Golem95: calculating tensor integrals with up to six external legs numerically

Gudrun Heinrich

University of Durham
Institute for Particle Physics Phenomenology

in collaboration with T.Binoth, J.-Ph.Guillet, E.Pilon, T.Reiter
Exploring the TeV scale

with LHC (or the Tevatron already?) we are entering a New Era in Particle Physics!
The LHC

- will shed light on the origin of mass ("Higgs mechanism")
- may discover supersymmetry/extra dimensions, provide information about dark matter
The LHC

- will shed light on the origin of mass ("Higgs mechanism")
- may discover supersymmetry/extra dimensions, provide information about dark matter
- 1 Terabyte of data every day
- $\sim 1000$ hadronic tracks in detector per event
  proton remnants or high energy interactions between quarks/gluons (QCD)
The LHC

- will shed light on the **origin of mass** ("Higgs mechanism")
- may discover **supersymmetry/extra dimensions**, provide information about **dark matter**
- 1 Terabyte of data every day
- \( \sim 1000 \) **hadronic tracks in detector per event**
  - proton remnants or high energy interactions between quarks/gluons (QCD)

<table>
<thead>
<tr>
<th>process</th>
<th>events/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD jets ( E_T &gt; 150 \text{ GeV} )</td>
<td>100</td>
</tr>
<tr>
<td>( W \rightarrow e\nu )</td>
<td>15</td>
</tr>
<tr>
<td>( t\bar{t} )</td>
<td>1</td>
</tr>
<tr>
<td>Higgs, ( m_H \sim 130 \text{ GeV} )</td>
<td>0.02</td>
</tr>
<tr>
<td>gluinos, ( m \sim 1 \text{ TeV} )</td>
<td>0.001</td>
</tr>
</tbody>
</table>

\( \Rightarrow \) **enormous backgrounds!**
preparing for the LHC

we might see very clear signatures

e.g. 4 highly energetic leptons
preparing for the LHC

- we might see very clear signatures
e.g. 4 highly energetic leptons

- we might have to work hard to claim a discovery
e.g. light Higgs boson $m_H \sim 120$ GeV
preparing for the LHC

- we might see very clear signatures
e.g. 4 highly energetic leptons

- we might have to work hard to claim a discovery
e.g. light Higgs boson \( m_H \sim 120 \text{ GeV} \)

⇒ be prepared!

- we first have to "rediscover" the Standard Model,
  control jet energy scale, underlying event, . . .

- maximal control of theory expectations
  for signals and backgrounds is required

- measuring the backgrounds is not always possible
  e.g. neutrinos in final state
preparing for the LHC

- we might see very clear signatures
e.g. 4 highly energetic leptons
- we might have to work hard to claim a discovery
e.g. light Higgs boson $m_H \sim 120$ GeV

$\Rightarrow$ be prepared!

- we first have to *rediscover* the Standard Model,
  control jet energy scale, underlying event, . . .
- maximal control of *theory expectations*
  for signals *and* backgrounds is required
- measuring the backgrounds is *not* always possible
e.g. neutrinos in final state
- need to have precise theory predictions

Golem95: calculating tensor integrals with up to six external legs numerically – p.
need High Precision for Hard Processes!
need High Precision for Hard Processes!

predictions based on Leading Order (LO) in perturbation theory are not sufficient
need High Precision for Hard Processes!

predictions based on Leading Order (LO) in perturbation theory are not sufficient

LHC: many interesting processes lead to multi-particle \((2 \to 3, 4, \ldots)\) final states, e.g.

\[
pp \to q q H \to W^+ W^- + 2 \text{jets}, \quad pp \to H + t\bar{t} \to b\bar{b} t\bar{t}, \ldots
\]
need High Precision for Hard Processes!

predictions based on Leading Order (LO) in perturbation theory are not sufficient

LHC: many interesting processes lead to multi-particle \((2 \rightarrow 3, 4, \ldots)\) final states, e.g.

\[ pp \rightarrow q q H \rightarrow W^+W^- + 2 \text{jets}, \quad pp \rightarrow H + t\bar{t} \rightarrow b\bar{b} t\bar{t}, \ldots \]

to calculate them at NLO: HT2 (\textsc{Hard Thinking for HEP Theorists})
need High Precision for Hard Processes!

predictions based on Leading Order (LO) in perturbation theory are not sufficient

LHC: many interesting processes lead to multi-particle \((2 \rightarrow 3, 4, \ldots)\) final states, e.g.

\[
pp \rightarrow q q H \rightarrow W^+ W^- + 2 \text{jets}, \quad pp \rightarrow H + t\bar{t} \rightarrow b\bar{b} t\bar{t}, \ldots
\]

to calculate them at NLO: HT2 (Hard Thinking for HEP Theorists)

⇒ better methods, more automation
Automation

lots of progress recently!

automated subtraction for NLO real radiation
Frederix/Gehrmann/Greiner, Hasegawa/Moch/Uwer, Tevlin/Seymour,
Gleisberg/Krauss 07-08
Automation

lots of progress recently!

- automated subtraction for NLO real radiation
  Frederix/Gehrmann/Greiner, Hasegawa/Moch/Uwer, Tevlin/Seymour,
  Gleisberg/Krauss 07-08

- new tools based on numerical implementation of unitarity cuts
  Ossola/Papadopoulos/Pittau (CutTools),
  Berger/Bern/Dixon/Febres-Cordero/Forde/Ita/Kosower/Maître (BlackHat),
  Giele/Zanderighi (Rocket),
  Ellis/Giele/Kunszt/Melnikov 07-08
Automation

lots of progress recently!

- automated subtraction for NLO real radiation
  Frederix/Gehrmann/Greiner, Hasegawa/Moch/Uwer, Tevlin/Seymour, Gleisberg/Krauss 07-08

- new tools based on numerical implementation of unitarity cuts
  Ossola/Papadopoulos/Pittau (CutTools), Berger/Bern/Dixon/Febres-Cordero/Forde/Ita/Kosower/Maître (BlackHat), Giele/Zanderighi (Rocket), Ellis/Giele/Kunszt/Melnikov 07-08

- new developments within methods based on Feynman diagrams
  Bredenstein/Denner/Dittmaier/Pozzorini, Hahn/Illana/Rauch (FeynArts/FormCalc/LoopTools), Golem, Passarino et al., Yuasa et al. (GraceNLO), . . .
Methods for one-loop amplitudes

- algebraic reduction
  (pioneered by Passarino/Veltman)
  generates factorial growth in complexity
Methods for one-loop amplitudes

- **algebraic reduction**
  (pioneered by Passarino/Veltman)
  generates factorial growth in complexity

- **fully numerical**
  (pioneered by D.Soper)
  needs extraction of poles beforehand
Methods for one-loop amplitudes

- **algebraic reduction**
  (pioneered by Passarino/Veltman)
  generates factorial growth in complexity

- **fully numerical**
  (pioneered by D.Soper)
  needs extraction of poles beforehand

- **unitarity-based ("string/twistor inspired")**
  (pioneered by Bern, Dixon, Dunbar, Kosower '94, Britto, Cachazo, Feng, Witten '04, Ossola, Papadopoulos, Pittau '06)
  needs special treatment of rational parts

Golem95: calculating tensor integrals with up to six external legs numerically – p.7
algebraic reduction

non-trivial tensor structure \Rightarrow scalar 6-point function + integrals with less legs

\[ \begin{align*}
\text{non-trivial tensor structure} & \quad \sum_{i=1}^{6} b_i \\
\text{scalar 6-point function} & \\
\text{... factorial growth in complexity!}
\end{align*} \]
algebraic reduction

non-trivial tensor structure \rightarrow \text{scalar 6-point function}

\[ = \sum_{i=1}^{6} b_i \]

\[ \ldots \] factorial growth in complexity!

reduction to set of basis integrals (4-, 3- and 2-point funcs.)

\[ A = C_4 + C_3 + C_2 + \mathcal{R} \]
reduction to scalar basis integrals

main problems with reduction based on Feynman diagrams:

- sheer complexity of the expressions
  ⇒ slow programs
reduction to scalar basis integrals

main problems with reduction based on Feynman diagrams:

- sheer **complexity** of the expressions
  ⇒ slow programs

- reduction coefficients \( C_i \) contain inverse determinants of kinematic variables
  ("Gram determinants" \( \det G \))
  if \( \det G \to 0 \) in certain phase space regions
  ⇒ numerical problems
Golem approach

one possible solution: **semi-numerical method**

[Binoth, Guillet, GH 00], [Binoth, GH, Kauer 03], [Binoth, Guillet, GH, Pilon, Schubert 05]

combine virtues of numerical and algebraic methods
Golem approach

one possible solution: semi-numerical method
[Binoth, Guillet, GH 00], [Binoth, GH, Kauer 03], [Binoth, Guillet, GH, Pilon, Schubert 05]

combine virtues of numerical and algebraic methods

- do tensor reduction numerically
- reduce to scalar integrals and use analytic expressions where inverse determinants are harmless ⇒ fast
- switch to numerical evaluation of boxes, triangles otherwise
Golem approach

one possible solution: semi-numerical method
[Binoth, Guillet, GH 00], [Binoth, GH, Kauer 03], [Binoth, Guillet, GH, Pilon, Schubert 05]

combine virtues of numerical and algebraic methods

- do tensor reduction numerically
- reduce to scalar integrals and use analytic expressions where inverse determinants are harmless ⇒ fast
- switch to numerical evaluation of boxes, triangles otherwise
- formalism valid for massive and massless particles, arbitrary number of legs
Golem approach

one possible solution: semi-numerical method
[Binoth, Guillet, GH 00], [Binoth, GH, Kauer 03], [Binoth, Guillet, GH, Pilon, Schubert 05]

combine virtues of numerical and algebraic methods

- do tensor reduction numerically
- reduce to scalar integrals and use analytic expressions where inverse determinants are harmless ⇒ fast
- switch to numerical evaluation of boxes, triangles otherwise
- formalism valid for massive and massless particles, arbitrary number of legs
- rational parts \( \mathcal{R} \) are for free!
  complexity of expressions greatly reduced if \( \mathcal{R} \) is projected out
form factor representation

\[
I_{N}^{n, \mu_{1} \ldots \mu_{r}}(S) = \sum_{l_{1} \ldots l_{r} \in S} p_{l_{1}}^{\mu_{1}} \cdots p_{l_{r}}^{\mu_{r}} A_{l_{1}, \ldots, l_{r}}^{N,r}(S)
+ \sum_{l_{1} \ldots l_{r-2} \in S} \left[ g \cdot p_{l_{1}} \cdots p_{l_{r-2}} \right] \{\mu_{1} \cdots \mu_{r}\} B_{l_{1}, \ldots, l_{r-2}}^{N,r}(S)
+ \sum_{l_{1} \ldots l_{r-4} \in S} \left[ g \cdot g \cdot p_{l_{1}} \cdots p_{l_{r-4}} \right] \{\mu_{1} \cdots \mu_{r}\} C_{l_{1}, \ldots, l_{r-4}}^{N,r}(S)
\]
form factor representation

\[
I^{n, \mu_1 \ldots \mu_r}_N (S) = \\
\sum_{l_1 \ldots l_r \in S} p_{l_1}^{\mu_1} \cdots p_{l_r}^{\mu_r} A_{l_1 \ldots , l_r}^{N,r} (S) \\
+ \sum_{l_1 \ldots l_{r-2} \in S} \left[ g \cdot p_{l_1} \cdots p_{l_{r-2}} \right] \left\{ \mu_1 \ldots \mu_r \right\} B_{l_1 \ldots , l_{r-2}}^{N,r} (S) \\
+ \sum_{l_1 \ldots l_{r-4} \in S} \left[ g \cdot g \cdot p_{l_1} \cdots p_{l_{r-4}} \right] \left\{ \mu_1 \ldots \mu_r \right\} C_{l_1 \ldots , l_{r-4}}^{N,r} (S)
\]

important: more than two metric tensors \( g^{\mu \nu} \) never occur!

reason: for \( N \geq 6 \): simultaneous reduction of rank \( r \) and number of legs \( N \)

\[
I^{n, \mu_1 \ldots \mu_r}_N (S) = - \sum_{j \in S} C_{j}^{\mu_1} I^{n, \mu_2 \ldots \mu_r}_{N-1} (S \setminus \{j\})
\]

\[
S_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2
\]

Golem95: calculating tensor integrals with up to six external legs numerically – p.1
Reduction algorithm schematically

- Diagram generation (e.g. QGRAF, FeynArts)

\[ A = \sum_i C_{\mu_1 \ldots \mu_r}^i I_{\mu_1 \ldots \mu_r} \]

\[ A = \sum \{l\} f_{\{l\}}(p_i \cdot p_j, p_i \cdot \epsilon_j, \epsilon_i \cdot \epsilon_j) \{ A^{N,r}_{\{l\}}, B^{N,r}_{\{l\}}, C^{N,r}_{\{l\}} \} \]

(Lorentz invariants × form factors)

- golem95

- Numerical evaluation
- Reduction to scalar integrals

Numbers (Laurent series in \( \epsilon \))
from tensor integrals to parameter integrals

\[ I_{N}^{\mu_{1}...\mu_{r}} \]

- \( N \geq 6 \) yes
  - \( r-1, N-1 \)
- \( N \leq 2 \)
- \( I_{2}^{n} (1|j_{1}|j_{2}) \),
  - \( I_{3}^{n} (1|j_{1}|j_{2}|j_{1}, j_{2}, j_{3}) \),
  - \( I_{3}^{n+2} (1|j_{1}) \)
  - \( I_{4}^{n+2} (1|j_{1}|j_{2}|j_{1}, j_{2}, j_{3}) \),
  - \( I_{4}^{n+4} (1|j_{1}) \)
- \( N = 5 \)
  - \( A^{5,r}, B^{5,r}, C^{5,r} \)
- \( N = 4 \)
  - \( A^{4,r}, B^{4,r}, C^{4,r} \)
- \( N = 3 \)
  - \( A^{3,r}, B^{3,r} \)
new: numerical integration based on one-dimensional parameter representation ⇒ fast and precise
The GOLEM project and the \textit{golem95} program

golem95 code:

- calculates form factors for tensor integrals numerically
- master integrals valid for \textbf{all kinematic regions}, but only massless \textit{internal} particles so far
- \textbf{no restriction} on masses of \textit{external} particles
- box with all 4 legs off-shell: no one-dimensional integral representation so far $\Rightarrow$ will always be reduced to scalar box
The GOLEM project and the golem95 program

golem95 code:

- calculates form factors for tensor integrals numerically
- master integrals valid for all kinematic regions, but only massless internal particles so far
- no restriction on masses of external particles
- box with all 4 legs off-shell: no one-dimensional integral representation so far ⇒ will always be reduced to scalar box

Golem project:

- include automated diagram generation, combine with real radiation, produce cross sections
  see T.Binoth’s talk
- combine with parton shower
golem95: installation and structure

installation:
download from http://lappweb.in2p3.fr/lapth/Golem/golem95.html and unpack
./configure.pl
[–install_path=mypath] [–compiler=mycompiler]
make
make install
golem95: installation and structure

installation:
download from http://lappweb.in2p3.fr/lapth/Golem/golem95.html and unpack
./configure.pl
[–install_path=mypath] [–compiler=mycompiler]
make
make install

golem95 subdirectories:
  src:  source files
  doc:  documentation
  demos: 8 demo programs
  test:  user interface for tests etc.
demo programs

typing configure.pl produces:

Choose which demo program you want to run:

1. three-point functions
2. four-point functions
3. five-point functions
4. six-point functions
5. calculation of 4-photon helicity amplitudes
6. numerical stability demo: $\det G \to 0$
7. numerical stability demo: $\det S \to 0$
8. Golem $\leftrightarrow$ LoopTools conventions
demo 3: rank 5 five-point

choosing option 3 will produce the following output:

you have chosen option 3: five-point functions
The Makefile has been created
Please run:
make
./comp.exe

running comp.exe will prompt for the rank:

Choose what the program should compute:
0) form factor for five-point function, rank 0
1) form factor for five-point function, rank 3 \((z_1 z_2 z_4)\)
2) form factor for five-point function, rank 5 \((z_1 z_2 z_3 z_4 z_5)\)
3) form factor for diagram with propagator 3 pinched, rank 0
4) form factor for diagram with propagators 1 and 4 pinched, rank 0

choosing option 2 will produce the result in about \(8 \times 10^{-3}\) seconds

the result written to test5point.txt looks as follows:
demo 3: rank 5 five-point

\[ S(1, 3) = (p_2 + p_3)^2 = -3. \]
\[ S(2, 4) = (p_3 + p_4)^2 = 6. \]
\[ S(2, 5) = (p_1 + p_2)^2 = 15. \]
\[ S(3, 5) = (p_4 + p_5)^2 = 2. \]
\[ S(1, 4) = (p_1 + p_5)^2 = -4. \]
\[ S(1, 2) = p_2^2 = 0. \]
\[ S(2, 3) = p_3^2 = 0. \]
\[ S(3, 4) = p_4^2 = 0. \]
\[ S(4, 5) = p_5^2 = 0. \]
\[ S(1, 5) = p_1^2 = 0. \]

A factor \( \Gamma(1 + \epsilon)\Gamma(1 - \epsilon)^2 / \Gamma(1 - 2\epsilon) (4\pi \mu^2)^\epsilon \) is factored out from the result.

result = \[
\frac{1}{\epsilon^2} * (0.0000000000E+00 + I* 0.0000000000E+00) \\
+ \frac{1}{\epsilon} * (0.0000000000E+00 + I* 0.0000000000E+00) \\
+ (-.8615520644E-04 + I* 0.1230709464E-03)
\]

CPU time = 7.999000000000001E-003
demo 6: Gram determinants

- reduction $N \geq 5 \rightarrow N = 4$: inverse Gram determinants completely absent
- reduction of $N \leq 4$ tensor integrals: introduces spurious $1/\text{det}(G)$

\[
I_{4}^{n+2}(j_1; S) = \frac{1}{B} \left\{ b_{j_1} I_{4}^{n+2}(S) + \frac{1}{2} \sum_{j_2 \in S} S_{j_1 j_2}^{-1} I_{3}^{m}(S \setminus \{j_2\}) \right. \\
\left. - \frac{1}{2} \sum_{j_2 \in S \setminus \{j_1\}} b_{j_2} I_{3}^{m}(j_1; S \setminus \{j_2\}) \right\}
\]

\[
I_{4}^{n+2}(j_1, j_2; S) \sim \frac{1}{B^2}, \quad I_{4}^{n+2}(j_1, j_2, j_3; S) \sim \frac{1}{B^3} \ldots
\]

\[
B = \frac{\text{det}(G)/\text{det}(S)}{(-1)^{N+1}}
\]

\[
S_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2 \quad ; \quad G_{ij} = 2 r_i \cdot r_j
\]
Gram determinants

to avoid spurious $1/\det(G)$ terms: do not reduce

goem95:
define dimensionless quantity $\hat{B} = B \times \text{(largest entry of } S\text{)}$

if $\hat{B} < \hat{B}^{\text{cut}}$ : switch to direct numerical evaluation
(default: $\hat{B}^{\text{cut}} = 0.005$)

file demo_detg.f90 contains example where $\hat{B} \to 0$
in rank 3 box integral $I^{n+2}_4(1, 2, 2; S)$ with two massive legs
Real part for $B \rightarrow 0$
Imaginary part for $B \to 0$

\[ \text{Im} \, I_{4}^{(n+2)}(z1 \cdot z2^2) \]
demo 8: comparison to LoopTools

if all external legs are off-shell: master integrals IR finite
⇒ direct comparison to LoopTools possible

box integrals:
demo 8: comparison to LoopTools

if all external legs are off-shell: master integrals IR finite
⇒ direct comparison to LoopTools possible

box integrals:

pentagon integrals:

note: for $N \geq 5$ metric $g^{\mu\nu}$ can be expressed by external vectors, so definition of $A_5, B_5, C_5$ not unique anymore

⇒ comparison of contracted tensor integrals rather than individual form factors
user-defined tests

if you would like to

- calculate certain selected numerators of a tensor form factor, or
- calculate all different numerators of a tensor form factor
- define the numerical point to be calculated

- go to subdirectory test
- edit the file param.input
- define the numerical point in file momenta.dat
- type perl maketest.pl

example: all possible form factors for rank two 6-point
user-defined tests

numerical point: \( (p_i = (E_i, x_i, y_i, z_i)):\)

\[
\begin{align*}
p_1 &= (0.5, 0., 0., 0.5) \\
p_2 &= (0.5, 0., 0., -0.5) \\
p_3 &= (-0.19178191, -0.12741180, -0.08262477, -0.11713105) \\
p_4 &= (-0.33662712, 0.06648281, 0.31893785, 0.08471424) \\
p_5 &= (-0.21604814, 0.20363139, -0.04415762, -0.05710657) \\
p_6 &= (-0.2555428, -0.14270241, -0.19215546, 0.08952338) \\
\end{align*}
\]
input parameters

- number of legs (only 3, 4, 5, 6 are possible): 6
- rank: 2
- type of form factor: A, B or C
  (note: type B exists only for rank $\geq 2$, type C exists only for rank $\geq 4$): A
- labels of Feynman parameters in the numerator
  (separated by commas):
  example: put 2, 2, 3 for a rank 3 integral with $z_2^2 z_3$ in the numerator
  put "all" if you want to calculate all possible numerators
  all
- name of the file containing the momenta for the numerical point to be calculated: momenta.dat
- label to distinguish different numerical points: 1
Summary

we are on our way towards the automation of NLO calculations for multi-particle processes
Summary

- we are on our way towards the automation of NLO calculations for multi-particle processes
- golem95 is a nice tool to have in the toolbox
  - numerically robust due to convenient basis integrals
  - can also be used as a library for master integrals
  - contains switch to compilation in quadruple precision
  - flag to calculate rational parts only
Summary

- we are on our way towards the **automation** of NLO calculations for **multi-particle** processes

- **golem95** is a nice tool to have in the toolbox
  - numerically robust due to **convenient basis integrals**
  - can also be used as a **library for master integrals**
  - contains switch to compilation in **quadruple precision**
  - flag to calculate **rational parts** only

- **ToDo:**
  - add basis integrals for internal masses
  - public automated interface to amplitude generation
  - combine with automated treatment of real radiation
Golem will evolve more and more towards automation!

...but be careful: Stanislaw Lem’s Golem XIV was so advanced that he refused to interact with those "stupid humans"...
demo 7: scattering singularity

\[ \det S \sim (\det G)^2 \to 0 \]

pentagon with \( s_5 \neq 0 \), else \( s_j = 0 \):

\[
\det S = 2 s_{12} s_{23} s_{34} (s_{15} s_{45} - s_5 s_{23})
\]

box (1,23,4,5):

\[
\det G = 2 s_{14} (s_{15} s_{45} - s_5 s_{23})
\]

using momentum parametrisation for 1+4 → 2+3+5

\[
\det S = 2 s_{12} s_{23} s_{34} s_{14} p_{T,5}^2, \quad \det G = 2 s_{14}^2 p_{T,5}^2
\]

\( p_{T,5} \): transverse momentum of particle 5 or the system 34 relative to the beam axis (z-axis)

rotation of 2,3,5 around the z-axis is evaluated to check for stability in the limit \( p_{T,5} \to 0 \)
scattering singularity
## N(N)LO wishlist for LHC (Les Houches 07)

<table>
<thead>
<tr>
<th>process ((V \in {Z, W, \gamma}))</th>
<th>relevant for</th>
<th>status</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (pp \rightarrow ZZ) jet</td>
<td>(t\bar{t}H), new physics</td>
<td>done</td>
</tr>
<tr>
<td>2. (pp \rightarrow t\bar{t}bb)</td>
<td>(t\bar{t}H)</td>
<td>in progress</td>
</tr>
<tr>
<td>3. (pp \rightarrow t\bar{t} + 2) jets</td>
<td>(t\bar{t}H)</td>
<td>done</td>
</tr>
<tr>
<td>4. (pp \rightarrow WW)</td>
<td>SUSY trilepton</td>
<td>in progress</td>
</tr>
<tr>
<td>5. (pp \rightarrow VVb\bar{b})</td>
<td>VBF, new physics</td>
<td>in progress</td>
</tr>
<tr>
<td>6. (pp \rightarrow VV + 2) jets</td>
<td>VBF</td>
<td>in progress</td>
</tr>
<tr>
<td>7. (pp \rightarrow V + 3) jets</td>
<td>new physics</td>
<td>in progress</td>
</tr>
<tr>
<td>8. (pp \rightarrow b\bar{b}\bar{b}\bar{b})</td>
<td>(H), SUSY searches</td>
<td>in progress</td>
</tr>
<tr>
<td>10. (\mathcal{O}(\alpha^2\alpha_s^3)) (gg \rightarrow WW)</td>
<td>EW sector</td>
<td>in progress</td>
</tr>
<tr>
<td>11. NNLO for (t\bar{t})</td>
<td>benchmark, (H) coupl.</td>
<td>in progress</td>
</tr>
<tr>
<td>12. NNLO to VBF, (Z/\gamma+j)</td>
<td>(H) coupl., benchmark</td>
<td>in progress</td>
</tr>
</tbody>
</table>