus, the third (one before the last) item of the sequence becomes highly influential on the decision when the last element is not informative, but its weight is independent of the information value of prior elements of the sequence.

3.5. Cognitive architecture and task context

We previously showed that performance in the symbolic task reached an asymptote near 85% accuracy for SOA values between 400 ms and 700 ms. This result is intriguing since addends are small digits and thus it is expected that at SOA values close to 700 ms performance should be better than for SOA of 400 ms.

We hypothesized that participants may commit to a strategy of approximate calculation, sufficient for the whole range of SOA values used in the experiment, and that an exact, algebraic strategy could be obtained if the context of SOA values were changed. To examine this hypothesis, we conducted an experiment (Experiment 5) where the SOA values were [550, 700, 1000, 1250] ms. Accuracy for the two SOA values present in this and in the main experiment (550 and 700 ms, yellow shade in Fig. 3(d)) was significantly higher when presented in a large-SOA context, confirming our hypothesis that task context had an effect on task performance and might be partially responsible for the early and low asymptote achieved in the main experiment (an ANOVA with factors of SOA and Context for the two shared SOA values revealed a significant effect of Context ($p < 0.001$) but no effect of SOA ($p > 0.5$) or their interaction ($p > 0.5$)).

4. Discussion

We explored human ability to perform rapid series of arithmetic operations over numeric quantities presented in symbolic and non-symbolic notations and for varying speeds. Increasing the time between successive operations resulted in a change in accuracy from almost chance to almost perfect for the symbolic condition, while no

effect of SOA was observed in the non-symbolic condition. For fast presentations or non-symbolic addition, the bulk of the decision relied almost exclusively on the last addend, reflecting a last-come first-served policy.

The last-come first-serve task schedule establishes a prediction which we verified by re-examining the experimental data: the addend in the third position should be determinant of the decision when the next (fourth) addend is equal to zero (i.e., when it is the last significant item of the decision) and its weight unaffected if the previous (second) addend was zero. This shows that the difference between two quantities can be extracted even at very fast tempos. We also showed that perceptual masking does not play a major role in the mild influence of early addends at short SOA values since: (a) performance is utterly unaffected by the presentation of a visual mask after the last display and (b) participants can accurately report whether the first addend was positive or negative even when stimuli were presented at a rate of 10 Hz. A consideration which must be taken into account is that, to assure that the total sum did not add to large numbers, each addend resulted from the difference between two numbers, presented on the left and right visual fields. In the future, other experiments in which additions involve a sole location in space will be studied to address whether the strict last-come first served strategy results from the inability of accessing information at different spatial locations.

The time between successive items had no influence on performance when quantities had to be estimated from sets of dots, and then the decision was mainly determined by the last addend in the sequence. People without basic mathematical training or a symbolic representation of numbers can still perform approximate arithmetic operations (Pica *et al.*, 2004), and therefore is not clear why SOA had no effect on the weight of early addends. Despite the capacity of humans and other animals to extract approximate numerical quantities from sets of items (Nieder & Dehaene, 2009), the error with which quantities are represented increase with the magnitude to be represented, and exact numerosity can be estimated from sets containing very few items, typically up to three or four (Dehaene, 1997). Accordingly, one possibility for the small contribution of early addends is that errors committed early in the calculation are amplified at later stages (Von Neumann, 1958), while symbols overcome the amplification of early variability by providing an error-free numerical code (Dehaene, 1997). Although speculative, this proposal is supported by our data as the contribution of early addends, while small, was a monotonic function its position in the sequence ($4 > 3 > 2 > 1$) (Fig. 2(b)), as expected from a process where noise add up during long calculations. A prediction of this proposal, which could be tested in future studies, is that the contribution of each addend should be similar for symbols and non-symbols if the quantities presented are below four, where quantities can be precisely estimated.

The contribution of individual addends in our sequential task revealed a strong deviation from the first-come first-serve policy traditionally used to explain results from dual-task experiments. This finding adds to a growing literature suggesting that serial operations selected on a first-come first-serve basis might only be a coarse

approximation to the dynamics of cognitive processing in overlapping tasks. Despite several methodological differences, some of the theories developed for the AB and the PRP are useful to understand our results. Chun & Potter (1995) conducted an attentional blink experiment where three targets embedded in an array of distracting stimuli had to be reported. They observed that the probability of retrieving a target was smaller when the previous target was correctly identified, compatible with dual-stage and bottleneck theories of serial processing (Chun & Potter, 1995). An interesting exception to the AB occurs when the second target immediately follows the first (*lag-1 sparing*). Recent studies have shown that many successive targets presented within a rapid sequence can be accurately identified if presented without intertwining distractors (Di Lollo *et al.*, 2005). This extended sparing has proven difficult to explain by capacity limited theories of the AB (but see for instance (Dux, Asplund & Marois, 2008)), favoring instead theories where a target initiates a processing *event* capable of processing many targets until it is interrupted by a distractor (Olivers & Meeter, 2008). Important for our results, these theories distinguish two types of interference (Wyble, Bowman & Nieuwenstein, 2009). Firstly, closing an attentional event and starting a new one is a slow process which gives rise to the AB when targets are presented within 200–500 ms. Secondly, within a single attentional event different items compete which gives rise to backwards masking and swaps in the perceived order of the items, as generally observed at lag-1 in the AB (Hommel & Akyurek, 2005). Within each episode, the strength of unconsolidated items may passively decay as in partial report experiments (Sperling, 1960), and memory consolidation would favor more recent items. Our results could then be accounted by the inability to treat each addend as a different event. While speculative, this proposal makes a concrete prediction which could be tested in future experiments. If the presentation of a distractor forces the closing of an attentional episode (as suggested in (Olivers & Meeter, 2008)), then intertwining distractors between successive addends should shift the pattern of interference from *last-come first-serve* to the *first-come first-serve* typical of PRP and AB experiments.

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