1.1: $1\,\text{mi}\times(5280\,\text{ft/mi})\times(12\,\text{in./ft})\times(2.54\,\text{cm/in.})\times(1\,\text{km}/10^5\,\text{cm})=1.61\,\text{km}$

Although rounded to three figures, this conversion is exact because the given conversion from inches to centimeters defines the inch.

1.2: $$0.473\,\text{L}\times\left(\frac{1000\,\text{cm}^3}{1\,\text{L}}\right)\times\left(\frac{1\,\text{in}}{2.54\,\text{cm}}\right)^3=28.9\,\text{in}^3.$$

1.3: The time required for light to travel any distance in a vacuum is the distance divided by the speed of light;

$$\frac{10^3\,\text{m}}{3.00\times10^8\,\text{m/s}}=3.33\times10^{-6}\,\text{s}=3.33\times10^3\,\text{ns}.$$

1.4: $$11.3\,\frac{\text{g}}{\text{cm}^3}\times\left(\frac{1\,\text{kg}}{1000\,\text{g}}\right)\times\left(\frac{100\,\text{cm}}{1\,\text{m}}\right)^3=1.13\times10^4\,\frac{\text{kg}}{\text{m}^3}.$$

1.5: $(327\,\text{in}^3)\times(2.54\,\text{cm/in})^3\times(1\,\text{L}/1000\,\text{cm}^3)=5.36\,\text{L}.$

1.6: $$1\,\text{m}^3\times\left(\frac{1000\,\text{L}}{1\,\text{m}^3}\right)\times\left(\frac{1\,\text{gal}}{3.788\,\text{L}}\right)\times\left(\frac{128\,\text{oz.}}{1\,\text{gal}}\right)\times\left(\frac{1\,\text{bottle}}{16\,\text{oz.}}\right).$$

$$=2111.9\,\text{bottles}\approx2112\,\text{bottles}$$

The daily consumption must then be

$$2.11\times10^3\,\frac{\text{bottles}}{\text{yr}}\times\left(\frac{1\,\text{yr}}{365.24\,\text{da}}\right)=5.78\,\frac{\text{bottles}}{\text{da}}.$$

1.7: $(1450\,\text{mi/hr})\times(1.61\,\text{km/mi})=2330\,\text{km/hr}.$

$2330\,\text{km/hr}\times(10^3\,\text{m/km})\times(1\,\text{hr}/3600\,\text{s})=648\,\text{m/s}.$

1.8: $$180{,}000\,\frac{\text{furlongs}}{\text{fortnight}}\times\left(\frac{1\,\text{mile}}{8\,\text{furlongs}}\right)\times\left(\frac{1\,\text{fortnight}}{14\,\text{day}}\right)\times\left(\frac{1\,\text{day}}{24\,\text{h}}\right)=67\,\frac{\text{mi}}{\text{h}}.$$

1.9: $$15.0\,\frac{\text{km}}{\text{L}}\times\left(\frac{1\,\text{mi}}{1.609\,\text{km}}\right)\times\left(\frac{3.788\,\text{L}}{1\,\text{gal}}\right)=35.3\,\frac{\text{mi}}{\text{gal}}.$$

1.10: a) $\left(60\dfrac{\text{mi}}{\text{hr}}\right)\left(\dfrac{1\text{h}}{3600\text{s}}\right)\left(\dfrac{5280\,\text{ft}}{1\,\text{mi}}\right)=88\dfrac{\text{ft}}{\text{s}}$

b) $\left(32\dfrac{\text{ft}}{\text{s}^2}\right)\left(\dfrac{30.48\,\text{cm}}{1\,\text{ft}}\right)\left(\dfrac{1\,\text{m}}{100\,\text{cm}}\right)=9.8\dfrac{\text{m}}{\text{s}^2}$

c) $\left(1.0\dfrac{\text{g}}{\text{cm}^3}\right)\left(\dfrac{100\,\text{cm}}{1\,\text{m}}\right)^3\left(\dfrac{1\,\text{kg}}{1000\,\text{g}}\right)=10^3\dfrac{\text{kg}}{\text{m}^3}$

1.11: The density is mass per unit volume, so the volume is mass divided by density.
$V=\left(60\times10^3\,\text{g}\right)/\left(19.5\,\text{g}/\text{cm}^3\right)=3077\,\text{cm}^3$

Use the formula for the volume of a sphere, $V=\dfrac{4}{3}\pi r^3$,

to calculate $r: r=\left(3V/4\pi\right)^{1/3}=9.0\,\text{cm}$

1.12: $(3.16\times10^7\,\text{s}-\pi\times10^7\,\text{s})/(3.16\times10^7\,\text{s})\times100=0.58\%$

1.13: a) $\dfrac{10\,\text{m}}{890\times10^3\,\text{m}}=1.1\times10^{-3}\%.$

b) Since the distance was given as 890 km, the total distance should be 890,000 meters.
To report the total distance as 890,010 meters, the distance should be given as 890.01 km.

1.14: a) $\left(12\,\text{mm}\right)\times\left(5.98\,\text{mm}\right)=72\,\text{mm}^2$ (two significant figures).
b) $\frac{5.98\,\text{mm}}{12\,\text{mm}}=0.50$ (also two significant figures).
c) 36 mm (to the nearest millimeter).
d) 6 mm.
e) 2.0.

1.15: a) If a meter stick can measure to the nearest millimeter, the error will be about 0.13%. b) If the chemical balance can measure to the nearest milligram, the error will be about $8.3\times10^{-3}\%$. c) If a handheld stopwatch (as opposed to electric timing devices) can measure to the nearest tenth of a second, the error will be about $2.8\times10^{-2}\%$.

1.16: The area is $9.69 \pm 0.07\ \text{cm}^2$, where the extreme values in the piece's length and width are used to find the uncertainty in the area. The fractional uncertainty in the area is $\frac{0.07\ \text{cm}^2}{9.69\ \text{cm}^2} = 0.72\%$, and the fractional uncertainties in the length and width are $\frac{0.01\ \text{cm}}{5.10\ \text{cm}} = 0.20\%$ and $\frac{0.01\ \text{cm}}{1.9\ \text{cm}} = 0.53\%$.

1.17: a) The average volume is

$$\pi \frac{(8.50\ \text{cm})^2}{4}(0.050\ \text{cm}) = 2.8\ \text{cm}^3$$

(two significant figures) and the uncertainty in the volume, found from the extreme values of the diameter and thickness, is about $0.3\ \text{cm}^3$, and so the volume of a cookie is $2.8 \pm 0.3\ \text{cm}^3$. (This method does not use the usual form for progation of errors, which is not addressed in the text. The fractional uncertainty in the thickness is so much greater than the fractional uncertainty in the diameter that the fractional uncertainty in the volume is 10%, reflected in the above answer.)

b) $\frac{8.50}{.05} = 170 \pm 20.$

1.18: (Number of cars $\times$ miles/car$\cdot$day)/mi/gal = gallons/day
(2×10^8 cars $\times$ 10000 mi/yr/car $\times$ 1 yr/365 days)/(20 mi/gal) = 2.75×10^8 gal/day

1.19: Ten thousand; if it were to contain ten million, each sheet would be on the order of a millionth of an inch thick.

1.20: If it takes about four kernels to fill 1 cm^3, a 2-L bottle will hold about 8000 kernels.

1.21: Assuming the two-volume edition, there are approximately a thousand pages, and each page has between 500 and a thousand words (counting captions and the smaller print, such as the end-of-chapter exercise and problems), so an estimate for the number of words is about 10^6.

1.22: Assuming about 10 breaths per minutes, 24×60 minutes per day, 365 days per year, and a lifespan of fourscore (80) years, the total volume of air breathed in a lifetime is about $2 \times 10^5\ \mathrm{m}^3$. This is the volume of a room $100\,\mathrm{m} \times 100\,\mathrm{m} \times 20\,\mathrm{m}$, which is kind of tight for a major-league baseball game, but it's the same order of magnitude as the volume of the Astrodome.

1.23: This will vary from person to person, but should be of the order of 1×10^5.

1.24: With a pulse rate of a bit more than one beat per second, a heart will beat 10^5 times per day. With 365 days in a year and the above lifespan of 80 years, the number of beats in a lifetime is about 3×10^9. With $\frac{1}{20}$ L (50 cm^3) per beat, and about $\frac{1}{4}$ gallon per liter, this comes to about 4×10^7 gallons.

1.25: The shape of the pile is not given, but gold coins stacked in a pile might well be in the shape of a pyramid, say with a height of $2\ \mathrm{m}$ and a base $3\ \mathrm{m} \times 3\ \mathrm{m}$. The volume of such a pile is $6\ \mathrm{m}^3$, and the calculations of Example 1-4 indicate that the value of this volume is $\$6 \times 10^8$.

1.26: The surface area of the earth is about $4\pi R^2 = 5 \times 10^{14}\ \mathrm{m}^2$, where R is the radius of the earth, about $6 \times 10^6\ \mathrm{m}$, so the surface area of all the oceans is about $4 \times 10^{14}\ \mathrm{m}^2$. An average depth of about 10 km gives a volume of $4 \times 10^{18}\ \mathrm{m}^3 = 4 \times 10^{24}\ \mathrm{cm}^3$. Characterizing the size of a "drop" is a personal matter, but $25\ \mathrm{drops/cm}^3$ is reasonable, giving a total of 10^{26} drops of water in the oceans.

1.27: This will of course depend on the size of the school and who is considered a "student". A school of thousand students, each of whom averages ten pizzas a year (perhaps an underestimate) will total 10^4 pizzas, as will a school of 250 students averaging 40 pizzas a year each.

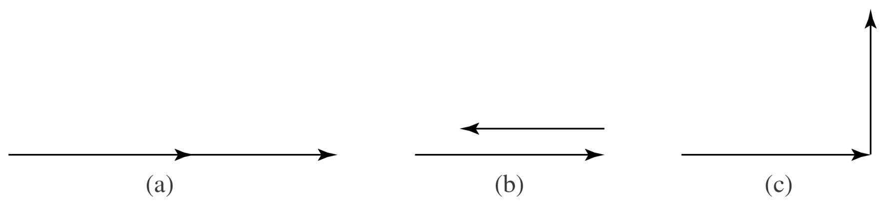
1.28: The moon is about $4 \times 10^8 \text{ m} = 4 \times 10^{11} \text{ mm}$ away. Depending on age, dollar bills can be stacked with about 2-3 per millimeter, so the number of bills in a stack to the moon would be about 10^{12}. The value of these bills would be $1 trillion (1 terabuck).

1.29: $(\text{Area of USA})/(\text{Area/bill}) = \text{number of bills.}$

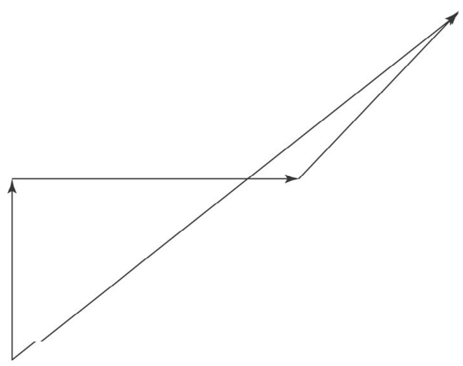
$$(9{,}372{,}571 \text{ km}^2 \times 10^6 \text{ m}^2/\text{km}^2)/(15.6 \text{ cm} \times 6.7 \text{ cm} \times 1 \text{ m}^2/10^4 \text{ cm}^2) = 9 \times 10^{14} \text{ bills}$$

$$9 \times 10^{14} \text{ bills}/2.5\ 10^8 \text{ inhabitants} = \$3.6 \text{ million}/\text{inhabitant}.$$

1.30:

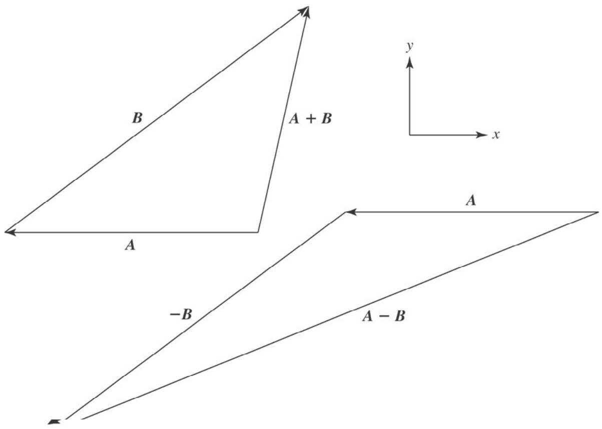


1.31:



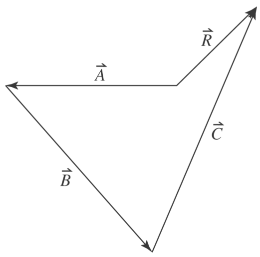
$7.8 \text{ km}, 38^\circ$ north of east

1.32:



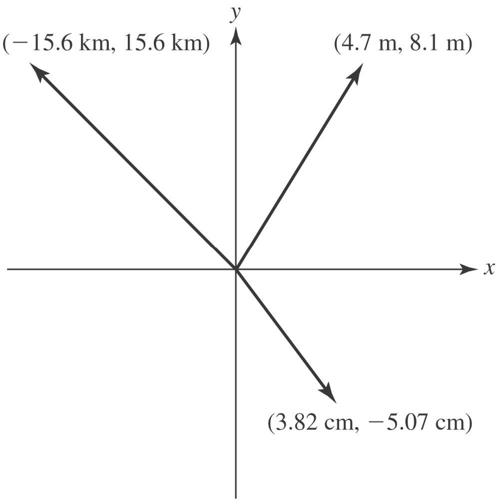
a) 11.1 m @ 77.6°
b) 28.5 m @ 202°
c) 11.1 m @ 258°
d) 28.5 m @ 22°

1.33:



144 m, 41° south of west.

1.34:



1.35: $\vec{A}; A_x = (12.0\ \text{m})\sin 37.0^\circ = 7.2\ \text{m},\ A_y = (12.0\ \text{m})\cos 37.0^\circ = 9.6\ \text{m}.$

$\vec{B}; B_x = (15.0\ \text{m})\cos 40.0^\circ = 11.5\ \text{m},\ B_y = -(15.0\ \text{m})\sin 40.0^\circ = -9.6\ \text{m}.$

$\vec{C}; C_x = -(6.0\ \text{m})\cos 60.0^\circ = -3.0\ \text{m},\ C_y = -(6.0\ \text{m})\sin 60.0^\circ = -5.2\ \text{m}.$

1.36: (a) $$\tan\theta = \frac{A_y}{A_x} = \frac{-1.00\ \text{m}}{2.00\ \text{m}} = -0.500$$

$$\theta = \tan^{-1}(-0.500) = 360^\circ - 26.6^\circ = 333^\circ$$

(b) $$\tan\theta = \frac{A_y}{A_x} = \frac{1.00\ \text{m}}{2.00\ \text{m}} = 0.500$$

$$\theta = \tan^{-1}(0.500) = 26.6^\circ$$

(*c*) $$\tan\theta = \frac{A_y}{A_x} = \frac{1.00\ \text{m}}{-2.00\ \text{m}} = -0.500$$

$$\theta = \tan^{-1}(-0.500) = 180^\circ - 26.6^\circ = 153^\circ$$

(d) $$\tan\theta = \frac{A_y}{A_x} = \frac{-1.00\ \text{m}}{-2.00\ \text{m}} = 0.500$$

$$\theta = \tan^{-1}(0.500) = 180^\circ + 26.6^\circ = 207^\circ$$

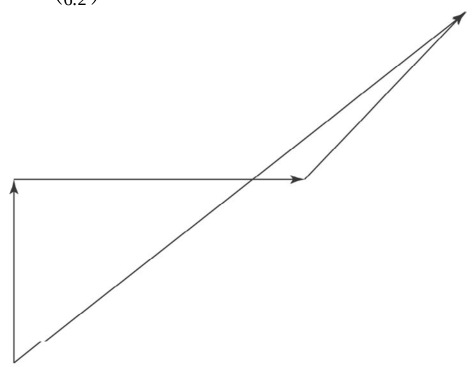
1.37: Take the +*x*-direction to be forward and the +*y*-direction to be upward. Then the second force has components $F_{2x} = F_2 \cos 32.4^\circ = 433\,\mathrm{N}$ and $F_{2y} = F_2 \sin 32.4^\circ = 275\,\mathrm{N}$. The first force has components $F_{1x} = 725\,\mathrm{N}$ and $F_{1y} = 0$.

$F_x = F_{1x} + F_{2x} = 1158\,\mathrm{N}$ and $F_y = F_{1y} + F_{2y} = 275\,\mathrm{N}$

The resultant force is 1190 N in the direction 13.4° above the forward direction.

1.38: (The figure is given with the solution to Exercise 1.31).

The net northward displacement is (2.6 km) + (3.1 km) sin 45° = 4.8 km, and the net eastward displacement is (4.0 km) + (3.1 km) cos 45° = 6.2 km. The magnitude of the resultant displacement is $\sqrt{(4.8\ \mathrm{km})^2 + (6.2\ \mathrm{km})^2}$ = 7.8 km, and the direction is arctan $\left(\frac{4.8}{6.2}\right)$ = 38° north of east.



1.39: Using components as a check for any graphical method, the components of $\vec{B}$ are $B_x = 14.4\ \text{m}$ and $B_y = 10.8\ \text{m}$, $\vec{A}$ has one component, $A_x = -12\ \text{m}$.

a) The x- and y-components of the sum are 2.4 m and 10.8 m, for a magnitude of $\sqrt{(2.4\ \text{m})^2 + (10.8\ \text{m})^2} = 11.1\ \text{m}$, , and an angle of $\left(\frac{10.8}{2.4}\right) = 77.6^\circ$.

b) The magnitude and direction of **A + B** are the same as **B + A**.

c) The *x*- and *y*-components of the vector difference are – 26.4 m and $-10.8\ \text{m}$, for a magnitude of $28.5\ \text{m}$ and a direction arctan $\left(\frac{-10.8}{-26.4}\right) = 202^\circ$. Note that 180° must be added to $\arctan\left(\frac{-10.8}{-26.4}\right) = \arctan\left(\frac{10.8}{26.4}\right) = 22^\circ$ in order to give an angle in the third quadrant.

d) $\vec{B} - \vec{A} = 14.4\ \text{m}\hat{i} + 10.8\ \text{m}\hat{j} + 12.0\ \text{m}\hat{i} = 26.4\ \text{m}\hat{i} + 10.8\ \text{m}\hat{j}$.

$\text{Magnitude} = \sqrt{(26.4\ \text{m})^2 + (10.8\ \text{m})^2} = 28.5\ \text{m}$ at and angle of $\arctan\left(\frac{10.8}{26.4}\right) = 22.2^\circ$.

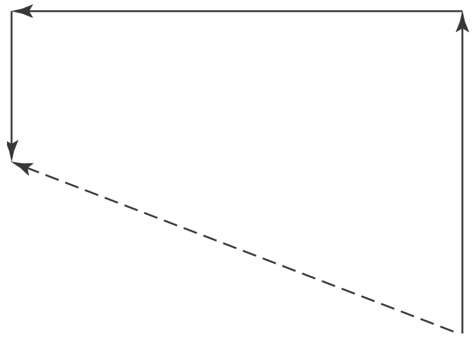
1.40: Using Equations (1.8) and (1.9), the magnitude and direction of each of the given vectors is:

a) $\sqrt{(-8.6\ \text{cm})^2 + (5.20\ \text{cm})^2}$ = 10.0 cm, arctan $\left(\frac{5.20}{-8.60}\right)$ = 148.8° (which is 180° – 31.2°).

b) $\sqrt{(-9.7\ \text{m})^2 + (-2.45\ \text{m})^2}$ = 10.0 m, arctan $\left(\frac{-2.45}{-9.7}\right)$ = 14° + 180° = 194°.

c) $\sqrt{(7.75\ \text{km})^2 + (-2.70\ \text{km})^2}$ = 8.21 km, arctan $\left(\frac{-2.7}{7.75}\right)$ = 340.8° (which is 360° – 19.2°).

1.41:



The total northward displacement is $3.25\,\text{km} - 1.50\,\text{km} = 1.75\,\text{km}$, , and the total westward displacement is $4.75\ \text{km}$. The magnitude of the net displacement is $\sqrt{(1.75\,\text{km})^2 + (4.75\,\text{km})^2} = 5.06\,\text{km}.$ The south and west displacements are the same, so The direction of the net displacement is 69.80° West of North.

1.42: a) The *x*- and *y*-components of the sum are 1.30 cm + 4.10 cm = 5.40 cm, 2.25 cm + (–3.75 cm) = –1.50 cm.

b) Using Equations (1-8) and (1-9),

$$\sqrt{(5.40\,\text{cm})^2\ (-1.50\ \text{cm})^2}\ = 5.60\ \text{cm, arctan}\ \left(\tfrac{-1.50}{+5.40}\right) = 344.5^\circ\ \text{ccw.}$$

c) Similarly, 4.10 cm – (1.30 cm) = 2.80 cm, –3.75 cm – (2.25 cm) = –6.00 cm.

d) $\sqrt{(2.80\,\text{cm})^2\ + (-6.0\,\text{cm})^2}\ = 6.62$ cm, arctan $\left(\tfrac{-6.00}{2.80}\right) = 295^\circ$ (which is 360° – 65

1.43: a) The magnitude of $\vec{A}+\vec{B}$ is

$$\sqrt{\begin{pmatrix}\left((2.80\text{ cm})\cos 60.0^\circ + (1.90\text{ cm})\cos 60.0^\circ\right)^2 \\ +\left((2.80\text{ cm})\sin 60.0^\circ - (1.90\text{ cm})\sin 60.0^\circ\right)^2\end{pmatrix}} = 2.48\text{ cm}$$

and the angle is

$$\arctan\left(\frac{(2.80\text{ cm})\sin 60.0^\circ - (1.90\text{ cm})\sin 60.0^\circ}{(2.80\text{ cm})\cos 60.0^\circ + (1.90\text{ cm})\cos 60.0^\circ}\right) = 18^\circ$$

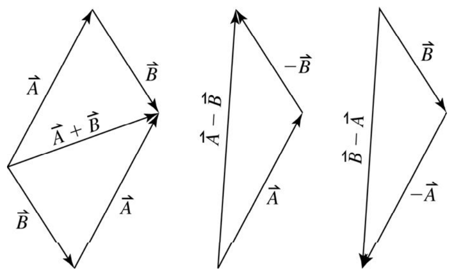
b) The magnitude of $\vec{A}-\vec{B}$ is

$$\sqrt{\begin{pmatrix}\left((2.80\text{ cm})\cos 60.0^\circ - (1.90\text{ cm})\cos 60.0^\circ\right)^2 \\ +\left((2.80\text{ cm})\sin 60.0^\circ + (1.90\text{ cm})\sin 60.0^\circ\right)^2\end{pmatrix}} = 4.10\text{ cm}$$

and the angle is

$$\arctan\left(\frac{(2.80\text{ cm})\sin 60.0^\circ + (1.90\text{ cm})\sin 60.0^\circ}{(2.80\text{ cm})\cos 60.0^\circ - (1.90\text{ cm})\cos 60.0^\circ}\right) = 84^\circ$$

c) $\vec{B}-\vec{A} = -(\vec{A}-\vec{B})$; the magnitude is 4.10 cm and the angle is $84^\circ + 180^\circ = 264^\circ$.



1.44: $\vec{A}$ = (–12.0 m) $\hat{\imath}$. More precisely,

$$\vec{A} = (12.0\text{ m})(\cos 180^\circ)\hat{\imath} + (12.0\text{ m})(\sin 180^\circ)\hat{\jmath}.$$

$$\vec{B} = (18.0\text{ m})(\cos 37^\circ)\hat{\imath} + (18.0\text{ m})(\sin 37^\circ)\hat{\jmath} = (14.4\text{ m})\hat{\imath} + (10.8\text{ m})\hat{\jmath}$$

1.45:

$$\vec{A} = (12.0\text{ m})\sin 37.0^\circ\,\hat{\imath} + (12.0\text{ m})\cos 37.0^\circ\,\hat{\jmath} = (7.2\text{ m})\hat{\imath} + (9.6\text{ m})\hat{\jmath}$$
$$\vec{B} = (15.0\text{ m})\cos 40.0^\circ\,\hat{\imath} - (15.0\text{ m})\sin 40.0^\circ\,\hat{\jmath} = (11.5\text{ m})\hat{\imath} - (9.6\text{ m})\hat{\jmath}$$
$$\vec{C} = -(6.0\text{ m})\cos 60.0^\circ\,\hat{\imath} - (6.0\text{ m})\sin 60.0^\circ\,\hat{\jmath} = -(3.0\text{ m})\hat{\imath} - (5.2\text{ m})\hat{\jmath}$$

1.46: a) $\vec{\boldsymbol{A}} = (3.60\text{ m})\cos 70.0^\circ\,\hat{\boldsymbol{i}} + (3.60\text{ m})\sin 70.0^\circ\,\hat{\boldsymbol{j}} = (1.23\text{ m})\hat{\boldsymbol{i}} + (3.38\text{ m})\hat{\boldsymbol{j}}$

$\vec{\boldsymbol{B}} = -(2.40\text{ m})\cos 30.0^\circ\,\hat{\boldsymbol{i}} - (2.40\text{ m})\sin 30.0^\circ\,\hat{\boldsymbol{j}} = (-2.08\text{ m})\hat{\boldsymbol{i}} + (-1.20\text{ m})\hat{\boldsymbol{j}}$

b)

$$\begin{aligned}\vec{\boldsymbol{C}} &= (3.00)\,\vec{\boldsymbol{A}} - (4.00)\vec{\boldsymbol{B}} \\ &= (3.00)(1.23\text{ m})\hat{\boldsymbol{i}} + (3.00)(3.38\text{ m})\hat{\boldsymbol{j}} - (4.00)(-2.08\text{ m})\hat{\boldsymbol{i}} - (4.00)(-1.20\text{ m})\hat{\boldsymbol{j}} \\ &= (12.01\text{ m})\hat{\boldsymbol{i}} + (14.94)\hat{\boldsymbol{j}}\end{aligned}$$

(Note that in adding components, the fourth figure becomes significant.)

c) From Equations (1.8) and (1.9),

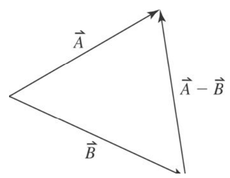
$$C = \sqrt{(12.01\text{ m})^2 + (14.94\text{ m})^2} = 19.17\text{ m, arctan}\left(\frac{14.94\text{ m}}{12.01\text{ m}}\right) = 51.2^\circ$$

1.47: a) $A = \sqrt{(4.00)^2 + (3.00)^2} = 5.00,\ B = \sqrt{(5.00)^2 + (2.00)^2} = 5.39$

b) $\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = (4.00 - 3.00)\hat{\boldsymbol{i}} + (5.00 - (-2.00))\hat{\boldsymbol{j}} = (-1.00)\hat{\boldsymbol{i}} + (5.00)\hat{\boldsymbol{j}}$

c) $\sqrt{(1.00)^2 + (5.00)^2} = 5.10, \quad \text{arctan}\left(\frac{5.00}{-1.00}\right) = 101.3^\circ$

d)



1.48: a) $\left|\hat{\boldsymbol{i}}+\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right| = \sqrt{1^2+1^2+1^2} = \sqrt{3} \neq 1$ so it is not a unit vector

b) $$\left|\vec{\boldsymbol{A}}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

If any component is greater than + 1 or less than –1, $\left|\vec{\boldsymbol{A}}\right| \geq 1$, so it cannot be a unit vector. $\vec{\boldsymbol{A}}$ can have negative components since the minus sign goes away when the component is squared.

c)

$$\left|\vec{\boldsymbol{A}}\right| = 1$$

$$\sqrt{a^2(3.0)^2 + a^2(4.0)^2} = 1$$

$$\sqrt{a^2}\sqrt{25} = 1$$

$$a = \pm\frac{1}{5.0} = \pm 0.20$$

1.49: a) Let $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}}$, $\vec{\boldsymbol{B}} = B_x\hat{\boldsymbol{i}} + B_y\hat{\boldsymbol{j}}$.

$$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = (A_x + B_x)\hat{\boldsymbol{i}} + (A_y + B_y)\hat{\boldsymbol{j}}$$
$$\vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} = (B_x + A_x)\hat{\boldsymbol{i}} + (B_y + A_y)\hat{\boldsymbol{j}}$$

Scalar addition is commutative, so $\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}$

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = A_xB_x + A_yB_y$$
$$\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = B_xA_x + B_yA_y$$

Scalar multiplication is commutative, so $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}}$

b) $$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = (A_yB_z - A_zB_y)\hat{\boldsymbol{i}} + (A_zB_x - A_xB_z)\hat{\boldsymbol{j}} + (A_xB_y - A_yB_x)\hat{\boldsymbol{k}}$$
$$\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}} = (B_yA_z - B_zA_y)\hat{\boldsymbol{i}} + (B_zA_x - B_xA_z)\hat{\boldsymbol{j}} + (B_xA_y - B_yA_x)\hat{\boldsymbol{k}}$$

Comparison of each component in each vector product shows that one is the negative of the other.

1.50: Method 1: $\left(\text{Product of magnitudes} \times \cos\theta\right)$

$\text{AB}\cos\theta = \left(12\text{ m} \times 15\text{ m}\right)\cos 93^\circ = -9.4\text{ m}^2$

$\text{BC}\cos\theta = \left(15\text{ m} \times 6\text{ m}\right)\cos 80^\circ = 15.6\text{ m}^2$

$\text{AC}\cos\theta = \left(12\text{ m} \times 6\text{ m}\right)\cos 187^\circ = -71.5\text{ m}^2$

Method 2: (Sum of products of components)

$\mathbf{A} \cdot \mathbf{B} = \left(7.22\right)(11.49) + (9.58)(-9.64) = -9.4\text{ m}^2$

$\mathbf{B} \cdot \mathbf{C} = (11.49)(-3.0) + (-9.64)(-5.20) = 15.6\text{ m}^2$

$\mathbf{A} \cdot \mathbf{C} = (7.22)(-3.0) + (9.58)(-5.20) = -71.5\text{ m}^2$

1.51: a) From Eq.(1.21),

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = (4.00)(5.00) + (3.00)(-2.00) = 14.00.$$

b) $\mathbf{A} \cdot \mathbf{B} = \text{AB}\cos\theta$, so $\theta = \arccos\left[(14.00)/(5.00 \times 5.39)\right] = \arccos(.5195) = 58.7^\circ$.

1.52: For all of these pairs of vectors, the angle is found from combining Equations (1.18) and (1.21), to give the angle ϕ as

$$\phi = \arccos\left(\frac{\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}}{AB}\right) = \arccos\left(\frac{A_x B_x + A_y B_y}{AB}\right).$$

In the intermediate calculations given here, the significant figures in the dot products and in the magnitudes of the vectors are suppressed.

a) $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = -22,\ A = \sqrt{40},\ B = \sqrt{13},$ **and so**

$$\phi = \arccos\left(\frac{-22}{\sqrt{40}\ \sqrt{13}}\right) = 165^\circ.$$

b) $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = 60,\ A = \sqrt{34},\ B = \sqrt{136},\quad \phi = \arccos\left(\frac{60}{\sqrt{34}\sqrt{136}}\right) = 28^\circ.$

c) $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = 0,\ \phi = 90.$

1.53: Use of the right-hand rule to find cross products gives (a) out of the page and b) into the page.

1.54: a) From Eq. (1.22), the magnitude of the cross product is

$$(12.0\,\text{m})(18.0\,\text{m})\sin(180^\circ - 37^\circ) = 130\,\text{m}^2$$

The right-hand rule gives the direction as being into the page, or the – z-direction. Using Eq. (1.27), the only non-vanishing component of the cross product is

$$C_z = A_x B_y = (-12\,\text{m})((18.0\,\text{m})\sin 37^\circ) = -130\,\text{m}^2$$

b) The same method used in part (a) can be used, but the relation given in Eq. (1.23) gives the result directly: same magnitude (130 m^2), but the opposite direction (+z-direction).

1.55: In Eq. (1.27), the only non-vanishing component of the cross product is

$$C_z = A_x B_y - A_y B_x = (4.00)(-2.00) - (3.00)(5.00) = -23.00,$$

so $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = -(23.00)\hat{\boldsymbol{k}}$, and the magnitude of the vector product is 23.00.

1.56: a) From the right-hand rule, the direction of $\vec{A} \times \vec{B}$ is into the page (the $-z$-direction). The magnitude of the vector product is, from Eq. (1.22),

$$AB \sin \phi = (2.80 \text{ cm})(1.90 \text{ cm}) \sin 120^\circ = 4.61 \text{ cm}^2.$$

Or, using Eq. (1.27) and noting that the only non-vanishing component is

$$\begin{aligned} C_z &= A_x B_y - A_y B_x \\ &= (2.80 \text{ cm}) \cos 60.0^\circ (-1.90 \text{ cm}) \sin 60^\circ \\ &\quad - (2.80 \text{ cm}) \sin 60.0^\circ (1.90 \text{ cm}) \cos 60.0^\circ \\ &= -4.61 \text{ cm}^2 \end{aligned}$$

gives the same result.

b) Rather than repeat the calculations, Eq. (1-23) may be used to see that $\vec{B} \times \vec{A}$ has magnitude 4.61 cm^2 and is in the $+z$-direction (out of the page).

1.57: a) The area of one acre is $\frac{1}{8} \text{ mi} \times \frac{1}{80} \text{ mi} = \frac{1}{640} \text{ mi}^2$, so there are 640 acres to a square mile.

b) $$(1 \text{ acre}) \times \left(\frac{1 \text{ mi}^2}{640 \text{ acre}} \right) \times \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right)^2 = 43{,}560 \text{ ft}^2$$

(all of the above conversions are exact).

c) $$(1 \text{ acre-foot}) = (43{,}560 \text{ ft}^3) \times \left(\frac{7.477 \text{ gal}}{1 \text{ ft}^3} \right) = 3.26 \times 10^5 \text{ gal},$$

which is rounded to three significant figures.

1.58: a) $(\$4{,}950{,}000/102 \text{ acres}) \times (1 \text{ acre}/43560 \text{ ft}^2) \times (10.77 \text{ ft}^2/\text{m}^2) = \$12/\text{m}^2$.

b) $(\$12/\text{m}^2) \times (2.54 \text{ cm}/\text{in})^2 \times (1 \text{ m}/100 \text{ cm})^2 = \$.008/\text{in}^2$.

c) $\$.008/\text{in}^2 \times (1 \text{ in} \times 7/8 \text{ in}) = \$.007$ for postage stamp sized parcel.

1.59: a) To three significant figures, the time for one cycle is

$$\frac{1}{1.420\times10^{9}\ \text{Hz}}=7.04\times10^{-10}\ \text{s}.$$

b) $$\left(1.420\times10^{9}\ \frac{\text{cycles}}{\text{s}}\right)\times\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)=5.11\times10^{12}\ \frac{\text{cycles}}{\text{h}}$$

c) Using the conversion from years to seconds given in Appendix F,

$$\left(1.42\times10^{9}\ \text{Hz}\right)\times\left(\frac{3.156\times10^{7}\ \text{s}}{1\,\text{y}}\right)\times\left(4.600\times10^{9}\ \text{y}\right)=2.06\times10^{26}.$$

d) $4.600\times10^{9}\ \text{y}=\left(4.60\times10^{4}\right)\left(1.00\times10^{5}\ \text{y}\right)$, so the clock would be off by 4.60×10^{4} s.

1.60: Assume a 70-kg person, and the human body is mostly water. Use Appendix D to find the mass of one H_2O molecule: 18.015 u × 1.661 × 10^{-27} kg/u = 2.992 × 10^{-26} kg/molecule. (70 kg/2.992 × 10^{-26} kg/molecule) = 2.34 × 10^{27} molecules. (Assuming carbon to be the most common atom gives 3 × 10^{27} molecules.

1.61: a) Estimate the volume as that of a sphere of diameter 10 cm:

$$V=\frac{4}{3}\pi r^{3}=5.2\times10^{-4}\,\text{m}^{3}$$

Mass is density times volume, and the density of water is $1000\,\text{kg/m}^{3}$, so

$$m=(0.98)\left(1000\,\text{kg/m}^{3}\right)\left(5.2\times10^{-4}\,\text{m}^{3}\right)=0.5\,\text{kg}$$

b) Approximate as a sphere of radius $r=0.25\mu\text{m}$ (probably an over estimate)

$$V=\frac{4}{3}\pi r^{3}=6.5\times10^{-20}\,\text{m}^{3}$$

$$m=(0.98)\left(1000\,\text{kg/m}^{3}\right)\left(6.5\times10^{-20}\,\text{m}^{3}\right)=6\times10^{-17}\,\text{kg}=6\times10^{-14}\,\text{g}$$

c) Estimate the volume as that of a cylinder of length 1 cm and radius 3 mm:

$$V=\pi r^{2}l=2.8\times10^{-7}\,\text{m}^{3}$$

$$m=(0.98)\left(1000\,\text{kg/m}^{3}\right)\left(2.8\times10^{-7}\,\text{m}^{3}\right)=3\times10^{-4}\,\text{kg}=0.3\,\text{g}$$

1.62: a)

$$\rho = \frac{M}{V}, \text{so}\, V = \frac{M}{\rho}$$

$$x^3 = \frac{0.200\,\text{kg}}{7.86 \times 10^3\ \text{kg/m}^3} = 2.54 \times 10^{-5}\,\text{m}^3$$

$$x = 2.94 \times 10^{-2}\,\text{m} = 2.94\,\text{cm}$$

b)

$$\frac{4}{3}\pi R^3 = 2.54 \times 10^{-5}\,\text{m}^3$$

$$R = 1.82 \times 10^{-2}\,\text{m} = 1.82\,\text{cm}$$

1.63: Assume each person sees the dentist twice a year for checkups, for 2 hours. Assume 2 more hours for restorative work. Assuming most dentists work less than 2000 hours per year, this gives $2000\,\text{hours}/4\ \text{hours per patient} = 500\,\text{patients per dentist}$. Assuming only half of the people who should go to a dentist do, there should be about 1 dentist per 1000 inhabitants. Note: A dental assistant in an office with more than one treatment room could increase the number of patients seen in a single dental office.

1.64: a) $$(6.0 \times 10^{24}\,\text{kg}) \times \left(\frac{6.0 \times 10^{23}\ \frac{\text{atoms}}{\text{mole}}}{14 \times 10^{-3}\ \frac{\text{kg}}{\text{mole}}} \right) = 2.6 \times 10^{50}\,\text{atoms.}$$

b) The number of neutrons is the mass of the neutron star divided by the mass of a neutron:

$$\frac{(2)(2.0 \times 10^{30}\ \text{kg})}{(1.7 \times 10^{-27}\ \text{kg/neutron})} = 2.4 \times 10^{57}\ \text{neutrons.}$$

c) The average mass of a particle is essentially $\frac{2}{3}$ the mass of either the proton or the neutron, 1.7×10^{-27} kg. The total number of particles is the total mass divided by this average, and the total mass is the volume times the average density. Denoting the density by ρ (the notation introduced in Chapter 14).

$$\frac{M}{m_{\text{ave}}} = \frac{\frac{4}{3}\pi R^3 \rho}{\frac{2}{3} m_{\text{p}}} = \frac{(2\pi)(1.5 \times 10^{11}\ \text{m})^3\ (10^{18}\ \text{kg/m}^3)}{(1.7 \times 10^{-27}\ \text{kg})} = 1.2 \times 10^{79}.$$

Note the conversion from g/cm^3 to kg/m^3.

1.65: Let $\vec{D}$ be the fourth force.

$$\vec{A}+\vec{B}+\vec{C}+\vec{D}=0, \text{so } \vec{D}=-\left(\vec{A}+\vec{B}+\vec{C}\right)$$

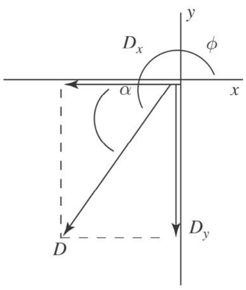
$$A_x = +A\cos 30.0^\circ = +86.6\,\text{N}, \quad A_y = +A\cos 30.0^\circ = +50.00\,\text{N}$$

$$B_x = -B\sin 30.0^\circ = -40.00\,\text{N}, \; B_y = +B\cos 30.0^\circ = +69.28\,\text{N}$$

$$C_x = +C\cos 53.0^\circ = -24.07\,\text{N}, \; C_y = -C\sin 53.0^\circ = -31.90\,\text{N}$$

Then $D_x = -22.53$ N, $\qquad D_y = -87.34$ N

$$D=\sqrt{D_x^2+D_y^2}=90.2\ \text{N};$$

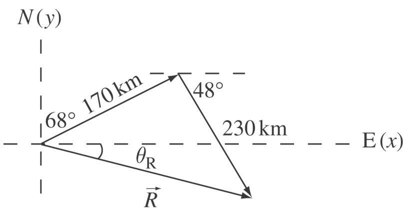


$$\tan\alpha = \left|D_y/D_x\right| = 87.34/22.53$$

$$\alpha = 75.54^\circ$$

$$\varphi = 180^\circ + \alpha = 256^\circ, \text{counterclockwise from} + x\text{-axis}$$

1.66:



$$R_x = A_x + B_x = (170\,\text{km})\sin 68^\circ + (230\,\text{km})\cos 48^\circ = 311.5\,\text{km}$$

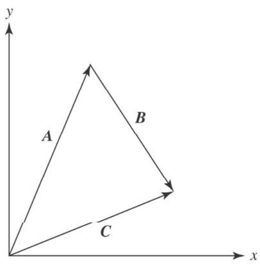
$$R_y = A_y + B_y = (170\,\text{km})\cos 68^\circ - (230\,\text{km})\sin 48^\circ = -107.2\,\text{km}$$

$$R=\sqrt{R_x^2+R_y^2}=\sqrt{(311.5\,\text{km})^2+(-107.2\,\text{km})^2}=330\,\text{km}$$

$$\tan\theta_R = \left|\frac{R_y}{R_x}\right| = \frac{107.2\,\text{km}}{311.5\,\text{km}} = 0.344$$

$\theta_R = 19^\circ$ south of east

1.67: a)



b) Algebraically, $\vec{\boldsymbol{A}} = \vec{\boldsymbol{C}} - \vec{\boldsymbol{B}}$, and so the components of $\vec{\boldsymbol{A}}$ are

$$A_x = C_x - B_x = (6.40\text{ cm})\cos 22.0^\circ - (6.40\text{ cm})\cos 63.0^\circ = 3.03\text{ cm}$$

$$A_y = C_y - B_y = (6.40\text{ cm})\sin 22.0^\circ + (6.40\text{ cm})\sin 63.0^\circ = 8.10\text{ cm.}$$

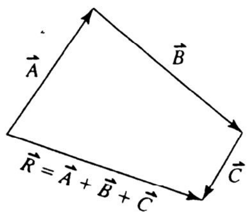
c) $A = \sqrt{(3.03\text{ cm})^2 + (8.10\text{ cm})^2} = 8.65\text{ cm},\quad \arctan\left(\dfrac{8.10\text{ cm}}{3.03\text{ cm}}\right) = 69.5^\circ$

1.68: a) $R_x = A_x + B_x + C_x$

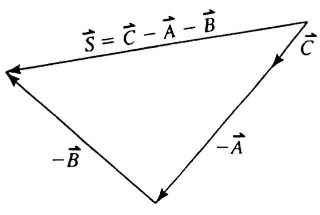
$$= (12.0\ \text{m})\cos(90^\circ - 37^\circ) + (15.00\ \text{m})\cos(-40^\circ) + (6.0\ \text{m})\cos(180^\circ + 60^\circ)$$
$$= 15.7\ \text{m, and}$$

$$R_y = A_y + B_y + C_y$$
$$= (12.0\ \text{m})\sin(90^\circ - 37^\circ) + (15.00\ \text{m})\sin(-40^\circ) + (6.0\ \text{m})\sin(180^\circ + 60^\circ)$$
$$= -5.3\ \text{m}.$$

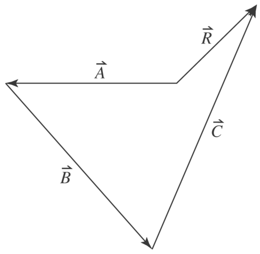
The magnitude of the resultant is $R = \sqrt{R_x^2 + R_y^2} = 16.6\ \text{m}$, and the direction from the positive x-axis is arctan $\left(\frac{-5.3}{15.7}\right) = -18.6^\circ$. Keeping extra significant figures in the intermediate calculations gives an angle of $-18.49°$, which when considered as a positive counterclockwise angle from the positive x-axis and rounded to the nearest degree is 342°.



b)
$$S_x = -3.00\ \text{m} - 7.22\ \text{m} - 11.49\ \text{m} = -21.71\ \text{m};$$
$$S_y = -5.20\ \text{m} - (-9.64\ \text{m}) - 9.58\ \text{m} = -5.14\ \text{m};$$
$$\theta = \arctan\left[\frac{(-5.14)}{(-21.71)}\right] = 13.3^\circ$$
$$S = \sqrt{(-21.71\ \text{m})^2 + (-5.14\ \text{m})^2} = 22.3\ \text{m}$$



1.69:



Take the east direction to be the x - direction and the north direction to be the y - direction. The *x*- and *y*-components of the resultant displacement of the first three displacements are then

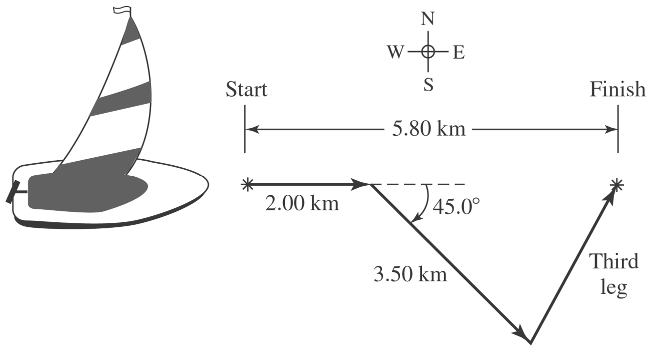
$$(-180\ \text{m}) + (210\ \text{m})\sin 45^\circ + (280\ \text{m})\sin 30^\circ = 108\ \text{m},$$
$$-(210\ \text{m})\cos 45^\circ + (280\ \text{m})\cos 30^\circ = +94.0\ \text{m},$$

keeping an extra significant figure. The magnitude and direction of this net displacement are

$$\sqrt{(108\ \text{m})^2 + (94.0\ \text{m})^2} = 144\ \text{m}, \quad \arctan\left(\frac{94\ \text{m}}{108\ \text{m}}\right) = 40.9^\circ.$$

The fourth displacement must then be 144 m in a direction 40.9° south of west.

1.70:



The third leg must have taken the sailor east a distance

$$(5.80\text{ km})-(3.50\text{ km})\cos 45^{\circ}-(2.00\text{ km})=1.33\text{ km}$$

and a distance north

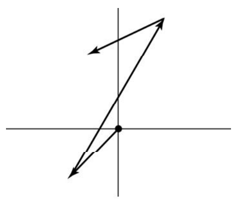
$$(3.5\text{ km})\sin 45^{\circ}=(2.47\text{ km})$$

The magnitude of the displacement is

$$\sqrt{(1.33\text{ km})^2+(2.47\text{ km})^2}=2.81\text{ km}$$

and the direction is arctan $\left(\frac{2.47}{1.33}\right)=62°$ north of east, which is $90°-62°=28°$ east of north. A more precise answer will require retaining extra significant figures in the intermediate calculations.

1.71: a)



b) The net east displacement is $-(2.80\text{ km})\sin 45^{\circ}+(7.40\text{ km})\cos 30^{\circ}-(3.30\text{ km})\cos 22^{\circ}=1.37\text{ km},$ and the net north displacement is $-(2.80\text{ km})\cos 45^{\circ}+(7.40\text{ km})\sin 30^{\circ}-(3.30\text{ km})\sin 22.0^{\circ}=0.48\text{ km},$ and so the distance traveled is $\sqrt{(1.37\text{ km})^2+(0.48\text{ km})^2}=1.45\text{ km}.$

1.72: The eastward displacement of Manhattan from Lincoln is

$$(147\text{ km})\sin 85^{\circ}+(106\text{ km})\sin 167^{\circ}+(166\text{ km})\sin 235^{\circ}=34.3\text{ km}$$

and the northward displacement is

$$(147\text{ km})\cos 85^{\circ}+(106\text{ km})\cos 167^{\circ}+(166\text{ km})\cos 235^{\circ}=-185.7\text{ km}$$

(A negative northward displacement is a southward displacement, as indicated in Fig. (1.33). Extra figures have been kept in the intermediate calculations.)

a) $$\sqrt{(34.3\text{ km})^2+(185.7\text{ km})^2}=189\text{ km}$$

b) The direction from Lincoln to Manhattan, relative to the north, is

$$\arctan\left(\frac{34.3\text{ km}}{-185.7\text{ km}}\right)=169.5^{\circ}$$

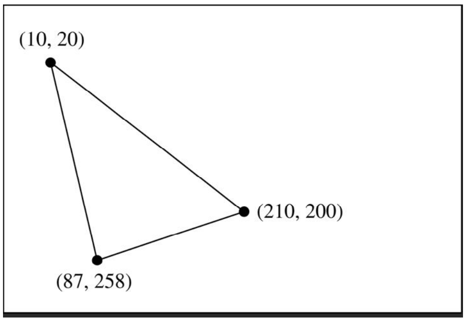
and so the direction to fly in order to return to Lincoln is $169.5^{\circ}+180^{\circ}+349.5^{\circ}$.

1.73: a) Angle of first line is $\theta = \tan^{-1}\left(\frac{200-20}{210-10}\right) = 42^\circ$. Angle of second line is $42^\circ + 30^\circ = 72^\circ$. Therefore

$$X = 10 + 250\cos 72^\circ = 87$$
$$Y = 20 + 250\sin 72^\circ = 258$$

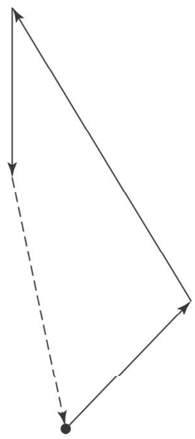
for a final point of (87,258).

b) The computer screen now looks something like this:



The length of the bottom line is $\sqrt{(210-87)^2 + (200-258)^2} = 136$ and its direction is $\tan^{-1}\left(\frac{258-200}{210-87}\right) = 25^\circ$ below straight left.

1.74: a)



b) To use the method of components, let the east direction be the *x*-direction and the north direction be the *y*-direction. Then, the explorer's net *x*-displacement is, in units of his step size,

$$(40)\cos 45^\circ - (80)\cos 60^\circ = -11.7$$

and the *y*-displacement is

$$(40)\sin 45^\circ + (80)\sin 60^\circ - 50 = 47.6.$$

The magnitude and direction of the displacement are

$$\sqrt{(-11.7)^2 + (47.6)^2} = 49, \quad \arctan\left(\frac{47.6}{-11.7}\right) = 104^\circ.$$

(More precision in the angle is not warranted, as the given measurements are to the nearest degree.) To return to the hut, the explorer must take 49 steps in a direction $104^\circ - 90^\circ = 14^\circ$ east of south.

1.75: Let $+x$ be east and $+y$ be north. Let $\vec{A}$ be the displacement 285 km at 40.0° north of west and let $\vec{B}$ be the unknown displacement.

$\vec{A} + \vec{B} = \vec{R}$, where $\vec{R} = 115\,\text{km}$, east

$\vec{B} = \vec{R} - \vec{A}$

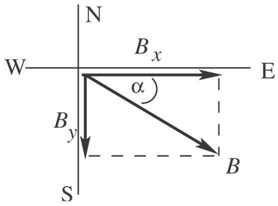
$B_x = R_x - A_x,\ B_y = R_y - A_y$

$A_x = -A\cos 40.0^\circ = -218.3\,\text{km},\ A_y = +A\sin 40.0^\circ = +183.2\,\text{km}$

$R_x = 115\,\text{km},\ R_y = 0$

Then $B_x = 333.3\,\text{km},\ B_y = -183.2\,\text{km}.$

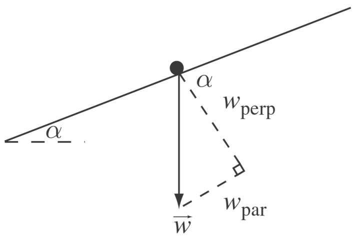
$B = \sqrt{B_x^2 + B_y^2} = 380\,\text{km}.$



$\tan\alpha = \left|B_y / B_x\right| = (183.2\,\text{km})/(333.3\,\text{km})$

$\alpha = 28.8^\circ$, south of east

1.76:



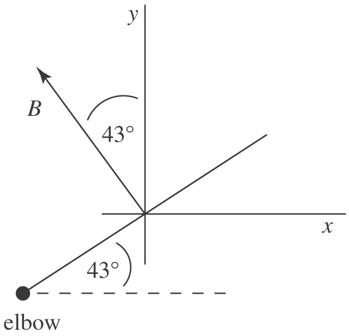
(a) $\omega_{\text{par}} = \omega \sin\alpha$

(b) $\omega_{\text{perp}} = \omega \cos\alpha$

(c) $\omega_{\text{par}} = \omega \sin\alpha$

$$\omega = \frac{\omega_{\text{par}}}{\sin\alpha} = \frac{550\ \text{N}}{\sin 35.0^0} = 960\,\text{N}$$

1.77:



$\vec{B}$ is the force the biceps exerts.

$\vec{E}$ is the force the elbow exerts.

$\vec{E} + \vec{B} = \vec{R}$, where $R = 132.5$ N and is upward.

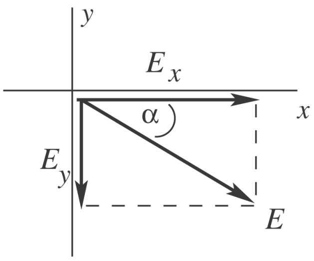
$E_x = R_x - B_x, \quad E_y = R_y - B_y$

$B_x = -B \sin 43^\circ = -158.2$ N, $B_y = +B \cos 43^\circ = +169.7$ N

$R_x = 0$, $R_y = +132.5$ N

Then $E_x = +158.2$ N, $E_y = -37.2$ N

$E = \sqrt{E_x^2 + E_y^2} = 160$ N;



$\tan\alpha = \left|E_y / E_x\right| = 37.2/158.2$

$\alpha = 13^\circ$, below horizontal

1.78: (a) Take the beginning of the journey as the origin, with north being the *y*-direction, east the *x*-direction, and the *z*-axis vertical. The first displacement is then $-30\hat{\boldsymbol{k}}$, the second is $-15\hat{\boldsymbol{j}}$, the third is $200\hat{\boldsymbol{i}}$ $(0.2\,\text{km} = 200\,\text{m})$, and the fourth is $100\hat{\boldsymbol{j}}$. Adding the four:

$$-30\hat{\boldsymbol{k}} - 15\hat{\boldsymbol{j}} + 200\hat{\boldsymbol{i}} + 100\hat{\boldsymbol{j}} = 200\hat{\boldsymbol{i}} + 85\hat{\boldsymbol{j}} - 30\hat{\boldsymbol{k}}$$

(b) The total distance traveled is the sum of the distances of the individual segments: 30 + 15 + 200 + 100 = 345 m. The magnitude of the total displacement is:

$$D = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{200^2 + 85^2 + (-30)^2} = 219\,\text{m}$$

1.79: Let the displacement from your camp to the store be $\vec{\boldsymbol{A}}$.

$A = 240\,\text{m}, 32^\circ$ south of east

$\vec{\boldsymbol{B}}$ is 32° south of west and $\vec{\boldsymbol{C}}$ is 62° south of west

Let $+x$ be east and $+y$ be north

$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} + \vec{\boldsymbol{C}} = \boldsymbol{0}$

$A_x + B_x + C_x = 0$, so $A\cos 32^\circ - B\cos 48^\circ - C\cos 62^\circ = 0$

$A_y + B_y + C_y = 0$, so $-A\sin 32^\circ + B\sin 48^\circ - C\sin 62^\circ = 0$

A is known so we have two equations in the two unknowns *B* and *C*. Solving gives B = 255 m and C = 70 m.

1.80: Take your tent's position as the origin. The displacement vector for Joe's tent is $(21\cos 23^\circ)\hat{\boldsymbol{i}} - (21\sin 23^\circ)\hat{\boldsymbol{j}} = 19.33\hat{\boldsymbol{i}} - 8.205\hat{\boldsymbol{j}}$. The displacement vector for Karl's tent is $(32\cos 37^\circ)\hat{\boldsymbol{i}} + (32\sin 37^\circ)\hat{\boldsymbol{j}} = 25.56\hat{\boldsymbol{i}} + 19.26\hat{\boldsymbol{j}}$. The difference between the two displacements is:

$$(19.33 - 25.56)\hat{\boldsymbol{i}} + (-8.205 - 19.25)\hat{\boldsymbol{j}} = -6.23\hat{\boldsymbol{i}} - 27.46\hat{\boldsymbol{j}}.$$

The magnitude of this vector is the distance between the two tents:

$$D = \sqrt{(-6.23)^2 + (-27.46)^2} = 28.2\,\text{m}$$

1.81: a) With $A_z = B_z = 0$, Eq. (1.22) becomes

$$
\begin{aligned}
A_x B_x + A_y B_y &= (A\cos\,\theta_A)(\mathrm{B}\cos\,\theta_B) + (A\sin\,\theta_A)(B\sin\,\theta_B) \\
&= AB(\cos\,\theta_A \cos\,\theta_B + \sin\,\theta_A \sin\,\theta_B) \\
&= AB\cos(\,\theta_A - \theta_B) \\
&= AB\cos\phi
\end{aligned}
$$

where the expression for the cosine of the difference between two angles has been used (see Appendix B).

b) With $A_z = B_z = 0$, $\vec{C} = C_z \hat{k}$ and $C = |C_z|$. From Eq. (1.27),

$$
\begin{aligned}
|C| &= |A_x B_x - A_y B_x| \\
&= |(A\cos\theta_A)(B\cos\theta_B) - (A\sin\theta_A)(B\cos\theta_A)| \\
&= AB|\cos\theta_A \sin\theta_B - \sin\theta_A \cos\theta_B| \\
&= AB|\sin(\theta_B - \theta_A)| \\
&= AB\sin\phi
\end{aligned}
$$

2: a) The angle between the vectors is $210^\circ - 70^\circ = 140^\circ$, and so Eq. (1.18) gives $\vec{A}\cdot\vec{B} = (3.60\ \mathrm{m})(2.40\ \mathrm{m})\cos 140^\circ = -6.62\ \mathrm{m}^2$ Or, Eq. (1.21) gives

$$
\begin{aligned}
\vec{A}\cdot\vec{B} &= A_x B_x + A_y B_y \\
&= (3.60\ \mathrm{m})\cos 70^\circ (2.4\ \mathrm{m})\cos 210^\circ + (3.6\ \mathrm{m})\sin 70^\circ (2.4\ \mathrm{m})\sin 210^\circ \\
&= -6.62\ \mathrm{m}^2
\end{aligned}
$$

b) From Eq. (1.22), the magnitude of the cross product is

$$(3.60\ \mathrm{m})(2.40\ \mathrm{m})\sin 140^\circ = 5.55\ \mathrm{m}^2,$$

and the direction, from the right-hand rule, is out of the page (the +*z*-direction). From Eq. (1-30), with the *z*-components of $\vec{A}$ and $\vec{B}$ vanishing, the *z*-component of the cross product is

$$
\begin{aligned}
A_x B_y - A_y B_x &= (3.60\ \mathrm{m})\cos 70^\circ (2.40\ \mathrm{m})\sin 210^\circ \\
&\quad - (3.60\ \mathrm{m})\sin 70^\circ (2.40\ \mathrm{m})\cos 210^\circ \\
&= 5.55\ \mathrm{m}^2
\end{aligned}
$$

1.83: a) Parallelogram area $= 2 \times$ area of triangle ABC

Triangle area $= 1/2\,(\text{base})(\text{height}) = 1/2\,(\text{B})(\text{A}\sin\theta)$

Parellogram area $= BA\sin\theta$

b) 90°

1.84: With the +*x*-axis to the right, +*y*-axis toward the top of the page, and +*z*-axis out of the page, $(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}})_x = 87.8\ \text{cm}^2$, $(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}})_y = 68.9\ \text{cm}^2$, $(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}})_z = 0$.

1.85: a)

$$A = \sqrt{(2.00)^2 + (3.00)^2 + (4.00)^2} = 5.39.$$

$$B = \sqrt{(3.00)^2 + (1.00)^2 + (3.00)^2} = 4.36.$$

b)

$$\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = (A_x - B_x)\hat{\boldsymbol{i}} + (A_y - B_y)\hat{\boldsymbol{j}} + (A_z - B_z)\hat{\boldsymbol{k}}$$

$$= (-5.00)\hat{\boldsymbol{i}} + (2.00)\hat{\boldsymbol{j}} + (7.00)\hat{\boldsymbol{k}}$$

c) $\sqrt{(5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83,$

and this will be the magnitude of $\vec{\boldsymbol{B}} - \vec{\boldsymbol{A}}$ as well.

1.86: The direction vectors each have magnitude $\sqrt{3}$, and their dot product is (1) (1) + (1) (–1) + (1) (–1) = –1, so from Eq. (1-18) the angle between the bonds is arccos $= \left(\frac{-1}{\sqrt{3}\sqrt{3}}\right) =$ arccos $\left(-\frac{1}{3}\right) = 109^\circ$.

1.87: The best way to show these results is to use the result of part (a) of Problem 1-65, a restatement of the law of cosines. We know that

$$C^2 = A^2 + B^2 + 2AB\cos\phi,$$

where ϕ is the angle between $\vec{A}$ and $\vec{B}$.

a) If $C^2 = A^2 + B^2$, $\cos\phi = 0$, and the angle between $\vec{A}$ and $\vec{B}$ is 90° (the vectors are perpendicular).

b) If $C^2 < A^2 + B^2$, $\cos\phi < 0$, and the angle between $\vec{A}$ and $\vec{B}$ is greater than 90°.

c) If $C^2 > A^2 + B^2$, $\cos\phi > 0$, and the angle between $\vec{A}$ and $\vec{B}$ is less than 90°.

1.88: a) This is a statement of the law of cosines, and there are many ways to derive it. The most straightforward way, using vector algebra, is to assume the linearity of the dot product (a point used, but not explicitly mentioned in the text) to show that the square of the magnitude of the sum $\vec{A} + \vec{B}$ is

$$\begin{aligned}
\left(\vec{A} + \vec{B}\right)\cdot\left(\vec{A} + \vec{B}\right) &= \vec{A}\cdot\vec{A} + \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B} \\
&= \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} + 2\,\vec{A}\cdot\vec{B} \\
&= A^2 + B^2 + 2\vec{A}\cdot\vec{B} \\
&= A^2 + B^2 + 2AB\cos\phi
\end{aligned}$$

Using components, if the vectors make angles θ_A and θ_B with the x-axis, the components of the vector sum are $A\cos\theta_A + B\cos\theta_B$ and $A\sin\theta_A + B\sin\theta_B$, and the square of the magnitude is

$$\begin{aligned}
&(A\cos\theta_A + B\cos\theta_B)^2 + (A\sin\theta_A + B\sin\theta_B)^2 \\
&\quad = A^2\left(\cos^2\theta_A + \sin^2\theta_A\right) + B^2\left(\cos^2\theta_B + \sin^2\theta_B\right) \\
&\qquad\qquad + 2AB\left(\cos\theta_A\cos\theta_B + \sin\theta_A\sin\theta_B\right) \\
&\quad = A^2 + B^2 + 2AB\cos(\theta_A - \theta_B) \\
&\quad = A^2 + B^2 + 2AB\cos\phi
\end{aligned}$$

where $\phi = \theta_A - \theta_B$ is the angle between the vectors.

b) A geometric consideration shows that the vectors $\vec{A}$, $\vec{B}$ and the sum $\vec{A} + \vec{B}$ must be the sides of an equilateral triangle. The angle *between* $\vec{A}$, and $\vec{B}$ is 120°, since one vector must shift to add head-to-tail. Using the result of part (a), with $A = B$, the condition is that $A^2 = A^2 + A^2 + 2A^2\cos\phi$, which solves for $1 = 2 + 2\cos\phi$, $\cos\phi = -\frac{1}{2}$, and $\phi = 120^\circ$.

c) Either method of derivation will have the angle ϕ replaced by $180^\circ - \phi$, so the cosine will change sign, and the result is $\sqrt{A^2 + B^2 - 2AB\cos\phi}$.

d) Similar to what is done in part (b), when the vector *difference* has the same magnitude, the angle between the vectors is 60°. Algebraically, ϕ is obtained from $1 = 2 - 2\cos\phi$, so $\cos\phi = \frac{1}{2}$ and $\phi = 60^\circ$.

1.89: Take the length of a side of the cube to be L, and denote the vectors from a to b, a to c and a to d as $\vec{\boldsymbol{B}}, \vec{\boldsymbol{C}}$, and $\vec{\boldsymbol{D}}$. In terms of unit vectors,

$$\vec{\boldsymbol{B}} = L\hat{\boldsymbol{k}}, \qquad \vec{\boldsymbol{C}} = L\left(\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right), \qquad \vec{\boldsymbol{D}} = L\left(\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right)$$

Using Eq. (1.18),

$$\arccos\left(\frac{\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{D}}}{BD}\right) = \arccos\left(\frac{L^2}{(L)\left(L\sqrt{3}\right)}\right) = 54.7^\circ,$$

$$\arccos\left(\frac{\vec{\boldsymbol{C}}\cdot\vec{\boldsymbol{D}}}{CD}\right) = \arccos\left(\frac{2L^2}{\left(L\sqrt{2}\right)\left(L\sqrt{3}\right)}\right) = 35.3^\circ.$$

1.90: From Eq. (1.27), the cross product is

$$(-13.00)\ \hat{\boldsymbol{i}} + (6.00)\ \hat{\boldsymbol{j}} + (-11.00)\ \hat{\boldsymbol{k}} = 13\left[-(1.00)\ \hat{\boldsymbol{i}} + \left(\frac{6.00}{13.00}\right)\hat{\boldsymbol{j}} - \frac{11.00}{13.00}\hat{\boldsymbol{k}}\right].$$

The magnitude of the vector in square brackets is $\sqrt{1.93},$ and so a unit vector in this direction (which is necessarily perpendicular to both $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$) is

$$\left[\frac{-(1.00)\hat{\boldsymbol{i}} + (6.00/13.00)\,\hat{\boldsymbol{j}} - (11.00/13)\hat{\boldsymbol{k}}}{\sqrt{1.93}}\right].$$

The negative of this vector,

$$\left[\frac{(1.00)\hat{\boldsymbol{i}} - (6.00/13.00)\,\hat{\boldsymbol{j}} + (11.00/13)\hat{\boldsymbol{k}}}{\sqrt{1.93}}\right],$$

is also a unit vector perpendicular to $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

1.91: $\vec{A}$ and $\vec{C}$ are perpendicular, so $\vec{A}\cdot\vec{C}=0$. $A_x C_x + A_y C_y = 0$, , which gives $5.0C_x - 6.5C_y = 0$.

$\vec{B}\cdot\vec{C} = 15.0$, so $-3.5C_x + 7.0C_y = 15.0$

We have two equations in two unknowns C_x and C_y. Solving gives $C_x = 8.0$ and $C_y = 6.1$.

1.92:

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

$$\sin\theta = \frac{\left|\vec{A}\times\vec{B}\right|}{AB} = \frac{\sqrt{(-5.00)^2+(2.00)^2}}{(3.00)(3.00)} = 0.5984$$

$$\theta = \sin^{-1}(0.5984) = 36.8^\circ$$

1.93: a) Using Equations (1.21) and (1.27), and recognizing that the vectors $\vec{A}, \vec{B}$, and $\vec{C}$ do not have the same meanings as they do in those equations,

$$\begin{aligned}(\vec{A}\times\vec{B})\cdot\vec{C} &= \left((A_y B_z - A_z B_y)\hat{\boldsymbol{i}} + (A_z B_x - A_x B_z)\hat{\boldsymbol{j}} + (A_x B_y - A_y B_x)\hat{\boldsymbol{k}}\right)\cdot\vec{C} \\ &= A_y B_z C_x - A_z B_y C_x + A_z B_x C_y - A_x B_z C_y + A_x B_y C_z - A_y B_x C_z.\end{aligned}$$

A similar calculation shows that

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = A_x B_y C_z - A_x B_z C_y + A_y B_z C_x - A_y B_x C_z + A_z B_x C_y - A_z B_y C_x$$

and a comparison of the expressions shows that they are the same.

b) Although the above expression could be used, the form given allows for ready compuation of $\vec{A}\times\vec{B}$ the magnitude is $AB\sin\phi = (20.00)\sin 37.0^\circ$ and the direction is, from the right-hand rule, in the +z-direction, and so

$$(\vec{A}\times\vec{B})\cdot\vec{C} = +(20.00)\sin 37.0^\circ(6.00) = +72.2.$$

1.94: a) The maximum and minimum areas are

$$(L + l)(W + w) = LW + lW + Lw, \quad (L - l)(W - w) = LW - lW - Lw,$$

where the common terms wl have been omitted. The area and its uncertainty are then $WL \pm (lW + Lw)$, so the uncertainty in the area is $a = lW + Lw$.

b) The fractional uncertainty in the area is

$$\frac{a}{A} = \frac{lW + Wl}{WL} = \frac{l}{L} + \frac{w}{W},$$

the sum of the fractional uncertainties in the length and width.

c) The similar calculation to find the uncertainty v in the volume will involve neglecting the terms lwH, lWh and Lwh as well as lwh; the uncertainty in the volume is $v = lWH + LwH + LWh$, and the fractional uncertainty in the volume is

$$\frac{v}{V} = \frac{lWH + LwH + LWh}{LW\ H} = \frac{l}{L} + \frac{w}{W} + \frac{h}{H},$$

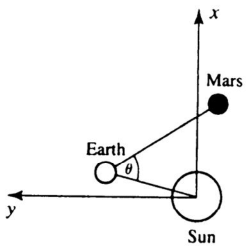
the sum of the fractional uncertainties in the length, width and height.

1.95: The receiver's position is

$$(+1.0 + 9.0 - 6.0 + 12.0)\hat{\boldsymbol{i}} + (-5.0 + 11.0 + 4.0 + 18.0)\hat{\boldsymbol{j}} = (16.0)\hat{\boldsymbol{i}} + (28.0)\hat{\boldsymbol{j}}.$$

The vector from the quarterback to the receiver is the receiver's position minus the quarterback's position, or $(16.0)\hat{\boldsymbol{i}} + (35.0)\hat{\boldsymbol{j}}$, a vector with magnitude $\sqrt{(16.0)^2 + (35.0)^2} = 38.5$, given as being in yards. The angle is $\arctan\left(\frac{16.0}{35.0}\right) = 24.6^\circ$ to the right of downfield.

1.96: a)



b) i) In AU, $\sqrt{(0.3182)^2 + (0.9329)^2} = 0.9857.$

ii) In AU, $\sqrt{(1.3087)^2 + (-.4423)^2 + (-.0414)^2} = 1.3820$

iii) In AU,

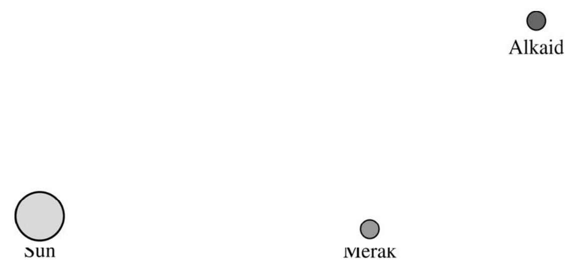
$$\sqrt{(0.3182 - (1.3087))^2 + (0.9329 - (-.4423))^2 + (0.0414)^2} = 1.695.$$

c) The angle between the directions from the Earth to the Sun and to Mars is obtained from the dot product. Combining Equations (1-18) and (1.21),

$$\phi = \arccos\left(\frac{(-0.3182)(1.3087 - 0.3182) + (-0.9329)(-0.4423 - 0.9329)}{(0.9857)(1.695)}\right.$$

d) Mars could not have been visible at midnight, because the Sun-Mars angle is less than 90°.

1.97: a)



The law of cosines (see Problem 1.88) gives the distance as

$$\sqrt{(138\,\text{ly})^2 + (77\,\text{ly})^2 + 2(138\,\text{ly})(77\,\text{ly})\cos 154.4^\circ} = 76.2\,\text{ly},$$

where the supplement of 25.6° has been used for the angle between the direction vectors.

b) Although the law of cosines could be used again, it's far more convenient to use the law of sines (Appendix B), and the angle is given by

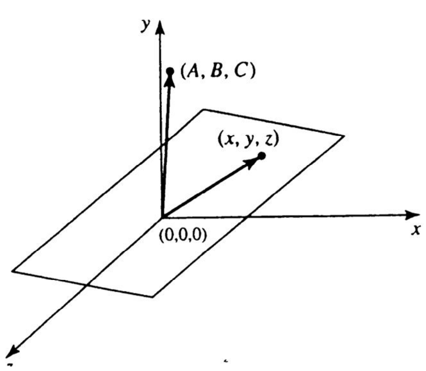
$$\arcsin\left(\frac{\sin 25.6^\circ}{76.2\,\text{ly}}138\,\text{ly}\right) = 51.5^\circ,\ 180^\circ - 51.5^\circ = 129^\circ,$$

where the appropriate angle in the second quadrant is used.

1.98: Define $\vec{S} = A\hat{i} + B\hat{j} + C\hat{k}$

$$\begin{aligned}\vec{r} \cdot \vec{S} &= (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (A\hat{i} + B\hat{j} + C\hat{k}) \\ &= Ax + By + Cz\end{aligned}$$

If the points satisfy $Ax + By + Cz = 0$, then $\vec{r} \cdot \vec{S} = 0$ and all points $\vec{r}$ are perpendicular to $\vec{S}$.



2.1: a) During the later 4.75-s interval, the rocket moves a distance $1.00 \times 10^3 \text{ m} - 63 \text{ m}$, and so the magnitude of the average velocity is

$$\frac{1.00 \times 10^3 \text{ m} - 63 \text{ m}}{4.75 \text{ s}} = 197 \text{ m/s}.$$

b) $\frac{1.00 \times 10^3 \text{ m}}{5.90 \text{ s}} = 169 \text{ m/s}$

2.2: a) The magnitude of the average velocity on the return flight is

$$\frac{(5150 \times 10^3 \text{ m})}{(13.5 \text{ da})(86,\ 400 \text{ s/da})} = 4.42 \text{ m/s}.$$

The direction has been defined to be the –*x*-direction $(-\hat{\boldsymbol{i}})$.

b) Because the bird ends up at the starting point, the average velocity for the round trip is **0**.

2.3: Although the distance could be found, the intermediate calculation can be avoided by considering that the time will be inversely proportional to the speed, and the extra time will be

$$(140 \text{ min})\left(\frac{105 \text{ km/hr}}{70 \text{ km/hr}} - 1\right) = 70 \text{ min}.$$

2.4: The eastward run takes $(200 \text{ m}/5.0 \text{ m/s}) = 40.0 \text{ s}$ and the westward run takes $(280 \text{ m}/4.0 \text{ m/s}) = 70.0 \text{ s}$. a) $(200 \text{ m} + 280 \text{ m})/(40.0 \text{ s} + 70.0 \text{ s}) = 4.4 \text{ m/s}$ to two significant figures. b) The net displacement is 80 m west, so the average velocity is $(80 \text{ m}/110.0 \text{ s}) = 0.73 \text{ m/s}$ in the –*x*-direction $(-\hat{\boldsymbol{i}})$.

2.5: In time *t* the fast runner has traveled 200 m farther than the slow runner:
$(5.50 \text{ m/s})t + 200 \text{ m} = (6.20 \text{ m/s})t$, so $t = 286 \text{ s}$.
Fast runner has run $(6.20 \text{ m/s})t = 1770 \text{ m}$.
Slow runner has run $(5.50 \text{ m/s})t = 1570 \text{ m}$.

2.6: The s-waves travel slower, so they arrive 33 s after the p-waves.

$$t_s = t_p + 33\text{s}$$
$$d = vt \rightarrow t = \frac{d}{v}$$
$$\frac{d}{v_s} = \frac{d}{v_p} + 33\text{s}$$
$$\frac{d}{3.5\,\frac{\text{km}}{\text{s}}} = \frac{d}{6.5\,\frac{\text{km}}{\text{s}}} + 33\text{s}$$
$$d = 250\,\text{km}$$

2.7: a) The van will travel 480 m for the first 60 s and 1200 m for the next 60 s, for a total distance of 1680 m in 120 s and an average speed of $14.0\ \text{m/s}$. b) The first stage of the journey takes $\frac{240\ \text{m}}{8.0\ \text{m/s}} = 30\ \text{s}$ and the second stage of the journey takes $(240\ \text{m}/20\ \text{m/s}) = 12\,\text{s}$, so the time for the 480-m trip is 42 s, for an average speed of $11.4\ \text{m/s}$. c) The first case (part (a)); the average speed will be the numerical average only if the time intervals are the same.

2.8: From the expression for $x(t)$, $x(0) = 0$, $x(2.00\ \text{s}) = 5.60$ m and $x(4.00\ \text{s}) = 20.8$ m. a) $\frac{5.60\ \text{m}-0}{2.00\ \text{s}} = 2.80\ \text{m/s}$ b) $\frac{20.8\ \text{m}-0}{4.00\ \text{s}} = 5.2\ \text{m/s}$ c) $\frac{20.8\ \text{m}-5.60\ \text{m}}{2.00\ \text{s}} = 7.6\ \text{m/s}$

2.9: a) At $t_1 = 0,\ x_1 = 0$, so Eq (2.2) gives

$$v_{av} = \frac{x_2}{t_2} = \frac{(2.4\ \text{m/s}^2)(10.0\,\text{s})^2 - (0.120\ \text{m/s}^3)(10.0\,\text{s})^3}{(10.0\,\text{s})} = 12.0\ \text{m/s}.$$

b) From Eq. (2.3), the instantaneous velocity as a function of time is

$$v_x = 2bt - 3ct^2 = (4.80\ \text{m/s}^2)t - (0.360\ \text{m/s}^3)t^2,$$

so i) $v_x(0) = 0$,

ii) $v_x(5.0\,\text{s}) = (4.80\ \text{m/s}^2)(5.0\,\text{s}) - (0.360\ \text{m/s}^3)(5.0\,\text{s})^2 = 15.0\ \text{m/s}$,

and iii) $v_x(10.0\,\text{s}) = (4.80\ \text{m/s}^2)(10.0\,\text{s}) - (0.360\ \text{m/s}^3)(10.0\,\text{s})^2 = 12.0\,\text{m/s}$.

c) The car is at rest when $v_x = 0$. Therefore $(4.80\ \text{m/s}^2)t - (0.360\ \text{m/s}^3)t^2 = 0$. The only time after $t = 0$ when the car is at rest is $t = \frac{4.80\ \text{m/s}^2}{0.360\ \text{m/s}^3} = 13.3\,\text{s}$

2.10: a) IV: The curve is horizontal; this corresponds to the time when she stops. b) I: This is the time when the curve is most nearly straight and tilted upward (indicating postive velocity). c) V: Here the curve is plainly straight, tilted downward (negative velocity). d) II: The curve has a postive slope that is increasing. e) III: The curve is still tilted upward (positive slope and positive velocity), but becoming less so.

2.11:

Time (s)	0	2	4	6	8	10	12	14	16
Acceleration (m/s^2)	0	1	2	2	3	1.5	1.5	0	

a) The acceleration is not constant, but is approximately constant between the times $t = 4\,\text{s}$ and $t = 8\,\text{s}$.

2.12: The cruising speed of the car is 60 km/hr = 16.7 m/s. a) $\frac{16.7\,\text{m/s}}{10\,\text{s}} = 1.7\,\text{m/s}^2$ (to two significant figures). b) $\frac{0-16.7\,\text{m/s}}{10\,\text{s}} = -1.7\,\text{m/s}^2$ c) No change in speed, so the acceleration is zero. d) The final speed is the same as the initial speed, so the average acceleration is zero.

2.13: a) The plot of the velocity seems to be the most curved upward near t = 5 s. b) The only negative acceleration (downward-sloping part of the plot) is between t = 30 s and t = 40 s. c) At t = 20 s, the plot is level, and in Exercise 2.12 the car is said to be cruising at constant speed, and so the acceleration is zero. d) The plot is very nearly a straight line, and the acceleration is that found in part (b) of Exercise 2.12, $-1.7\,\text{m/s}^2$.
e)

2.14: (a) The displacement vector is:

$$\vec{r}(t) = -(5.0\ \text{m/s})t\hat{\boldsymbol{i}} + (10.0\ \text{m/s})t\hat{\boldsymbol{j}} + \left((7.0\ \text{m/s})t - (3.0\ \text{m/s}^2)t^2\right)\hat{\boldsymbol{k}}$$

The velocity vector is the time derivative of the displacement vector:

$$\frac{d\vec{r}(t)}{dt} = (-5.0\ \text{m/s})\hat{\boldsymbol{i}} + (10.0\ \text{m/s})\hat{\boldsymbol{j}} + (7.0\ \text{m/s} - 2(3.0\ \text{m/s}^2)t)\,\hat{\boldsymbol{k}}$$

and the acceleration vector is the time derivative of the velocity vector:

$$\frac{d^2\vec{r}(t)}{dt^2} = -6.0\ \text{m/s}^2\,\hat{\boldsymbol{k}}$$

At $t = 5.0$ s:

$$\begin{aligned}
\vec{r}(t) &= -(5.0\ \text{m/s})(5.0\,\text{s})\hat{\boldsymbol{i}} + (10.0\ \text{m/s})(5.0\,\text{s})\,\hat{\boldsymbol{j}} + \left((7.0\,\text{m/s})(5.0\,\text{s}) - (-3.0\ \text{m/s}^2)(25.0\,\text{s}^2)\right)\hat{\boldsymbol{k}} \\
&= (-25.0\,\text{m})\hat{\boldsymbol{i}} + (50.0\,\text{m})\,\hat{\boldsymbol{j}} - (40.0\,\text{m})\hat{\boldsymbol{k}} \\
\frac{d\,\vec{r}(t)}{dt} &= (-5.0\ \text{m/s})\hat{\boldsymbol{i}} + (10.0\ \text{m/s})\,\hat{\boldsymbol{j}} + ((7.0\ \text{m/s} - (6.0\ \text{m/s}^2)(5.0\,\text{s}))\,\hat{\boldsymbol{k}} \\
&= (-5.0\ \text{m/s})\hat{\boldsymbol{i}} + (10.0\ \text{m/s})\,\hat{\boldsymbol{j}} - (23.0\ \text{m/s})\,\hat{\boldsymbol{k}} \\
\frac{d^2\vec{r}(t)}{dt^2} &= -6.0\ \text{m/s}^2\,\hat{\boldsymbol{k}}
\end{aligned}$$

(b) The velocity in both the *x*- and the *y*-directions is constant and nonzero; thus the overall velocity can never be zero.

(c) The object's acceleration is constant, since *t* does not appear in the acceleration vector.

2.15: $v_x = \dfrac{dx}{dt} = 2.00\ \text{cm/s} - (0.125\ \text{cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125\ \text{cm/s}^2$

a) At $t = 0, x = 50.0\,\text{cm}, v_x = 2.00\,\text{cm/s}$, $a_x = -0.125\,\text{cm/s}^2$.

b) Set $v_x = 0$ and solve for $t : t = 16.0\,\text{s}$.

c) Set x = 50.0 cm and solve for t. This gives $t = 0$ and $t = 32.0\,\text{s}$. The turtle returns to the starting point after 32.0 s.

d) Turtle is 10.0 cm from starting point when x = 60.0 cm or x = 40.0 cm.
Set $x = 60.0\,\text{cm}$ and solve for $t : t = 6.20\,\text{s}$ and $t = 25.8\,\text{s}$.

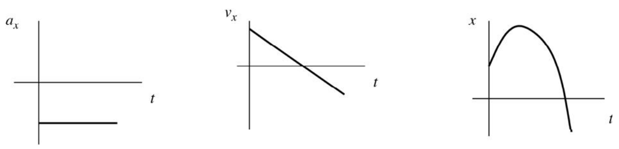
At $t = 6.20\,\text{s}, v_x = +1.23\ \text{cm/s}$.

At $t = 25.8\,\text{s}, v_x = -1.23\ \text{cm/s}$.

Set $x = 40.0\,\text{cm}$ and solve for $t : t = 36.4\,\text{s}$ (other root to the quadratic equation is negative and hence nonphysical).

At $t = 36.4\,\text{s}$, $v_x = -2.55\ \text{cm/s}$.

e)



2.16: Use of Eq. (2.5), with Δt = 10 s in all cases,

a) $\left((5.0\ \text{m/s}) - (15.0\ \text{m/s})\right)/(10\,\text{s}) = -1.0\ \text{m/s}^2$

b) $\left((-15.0\ \text{m/s}) - (-5.0\ \text{m/s})\right)/(10\,\text{s}) = -1.0\ \text{m/s}^2$

c) $\left((-15.0\ \text{m/s}) - (-15.0\ \text{m/s})\right)/(10\,\text{s}) = -3.0\ \text{m/s}^2$.

In all cases, the negative acceleration indicates an acceleration to the left.

2.17: a) Assuming the car comes to rest from 65 mph (29 m/s) in 4 seconds, $a_x = (29\ \text{m/s} - 0)/(4\,\text{s}) = 7.25\ \text{m/s}^2$.

b) Since the car is coming to a stop, the acceleration is in the direction opposite to the velocity. If the velocity is in the positive direction, the acceleration is negative; if the velocity is in the negative direction, the acceleration is positive.

2.18: a) The velocity at $t = 0$ is

$$(3.00\ \text{m/s}) + (0.100\ \text{m/s}^3)\ (0) = 3.00\ \text{m/s},$$

and the velocity at $t = 5.00$ s is

$$(3.00\ \text{m/s}) + (0.100\ \text{m/s}^3)\ (5.00\ \text{s})^2 = 5.50\ \text{m/s},$$

so Eq. (2.4) gives the average acceleration as

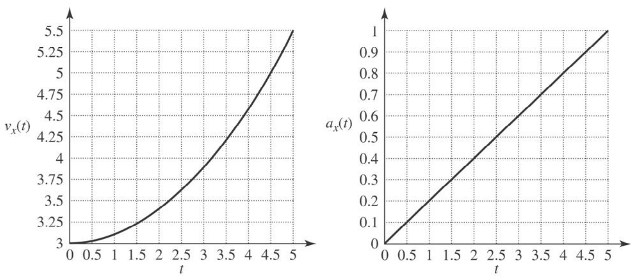
$$\frac{(5.50\ \text{m/s}) - (3.00\ \text{m/s})}{(5.00\ \text{s})} = .50\ \text{m/s}^2.$$

b) The instantaneous acceleration is obtained by using Eq. (2.5),

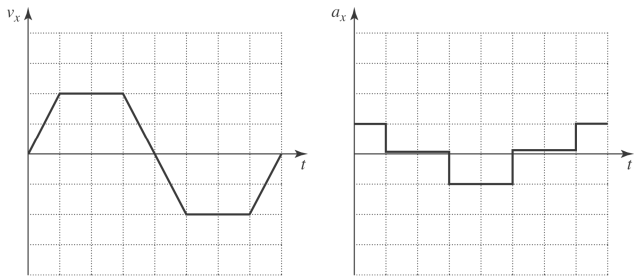
$$a_x = \frac{dv}{dt} = 2\beta t = (0.2\ \text{m/s}^3)t.$$

Then, i) at $t = 0$, $a_x = (0.2\ \text{m/s}^3)\ (0) = 0$, and

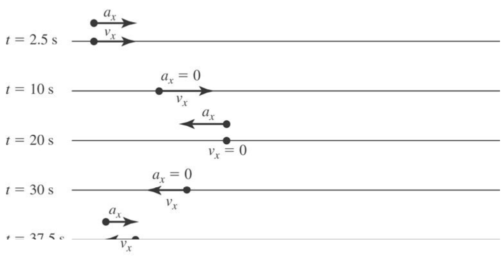
ii) at $t = 5.00$ s, $a_x = (0.2\ \text{m/s}^3)\ (5.00\ \text{s}) = 1.0\ \text{m/s}^2$.



2.19: a)



b)



2.20: a) The bumper's velocity and acceleration are given as functions of time by

$$v_x = \frac{dx}{dt} = (9.60\ \text{m/s}^2)t - (0.600\ \text{m/s}^6)t^5$$

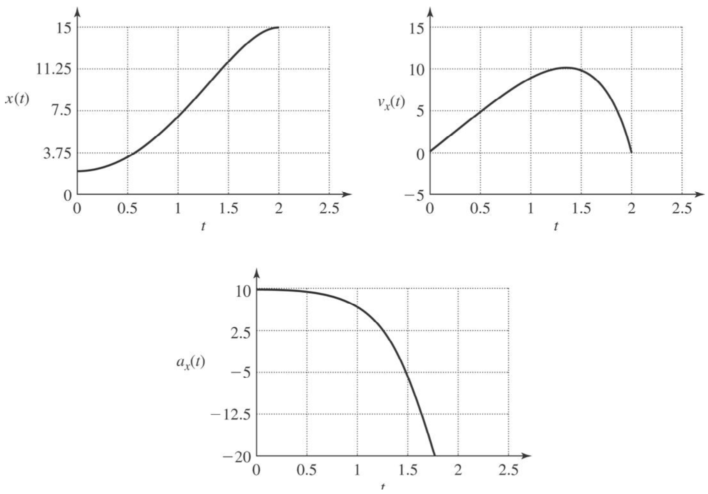
$$a_x = \frac{dv}{dt} = (9.60\ \text{m/s}^2) - (3.000\ \text{m/s}^6)t^4.$$

There are two times at which $v = 0$ (three if negative times are considered), given by $t =$ 0 and $t^4 = 16\ \text{s}^4$. At $t = 0$, $x = 2.17$ m and $a_x = 9.60\ \text{m/s}^2$. When $t^4 = 16\ \text{s}^4$,

$$x = (2.17\ \text{m}) + (4.80\ \text{m/s}^2)\ \sqrt{(16\ \text{s}^4)} - (0.100)\ \text{m/s}^6)(16\ \text{s}^4)^{3/2} = 14.97\ \text{m},$$

$$a_x = (9.60\ \text{m/s}^2) - (3.000\ \text{m/s}^6)(16\ \text{s}^4) = -38.4\ \text{m/s}^2.$$

b)



2.21: a) Equating Equations (2.9) and (2.10) and solving for v_0,

$$v_{0x} = \frac{2(x - x_0)}{t} - v_x = \frac{2(70\,\text{m})}{7.00\,\text{s}} - 15.0\ \text{m/s} = 5.00\ \text{m/s}.$$

b) The above result for v_{0x} may be used to find

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{15.0\ \text{m/s} - 5.00\ \text{m/s}}{7.00\,\text{s}} = 1.43\ \text{m/s}^2,$$

or the intermediate calculation can be avoided by combining Eqs. (2.8) and (2.12) to eliminate v_{0x} and solving for a_x,

$$a_x = 2\left(\frac{v_x}{t} - \frac{x - x_0}{t^2}\right) = 2\left(\frac{15.0\ \text{m/s}}{7.00\,\text{s}} - \frac{70.0\,\text{m}}{(7.00\,\text{s})^2}\right) = 1.43\ \text{m/s}^2.$$

2.22: a) The acceleration is found from Eq. (2.13), which $v_{0x} = 0$;

$$a_x = \frac{v_x^2}{2(x - x_0)} = \frac{\left((173\ \text{mi/hr})\left(\frac{0.4470\ \text{m/s}}{1\ \text{mi/hr}}\right)\right)^2}{2\left((307\ \text{ft})\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)\right)} = 32.0\ \text{m/s}^2,$$

where the conversions are from Appendix E.

b) The time can be found from the above acceleration,

$$t = \frac{v_x}{a_x} = \frac{(173\ \text{mi/hr})\left(\frac{0.4470\ \text{m/s}}{1\ \text{mi/hr}}\right)}{32.0\ \text{m/s}^2} = 2.42\ \text{s}.$$

The intermediate calculation may be avoided by using Eq. (2.14), again with $v_{0x} = 0$,

$$t = \frac{2(x - x_0)}{v_x} = \frac{2\left((307\ \text{ft}\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)\right)}{(173\ \text{mi/hr})\left(\frac{0.4470\ \text{m/s}}{1\ \text{mi/hr}}\right)} = 2.42\ \text{s}.$$

2.23: From Eq. (2.13), with $v_x = 0$, $a_x = \frac{v_0^2}{2(x - x_0)} < a_{\text{max}}$. Taking $x_0 = 0$,

$$x > \frac{v_{0x}^2}{2a_{\text{max}}} = \frac{((105\ \text{km/hr})(1\ \text{m/s})(3.6\ \text{km/hr}))^2}{2(250\ \text{m/s}^2)} = 1.70\ \text{m}.$$

2.24: In Eq. (2.14), with $x - x_0$ being the length of the runway, and $v_{0x} = 0$ (the plane starts from rest), $v_x = 2\frac{x - x_0}{t} = 2\frac{280\ \text{m}}{8\ \text{s}} = 70.0\ \text{m/s}$.

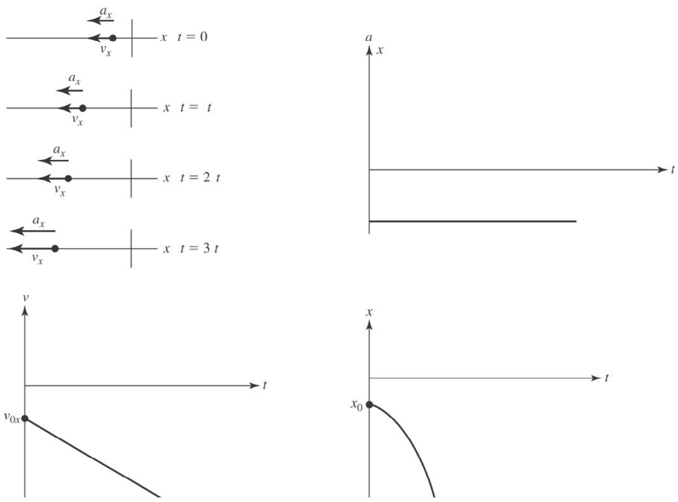
2.25: a) From Eq. (2.13), with $v_{0x} = 0$,

$$a_x = \frac{v_x^2}{2(x - x_0)} = \frac{(20\ \text{m/s})^2}{2(120\ \text{m})} = 1.67\ \text{m/s}^2.$$

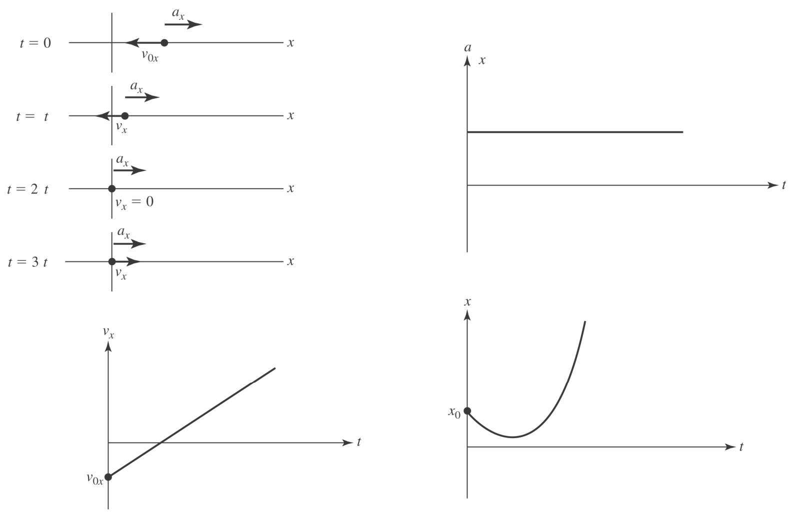
b) Using Eq. (2.14), $t = 2(x - x_0)/v = 2(120\ \text{m})/(20\ \text{m/s}) = 12\ \text{s}$.

c) $(12\ \text{s})(20\ \text{m/s}) = 240\ \text{m}$.

2.26: a) $x_0 < 0$, $v_{0x} < 0$, $a_x < 0$



b) $x_0 > 0$, $v_{0x} < 0$, $a_x > 0$



c) $x_0 > 0$, $v_{0x} > 0$, $a_x < 0$

2.27: a) speeding up:

$x - x_0 = 1320\,\text{ft}, v_{0x} = 0,\ t = 19.9\,\text{s}, a_x = ?$
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = 6.67\ \text{ft/s}^2$

slowing down:

$x - x_0 = 146\,\text{ft}, v_{0x} = 88.0\ \text{ft/s},\ v_x = 0,\ a_x = ?$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = -26.5\ \text{ft/s}^2$.

b) $x - x_0 = 1320\,\text{ft}, v_{0x} = 0, a_x = 6.67\ \text{ft/s}^2,\ v_x = ?$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 133\ \text{ft/s} = 90.5\,\text{mph}$.
a_x must not be constant.

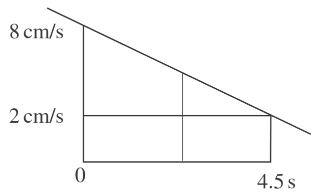
c) $v_{0x} = 88.0\ \text{ft/s},\ a_x = -26.5\ \text{ft/s}^2,\ v_x = 0,\ t = ?$
$v_x = v_{0x} + a_x t$ gives $t = 3.32\,\text{s}$.

2.28: a) Interpolating from the graph:

$$\text{At } 4.0\,\text{s},\ v = +2.7\ \text{cm/s (to the right)}$$
$$\text{At } 7.0\,\text{s},\ v = -1.3\ \text{cm/s (to the left)}$$

b) $a = \text{slope of } v\text{-}t \text{ graph} = -\frac{8.0\ \text{cm/s}}{6.0\ \text{s}} = -1.3\,\text{cm/s}^2$ which is constant

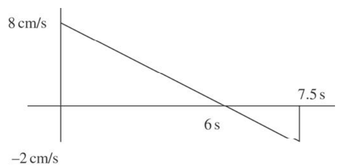
c) $\Delta x = $ area under v-t graph



First 4.5 s:

$$\Delta x = A_{\text{Rectangle}} + A_{\text{Triangle}}$$
$$= (4.5\,\text{s})\left(2\frac{\text{cm}}{\text{s}}\right) + \frac{1}{2}(4.5\,\text{s})\left(6\frac{\text{cm}}{\text{s}}\right) = 22.5\,\text{cm}$$

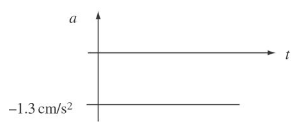
From 0 to 7.5 s:

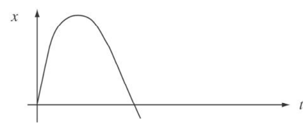


The *distance* is the sum of the magnitudes of the areas.

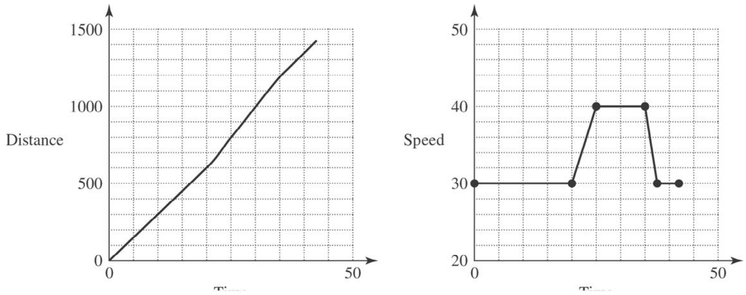
$$d = \frac{1}{2}(6\,\text{s})\left(8\frac{\text{cm}}{\text{s}}\right) + \frac{1}{2}(1.5\,\text{s})\left(2\frac{\text{cm}}{\text{s}}\right) = 25.5\,\text{cm}$$

d)

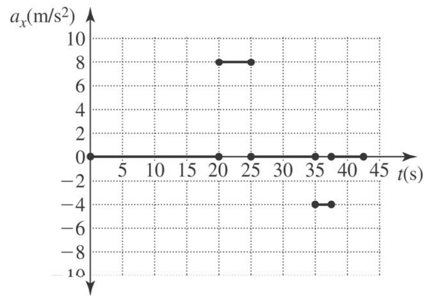




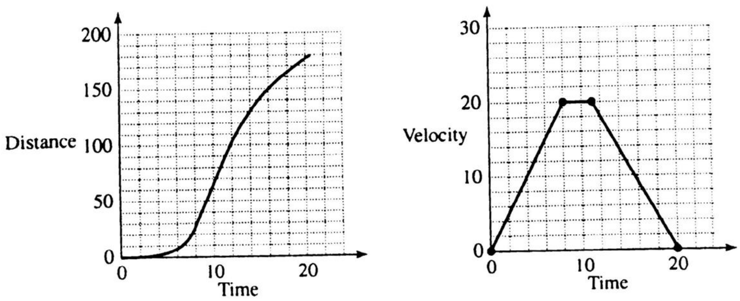
2.29: a)


1500
1000
Distance
500
0
0
50
50
40
Speed
30
20
0
50

b)


a_x(m/s^2)
10
8
6
4
2
0
−2
−4
−6
−8
5 10 15 20 25 30 35 40 45 t(s)

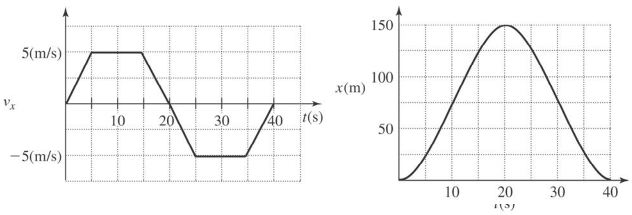
2.30: a)


200
150
Distance 100
50
0
0
10
20
Time
30
20
Velocity
10
0
0
10
20
Time

2.31: a) At t = 3 s the graph is horizontal and the acceleration is 0. From t = 5 s to t = 9 s, the acceleration is constant (from the graph) and equal to $\frac{45\,\mathrm{m/s}-20\mathrm{m/s}}{4\,\mathrm{s}} = 6.3\,\mathrm{m/s}^2$. From t = 9 s to t = 13 s the acceleration is constant and equal to $\frac{0-45\,\mathrm{m/s}}{4\,\mathrm{s}} = -11.2\ \mathrm{m/s}^2$.

b) In the first five seconds, the area under the graph is the area of the rectangle, (20 m)(5 s) = 100 m. Between t = 5 s and t = 9 s, the area under the trapezoid is (1/2)(45 m/s + 20 m/s)(4 s) = 130 m (compare to Eq. (2.14)), and so the total distance in the first 9 s is 230 m. Between t = 9 s and t = 13 s, the area under the triangle is $(1/2)(45\,\mathrm{m/s})(4\,\mathrm{s}) = 90\,\mathrm{m}$, and so the total distance in the first 13 s is 320 m.

2.32:



2.33: a) The maximum speed will be that after the first 10 min (or 600 s), at which time the speed will be

$$(20.0\ \mathrm{m/s}^2)(900\,\mathrm{s}) = 1.8\times 10^4\ \mathrm{m/s} = 18\ \mathrm{km/s}.$$

b) During the first 15 minutes (and also during the last 15 minutes), the ship will travel $(1/2)(18\ \mathrm{km/s})(900\,\mathrm{s}) = 8100\,\mathrm{km}$, so the distance traveled at non-constant speed is 16,200 km and the fraction of the distance traveled at constant speed is

$$1 - \frac{16{,}200\,\mathrm{km}}{384{,}000\,\mathrm{km}} = 0.958,$$

keeping an extra significant figure.

c) The time spent at constant speed is $\frac{384{,}000\ \mathrm{km} - 16{,}200\ \mathrm{km}}{18\,\mathrm{km/s}} = 2.04\times 10^4\,\mathrm{s}$ and the time spent during both the period of acceleration and deceleration is 900 s, so the total time required for the trip is $2.22\times 10^4\,\mathrm{s}$, about 6.2 hr.

2.34: After the initial acceleration, the train has traveled

$$\frac{1}{2}(1.60\ \text{m/s}^2)(14.0\ \text{s})^2 = 156.8\ \text{m}$$

(from Eq. (2.12), with $x_0 = 0$, $v_{0x} = 0$), and has attained a speed of

$$(1.60\ \text{m/s}^2)(14.0\ \text{s}) = 22.4\ \text{m/s}.$$

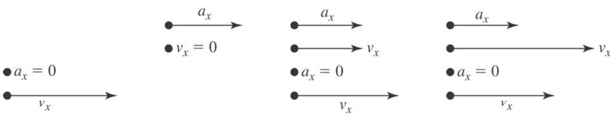
During the 70-second period when the train moves with constant speed, the train travels $(22.4\ \text{m/s})(70\ \text{s}) = 1568\ \text{m}$. The distance traveled during deceleration is given by Eq. (2.13), with $v_x = 0, v_{0x} = 22.4\ \text{m/s}$ and $a_x = -3.50\ \text{m/s}^2$, so the train moves a distance $x - x_0 = \frac{-(22.4\ \text{m/s})^2}{2(-3.50\ \text{m/s}^2)} = 71.68\ \text{m}$. The total distance covered in then 156.8 m + 1568 m + 71.7 m = 1.8 km.

In terms of the initial acceleration a_1, the initial acceleration time t_1, the time t_2 during which the train moves at constant speed and the magnitude a_2 of the final acceleration, the total distance x_T is given by

$$x_\text{T} = \frac{1}{2}a_1 t_1^2 + (a_1 t_1)t_2 + \frac{1}{2}\frac{(a_1 t_1)^2}{|a_2|} = \left(\frac{a_1 t_1}{2}\right)\left(t_1 + 2t_2 + \frac{a_1 t_1}{|a_2|}\right),$$

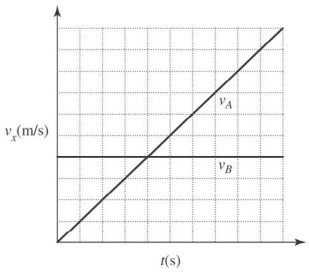
which yields the same result.

2.35: a)



b) From the graph (Fig. (2.35)), the curves for *A* and *B* intersect at $t = 1$ s and $t = 3$ s.

c)



d) From Fig. (2.35), the graphs have the same slope at $t = 2$ s . e) Car *A* passes car *B* when they have the same position and the slope of curve *A* is greater than that of curve *B* in Fig. (2.30); this is at $t = 3$ s. f) Car *B* passes car *A* when they have the same position and the slope of curve *B* is greater than that of curve *A*; this is at $t = 1$ s.

2.36: a) The truck's position as a function of time is given by $x_T = v_T t$, with v_T being the truck's constant speed, and the car's position is given by $x_C = (1/2)\, a_C t^2$. Equating the two expressions and dividing by a factor of t (this reflects the fact that the car and the truck are at the same place at $t = 0$) and solving for t yields

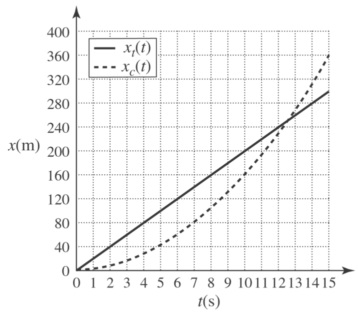
$$t = \frac{2v_T}{a_C} = \frac{2(20.0 \text{ m/s})}{3.20 \text{ m/s}^2} = 12.5 \text{ s}$$

and at this time

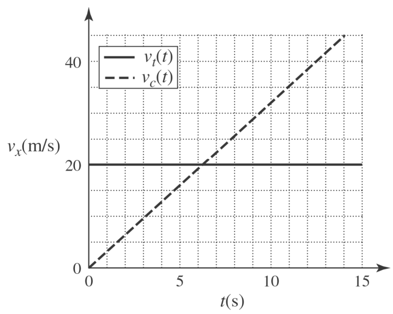
$$x_T = x_C = 250 \text{ m}.$$

b) $a_C t = (3.20 \text{ m/s}^2)(12.5 \text{ s}) = 40.0$ m/s (See Exercise 2.37 for a discussion of why the car's speed at this time is twice the truck's speed.)

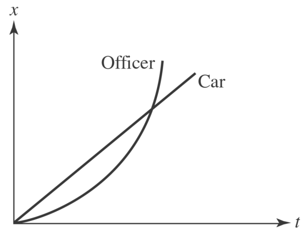
c)



d)



2.37: a)



The car and the motorcycle have gone the same distance during the same time, so their average speeds are the same. The car's average speed is its constant speed v_C, and for constant acceleration from rest, the motorcycle's speed is always twice its average, or $2v_C$. b) From the above, the motorcyle's speed will be v_C after half the time needed to catch the car. For motion from rest with constant acceleration, the distance traveled is proportional to the square of the time, so for half the time one-fourth of the total distance has been covered, or $d/4$.

2.38: a) An initial height of 200 m gives a speed of 60 m/s when rounded to one significant figure. This is approximately 200 km/hr or approximately 150 mi/hr. (Different values of the approximate height will give different answers; the above may be interpreted as slightly better than order of magnitude answers.) b) Personal experience will vary, but speeds on the order of one or two meters per second are reasonable. c) Air resistance may certainly not be neglected.

2.39: a) From Eq. (2.13), with $v_y = 0$ and $a_y = -g$,

$$v_{0y} = \sqrt{2g(y - y_0)} = \sqrt{2(9.80\ \text{m/s}^2)(0.440\ \text{m})} = 2.94\ \text{m/s},$$

which is probably too precise for the speed of a flea; rounding down, the speed is about $2.9\ \text{m/s}$.

b) The time the flea is rising is the above speed divided by g, and the total time is twice this; symbolically,

$$t = 2\frac{\sqrt{2g(y - y_0)}}{g} = 2\sqrt{\frac{2(y - y_0)}{g}} = 2\sqrt{\frac{2(0.440\ \text{m})}{(9.80\ \text{m/s}^2)}} = 0.599\ \text{s},$$

or about 0.60 s.

2.40: Using Eq. (2.13), with downward velocities and accelerations being positive, v_y^2 = $(0.8\ \text{m/s})^2 + 2(1.6\ \text{m/s}^2)(5.0\ \text{m}) = 16.64\ \text{m}^2/\text{s}^2$ (keeping extra significant figures), so v_y = $4.1\ \text{m/s}$.

2.41: a) If the meter stick is in free fall, the distance d is related to the reaction time t by $d = (1/2)gt^2$, so $t = \sqrt{2d/g}$. If d is measured in centimeters, the reaction time is

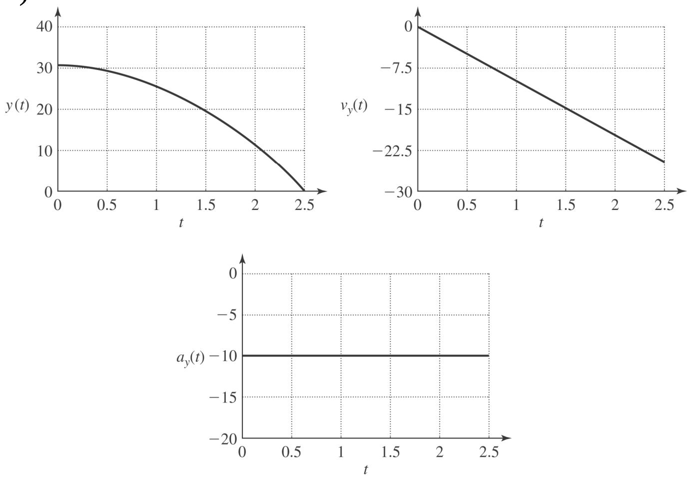
$$t = \sqrt{\frac{2}{g}}\sqrt{d} = \sqrt{\frac{2}{980\ \text{cm/s}^2}}\sqrt{d} = (4.52\times10^{-2}\ \text{s})\sqrt{d/(1\,\text{cm})}.$$

b) Using the above result, $(4.52\times10^{-2}\,\text{s})\sqrt{17.6} = 0.190\,\text{s}$.

2.42: a) $(1/2)gt^2 = (1/2)(9.80\ \text{m/s}^2)(2.5\,\text{s})^2 = 30.6\,\text{m}$.

b) $gt = (9.80\,\text{m/s}^2)(2.5\,\text{s}) = 24.5.\text{m/s}$.

c)



2.43: a) Using the method of Example 2.8, the time the ring is in the air is

$$t = \frac{v_{0y} + \sqrt{v_{0y}^2 - 2g(y - y_0)}}{g}$$

$$= \frac{(5.00 \text{ m/s}) + \sqrt{(5.00 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(-12.0 \text{ m})}}{(9.80 \text{ m/s}^2)}$$

$$= 2.156 \text{ s},$$

keeping an extra significant figure. The average velocity is then $\frac{12.0 \text{ m}}{2.156 \text{ s}} = 5.57 \text{ m/s}$, down. As an alternative to using the quadratic formula, the speed of the ring when it hits the ground may be obtained from $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, and the average velocity found from $\frac{v_y + v_{0y}}{2}$; this is algebraically identical to the result obtained by the quadratic formula.

b) While the ring is in free fall, the average acceleration is the constant acceleration due to gravity, 9.80 m/s^2 down.

c)
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

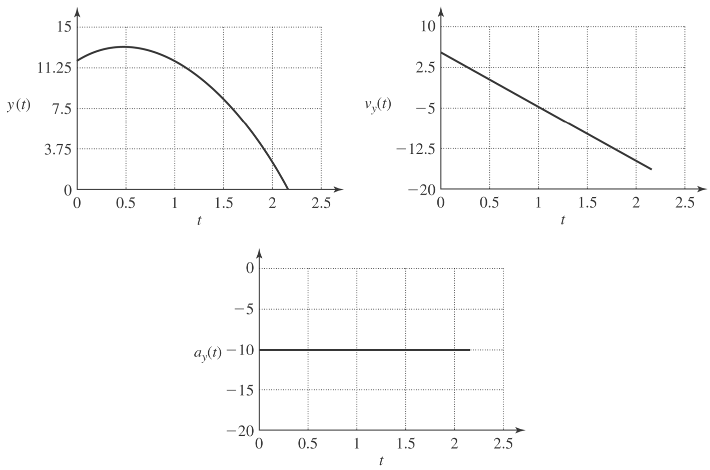
$$0 = 12.0 \text{ m} + (5.00 \text{ m/s})t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2$$

Solve this quadratic as in part a) to obtain t = 2.156 s.

d) $v_y^2 = v_{0y}^2 - 2g(y - y_0) = (5.00 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(-12.0 \text{ m})$

$|v_y| = 16.1 \text{ m/s}$

e)



2.44: a) Using $a_y = -g$, $v_{0y} = 5.00 \text{ m/s}$ and $y_0 = 40.0$ m in Eqs. (2.8) and (2.12) gives i) at $t = 0.250$ s,

$$y = (40.0 \text{ m}) + (5.00 \text{ m/s})(0.250 \text{ s}) - (1/2)(9.80 \text{ m/s}^2)(0.250 \text{ s})^2 = 40.9 \text{ m},$$

$$v_y = (5.00 \text{ m/s}) - (9.80 \text{ m/s}^2)(0.250 \text{ s}) = 2.55 \text{ m/s}$$

and ii) at $t = 1.00$ s,

$$y = (40.0 \text{ m}) + (5.00 \text{ m/s})(1.00 \text{ s}) - (1/2)(9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = 40.1 \text{ m},$$

$$v_y = (5.00 \text{ m/s}) - (9.80 \text{ m/s}^2)(1.00 \text{ s}) = -4.80 \text{ m/s}.$$

b) Using the result derived in Example 2.8, the time is

$$t = \frac{(5.00 \text{ m/s}) + \sqrt{(5.00 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(0 - 40.0 \text{ m})}}{(9.80 \text{ m/s}^2)} = 3.41 \text{ s}.$$

c) Either using the above time in Eq. (2.8) or avoiding the intermediate calculation by using Eq. (2.13),

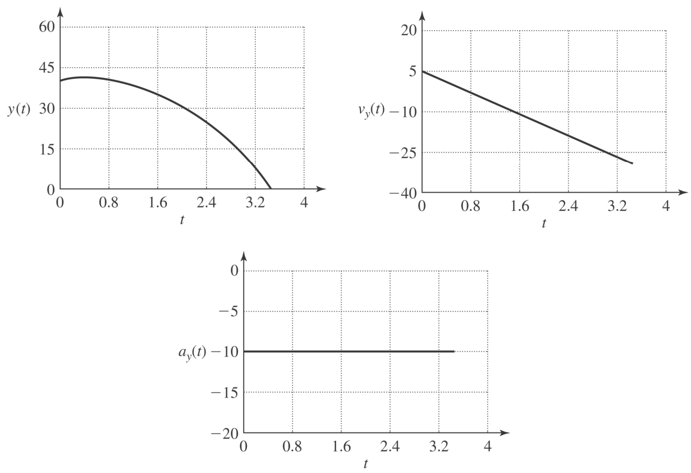
$$v_y^2 = v_{0y}^2 - 2g(y - y_0) = (5.00 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(-40.0 \text{ m}) = 809 \text{ m}^2/\text{s}^2,$$

$$v_y = 28.4 \text{ m/s}.$$

d) Using $v_y = 0$ in Eq. (2.13) gives

$$y = \frac{v_0^2}{2g} + y_0 = \frac{(5.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} + 40.0 \text{ m} = 41.2 \text{ m}.$$

e)



2.45: a) $v_y = v_{0y} - gt = (-6.00\ \mathrm{m/s}) - (9.80\ \mathrm{m/s^2})(2.00\,\mathrm{s}) = -25.6\ \mathrm{m/s}$, so the speed is $25.6\ \mathrm{m/s}$.

b) $y = v_{0y}t - \frac{1}{2}gt^2 = (-6.00\ \mathrm{m/s})(2.00\,\mathrm{s}) - \frac{1}{2}(9.80\ \mathrm{m/s^2})(2.00\,\mathrm{s})^2 = -31.6\,\mathrm{m}$, with the minus sign indicating that the balloon has indeed fallen.

c)
$v_y^2 = v_{0y}^2 - 2g(y_0 - y) = (6.00\ \mathrm{m/s})^2 - 2(9.80\ \mathrm{m/s^2})(-10.0\,\mathrm{m}) = 232\ \mathrm{m^2/s^2}$, so $v_y = 15.2$ m

2.46: a) The vertical distance from the initial position is given by

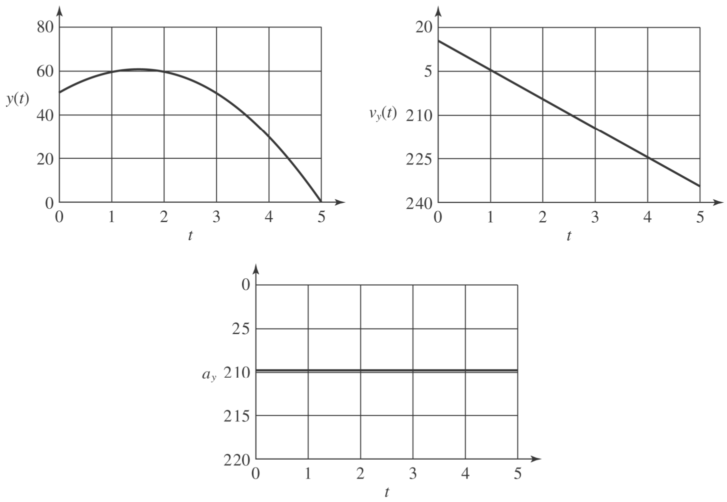
$$y = v_{0y}t - \frac{1}{2}gt^2;$$

solving for v_{0y},

$$v_{0y} = \frac{y}{t} + \frac{1}{2}gt = \frac{(-50.0\ \mathrm{m})}{(5.00\ \mathrm{s})} + \frac{1}{2}(9.80\ \mathrm{m/s^2})(5.00\ \mathrm{s}) = 14.5\ \mathrm{m/s}.$$

b) The above result could be used in $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, with $v = 0$, to solve for $y - y_0 = 10.7$ m (this requires retention of two extra significant figures in the calculation for v_{0y}). c) 0 d) $9.8\ \mathrm{m/s^2}$, down.

e) Assume the top of the building is 50 m above the ground for purposes of graphing:



2.47: a) $(224\ \text{m/s})/(0.9\ \text{s}) = 249\ \text{m/s}^2$. b) $\frac{249\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 25.4$. c) The most direct way to find the distance is $v_{\text{ave}}t = ((224\ \text{m/s})/2)(0.9\ \text{s}) = 101\,\text{m}$.

d) $(283\ \text{m/s})/(1.40\ \text{s}) = 202\ \text{m/s}^2$ but $40g = 392\ \text{m/s}^2$, so the figures are not consistent.

2.48: a) From Eq. (2.8), solving for t gives $(40.0\ \text{m/s} - 20.0\ \text{m/s})/9.80\ \text{m/s}^2 = 2.04$ s.

b) Again from Eq. (2.8),

$$\frac{40.0\ \text{m/s} - (-20.0\ \text{m/s})}{9.80\ \text{m/s}^2} = 6.12\ \text{s}.$$

c) The displacement will be zero when the ball has returned to its original vertical position, with velocity opposite to the original velocity. From Eq. (2.8),

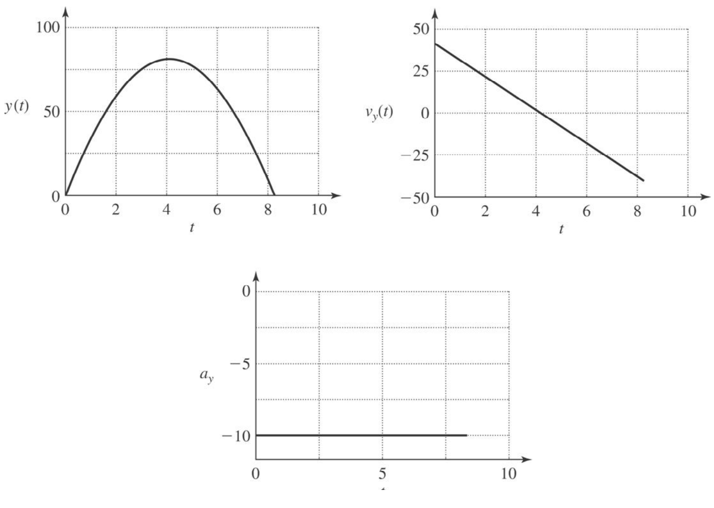
$$\frac{40\ \text{m/s} - (-40\ \text{m/s})}{9.80\ \text{m/s}^2} = 8.16\ \text{s}.$$

(This ignores the $t = 0$ solution.)

d) Again from Eq. (2.8), $(40\ \text{m/s})/(9.80\ \text{m/s}^2) = 4.08$ s. This is, of course, half the time found in part (c).

e) $9.80\ \text{m/s}^2$, down, in all cases.

f)



2.49: a) For a given initial upward speed, the height would be inversely proportional to the magnitude of g, and with g one-tenth as large, the height would be ten times higher, or 7.5 m. b) Similarly, if the ball is thrown with the same upward speed, it would go ten times as high, or 180 m. c) The maximum height is determined by the speed when hitting the ground; if this speed is to be the same, the maximum height would be ten times as large, or 20 m.

2.50: a) From Eq. (2.15), the velocity v_2 at time t

$$v_2 = v_1 + \int_{t_1}^{t} \alpha t \, dt$$

$$= v_1 + \frac{\alpha}{2}\,(t^2 - t_1^2)$$

$$= v_1 - \frac{\alpha}{2}t_1^2 + \frac{\alpha}{2}t^2$$

$$= (5.0 \text{ m/s}) - (0.6 \text{ m/s}^3)(1.0 \text{ s})^2 + (0.6 \text{ m/s}^3)\, t^2$$

$$= (4.40 \text{ m/s}) + (0.6 \text{ m/s}^3)\, t^2.$$

At $t_2 = 2.0$ s, the velocity is $v_2 = (4.40 \text{ m/s}) + (0.6 \text{ m/s}^3)(2.0 \text{ s})^2 = 6.80$ m/s, or 6.8 m/s to two significant figures.

b) From Eq. (2.16), the position x_2 as a function of time is

$$x_2 = x_1 + \int_{t_1}^{t} v_x \, dt$$

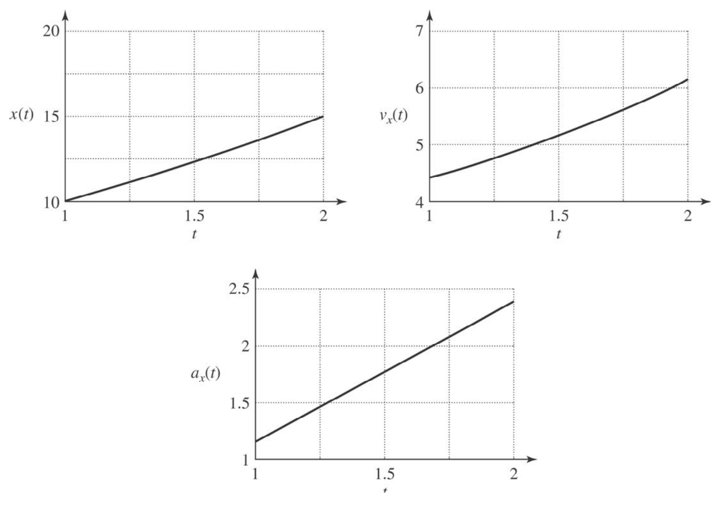
$$= (6.0 \text{ m}) + \int_{t_1}^{t} ((4.40 \text{ m/s}) + (0.6 \text{ m/s}^3)t^2)dt$$

$$= (6.0 \text{ m}) + (4.40 \text{ m/s})(t - t_1) + \frac{(0.6 \text{ m/s}^3)}{3}\,(t^3 - t_1^3).$$

At $t = 2.0$ s, and with $t_1 = 1.0$ s,

$$x = (6.0 \text{ m}) + (4.40 \text{ m/s})((2.0 \text{ s}) - (1.0 \text{ s})) + (0.20 \text{ m/s}^3)((2.0 \text{ s})^3 - (1.0 \text{ s})^3)$$
$$= 11.8 \text{ m}.$$

c)



2.51: a) From Eqs. (2.17) and (2.18), with v_0=0 and x_0=0,

$$v_x = \int_0^t (At - Bt^2)\,dt = \frac{A}{2}t^2 - \frac{B}{3}t^2 = (0.75\ \mathrm{m/s^3})t^3 - (0.040\ \mathrm{m/s^4})t^3$$

$$x = \int_0^t \left(\frac{A}{2}t^2 - \frac{B}{3}t^3\right) dt = \frac{A}{6}t^3 - \frac{B}{12}t^4 = (0.25\,\mathrm{m/s^3})t^3 - (0.010\ \mathrm{m/s^4})t^4.$$

b) For the velocity to be a maximum, the acceleration must be zero; this occurs at t=0 and $t = \frac{A}{B} = 12.5\,\mathrm{s}$. At t=0 the velocity is a minimum, and at t=12.5 s the velocity is

$$v_x = (0.75\,\mathrm{m/s^3})(12.5\,\mathrm{s})^2 - (0.040\ \mathrm{m/s^4})(12.5\,\mathrm{s})^3 = 39.1\,\mathrm{m/s}.$$

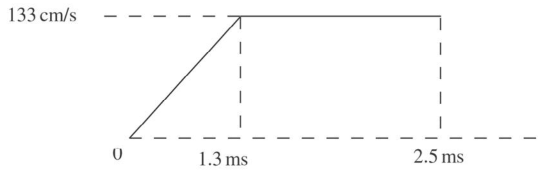
2.52: a) Slope $= a = 0$ for $t \geq 1.3\,\text{ms}$

b) h_{max} = Area under $v - t$ graph

$$\approx A_{\text{Triangle}} + A_{\text{Rectangle}}$$

$$\approx \frac{1}{2}(1.3\,\text{ms})\left(133\frac{\text{cm}}{\text{s}}\right) + (2.5\,\text{ms} - 1.3\,\text{ms})(133\,\text{cm/s})$$

$$\approx 0.25\,\text{cm}$$



c) a = slope of v– t graph

$$a(0.5\,\text{ms}) \approx a(1.0\,\text{ms}) \approx \frac{133\,\text{cm/s}}{1.3\,\text{ms}} = 1.0 \times 10^5\,\text{cm/s}^2$$

$$a(1.5\,\text{ms}) = 0 \text{ because the slope is zero.}$$

d) h = area under v– t graph

$$h(0.5\,\text{ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(0.5\,\text{ms})\left(33\frac{\text{cm}}{\text{s}}\right) = 8.3 \times 10^{-3}\,\text{cm}$$

$$h(1.0\,\text{ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(1.0\,\text{ms})(100\,\text{cm/s}) = 5.0 \times 10^{-2}\,\text{cm}$$

$$h(1.5\,\text{ms}) \approx A_{\text{Triangle}} + A_{\text{Rectangle}} = \frac{1}{2}(1.3\,\text{ms})\left(133\frac{\text{cm}}{\text{s}}\right) + (0.2\,\text{ms})(1.33)$$

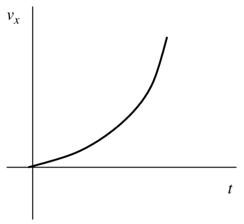
$$= 0.11\,\text{cm}$$

2.53: a) The change in speed is the area under the a_x versus t curve between vertical lines at $t = 2.5\,\text{s}$ and $t = 7.5\,\text{s}$. This area is

$\frac{1}{2}(4.00\ \text{cm/s}^2 + 8.00\,\text{cm/s}^2)(7.5\,\text{s} - 2.5\,\text{s}) = 30.0\ \text{cm/s}$

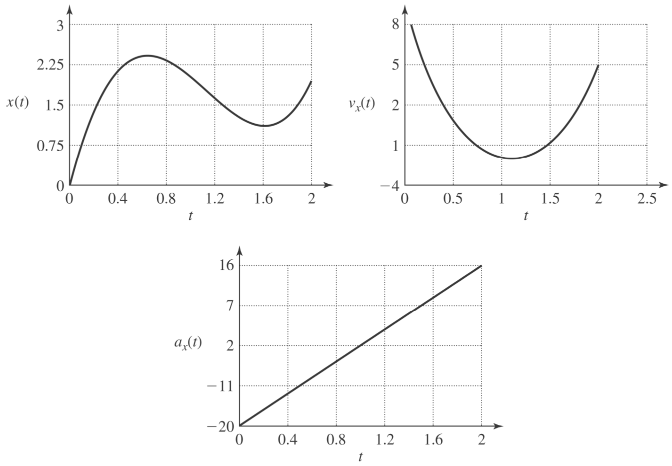
This acceleration is positive so the change in velocity is positive.

b) Slope of v_x versus t is positive and increasing with t.



2.54: a) To average $4\ \text{mi/hr}$, the total time for the twenty-mile ride must be five hours, so the second ten miles must be covered in 3.75 hours, for an average of 2.7 mi/hr. b) To average $12\ \text{mi/hr}$, the second ten miles must be covered in 25 minutes and the average speed must be $24\ \text{mi/hr}$. c) After the first hour, only ten of the twenty miles have been covered, and $16\ \text{mi/hr}$ is not possible as the average speed.

2.55: a)



The velocity and acceleration of the particle as functions of time are

$$v_x(t) = (9.00\ \mathrm{m/s^3})t^2 - (20.00\,\mathrm{m/s^2})t + (9.00\ \ \mathrm{m/s})$$
$$a_x(t) = (18.0\ \mathrm{m/s^3})t - (20.00\ \mathrm{m/s^2}).$$

b) The particle is at rest when the velocity is zero; setting $v = 0$ in the above expression and using the quadratic formula to solve for the time t,

$$t = \frac{(20.0\ \mathrm{m/s^3}) \pm \sqrt{(20.0\ \mathrm{m/s^3})^2 - 4(9.0\ \mathrm{m/s^3})(9.0\ \mathrm{m/s})}}{2(9.0\ \mathrm{m/s^3})}$$

and the times are 0.63 s and 1.60 s. c) The acceleration is negative at the earlier time and positive at the later time. d) The velocity is instantaneously not changing when the acceleration is zero; solving the above expression for $a_x(t) = 0$ gives

$$\frac{20.00\ \mathrm{m/s^2}}{18.00\ \mathrm{m/s^3}} = 1.11\,\mathrm{s}.$$

Note that this time is the numerical average of the times found in part (c). e) The greatest distance is the position of the particle when the velocity is zero and the acceleration is negative; this occurs at 0.63 s, and at that time the particle is at

$$(3.00\,\mathrm{m/s^3})(0.63\,\mathrm{s})^3 - (10.0\,\mathrm{m/s^2})(0.63\,\mathrm{s})^2 + (9.00\ \mathrm{m/s})(0.63\,\mathrm{s}) = 2.45\,\mathrm{m}.$$

(In this case, retaining extra significant figures in evaluating the roots of the quadratic equation does not change the answer in the third place.) f) The acceleration is negative at $t = 0$ and is increasing, so the particle is speeding up at the greatest rate at $t = 2.00$ s and slowing down at the greatest rate at $t = 0$. This is a situation where the extreme values of a function (in the case the acceleration) occur not at times when $\dfrac{da}{dt} = 0$ but at the endpoints of the given range.

2.56: a) $\frac{25.0\,\text{m}}{20.0\,\text{s}} = 1.25\,\text{m/s}$.

b) $\frac{25\,\text{m}}{15\,\text{s}} = 1.67\,\text{m/s}$.

c) Her net displacement is zero, so the average velocity has zero magnitude.

d) $\frac{50.0\,\text{m}}{35.0\,\text{s}} = 1.43\,\text{m/s}$. Note that the answer to part (d) is the *harmonic* mean, not the arithmetic mean, of the answers to parts (a) and (b). (See Exercise 2.5).

2.57: Denote the times, speeds and lengths of the two parts of the trip as t_1 and t_2, v_1 and v_2, and l_1 and l_2.

a) The average speed for the whole trip is

$$\frac{l_1 + l_2}{t_1 + t_2} = \frac{l_1 + l_2}{(l_1/v_1) + (l_2/v_2)} = \frac{(76\,\text{km}) + (34\,\text{km})}{\left(\frac{76\,\text{km}}{88\,\text{km/h}}\right) + \left(\frac{34\,\text{km}}{72\,\text{km/h}}\right)} = 82\,\text{km/h},$$

or 82.3 km/h, keeping an extra significant figure.

b) Assuming nearly straight-line motion (a common feature of Nebraska highways), the total distance traveled is $l_1 – l_2$ and

$$|v_{\text{ave}}| = \frac{l_1 - l_2}{t_1 + t_2} = \frac{(76\,\text{km}) - (34\,\text{km})}{\left(\frac{76\,\text{km}}{88\,\text{km/h}}\right) + \left(\frac{34\,\text{km}}{72\,\text{km/h}}\right)} = 31\,\text{km/h}.$$

($31.4\,\text{km/hr}$ to three significant figures.)

2.58: a) The space per vehicle is the speed divided by the frequency with which the cars pass a given point;

$$\frac{96\ \text{km/h}}{2400\ \text{vehicles/h}} = 40\ \text{m/vehicle}.$$

An average vehicle is given to be 4.5 m long, so the average spacing is 40.0 m – 4.6 m = 35.4 m.

b) An average spacing of 9.2 m gives a space per vehicle of 13.8 m, and the traffic flow rate is

$$\frac{96000\ \text{m/h}}{13.8\ \text{m/vehicle}} = 6960\ \text{vehicle/h}.$$

2.59: (a) Denote the time for the acceleration (4.0 s) as t_1 and the time spent running at constant speed (5.1 s) as t_2. The constant speed is then at_1, where a is the unknown acceleration. The total l is then given in terms of a, t_1 and t_2 by

$$l = \frac{1}{2}at_1^2 + at_1t_2,$$

and solving for a gives

$$a = \frac{l}{(1/2)t_1^2 + t_1t_2} = \frac{(100\,\text{m})}{(1/2)(4.0\,\text{s})^2 + (4.0\,\text{s})(5.1\,\text{s})} = 3.5\ \text{m/s}^2.$$

(b) During the 5.1 s interval, the runner is not accelerating, so $a = 0$.

(c) $\Delta v/\Delta t = [(3.5\ \text{m/s}^2)(4\,\text{s})]/(9.1\,\text{s}) = 1.54\ \text{m/s}^2$.

(d) The runner was moving at constant velocity for the last 5.1 s.

2.60: a) Simple subtraction and division gives average speeds during the 2-second intervals as 5.6, 7.2 and 8.8 m/s.

b) The average speed increased by 1.6 m/s during each 2-second interval, so the acceleration is 0.8 m/s^2.

c) From Eq. (2.13), with $v_0 = 0$, $v = \sqrt{2(0.8\ \text{m/s}^2)(14.4\ \text{m})} = 4.8\ \text{m/s}$. Or, recognizing that for constant acceleration the average speed of 5.6 m/s is the speed one second after passing the 14.4-m mark, $5.6\ \text{m/s} - (0.8\ \text{m/s}^2)(1.0\ \text{s}) = 4.8\ \text{m/s}$.

d) With both the acceleration and the speed at the 14.4-m known, either Eq. (2.8) or Eq. (2.12) gives the time as 6.0 s.

e) From Eq. (2.12), $x - x_0 = (4.8\ \text{m/s})(1.0\ \text{s}) + \frac{1}{2}(0.8\ \text{m/s}^2)(1.0\ \text{s})^2 = 5.2\ \text{m}$. This is also the average velocity $(1/2)(5.6\ \text{m/s} + 4.8\ \text{m/s})$ times the time interval of 1.0 s.

2.61: If the driver steps on the gas, the car will travel

$$(20\,\text{m/s})(3.0\,\text{s}) + (1/2)(2.3\ \text{m/s}^2)(3.0\,\text{s})^2 = 70.4\,\text{m}.$$

If the brake is applied, the car will travel

$$(20\,\text{m/s})(3.0\,\text{s}) + (1/2)(-3.8\ \text{m/s}^2)(3.0\,\text{s})^2 = 42.9\,\text{m},$$

so the driver should apply the brake.

2.62: a) $$d = ct = (3.0\times10^{8}\ \frac{\text{m}}{\text{s}})(1\,\text{y})\left(\frac{365\frac{1}{4}\,\text{d}}{1\,\text{y}}\right)\left(\frac{24\,\text{h}}{1\,\text{d}}\right)\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)$$
$$= 9.5\times10^{15}\ \text{m}$$

b) $$d = ct = (3.0\times10^{8}\ \frac{\text{m}}{\text{s}})(10^{-9}\ \text{s}) = 0.30\ \text{m}$$

c) $$t = \frac{d}{c} = \frac{1.5\times10^{11}\ \text{m}}{3.0\times10^{8}\ \text{m/s}} = 500\ \text{s} = 8.33\ \text{min}$$

d) $$t = \frac{d}{c} = \frac{2(3.84\times10^{8}\ \text{m})}{3.0\times10^{8}\ \text{m/s}} = 2.6\ \text{s}$$

e) $$t = \frac{d}{c} = \frac{3\times10^{9}\ \text{mi}}{186{,}000\ \text{mi/s}} = 16{,}100\ \text{s} = 4.5\ \text{h}$$

2.63: a) $v = 2\pi R_{\text{E}}/t = 464\ \text{m/s}$

b) $v = 2\pi r/t = 2.99\times10^{4}\ \text{m/s}$ (r is the radius of the earth's orbit)

c) Let c be the speed of light, then in one second light travels a distance $c(1.00\,\text{s})$. The number of times around the earth to which this corresponds is $c(1.00\,\text{s})/2\pi R_{\text{E}} = 7.48$

2.64: Taking the start of the race as the origin, runner A's speed at the end of 30 m can be found from:

$$v_A^2 = v_{0A}^2 + 2a_A(x - x_0) = 0 + 2(1.6\ \text{m/s}^2)(30\ \text{m}) = 96\ \text{m}^2/\text{s}^2$$
$$v_A = \sqrt{96\ \text{m}^2/\text{s}^2} = 9.80\ \text{m/s}$$

A's time to cover the first 30 m is thus:

$$t = \frac{v_A - v_{0A}}{a_A} = \frac{9.80\ \text{m/s}}{1.6\ \text{m/s}^2} = 6.13\ \text{s}$$

and A's total time for the race is:

$$6.13\ \text{s} + \frac{(350 - 30)\ \text{m}}{9.80\ \text{m/s}} = 38.8\ \text{s}$$

B's speed at the end of 30 m is found from:

$$v_B^2 = v_{0B}^2 + 2a_B(x - x_0) = 0 + 2(2.0\ \text{m/s}^2)(30\ \text{m}) = 120\ \text{m}^2/\text{s}^2$$
$$v_B = \sqrt{120\ \text{m}^2/\text{s}^2} = 10.95\ \text{m/s}$$

B's time for the first 30 m is thus

$$t = \frac{v_B - v_{0B}}{a_B} = \frac{10.95\ \text{m/s}}{2.0\ \text{m/s}^2} = 5.48\ \text{s}$$

and B's total time for the race is:

$$5.48\ \text{s} + \frac{(350 - 30)\ \text{m}}{10.95\ \text{m/s}} = 34.7\ \text{s}$$

B can thus nap for $38.8 - 34.7 = 4.1\ \text{s}$ and still finish at the same time as A.

2.65: For the first 5.0 s of the motion, $v_{0x} = 0,\ t = 5.0\ \text{s}$.

$v_x = v_{0x} + a_x t$ gives $v_x = a_x(5.0\ \text{s})$.

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:

$v_{0x} = a_x(5.0\ \text{s}),\ t = 5.0\ \text{s},\ x - x_0 = 150\ \text{m}$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $150\ \text{m} = (25\ \text{s}^2)a_x + (12.5\ \text{s}^2)a_x$ and $a_x = 4.0\ \text{m/s}^2$

Use this a_x and consider the first 5.0 s of the motion:

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(4.0\ \text{m/s}^2)(5.0\ \text{s})^2 = 50.0\ \text{m}$.

2.66: a) The simplest way to do this is to go to a frame in which the freight train (which moves with constant velocity) is stationary. Then, the passenger train has an initial relative velocity of $v_{\text{rel},0} = 10\ \text{m/s}$. This relative speed would be decreased to zero after the relative separation had decreased to $\frac{v_{\text{rel},0}^2}{2a_{\text{rel}}} = +500\ \text{m}$. Since this is larger in magnitude than the original relative separation of 200 m, there will be a collision. b) The time at which the relative separation goes to zero (*i.e.*, the collision time) is found by solving a quadratic (see Problems 2.35 & 2.36 or Example 2.8). The time is given by

$$\begin{aligned} t &= \frac{1}{a}\left(v_{\text{rel},0} - \sqrt{v_{\text{rel},0}^2 + 2ax_{\text{rel},0}}\ \right) \\ &= (10\ \text{s}^2/\text{m})(10\ \text{m/s} - \sqrt{100\ \text{m}^2/\text{s}^2 - 40\ \text{m}^2/\text{s}^2}) \\ &= (100\ \text{s})\left((1 - \sqrt{0.6}\right). \end{aligned}$$

Substitution of this time into Eq. (2.12), with $x_0 = 0$, yields 538 m as the distance the passenger train moves before the collision.

2.67: The total distance you cover is $1.20\,\text{m} + 0.90\,\text{m} = 2.10\,\text{m}$ and the time available is $\frac{1.20\,\text{m}}{1.50\,\text{m/s}} = 0.80\,\text{s}$. Solving Eq. (2.12) for a_x,

$$a_x = 2\frac{(x - x_0) - v_{0x}t}{t^2} = 2\frac{(2.10\,\text{m}) - (0.80\,\text{m/s})(0.80\,\text{s})}{(0.80\,\text{s})^2} = 4.56\ \text{m/s}^2.$$

2.68: One convenient way to do the problem is to do part (b) first; the time spent accelerating from rest to the maximum speed is $\frac{20\,\text{m/s}}{2.5\,\text{m/s}^2} = 80$ s.

At this time, the officer is

$$x_1 = \frac{v_1^2}{2a} = \frac{(20\ \text{m/s})^2}{2(2.5\ \text{m/s}^2)} = 80.0\ \text{m}.$$

This could also be found from $(1/2)a_1t_1^2$, where t_1 is the time found for the acceleration. At this time the car has moved $(15\ \text{m/s})(8.0\ \text{s}) = 120$ m, so the officer is 40 m behind the car.

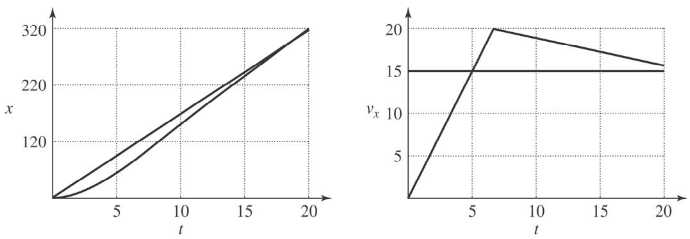
a) The remaining distance to be covered is 300 m – x_1 and the average speed is $(1/2)(v_1 + v_2) = 17.5\ \text{m/s}$, so the time needed to slow down is

$$\frac{360\ \text{m} - 80\ \text{m}}{17.5\ \text{m/s}} = 16.0\ \text{s},$$

and the total time is 24.0 s.

c) The officer slows from $20\ \text{m/s}$ to $15\ \text{m/s}$ in 16.0 s (the time found in part (a)), so the acceleration is $-0.31\ \text{m/s}^2$.

d), e)



2.69: a) $x_T = (1/2)a_T t^2$, and with $x_T = 40.0\ \text{m}$, solving for the time gives $t = \sqrt{\frac{2(40.0\ \text{m})}{(2.10\ \text{m/s})}} = 6.17\ \text{s}$

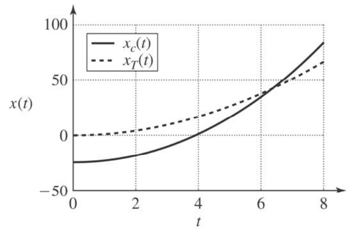
b) The car has moved a distance

$$\frac{1}{2}a_C t^2 = \frac{a_C}{a_T}x_1 = \frac{3.40\ \text{m/s}^2}{2.10\ \text{m/s}^2}40.0\ \text{m} = 64.8\ \text{m},$$

and so the truck was initially 24.8 m in front of the car.

c) The speeds are $a_T t = 13\ \text{m/s}$ and $a_C t = 21\ \text{m/s}$.

d)



2.70: The position of the cars as functions of time (taking $x_1 = 0$ at $t = 0$) are

$$x_1 = \frac{1}{2}at^2, \qquad x_2 = D - v_0 t.$$

The cars collide when $x_1 = x_2$; setting the expressions equal yields a quadratic in t,

$$\frac{1}{2}at^2 + v_0 t - D = 0,$$

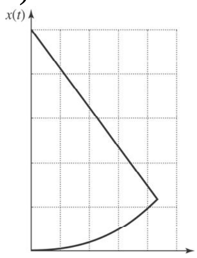
the solutions to which are

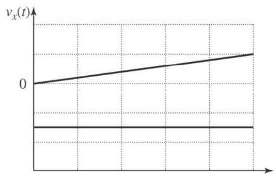
$$t = \frac{1}{a}\left(\sqrt{v_0^2 + 2aD} - v_0\right), \qquad t = \frac{1}{a}\left(-\sqrt{v_0^2 + 2aD} - v_0\right).$$

The second of these times is negative and does not represent the physical situation.

b) $$v_1 = at = \left(\sqrt{v_0^2 + 2aD} - v_0\right)$$

c)





2.71: a) Travelling at $20\ \text{m/s}$, Juan is $x_1 = 37\ \text{m} - (20\ \text{m/s})(0.80\ \text{s}) = 21\ \text{m}$ from the spreader when the brakes are applied, and the magnitude of the acceleration will be $a = \frac{v_1^2}{2x_1}$. Travelling at $25\ \text{m/s}$, Juan is $x_2 = 37\ \text{m} - (25\ \text{m/s})(0.80\ \text{s}) = 17\ \text{m}$ from the spreader, and the speed of the car (and Juan) at the collision is obtained from

$$v_x^2 = v_{0x}^2 - 2a_x x_2 = v_{0x}^2 - 2\left(\frac{v_1^2}{2x_1}\right)x_2 = v_{0x}^2 - v_1^2\left(\frac{x_2}{x_1}\right) = (25\ \text{m/s})^2 - (20\ \text{m/s})^2\left(\frac{17\ \text{m}}{21\ \text{m}}\right)$$
$$= 301\ \text{m}^2/\text{s}^2$$

and so $v_x = 17.4\ \text{m/s}$.

b) The time is the reaction time plus the magnitude of the change in speed $(v_0 - v)$ divided by the magnitude of the acceleration, or

$$t_{\text{flash}} = t_{\text{reaction}} + 2\frac{v_0 - v}{v_0^2}x_1 = (0.80\ \text{s}) + 2\frac{25\ \text{m/s} - 17.4\ \text{m/s}}{(20\ \text{m/s})^2}(21\ \text{m}) = 1.60\ \text{s}.$$

2.72: a) There are many ways to find the result using extensive algebra, but the most straightforward way is to note that between the time the truck first passes the police car and the time the police car catches up to the truck, both the truck and the car have travelled the same distance in the same time, and hence have the same average velocity over that time. Since the truck had initial speed $\frac{3}{2}v_{\text{p}}$ and the average speed is v_{p}, the truck's final speed must be $\frac{1}{2}v_{\text{p}}$.

2.73: a) The most direct way to find the time is to consider that the truck and the car are initially moving at the same speed, and the time of the acceleration must be that which gives a difference between the truck's position and the car's position as $24\ \text{m} + 21\ \text{m} + 26\ \text{m} + 4.5\ \text{m} = 75.5\ \text{m}$, or $t = \sqrt{2(75.5\ \text{m})/(0.600\ \text{m/s}^2)} = 15.9\ \text{s}$.

b) $v_{0x}t + (1/2)a_x t^2 = (20.0\ \text{m/s})(15.9\ \text{s}) + (1/2)(0.600\ \text{m/s}^2)(15.9\ \text{s})^2 = 394\ \text{m}$.

c) $v_{0x} + a_x t = (20.0\ \text{m/s}) + (0.600\ \text{m/s}^2)(15.9\ \text{s}) = 29.5\ \text{m/s}$.

2.74: a) From Eq. (2.17), $x(t) = \alpha t - \frac{\beta}{3}t^3 = (4.00\ \text{m/s})t - (0.667\ \text{m/s}^3)t^3$. From Eq. (2.5), the acceleration is $a(t) = -2\beta t = (-4.00\ \text{m/s}^3)t$.

b) The velocity is zero at $t = \pm\sqrt{\frac{\alpha}{\beta}}$ ($a = 0$ at $t = 0$, but this is an inflection point, not an extreme). The extreme values of x are then

$$x = \pm\left(\alpha\sqrt{\frac{\alpha}{\beta}} - \frac{\beta}{3}\sqrt{\frac{\alpha^3}{\beta^3}}\right) = \pm\frac{2}{3}\sqrt{\frac{\alpha^3}{\beta}}.$$

The positive value is then

$$x = \frac{2}{3}\left(\frac{(4.00\ \text{m/s})^3}{2.00\ \text{m/s}^3}\right)^{\frac{1}{2}} = \frac{2}{3}\sqrt{32\ \text{m}^2} = 3.77\ \text{m}.$$

2.75: a) The particle's velocity and position as functions of time are

$$v_x(t) = v_{0x} + \int_0^t ((-2.00\ \text{m/s}^2) + (3.00\ \text{m/s}^3)t)\,dt$$

$$= v_{0x} - (2.00\ \text{m/s}^2)t + \left(\frac{3.00\ \text{m/s}^3}{2}\right)t^2,$$

$$x(t) = \int_0^t v_x(t)dt = v_{0x}t - (1.00\ \text{m/s}^2)t^2 + (0.50\ \text{m/s}^3)t^3$$

$$= t(v_{0x} - (1.00\ \text{m/s}^2)t + (0.50\ \text{m/s}^3)t^2),$$

where x_0 has been set to 0. Then, $x(0) = 0$, and to have $x(4\ \text{s}) = 0$,

$$v_{0x} - (1.00\ \text{m/s}^2)(4.00\ \text{s}) + (0.50\ \text{m/s}^3)(4.00\ \text{s})^2 = 0,$$

which is solved for $v_{0x} = -4.0\ \text{m/s}$. b) $v_x(4\ \text{s}) = 12.0\ \text{m/s}$.

2.76: The time needed for the egg to fall is

$$t = \sqrt{\frac{2\Delta h}{9}} = \sqrt{\frac{2(46.0\ \text{m} - 1.80\ \text{m})}{(9.80\ \text{m/s}^2)}} = 3.00\ \text{s},$$

and so the professor should be a distance $v_y t = (1.20\ \text{m/s})(3.00\ \text{s}) = 3.60\ \text{m}$.

2.77: Let t_1 be the fall for the watermelon, and t_2 be the travel time for the sound to return. The total time is $T = t_1 + t_2 = 2.5\,\text{s}$. Let y be the height of the building, then, $y = \frac{1}{2} g t_1^2$ and $y = v_s t_2$. There are three equations and three unknowns. Eliminate t_2, solve for t_1, and use the result to find y. A quadratic results: $\frac{1}{2} g t_1^2 + v_s t_1 - v_s T = 0$. If $at^2 + bt + c = 0$, then $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Here, $t = t_1$, $a = 1/2\,g = 4.9\ \text{m/s}^2$, $b = v_s = 340\ \text{m/s}$, and $c = -v_s T = -(340\ \text{m/s})(2.5\,\text{s}) = -850\,\text{m}$

Then upon substituting these values into the quadratic formula,

$$t_1 = \frac{-(340\ \text{m/s}) \pm \sqrt{(340\,\text{m/s})^2 - 4(4.9\ \text{m/s}^2)(-850\,\text{m})}}{2(4.9\ \text{m/s}^2)}$$

$t_1 = \frac{-(340\ \text{m/s}) \pm (363.7\ \text{m/s})}{2\left(4.9\ \text{m/s}^2\right)} = 2.42\,\text{s}$. The other solution, –71.8 s has no real physical meaning. Then, $y = \frac{1}{2} g t_1^2 = \frac{1}{2}(9.8\ \text{m/s}^2)(2.42\,\text{s})^2 = 28.6\,\text{m}$. Check: $(28.6\,\text{m})/(340\ \text{m/s}) = .08\,\text{s}$, the time for the sound to return.

2.78: The elevators to the observation deck of the Sears Tower in Chicago move from the ground floor to the 103rd floor observation deck in about 70 s. Estimating a single floor to be about 3.5 m (11.5 ft), the average speed of the elevator is $\frac{(103)(3.5\,\text{m})}{70\,\text{s}} = 5.15\ \text{m/s}$. Estimating that the elevator must come to rest in the space of one floor, the acceleration is about $\frac{0^2 - (5.15\,\text{m/s})^2}{2(3.5\,\text{m})} = -3.80\,\text{m/s}^2$.

2.79: a) $v = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(21.3\,\text{m})} = 20.4\ \text{m/s}$; the announcer is mistaken.

b) The required speed would be

$$v_0 = \sqrt{v^2 + 2g(y - y_0)} = \sqrt{(25\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(-21.3\text{m})} = 14.4\ \text{m/s},$$

which is not possible for a leaping diver.

2.80: If the speed of the flowerpot at the top of the window is v_0, height h of the window is

$$h = v_{\text{ave}}t = v_0t + (1/2)gt^2, \text{ or } \quad v_0 = \frac{h}{t} - (1/2)gt.$$

The distance l from the roof to the top of the window is then

$$l = \frac{v_0^2}{2g} = \frac{((1.90 \text{ m})/(0.420 \text{ s}) - (1/2)(9.80 \text{ m/s}^2)(0.420 \text{ s}))^2}{2(9.80 \text{ m/s}^2)} = 0.310 \text{ m}.$$

An alternative but more complicated algebraic method is to note that t is the difference between the times taken to fall the heights $l + h$ and h, so that

$$t = \sqrt{\frac{2(l+h)}{g}} - \sqrt{\frac{2l}{g}}, \; \sqrt{gt^2/2} + \sqrt{l} = \sqrt{l + h}.$$

Squaring the second expression allows cancelation of the l terms,

$$(1/2)gt^2 + 2\sqrt{gt^2l/2} = h,$$

which is solved for

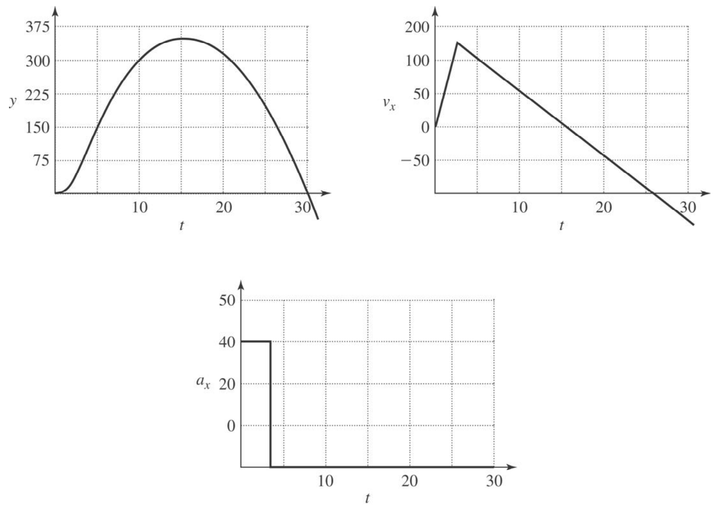
$$l = \frac{1}{2g}\left(\frac{h}{t} - (1/2)gt\right)^2,$$

which is the same as the previous expression.

2.81: a) The football will go an additional $\frac{v^2}{2g} = \frac{(5.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 1.27 \text{ m}$ above the window, so the greatest height is 13.27 m or 13.3 m to the given precision.

b) The time needed to reach this height is $\sqrt{2(13.3\text{m})/(9.80 \text{ m/s}^2)} = 1.65 \text{s}.$

2.82: a)



2.83: a) From Eq. (2.14), with v_0=0,

$$v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(45.0\ \text{m/s}^2)(0.640\,\text{m})} = 7.59\,\text{m/s}.$$

b) The height above the release point is also found from Eq. (2.14), with $v_{0y} = 7.59\,\text{m/s}, v_y = 0$ and $a_y = -g$,

$$h = \frac{v_{0y}^2}{2g} = \frac{(7.59\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 2.94\,\text{m}$$

(Note that this is also (64.0 cm)$\left(\frac{45\ \text{m/s}^2}{g}\right)$.The height above the ground is then 5.14 m.

c) See Problems 2.46 & 2.48 or Example 2.8: The shot moves a total distance 2.20 m –1.83 m = 0.37 m, and the time is

$$\frac{(7.59\ \text{m/s}) + \sqrt{(7.59\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(0.37\,\text{m})}}{(9.80\ \text{m/s}^2)} = 1.60\,\text{s}.$$

2.84: a) In 3.0 seconds the teacher falls a distance

$$y = \frac{1}{2}gt^2 = \frac{1}{2}(9.8\ \text{m/s}^2)(9.0\ \text{s}^2) = 44.1\ \text{m}$$

To reach her ears after 3.0 s, the sound must therefore have traveled a total distance of $h + (h - 44.1)\ \text{m} = 2h - 44.1\ \text{m}$,where h is the height of the cliff. Given 340 m/s for the speed of sound: $2h - 44.1\ \text{m} = (340\ \text{m/s})(3.0\ \text{s}) = 1020\ \text{m}$, which gives $h = 532\ \text{m}$ or $530\ \text{m}$ to the given precision.

b) We can use $v_y^2 = v_{0y}^2 + 2g(y - y_0)$ to find the teacher's final velocity. This gives $v_y^2 = 2(9.8\ \text{m/s}^2)(532\ \text{m}) = 10427\ \text{m}^2/\text{s}^2$ and $v_y = 102\ \text{m/s}$.

2.85: a) Let $+y$ be upward.

At ceiling, $v_y = 0, y - y_0 = 3.0\ \text{m}, a_y = -9.80\ \text{m/s}^2$. Solve for v_{0y}.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = 7.7\ \text{m/s}$.

b) $v_y = v_{0y} + a_y t$ with the information from part (a) gives $t = 0.78\ \text{s}$.

c) Let the first ball travel downward a distance d in time t. It starts from its maximum height, so $v_{0y} = 0$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $d = (4.9\ \text{m/s}^2)t^2$

The second ball has $v_{0y} = \frac{1}{3}(7.7\ \text{m/s}) = 5.1\ \text{m/s}$. In time t it must travel upward $3.0\ \text{m} - d$ to be at the same place as the first ball.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $3.0\ \text{m} - d = (5.1\ \text{m/s})t - (4.9\ \text{m/s}^2)t^2$.

We have two equations in two unknowns, d and t. Solving gives $t = 0.59\ \text{s}$ and $d = 1.7\ \text{m}$.

d) $3.0\ \text{m} - d = 1.3\ \text{m}$

2.86: a) The helicopter accelerates from rest for 10.0 s at a constant $5.0\ \text{m/s}^2$. It thus reaches an upward velocity of

$$v_y = v_{0y} + a_y t = (5.0\ \text{m/s}^2)(10.0\ \text{s}) = 50.0\ \text{m/s}$$

and a height of $y = \frac{1}{2} a_y t^2 = \frac{1}{2}(5.0\ \text{m/s}^2)(10.0\ \text{s})^2 = 250\ \text{m}$ at the moment the engine is shut off. To find the helicopter's maximum height use

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

Taking $y_0 = 250\ \text{m}$, where the engine shut off, and since $v_y^2 = 0$ at the maximum height:

$$y_{\text{max}} - y_0 = \frac{-v_{0y}^2}{2g}$$

$$y_{\text{max}} = 250\ \text{m} - \frac{(50.0\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 378\ \text{m}$$

or 380 m to the given precision.

b) The time for the helicopter to crash from the height of 250 m where Powers stepped out and the engine shut off can be found from:

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 = 250\ \text{m} + (50.0\ \text{m/s})t + \tfrac{1}{2}(-9.8\ \text{m/s}^2)t^2 = 0$$

where we now take the ground as $y = 0$. The quadratic formula gives solutions of $t = 3.67\ \text{s}$ and 13.88 s, of which the first is physically impossible in this situation. Powers' position 7.0 seconds after the engine shutoff is given by:

$$y = 250\ \text{m} + (50.0\ \text{m/s})(7.0\ \text{s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(49.0\ \text{s}^2) = 359.9\ \text{m}$$

at which time his velocity is

$$v_y = v_{0y} + gt = 50.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(7.0\ \text{s}) = -18.6\ \text{m/s}$$

Powers thus has $13.88 - 7.0 = 6.88\text{s}$ more time to fall before the helicopter crashes, at his constant downward acceleration of $2.0\,\text{m/s}^2$. His position at crash time is thus:

$$\begin{aligned} y &= y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \\ &= 359.9\ \text{m} + (-18.6\ \text{m/s})(6.88\ \text{s}) + \frac{1}{2}(-2.0\ \text{m/s}^2)(6.88\text{s})^2 \\ &= 184.6\ \text{m} \end{aligned}$$

or 180 m to the given precision.

2.87: Take $+y$ to be downward.
Last 1.0 s of fall:
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h/4 = v_{0y}(1.0\text{ s}) + (4.9\text{ m/s}^2)(1.0\text{ s})^2$
v_{0y} is his speed at the start of this time interval.
Motion from roof to $y - y_0 = 3h/4$:
$v_{0y} = 0, v_y = ?$
$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(9.80\text{ m/s}^2)(3h/4)} = 3.834\sqrt{h}\text{ m/s}$
This is v_y for the last 1.0 s of fall. Using this in the equation for the first 1.0 s gives
$h/4 = 3.834\sqrt{h} + 4.9$
Let $h = u^2$ and solve for $u : u = 16.5$. Then $h = u^2 = 270\text{ m}$.

2.88: a)

$$t_{\text{fall}} + t_{\text{sound return}} = 10.0\text{ s}$$
$$t_{\text{f}} + t_{\text{s}} = 10.0\text{ s} \qquad (1)$$

$$d_{\text{Rock}} = d_{\text{Sound}}$$
$$\frac{1}{2}gt_{\text{f}}^2 = v_{\text{s}}t_{\text{s}}$$
$$\frac{1}{2}(9.8\text{ m/s}^2)t_{\text{f}}^2 = (330\text{ m/s})t_{\text{s}} \qquad (2)$$

Combine (1) and (2): $t_{\text{f}} = 8.84\text{ s}, t_{\text{s}} = 1.16\text{ s}$

$$h = v_{\text{s}}t_{\text{s}} = (330\frac{\text{m}}{\text{s}})(1.16\text{ s}) = 383\text{ m}$$

b) You would think that the rock fell for 10 s, not 8.84 s, so you would have thought it fell farther. Therefore your answer would be an *overestimate* of the cliff's height.

2.89: a) Let $+y$ be upward.
$y - y_0 = -15.0\text{ m},\ t = 3.25\text{ s},\ a_y = -9.80\text{ m/s}^2, v_{0y} = ?$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = 11.31\text{ m/s}$
Use this v_{0y} in $v_y = v_{0y} + a_y t$ to solve for $v_y : v_y = -20.5\text{ m/s}$

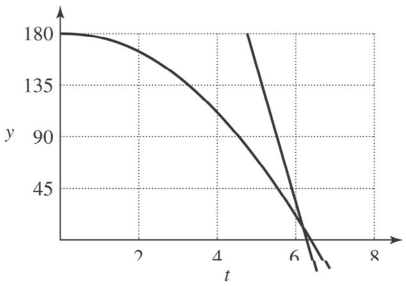
b) Find the maximum height of the can, above the point where it falls from the scaffolding:
$v_y = 0,\ v_{0y} = +11.31\text{ m/s},\ a_y = -9.80\text{ m/s}^2,\ y - y_0 = ?$
$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 6.53\text{ m}$
The can will pass the location of the other painter. Yes, he gets a chance.

2.90: a) Suppose that Superman falls for a time t, and that the student has been falling for a time t_0 before Superman's leap (in this case, $t_0 = 5$ s). Then, the height h of the building is related to t and t_0 in two different ways:

$$-h = v_{0y}t - \frac{1}{2}gt^2$$
$$= -\frac{1}{2}g(t + t_0)^2,$$

where v_{0y} is Superman's initial velocity. Solving the second t gives $t = \sqrt{\frac{2h}{g}} - t_0$. Solving the first for v_{0y} gives $v_{0y} = -\frac{h}{t} + \frac{g}{2}t,$ and substitution of numerical values gives $t = 1.06$ s and $v_{0y} = -165\ \text{m/s}$, with the minus sign indicating a downward initial velocity.

b)



c) If the skyscraper is so short that the student is already on the ground, then $h = \frac{1}{2}gt_0^2 = 123$ m.

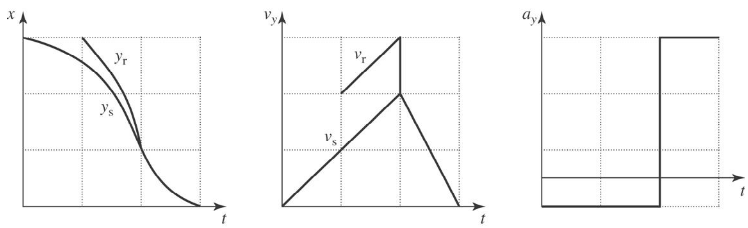
2.91: a) The final speed of the first part of the fall (free fall) is the same as the initial speed of the second part of the fall (with the Rocketeer supplying the upward acceleration), and assuming the student is a rest both at the top of the tower and at the ground, the distances fallen during the first and second parts of the fall are $\frac{v_1^2}{2g}$ and $\frac{v_1^2}{10g}$, where v_1 is the student's speed when the Rocketeer catches him. The distance fallen in free fall is then five times the distance from the ground when caught, and so the distance above the ground when caught is one-sixth of the height of the tower, or 92.2 m. b) The student falls a distance $5H/6$ in time $t = \sqrt{5H/3g}$, and the Rocketeer falls the same distance in time t–t_0, where t_0=5.00 s (assigning three significant figures to t_0 is more or less arbitrary). Then,

$$\frac{5H}{6} = v_0(t - t_0) + \frac{1}{2}g(t - t_0)^2, \text{or}$$

$$v_0 = \frac{5H/6}{(t - t_0)} - \frac{1}{2}g(t - t_0).$$

At this point, there is no great advantage in expressing t in terms of H and g algebraically; $t - t_0 = \sqrt{5(553\,\text{m})/29.40\ \text{m/s}^2} - 5.00\,\text{s} = 4.698\,\text{s}$, from which $v_0 = 75.1\text{m/s}$.

c)



2.92: a) The time is the initial separation divided by the initial relative speed, H/v_0. More precisely, if the positions of the balls are described by

$$y_1 = v_0 t - (1/2)gt^2, \quad y_2 = H - (1/2)gt^2,$$

setting $y_1 = y_2$ gives $H = v_0 t$. b) The first ball will be at the highest point of its motion if at the collision time t found in part (a) its velocity has been reduced from v_0 to 0, or $gt = gH/v_0 = v_0$, or $H = v_0^2/g$.

2.93: The velocities are $v_A = \alpha + 2\beta t$ and $v_B = 2\gamma t - 3\delta t^2$ a) Since v_B is zero at $t = 0$, car A takes the early lead. b) The cars are both at the origin at $t = 0$. The non-trivial solution is found by setting $x_A = x_B$, cancelling the common factor of t, and solving the quadratic for

$$t = \frac{1}{2\delta}\left[(\beta - \gamma) \pm \sqrt{(\beta - \gamma)^2 - 4\alpha\delta}\right]$$

Substitution of numerical values gives 2.27 s, 5.73 s. The use of the term "starting point" can be taken to mean that negative times are to be neglected. c) Setting $v_A = v_B$ leads to a different quadratic, the positive solution to which is

$$t = -\frac{1}{6\delta}\left[(2\beta - 2\gamma) - \sqrt{(2\beta - 2\gamma)^2 - 12\alpha\delta}\right]$$

Substitution of numerical results gives 1.00 s and 4.33 s.

d) Taking the second derivative of x_A and x_B and setting them equal, yields, $2\beta = 2\gamma - 6\delta t$. Solving, $t = 2.67\,\text{s}$.

2.94: a) The speed of any object falling a distance $H - h$ in free fall is $\sqrt{2g(H - h)}$. b) The acceleration needed to bring an object from speed v to rest over a distance h is $\frac{v^2}{2h} = \frac{2g(H - h)}{2h} = g\left(\frac{H}{h} - 1\right)$.

2.95: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then

$$x_1 = v_0 t, \qquad x_2 = x_0 + (1/2)at^2.$$

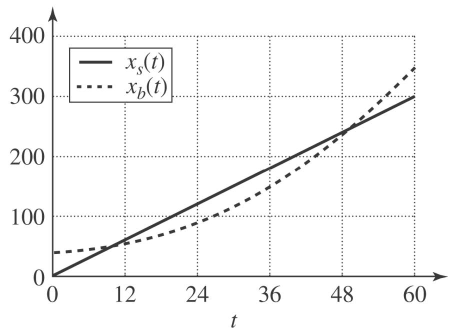
a) Setting $x_1 = x_2$ and solving for the times t gives

$$\begin{aligned} t &= \frac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right) \\ &= \frac{1}{(0.170\ \text{m/s}^2)}\left((5.0\ \text{m/s}) \pm \sqrt{(5.0\ \text{m/s})^2 - 2(0.170\ \text{m/s}^2)(40.0\ \text{m})}\right) \\ &= 9.55\ \text{s},\ 49.3\ \text{s}. \end{aligned}$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5\ \text{m/s})(9.55\ \text{s}) = 47.8\ \text{m}$.

b) The speed of the bus is $(0.170\,\text{m/s}^2)(9.55\,\text{s}) = 1.62\,\text{m/s}$.

c) The results can be verified by noting that the x lines for the student and the bus intersect at two points:



d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170\,\text{m/s}^2)(49.3\ \text{s}) = 8.38\,\text{m/s}$.

2.96: The time spent above $y_{\max}/2$ is $\frac{1}{\sqrt{2}}$ the total time spent in the air, as the time is proportional to the square root of the change in height. Therefore the ratio is

$$\frac{1/\sqrt{2}}{1-1/\sqrt{2}}=\frac{1}{\sqrt{2}-1}=2.4.$$

2.97: For the purpose of doing all four parts with the least repetition of algebra, quantities will be denoted symbolically. That is,

let $y_1 = h + v_0 t - \frac{1}{2}gt^2, y_2 = h - \frac{1}{2}g(t - t_0)^2$. In this case, $t_0 = 1.00\,\text{s}$. Setting $y_1 = y_2 = 0$, expanding the binomial $(t - t_0)^2$ and eliminating the common term $\frac{1}{2}gt^2$ yields $v_0 t = gt_0 t - \frac{1}{2}gt_0^2$, which can be solved for *t*;

$$t = \frac{\frac{1}{2}gt_0^2}{gt_0 - v_0} = \frac{t_0}{2}\frac{1}{1 - \frac{v_0}{gt_0}}.$$

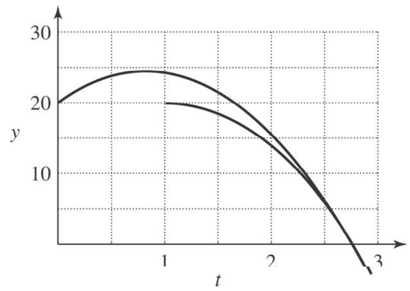
Substitution of this into the expression for y_1 and setting $y_1 = 0$ and solving for *h* as a function of v_0 yields, after some algebra,

$$h = \frac{1}{2}gt_0^2 \frac{\left(\frac{1}{2}gt_0 - v_0\right)^2}{(gt_0 - v_0)^2}.$$

a) Using the given value $t_0 = 1.00\,\text{s}$ and $g = 9.80\,\text{m/s}^2$,

$$h = 20.0\,\text{m} = (4.9\,\text{m})\left(\frac{4.9\,\text{m/s} - v_0}{9.8\,\text{m/s} - v_0}\right)^2.$$

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for v_0, and yields $8.2\,\text{m/s}$.



b) The above expression gives for i), 0.411 m and for ii) 1.15 km. c) As v_0 approaches $9.8\,\text{m/s}$, the height *h* becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8\,\text{m/s}$, the first ball can never catch the second ball. d) As v_0 approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 > 4.9\,\text{m/s}$, the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

2.98: a) Let the height be h and denote the 1.30-s interval as Δt; the simultaneous equations $h = \frac{1}{2}gt^2$, $\frac{2}{3}h = \frac{1}{2}g(t - \Delta t)^2$ can be solved for t. Eliminating h and taking the square root, $\frac{t}{t-\Delta t} = \sqrt{\frac{3}{2}}$, and $t = \frac{\Delta t}{1-\sqrt{2/3}}$, and substitution into $h = \frac{1}{2}gt^2$ gives h = 246 m. This method avoids use of the quadratic formula; the quadratic formula is a generalization of the method of "completing the square", and in the above form, $\frac{2}{3}h = \frac{1}{2}g(t - \Delta t)^2$, the square is already completed.

b) The above method assumed that t >0 when the square root was taken. The negative root (with Δt = 0) gives an answer of 2.51 m, clearly not a "cliff". This would correspond to an object that was initially near the bottom of this "cliff" being thrown upward and taking 1.30 s to rise to the top and fall to the bottom. Although physically possible, the conditions of the problem preclude this answer.

3.1: a)

$$v_{x,\text{ave}} = \frac{(5.3\,\text{m}) - (1.1\,\text{m})}{(3.0\,\text{s})} = 1.4\ \text{m/s},$$
$$v_{y,\text{ave}} = \frac{(-0.5\,\text{m}) - (3.4\,\text{m})}{(3.0\,\text{s})} = -1.3\ \text{m/s}.$$

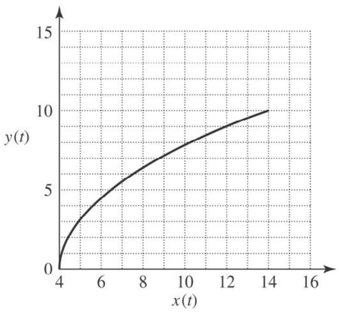
b) $v_{\text{ave}} = \sqrt{(1.4\ \text{m/s})^2 + (-1.3\ \text{m/s})^2} = 1.91\ \text{m/s}$, or $1.9\ \text{m/s}$ to two significant figures,
$\theta = \arctan\left(\frac{-1.3}{1.4}\right) = -43°$.

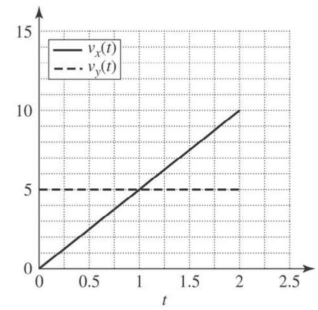
3.2: a)

$$x = (v_{x,\text{ave}})\Delta t = (-3.8\ \text{m/s})(12.0\,\text{s}) = -45.6\,\text{m} \ \text{ and}$$
$$y = (v_{y,\text{ave}})\Delta t = (4.9\ \text{m/s})(12.0\,\text{s}) = 58.8\,\text{m}.$$

b) $r = \sqrt{x^2 + y^2} = \sqrt{(-45.6\,\text{m})^2 + (58.8\,\text{m})^2} = 74.4\,\text{m}.$

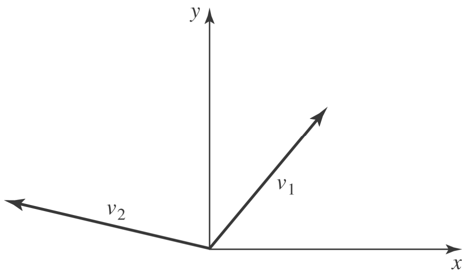
3.3: The position is given by $\vec{r} = [4.0\ \text{cm} + (2.5\ \text{cm/s}^2)t^2]\hat{\boldsymbol{i}} + (5.0\ \text{cm/s})t\hat{\boldsymbol{j}}$.
(a) $\mathbf{r}(0) = [4.0\,\text{cm}]\hat{\boldsymbol{i}}$, and
$\mathbf{r}(2\text{s}) = [4.0\,\text{cm} + (2.5\ \text{cm/s}^2)(2\,\text{s})^2]\hat{\boldsymbol{i}} + (5.0\ \text{cm/s})(2\,\text{s})\hat{\boldsymbol{j}} = (14.0\,\text{cm})\hat{\boldsymbol{i}} + (10.0\,\text{cm})\hat{\boldsymbol{j}}$. Then using the definition of average velocity, $\vec{\boldsymbol{v}}_{\text{ave}} = \frac{(14\,\text{cm} - 4\,\text{cm})\hat{i} + (10\,\text{cm} - 0)\hat{j}}{2\,\text{s}} = (5\ \text{cm/s})\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}}.$
$v_{\text{ave}} = 7.1\,\text{cm/s}$ at an angle of $45°$.
b) $\vec{\boldsymbol{v}} = \frac{d\vec{r}}{dt} = (2)(2.5\ \text{cm/s})t\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}} = (5\ \text{cm/s})t\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}}$. Substituting for $t = 0{,}1\text{s}$, and 2 s, gives:
$\vec{\boldsymbol{v}}(0) = (5\ \text{cm/s})\hat{\boldsymbol{j}}, \vec{\boldsymbol{v}}(1\text{s}) = (5\ \text{cm/s})\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}}$, and $\vec{\boldsymbol{v}}(2\text{s}) = (10\ \text{cm/s})\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}}$.
The magnitude and direction of $\vec{\boldsymbol{v}}$ at each time therefore are: $t = 0 : 5.0\ \text{cm/s}$ at $90°$;
$t = 1.05 : 7.1\ \text{cm/s}$ at $45°$; $t = 2.05 : 11\,\text{cm/s}$ at $27°$.
c)





3.4: $\vec{v} = 2bt\hat{\boldsymbol{i}} + 3ct^2\hat{\boldsymbol{j}}$. This vector will make a $45°$-angle with both axes when the *x*- and *y*-components are equal; in terms of the parameters, this time is $2b/3c$.

3.5: a)



b)
$$a_{x,\text{ave}} = \frac{(-170\ \text{m/s}) - (90\ \text{m/s})}{(30.0\,\text{s})} = -8.7\ \text{m/s}^2,$$
$$a_{y,\text{ave}} = \frac{(40\ \text{m/s}) - (110\,\text{m/s})}{(30.0\,\text{s})} = -2.3\ \text{m/s}^2.$$

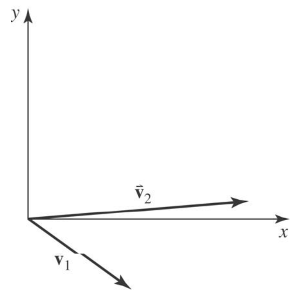
c) $\sqrt{(-8.7\ \text{m/s}^2)^2 + (-\,2.3\,\text{m/s}^2)^2} = 9.0\ \text{m/s}^2$, $\arctan\left(\frac{-2.3}{-8.7}\right) = 14.8° + 180° = 195°.$

3.6: a) $a_x = (0.45\ \text{m/s}^2)\cos 31.0° = 0.39\ \text{m/s}^2$, $a_y = (0.45\ \text{m/s}^2)\sin 31.0° = 0.23\ \text{m/s}^2$, so $v_x = 2.6\ \text{m/s} + (0.39\ \text{m/s}^2)(10.0\,\text{s}) = 6.5\ \text{m/s}$ and

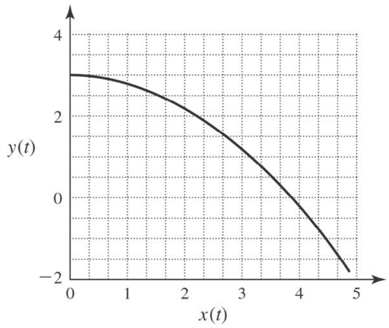
$v_y = -1.8\ \text{m/s} + (0.23\ \text{m/s}^2)(10.0\,\text{s}) = 0.52\ \text{m/s}$.

b) $v = \sqrt{(6.5\ \text{m/s})^2 + (0.52\ \text{m/s})^2} = 6.48\ \text{m/s}$, at an angle of $\arctan\left(\frac{0.52}{6.5}\right) = 4.6°$ above the horizontal.

c)



3.7: a)



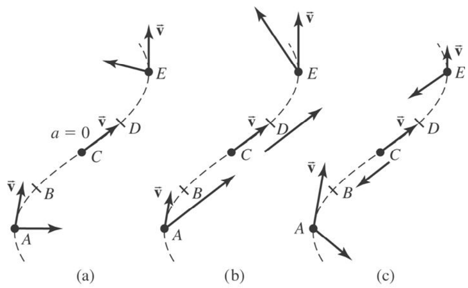
b)

$$\vec{v} = \alpha\hat{i} - 2\beta t\hat{j} = (2.4\,\text{m/s})\hat{i} - [(2.4\,\text{m/s}^2)t]\hat{j}$$

$$\vec{a} = -2\beta\hat{j} = (-2.4\,\text{m/s}^2)\,\hat{j}.$$

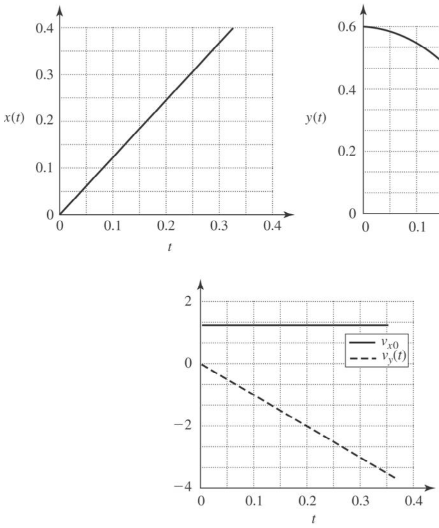
c) At $t = 2.0\,\text{s}$, the velocity is $\vec{v} = (2.4\text{ m/s})\hat{i} - (4.8\text{ m/s})\hat{j}$; the magnitude is $\sqrt{(2.4\text{ m/s})^2 + (-4.8\text{ m/s})^2} = 5.4\text{ m/s}$, and the direction is $\arctan\left(\frac{-4.8}{2.4}\right) = -63°$. The acceleration is constant, with magnitude 2.4 m/s^2 in the $-y$-direction. d) The velocity vector has a component parallel to the acceleration, so the bird is speeding up. The bird is turning toward the $-y$ -direction, which would be to the bird's right (taking the $+z$ -direction to be vertical).

3.8:



3.9: a) Solving Eq. (3.18) with $y = 0$, $v_{0y} = 0$ and $t = 0.350\,\text{s}$ gives $y_0 = 0.600\,\text{m}$.

b) $v_x t = 0.385\,\text{m}$ c) $v_x = v_{0x} = 1.10\ \text{m/s}$, $v_y = -gt = -3.43\ \text{m/s}$, $v = 3.60\ \text{m/s}, 72.2°$ below the horizontal.

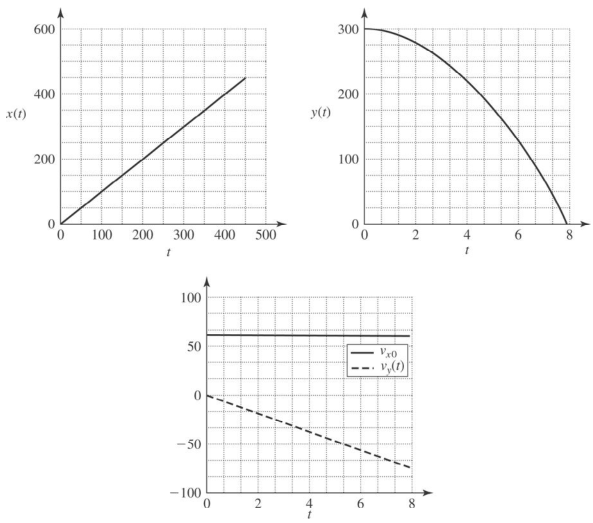


3.10: a) The time *t* is given by $t = \sqrt{\frac{2h}{g}} = 7.82\,\text{s}$.

b) The bomb’s constant horizontal velocity will be that of the plane, so the bomb travels a horizontal distance $x = v_x t = (60\ \text{m/s})(7.82\,\text{s}) = 470\,\text{m}$.

c) The bomb’s horizontal component of velocity is 60 m/s, and its vertical component is $-gt = -76.7\ \text{m/s}$.

d)



e) Because the airplane and the bomb always have the same *x*-component of velocity *and* position, the plane will be 300 m above the bomb at impact.

3.11: Take $+y$ to be upward.
Use Chirpy's motion to find the height of the cliff.
$v_{0y} = 0,\ a_y = -9.80 \text{ m/s}^2,\ y - y_0 = -h,\ t = 3.50\,\text{s}$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = 60.0\,\text{m}$
Milada: Use vertical motion to find time in the air.
$v_{0y} = v_0 \sin 32.0°,\ y - y_0 = -60.0\,\text{m},\ a_y = -9.80 \text{ m/s}^2,\ t = ?$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 3.55\,\text{s}$
Then $v_{0x} = v_0 \cos 32.0°,\ a_x = 0,\ t = 3.55\,\text{s}$ gives $x - x_0 = 2.86\,\text{m}$.

3.12: Time to fall 9.00 m from rest:

$$y = \frac{1}{2}gt^2$$
$$9.00\,\text{m} = \frac{1}{2}(9.8\,\text{m/s}^2)t^2$$
$$t = 1.36\,\text{s}$$

Speed to travel 1.75 m horizontally:

$$x = v_0 t$$
$$1.75\,\text{m} = v_0(1.36\,\text{s})$$
$$v_0 = 1.3\,\text{m/s}$$

3.13: Take $+y$ to be upward.
Use the vertical motion to find the time in the air:
$v_{0y} = 0,\ a_y = -9.80 \text{ m/s}^2,\ y - y_0 = -(21.3\,\text{m} - 1.8\,\text{m}) = -19.5\,\text{m},\ t = ?$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 1.995\,\text{s}$
Then $x - x_0 = 61.0\,\text{m},\ a_x = 0,\ t = 1.995\,\text{s}, v_{0x} = ?$
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_{0x} = 30.6 \text{ m/s}$.
b) $v_x = 30.6 \text{ m/s}$ since $a_x = 0$
$v_y = v_{0y} + a_y t = -19.6 \text{ m/s}$
$v = \sqrt{v_x^2 + v_y^2} = 36.3 \text{ m/s}$

3.14: To make this prediction, the student needs the ball's horizontal velocity at the moment it leaves the tabletop and the time it will take for the ball to reach the floor (or rather, the rim of the cup). The latter can be determined simply by measuring the height of the tabletop above the rim of the cup and using $y = \frac{1}{2}gt^2$ to calculate the falling time. The horizontal velocity can be determined (although with significant uncertainty) by timing the ball's roll for a measured distance before it leaves the table, assuming that its speed doesn't change much on the hard tabletop. The horizontal distance traveled while the ball is in flight will simply be horizontal velocity $\times$ falling time. The cup should be placed at this distance (or a slightly shorter distance, to allow for the slowing of the ball on the tabletop and to make sure it clears the rim of the cup) from a point vertically below the edge of the table.

3.15: a) Solving Eq. (3.17) for $v_y = 0$, with $v_{0y} = (15.0\ \text{m/s})\sin 45.0°$,

$$T = \frac{(15.0\ \text{m/s})\sin 45°}{9.80\ \text{m/s}^2} = 1.08\ \text{s}.$$

b) Using Equations (3.20) and (3.21) gives at t_1, $(x, y) = (6.18\,\text{m}, 4.52\,\text{m})$: t_2, $(11.5\,\text{m}, 5.74\ \text{m})$: t_3, $(16.8\ \text{m}, 4.52\ \text{m})$.

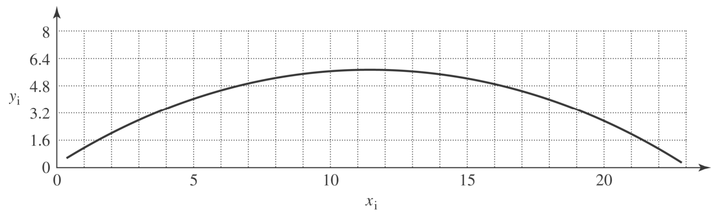
c) Using Equations (3.22) and (3.23) gives at t_1, $(v_x, v_y) = (10.6\ \text{m/s}, 4.9\ \text{m/s})$: t_2, $(10.6\ \text{m/s}, 0)$ t_3 : $(10.6\ \text{m/s}, -4.9\ \text{m/s})$, for velocities, respectively, of $11.7\ \text{m/s}\ @\ 24.8°$, $10.6\ \text{m/s}\ @\ 0°$ and $11.7\ \text{m/s}\ @\ -24.8°$. Note that v_x is the same for all times, and that the *y*-component of velocity at t_3 is negative that at t_1.

d) The parallel and perpendicular components of the acceleration are obtained from

$$\vec{\boldsymbol{a}}_{\parallel} = \frac{(\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{v}})\vec{\boldsymbol{v}}}{v^2}, \left|\vec{\boldsymbol{a}}_{\parallel}\right| = \frac{\left|\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{v}}\right|}{v}, \left|\vec{\boldsymbol{a}}_{\perp}\right| = \sqrt{\left|\vec{\boldsymbol{a}}\right| - \left|\vec{\boldsymbol{a}}_{\parallel}\right|}.$$

For projectile motion, $\vec{\boldsymbol{a}} = -g\hat{\boldsymbol{j}}$, so $\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{v}} = -gv_y$, and the components of acceleration parallel and perpendicular to the velocity are $t_1 : -4.1\ \text{m/s}^2, 8.9\ \text{m/s}^2$. $t_2 : 0, 9.8\ \text{m/s}^2$. $t_3 : 4.1\ \text{m/s}^2, 8.9\ \text{m/s}^2$.

e)



f) At t_1, the projectile is moving upward but slowing down; at t_2 the motion is instantaneously horizontal, but the vertical component of velocity is decreasing; at t_3, the projectile is falling down and its speed is increasing. The horizontal component of velocity is constant.

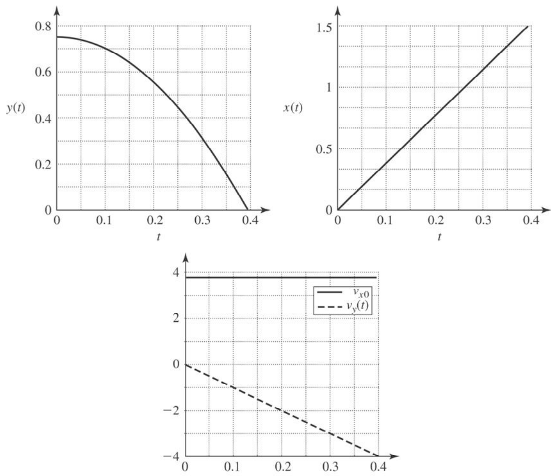
3.16: a) Solving Eq. (3.18) with $y = 0,\ y_0 = 0.75\,\text{m}$ gives $t = 0.391\,\text{s}$.

b) Assuming a horizontal tabletop, $v_{0y} = 0$, and from Eq. (3.16),

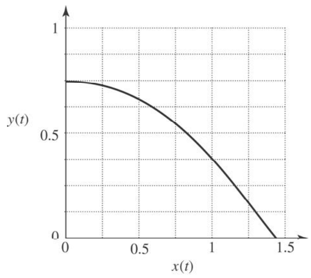
$v_{0x} = (x - x_0)/t = 3.58\,\text{m/s}$.

c) On striking the floor, $v_y = -gt = -\sqrt{2gy_0} = -3.83\,\text{m/s}$, and so the ball has a velocity of magnitude $5.24\,\text{m/s}$, directed $46.9°$ below the horizontal.

d)



Although not asked for in the problem, this *y* vs. *x* graph shows the trajectory of the tennis ball as viewed from the side.



3.17: The range of a projectile is given in Example 3.11, $R = v_0^2 \sin 2\alpha_0 / g$.

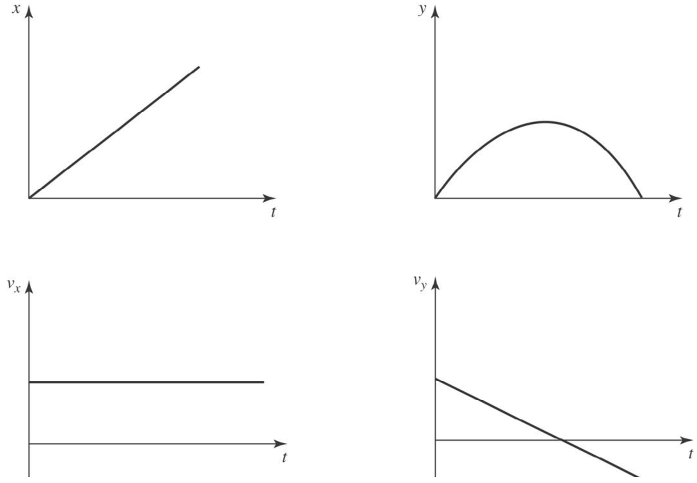
a) $(120\ \text{m/s})^2 \sin 110° / (9.80\ \text{m/s}^2) = 1.38\ \text{km}$ .b) $(120\ \text{m/s})^2 \sin 110° / (1.6\ \text{m/s}^2) = 8.4\ \text{km}$.

3.18: a) The time t is $\frac{v_{y0}}{g} = \frac{16.0\ \text{m/s}}{9.80\ \text{m/s}^2} = 1.63\ \text{s}$.

b) $\frac{1}{2}gt^2 = \frac{1}{2}v_{y0}t = \frac{v_{y0}^2}{2g} = 13.1\ \text{m}$.

c) Regardless of how the algebra is done, the time will be twice that found in part (a), or 3.27 s d) v_x is constant at $20.0\ \text{m/s}$, so $(20.0\ \text{m/s})(3.27\ \text{s}) = 65.3\ \text{m}$.

e)



3.19: a) $v_{0y} = (30.0\ \text{m/s})\sin 36.9° = 18.0\ \text{m/s}$; solving Eq. (3.18) for t with $y_0 = 0$ and $y = 10.0\ \text{m}$ gives

$$t = \frac{(18.0\ \text{m/s}) \pm \sqrt{(18.0\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(10.0\ \text{m})}}{9.80\ \text{m/s}^2} = 0.68\ \text{s}, 2.99\ \text{s}$$

b) The x-component of velocity will be $(30.0\ \text{m/s})\cos 36.9° = 24.0\ \text{m/s}$ at all times. The y-component, obtained from Eq. (3.17), is $11.3\ \text{m/s}$ at the earlier time and $-11.3\ \text{m/s}$ at the later.

c) The magnitude is the same, $30.0\ \text{m/s}$, but the direction is now $36.9°$ below the horizontal.

3.20: a) If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80 \text{ m/s}^2$ vertically.

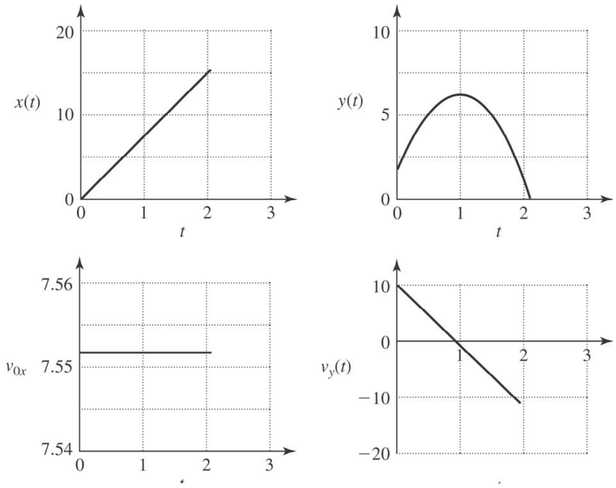
b) The *x*-component of velocity is constant at $v_x = (12.0 \text{ m/s})\cos 51.0° = 7.55 \text{ m/s}$. The *y*-component is $v_{0y} = (12.0 \text{ m/s})\sin 51.0° = 9.32 \text{ m/s}$ at release and $v_{0y} - gt = (10.57 \text{ m/s}) - (9.80 \text{ m/s}^2)(2.08 \text{ s}) = -11.06 \text{ m/s}$ when the shot hits.

c) $v_{0x}t = (7.55 \text{ m/s})(2.08 \text{ s}) = 15.7 \text{ m}$.

d) The initial and final heights are not the same.

e) With $y = 0$ and v_{0y} as found above, solving Eq. (3.18) for $y_0 = 1.81 \text{m}$.

f)



3.21: a) The time the quarter is in the air is the horizontal distance divided by the horizontal component of velocity. Using this time in Eq. (3.18),

$$\begin{aligned} y - y_0 &= v_{0y}\frac{x}{v_{0x}} - \frac{gx^2}{2v_{0x}^2} \\ &= \tan\alpha_0 x - \frac{gx^2}{v_0^2 2\cos^2\alpha_0} \\ &= \tan 60°(2.1 \text{m}) - \frac{(9.80 \text{ m/s}^2)(2.1 \text{ m})^2}{2(6.4 \text{ m/s})^2\cos^2 60°} = 1.53 \text{ m rounded.} \end{aligned}$$

b) Using the same expression for the time in terms of the horizontal distance in Eq. (3.17),

$$v_y = v_0\sin\alpha_0 - \frac{gx}{v_0\cos\alpha_0} = (6.4 \text{ m/s})\sin 60° - \frac{(9.80 \text{ m/s}^2)(2.1 \text{ m})}{(6.4 \text{ m/s})\cos 60°} = -0.89 \text{ m/s}.$$

3.22: Substituting for *t* in terms of *d* in the expression for y_{dart} gives

$$y_{\text{dart}} = d\left(\tan\alpha_0 - \frac{gd}{2v_0^2\cos^2\alpha_0}\right).$$

Using the given values for *d* and α_0 to express this as a function of v_0,

$$y = (3.00\text{ m})\left(0.90 - \frac{26.62\text{ m}^2/\text{s}^2}{v_0^2}\right).$$

Then, a) $y = 2.14\text{m}$, b) $y = 1.45\text{m}$, c) $y = -2.29\text{m}$. In the last case, the dart was fired with so slow a speed that it hit the ground before traveling the 3-meter horizontal distance.

3.23: a) With $v_y = 0$ in Eq. (3.17), solving for *t* and substituting into Eq. (3.18) gives

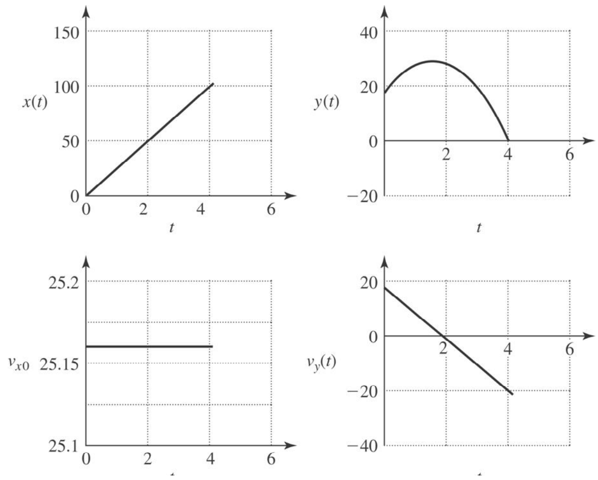
$$(y - y_0) = \frac{v_{0y}^2}{2g} = \frac{v_0^2\sin^2\alpha_0}{2g} = \frac{(30.0\text{ m/s})^2\sin^2 33.0°}{2(9.80\text{ m/s}^2)} = 13.6\text{ m}$$

b) Rather than solving a quadratic, the above height may be used to find the time the rock takes to fall from its greatest height to the ground, and hence the vertical component of velocity, $v_y = \sqrt{2yg} = \sqrt{2(28.6\text{ m})(9.80\text{ m/s}^2)} = 23.7\text{ m/s}$, and so the speed of the rock is $\sqrt{(23.7\text{ m/s})^2 + ((30.0\text{ m/s})(\cos 33.0°))^2} = 34.6\text{ m/s}$.

c) The time the rock is in the air is given by the change in the vertical component of velocity divided by the acceleration $-g$; the distance is the constant horizontal component of velocity multiplied by this time, or

$$x = (30.0\text{ m/s})\cos 33.0° \frac{(-23.7\text{ m/s} - ((30.0\text{ m/s})\sin 33.0°))}{(-9.80\text{ m/s}^2)} = 103\text{m}.$$

d)



3.24: a)

$$v_0 \cos\alpha t = 45.0\,\text{m}$$

$$\cos\alpha = \frac{45.0\,\text{m}}{(25.0\,\text{m/s})(3.00\,\text{s})} = 0.600$$

$$\alpha = 53.1°$$

b)

$$v_x = (25.0\,\text{m/s})\cos 53.1° = 15.0\ \text{m/s}$$
$$v_y = 0$$
$$v = 15.0\,\text{m/s}$$
$$a = 9.80\,\text{m/s}^2\ \text{downward}$$

c) Find y when $t = 3.00\,\text{s}$

$$y = v_0 \sin\alpha t - \frac{1}{2}gt^2$$
$$= (25.0\,\text{m/s})(\sin 53.1°)(3.00\text{s}) - \frac{1}{2}(9.80\,\text{m/s}^2)(3.00\,\text{s})^2$$
$$= 15.9\,\text{m}$$
$$v_x = 15.0\ \text{m/s} = \text{constant}$$
$$v_y = v_0 \sin\alpha - gt = (25.0\,\text{m/s})(\sin 53.1°) - (9.80\,\text{m/s}^2)(3.00\,\text{s}) = -9.41$$
$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{(15.0\,\text{m/s})^2 + (-9.41\,\text{m/s}^2} = 17.7\,\text{m/s}$$

3.25: Take $+y$ to be downward.

a) Use the vertical motion of the rock to find the initial height.

$t = 6.00\,\text{s},\ v_{0y} = +20.0\,\text{s},\ a_y = +9.80\,\text{m/s}^2,\ y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 296\,\text{m}$

b) In 6.00 s the balloon travels downward a distance $y - y_0 = (20.0\,\text{s})(6.00\,\text{s}) = 120\,\text{m}$. So, its height above ground when the rock hits is $296\,\text{m} - 120\,\text{m} = 176\,\text{m}$.

c) The horizontal distance the rock travels in 6.00 s is 90.0 m. The vertical component of the distance between the rock and the basket is 176 m, so the rock is $\sqrt{(176\,\text{m})^2 + (90\,\text{m})^2} = 198\,\text{m}$ from the basket when it hits the ground.

d) (i) The basket has no horizontal velocity, so the rock has horizontal velocity 15.0 m/s relative to the basket.

Just before the rock hits the ground, its vertical component of velocity is

$v_y = v_{0y} + a_y t = 20.0\,\text{s} + (9.80\,\text{m/s}^2)(6.00\,\text{s}) = 78.8\,\text{m/s}$, downward, relative to the ground.

The basket is moving downward at 20.0 m/s, so relative to the basket the rock has downward component of velocity 58.8 m/s.

e) horizontal: 15.0 m/s; vertical: 78.8 m/s

3.26: a) horizontal motion: $x - x_0 = v_{0x}t$ so $t = \frac{60.0\,\text{m}}{(v_0 \cos\,43°)t}$

vertical motion (take $+y$ to be upward):

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $25.0\,\text{m} = (v_0 \sin 43.0°)t + \frac{1}{2}(-9.80\,\text{m/s}^2)t^2$

Solving these two simultaneous equations for v_0 and t gives $v_0 = 3.26\,\text{m/s}$ and $t = 2.51\,\text{s}$.

b) v_y when shell reaches cliff:

$$v_y = v_{0y} + a_y t = (32.6\,\text{m/s})\sin 43.0° - (9.80\,\text{m/s}^2)(2.51\text{s}) = -2.4\,\text{m/s}$$

The shell is traveling downward when it reaches the cliff, so it lands right at the edge of the cliff.

3.27: Take $+y$ to be upward.

Use the vertical motion to find the time it takes the suitcase to reach the ground:

$v_{0y} = v_0 \sin 23°,\ a_y = -9.80\,\text{m/s}^2,\ y - y_0 = -114\,\text{m},\ t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 9.60\,\text{s}$

The distance the suitcase travels horizontally is $x - x_0 = v_{0x} = (v_0 \cos 23.0°)t = 795\,\text{m}$

3.28: For any item in the washer, the centripetal acceleration will be proportional to the square of the frequency, and hence inversely proportional to the square of the rotational period; tripling the centripetal acceleration involves decreasing the period by a factor of $\sqrt{3}$, so that the new period T' is given in terms of the previous period T by $T' = T/\sqrt{3}$.

3.29: Using the given values in Eq. (3.30),

$$a_{\text{rad}} = \frac{4\pi^2(6.38\times10^6\ \text{m})}{((24\,\text{h})(3600\,\text{s/h}))^2} = 0.034\ \text{m/s}^2 = 3.4\times10^{-3}g.$$

(Using the time for the siderial day instead of the solar day will give an answer that differs in the third place.) b) Solving Eq. (3.30) for the period T with $a_{\text{rad}} = g$,

$$T = \sqrt{\frac{4\pi^2(6.38\times10^6\,\text{m})}{9.80\,\text{m/s}^2}} = 5070\,\text{s} \sim 1.4\,\text{h}.$$

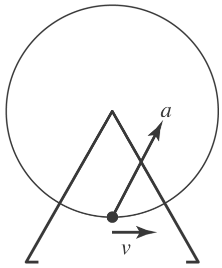
3.30: $550\,\text{rev/min} = 9.17\,\text{rev/s}$, corresponding to a period of 0.109 s. a) From Eq. (3.29), $v = \frac{2\pi R}{T} = 196\,\text{m/s}$. b) From either Eq. (3.30) or Eq. (3.31), $a_{\text{rad}} = 1.13\times10^4\ \text{m/s}^2 = 1.15\times10^3\ \text{g}$.

3.31: Solving Eq. (3.30) for T in terms of R and a_{rad},
a) $\sqrt{4\pi^2(7.0\,\text{m})/(3.0)(9.80\,\text{m/s}^2)} = 3.07\,\text{s}$. b) 1.68 s.

3.32: a) Using Eq. (3.31), $\frac{2\pi R}{T} = 2.97\times10^4\ \text{m/s}$. b) Either Eq. (3.30) or Eq. (3.31) gives $a_{\text{rad}} = 5.91\times10^{-3}\ \text{m/s}^2$. c) $v = 4.78\times10^4\ \text{m/s}$, and $a = 3.97\times10^{-2}\ \text{m/s}^2$.

3.33: a) From Eq. (3.31), $a = (7.00\,\text{m/s})^2/(15.0\,\text{m}) = 3.50\,\text{m/s}^2$. The acceleration at the bottom of the circle is toward the center, up.
b) $a = 3.50\,\text{m/s}^2$, the same as part (a), but is directed *down*, and still towards the center.
c) From Eq. (3.29), $T = 2\pi R/v = 2\pi(15.0\,\text{m})/(7.00\,\text{m/s}) = 12.6\,\text{s}$.

3.34: a) $a_{\text{rad}} = (3\,\text{m/s})^2/(14\,\text{m}) = 0.643\,\text{m/s}^2$, and $a_{\tan} = 0.5\,\text{m/s}^2$. So,
$a = ((0.643\,\text{m/s}^2)^2 + (0.5\,\text{m/s}^2)^2)^{1/2} = 0.814\,\text{m/s}^2, 37.9°$ to the right of vertical.
b)



3.35: b) No. Only in a circle would a_{rad} point to the center (See planetary motion in Chapter 12).
c) Where the car is farthest from the center of the ellipse.

3.36: Repeated use of Eq. (3.33) gives a) $5.0 = \text{m/s}$ to the right, b) 16.0 m/s to the left, and c) $13.0 = \text{m/s}$ to the left.

3.37: a) The speed relative to the ground is $1.5\,\text{m/s} + 1.0\,\text{m/s} = 2.5\,\text{m/s}$, and the time is $35.0\,\text{m}/2.5\,\text{m/s} = 14.0\,\text{s}$. b) The speed relative to the ground is 0.5 m/s, and the time is 70 s.

3.38: The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min). The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream, so the total time the rower takes is

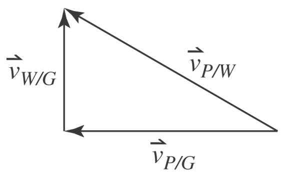
$$\frac{1.5\,\text{km}}{6.8\,\text{km/h}} + \frac{1.5\,\text{km}}{1.2\,\text{km/h}} = 1.47\,\text{hr} = 88\,\text{min}.$$

3.39: The velocity components are

$$-0.50\,\text{m/s} + (0.40\,\text{m/s})/\sqrt{2}\,\text{east} \quad \text{and} \quad (0.40\,\text{m/s})/\sqrt{2}\,\text{south},$$

for a velocity relative to the earth of 0.36 m/s, $52.5°$ south of west.

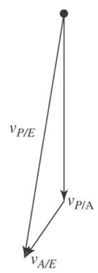
3.40: a) The plane's northward component of velocity relative to the air must be 80.0 km/h, so the heading must be $\arcsin \frac{80.8}{320} = 14°$ north of west. b) Using the angle found in part (a), $(320\,\text{km/h})\cos 14° = 310\,\text{km/h}$. Equivalently,
$\sqrt{(320\,\text{km/h})^2 - (80.0\,\text{km/h})^2} = 310\,\text{km/h}$.



3.41: a) $\sqrt{(2.0\,\text{m/s})^2 + (4.2\,\text{m/s})^2} = 4.7\,\text{m/s}, \arctan\frac{2.0}{4.2} = 25.5°$, south of east.
b) $800\,\text{m}/4.2\,\text{m/s} = 190\,\text{s}$.
c) $2.0\,\text{m/s} \times 190\,\text{s} = 381\,\text{m}$.

3.42: a) The speed relative to the water is still 4.2 m/s; the necessary heading of the boat is $\arcsin\frac{2.0}{4.2} = 28°$ north of east. b) $\sqrt{(4.2\,\text{m/s})^2 - (2.0\,\text{m/s})^2} = 3.7\,\text{m/s}$, east. d) $800\,\text{m}/3.7\,\text{m/s} = 217\,\text{s}$, rounded to three significant figures.

3.43: a)



b) $x: -(10\,\text{m/s})\cos 45° = -7.1\,\text{m/s}.\ y := -(35\,\text{m/s}) - (10\,\text{m/s})\sin 45° = -42.1\,\text{m/s}$.
c) $\sqrt{(-7.1\,\text{m/s})^2 + (-42.1\,\text{m/s})^2} = 42.7\,\text{m/s}, \arctan\frac{-42.1}{-7.1} = 80°$, south of west.

3.44: a) Using generalizations of Equations 2.17 and 2.18,
$v_x = v_{0x} + \frac{\alpha}{3}t^3, v_y = v_{0y} + \beta t - \frac{\gamma}{2}t^2$, and $x = v_{0x}t + \frac{\alpha}{12}t^4, y = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$. b) Setting $v_y = 0$ yields a quadratic in $t, 0 = v_{0y} + \beta t - \frac{\gamma}{2}t^2$, which has as the positive solution

$$t = \frac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2v_0\gamma}\right] = 13.59\,\text{s},$$

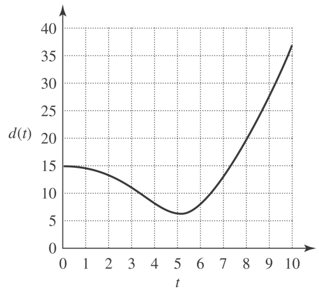
keeping an extra place in the intermediate calculation. Using this time in the expression for $y(t)$ gives a maximum height of 341 m.

3.45: a) The $a_x = 0$ and $a_y = -2\beta$, so the velocity and the acceleration will be perpendicular only when $v_y = 0$, which occurs at $t = 0$.

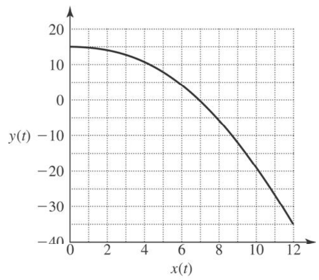
b) The speed is $v = (\alpha^2 + 4\beta^2 t^2)^{1/2}$, $dv/dt = 0$ at $t = 0$. (See part d below.)

c) r and v are perpendicular when their dot product is 0:
$(\alpha t)(\alpha) + (15.0\text{ m} - \beta t^2) \times (-2\beta t) = \alpha^2 t - (30.0\text{ m})\beta t + 2\beta^2 t^3 = 0$. Solve this for t:
$t = \pm\sqrt{\frac{(30.0\text{ m})(0.500\text{ m/s}^2) - (1.2\text{ m/s})^2}{2(0.500\text{ m/s}^2)^2}} = +5.208\text{ s}$, and 0 s, at which times the student is at (6.25 m, 1.44 m) and (0 m, 15.0 m), respectively.

d) At $t = 5.208\text{ s}$, the student is 6.41 m from the origin, at an angle of $13°$ from the x-axis. A plot of $d(t) = (x(t)^2 + y(t)^2)^{1/2}$ shows the minimum distance of 6.41 m at 5.208 s:



e) In the x - y plane the student's path is:



3.46: a) Integrating, $\vec{r} = (\alpha t - \frac{\beta}{3}t^3)\hat{i} + (\frac{\gamma}{2}t^2)\hat{j}$. Differentiating, $\vec{a} = (-2\beta)\hat{i} + \gamma\hat{j}$.

b) The positive time at which $x = 0$ is given by $t^2 = 3\alpha/\beta$. At this time, the y-coordinate is

$$y = \frac{\gamma}{2}t^2 = \frac{3\alpha\gamma}{2\beta} = \frac{3(2.4\text{ m/s})(4.0\text{ m/s}^2)}{2(1.6\text{ m/s}^3)} = 9.0\text{ m}$$

.

3.47: a) The acceleration is

$$a = \frac{v^2}{2x} = \frac{((88\ \text{km/h})(1\ \text{m/s})/(3.6\ \text{km/h}))^2}{2(300\ \text{m})} = 0.996\ \text{m/s}^2 \approx 1\ \text{m/s}^2$$

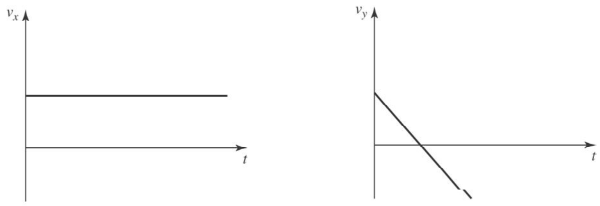
b) $\arctan\left(\frac{15\ \text{m}}{460\ \text{m} - 300\ \text{m}}\right) = 5.4°$. c) The vertical component of the velocity is $(88\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\frac{15\ \text{m}}{160\ \text{m}} = 2.3\ \text{m/s}$. d) The average speed for the first 300 m is 44 km/h, so the elapsed time is

$$\frac{300\ \text{m}}{(44\ \text{km/h})(1\ \text{m/s})(3.6\ \text{km/h})} + \frac{160\ \text{m}}{(88\ \text{km/h})(1\ \text{m/s})\cos 5.4°/(3.6\ \text{km/h})} = 31.1\ \text{s},$$

or 31 s to two places.

3.48:

a)



The equations of motions are:

$$y = h + (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

$$x = (v_0 \cos\alpha)t$$

$$v_y = v_0 \sin\alpha - gt$$

$$v_x = v_0 \cos\alpha$$

Note that the angle of 36.9° results in $\sin 36.9° = 3/5$ and $\cos 36.9° = 4/5$.

b) At the top of the trajectory, $v_y = 0$. Solve this for *t* and use in the equation for *y* to find the maximum height: $t = \frac{v_0 \sin\alpha}{g}$. Then, $y = h + (v_0 \sin\alpha)\left(\frac{v_0 \sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\alpha}{g}\right)^2$, which reduces to $y = h + \frac{v_0{}^2 \sin^2\alpha}{2g}$. Using $v_0 = \sqrt{25gh/8}$, and $\sin\alpha = 3/5$, this becomes $y = h + \frac{(25gh/8)(3/5)^2}{2g} = h + \frac{9}{16}h$, or $y = \frac{25}{16}h$. Note: This answer assumes that $y_0 = h$. Taking $y_0 = 0$ will give a result of $y = \frac{9}{16}h$ (above the roof).

c) The total time of flight can be found from the *y* equation by setting $y = 0$, assuming $y_0 = h$, solving the quadratic for *t* and inserting the total flight time in the *x* equation to find the range. The quadratic is $\frac{1}{2}gt^2 - \frac{3}{5}v_0 - h = 0$. Using the quadratic formula gives $t = \frac{(3/5)v_0 \pm \sqrt{(-(3/5)v_0)^2 - 4(\frac{1}{2}g)(-h)}}{2(\frac{1}{2}g)}$. Substituting $v_0 = \sqrt{25gh/8}$ gives $t = \frac{(3/5)\sqrt{25gh/8} \pm \sqrt{\frac{9}{25} \bullet \frac{25gh}{8} + \frac{16gh}{8}}}{g}$. Collecting terms gives *t*: $t = \frac{1}{2}\left(\sqrt{\frac{9h}{2g}} \pm \sqrt{\frac{25h}{2g}}\right) = \frac{1}{2}\left(3\sqrt{\frac{h}{2g}} \pm 5\sqrt{\frac{h}{2g}}\right)$. Only the positive root is meaningful and so $t = 4\sqrt{\frac{h}{2g}}$. Then, using $x = (v_0 \cos\alpha)t$, $x = \sqrt{\frac{25gh}{8}}\left(\frac{4}{5}\right)\left(4\sqrt{\frac{h}{2g}}\right) = 4h$.

3.49: The range for a projectile that lands at the same height from which it was launched is $R = \frac{v_0^2 \sin 2\alpha}{g}$. Assuming $\alpha = 45°$, and $R = 50$ m, $v_0 = \sqrt{gR} = 22$ m/s.

3.50: The bird's tangential velocity can be found from

$$v_x = \frac{\text{circumference}}{\text{time of rotation}} = \frac{2\pi(8.00\ \text{m})}{5.00\ \text{s}} = \frac{50.27\ \text{m}}{5.00\ \text{s}} = 10.05\ \text{m/s}$$

Thus its velocity consists of the components $v_x = 10.05\ \text{m/s}$ and $v_y = 3.00\ \text{m/s}$. The speed relative to the ground is then

$$v = v_x^2 + v_y^2 = 10.052 + 3.002 = 10.49\ \text{m/s or } 10.5\ \text{m/s}$$

(b) The bird's speed is constant, so its acceleration is strictly centripetal–entirely in the horizontal direction, toward the center of its spiral path–and has magnitude

$$a_\text{c} = \frac{v_x^2}{r} = \frac{(10.05\ \text{m/s})^2}{8.00\ \text{m}} = 12.63\ \text{m/s}^2 \quad \text{or} \quad 12.6\ \text{m/s}^2$$

(c) Using the vertical and horizontal velocity components:

$$\theta = \tan^{-1}\frac{3.00\ \text{m/s}}{10.05\ \text{m/s}} = 16.6°$$

3.51: Take $+y$ to be downward.

Use the vertical motion to find the time in the air:

$v_{0y} = 0,\ a_y = 9.80\ \text{m/s}^2,\ y - y_0 = 25\ \text{m},\ t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.259\ \text{s}$

During this time the dart must travel 90 m horizontally, so the horizontal component of its velocity must be

$$v_{0x} = \frac{x - x_0}{t} = \frac{90\ \text{m}}{2.25\ \text{s}} = 40\ \text{m/s}$$

3.52: a) Setting $y = -h$ in Eq. (3.27) (h being the stuntwoman's initial height above the ground) and rearranging gives

$$x^2 - \frac{2v_0^2 \sin\alpha_0 \cos\alpha_0}{g} x - \frac{2v_{0x}^2}{g} h = 0,$$

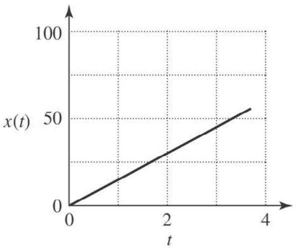
The easier thing to do here is to recognize that this can be put in the form

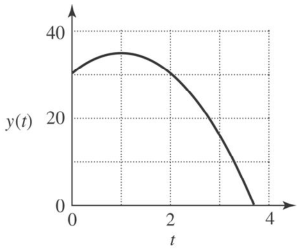
$$x^2 - \frac{2v_{0x}v_{0y}}{g} x - \frac{2v_{0x}^2}{g} h = 0,$$

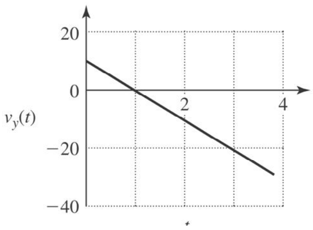
the solution to which is

$$x = \frac{v_{0x}}{g}\left[v_{0y} + \sqrt{v_{0y}^2 + 2gh}\right] = 55.5\ \text{m (south)}.$$

b) The graph of $v_x(t)$ is a horizontal line.







3.53: The distance is the horizontal speed times the time of free fall,

$$v_x\sqrt{\frac{2y}{g}} = (64.0\ \text{m/s})\sqrt{\frac{2(90\ \text{m})}{(9.80\ \text{m/s}^2)}} = 274\ \text{m}.$$

3.54: In terms of the range R and the time t that the balloon is in the air, the car's original distance is $d = R + v_{\text{car}}t$. The time t can be expressed in terms of the range and the horizontal component of velocity, $t = \frac{R}{v_0 \cos\alpha_0}$, so $d = R\left(1 + \frac{v_{\text{car}}}{v_0 \cos\alpha_0}\right)$. Using $R = v_0^2 \sin 2\alpha_0 / g$ and the given values yields $d = 29.5\ \text{m}$.

3.55: a) With $\alpha_0 = 45°$, Eq. (3.27) is solved for $v_0^2 = \frac{gx^2}{x-y}$. In this case, $y = -0.9\text{ m}$ is the change in height. Substitution of numerical values gives $v_0 = 42.8\text{ m/s}$. b) Using the above algebraic expression for v_0 in Eq. (3.27) gives

$$y = x - \left(\frac{x}{188\text{ m}}\right)^2 (188.9\text{ m})$$

Using $x = 116\text{ m}$ gives $y = 44.1\text{ m}$ above the initial height, or 45.0 m above the ground, which is 42.0 m above the fence.

3.56: The equations of motions are $y = (v_0 \sin\alpha)t - 1/2gt^2$ and $x = (v_0 \cos\alpha)t$, assuming the match starts out at $x = 0$ and $y = 0$. When the match goes in the wastebasket for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the match goes in the wastebasket for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin\alpha = \cos\alpha = \sqrt{2}/2.$.

To reach the *minimum* distance: $6D = \frac{\sqrt{2}}{2}v_0 t$, and $2D = \frac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for *t* gives $t = \frac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \frac{1}{2}g\left(\frac{6D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \frac{\sqrt{2}}{2}v_0 t$, and $2D = \frac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for *t* gives $t = \frac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2}g\left(\frac{7D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

3.57: The range for a projectile that lands at the same height from which it was launched is $R = \frac{v_0^2 \sin 2\alpha}{g}$, and the maximum height that it reaches is $H = \frac{v_0^2 \sin^2\alpha}{2g}$. We must find *R* when $H = D$ and $v_0 = \sqrt{6gD}$. Solving the height equation for $\sin\alpha$, $D = \frac{6gD\sin^2\alpha}{2g}$, or $\sin\alpha = (1/3)^{1/2}$. Then, $R = \frac{6gD\sin(70.72°)}{g}$, or $R = 5.6569D = 4\sqrt{2}D$.

3.58: Equation 3.27 relates the vertical and horizontal components of position for a given set of initial values.

a) Solving for v_0 gives

$$v_0^2 = \frac{gx^2 / 2\cos^2\alpha_0}{x\tan\alpha_0 - y}.$$

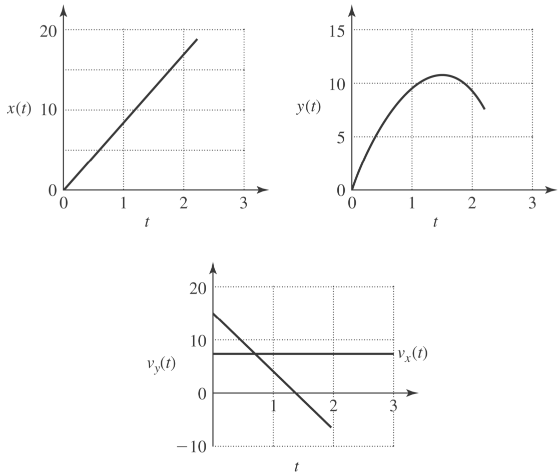
Insertion of numerical values gives $v_0 = 16.6\ \text{m/s}$.

b) Eliminating *t* between Equations 3.20 and 3.23 gives v_y as a function of x,

$$v_y = v_0\sin\alpha_0 - \frac{gx}{v_0\cos\alpha_0}.$$

Using the given values yields $v_x = v_0\cos\alpha_0 = 8.28\ \text{m/s}, v_y = -6.98\ \text{m/s}$, so $v = \sqrt{(8.28\,\text{m/s})^2 + (-6.98\ \text{m/s})^2} = 10.8\ \text{m/s}$, at an angle of $\arctan\left(\frac{-6.98}{8.24}\right) = -40.1°$, with the negative sign indicating a direction *below* the horizontal.

c) The graph of $v_x(t)$ is a horizontal line.

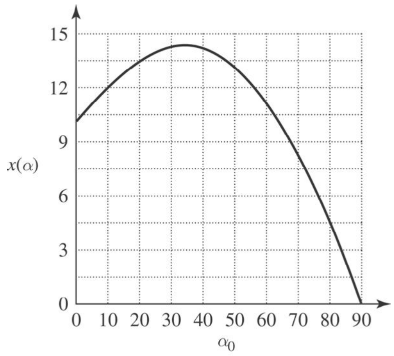


3.59: a) In Eq. (3.27), the change in height is $y = -h$. This gives a quadratic equation in x, the solution to which is

$$x = \frac{v_0^2 \cos \alpha_0}{g}\left[\tan^2 \alpha_0 + \frac{2gh}{v_0^2 \cos \alpha_0}\right]$$
$$= \frac{v_0 \cos \alpha_0}{g}\left[v_0 \sin \alpha_0 + \sqrt{v_0^2 \sin^2 \alpha_0 + 2gh}\right]$$

If $h = 0$, the square root reduces to $v_0 \sin \alpha_0$, and $x = R$. b) The expression for x becomes $x = (10.2\ \text{m}) \cos \alpha_0 + [\sin^2 \alpha_0 + \sqrt{\sin^2 \alpha_0 + 0.98}]$

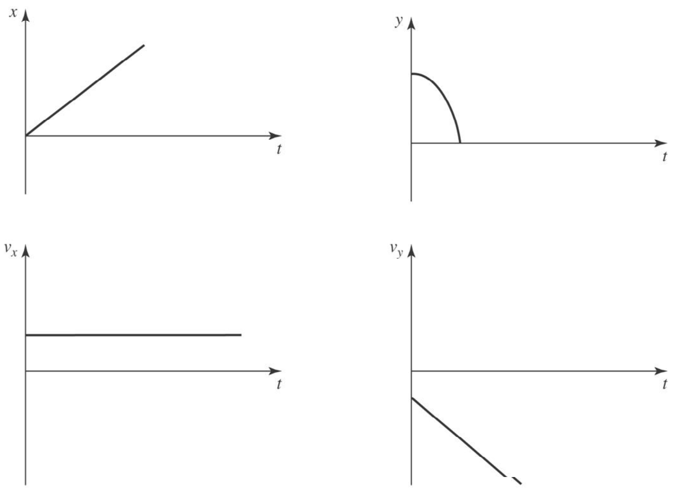
The angle $\alpha_0 = 90°$ corresponds to the projectile being launched straight up, and there is no horizontal motion. If $\alpha_0 = 0$, the projectile moves horizontally until it has fallen the distance h.



c) The maximum range occurs for an angle less than $45°$, and in this case the angle is about $36°$.

3.60: a) This may be done by a direct application of the result of Problem 3.59; with $\alpha_0 = -40°$, substitution into the expression for *x* gives 6.93 m.

b)



c) Using $(14.0\ \text{m} - 1.9\ \text{m})$ instead of *h* in the above calculation gives $x = 6.3\ \text{m}$, so the man will not be hit.

3.61: a) The expression for the range, as derived in Example 3.10, involves the sine of twice the launch angle, and

$$\sin(2(90° - \alpha_0)) = \sin(180° - 2\alpha_0) = \sin 180° \cos 2\alpha_0 - \cos 180° \sin 2\alpha_0 = \sin 2\alpha_0,$$

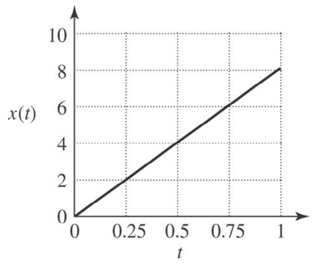
and so the range is the same. As an alternative, using $\sin(90° - \alpha_0) = \cos\alpha$ and $\cos(90° - \alpha_0) = \sin\alpha_0$ in the expression for the range that involves the product of the sine and cosine of α_0 gives the same result.

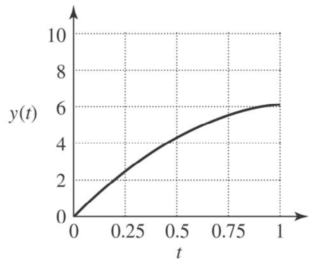
b) The range equation is $R = \frac{v_0^2 \sin 2\alpha}{g}$. In this case, $v_0 = 2.2\ \text{m/s}$ and $R = 0.25\ \text{m}$. Hence $\sin 2\alpha = (9.8\ \text{m/s}^2)(0.25\ \text{m})/(2.2\ \text{m/s}^2)$, or $\sin 2\alpha = 0.5062$; and $\alpha = 15.2°$ or $74.8°$.

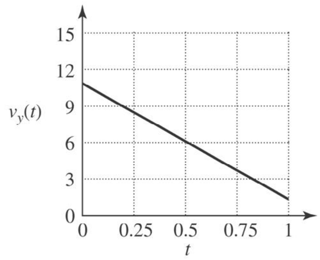
3.62: a) Using the same algebra as in Problem 3.58(a), $v_0 = 13.8\ \text{m/s}$.

b) Again, the algebra is the same as that used in Problem 3.58; $v = 8.4\ \text{m/s}$, at an angle of $9.1°$, this time above the horizontal.

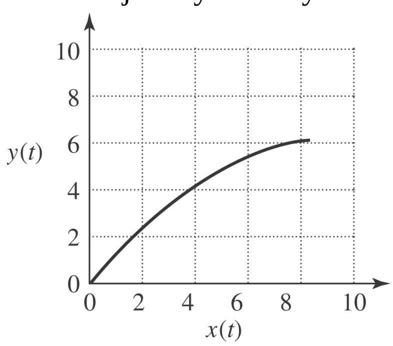
c) The graph of $v_x(t)$ is a horizontal line.







A graph of $y(t)$ vs. $x(t)$ shows the trajectory of Mary Belle as viewed from the side:



d) In this situation it's convenient to use Eq. (3.27), which becomes $y = (1.327)x - (0.071115\ \text{m}^{-1})x^2$. Use of the quadratic formula gives $x = 23.8\ \text{m}$.

3.63: a) The algebra is the same as that for Problem 3.58,

$$v_0^2 = \frac{gx^2}{2\cos^2\alpha_0(x\tan\alpha_0 - y)}.$$

In this case, the value for y is $-15.0\ \text{m}$, the change in height. Substitution of numerical values gives 17.8 m/s. b) 28.4 m from the near bank (i.e., in the water!).

3.64: Combining equations 3.25, 3.22 and 3.23 gives

$$\begin{aligned} v^2 &= v_0^2 \cos^2 \alpha_0 + (v_0 \sin \alpha_0 - gt)^2 \\ &= v_0^2 (\sin^2 \alpha_0 + \cos^2 \alpha_0) - 2v_0 \sin \alpha_0 gt + (gt)^2 \\ &= v_0^2 - 2g(v_0 \sin \alpha_0 t - \frac{1}{2} gt^2) \\ &= v_0^2 - 2gy, \end{aligned}$$

where Eq. (3.21) has been used to eliminate *t* in favor of *y*. This result, which will be seen in the chapter dealing with conservation of energy (Chapter 7), is valid for any *y*, positive, negative or zero, as long as $v^2 > 0$. For the case of a rock thrown from the roof of a building of height *h*, the speed at the ground is found by substituting $y = -h$ into the above expression, yielding $v = \sqrt{v_0^2 + 2gh}$, which is independent of α_0.

3.65: Take $+y$ to be upward. The vertical motion of the rocket is unaffected by its horizontal velocity.

a) $v_y = 0$ (at maximum height), $v_{0y} = +40.0\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 81.6\text{ m}$

b) Both the cart and the rocket have the same constant horizontal velocity, so both travel the same horizontal distance while the rocket is in the air and the rocket lands in the cart.

c) Use the vertical motion of the rocket to find the time it is in the air.

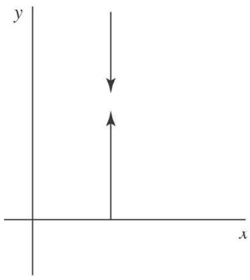
$v_{0y} = 40\text{ m/s}, a_y = -9.80\text{ m/s}^2, v_y = -40\text{ m/s}, t = ?$

$v_y = v_{0y} + a_y t$ gives $t = 8.164\text{ s}$

Then $x - x_0 = v_{0x}t = (30.0\text{ m/s})(8.164\text{ s}) = 245\text{ m}$.

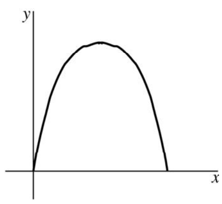
d) Relative to the ground the rocket has initial velocity components $v_{0x} = 30.0\text{ m/s}$ and $v_{0y} = 40.0\text{ m/s}$, so it is traveling at $53.1°$ above the horizontal.

e) (i)



Relative to the cart, the rocket travels straight up and then straight down

(ii)



Relative to the ground the rocket travels in a parabola.

3.66: (a)

$$v_x(\text{runner}) = v_x(\text{ball})$$
$$6.00\ \text{m/s} = (20.0\ \text{m/s})\cos\theta$$
$$\cos\theta = 0.300$$
$$\theta = 72.5°$$

Time the ball is in the air:

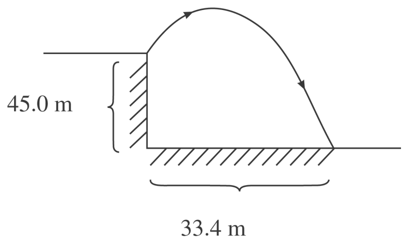
$$y = v_0 \sin\theta t - \tfrac{1}{2} g t^2$$
$$-45.0\ \text{m} = (20.0\ \text{m/s})(\sin 72.5°)t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

Solve for t: $t = 5.549\ \text{s}$.

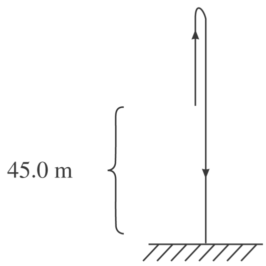
$$x = v_0 \cos\theta t = (20.0\ \text{m/s})(\cos 72.5°)(5.549\ \text{s})$$
$$= 33.4\ \text{m}$$

(b)

(i)



(ii)



3.67: Take $+y$ to be downward.

a) Use the vertical motion of the boulder to find the time it takes it to fall 20 m to the level of the surface of the water.

$v_{0y} = 0, a_y = 9.80\ \text{m/s}^2, y - y_0 = 20\ \text{m}, t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.02\ \text{s}$

The rock must travel 100 m horizontally during this time, so

$$v_{0x} = \frac{x - x_0}{t} = \frac{100\ \text{m}}{2.20\ \text{s}} = 49\ \text{m/s}.$$

b) The rock travels downward 45 m in going from the cliff to the plain. Use this vertical motion to find the time:

$v_{0y} = 0, a_y = 9.80\ \text{m/s}^2, y - y_0 = 45\ \text{m}, t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 3.03\ \text{s}$

During this time the rock travels horizontally

$x - x_0 = v_{0x}t = (49\ \text{m/s})(3.03\ \text{s}) = 150\ \text{m}$

The rock lands 50 m past the foot of the dam.

3.68: (a) When she catches the bagels, Henrietta has been jogging for 9.00 s plus the time for the bagels to fall 43.9 m from rest. Get the time to fall:

$$y = \frac{1}{2}gt^2$$

$$43.9\ \text{m} = \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

$$t = 2.99\ \text{s}$$

So she has been jogging for $9.00\ \text{s} + 2.99\ \text{s} = 12.0\ \text{s}$. During this time she has gone $x = vt = (3.05\ \text{m/s})(12.0\ \text{s}) = 36.6\ \text{m}$. Bruce must throw the bagels so they travel 36.6 m horizontally in 2.99 s

$$x = vt$$

$$36.6\ \text{m} = v(2.99\ \text{s})$$

$$v = 12.2\ \text{m/s}$$

(b) 36.6 m from the building.

3.69: Take $+y$ to be upward.

a) The vertical motion of the shell is unaffected by the horizontal motion of the tank. Use the vertical motion of the shell to find the time the shell is in the air:

$v_{0y} = v_0 \sin\alpha = 43.4\ \text{m/s}, a_y = -9.80\ \text{m/s}^2, y - y_0 = 0$ (returns to initial height), $t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 8.86\ \text{s}$

Relative to tank #1 the shell has a constant horizontal velocity $v_0 \cos\alpha = 246.2\ \text{m/s}$. Relative to the ground the horizontal velocity component is $246.2\ \text{m/s} + 15.0\ \text{m/s} = 261.2\ \text{m/s}$. Relative to tank #2 the shell has horizontal velocity component $261.2\ \text{m/s} - 35.0\ \text{m/s} = 226.2\ \text{m/s}$. The distance between the tanks when the shell was fired is the $(226.2\ \text{m/s})(8.86\ \text{s}) = 2000\ \text{m}$ that the shell travels relative to tank #2 during the 8.86 s that the shell is in the air.

b) The tanks are initially 2000 m apart. In 8.86 s tank #1 travels 133 m and tank #2 travels 310 m, in the same direction. Therefore, their separation increases by $310\ \text{m} - 183\ \text{m} = 177\ \text{m}$. So, the separation becomes 2180 m (rounding to 3 significant figures).

3.70: The firecracker's falling time can be found from the usual

$$t = \sqrt{\frac{2h}{g}}$$

The firecracker's horizontal position at any time t (taking the student's position as $x = 0$) is $x = vt - \frac{1}{2}at^2 = 0$ when cracker hits the ground, from which we can find that $t = 2v/a$. Combining this with the expression for the falling time:

$$\frac{2v}{a} = \sqrt{\frac{2h}{g}}$$

so

$$h = \frac{2v^2 g}{a^2}$$

3.71: a) The height above the player's hand will be $\frac{v_{0y}^2}{2g} = \frac{v_0^2 \sin^2 \alpha_0}{2g} = 0.40\ \text{m}$, so the maximum height above the floor is 2.23 m. b) Use of the result of Problem 3.59 gives 3.84 m. c) The algebra is the same as that for Problems 3.58 and 3.62. The distance *y* is $3.05\ \text{m} - 1.83\ \text{m} = 1.22\ \text{m}$, and

$$v_0 = \sqrt{\frac{(9.80\ \text{m/s}^2)(4.21\ \text{m})^2}{2\cos^2 35°((4.21\ \text{m})\tan 35° - 1.22\ \text{m})}} = 8.65\ \text{m/s}.$$

d) As in part (a), but with the larger speed,

$$1.83\ \text{m} + (8.65\ \text{m/s})^2 \sin^2 35° / 2(9.80\ \text{m/s}^2) = 3.09\ \text{m}.$$

The distance from the basket is the distance from the foul line to the basket, minus half the range, or

$$4.21\ \text{m} - (8.655\ \text{m/s})^2 \sin 70° / 2(9.80\ \text{m/s}^2) = 0.62\ \text{m}.$$

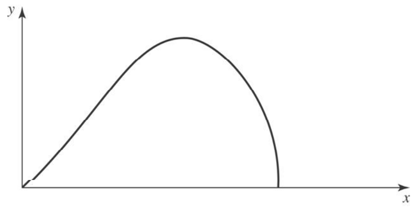
Note that an extra figure in the intermediate calculation was kept to avoid roundoff error.

3.72: The initial *y*-component of the velocity is $v_{0y} = \sqrt{2gy}$, and the time the pebble is in flight is $t = \sqrt{2y/g}$. The initial *x*-component is $v_{0x} = x/t = \sqrt{x^2 g / 2y}$. The magnitude of the initial velocity is then

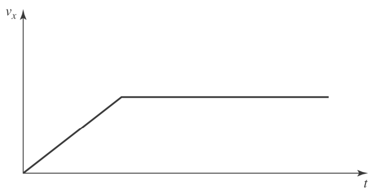
$$v_0 = \sqrt{2gy + \frac{x^2 g}{2y}} = \sqrt{2gy}\sqrt{1 + \left(\frac{x}{2y}\right)^2},$$

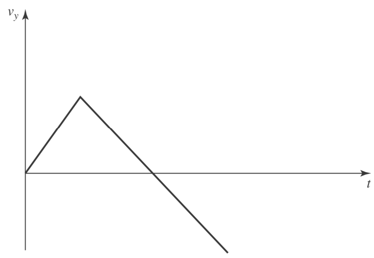
and the angle is $\arctan\left(\frac{v_{0y}}{v_{0x}}\right) = \arctan(2y/x)$.

3.73: a) The acceleration is given as g at an angle of $53.1°$ to the horizontal. This is a 3-4-5 triangle, and thus, $a_x = (3/5)g$ and $a_y = (4/5)g$ during the "boost" phase of the flight. Hence this portion of the flight is a straight line at an angle of $53.1°$ to the horizontal. After time *T*, the rocket is in free flight, the acceleration is $a_x = 0$ and $a_y = g$, and the familiar equations of projectile motion apply. During this coasting phase of the flight, the trajectory is the familiar parabola.



b) During the boost phase, the velocities are: $v_x = (3/5)gt$ and $v_y = (4/5)gt$, both straight lines. After $t = T$, the velocities are $v_x = (3/5)gT$, a horizontal line, and $v_y = (4/5)gT - g(t - T)$, a negatively sloping line which crosses the axis at the time of the maximum height.





c) To find the maximum height of the rocket, set $v_y = 0$, and solve for *t*, where $t = 0$ when the engines are cut off, use this time in the familiar equation for *y*. Thus, using $t = (4/5)T$ and

$y_{\max} = y_0 + v_{0y}t - \frac{1}{2}gt^2$, $y_{\max} = \frac{2}{5}gT^2 + \frac{4}{5}gT(\frac{4}{5}T) - \frac{1}{2}g(\frac{4}{5}T)^2$, $y_{\max} = \frac{2}{5}gT^2 + \frac{16}{25}gT^2 - \frac{8}{25}g$

Combining terms, $y_{\max} = \frac{18}{25}gT^2$.

d) To find the total horizontal distance, break the problem into three parts: The boost phase, the rise to maximum, and the fall back to earth. The fall time back to earth can be found from the answer to part (c), $(18/25)gT^2 = (1/2)gt^2$, or $t = (6/5)T$. Then, multiplying these times and the velocity, $x = \frac{3}{10}gT^2 + (\frac{3}{5}gT)(\frac{4}{5}T) + (\frac{3}{5}gT)(\frac{6}{5}T)$, or $x = \frac{3}{10}gT^2 + \frac{12}{25}gT^2 + \frac{18}{25}gT^2$. Combining terms gives $x = \frac{3}{2}gT^2$.

3.74: In the frame of the hero, the range of the object must be the initial separation plus the amount the enemy has pulled away in that time. Symbolically, $R = x_0 + v_{\mathrm{E/H}} t = x_0 + v_{\mathrm{E/H}} \frac{R}{v_{0x}}$, where $v_{\mathrm{E/H}}$ is the velocity of the enemy relative to the hero, t is the time of flight, $v_{0\mathrm{x}}$ is the (constant) x-component of the grenade's velocity, as measured by the hero, and R is the range of the grenade, also as measured by the hero. Using Eq. (3-29) for R, with $\sin 2\alpha_0 = 1$ and $v_{0x} = v_0 / \sqrt{2}$,

$$\frac{v_0^2}{g} = x_0 + v_{\mathrm{E/H}} \frac{v_0}{g} \sqrt{2}, \quad \text{or} \quad v_0^2 - (\sqrt{2} v_{\mathrm{E/H}}) v_0 - g x_0 = 0.$$

This quadratic is solved for

$$v_0 = \frac{1}{2}(\sqrt{2} v_{\mathrm{E/H}} + \sqrt{2 v_{\mathrm{E/H}}^2 + 4 g x_0}) = 61.1\,\mathrm{km/h},$$

where the units for g and x_0 have been properly converted. Relative to the earth, the x-component of velocity is $90.0\,\mathrm{km/h} + (61.1\,\mathrm{km/h})\cos 45° = 133.2\,\mathrm{km/h}$, the y-component, the same in both frames, is $(61.1\,\mathrm{km/h})\sin 45° = 43.2\,\mathrm{km/h}$, and the magnitude of the velocity is then 140 km/h.

3.75: a) $x^2 + y^2 = (R\cos\omega t)^2 + (R\sin\omega t)^2 = R^2(\cos^2\omega t + \sin^2\omega t) = R^2$, so the radius is R.

b)
$$v_x = -\omega R \sin\omega t, \; v_y = \omega R \cos\omega t,$$

and so the dot product

$$\begin{aligned}\vec{r} \cdot \vec{v} &= x v_x + y v_y \\ &= (R\cos\omega t)(-\omega R \sin\omega t) + (R\sin\omega t)(\omega R\cos\omega t) \\ &= \omega R(-\cos\omega t \sin\omega t + \sin\omega t \cos\omega t) \\ &= 0.\end{aligned}$$

c) $a_x = -\omega^2 R\cos\omega t = -\omega^2 x, \quad a_y = \omega^2 R \sin\omega t = -\omega^2 y,$
and so $\vec{a} = -\omega^2 \vec{r}$ and $a = \omega^2 R.$

d) $v^2 = v_x^2 + v_y^2 = (-\omega R\sin\omega t)^2 + (\omega R\cos\omega t)^2 = \omega^2 R^2(\sin^2\omega t + \cos^2\omega t) = \omega^2 R^2$, and so $v = \omega R$.

e)
$$a = \omega^2 R = \frac{(\omega R)^2}{R} = \frac{v^2}{R}.$$

3.76: a)

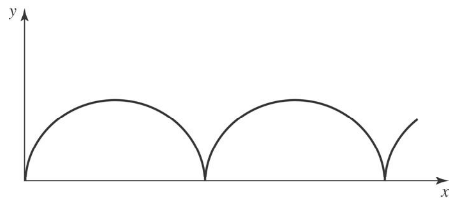
$$\frac{dv}{dt} = \frac{d}{dt}\sqrt{v_x^2 + v_y^2}$$
$$= \frac{(1/2)\frac{d}{dt}(v_x^2 + v_y^2)}{\sqrt{v_x^2 + v_y^2}}$$
$$= \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}}.$$

b) Using the numbers from Example 3.1 and 3.2,

$$\frac{dv}{dt} = \frac{(-1.0\ \text{m/s})(-0.50\ \text{m/s}^2) + (1.3\ \text{m/s})(0.30\ \text{m/s}^2)}{\sqrt{(-1.0\ \text{m/s})^2 + (1.3\ \text{m/s})^2}} = 0.54\ \text{m/s}.$$

The acceleration is due to changing both the magnitude and direction of the velocity. If the direction of the velocity is changing, the magnitude of the acceleration is larger than the rate of change of speed. c) $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{a}} = v_x a_x + v_y a_y$, $v = \sqrt{v_x^2 + v_y^2}$, and so the above form for $\frac{dv}{dt}$ is seen to be $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{a}} / v$.

3.77: a) The path is a cycloid.



b) To find the velocity components, take the derivative of x and y with respect to time: $v_x = R\omega(1 - \cos\omega t)$, and $v_y = R\omega \sin \omega t$. To find the acceleration components, take the derivative of v_x and v_y with respect to time: $a_x = R\omega^2 \sin\omega t$, and $a_y = R\omega^2 \cos\omega t$.

c) The particle is at rest $(v_y = v_x = 0)$ every period, namely at $t = 0, 2\pi/\omega, 4\pi/\omega, \ldots.$ At that time, $x = 0, 2\pi R, 4\pi R, \ldots;$ and $y = 0.$ The acceleration is $a = R\omega^2$ in the $+y$-direction.

d) No, since $a = \left[\left(R\omega^2 \sin\omega t\right)^2 + \left(R\omega^2 \cos\omega t\right)^2\right]^{1/2} = R\omega^2.$

3.78: A direct way to find the angle is to consider the velocity relative to the air and the velocity relative to the ground as forming two sides of an isosceles triangle. The wind direction relative to north is half of the included angle, or $\arcsin(10/50) = 11.53°$, east of north.

3.79: Finding the infinite series consisting of the times between meeting with the brothers is possible, and even entertaining, but hardly necessary. The relative speed of the brothers is 70 km/h, and as they are initially 42 km apart, they will reach each other in six-tenths of an hour, during which time the pigeon flies 30 km.

3.80: a) The drops are given as falling vertically, so their horizontal component of velocity with respect to the earth is zero. With respect to the train, their horizontal component of velocity is 12.0 m/s, west (as the train is moving eastward). b) The vertical component, in either frame, is $(12.0\ \text{m/s})/(\tan 30°) = 20.8\ \text{m/s},$ and this is the magnitude of the velocity in the frame of the earth. The magnitude of the velocity in the frame of the train is $\sqrt{(12.0\ \text{m/s})^2 + (20.8\ \text{m/s})^2} = 24\ \text{m/s}.$ This is, of course, the same as $(12.0\ \text{m/s})/\sin 30°.$

3.81: a) With no wind, the plane would be 110 km west of the starting point; the wind has blown the plane 10 km west and 20 km south in half an hour, so the wind velocity is $\sqrt{(20\ \text{km/h})^2 + (40\ \text{km/h})^2} = 44.7\ \text{km/h}$ at a direction of $\arctan(40/20) = 63°$ south of west. b) $\arcsin(40/220) = 10.5°$ north of west.

3.82: a) $2D/v$ b) $2Dv/(v^2 - w^2)$ c) $2D/\sqrt{v^2 - w^2}$ d) 1.50 h, 1.60 h, 1.55 h.

3.83: a) The position of the bolt is $3.00\ \text{m} + (2.50\ \text{m/s})t - 1/2(9.80\ \text{m/s}^2)t^2,$ and the position of the floor is (2.50 m/s)*t*. Equating the two, $3.00\ \text{m} = (4.90\ \text{m/s}^2)t^2.$ Therefore $t = 0.782\ \text{s}.$ b) The velocity of the bolt is $2.50\ \text{m/s} - (9.80\ \text{m/s}^2)(0.782\ \text{s}) = -5.17\ \text{m/s}$ relative to Earth, therefore, relative to an observer in the elevator $v = -5.17\ \text{m/s} - 2.50\ \text{m/s} = -7.67\ \text{m/s}.$ c) As calculated in part (b), the speed relative to Earth is 5.17 m/s. d) Relative to Earth, the distance the bolt travelled is $(2.50\ \text{m/s})t - 1/2(9.80\ \text{m/s}^2)t^2 = (2.50\ \text{m/s})(0.782\ \text{s}) - (4.90\ \text{m/s}^2)(0.782\ \text{s})^2 = -1.04\ \text{m}$

3.84: Air speed of plane $= \frac{5310\ \text{km}}{6.60\ \text{h}} = 804.5\ \text{km/h}$

With wind from A to B:

$$t_{\text{AB}} + t_{\text{BA}} = 6.70\ \text{h}$$

Same distance both ways:

$$(804.5\ \text{km/h} + v_{\text{w}})t_{\text{AB}} = \frac{5310\ \text{km}}{2} = 2655\ \text{km}$$

$$(804.5\ \text{km/h} + v_{\text{w}})t_{\text{BA}} = 2655\ \text{km}$$

Solve (1), (2), and (3) to obtain wind speed v_{w} :

$$v_{\text{w}} = 98.1\ \text{km/h}$$

3.85: The three relative velocities are:

$\vec{v}_{\mathrm{J/G,}}$ Juan relative to the ground. This velocity is due north and has magnitude $v_{\mathrm{J/G}} = 8.00\,\mathrm{m/s}$.

$\vec{v}_{\mathrm{B/G,}}$ the ball relative to the ground. This vector is $37.0°$ east of north and has magnitude $v_{\mathrm{B/G}} = 12.0\,\mathrm{m/s}$.

$\vec{v}_{\mathrm{B/J,}}$ the ball relative to Juan. We are asked to find the magnitude and direction of this vector.

The relative velocity addition equation is $\vec{v}_{\mathrm{B/G}} = \vec{v}_{\mathrm{B/J}} + \vec{v}_{\mathrm{J/G,}}$ so $\vec{v}_{\mathrm{B/J}} = \vec{v}_{\mathrm{B/G}} - \vec{v}_{\mathrm{J/G}}$.

Take $+y$ to be north and $+x$ to be east.

$v_{\mathrm{B/J}x} = +v_{\mathrm{B/G}} \sin 37.0° = 7.222\,\mathrm{m/s}$

$v_{\mathrm{B/J}y} = +v_{\mathrm{B/G}} \cos 37.0° - v_{J/G} = 1.584\,\mathrm{m/s}$

These two components give $v_{B/J} = 7.39\,\mathrm{m/s}$ at $12.4°$ north of east.

3.86: a) $v_{0y} = \sqrt{2gh} = \sqrt{2(9.80\,\mathrm{m/s^2})(4.90\,\mathrm{m})} = 9.80\,\mathrm{m/s}$. b) $v_{0y}/g = 1.00\,\mathrm{s}$. c) The speed relative to the man is $\sqrt{(10.8\,\mathrm{m/s})^2 - (9.80\,\mathrm{m/s})^2} = 4.54\,\mathrm{m/s},$ and the speed relative to the hoop is 13.6 m/s (rounding to three figures), and so the man must be 13.6 m in front of the hoop at release. d) Relative to the flat car, the ball is projected at an angle $\theta = \tan^{-1}\left(\frac{9.80\ \mathrm{m/s}}{4.54\ \mathrm{m/s}}\right) = 65°$. Relative to the ground the angle is $\theta = \tan^{-1}\left(\frac{9.80\ \mathrm{m/s}}{4.54\ \mathrm{m/s} + 9.10\ \mathrm{m/s}}\right) = 35.7°$

3.87: a) $(150\,\mathrm{m/s})^2 \sin 2° / 9.80\,\mathrm{m/s^2} = 80\,\mathrm{m}$.

b) $1000 \times \frac{\pi(10\times10^{-2}\ \mathrm{m})^2}{\pi(80\ \mathrm{m})^2} = 1.6\times10^{-3}$.

c) The slower rise will tend to reduce the time in the air and hence reduce the radius. The slower horizontal velocity will also reduce the radius. The lower speed would tend to increase the time of descent, hence increasing the radius. As the bullets fall, the friction effect is smaller than when they were rising, and the overall effect is to decrease the radius.

3.88: Write an expression for the square of the distance (D^2) from the origin to the particle, expressed as a function of time. Then take the derivative of D^2 with respect to *t*, and solve for the value of *t* when this derivative is zero. If the discriminant is zero or negative, the distance *D* will never decrease. Following this process, $\sin^{-1}\sqrt{8/9} = 70.5°$.

3.89: a) The trajectory of the projectile is given by Eq. (3.27), with $\alpha_0 = \theta + \varphi,$ and the equation describing the incline is $y = x \tan\theta.$ Setting these equal and factoring out the $x = 0$ root (where the projectile is on the incline) gives a value for $x_0;$ the range measured along the incline is

$$x/\cos\theta = \left[\frac{2v_0^2}{g}\right][\tan(\theta+\varphi) - \tan\theta]\left[\frac{\cos^2(\theta+\varphi)}{\cos\theta}\right].$$

b) Of the many ways to approach this problem, a convenient way is to use the same sort of "trick", involving double angles, as was used to derive the expression for the range along a horizontal incline. Specifically, write the above in terms of $\alpha = \theta + \varphi,$ as

$$R = \left[\frac{2v_0^2}{g\cos^2\theta}\right][\sin\alpha\cos\alpha\cos\theta - \cos^2\alpha\sin\theta].$$

The dependence on α and hence φ is in the second term. Using the identities $\sin\alpha\cos\alpha = (1/2)\sin 2\alpha$ and $\cos^2\alpha = (1/2)(1+\cos 2\alpha),$ this term becomes

$$(1/2)[\cos\theta\sin 2\alpha - \sin\theta\cos 2\alpha - \sin\theta] = (1/2)[\sin(2\alpha-\theta) - \sin\theta].$$

This will be a maximum when $\sin(2\alpha - \theta)$ is a maximum, at $2\alpha - \theta = 2\varphi + \theta = 90°,$ or $\varphi = 45° - \theta/2.$ Note that this reduces to the expected forms when $\theta = 0$ (a flat incline, $\varphi = 45°$ and when $\theta = -90°$ (a vertical cliff), when a horizontal launch gives the greatest distance).

3.90: As in the previous problem, the horizontal distance x in terms of the angles is

$$\tan\theta = \tan(\theta+\phi) - \left(\frac{gx}{2v_0^2}\right)\frac{1}{\cos^2(\theta+\phi)}.$$

Denote the dimensionless quantity $gx/2v_0^2$ by β; in this case

$$\beta = \frac{(9.80\ \text{m/s}^2)(60.0\ \text{m})\cos 30.0°}{2(32.0\ \text{m/s})^2} = 0.2486.$$

The above relation can then be written, on multiplying both sides by the product $\cos\theta\cos(\theta+\phi)$,

$$\sin\theta\cos(\theta+\phi) = \sin(\theta+\phi)\cos\theta - \frac{\beta\cos\theta}{\cos(\theta+\phi)},$$

and so

$$\sin(\theta+\phi)\cos\theta - \cos(\theta+\phi)\sin\theta = \frac{\beta\cos\theta}{\cos(\theta+\phi)}.$$

The term on the left is $\sin((\theta+\phi)-\theta) = \sin\phi,$ so the result of this combination is

$$\sin\phi\cos(\theta+\phi) = \beta\cos\theta.$$

Although this can be done numerically (by iteration, trial-and-error, or other methods), the expansion $\sin a\cos b = \frac{1}{2}(\sin(a+b)+\sin(a-b))$ allows the angle ϕ to be isolated; specifically, then

$$\frac{1}{2}(\sin(2\phi+\theta)+\sin(-\theta)) = \beta\cos\theta,$$

with the net result that

$$\sin(2\phi+\theta) = 2\beta\cos\theta + \sin\theta.$$

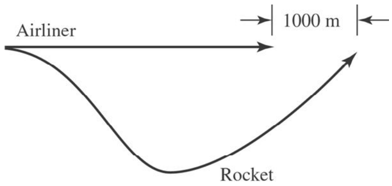
a) For $\theta = 30°$, and β as found above, $\phi = 19.3°$ and the angle above the horizontal is $\theta+\phi = 49.3°.$ For level ground, using $\beta = 0.2871,$ gives $\phi = 17.5°.$ b) For $\theta = -30°$, the same β as with $\theta = 30°$ may be used $(\cos 30° = \cos(-30°))$, giving $\phi = 13.0°$ and $\phi+\theta = -17.0°.$

3.91: In a time Δt, the velocity vector has moved through an angle (in radians) $\Delta\phi = \frac{v\Delta t}{R}$ (see Figure 3.23). By considering the isosceles triangle formed by the two velocity vectors, the magnitude $|\Delta\vec{v}|$ is seen to be $2v\sin(\phi/2)$, so that

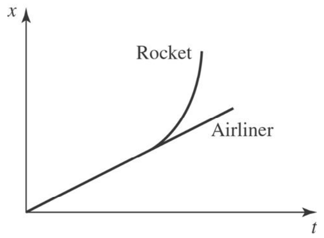
$$|\vec{a}_{\text{ave}}| = 2\frac{v}{\Delta t}\sin\left(\frac{v\Delta t}{2R}\right) = \frac{10\,\text{m/s}}{\Delta \text{t}}\sin(1.0/\text{s}\cdot\Delta\text{t})$$

Using the given values gives magnitudes of $9.59\,\text{m/s}^2, 9.98\,\text{m/s}^2$ and $10.0\,\text{m/s}^2$. The instantaneous acceleration magnitude, $v^2/R = (5.00\,\text{m/s})^2/(2.50\,\text{m}) = 10.0\,\text{m/s}^2$ is indeed approached in the limit at $\Delta t \to 0$. The changes in direction of the velocity vectors are given by $\Delta\theta = \frac{v\Delta t}{R}$ and are, respectively, 1.0 rad, 0.2 rad, and 0.1 rad. Therefore, the angle of the average acceleration vector with the original velocity vector is $\frac{\pi+\Delta\theta}{2} = \pi/2 + 1/2\,\text{rad}\,(118.6°), \pi/2 + 0.1\,\text{rad}\,(95.7°)$, and $\pi/2 + 0.05\,\text{rad}\,(92.9°)$.

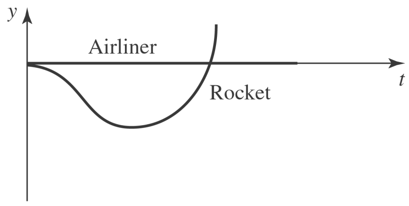
3.92:



The x-position of the plane is $(236\,\text{m/s})t$ and the x-position of the rocket is $(236\,\text{m/s})t + 1/2(3.00)(9.80\,\text{m/s}^2)\cos 30°(t-T)^2$. The graphs of these two have the form,



If we take $y = 0$ to be the altitude of the airliner, then $y(t) = -1/2gT^2 - gT(t-T) + 1/2(3.00)(9.80\,\text{m/s}^2)(\sin 30°)(t-T)^2$ for the rocket. This graph looks like



By setting $y = 0$ for the rocket, we can solve for t in terms of $T, 0 = -(4.90\,\text{m/s}^2)T^2 - (9.80\,\text{m/s}^2)T(t-T) + (7.35\,\text{m/s}^2)(t-T)^2$. Using the quadratic formula for the variable $x = t - T$, we find

$$x = t - T = \frac{(9.80\text{ m/s}^2)T + \sqrt{(9.80\text{ m/s}^2 T)^2 + (4)(7.35\text{ m/s}^2)(4.9)T^2}}{2(7.35\text{ m/s}^2)}$$ or $t = 2.72\,T$. Now, using the condition that $x_{\text{rocket}} - x_{\text{plane}} = 1000\,\text{m}$, we find $(236\,\text{m/s})t + (12.7\,\text{m/s}^2) \times (t-T)^2 - (236\,\text{m/s})t = 1000\,\text{m}$, or $(1.72T)^2 = 78.6\,\text{s}^2$. Therefore $T = 5.15\,\text{s}$.

3.93: a) Taking all units to be in km and h, we have three equations. We know that heading upstream $v_{c/w} - v_{w/G} = 2$ where $v_{c/w}$ is the speed of the curve relative to water and $v_{w/G}$ is the speed of the water relative to the ground. We know that heading downstream for a time $t, (v_{c/w} + v_{w/G})t = 5.$ We also know that for the bottle $v_{w/G}(t+1) = 3.$ Solving these three equations for $v_{w/G} = x, v_{c/w} = 2 + x,$ therefore $(2 + x + x)t = 5$ or $(2 + 2x)t = 5.$ Also $t = 3/x - 1,$ so $(2 + 2x)(\frac{3}{x} - 1) = 5$ or $2x^2 + x - 6 = 0.$ The positive solution is $x = v_{w/G} = 1.5\,\text{km/h}.$

b) $v_{c/w} = 2\,\text{km/h} + v_{w/G} = 3.5\,\text{km/h}.$

4.1: a) For the magnitude of the sum to be the sum of the magnitudes, the forces must be parallel, and the angle between them is zero. b) The forces form the sides of a right isosceles triangle, and the angle between them is $90°$. Alternatively, the law of cosines may be used as

$$F^2 + F^2 = \left(\sqrt{2}F\right)^2 - 2F^2 \cos\theta,$$

from which $\cos\theta = 0$, and the forces are perpendicular. c) For the sum to have 0 magnitude, the forces must be antiparallel, and the angle between them is $180°$.

4.2: In the new coordinates, the 120-N force acts at an angle of $53°$ from the $-x$-axis, or $233°$ from the $+x$-axis, and the 50-N force acts at an angle of $323°$ from the $+x$-axis.

a) The components of the net force are

$$R_x = (120\ \text{N})\cos 233° + (50\ \text{N})\cos 323° = -32\ \text{N}$$

$$R_y = (250\ \text{N}) + (120\ \text{N})\sin 233° + (50\ \text{N})\sin 323° = 124\ \text{N}.$$

b) $R = \sqrt{R_x^2 + R_y^2} = 128\ \text{N}$, $\arctan\left(\frac{124}{-32}\right) = 104°$. The results have the same magnitude, and the angle has been changed by the amount $(37°)$ that the coordinates have been rotated.

4.3: The horizontal component of the force is $(10\ \text{N})\cos\ 45° = 7.1\ \text{N}$ to the right and the vertical component is $(10\ \text{N})\sin\ 45° = 7.1\ \text{N}$ down.

4.4: a) $F_x = F\cos\theta$, where θ is the angle that the rope makes with the ramp ($\theta = 30°$ in this problem), so $F = \left|\vec{\boldsymbol{F}}\right| = \frac{F_x}{\cos\theta} = \frac{60.0\ \text{N}}{\cos\ 30°} = 69.3\ \text{N}.$

b) $F_y = F\sin\theta = F_x \tan\theta = 34.6\ \text{N}.$

4.5: Of the many ways to do this problem, two are presented here.

Geometric: From the law of cosines, the magnitude of the resultant is

$$R=\sqrt{(270\ \mathrm{N})^2+(300\ \mathrm{N})^2+2(270\ \mathrm{N})(300\ \mathrm{N})\cos 60^\circ}=494\ \mathrm{N}.$$

The angle between the resultant and dog A's rope (the angle opposite the side corresponding to the 250-N force in a vector diagram) is then

$$\arcsin\left(\frac{\sin 120^\circ(300\ \mathrm{N})}{(494\ \mathrm{N})}\right)=31.7^\circ.$$

Components: Taking the $+x$-direction to be along dog A's rope, the components of the resultant are

$$R_x=(270\ \mathrm{N})+(300\ \mathrm{N})\cos 60^\circ=420\ \mathrm{N}$$
$$R_y=(300\ \mathrm{N})\sin 60^\circ=259.8\ \mathrm{N},$$

so $R=\sqrt{(420\ \mathrm{N})^2+(259.8\ \mathrm{N})^2}=494\ \mathrm{N}, \theta=\arctan\left(\frac{259.8}{420}\right)=31.7^\circ.$

4.6: a) $F_{1x}+F_{2x}=(9.00\ \mathrm{N})\cos 120^\circ+(6.00\ \mathrm{N})\cos(-126.9^\circ)=-8.10\ \mathrm{N}$

$F_{1y}+F_{2y}=(9.00\ \mathrm{N})\sin 120^\circ+(6.00\ \mathrm{N})\sin(-126.9^\circ)=+3.00\ \mathrm{N}.$

b) $R=\sqrt{R_x^2+R_y^2}=\sqrt{(8.10\ \mathrm{N})^2+(3.00\ \mathrm{N})^2}=8.64\ \mathrm{N}.$

4.7: $a=F/m=(132\ \mathrm{N})/(60\ \mathrm{kg})=2.2\ \mathrm{m/s^2}$ (to two places).

4.8: $F=ma=(135\ \mathrm{kg})(1.40\ \mathrm{m/s^2})=189\ \mathrm{N}.$

4.9: $m=F/a=(48.0\ \mathrm{N})/(3.00\ \mathrm{m/s^2})=16.00\ \mathrm{kg}.$

4.10: a) The acceleration is $a=\frac{2x}{t^2}=\frac{2(11.0\mathrm{m})}{(5.00\ \mathrm{s})^2}=0.88\ \mathrm{m/s^2}$. The mass is then $m=\frac{F}{a}=\frac{80.0\ \mathrm{N}}{0.88\ \mathrm{m/s^2}}=90.9\ \mathrm{kg}.$

b) The speed at the end of the first 5.00 seconds is $at=4.4\ \mathrm{m/s}$, and the block on the frictionless surface will continue to move at this speed, so it will move another $vt=22.0\ \mathrm{m}$ in the next 5.00 s.

4.11: a) During the first 2.00 s, the acceleration of the puck is $F/m = 1.563\,\text{m/s}^2$ (keeping an extra figure). At $t = 2.00\,\text{s}$, the speed is $at = 3.13\text{m/s}$ and the position is $at^2/2 = vt/2 = 3.13\,\text{m}$. b) The acceleration during this period is also $1.563\,\text{m/s}^2$, and the speed at 7.00 s is $3.13\,\text{m/s} + (1.563\,\text{m/s}^2)(2.00\,\text{s}) = 6.26\,\text{m/s}$. The position at $t = 5.00\text{s}$ is $x = 3.13\,\text{m} + (3.13\text{m/s})(5.00\text{s} - 2.00\text{s}) = 125\,\text{m}$, and at $t = 7.00\,\text{s}$ is

$$12.5\,\text{m} + (3.13\,\text{m/s})(2.00\,\text{s}) + (1/2)(1.563\,\text{m/s}^2)(2.00\,\text{s})^2 = 21.89\,\text{m},$$

or 21.9 m to three places.

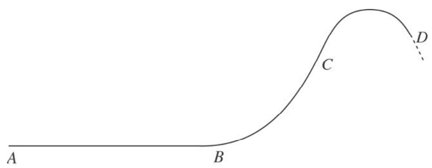
4.12: a) $a_x = F/m = 140\,\text{N}/32.5\,\text{kg} = 4.31\,\text{m/s}^2.$

b) With $v_{0x} = 0, x = \frac{1}{2}at^2 = 215\,\text{m}$.

c) With $v_{0x} = 0, v_x = a_x t = 2x/t = 43.0\,\text{m/s}$.

4.13: a) $\sum \vec{\boldsymbol{F}} = \mathbf{0}$

b), c), d)



4.14: a) With $v_{0x} = 0$,

$$a_x = \frac{v_x^2}{2x} = \frac{(3.00 \times 10^6\ \text{m/s})^2}{2(1.80 \times 10^{-2}\ \text{m})} = 2.50 \times 10^{14}\ \text{m/s}^2.$$

b) $t = \frac{v_x}{a_x} = \frac{3.00\times10^6\ \text{m/s}}{2.50\times10^{14}\ \text{m/s}^2} = 1.20 \times 10^{-8}\ \text{s}$. Note that this time is also the distance divided by the *average* speed.

c) $F = ma = (9.11 \times 10^{-31}\ \text{kg})(2.50 \times 10^{14}\ \text{m/s}^2) = 2.28 \times 10^{-16}\ \text{N}.$

4.15: $F = ma = w(a/g) = (2400\ \text{N})(12\ \text{m/s}^2)(9.80\ \text{m/s}^2) = 2.94 \times 10^3\ \text{N}.$

4.16:
$$a = \frac{F}{m} = \frac{F}{w/g} = \frac{F}{w}g = \left(\frac{160}{71.2}\right)(9.80\,\text{m/s}^2) = 22.0\,\text{m/s}^2.$$

4.17: a) $m = w/g = (44.0\ \text{N})/(9.80\,\text{m/s}^2) = 4.49\,\text{kg}$ b) The mass is the same, 4.49 kg, and the weight is $(4.49\,\text{kg})(1.81\,\text{m/s}^2) = 8.13\,\text{N}.$

4.18: a) From Eq. (4.9), $m = w/g = (3.20\,\mathrm{N})/(9.80\,\mathrm{m/s^2}) = 0.327\,\mathrm{kg}.$
b) $w = mg = (14.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 137\,\mathrm{N}.$

4.19: $F = ma = (55\,\mathrm{kg})(15\,\mathrm{m/s^2}) = 825\,\mathrm{N}.$ The net forward force on the sprinter is exerted by the blocks. (The sprinter exerts a backward force on the blocks.)

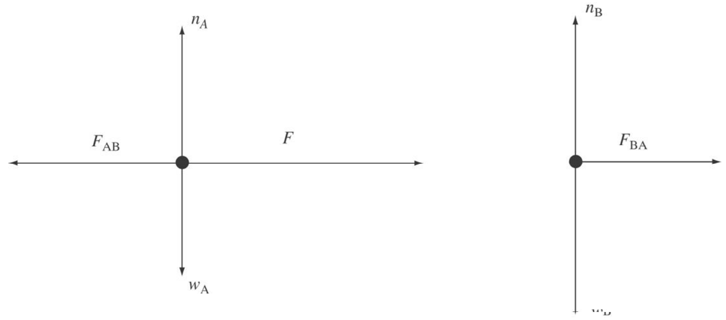
4.20: a) the earth (gravity) b) 4 N, the book c) no d) 4 N, the earth, the book, up e) 4 N, the hand, the book, down f) second g) third h) no i) no j) yes k) yes l) one (gravity) m) no

4.21: a) When air resistance is not neglected, the net force on the bottle is the weight of the bottle plus the force of air resistance. b) The bottle exerts an upward force on the earth, and a downward force on the air.

4.22: The reaction to the upward normal force on the passenger is the downward normal force, also of magnitude 620 N, that the passenger exerts on the floor. The reaction to the passenger's weight is the gravitational force that the passenger exerts on the earth, upward and also of magnitude 650 N. $\frac{\sum F}{m} = \frac{620\ \mathrm{N} - 650\ \mathrm{N}}{650\ \mathrm{N}/9.80\ \mathrm{m/s^2}} = -0.452\ \mathrm{m/s^2}.$ The passenger's acceleration is $0.452\,\mathrm{m/s^2}$, downward.

4.23: $a_{\mathrm{E}} = \frac{F}{m_{\mathrm{E}}} = \frac{mg}{m_{\mathrm{E}}} = \frac{(45\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{(6.0 \times 10^{24}\ \mathrm{kg})} = 7.4 \times 10^{-23}\ \mathrm{m/s^2}.$

4.24: (a) Each crate can be considered a single particle:

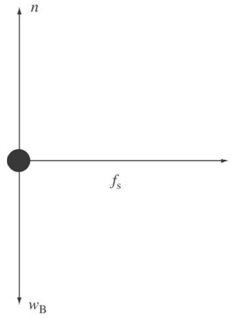


F_{AB} (the force on m_A due to m_B) and F_{BA} (the force on m_B due to m_A) form an action-reaction pair.

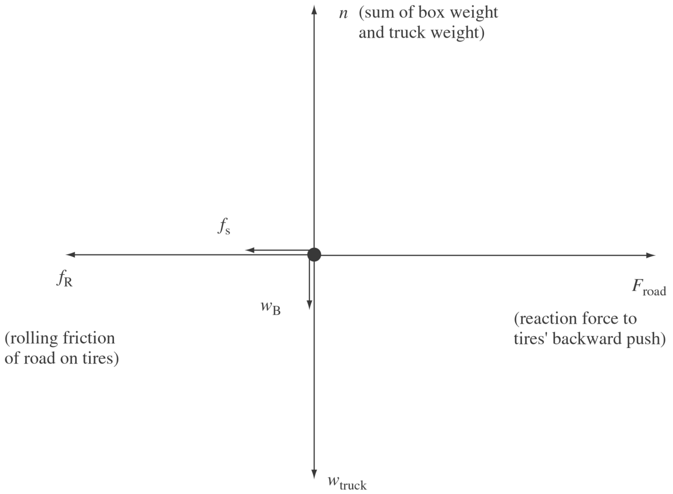
(b) Since there is no horizontal force opposing *F*, any value of *F*, no matter how small, will cause the crates to accelerate to the right. The weight of the two crates acts at a right angle to the horizontal, and is in any case balanced by the upward force of the surface on them.

4.25: The ball must accelerate eastward with the same acceleration as the train. There must be an eastward component of the tension to provide this acceleration, so the ball hangs at an angle relative to the vertical. The net force on the ball is not zero.

4.26: The box can be considered a single particle.

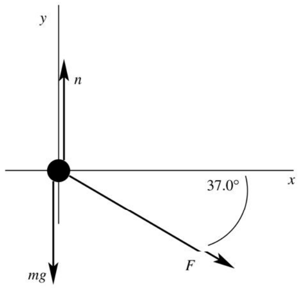


For the truck:



The box's friction force on the truck bed and the truck bed's friction force on the box form an action-reaction pair. There would also be some small air-resistance force action to the left, presumably negligible at this speed.

4.27: a)

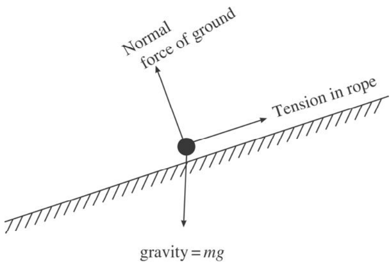


b) For the chair, $a_y = 0$ so $\sum F_y = ma_y$ gives

$n - mg - F\sin 37° = 0$

$n = 142\,\text{N}$

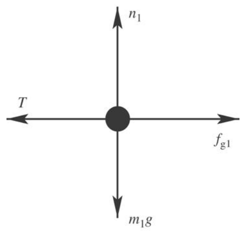
4.28: a)



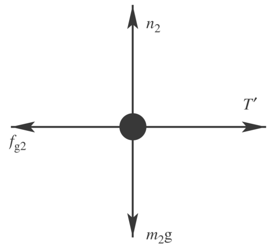
b)

$$\begin{aligned} T &= mg\sin\theta \\ &= (65.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 26.0° = 279\,\text{N} \end{aligned}$$

4.29: tricycle and Frank



T is the force exerted by the rope and f_g is the force the ground exerts on the tricycle.
spot and the wagon



T' is the force exerted by the rope. T and T' form a third-law action-reaction pair, $\vec{T} = -\vec{T}'$.

4.30: a) The stopping time is $\frac{x}{v_{ave}} = \frac{x}{(v_0/2)} = \frac{2(0.130\text{ m})}{350\text{ m/s}} = 7.43\times10^{-4}$ s.

b) $F = ma = (1.80\times10^{-3}\text{ kg})\frac{(350\text{ m/s})}{(7.43\times10^{-4}\text{ s})} = 848\,\text{N}$. (Using $a = v_0^2/2x$ gives the same result.)

4.31: Take the $+x$-direction to be along $\vec{F}_1$ and the $+y$-direction to be along $\vec{R}$. Then $F_{2x} = -1300\text{N}$ and $F_{2y} = 1300\,\text{N}$, so $F_2 = 1838\,\text{N}$, at an angle of $135°$ from $\vec{F}_1$.

4.32: Get g on X:

$$y = \frac{1}{2}gt^2$$
$$10.0\,\text{m} = \frac{1}{2}g(2.2\,\text{s})^2$$
$$g = 4.13\,\text{m/s}^2$$
$$w_{\text{X}} = mg_{\text{X}} = (0.100\,\text{kg})(4.03\,\text{m/s}^2) = 0.41\,\text{N}$$

4.33: a) The resultant must have no y-component, and so the child must push with a force with y-component $(140\text{ N})\sin 30° - (100\text{ N})\sin 60° = -16.6\text{ N}$. For the child to exert the smallest possible force, that force will have no x-component, so the smallest possible force has magnitude 16.6 N and is at an angle of $270°$, or $90°$ clockwise from the $+x$-direction.

b) $m = \frac{\sum F}{a} = \frac{100\text{ N}\cos 60° + 140\text{ N}\cos 30°}{2.0\text{ m/s}^2} = 85.6\text{ kg}$. $w = mg = (85.6\text{ kg})(9.80\text{ m/s}^2) = 840\text{ N}$. .

4.34: The ship would go a distance

$$\frac{v_0^2}{2a} = \frac{v_0^2}{2(F/m)} = \frac{mv_0^2}{2F} = \frac{(3.6\times 10^7\text{ kg})(1.5\text{ m/s})^2}{2(8.0\times 10^4\text{ N})} = 506.25\text{ m},$$

so the ship would hit the reef. The speed when the tanker hits the reef is also found from

$$v = \sqrt{v_0^2 - (2Fx/m)} = \sqrt{(1.5\text{ m/s})^2 - \frac{2(8.0\times 10^4\text{ N})(500\text{ m})}{(3.6\times 10^7\text{ kg})}} = 0.17\text{ m/s},$$

so the oil should be safe.

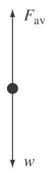
4.35: a) Motion after he leaves the floor: $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$.

$v_y = 0$ at the maximum height, $y - y_0 = 1.2\text{ m}$, $a_y = -9.80\text{ m/s}^2$, so

$v_{0y} = 4.85\text{ m/s}$.

b) $a_{\text{av}} = \Delta v/\Delta t = (4.85\text{ m/s})/(0.300\text{ s}) = 16.2\text{ m/s}^2$.

c)



$F_{\text{av}} - w = ma_{\text{av}}$

$F_{\text{av}} = w + ma_{\text{av}} = 890\text{ N} + (890\text{ N}/9.80\text{ m/s}^2)(16.2\text{ m/s}^2)$

$F_{\text{av}} = 2.36\times 10^3\text{ N}$

4.36:

$$F = ma = m\frac{v_0^2}{2x} = (850\text{ kg})\frac{(12.5\text{ m/s})^2}{2(1.8\times 10^{-2}\text{ m})} = 3.7\times 10^6\text{ N}.$$

4.37: a)

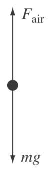
$$F_{net} = F - mg \text{ (upward)}$$

b) When the upward force has its maximum magnitude F_{max} (the breaking strength), the net upward force will be $F_{max} - mg$ and the upward acceleration will be

$$a = \frac{F_{max} - mg}{m} = \frac{F_{max}}{m} - g = \frac{75.0\ \text{N}}{4.80\ \text{kg}} - 9.80\ \text{m/s}^2 = 5.83\ \text{m/s}^2.$$

4.38: a) $w = mg = 539\ \text{N}$

b)



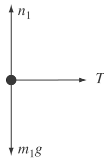
Downward velocity is decreasing so $\vec{a}$ is upward and the net force should be upward. $F_{air} > mg$, so the net force is upward.

c) Taking the upward direction as positive, the acceleration is

$$a = \frac{F}{m} = \frac{F_{air} - mg}{m} = \frac{F_{air}}{m} - 9.80\ \text{m/s}^2 = \frac{620\ \text{N}}{55.0\ \text{kg}} - 9.80\ \text{m/s}^2 = 1.47\ \text{m/s}^2.$$

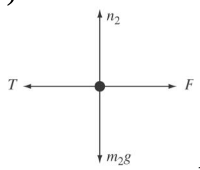
4.39: a) Both crates moves together, so $a = 2.50\,\text{m/s}^2$

b)



$T = m_1 a = (4.00\,\text{kg})(2.50\,\text{m/s}^2) = 10.0\,\text{N}$

c)



$F > T$ and the net force is to the right, in the direction of $\vec{a}$.

d) $F - T = m_2 a$

$F = T + m_2 a = 10.0\,\text{N} + (6.00\,\text{kg})(2.50\,\text{m/s}^2) = 25.0\,\text{N}$

4.40: a) The force the astronaut exerts on the rope and the force that the rope exerts on the astronaut are an action-reaction pair, so the rope exerts a force of 80.0 N on the astronaut. b) The cable is under tension. c) $a = \frac{F}{m} = \frac{80.0\ \text{N}}{105.0\ \text{kg}} = 0.762\,\text{m/s}^2$. d) There is no net force on the massless rope, so the force that the shuttle exerts on the rope must be 80.0 N (this is *not* an action-reaction pair). Thus, the force that the rope exerts on the shuttle must be 80.0 N. e) $a = \frac{F}{m} = \frac{80.0\ \text{N}}{9.05\times10^4\ \text{kg}} = 8.84\times10^{-4}\ \text{m/s}^2$.

4.41: a) $x\,(0.025\text{s}) = (9.0\times10^3\ \text{m/s}^2)(0.025\text{s})^2 - (8.0\times10^4\ \text{m/s}^3)(0.025\text{s})^3 = 4.4\,\text{m}.$

b) Differentiating, the velocity as a function of time is

$$v(t) = (1.80\times10^4\ \text{m/s}^2)t - (2.40\times10^5\ \text{m/s}^3)t^2, \text{ so}$$

$$v(0.025\text{s}) = (1.80\times10^4\ \text{m/s}^2)(0.025\text{s}) - (2.40\times10^5\ \text{m/s}^3)(0.025\text{s})^2$$
$$= 3.0\times10^2\ \text{m/s}.$$

c) The acceleration as a function of time is

$$a(t) = 1.80\times10^4\ \text{m/s}^2 - (4.80\times10^5\ \text{m/s}^3)t,$$

so (i) at $t = 0$, $a = 1.8\times10^4\ \text{m/s}^2$, and (ii) $a(0.025\text{s}) = 6.0\times10^3\ \text{m/s}^2$, and the forces are (i) $ma = 2.7\times10^4\ \text{N}$ and (ii) $ma = 9.0\times10^3\ \text{N}$.

4.42: a) The velocity of the spacecraft is downward. When it is slowing down, the acceleration is upward. When it is speeding up, the acceleration is downward.

b)

speeding up: $w > F$ and the net force is downward
slowing down: $w < F$ and the net force is upward

c) Denote the *y*-component of the acceleration when the thrust is F_1 by a_1 and the *y*-component of the acceleration when the thrust is F_2 by a_2. The forces and accelerations are then related by

$$F_1 - w = ma_1, \quad F_2 - w = ma_2.$$

Dividing the first of these by the second to eliminate the mass gives

$$\frac{F_1 - w}{F_2 - w} = \frac{a_1}{a_2},$$

and solving for the weight *w* gives

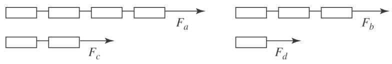
$$w = \frac{a_1 F_2 - a_2 F_1}{a_1 - a_2}.$$

In this form, it does not matter which thrust and acceleration are denoted by 1 and which by 2, and the acceleration due to gravity at the surface of Mercury need not be found. Substituting the given numbers, with $+y$ upward, gives

$$w = \frac{(1.20\,\mathrm{m/s^2})(10.0\times 10^3\ \mathrm{N}) - (-0.80\,\mathrm{m/s^2})(25.0\times 10^3\ \mathrm{N})}{1.20\,\mathrm{m/s^2} - (-0.80\,\mathrm{m/s^2})} = 16.0\times 10^3\ \mathrm{N}.$$

In the above, note that the upward direction is taken to be positive, so that a_2 is negative. Also note that although a_2 is known to two places, the sums in both numerator and denominator are known to three places.

4.43:



a) The engine is pulling four cars, and so the force that the engine exerts on the first car is $4m|\vec{a}|$. b), c), d): Similarly, the forces the cars exert on the car behind are $3m|\vec{a}|$, $2m|\vec{a}|$ and $-m|\vec{a}|$. e) The direction of the acceleration, and hence the direction of the forces, would change but the magnitudes would not; the answers are the same.

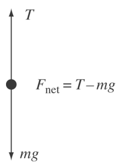
4.44: a) If the gymnast climbs at a constant rate, there is no net force on the gymnast, so the tension must equal the weight; $T = mg$.

b) No motion is no acceleration, so the tension is again the gymnast's weight.

c) $T - w = T - mg = ma = m|\vec{a}|$ (the acceleration is upward, the same direction as the tension), so $T = m(g + |\vec{a}|)$.

d) $T - w = T - mg = ma = -m|\vec{a}|$ (the acceleration is downward, the same opposite as the tension), so $T = m(g - |\vec{a}|)$.

4.45: a)



The maximum acceleration would occur when the tension in the cables is a maximum,

$$a = \frac{F_{net}}{m} = \frac{T - mg}{m} = \frac{T}{m} - g = \frac{28{,}000\ \text{N}}{2200\ \text{kg}} - 9.80\ \text{m/s}^2 = 2.93\ \text{m/s}^2.$$

b) $$\frac{28{,}000\ \text{N}}{2200\ \text{kg}} - 1.62\ \text{m/s}^2 = 11.1\ \text{m/s}^2.$$

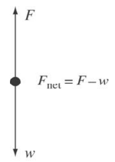
4.46: a) His speed as he touches the ground is

$$v = \sqrt{2gh} = \sqrt{2(9.80\,\mathrm{m/s^2})(3.10\,\mathrm{m})} = 7.80\,\mathrm{m/s}.$$

b) The acceleration while the knees are bending is

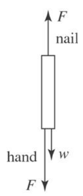
$$a = \frac{v^2}{2y} = \frac{(7.80\,\mathrm{m/s})^2}{2(0.60\,\mathrm{m})} = 50.6\,\mathrm{m/s^2}.$$

c)



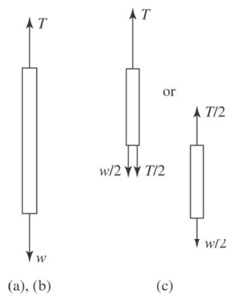
The net force that the feet exert on the ground is the force that the ground exerts on the feet (an action-reaction pair). This force is related to the weight and acceleration by $F - w = F - mg = ma,$ so $F = m(a+g) = (75.0\,\mathrm{kg})(50.6\,\mathrm{m/s^2} + 9.80\,\mathrm{m/s^2}) = 4532\,\mathrm{N}$. As a fraction of his weight, this force is $\frac{F}{mg} = \left(\frac{a}{g}+1\right) = 6.16$ (keeping an extra figure in the intermediate calculation of a). Note that this result is the same algebraically as $\left(\frac{3.10\ \mathrm{m}}{0.60\ \mathrm{m}}+1\right)$.

4.47: a)



b) The acceleration of the hammer head will be the same as the nail, $a = v_0^2/2x = (3.2\,\mathrm{m/s})^2/2(0.45\,\mathrm{cm}) = 1.138\times10^3\ \mathrm{m/s^2}$. The mass of the hammer head is its weight divided by g, $4.9\,\mathrm{N}/9.80\,\mathrm{m/s^2} = 0.50\,\mathrm{kg}$, and so the net force on the hammer head is $(0.50\,\mathrm{kg})(1.138\times10^3\ \mathrm{m/s^2}) = 570\,\mathrm{N}$. This is the sum of the forces on the hammer head; the upward force that the nail exerts, the downward weight and the downward 15-N force. The force that the nail exerts is then 590 N, and this must be the magnitude of the force that the hammer head exerts on the nail. c) The distance the nail moves is .12 m, so the acceleration will be $4267\,\mathrm{m/s^2}$, and the net force on the hammer head will be 2133 N. The magnitude of the force that the nail exerts on the hammer head, and hence the magnitude of the force that the hammer head exerts on the nail, is 2153 N, or about 2200 N.

4.48:



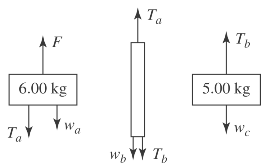
a) The net force on a point of the cable at the top is zero; the tension in the cable must be equal to the weight *w*.

b) The net force on the cable must be zero; the difference between the tensions at the top and bottom must be equal to the weight *w*, and with the result of part (a), there is no tension at the bottom.

c) The net force on the bottom half of the cable must be zero, and so the tension in the cable at the middle must be half the weight, $w/2$. Equivalently, the net force on the upper half of the cable must be zero. From part (a) the tension at the top is *w*, the weight of the top half is $w/2$ and so the tension in the cable at the middle must be $w - w/2 = w/2$.

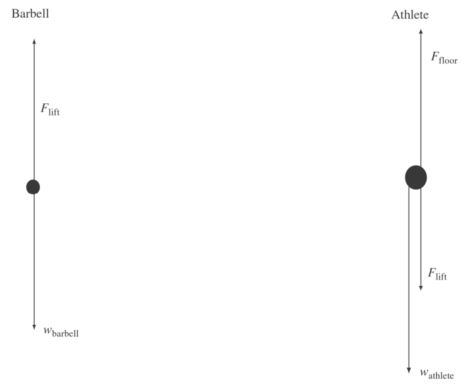
d) A graph of *T* vs. distance will be a negatively sloped line.

4.49: a)



b) The net force on the system is $200\ \text{N} - (15.00\ \text{kg})(9.80\ \text{m/s}^2) = 53.0\ \text{N}$ (keeping three figures), and so the acceleration is $(53.0\ \text{N})/(15.0\ \text{kg}) = 3.53\ \text{m/s}^2$, up. c) The net force on the 6-kg block is $(6.00\ \text{kg})(3.53\ \text{m/s}^2) = 21.2\ \text{N}$, so the tension is found from $F - T - mg = 21.2\ \text{N}$, or $T = (200\ \text{N}) - (6.00\ \text{kg})(9.80\ \text{m/s}^2) - 21.2\ \text{N} = 120\ \text{N}$. Equivalently, the tension at the top of the rope causes the upward acceleration of the rope and the bottom block, so $T - (9.00\ \text{kg})g = (9.00\ \text{kg})a$, which also gives $T = 120\ \text{N}$. d) The same analysis of part (c) is applicable, but using $6.00\ \text{kg} + 2.00\ \text{kg}$ instead of the mass of the top block, or 7.00 kg instead of the mass of the bottom block. Either way gives $T = 93.3\ \text{N}$.

4.50: a)



b) The athlete's weight is $mg = (90.0\,\text{kg})(9.80\,\text{m/s}^2) = 882\,\text{N}$. The acceleration of the barbell is found from $v_{av} = 0.60\,\text{m}/1.6\,\text{s} = 0.375\,\text{m/s}$. Its final velocity is thus $(2)(0.375\,\text{m/s}) = 0.750\,\text{m/s}$, and its acceleration is

$$a = \frac{v - v_0}{t} = \frac{0.750\,\text{m/s}}{1.65} = 0.469\,\text{m/s}^2$$

The force needed to lift the barbell is given by:

$$F_{net} = F_{lift} - w_{barbell} = ma$$

The barbell's mass is $(490\,\text{N})/(9.80\,\text{m/s}^2) = 50.0\,\text{kg}$, so

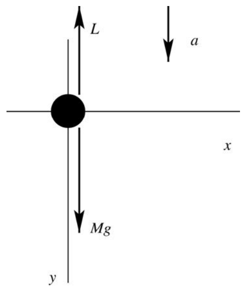
$$\begin{aligned} F_{lift} &= w_{barbell} + ma = 490\,\text{N} + (50.0\,\text{kg})(0.469\,\text{m/s}^2) \\ &= 490\,\text{N} + 23\,\text{N} = 513\,\text{N} \end{aligned}$$

The athlete is not accelerating, so:

$$F_{net} = F_{floor} - F_{lift} - w_{athlete} = 0$$

$$F_{floor} = F_{lift} + w_{athlete} = 513\,\text{N} + 882\,\text{N} = 1395\,\text{N}$$

4.51: a)



L is the lift force

b) $\sum F_y = ma_y$

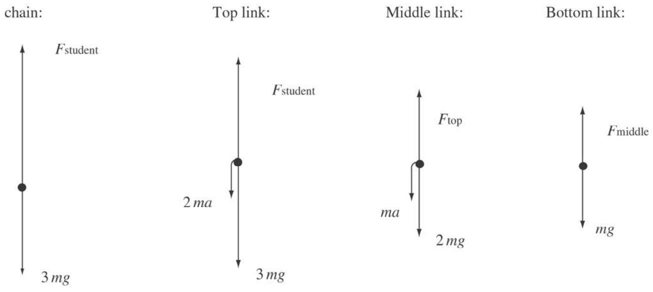
$Mg - L = M(g/3)$

$L = 2Mg/3$

c) $L - mg = m(g/2)$, where m is the mass remaining.

$L = 2Mg/3$, so $m = 4M/9$. Mass $5M/9$ must be dropped overboard.

4.52: a) $m =$ mass of one link



The downward forces of magnitude $2ma$ and ma for the top and middle links are the reaction forces to the upward force needed to accelerate the links below.

b) (i) The weight of each link is $mg = (0.300\,\text{kg})(9.80\,\text{m/s}^2) = 2.94\,\text{N}$. Using the free-body diagram for the whole chain:

$$a = \frac{F_{\text{net}}}{3m} = \frac{12\,\text{N} - 3(2.94\,\text{N})}{0.900\,\text{kg}} = \frac{3.18\,\text{N}}{0.900\,\text{kg}} = 3.53\,\text{m/s}^2 \text{ or } 3.5\,\text{m/s}^2$$

(ii) The second link also accelerates at $3.53\,\text{m/s}^2$, so:

$$F_{\text{net}} = F_{\text{top}} - ma - 2mg = ma$$
$$F_{\text{top}} = 2ma + 2mg = 2(0.300\text{ kg})(3.53\,\text{m/s}^2) + 2(2.94\text{ N})$$
$$= 2.12\,\text{N} + 5.88\,\text{N} = 8.0\,\text{N}$$

4.53: Differentiating twice, the acceleration of the helicopter as a function of time is

$$\vec{a} = (0.120\,\text{m/s}^3)t\hat{\boldsymbol{i}} - (0.12\,\text{m/s}^2)\hat{\boldsymbol{k}},$$

and at $t = 5.0\,\text{s}$, the acceleration is

$$\vec{a} = (0.60\,\text{m/s}^2)\hat{\boldsymbol{i}} - (0.12\,\text{m/s}^2)\hat{\boldsymbol{k}}.$$

The force is then

$$F = m\vec{a} = \frac{w}{g}\vec{a} = \frac{(2.75\times10^5\text{ N})}{(9.80\,\text{m/s}^2)}\left[(0.60\,\text{m/s}^2)\hat{\boldsymbol{i}} - (0.12\,\text{m/s}^2)\hat{\boldsymbol{k}}\right]$$
$$= (1.7\times10^4\text{ N})\hat{\boldsymbol{i}} - (3.4\times10^3\text{ N})\hat{\boldsymbol{k}}.$$

4.54: The velocity as a function of time is $v(t) = A - 3Bt^2$ and the acceleration as a function of time is $a(t) = -6Bt$, and so the Force as a function of time is $F(t) = ma(t) = -6mBt$.

4.55:

$$\vec{\boldsymbol{v}}(t) = \frac{1}{m}\int_0^t \vec{\boldsymbol{a}}\ dt = \frac{1}{m}\left(k_1 t\hat{\boldsymbol{i}} + \frac{k^2}{4}t^4\hat{\boldsymbol{j}}\right).$$

4.56: a) The equation of motion, $-Cv^2 = m\frac{dv}{dt}$ cannot be integrated with respect to time, as the unknown function $v(t)$ is part of the integrand. The equation must be *separated* before integration; that is,

$$-\frac{C}{m}dt = \frac{dv}{v^2}$$

$$-\frac{Ct}{m} = -\frac{1}{v} + \frac{1}{v_0},$$

where v_0 is the constant of integration that gives $v = v_0$ at $t = 0$. Note that this form shows that if $v_0 = 0$, there is no motion. This expression may be rewritten as

$$v = \frac{dx}{dt} = \left(\frac{1}{v_0} + \frac{Ct}{m}\right)^{-1},$$

which may be integrated to obtain

$$x - x_0 = \frac{m}{C}\ln\left[1 + \frac{Ctv_0}{m}\right].$$

To obtain *x* as a function of *v*, the time *t* must be eliminated in favor of *v*; from the expression obtained after the first integration, $\frac{Ctv_0}{m} = \frac{v_0}{v} - 1$, so

$$x - x_0 = \frac{m}{C}\ln\left(\frac{v_0}{v}\right).$$

b) By the chain rule,

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dv}{dt} = \frac{dv}{dx}v,$$

and using the given expression for the net force,

$$-Cv^2 = \left(v\frac{dv}{dx}\right)m$$

$$-\frac{C}{m}dx = \frac{dv}{v}$$

$$-\frac{C}{m}(x - x_0) = \ln\left(\frac{v}{v_0}\right)$$

$$x - x_0 = \frac{m}{C}\ln\left(\frac{v_0}{v}\right).$$

4.57: In this situation, the *x*-component of force depends explicitly on the *y*-component of position. As the *y*-component of force is given as an explicit function of time, v_y and *y* can be found as functions of time. Specifically, $a_y = (k_3/m)t$, so $v_y = (k_3/2m)t^2$ and $y = (k_3/6m)t^3$, where the initial conditions $v_{0y} = 0, y_0 = 0$ have been used. Then, the expressions for a_x, v_x and *x* are obtained as functions of time:

$$a_x = \frac{k_1}{m} + \frac{k_2 k_3}{6m^2} t^3$$

$$v_x = \frac{k_1}{m} t + \frac{k_2 k_3}{24m^2} t^4$$

$$x = \frac{k_1}{2m} t^2 + \frac{k_2 k_3}{120m^2} t^5.$$

In vector form,

$$\vec{\boldsymbol{r}} = \left(\frac{k_1}{2m} t^2 + \frac{k_2 k_3}{120m^2} t^5 \right) \hat{\boldsymbol{i}} + \left(\frac{k_3}{6m} t^3 \right) \hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{v}} = \left(\frac{k_1}{m} t + \frac{k_2 k_3}{24m^2} t^4 \right) \hat{\boldsymbol{i}} + \left(\frac{k_3}{2m} t^2 \right) \hat{\boldsymbol{j}}.$$

5.1: a) The tension in the rope must be equal to each suspended weight, 25.0 N. b) If the mass of the light pulley may be neglected, the net force on the pulley is the vector sum of the tension in the chain and the tensions in the two parts of the rope; for the pulley to be in equilibrium, the tension in the chain is twice the tension in the rope, or 50.0 N.

5.2: In all cases, each string is supporting a weight *w* against gravity, and the tension in each string is *w*. Two forces act on each mass**:** *w* down and $T(=w)$ up.

5.3: a) The two sides of the rope each exert a force with vertical component $T \sin\theta$, and the sum of these components is the hero's weight. Solving for the tension *T*,

$$T=\frac{w}{2\sin\theta}=\frac{(90.0\,\text{kg})\,(9.80\,\text{m/s}^2)}{2\sin 10.0°}=2.54\times10^3\ \text{N}.$$

b) When the tension is at its maximum value, solving the above equation for the angle θ gives

$$\theta=\arcsin\left(\frac{w}{2T}\right)=\arcsin\left(\frac{(90.0\,\text{kg})\,(9.80\,\text{m/s}^2}{2(2.50\times10^4\ \text{N})}\right)=1.01°.$$

5.4: The vertical component of the force due to the tension in each wire must be half of the weight, and this in turn is the tension multiplied by the cosine of the angle each wire makes with the vertical, so if the weight is $w, \frac{w}{2}=\frac{3w}{4}\cos\theta$ and $\theta=\arccos\frac{2}{3}=48°$.

5.5: With the positive *y*-direction up and the positive *x*-direction to the right, the free-body diagram of Fig. 5.4(b) will have the forces labeled *n* and *T* resolved into *x*- and *y*-components, and setting the net force equal to zero,

$$F_x=T\cos\alpha-n\sin\alpha=0$$

$$F_y=n\cos\alpha+T\sin\alpha-w=0.$$

Solving the first for $n=T\cot\alpha$ and substituting into the second gives

$$T\frac{\cos^2\alpha}{\sin\alpha}+T\sin\alpha=T\left(\frac{\cos^2\alpha}{\sin\alpha}+\frac{\sin^2\alpha}{\sin\alpha}\right)=\frac{T}{\sin\alpha}=w$$

and so $n=T\cot\alpha=w\sin\alpha\cot\alpha=w\cos\alpha$, as in Example 5.4.

5.6: $w\sin\alpha=mg\sin\alpha=(1390\,\text{kg})\,(9.80\,\text{m/s}^2)\sin 17.5°=4.10\times10^3\ \text{N}.$

5.7: a) $T_B \cos\theta = W$, or $T_B = W/\cos\theta = \frac{(4090\text{ kg})(9.8\text{ m/s}^2)}{\cos 40°} = 5.23\times10^4$ N.
b) $T_A = T_B \sin\theta = (5.23\times10^4\text{ N})\sin 40° = 3.36\times10^4$ N.

5.8: a) $T_C = w, T_A \sin 30° + T_B \sin 45° = T_C = w$, and $T_A \cos 30° - T_B \cos 45° = 0$. Since $\sin 45° = \cos 45°$, adding the last two equations gives $T_A(\cos 30° + \sin 30°) = w$, and so $T_A = \frac{w}{1.366} = 0.732w$. Then, $T_B = T_A \frac{\cos 30°}{\cos 45°} = 0.897w$.
b) Similar to part (a), $T_C = w, -T_A \cos 60° + T_B \sin 45° = w$, and $T_A \sin 60° - T_B \cos 45° = 0$. Again adding the last two, $T_A = \frac{w}{(\sin 60° - \cos 60°)} = 2.73w$, and $T_B = T_B \frac{\sin 60°}{\cos 45°} = 3.35w$.

5.9: The resistive force is $w \sin\alpha = (1600\text{ kg})(9.80\text{ m/s}^2)(200\text{ m}/6000\text{ m}) = 523$ N..

5.10: The magnitude of the force must be equal to the component of the weight along the incline, or $W \sin\theta = (180\text{ kg})(9.80\text{ m/s}^2)\sin 11.0° = 337$ N.

5.11: a) $W = 60\text{ N}, T\sin\theta = W$, so $T = (60\text{ N})/\sin 45°$, or $T = 85$ N.
b) $F_1 = F_2 = T\cos\theta, F_1 = F_2 = 85\text{ N}\cos 45° = 60$ N.

5.12: If the rope makes an angle θ with the vertical, then $\sin\theta = \frac{0.110}{1.51} = 0.073$ (the denominator is the sum of the length of the rope and the radius of the ball). The weight is then the tension times the cosine of this angle, or

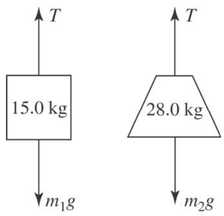
$$T = \frac{w}{\cos\theta} = \frac{mg}{\cos(\arcsin(.073))} = \frac{(0.270\text{ kg})(9.80\text{ m/s}^2)}{0.998} = 2.65 \text{ N}.$$

The force of the pole on the ball is the tension times $\sin\theta$, or $(0.073)T = 0.193$ N.

5.13: a) In the absence of friction, the force that the rope between the blocks exerts on block *B* will be the component of the weight along the direction of the incline, $T = w\sin\alpha$. b) The tension in the upper rope will be the sum of the tension in the lower rope and the component of block *A*'s weight along the incline, $w\sin\alpha + w\sin\alpha = 2w\sin\alpha$. c) In each case, the normal force is $w\cos\alpha$. d) When $\alpha = 0, n = w$, when $\alpha = 90°, n = 0$.

5.14: a) In level flight, the thrust and drag are horizontal, and the lift and weight are vertical. At constant speed, the net force is zero, and so $F = f$ and $w = L$. b) When the plane attains the new constant speed, it is again in equilibrium and so the new values of the thrust and drag, F' and f', are related by $F' = f'$; if $F' = 2F$, $f' = 2f$. c) In order to increase the magnitude of the drag force by a factor of 2, the speed must increase by a factor of $\sqrt{2}$.

5.15: a)



The tension is related to the masses and accelerations by

$$T - m_1 g = m_1 a_1$$

$$T - m_2 g = m_2 a_2.$$

b) For the bricks accelerating upward, let $a_1 = -a_2 = a$ (the counterweight will accelerate down). Then, subtracting the two equations to eliminate the tension gives

$$(m_2 - m_1)g = (m_1 + m_2)a, \text{ or}$$

$$a = g\frac{m_2 - m_1}{m_2 + m_1} = 9.80\ \text{m/s}^2\left(\frac{28.0\ \text{kg} - 15.0\ \text{kg}}{28.0\ \text{kg} + 15.0\text{kg}}\right) = 2.96\ \text{m/s}^2.$$

c) The result of part (b) may be substituted into either of the above expressions to find the tension $T = 191\ \text{N}$. As an alternative, the expressions may be manipulated to eliminate a algebraically by multiplying the first by m_2 and the second by m_1 and adding (with $a_2 = -a_1$) to give

$$T(m_1 + m_2) - 2m_1 m_2 g = 0, \quad \text{or}$$

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2(15.0\ \text{kg})\ (28.0\ \text{kg})\ (9.80\ \text{m/s}^2)}{(15.0\ \text{kg} + 28.0\ \text{kg})} = 191\ \text{N}.$$

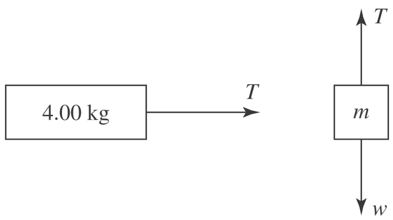
In terms of the weights, the tension is

$$T = w_1 \frac{2m_2}{m_1 + m_2} = w_2 \frac{2m_1}{m_1 + m_2}.$$

If, as in this case, $m_2 > m_1$, $2m_2 > m_1 + m_2$ and $2m_1 < m_1 + m_2$, so the tension is greater than w_1 and less than w_2; this must be the case, since the load of bricks rises and the counterweight drops.

5.16: Use Second Law and kinematics: $a = g\sin\theta$, $2ax = v^2$, solve for θ.
$g\sin\theta = v^2/2x$, or
$\theta = \arcsin\left(v^2/2gx\right) = \arcsin[(2.5\ \text{m/s})^2/[(2)(9.8\ \text{m/s}^2)(1.5\ \text{m})]], \theta = 12.3°.$

5.17: a)



b) In the absence of friction, the net force on the 4.00-kg block is the tension, and so the acceleration will be $(10.0\ \text{N})/(4.00\ \text{kg}) = 2.50\ \text{m/s}^2$. c) The net upward force on the suspended block is $T - mg = ma$, or $m = T/(g + a)$. The block is accelerating downward, so $a = -2.50\ \text{m/s}^2$, and so $m = (10.0\ \text{N})/(9.80\ \text{m/s}^2 - 2.50\ \text{m/s}^2) = 1.37\ \text{kg}$.
d) $T = ma + mg$, so $T < mg$, because $a < 0$.

5.18: The maximum net force on the glider combination is

$$12{,}000\ \text{N} - 2 \times 2500\ \text{N} = 7000\ \text{N},$$

so the maximum acceleration is $a_{\text{max}} = \frac{7000\ \text{N}}{1400\ \text{kg}} = 5.0\ \text{m/s}^2$.

a) In terms of the runway length L and takoff speed $v, a = \frac{v^2}{2L} < a_{\text{max}}$, so

$$L > \frac{v^2}{2a_{\text{max}}} = \frac{(40\ \text{m/s})^2}{2(5.0\ \text{m/s}^2)} = 160\ \text{m}.$$

b) If the gliders are accelerating at a_{max}, from
$T - F_{\text{drag}} = ma, T = ma + F_{\text{drag}} = (700\ \text{kg})(5.0\ \text{m/s}^2) + 2500\ \text{N} = 6000\ \text{N}$. Note that this is exactly half of the maximum tension in the towrope between the plane and the first glider.

5.19: Denote the scale reading as F, and take positive directions to be upward. Then,

$$F - w = ma = \frac{w}{g}a, \text{ or } a = g\left(\frac{F}{w} - 1\right).$$

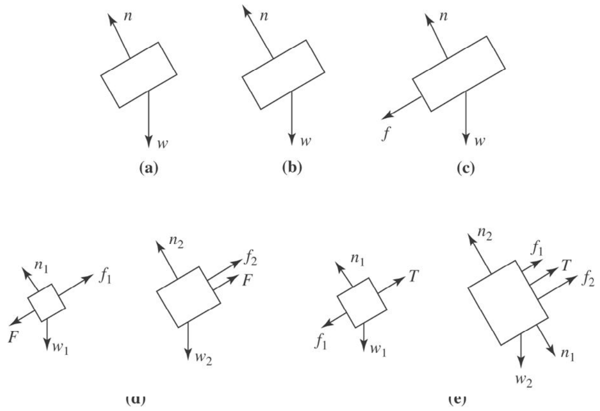
a) $a = (9.80\ \text{m/s}^2)((450\ \text{N})/(550\text{N}) - 1) = -1.78\ \text{m/s}^2$, down.
b) $a = (9.80\ \text{m/s}^2)((670\ \text{N})/(550\ \text{N}) - 1) = 2.14\ \text{m/s}^2$, up. c) If $F = 0, a = -g$ and the student, scale, and elevator are in free fall. The student should worry.

5.20: Similar to Exercise 5.16, the angle is $\arcsin(\frac{2L}{gt^2})$, , but here the time is found in terms of velocity along the table, $t = \frac{x}{v_0}, x$ being the length of the table and v_0 the velocity component along the table. Then,

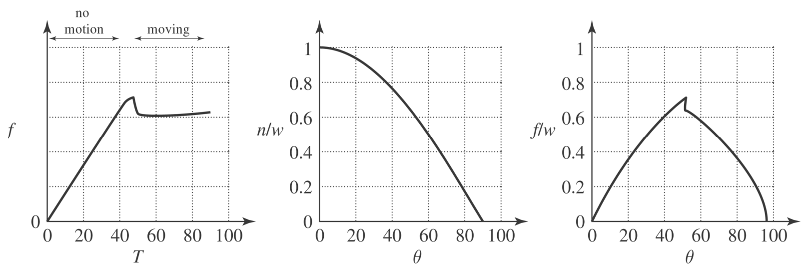
$$\arcsin\left(\frac{2L}{g(x/v_0)^2}\right) = \arcsin\left(\frac{2Lv_0^2}{gx^2}\right)$$

$$= \arcsin\left(\frac{2(2.50\times10^{-2}\,\text{m})(3.80\,\text{m/s})^2}{(9.80\,\text{m/s}^2)(1.75\,\text{m})^2}\right) = 1.38°.$$

5.21:



5.22:



5.23: a) For the net force to be zero, the applied force is

$$F = f_k = \mu_k n = \mu_k mg = (0.20)\,(11.2\,\text{kg})\,(9.80\,\text{m/s}^2) = 22.0\ \text{N}.$$

b) The acceleration is $\mu_k g$, and $2ax = v^2$, so $x = v^2/2\mu_k g$, or $x = 3.13\,\text{m}$.

5.24: a) If there is no applied horizontal force, no friction force is needed to keep the box in equilibrium. b) The maximum static friction force is, from Eq. (5.6), $\mu_s n = \mu_s w = (0.40)\,(40.0\ \text{N}) = 16.0\ \text{N},$ so the box will not move and the friction force balances the applied force of 6.0 N. c) The maximum friction force found in part (b), 16.0 N. d) From Eq. (5.5), $\mu_k n = (0.20)(40.0\ \text{N}) = 8.0\ \text{N}$ e) The applied force is enough to either start the box moving or to keep it moving. The answer to part (d), from Eq. (5.5), is independent of speed (as long as the box is moving), so the friction force is 8.0 N. The acceleration is $(F - f_k)/m = 2.45\ \text{m/s}^2$.

5.25: a) At constant speed, the net force is zero, and the magnitude of the applied force must equal the magnitude of the kinetic friction force,

$$\left|\vec{F}\right| = f_k = \mu_k n = \mu_k mg = (0.12)\,(6.00\ \text{kg})\,(9.80\ \text{m/s}^2) = 7\ \text{N}.$$

b) $\left|\vec{F}\right| - f_k = ma,$ so

$$\left|\vec{F}\right| = ma + f_k = ma = \mu_k mg = m(a + \mu_k g)$$

$$= (6.00\ \text{kg})(0.180\ \text{m/s}^2 + (0.12)9.80\ \text{m/s}^2) = 8\ \text{N}.$$

c) Replacing $g = 9.80\ \text{m/s}^2$ with $1.62\ \text{m/s}^2$ gives 1.2 N and 2.2 N.

5.26: The coefficient of kinetic friction is the ratio $\frac{f_k}{n}$, and the normal force has magnitude $85\ \text{N} + 25\ \text{N} = 110\ \text{N}$. The friction force, from $F_H - f_k = ma = w\frac{a}{g}$ is

$$f_k = F_H - w\frac{a}{g} = 20\ \text{N} - 85\ \text{N}\left(\frac{-0.9\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) = 28\ \text{N}$$

(note that the acceleration is negative), and so $\mu_k = \frac{28\ \text{N}}{110\ \text{N}} = 0.25$.

5.27: As in Example 5.17, the friction force is $\mu_k n = \mu_k w\cos\alpha$ and the component of the weight down the skids is $w\sin\alpha$. In this case, the angle α is $\arcsin(2.00/20.0) = 5.7°$. The ratio of the forces is $\frac{\mu_k \cos\alpha}{\sin\alpha} = \frac{\mu_k}{\tan\alpha} = \frac{0.25}{0.10} > 1,$ so the friction force holds the safe back, and another force is needed to move the safe down the skids.

b) The difference between the downward component of gravity and the kinetic friction force is

$$w\,(\sin\alpha - \mu_k \cos\alpha) = (260\ \text{kg})\,(9.80\ \text{m/s}^2)\,(\sin 5.7° - (0.25)\cos 5.7°) = -381\ \text{N}.$$

5.28: a) The stopping distance is

$$\frac{v^2}{2a} = \frac{v^2}{2\mu_k g} = \frac{(28.7\ \text{m/s})^2}{2(0.80)\,(9.80\ \text{m/s}^2)} = 53\ \text{m}.$$

b) The stopping distance is inversely proportional to the coefficient of friction and proportional to the square of the speed, so to stop in the same distance the initial speed should not exceed

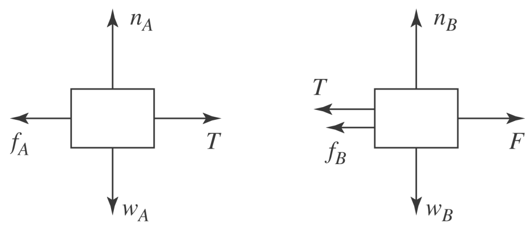
$$v\sqrt{\frac{\mu_{\text{k, wet}}}{\mu_{\text{k,dry}}}} = (28.7\ \text{m/s})\sqrt{\frac{0.25}{0.80}} == 16\ \text{m/s}.$$

5.29: For a given initial speed, the distance traveled is inversely proportional to the coefficient of kinetic friction. From Table 5.1, the ratio of the distances is then $\frac{0.44}{0.04} = 11$.

5.30: (a) If the block descends at constant speed, the tension in the connecting string must be equal to the hanging block's weight, w_B. Therefore, the friction force $\mu_k w_A$ on block A must be equal to w_B, and $w_B = \mu_k w_A$.

(b) With the cat on board, $a = g(w_B - \mu_k 2w_A)/(w_B + 2w_A)$.

5.31:



a) For the blocks to have no acceleration, each is subject to zero net force. Considering the horizontal components,

$$T = f_A, \left|\vec{\boldsymbol{F}}\right| = T + f_B, \text{ or}$$

$$\left|\vec{\boldsymbol{F}}\right| = f_A + f_B.$$

Using $f_A = \mu_k g m_A$ and $f_B = \mu_k g m_B$ gives $\left|\vec{\boldsymbol{F}}\right| = \mu_k g(m_A + m_B)$.

b) $T = f_A = \mu_k g m_A$.

5.32:

$$\mu_r = \frac{a}{g} = \frac{v_0^2 - v^2}{2Lg} = \frac{v_0^2 - \frac{1}{4}v_0^2}{2Lg} = \frac{3}{8}\frac{v_0^2}{Lg},$$

where L is the distance covered before the wheel's speed is reduced to half its original speed. Low pressure, $L = 18.1\ \text{m}$; $\frac{3}{8}\frac{(3.50\ \text{m/s})^2}{(18.1\ \text{m})(9.80\ \text{m/s}^2)} = 0.0259.$ High pressure,

$L = 92.9\ \text{m}$; $\frac{3}{8}\frac{(3.50\ \text{m/s})^2}{(92.9\text{m})(9.80\text{m/s}^2)} = 0.00505.$

5.33: Without the dolly: $n = mg$ and $F - \mu_k n = 0$ ($a_x = 0$ since speed is constant).

$$m = \frac{F}{\mu_k g} = \frac{160\ \text{N}}{(0.47)(9.80\ \text{m/s}^2)} = 34.74\ \text{kg}$$

With the dolly: the total mass is $34.7\ \text{kg} + 5.3\ \text{kg} = 40.04\ \text{kg}$ and friction now is rolling friction, $f_r = \mu_r mg$.

$$F - \mu_r mg = ma$$

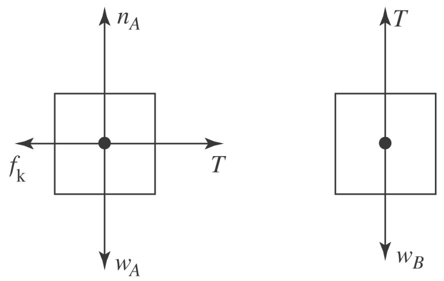
$$a = \frac{F - \mu_r mg}{m} = 3.82\ \text{m/s}^2$$

5.34: Since the speed is constant and we are neglecting air resistance, we can ignore the 2.4 m/s, and F_{net} in the horizontal direction must be zero. Therefore $f_r = \mu_r n =$ $= F_{\text{horiz}} = 200\ \text{N}$ before the weight and pressure changes are made. After the changes, $(0.81)(1.42n) = F_{\text{horiz}}$, because the speed is still constant and $F_{\text{net}} = 0$. We can simply divide the two equations**:**

$$\frac{(0.81\mu_r)(1.42n)}{\mu_r n} = \frac{F_{\text{horiz}}}{200\ \text{N.}}$$

$$(0.81)(1.42)(200\ \text{N}) = F_{\text{horiz}} = 230\ \text{N}$$

5.35: First, determine the acceleration from the freebody diagrams.



There are two equations and two unknowns, a and T:

$$-\mu_k m_A g + T = m_A a$$
$$m_B g - T = m_B a$$

Add and solve for $a: a = g(m_B - \mu_k m_A)/(m_B + m_A), a = 0.79 \text{ m/s}^2$.

(a) $v = (2ax)^{1/2} = 0.22 \text{ m/s}$.

(b) Solving either equation for the tension gives $T = 11.7$ N.

5.36: a) The normal force will be $w\cos\theta$ and the component of the gravitational force along the ramp is $w\sin\theta$. The box begins to slip when $w\sin\theta > \mu_s w\cos\theta$, or $\tan\theta > \mu_s = 0.35$, so slipping occurs at $\theta = \arctan(0.35) = 19.3°$, or $19°$ to two figures.

b) When moving, the friction force along the ramp is $\mu_k w\cos\theta$, the component of the gravitational force along the ramp is $w\sin\theta$, so the acceleration is

$$(w\sin\theta) - w\mu_k\cos\theta)/m = g(\sin\theta - \mu_k\cos\theta) = 0.92 \text{ m/s}^2.$$

(c) $2ax = v^2$, so $v = (2ax)^{1/2}$, or $v = [(2)(0.92 \text{ m/s}^2)(5 \text{ m})]^{1/2} = 3 \text{ m}$.

5.37: a) The magnitude of the normal force is $mg + |\vec{F}|\sin\theta$. The horizontal component of $\vec{F}, |\vec{F}|\cos\theta$ must balance the frictional force, so

$$|\vec{F}|\cos\theta = \mu_k(mg + |\vec{F}|\sin\theta);$$

solving for $|\vec{F}|$ gives

$$|\vec{F}| = \frac{\mu_k mg}{\cos\theta - \mu_k\sin\theta}$$

b) If the crate remains at rest, the above expression, with μ_s instead of μ_k, gives the force that must be applied in order to start the crate moving. If $\cot\theta < \mu_s$, the needed force is infinite, and so the critical value is $\mu_s = \cot\theta$.

5.38: a) There is no net force in the vertical direction, so $n + F\sin\theta - w = 0$, or $n = w - F\sin\theta = mg - F\sin\theta$. The friction force is $f_k = \mu_k n = \mu_k(mg - F\sin\theta)$. The net horizontal force is $F\cos\theta - f_k = F\cos\theta - \mu_k(mg - F\sin\theta)$, and so at constant speed,

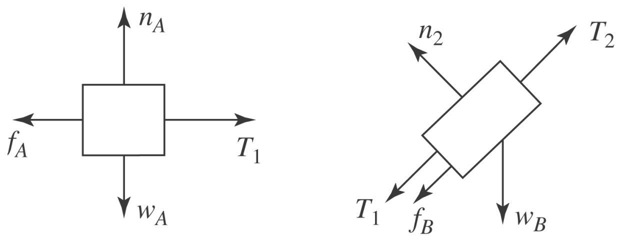
$$F = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

b) Using the given values,

$$F = \frac{(0.35)(90\,\text{kg})(9.80\,\text{m/s}^2)}{(\cos 25° + (0.35)\sin 25°)} = 293\ \text{N},$$

or 290 N to two figures.

5.39: a)



b) The blocks move with constant speed, so there is no net force on block *A*; the tension in the rope connecting *A* and *B* must be equal to the frictional force on block *A*, $\mu_k = (0.35)\,(25.0\ \text{N}) = 9\ \ \text{N}.$ c) The weight of block *C* will be the tension in the rope connecting *B* and *C*; this is found by considering the forces on block *B*. The components of force along the ramp are the tension in the first rope (9 N, from part (a)), the component of the weight along the ramp, the friction on block *B* and the tension in the second rope. Thus, the weight of block *C* is

$$\begin{aligned} w_C &= 9\ \text{N} + w_B(\sin 36.9° + \mu_k \cos 36.9°) \\ &= 9\ \text{N} + (25.0\ \text{N})(\sin 36.9° + (0.35)\cos 36.9°) = 31.0\ \text{N}, \end{aligned}$$

or 31 N to two figures. The intermediate calculation of the first tension may be avoided to obtain the answer in terms of the common weight *w* of blocks *A* and *B*,

$$w_C = w(\mu_k + (\sin\theta + \mu_k \cos\theta)),$$

giving the same result.

(d) Applying Newton's Second Law to the remaining masses (*B* and *C*) gives**:**

$$a = g(w_c - \mu_k w_B \cos\theta - w_B \sin\theta)/(w_B + w_c) = 1.54\,\text{m/s}^2.$$

5.40: Differentiating Eq. (5.10) with respect to time gives the acceleration

$$a = v_{\mathrm{t}}\left(\frac{k}{m}\right)e^{-(k/m)t} = ge^{-(k/m)t},$$

where Eq. (5.9), $v_{\mathrm{t}} = mg/k$ has been used.

Integrating Eq. (5.10) with respect to time with $y_0 = 0$ gives

$$\begin{aligned} y &= \int_0^t v_{\mathrm{t}}[1 - e^{-(k/m)t}]\,dt \\ &= v_{\mathrm{t}}\left[t + \left(\frac{m}{k}\right)e^{-(k/m)t}\right] - v_{\mathrm{t}}\left(\frac{m}{k}\right) \\ &= v_{\mathrm{t}}\left[t - \frac{m}{k}\left(1 - e^{-(k/m)t}\right)\right]. \end{aligned}$$

5.41: a) Solving for D in terms of v_t,

$$D = \frac{mg}{v_t^2} = \frac{(80\ \mathrm{kg})\ (9.80\ \mathrm{m/s^2})}{(42\ \mathrm{m/s})^2} = 0.44\ \mathrm{kg/m}.$$

b) $$v_{\mathrm{t}} = \sqrt{\frac{mg}{D}} = \sqrt{\frac{(45\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{(0.25\ \mathrm{kg/m})}} = 42\ \mathrm{m/s}.$$

5.42: At half the terminal speed, the magnitude of the frictional force is one-fourth the weight. a) If the ball is moving up, the frictional force is down, so the magnitude of the net force is (5/4)w and the acceleration is (5/4)g, down. b) While moving down, the frictional force is up, and the magnitude of the net force is (3/4)w and the acceleration is (3/4)g, down.

5.43: Setting F_{net} equal to the maximum tension in Eq. (5.17) and solving for the speed v gives

$$v = \sqrt{\frac{F_{\mathrm{net}}R}{m}} = \sqrt{\frac{(600\ \mathrm{N})(0.90\ \mathrm{m})}{(0.80\ \mathrm{kg})}} = 26.0\ \mathrm{m/s},$$

or 26 m/s to two figures.

5.44: This is the same situation as Example 5.23. Solving for μ_{s} yields

$$\mu_{\mathrm{s}} = \frac{v^2}{Rg} = \frac{(25.0\ \mathrm{m/s})^2}{(220\ \mathrm{m})(9.80\ \mathrm{m/s^2})} = 0.290.$$

5.45: a) The magnitude of the force F is given to be equal to 3.8w. "Level flight" means that the net vertical force is zero, so $F \cos\beta = (3.8)w\cos\beta = w$, , and $\beta = \arccos(1/3.8) = 75°$.

(b) The angle does not depend on speed.

5.46: a) The analysis of Example 5.22 may be used to obtain $\tan\beta = (v^2/gR)$, but the subsequent algebra expressing R in terms of L is not valid. Denoting the length of the horizontal arm as r and the length of the cable as $l, R = r + l\sin\beta$. The relation $v = \frac{2\pi R}{T}$ is still valid, so $\tan\beta = \frac{4\pi^2 R}{gT^2} = \frac{4\pi^2(r + l\sin\beta)}{gT^2}$. Solving for the period T,

$$T = \sqrt{\frac{4\pi^2(r + l\sin\beta)}{g\tan\beta}} = \sqrt{\frac{4\pi^2(3.00\,\text{m} + (5.00\,\text{m})\sin 30°)}{(9.80\,\text{m/s}^2)\tan 30°}} = 6.19\,\text{s}.$$

Note that in the analysis of Example 5.22, β is the angle that the support (string or cable) makes with the vertical (see Figure 5.30(b)). b) To the extent that the cable can be considered massless, the angle will be independent of the rider's weight. The tension in the cable will depend on the rider's mass.

5.47: This is the same situation as Example 5.22, with the lift force replacing the tension in the string. As in that example, the angle β is related to the speed and the turning radius by $\tan\beta = \frac{v^2}{gR}$. Solving for β,

$$\beta = \arctan\left(\frac{v^2}{gR}\right) = \arctan\left(\frac{(240\,\text{km/h} \times ((1\text{m/s})/(3.6\,\text{km/h})))^2}{(9.80\,\text{m/s}^2)(1200\,\text{m})}\right) = 20.7°.$$

5.48: a) This situation is equivalent to that of Example 5.23 and Problem 5.44, so $\mu_s = \frac{v^2}{Rg}$. Expressing v in terms of the period T, $v = \frac{2\pi R}{T}$, so $\mu_S = \frac{4\pi^2 R}{T^2 g}$. A platform speed of 40.0 rev/min corresponds to a period of 1.50 s, so

$$\mu_s = \frac{4\pi^2(0.150\,\text{m})}{(1.50\,\text{s})^2(9.80\,\text{m/s}^2)} = 0.269.$$

b) For the same coefficient of static friction, the maximum radius is proportional to the square of the period (longer periods mean slower speeds, so the button may be moved further out) and so is inversely proportional to the square of the speed. Thus, at the higher speed, the maximum radius is (0.150 m) $\left(\frac{40.0}{60.0}\right)^2 = 0.067\,\text{m}$.

5.49: a) Setting $a_{\text{rad}} = g$ in Eq. (5.16) and solving for the period T gives

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{400\ \text{m}}{9.80\ \text{m/s}^2}} = 40.1\,\text{s},$$

so the number of revolutions per minute is $(60\,\text{s/min})/(40.1\,\text{s}) = 1.5\,\text{rev/min}$.

b) The lower acceleration corresponds to a longer period, and hence a lower rotation rate, by a factor of the square root of the ratio of the accelerations, $T' = (1.5\ \text{rev/min}) \times \sqrt{3.70/9.8} = 0.92\ \text{rev/min.}.$

5.50: a) $2\pi R/T = 2\pi(50.0\,\text{m})/(60.0\,\text{s}) = 5.24\,\text{m/s}$. b) The magnitude of the radial force is $mv^2/R = m4\pi^2 R/T^2 = w(4\pi^2 R/gT^2) = 49\ \text{N}$ (to the nearest Newton), so the apparent weight at the top is $882\ \text{N} - 49\ \text{N} = 833\,\text{N}$, and at the bottom is $882\ \text{N} + 49\ \text{N} = 931\,\text{N}$. c) For apparent weightlessness, the radial acceleration at the top is equal to g in magnitude. Using this in Eq. (5.16) and solving for T gives

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{50.0\ \text{m}}{9.80\ \text{m/s}^2}} = 14\,\text{s}.$$

d) At the bottom, the apparent weight is twice the weight, or 1760 N.

5.51: a) If the pilot feels weightless, he is in free fall, and $a = g = v^2/R$, so $v = \sqrt{Rg} = \sqrt{(150\,\text{m})(9.80\,\text{m/s}^2)} = 38.3\,\text{m/s}$, or $138\,\text{km/h}$. b) The apparent weight is the sum of the net inward (upward) force and the pilot's weight, or

$$\begin{aligned} w + ma &= w\left(1 + \frac{a}{g}\right) \\ &= (700\ \text{N})\left(1 + \frac{(280\,\text{km/h})^2}{(3.6(\text{km/h})/(\text{m/s}))^2(9.80\,\text{m/s}^2)(150\,\text{m})}\right) \\ &= 3581\,\text{N}, \end{aligned}$$

or 3580 N to three places.

5.52: a) Solving Eq. (5.14) for R,

$$R = v^2/a = v^2/4g = (95.0\,\text{m/s})^2/(4 \times 9.80\,\text{m/s}^2) = 230\,\text{m}.$$

b) The apparent weight will be five times the actual weight,

$$5\,mg = 5\,(50.0\ \text{kg})\,(9.80\,\text{m/s}^2) = 2450\ \ \text{N}$$

to three figures.

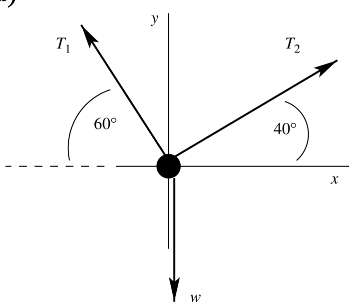
5.53: For no water to spill, the magnitude of the downward (radial) acceleration must be at least that of gravity; from Eq. (5.14), $v > \sqrt{gR} = \sqrt{(9.80\ \text{m/s}^2)(0.600\ \text{m})} = 2.42\ \text{m/s}$.

5.54: a) The inward (upward, radial) acceleration will be $\frac{v^2}{R} = \frac{(4.2\ \text{m/s})^2}{(3.80\ \text{m})} = 4.64\ \text{m/s}^2$. At the bottom of the circle, the inward direction is upward.

b) The forces on the ball are tension and gravity, so $T - mg = ma$,

$$T = m(a+g) = w\left(\frac{a}{g}+1\right) = (71.2\ \text{N})\left(\frac{4.64\ \text{m/s}^2}{9.80\ \text{m/s}^2}+1\right) = 105\ \text{N}.$$

5.55: a)



T_1 is more vertical so supports more of the weight and is larger.

You can also see this from $\sum F_x = ma_x$:

$$T_2 \cos 40° - T_1 \cos 60° = 0$$

$$T_1 = \left(\frac{\cos 40°}{\cos 60°}\right) T_2 = 1.532 T_2$$

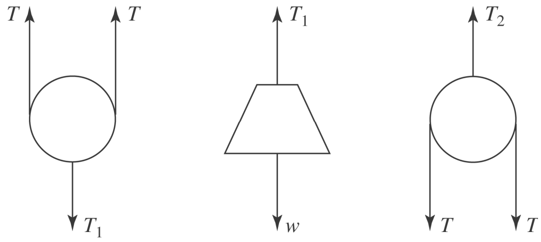
b) T_1 is larger so set $T_1 = 5000\ \text{N}$. Then $T_2 = T_1/1.532 = 3263.5\ \text{N}$,

$$\sum F_y = ma_y$$

$$T_1 \sin 60° + T_2 \sin 40° = w$$

$$w = 6400\ \text{N}$$

5.56:



The tension in the lower chain balances the weight and so is equal to *w*. The lower pulley must have no net force on it, so twice the tension in the rope must be equal to *w*, and so the tension in the rope is $w/2$. Then, the downward force on the upper pulley due to the rope is also *w*, and so the upper chain exerts a force *w* on the upper pulley, and the tension in the upper chain is also *w*.

5.57: In the absence of friction, the only forces along the ramp are the component of the weight along the ramp, $w \sin \alpha$, and the component of $\vec{F}$ along the ramp, $\left| \vec{F} \right| \cos \alpha = F \cos \alpha$. These forces must sum to zero, so $F = w \tan \alpha$.

Considering horizontal and vertical components, the normal force must have horizontal component equal to $n \sin \alpha$, which must be equal to *F*; the vertical component must balance the weight, $n \cos \alpha = w$. Eliminating *n* gives the same result.

5.58: The hooks exert forces on the ends of the rope. At each hook, the force that the hook exerts and the force due to the tension in the rope are an action-reaction pair.
The vertical forces that the hooks exert must balance the weight of the rope, so each hook exerts an upward vertical force of $w/2$ on the rope. Therefore, the downward force that the rope exerts at each end is $T_{\text{end}} \sin \theta = w/2$, so $T_{\text{end}} = w/(2 \sin \theta) = Mg/(2 \sin \theta)$.
b) Each half of the rope is itself in equilibrium, so the tension in the middle must balance the horizontal force that each hook exerts, which is the same as the horizontal component of the force due to the tension at the end; $T_{\text{end}} \cos \theta = T_{\text{middle}}$, so
$T_{\text{middle}} = Mg \cos \theta/(2 \sin \theta) = Mg/(2 \tan \theta)$.
(c) Mathematically speaking, $\theta \neq 0$ because this would cause a division by zero in the equation for T_{end} or T_{middle}. Physically speaking, we would need an infinite tension to keep a non-massless rope perfectly straight.

5.59: Consider a point a distance x from the top of the rope. The forces acting in this point are T up and $\left(M+\frac{m(L-x)}{L}\right)g$ downwards. Newton's Second Law becomes $T-\left(M+\frac{m(L-x)}{L}\right)g=\left(M+\frac{m(L-x)}{L}\right)a$. Since $a=\frac{F-(M+m)g}{M+m}, T=\left(M+\frac{m(L-x)}{L}\right)\left(\frac{F}{M+m}\right)$. At $x=0, T=F$, and at $x=L, T=\frac{MF}{M+m}=M(a+g)$ as expected.

5.60: a) The tension in the cord must be m_2g in order that the hanging block move at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so $m_2g=\left(m_1g\sin\alpha+\mu_k m_1 g\cos\alpha\right)$ and $m_2=m_1(\sin\alpha+\mu_k\cos\alpha)$.

b) In this case, the friction force acts in the same direction as the tension on the block of mass m_1, so $m_2g=(m_1g\sin\alpha-\mu_\text{k}m_1g\cos\alpha)$, or $m_2=m_1(\sin\alpha-\mu_\text{k}\cos\alpha)$.

c) Similar to the analysis of parts (a) and (b), the largest m_2 could be is $m_1(\sin\alpha+\mu_\text{s}\cos\alpha)$ and the smallest m_2 could be is $m_1(\sin\alpha-\mu_\text{s}\cos\alpha)$.

5.61: For an angle of $45.0°$, the tensions in the horizontal and vertical wires will be the same. a) The tension in the vertical wire will be equal to the weight $w=12.0\text{ N}$; this must be the tension in the horizontal wire, and hence the friction force on block A is also 12.0 N. b) The maximum frictional force is $\mu_\text{s}w_A=(0.25)(60.0\text{ N})=15\text{ N}$; this will be the tension in both the horizontal and vertical parts of the wire, so the maximum weight is 15 N.

5.62: a) The most direct way to do part (a) is to consider the blocks as a unit, with total weight 4.80 N. Then the normal force between block B and the lower surface is 4.80 N, and the friction force that must be overcome by the force F is $\mu_\text{k}n=(0.30)(4.80\text{ N})=1.440\text{ N}$, or 1.44 N, to three figures. b) The normal force between block B and the lower surface is still 4.80 N, but since block A is moving *relative to block B*, there is a friction force between the blocks, of magnitude $(0.30)(1.20\text{ N})=0.360\text{ N}$, so the total friction force that the force F must overcome is $1.440\text{ N}+0.360\text{ N}=1.80\text{ N}$. (An extra figure was kept in these calculations for clarity.)

5.63: (Denote $|\vec{\boldsymbol{F}}|$ by F.) a) The force normal to the surface is $n = F\cos\theta$; the vertical component of the applied force must be equal to the weight of the brush plus the friction force, so that $F\sin\theta = w + \mu_k F\cos\theta$, and

$$F = \frac{w}{\sin\theta - \mu_k\cos\theta} = \frac{12.00\ \text{N}}{\sin 53.1° - (0.51)\cos 53.1°} = 16.9\ \text{N},$$

keeping an extra figure. b) $F\cos\theta = (16.91\ \text{N})\cos 53.1° = 10.2\ \text{N}$.

5.64: a)

$$\begin{aligned}\sum F &= ma = m(62.5g) = 62.5mg\\ &= (62.5)(210\times10^{-6}\,\text{g})(980\,\text{cm/s}^2)\\ &= 13\,\text{dynes} = 1.3\times10^{-4}\ \text{N}\end{aligned}$$

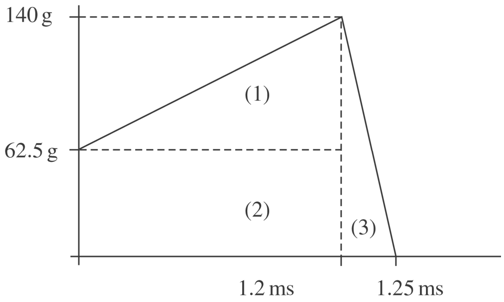
This force is 62.5 times the flea's weight.

b)

$$\begin{aligned}F_{\text{max}} &= ma_{\text{max}} = m(140g) = 140mg\\ &= 29\,\text{dynes} = 2.9\times10^{-4}\ \text{N}\end{aligned}$$

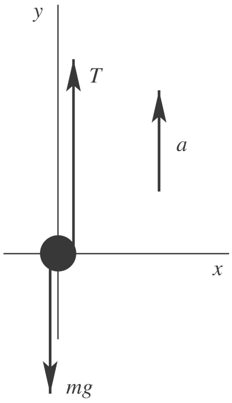
Occurs at approximately 1.2 ms.

c) $\Delta v = v - v_0 = v - 0 = v =$ area under *a-t* graph. Approximate area as shown:



$$\begin{aligned}A &= A(1) + A(2) + A(3)\\ &= \frac{1}{2}(1.2\ \text{ms})(77.5\,\text{g}) + (1.2\ \text{ms})(62.5\,\text{g})\\ &\quad + \frac{1}{2}(0.05\ \text{ms})(140\,\text{g})\\ &= 120\,\text{cm/s} = 1.2\ \text{m/s}\end{aligned}$$

5.65: a) The instrument has mass $m = w/g = 1.531\,\text{kg}$. Forces on the instrument:



$$\sum F_y = ma_y$$

$$T - mg = ma$$

$$a = \frac{T - mg}{m} = 13.07\ \text{m/s}^2$$

$v_{0y} = 0, v_y = 330\ \text{m/s}, a_y = 13.07\ \text{m/s}^2, t = ?$

$v_y = v_{0y} + a_y t$ gives $t = 25.3\,\text{s}$

Consider forces on the rocket; rocket has the same a_y. Let F be the thrust of the rocket engines.

$F - mg = ma$

$$F = m(g + a) = (25{,}000\,\text{kg})\,(9.80\ \text{m/s}^2 + 13.07\,\text{m/s}^2) = 5.72 \times 10^5\,\text{N}$$

b) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 4170\,\text{m}$.

5.66: The elevator's acceleration is:

$$a = \frac{dv(t)}{dt} = 3.0\,\text{m/s}^2 + 2(0.20\,\text{m/s}^3)t = 3.0\,\text{m/s}^2 + (0.40\,\text{m/s}^3)t$$

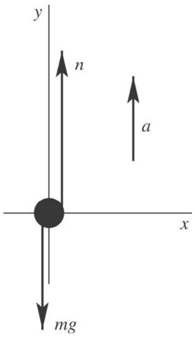
At $t = 4.0\,\text{s}, a = 3.0\,\text{m/s}^2 + (0.40\,\text{m/s}^3)(4.0\,\text{s}) = 4.6\ \text{m/s}^2$. From Newton's Second Law, the net force on you is

$$F_{\text{net}} = F_{\text{scale}} - w = ma$$

$$F_{\text{scale}} = \text{apparent weight} = w + ma = (72\,\text{kg})(9.8\ \text{m/s}^2) + (72\,\text{kg})(4.6\,\text{m/s}^2)$$

$$= 1036.8\ \text{N or } 1040\ \text{N}$$

5.67: Consider the forces on the person:



$$\sum F_y = ma_y$$

$$n - mg = ma$$

$$n = 1.6mg \text{ so } a = 0.60\ g = 5.88\ \text{m/s}^2$$

$$y - y_0 = 3.0\ \text{m},\ a_y = 5.88\ \text{m/s}^2,\ v_{0y} = 0, v_y = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } v_y = 5.0\ \text{m/s}$$

5.68: (a) Choosing upslope as the positive direction:

$$F_{\text{net}} = -mg\sin 37° - f_{\text{k}} = -mg\sin 37° - \mu_{\text{k}} mg\cos 37° = ma$$

and

$$a = -(9.8\ \text{m/s}^2)(0.602 + (0.30)(0.799)) = -8.25\ \text{m/s}^2$$

Since we know the length of the slope, we can use $v^2 = v_0^2 + 2a(x - x_0)$ with $x_0 = 0$ and $v = 0$ at the top.

$$v_0^2 = -2ax = -2(-8.25\ \text{m/s}^2)(8.0\ \text{m}) = 132\ \text{m}^2/\text{s}^2$$

$$v_0 = \sqrt{132\ \text{m}^2/\text{s}^2} = 11.5\ \text{m/s or } 11\ \text{m/s}$$

(b) For the trip back down the slope, gravity and the friction force operate in opposite directions**:**

$$F_{\text{net}} = -mg\sin 37° + \mu_{\text{k}} mg\cos 37° = ma$$

$$a = g(-\sin 37° + 0.30\cos 37°) = (9.8\ \text{m/s}^2)((-0.602) + (0.30)(0.799)) = -3.55\ \text{m/s}^2$$

Now

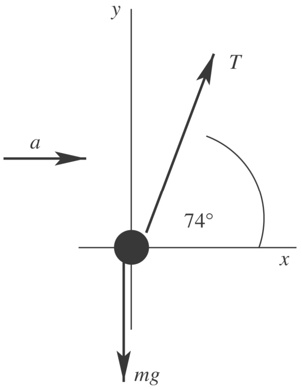
$$v_0 = 0, x_0 = -8.0\ \text{m}, x = 0, \text{ and}$$

$$v^2 = v_0^2 + 2a(x - x_0) = 0 + 2(-3.55\ \text{m/s}^2)(-8.0\ \text{m})$$

$$= 56.8\ \text{m}^2/\text{s}^2$$

$$v = \sqrt{56.8\ \text{m}^2/\text{s}^2} = 7.54\ \text{m/s or } 7.5\ \text{m/s}$$

5.69: Forces on the hammer**:**



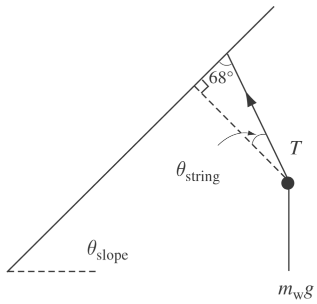
$\sum F_y = ma_y$ gives $T \sin 74° - mg = 0$ so $T \sin 74° = mg$

$\sum F_x = ma_x$ gives $T \cos 74° - ma$

Divide the second equation by the first**:**

$\dfrac{a}{g} = \dfrac{1}{\tan 74°}$ and $a = 2.8\ \mathrm{m/s^2}$

5.70:



It's interesting to look at the string's angle measured from the perpendicular to the top of the crate. This angle is of course $90°$—angle measured from the top of the crate. The free-body diagram for the washer then leads to the following equations, using Newton's Second Law and taking the upslope direction as positive:

$$-m_w g\sin\theta_{\text{slope}} + T\sin\theta_{\text{string}} = m_w a$$
$$-m_w g\cos\theta_{\text{slope}} + T\cos\theta_{\text{string}} = 0$$
$$T\sin\theta_{\text{string}} = m_w(a + g\sin\theta_{\text{slope}})$$
$$T\cos\theta_{\text{string}} = m_w g\cos\theta_{\text{slope}}$$

Dividing the two equations:

$$\tan\theta_{\text{string}} = \frac{a + g\sin\theta_{\text{slope}}}{g\cos 0_{\text{slope}}}$$

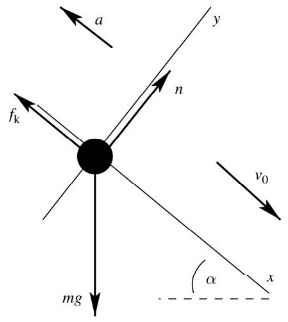
For the crate, the component of the weight along the slope is $-m_c g\sin\theta_{\text{slope}}$ and the normal force is $m_c g\cos\theta_{\text{slope}}$. Using Newton's Second Law again:

$$-m_c g\sin\theta_{\text{slope}} + \mu_k m_c g\cos\theta_{\text{slope}} = m_c a$$
$$\mu_k = \frac{a + g\sin\theta_{\text{slope}}}{g\cos\theta_{\text{slope}}}$$

which leads to the interesting observation that the string will hang at an angle whose tangent is equal to the coefficient of kinetic friction**:**

$$\mu_k = \tan\theta_{\text{string}} = \tan(90° - 68°) = \tan 22° = 0.40$$

5.71: a) Forces on you**:**



$$\sum F_y = ma_y \text{ gives } n = mg\ \cos\alpha$$
$$\sum F_x = ma_x$$
$$mg\sin\alpha - f_k = ma$$
$$a = g(\sin\alpha - \mu_k\cos\alpha) = -3.094\ \mathrm{m/s^2}$$

Find your stopping distance

$v_x = 0, a_x = -3.094\ \mathrm{m/s^2}, v_{0x} = 20\ \mathrm{m/s}, x - x_0 = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = 64.6\ \mathrm{m}$, which is greater than 40 m. You don't stop before you reach the hole, so you fall into it.

b) $a_x = -3.094\ \mathrm{m/s^2}, x - x_0 = 40\ \mathrm{m}, v_x = 0, v_{0x} = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_{0x} = 16\ \mathrm{m/s}$.

5.72: The key idea in solving this problem is to recognize that if the system is accelerating, the tension that block *A* exerts on the rope is different from the tension that block *B* exerts on the rope. (Otherwise the net force on the rope would be zero, and the rope couldn't accelerate.) Also, treat the rope as if it is just another object. Taking the "clockwise" direction to be positive, the Second Law equations for the three different parts of the system are**:**

Block *A* (The only horizontal forces on *A* are tension to the right, and friction to the left):

$$-\mu_k m_A g + T_A = m_A a.$$

Block *B* (The only vertical forces on *B* are gravity down, and tension up):

$$m_B g - T_B = m_B a.$$

Rope (The forces on the rope along the direction of its motion are the tensions at either end and the weight of the portion of the rope that hangs vertically):

$$m_R\left(\tfrac{d}{L}\right)g + T_B - T_A = m_R a.$$

To solve for *a* and eliminate the tensions, add the left hand sides and right hand sides of the three equations**:** $-\mu_k m_A g + m_B g + m_R\left(\frac{d}{L}\right)g = (m_A + m_B + m_R)a$, or $a = g\frac{m_B + m_R\frac{d}{L} - \mu_k m_A}{(m_A + m_B + m_R)}$.

(a) When $\mu_k = 0, a = g\frac{m_B + m_R\left(\frac{d}{L}\right)}{(m_A + m_B + m_R)}$. As the system moves, *d* will increase, approaching *L* as a limit, and thus the acceleration will approach a maximum value of $a = g\frac{m_B + m_R}{(m_A + m_B + m_R)}$.

(b) For the blocks to just begin moving, $a > 0$, so solve $0 = [m_B + m_R\left(\frac{d}{L}\right) - \mu_s m_A]$ for d. Note that we must use static friction to find *d* for when the block will *begin* to move. Solving for *d*, $d = \frac{L}{m_R}(\mu_s m_A - m_B)$, or $d = \frac{1.0\ \text{m}}{.160\ \text{kg}}(.25(2\ \text{kg}) - .4\ \text{kg}) = .63\ \text{m}.$.

(c) When $m_R = .04\ \text{kg}$, $d = \frac{1.0\ \text{m}}{.04\ \text{kg}}(.25(2\ \text{kg}) - .4\ \text{kg}) = 2.50\ \text{m}$. This is not a physically possible situation since $d > L$. The blocks won't move, no matter what portion of the rope hangs over the edge.

5.73: For a rope of length *L*, and weight *w*, assume that a length *rL* is on the table, so that a length $(1-r)L$ is hanging. The tension in the rope at the edge of the table is then $(1-r)w$, and the friction force on the part of the rope on the table is $f_s = \mu_s rw$. This must be the same as the tension in the rope at the edge of the table, so $\mu_s rw = (1-r)w$ and $r = 1/(1+\mu_s)$. Note that this result is independent of *L* and *w* for a uniform rope. The fraction that hangs over the edge is $1 - r = \mu_s/(1+\mu_s)$; note that if $\mu_s = 0, r = 1$ and $1 - r = 0$.

5.74: a) The normal force will be $mg\cos\alpha + F\sin\alpha$, and the net force along (up) the ramp is

$F\cos\alpha - mg\sin\alpha - \mu_s(mg\cos\alpha + F\sin\alpha) = F(\cos\alpha - \mu_s\sin\alpha) - mg(\sin\alpha + \mu_s\cos\alpha)$.

In order to move the box, this net force must be greater than zero. Solving for *F*,

$$F > mg\frac{\sin\alpha + \mu_s\cos\alpha}{\cos\alpha - \mu_s\sin\alpha}.$$

Since *F* is the magnitude of a force, *F* must be positive, and so the denominator of this expression must be positive, or $\cos\alpha > \mu_s\sin\alpha$, and $\mu_s < \cot\alpha$. b) Replacing μ_s with μ_k with in the above expression, and making the inequality an equality,

$$F = mg\frac{\sin\alpha + \mu_k\cos\alpha}{\cos\alpha - \mu_k\sin\alpha}.$$

5.75: a) The product $\mu_s g = 2.94\,\text{m/s}^2$ is greater than the magnitude of the acceleration of the truck, so static friction can supply sufficient force to keep the case stationary relative to the truck; the crate accelerates north at $2.20\,\text{m/s}^2$, due to the friction force of $ma = 66.0\,\text{N}$. b) In this situation, the static friction force is insufficient to maintain the case at rest relative to the truck, and so the friction force is the kinetic friction force, $\mu_k n = \mu_k mg = 59\,\text{N}$.

5.76: To answer the question, v_0 must be found and compared with $20\,\text{m/s}\,(72\,\text{km/hr})$. The kinematics relationship $2ax = -v_0^2$ is useful, but we also need *a*. The acceleration must be large enough to cause the box to begin sliding, and so we must use the force of static friction in Newton's Second Law: $-\mu_s mg \le ma$, or $a = -\mu_s g$. Then, $2(-\mu_s g)x = -v_0^2$, or $v_0 = \sqrt{2\mu_s gx} = \sqrt{2(.30)(9.8\,\text{m/s}^2)(47\,\text{m})}$. Hence, $v_0 = 16.6\,\text{m/s} = 60\,\text{km/h}$, which is less than 72 km/h, so do you not go to jail.

5.77: See Exercise 5.40. a) The maximum tension and the weight are related by

$$T_{\max}\cos\beta = \mu_k(w - T_{\max}\sin\beta),$$

and solving for the weight w gives

$$w = T_{\max}\left(\frac{\cos\beta}{\mu_k} + \sin\beta\right).$$

This will be a maximum when the quantity in parentheses is a maximum. Differentiating with respect to β,

$$\frac{d}{d\beta}\left(\frac{\cos\beta}{\mu_k} + \sin\beta\right) = -\frac{\sin\beta}{\mu_k} + \cos\beta = 0,$$

or $\tan\theta = \mu_k$, where θ is the value of β that maximizes the weight. Substituting for μ_k in terms of θ,

$$\begin{aligned} w &= T_{\max}\left(\frac{\cos\theta}{\sin\theta/\cos\theta} + \sin\theta\right) \\ &= T_{\max}\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta}\right) \\ &= \frac{T_{\max}}{\sin\theta}. \end{aligned}$$

b) In the absence of friction, any non-zero horizontal component of force will be enough to accelerate the crate, but slowly.

5.78: a) Taking components along the direction of the plane's descent, $f = w\sin\alpha$ and $L = w\cos\alpha$. b) Dividing one of these relations by the other cancels the weight, so $\tan\alpha = f/L$. c) The distance will be the initial altitude divided by the tangent of α. $f = L\tan\alpha$ and $L = w\cos\alpha$, therefore $\sin\alpha = f/w = \frac{1300\ \text{N}}{12{,}900\ \text{N}}g$ and so $\alpha = 5.78°$. This makes the horizontal distance $(2500\ \text{m})/\tan(5.78°) = 24.7\ \text{km}$. d) If the drag is reduced, the angle α is reduced, and the plane goes further.

5.79: If the plane is flying at a constant speed of $36.1\ \text{m/s}$, then $\sum F = 0$, or $T - w\sin\alpha - f = 0$. The rate of climb and the speed give the angle α, $\alpha = \arcsin(5/36.1) = 7.96°$. Then, $T = w\sin\alpha + f$. $T = (12{,}900\ \text{N})\sin 7.96° + 1300\ \text{N} = 3087\ \text{N}$. Note that in level flight $(\alpha = 0)$, the thrust only needs to overcome the drag force to maintain the constant speed of $36.1\ \text{m/s}$.

5.80: If the block were to remain at rest relative to the truck, the friction force would need to cause an acceleration of $2.20\,\text{m/s}^2$; however, the maximum acceleration possible due to static friction is $(0.19)(9.80\,\text{m/s}^2) = 1.86\,\text{m/s}^2$, and so the block will move relative to the truck; the acceleration of the box would be $\mu_\text{k} g = (0.15)(9.80\,\text{m/s}^2) = 1.47\,\text{m/s}^2$. The difference between the distance the truck moves and the distance the box moves (*i.e.*, the distance the box moves relative to the truck) will be 1.80 m after a time

$$t = \sqrt{\frac{2\Delta x}{a_\text{truck} - a_\text{box}}} = \sqrt{\frac{2(1.80\,\text{m})}{(2.20\,\text{m/s}^2 - 1.47\,\text{m/s}^2)}} = 2.22\,\text{s}.$$

In this time, the truck moves $\frac{1}{2} a_\text{truck} t^2 = \frac{1}{2}(2.20\,\text{m/s}^2)(2.221\,\text{s})^2 = 5.43\,\text{m}$. Note that an extra figure was kept in the intermediate calculation to avoid roundoff error.

5.81: The friction force *on* block *A* is $\mu_\text{k} w_A = (0.30)(1.40\,\text{N}) = 0.420\,\text{N}$, as in Problem 5-68. This is the magnitude of the friction force that block *A* exerts on block *B*, as well as the tension in the string. The force *F* must then have magnitude

$$\begin{aligned} F &= \mu_\text{k}(w_B + w_A) + \mu_\text{k} w_A + T = \mu_\text{k}(w_B + 3w_A) \\ &= (0.30)(4.20\,\text{N} + 3(1.40\,\text{N})) = 2.52\ \text{N}. \end{aligned}$$

Note that the normal force exerted on block *B* by the table is the sum of the weights of the blocks.

5.82: We take the upward direction as positive. The explorer's vertical acceleration is $-3.7\ \text{m/s}^2$ for the first 20 s. Thus at the end of that time her vertical velocity will be $v_y = at = (-3.7\ \text{m/s}^2)(20\ \text{s}) = -74\ \text{m/s}$. She will have fallen a distance

$$d = v_{\text{av}}t = \left(\frac{-74\ \text{m/s}}{2}\right)(20\ \text{s}) = -740\ \text{m}$$

and will thus be $1200 - 740 = 460\ \text{m}$ above the surface. Her vertical velocity must reach zero as she touches the ground; therefore, taking the ignition point of the PAPS as $y_0 = 0$,

$$v_y^2 = v_0^2 + 2a(y - y_0)$$

$$a = \frac{v_y^2 - v_0^2}{2(y - y_0)} = \frac{0 - (-74\ \text{m/s})^2}{-460} = 5.95\ \text{m/s}^2 \text{ or } 6.0\ \text{m/s}^2$$

which is the vertical acceleration that must be provided by the PAPS. The time it takes to reach the ground is given by

$$t = \frac{v - v_0}{a} = \frac{0 - (-74\ \text{m/s})}{5.95\ \text{m/s}^2} = 12.4\ \text{s}$$

Using Newton's Second Law for the vertical direction

$$F_{\text{PAPSv}} + mg = ma$$

$$F_{\text{PAPSv}} = ma - mg = m(a + g) = (150\text{kg})(5.95 - (-3.7))\ \text{m/s}^2$$

$$= 1447.5\ \text{N or } 1400\ \text{N}$$

which is the vertical component of the PAPS force. The vehicle must also be brought to a stop horizontally in 12.4 seconds; the acceleration needed to do this is

$$a = \frac{v - v_0}{t} = \frac{0 - 33\ \text{m/s}^2}{12.4\ \text{s}} = 2.66\ \text{m/s}^2$$

and the force needed is $F_{\text{PAPSh}} = ma = (150\ \text{kg})(2.66\ \text{m/s}^2) = 399\ \text{N}$ or 400 N, since there are no other horizontal forces.

5.83: Let the tension in the cord attached to block *A* be T_A and the tension in the cord attached to block *C* be T_C. The equations of motion are then

$$\begin{aligned} T_A - m_A g &= m_A a \\ T_C - \mu_k m_B g - T_A &= m_B a \\ m_C g - T_C &= m_C a. \end{aligned}$$

a) Adding these three equations to eliminate the tensions gives

$$a(m_A + m_B + m_C) = g(m_C - m_A - \mu_k m_B),$$

solving for m_C gives

$$m_C = \frac{m_A(a+g) + m_B(a + \mu_k g)}{g - a},$$

and substitution of numerical values gives $m_C = 12.9$ kg.

b) $T_A = m_A(g + a) = 47.2\ \text{N}, T_C = m_C(g - a) = 101\ \text{N}.$

5.84: Considering positive accelerations to be to the right (up and to the right for the left-hand block, down and to the right for the right-hand block), the forces along the inclines and the accelerations are related by $T - (100\,\text{kg})g\sin 30° = (100\,\text{kg})a, (50\,\text{kg})g\sin 53° - T = (50\,\text{kg})a,$ where *T* is the tension in the cord and *a* the mutual magnitude of acceleration. Adding these relations, $(50\,\text{kg}\sin 53° - 100\,\text{kg}\sin 30°)g = (50\,\text{kg} + 100\,\text{kg})a,$ or $a = -0.067\,g.$ a) Since *a* comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, *a* would be $+0.067\,g.$ b) $0.067(9.80\,\text{m/s}^2) = 0.658\,\text{m/s}^2.$

c) Substituting the value of *a* (including the proper sign, depending on choice of coordinates) into either of the above relations involving *T* yields 424 N.

5.85: Denote the magnitude of the acceleration of the block with mass m_1 as a; the block of mass m_2 will descend with acceleration $a/2$. If the tension in the rope is T, the equations of motion are then

$$T = m_1 a$$
$$m_2 g - 2T = m_2\, a/2.$$

Multiplying the first of these by 2 and adding to eliminate T, and then solving for a gives

$$a = \frac{m_2 g}{2m_1 + m_2/2} = g\frac{2m_2}{4m_1 + m_2}.$$

The acceleration of the block of mass m_2 is half of this, or $g\, m_2/(4m_1 + m_2)$.

5.86: Denote the common magnitude of the maximum acceleration as a. For block A to remain at rest with respect to block B, $a < \mu_s g$. The tension in the cord is then $T = (m_A + m_B)a + \mu_k g(m_A + m_B) = (m_A + m_B)(a + \mu_k g)$. This tension is related to the mass m_C by $T = m_C(g - a)$. Solving for a yields

$$a = g\frac{m_C - \mu_k(m_A + m_B)}{m_A + m_B + m_C} < \mu_s g.$$

Solving the inequality for m_C yields

$$m_C < \frac{(m_A + m_B)(\mu_s + \mu_k)}{1 - \mu_s}.$$

5.87: See Exercise 5.15 (Atwood's machine). The 2.00-kg block will accelerate upward at $g\frac{5.00\text{ kg} - 2.00\text{ kg}}{5.00\text{ kg} + 2.00\text{ kg}} = 3g/7$, and the 5.00-kg block will accelerate downward at $3g/7$. Let the initial height above the ground be h_0; when the large block hits the ground, the small block will be at a height $2h_0$, and moving upward with a speed given by $v_0^2 = 2ah_0 = 6gh_0/7$. The small block will continue to rise a distance $v_0^2/2g = 3h_0/7$, and so the maximum height reached will be $2h_0 + 3h_0/7 = 17h_0/7 = 1.46\text{ m}$, which is 0.860 m above its initial height.

5.88: The floor exerts an upward force n on the box, obtained from $n - mg = ma$, or $n = m(a+g)$. The friction force that needs to be balanced is

$$\mu_{\mathrm{k}} n = \mu_{\mathrm{k}} m(a+g)\,(0.32)(28.0\ \mathrm{kg})(1.90\ \mathrm{m/s^2} + 9.80\ \mathrm{m/s^2}) = 105\ \mathrm{N}.$$

5.89: The upward friction force must be $f_{\mathrm{s}} = \mu_{\mathrm{s}} n = m_A g$, and the normal force, which is the only horizontal force on block *A*, must be $n = m_A a$, and so $a = g/\mu_{\mathrm{s}}$. An observer on the cart would "feel" a backwards force, and would say that a similar force acts on the block, thereby creating the need for a normal force.

5.90: Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; *i.e.*, the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00\ \mathrm{kg})g(\sin 30° - (0.25)\cos 30°) - T = (4.00\ \mathrm{kg})a$, or $11.11\ \mathrm{N} - T = (4.00\ \mathrm{kg})a$, and similarly for the larger, $15.44\ \mathrm{N} + T = (8.00\ \mathrm{kg})a$, a) Adding these two relations, $26.55\ \mathrm{N} = (12.00\ \mathrm{kg})a, a = 2.21\ \mathrm{m/s^2}$ (note that an extra figure was kept in the intermediate calculation to avoid roundoff error). b) Substitution into either of the above relations gives $T = 2.27\ \mathrm{N}$. Equivalently, dividing the second relation by 2 and subtracting from the first gives $\frac{3}{2}T = 11.11\ \mathrm{N} - \frac{15.44\ \mathrm{N}}{2}$, giving the same result. c) The string will be slack. The 4.00-kg block will have $a = 2.78\ \mathrm{m/s^2}$ and the 8.00-kg block will have $a = 1.93\ \mathrm{m/s^2}$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.

5.91: a) Let n_B be the normal force between the plank and the block and n_A be the normal force between the block and the incline. Then, $n_B = w\cos\theta$ and $n_A = n_B + 3w\cos\theta = 4w\cos\theta.$ The net frictional force on the block is $\mu_{\mathrm{k}}(n_A + n_B) = \mu_{\mathrm{k}} 5w\cos\theta$. To move at constant speed, this must balance the component of the block's weight along the incline, so $3w\sin\theta = \mu_{\mathrm{k}} 5w\cos\theta$, and $\mu_{\mathrm{k}} = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 37° = 0.452.$

5.92: (a) There is a contact force *n* between the man (mass *M*) and the platform (mass *m*). The equation of motion for the man is $T + n - Mg = Ma,$ where *T* is the tension in the rope, and for the platform, $T - n - mg = ma$. Adding to eliminate *n*, and rearranging, $T = \frac{1}{2}(M + m)(a + g).$ This result could be found directly by considering the man-platform combination as a unit, with mass $m + M,$ being pulled upward with a force 2*T* due to the *two* ropes on the combination. The tension *T* in the rope is the same as the force that the man applies to the rope. Numerically,

$$T = \frac{1}{2}(70.0\,\text{kg} + 25.0\,\text{kg})(1.80\,\text{m/s}^2 + 9.80\,\text{m/s}^2) = 551\,\text{N}.$$

(b) The end of the rope moves downward 2 m when the platform moves up 1 m, so $a_{\text{rope}} = -2a_{\text{platform}}$. Relative to the man, the acceleration of the rope is $3a = 5.40\,\text{m/s}^2$, downward.

5.93: a) The only horizontal force on the two-block combination is the horizontal component of $\vec{F}, F\cos\alpha$. The blocks will accelerate with $a = F\cos\alpha/(m_1 + m_2).$ b) The normal force between the blocks is $m_1 g + F\sin\alpha,$ for the blocks to move together, the product of this force and μ_s must be greater than the horizontal force that the lower block exerts on the upper block. That horizontal force is one of an action-reaction pair; the reaction to this force accelerates the lower block. Thus, for the blocks to stay together, $m_2 a \le \mu_s(m_1 g + F\sin\alpha).$ Using the result of part (a),

$$m_2\frac{F\cos\alpha}{\text{m}_1 + m_2} \le \mu_s(m_1 g + F\sin\alpha).$$

Solving the inequality for *F* gives the desired result.

5.94: The banked angle of the track has the same form as that found in Example 5.24, $\tan\beta = \frac{v_0^2}{gR}$, where v_0 is the ideal speed, $20\ \text{m/s}$ in this case. For speeds larger than v_0, a frictional force is needed to keep the car from skidding. In this case, the inward force will consist of a part due to the normal force *n* and the friction force $f; n\sin\beta + f\cos\beta = ma_{\text{rad}}$. The normal and friction forces both have vertical components; since there is no vertical acceleration, $n\cos\beta - f\sin\beta = mg$. Using $f = \mu_s n$ and $a_{\text{rad}} = \frac{v^2}{R} = \frac{(1.5v_0)^2}{R} = 2.25\,g\tan\beta$, these two relations become

$$n\sin\beta + \mu_s n\cos\beta = 2.25\,mg\tan\beta,$$
$$n\cos\beta - \mu_s n\sin\beta = mg.$$

Dividing to cancel *n* gives

$$\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = 2.25\tan\beta.$$

Solving for μ_s and simplifying yields

$$\mu_s = \frac{1.25\sin\beta\cos\beta}{1 + 1.25\sin^2\beta}.$$

Using $\beta = \arctan\left(\frac{(20\ \text{m/s})^2}{(9.80\ \text{m/s}^2)(120\ \text{m})}\right) = 18.79°$ gives $\mu_s = 0.34$.

5.95: a) The same analysis as in Problem 5.90 applies, but with the speed *v* an unknown. The equations of motion become

$$n\sin\beta + \mu_s n\cos\beta = mv^2/R,$$
$$n\cos\beta - \mu_s n\sin\beta = mg.$$

Dividing to cancel *n* gives

$$\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = \frac{v^2}{Rg}.$$

Solving for *v* and substituting numerical values gives $v = 20.9\ \text{m/s}$ (note that the value for the coefficient of static friction must be used).

b) The same analysis applies, but the friction force must be directed up the bank; this has the same algebraic effect as replacing f with $-f$, or replacing μ_s with $-\mu_s$ (although coefficients of friction may certainly never be negative). The result is

$$v^2 = (gR)\frac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta},$$

and substitution of numerical values gives $v = 8.5\ \text{m/s}$.

5.96: (a) $80\,\text{mi/h}$ is $35.7\,\text{m/s}$ in SI units. The centripetal force needed to keep the car on the road is provided by friction; thus

$$\mu_s mg = \tfrac{mv^2}{r}$$

$$r = \frac{v^2}{\mu_s g} = \frac{(35.7\,\text{m/s})^2}{(0.76)(9.8\,\text{m/s}^2)} = 171\,\text{m} \text{ or } 170\,\text{m}$$

(b) If $\mu_s = 0.20$:

$$v^2 = r\mu_s g = (171\,\text{m})\,(0.20)\,(9.8\,\text{m/s}^2) = 335.2\,\text{m}^2/\text{s}^2$$

$$v = 18.3\,\text{m/s or about } 41\,\text{mi/h}$$

(c) If $\mu_s = 0.37$:

$$v^2 = (171\,\text{m})\,(0.37)\,(9.8\,\text{m/s}^2) = 620\,\text{m}^2/\text{s}^2$$

$$v = 24.9\,\text{m/s or about } 56\,\text{mi/h}$$

The speed limit is evidently designed for these conditions.

5.97: a) The static friction force between the tires and the road must provide the centripetal acceleration for motion in the circle.

$$\mu_s mg = m\frac{v^2}{r}$$

m, g, and r are constant so $\dfrac{v_1}{\sqrt{\mu_{s1}}} = \dfrac{v_2}{\sqrt{\mu_{s2}}}$, where 1 refers to dry road and 2 to wet road.

$\mu_{s2} = \frac{1}{2}\mu_{s1}$, so $v_2 = (27\,\text{m/s})/\sqrt{2} = 19\,\text{m/s}$

b) Calculate the time it takes you to reach the curve

$v_{0x} = 27\,\text{m/s},\ v_x = 19\,\text{m/s},\ x - x_0 = 800\,\text{m},\ t = ?$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t \text{ gives } t = 34.7\,\text{s}$$

During this time the other car will travel $x - x_0 = v_{0x}t = (36\,\text{m/s})\,(34.7\text{s}) = 1250\,\text{m}.$ The other car will be 50 m behind you as you enter the curve, and will be traveling at nearly twice your speed, so it is likely it will skid into you.

5.98: The analysis of this problem is the same as that of Example 5.22; solving for v in terms of β and $R, v = \sqrt{gR\tan\beta} = \sqrt{(9.80\,\text{m/s}^2)\,(50.0)\tan 30.0°} = 16.8\,\text{m/s},$ about $60.6\,\text{km/h}.$

5.99: The point to this problem is that the monkey and the bananas have the same weight, and the tension in the string is the same at the point where the bananas are suspended and where the monkey is pulling; in all cases, the monkey and bananas will have the same net force and hence the same acceleration, direction and magnitude. a) The bananas move up. b) The monkey and bananas always move at the same velocity, so the distance between them stays the same. c) Both the monkey and bananas are in free fall, and as they have the same initial velocity, the distance bewteen them doesn't change. d) The bananas will slow down at the same rate as the monkey; if the monkey comes to a stop, so will the bananas.

5.100: The separated equation of motion has a lower limit of $3v_{\mathrm{t}}$ instead of 0; specifically,

$$\int_{3v_{\mathrm{t}}}^{v} \frac{dv}{v - v_{\mathrm{t}}} = \ln \frac{v_{\mathrm{t}} - v}{-2v_{\mathrm{t}}} = \ln\left(\frac{v}{2v_{\mathrm{t}}} - \frac{1}{2}\right) = -\frac{k}{m}t, \text{ or}$$

$$v = 2v_{\mathrm{t}}\left[\frac{1}{2} + e^{-(k/m)t}\right].$$

Note that the speed is always greater than v_{t}.

5.101: a) The rock is released from rest, and so there is initially no resistive force and $a_0 = (18.0\text{ N})/(3.00\text{ kg}) = 6.00\text{ m/s}^2.$

b) $(18.0\text{ N} - (2.20\text{ N}\cdot\text{s/m})\,(3.00\text{ m/s}))/(3.00\text{ kg}) = 3.80\text{ m/s}^2.$ c) The net force must be 1.80 N, so $kv = 16.2\ \text{ N}$ and $v = (16.2\ \text{ N})/(2.20\text{ N}\cdot\text{s/m}) = 7.36\text{ m/s}.$ d) When the net force is equal to zero, and hence the acceleration is zero, $kv_{\text{t}} = 18.0\ \text{ N}$ and $v_{\text{t}} = (18.0\ \text{ N})/(2.20\text{ N}\cdot\text{s/m}) = 8.18\text{ m/s}.$ e) From Eq. (5.12),

$$y = (8.18\text{ m/s})\left[(2.00\text{ s}) - \frac{3.00\text{ kg}}{2.20\text{ N}\cdot\text{s/m}}(1 - e^{-((2.20\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})})\right]$$
$$= +7.78\text{ m}.$$

From Eq. (5.10),

$$v = (8.18\text{ m/s})[1 - e^{-((2.2\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})}]$$
$$= 6.29\text{ m/s}.$$

From Eq. (5.11), but with a_0 instead of g,

$$a = (6.00\text{ m/s}^2)e^{-((2.20\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})} = 1.38\text{ m/s}^2.$$

.

f)

$$1 - \frac{v}{v_{\text{t}}} = 0.1 = e^{-(k/m)t},\ \text{so}$$
$$t = \frac{m}{k}\ln(10) = 3.14\text{ s}.$$

5.102: (a) The retarding force of the surface is the only horizontal force acting. Thus

$$a = \frac{F_{\text{net}}}{m} = \frac{F_R}{m} = \frac{-kv^{1/2}}{m} = \frac{dv}{dt}$$

$$\frac{dv}{v^{1/2}} = -\frac{k}{m}dt$$

$$\int_{v_0}^{v} \frac{dv}{v^{1/2}} = \frac{k}{m}\int_0^t dt$$

$$2v^{1/2}\Big|_{v_0}^{v} = -\frac{kt}{m}$$

which gives

$$v = v_0 - \frac{v_0^{1/2}kt}{m} + \frac{k^2t^2}{4m^2}$$

For the rock's position**:**

$$\frac{dx}{dt} = v_0 - \frac{v_0^{1/2}kt}{m} + \frac{k^2t^2}{4m^2}$$

$$dx = v_0 dt - \frac{v_0^{1/2}ktdt}{m} + \frac{k^2t^2dt}{4m^2}$$

and integrating gives

$$x = v_0 t - \frac{v_0^{1/2}kt^2}{2m} + \frac{k^2t^3}{12m^2}$$

(b)

$$v = 0 = v_0 - \frac{v_0^{1/2}kt}{m} + \frac{k^2t^2}{2m^2}$$

This is a quadratic equation in t; from the quadratic formula we can find the single solution**:**

$$t = \frac{2mv_0^{1/2}}{k}$$

(c) Substituting the expression for t into the equation for x**:**

$$x = v_0 \cdot \frac{2mv_0^{1/2}}{k} - \frac{v_0^{1/2}k}{2m}\cdot\frac{4m^2v_0}{k^2} + \frac{k^2}{12m^2}\cdot\frac{8m^3v_0^{3/2}}{k^3}$$

$$= \frac{2mv_0^{3/2}}{3k}$$

5.103: Without buoyancy, $kv_{\mathrm{t}} = mg$, so $k = \dfrac{mg}{v_{\mathrm{t}}} = \dfrac{mg}{0.36\,\mathrm{s}}$.

With buoyancy included there is the additional upward buoyancy force *B*, so

$$B - kv_{\mathrm{t}} = mg$$

$$B = mg - kv_{\mathrm{t}} = mg\left(1 - \frac{0.24\,\mathrm{m/s}}{0.36\,\mathrm{m/s}}\right) = mg/3$$

5.104: Recognizing the geometry of a 3-4-5 right triangle simplifies the calculation. For instance, the radius of the circle of the mass' motion is 0.75 m.

a) Balancing the vertical force, $T_{\mathrm{U}}\,\frac{4}{5} - T_{\mathrm{L}}\,\frac{4}{5} = w,$ so

$$T_{\mathrm{L}} = T_{\mathrm{U}} - \frac{5}{4}w = 80.0\ \mathrm{N} - \frac{5}{4}(4.00\,\mathrm{kg})\,(9.80\,\mathrm{m/s^2}) = 31.0\ \mathrm{N}.$$

b) The net inward force is $F = \frac{3}{5}T_{\mathrm{U}} + \frac{3}{5}T_{\mathrm{L}} = 66.6\ \mathrm{N}$. Solving $F = ma_{\mathrm{rad}} = m\frac{4\pi^2 R}{T^2}$ for the period *T*,

$$T = 2\pi\sqrt{\frac{mR}{F}} = 2\pi\sqrt{\frac{(4.00\,\mathrm{kg})\,(0.75\,\mathrm{m})}{(66.6\,\mathrm{N})}} = 1.334\,\mathrm{s},$$

or 0.02223 min, so the system makes 45.0 rev/min. c) When the lower string becomes slack, the system is the same as the conical pendulum considered in Example 5.22. With $\cos\beta = 0.800$, the period is $T = 2\pi\sqrt{(1.25\,\mathrm{m})\,(0.800)/(9.80\,\mathrm{m/s^2})} = 2.007\,\mathrm{s}$, which is the same as $29.9\ \mathrm{rev/min}$. d) The system will still be the same as a conical pendulum, but the block will drop to a smaller angle.

5.105: a) Newton's 2nd law gives

$$m\frac{dv_y}{dt} = mg - kv_y, \text{ where } \frac{mg}{k} = v_{\rm t}$$

$$\int_{v_0}^{v_y} \frac{dv_y}{v_y - v_t} = -\frac{k}{m}\int_0^t dt$$

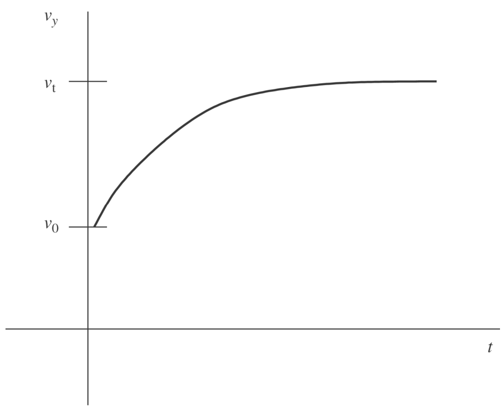
This is the same expression used in the derivation of Eq. (5.10), except the lower limit in the velocity integral is the initial speed v_0 instead of zero.

Evaluating the integrals and rearranging gives

$$v = v_0 e^{-kt/m} + v_{\rm t}(1 - e^{-kt/m})$$

Note that at $t = 0$ this expression says $v_y = v_0$ and at $t \to \alpha$ it says $v_y \to v_{\rm t}$.

b) The downward gravity force is larger than the upward fluid resistance force so the acceleration is downward, until the fluid resistance force equals gravity when the terminal speed is reached. The object speeds up until $v_y = v_{\rm t}$. Take $+y$ to be downward.



c) The upward resistance force is larger than the downward gravity force so the acceleration is upward and the object slows down, until the fluid resistance force equals gravity when the terminal speed is reached. Take $+y$ to be downward.

5.106: (a) To find find the maximum height and time to the top without fluid resistance**:**

$$v_y^2 = v_{0y}^2 + 2a(y - y_0)$$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a} = \frac{0 - (6.0\ \mathrm{m/s})^2}{2(-9.8\ \mathrm{m/s^2})} = 1.84\ \mathrm{m\ or\ } 1.8\ \mathrm{m}$$

$$t = \frac{v - v_0}{a} = \frac{0 - 6.0\ \mathrm{m/s}}{-9.8\ \mathrm{m/s^2}} = 0.61\ \mathrm{s}$$

(b) Starting from Newton's Second Law for this situation

$$m\frac{dv}{dt} = mg - kv$$

we rearrange and integrate, taking downward as positive as in the text and noting that the velocity at the top of the rock's "flight" is zero**:**

$$\int_v^0 \frac{dv}{v - v_\mathrm{t}} = -\frac{k}{m}t$$

$$\ln(v - v_\mathrm{t})\Big|_v^0 = \ln\frac{-v_\mathrm{t}}{v - v_\mathrm{t}} = \ln\frac{-2.0\ \mathrm{m/s}}{-6.0\ \mathrm{m/s} - 2.0\ \mathrm{m/s}} = \ln(0.25) = -1.386$$

From Eq. 5.9, $m/k = v_\mathrm{t}/g = (2.0\ \mathrm{m/s^2})/(9.8\ \mathrm{m/s^2}) = 0.204\ \mathrm{s}$, and

$t = -\frac{m}{k}(-1.386) = (0.204\ \mathrm{s})\,(1.386) = 0.283\ \mathrm{s}$ to the top. Equation 5.10 in the text gives us

$$\frac{dx}{dt} = v_\mathrm{t}(1 - e^{-(\mathrm{k/m})t}) = v_\mathrm{t} - v_\mathrm{t}e^{-(\mathrm{k/m})t}$$

$$\int_0^x dx = \int_0^t v_\mathrm{t}dt - \int_0^t v_\mathrm{t}e^{-(\mathrm{k/m})t}dt$$

$$= v_\mathrm{t}t + \frac{v_\mathrm{t}m}{k}(e^{-(\mathrm{k/m})t} - 1)$$

$$= (2.0\ \mathrm{m/s})\,(0.283\ \mathrm{s}) + (2.0\ \mathrm{m/s})\,(0.204\ \mathrm{s})(\mathrm{e}^{-1.387} - 1)$$

$$= 0.26\ \mathrm{m}$$

5.107: a) The forces on the car are the air drag force $f_D = Dv^2$ and the rolling friction force $\mu_r mg$. Take the velocity to be in the $+x$-direction. The forces are opposite in direction to the velocity. $\sum F_x = ma_x$ gives

$$-Dv^2 - \mu_r mg = ma$$

We can write this equation twice, once with $v = 32\ \text{m/s}$ and $a = -0.42\ \text{m/s}^2$ and once with $v = 24\ \text{m/s}$ and $a = -0.30\ \text{m/s}^2$. Solving these two simultaneous equations in the unknowns D and μ_r gives $\mu_r = 0.015$ and $D = 0.36\ \text{N}\cdot\text{s}^2/\text{m}^2$.

b) $n = mg\cos\beta$ and the component of gravity parallel to the incline is $mg\sin\beta$, where $\beta = 2.2°$.

For constant speed, $mg\sin 2.2° - \mu_r mg\cos 2.2° - Dv^2 = 0$.

Solving for v gives $v = 29\ \text{m/s}$.

c) For angle $\beta, mg\sin\beta - \mu_r mg\cos\beta - Dv^2 = 0$

and $v = \sqrt{\dfrac{mg(\sin\beta - \mu_r\cos\beta)}{D}}$

The terminal speed for a falling object is derived from $Dv_t^2 - mg = 0$, so

$v_t = \sqrt{mg/D}.$

$v/v_t = \sqrt{\sin\beta - \mu_r\cos\beta}$

And since $\mu_r = 0.015, v/v_t = \sqrt{\sin\beta - (0.015)\cos\beta}$

5.108: (a) One way of looking at this is that the apparent weight, which is the same as the upward force on the person, is the actual weight of the person minus the centripetal force needed to keep him moving in its circular path**:**

$$w_{\text{app}} = mg - \frac{mv^2}{R} = (70\ \text{kg})\left[(9.8\ \text{m/s}^2) - \frac{(12\ \text{m/s})^2}{40\ \text{m}}\right]$$

$$= 434\ \text{N}$$

(b) The cart will lose contact with the surface when its apparent weight is zero; i.e., when the road no longer has to exert any upward force on it**:**

$$mg - \frac{mv^2}{R} = 0$$

$$v^2 = Rg = (40\ \text{m})(9.8\ \text{m/s}^2) = 392\ \text{m}^2/\text{s}^2$$

$$v = 19.8\ \text{m/s or } 20\ \text{m/s}$$

The answer doesn't depend on the cart's mass, because the centripetal force needed to hold it on the road is proportional to its mass and so is its weight, which provides the centripetal force in this situation.

5.109: a) For the same rotation rate, the magnitude of the radial acceleration is proportional to the radius, and for twins of the same mass, the needed force is proportional to the radius; Jackie is twice as far away from the center, and so must hold on with twice as much force as Jena, or 120 N.

b) $\sum F_{\text{Jackie}} = mv^2/r$.

$$v = \sqrt{\frac{(120\ \text{N})\,(3.6\ \text{m})}{30\ \text{kg}}} = 3.8\ \text{m/s}.$$

5.110: The passenger's velocity is $v = 2\pi R/t = 8.80\ \text{m/s}$. The vertical component of the seat's force must balance the passenger's weight and the horizontal component must provide the centripetal force. Therefore**:**

$$F_{\text{seat}} \sin\theta = mg = 833\ \text{N}$$

$$F_{\text{seat}} \cos\theta = \frac{mv^2}{R} = 188\ \text{N}$$

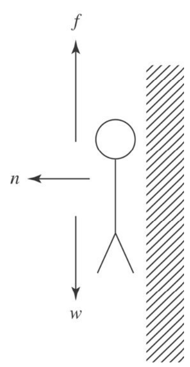
Therefore $\tan\theta = 833\ \text{N}/188\ \text{N} = 4.43; \theta = 77.3°$ above the horizontal. The magnitude of the net force exerted by the seat (note that this is not the net force on the passenger) is

$$F_{\text{seat}} = \sqrt{(mg)^2 + \left(\frac{mv^2}{R}\right)^2} = \sqrt{(833\ \text{N})^2 + (188\ \text{N})^2}$$

$$= 854\ \text{N}$$

(b) The magnitude of the force is the same, but the horizontal component is reversed.

5.111: a)



b) The upward friction force must be equal to the weight, so $\mu_s n = \mu_s m(4\pi^2 R/T^2) \ge mg$ and

$$\mu_s > \frac{gT^2}{4\pi^2 R} = \frac{(9.80\ \text{m/s}^2)\,(1\text{s}/0.60\ \text{rev})^2}{4\pi^2 (2.5\ \text{m})} = 0.28.$$

c) No; both the weight and the required normal force are proportional to the rider's mass.

5.112: a) For the tires not to lose contact, there must be a downward force on the tires. Thus, the (downward) acceleration at the top of the sphere must exceed *mg*, so $m\frac{v^2}{R} > mg$, and $v > \sqrt{gR} = \sqrt{(9.80\ \mathrm{m/s^2})\,(13.0\ \mathrm{m})} = 11.3\ \mathrm{m/s}$.

b) The (upward) acceleration will then be 4*g*, so the upward normal force must be $5\,mg = 5(110\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2}) = 5390\ \ \mathrm{N}$.

5.113: a) What really happens (according to a nosy observer on the ground) is that you slide closer to the passenger by turning to the right. b) The analysis is the same as that of Example 5.23. In this case, the friction force should be insufficient to provide the inward radial acceleration, and so $\mu_s mg < mv^2/R$, or

$$R < \frac{v^2}{\mu_s g} = \frac{(20\ \mathrm{m/s})^2}{(0.35)\,(9.80\ \mathrm{m/s^2})} = 120\ \mathrm{m}$$

to two places. Why the passenger is not wearing a seat belt is another question.

5.114: The tension *F* in the string must be the same as the weight of the hanging block, and must also provide the resultant force necessary to keep the block on the table in uniform circular motion; $Mg = F = m\frac{v^2}{r}$, so $v = \sqrt{gr\,M/m}$.

5.115: a) The analysis is the same as that for the conical pendulum of Example 5.22, and so

$$\beta = \arccos\left(\frac{gT^2}{4\pi^2 L}\right) = \arccos\left(\frac{(9.80\ \mathrm{m/s^2})(1/4.00\ \mathrm{s})^2}{4\pi^2(0.100\ \mathrm{m})}\right) = 81.0°.$$

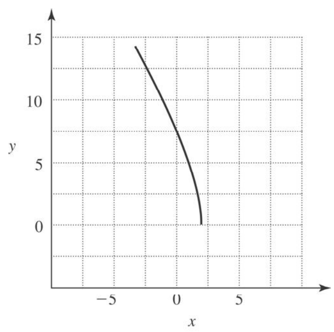
b) For the bead to be at the same elevation as the center of the hoop, $\beta = 90°$ and $\cos\beta = 0$, which would mean $T = 0$, the speed of the bead would be infinite, and this is not possible. c) The expression for $\cos\beta$ gives $\cos\beta = 2.48$, which is not possible. In deriving the expression for $\cos\beta$, a factor of $\sin\beta$ was canceled, precluding the possibility that $\beta = 0$. For this situation, $\beta = 0$ is the only physical possibility.

5.116: a) Differentiating twice, $a_x = -6\beta t$ and $a_y = -2\delta,$ so

$$F_x = ma_x = (2.20\ \text{kg})\,(-0.72\ \text{N/s})t = -(1.58\ \text{N/s})t$$

$$F_y = ma_y = (2.20\ \text{kg})\,(-2.00\ \text{m/s}^2) = -4.40\ \text{N}.$$

b)

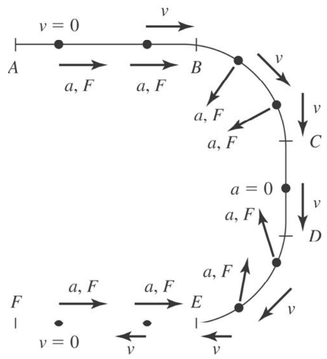


c) At $t = 3.00\ \text{s}, F_x = -4.75\ \text{N}$ and $F_y = -4.40\ \text{N},$ so

$$F = \sqrt{(-4.75\ \text{N})^2 + (-4.40\ \text{N})^2} = 6.48\ \text{N},$$

at an angle of $\arctan\left(\frac{-4.40}{-4.75}\right) = 223°.$

5.117:



5.118: See Example 5.25.

a) $F_A = m\left(g + \frac{v^2}{R}\right) = (1.60\ \text{kg})\left(9.80\ \text{m/s}^2 + \frac{(12.0\ \text{m/s})^2}{5.00\ \text{m}}\right) = 61.8\ \text{N}.$

b) $F_B = m\left(g - \frac{v^2}{R}\right) = (1.60\ \text{kg})\left(9.80\ \text{m/s}^2 - \frac{(12.0\ \text{m/s})^2}{5.00\ \text{m}}\right) = -30.4\ \text{N}.$, where the minus sign indicates that the track pushes *down* on the car. The magnitude of this force is 30.4 N.

5.119: The analysis is the same as for Problem 5.95; in the case of the cone, the speed is related to the period by $v = 2\pi R/T = 2\pi h \tan\beta/T$, or $T = 2\pi h \tan\beta/v$. The maximum and minimum speeds are the same as those found in Problem 5.95,

$$v_{\text{max}} = \sqrt{gh\tan\beta\frac{\cos\beta + \mu_s \sin\beta}{\sin\beta - \mu_s \cos\beta}}$$

$$v_{\text{min}} = \sqrt{gh\tan\beta\frac{\cos\beta - \mu_s \sin\beta}{\sin\beta + \mu_s \cos\beta}}.$$

The minimum and maximum values of the period T are then

$$T_{\text{min}} = 2\pi\sqrt{\frac{h\tan\beta}{g}\frac{\sin\beta - \mu_s \cos\beta}{\cos\beta + \mu_s \sin\beta}}$$

$$T_{\text{max}} = 2\pi\sqrt{\frac{h\tan\beta}{g}\frac{\sin\beta + \mu_s \cos\beta}{\cos\beta - \mu_s \sin\beta}}.$$

5.120: a) There are many ways to do these sorts of problems; the method presented is fairly straightforward in terms of application of Newton's laws, but involves a good deal of algebra. For both parts, take the *x*-direction to be horizontal and positive to the right, and the *y*-direction to be vertical and positive upward. The normal force between the block and the wedge is *n*; the normal force between the wedge and the horizontal surface will not enter, as the wedge is presumed to have zero vertical acceleration. The horizontal acceleration of the wedge is *A*, and the components of acceleration of the block are a_x and a_y. The equations of motion are then

$$MA = -n \sin \alpha$$

$$ma_x = n \sin \alpha$$

$$ma_y = n \cos \alpha - mg.$$

Note that the normal force gives the wedge a negative acceleration; the wedge is expected to move to the left. These are three equations in four unknowns, A, a_x, a_y and n. Solution is possible with the imposition of the relation between A, a_x and a_y.

An observer on the wedge is not in an inertial frame, and should not apply Newton's laws, but the kinematic relation between the components of acceleration are not so restricted. To such an observer, the vertical acceleration of the block is a_y, but the horizontal acceleration of the block is $a_x - A$. To this observer, the block descends at an angle α, so the relation needed is

$$\frac{a_y}{a_x - A} = -\tan \alpha.$$

At this point, algebra is unavoidable. Symbolic-manipulation programs may save some solution time. A possible approach is to eliminate a_x by noting that $a_x = -\frac{M}{m} A$ (a result that anticipates conservation of momentum), using this in the kinematic constraint to eliminate a_y and then eliminating *n*. The results are**:**

$$A = \frac{-gm}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_x = \frac{gM}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_y = \frac{-g(M+\mathrm{m})\tan\alpha}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

(b) When $M >> m, A \to 0,$ as expected (the large block won't move). Also, $a_x \to \frac{g}{\tan\alpha + (1/\tan\alpha)} = g\frac{\tan\alpha}{\tan^2\alpha + 1} = g\sin\alpha\cos\alpha,$ which is the acceleration of the block ($g\sin\alpha$ in this case), with the factor of $\cos\alpha$ giving the horizontal component. Similarly, $a_y \to -g\sin^2\alpha.$

(c) The trajectory is a spiral.

5.121: If the block is not to move vertically, the acceleration must be horizontal. The common acceleration is $a = g \tan\theta$, so the applied force must be $(M + m)a = (M + m)g \tan\theta$.

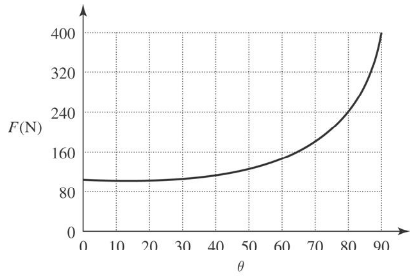
5.122: The normal force that the ramp exerts on the box will be $n = w \cos\alpha - T \sin\theta$. The rope provides a force of $T \cos\theta$ up the ramp, and the component of the weight down the ramp is $w \sin\alpha$. Thus, the net force up the ramp is

$$F = T\cos\theta - w\sin\alpha - \mu_k(w\cos\alpha - T\sin\theta)$$
$$= T(\cos\theta + \mu_k \sin\theta) - w(\sin\alpha + \mu_k \cos\alpha).$$

The acceleration will be the greatest when the first term in parantheses is greatest; as in Problems 5.77 and 5.123, this occurs when $\tan\theta = \mu_k$.

5.123: a) See Exercise 5.38; $F = \mu_k w/(\cos\theta + \mu_k \sin\theta)$.

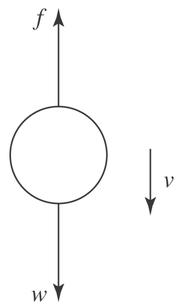
b)



c) The expression for F is a minimum when the denominator is a maximum; the calculus is identical to that of Problem 5.77 (maximizing w for a given F gives the same result as minimizing F for a given w), and so F is minimized at $\tan\theta = \mu_k$. For $\mu_k = 0.25, \theta = 14.0°$, keeping an extra figure.

5.124: For convenience, take the positive direction to be down, so that for the baseball released from rest, the acceleration and velocity will be positive, and the speed of the baseball is the same as its positive component of velocity. Then the resisting force, directed against the velocity, is upward and hence negative.

a)



b) Newton's Second Law is then $ma = mg - Dv^2$. Initially, when $v = 0$, the acceleration is g, and the speed increases. As the speed increases, the resistive force increases and hence the acceleration decreases. This continues as the speed approaches the terminal speed. c) At terminal velocity, $a = 0$, so $v_t = \sqrt{\frac{mg}{D}}$, in agreement with Eq. (5.13). d) The equation of motion may be rewritten as $\frac{dv}{dt} = \frac{g}{v_t^2}(v_t^2 - v^2)$. This is a separable equation and may be expressed as

$$\int \frac{dv}{v_t^2 - v^2} = \frac{g}{v_t^2}\int dt, \quad \text{or}$$

$$\frac{1}{v_t}\operatorname{arctanh}\left(\frac{v}{v_t}\right) = \frac{gt}{v_t^2},$$

so $v = v_t \tanh(gt/v_t)$.

Note**:** If inverse hyperbolic functions are unknown or undesirable, the integral can be done by partial fractions, in that

$$\frac{1}{v_t^2 - v^2} = \frac{1}{2v_t}\left[\frac{1}{v_t - v} + \frac{1}{v_t + v}\right],$$

and the resulting logarithms in the integrals can be solved for $v(t)$ in terms of exponentials.

5.125: Take all accelerations to be positive downward. The equations of motion are straightforward, but the kinematic relations between the accelerations, and the resultant algebra, are not immediately obvious. If the acceleration of pulley *B* is a_B, then $a_B = -a_3$, and a_B is the average of the accelerations of masses 1 and 2, or $a_1 + a_2 = 2a_B = -2a_3$. There can be no net force on the massless pulley *B*, so $T_C = 2T_A$. The five equations to be solved are then

$$m_1 g - T_A = m_1 a_1$$
$$m_2 g - T_A = m_2 a_2$$
$$m_3 g - T_C = m_3 a_3$$
$$a_1 + a_2 + 2a_3 = 0$$
$$2T_A - T_C = 0.$$

These are five equations in five unknowns, and may be solved by standard means. A symbolic-manipulation program is of great use here.

a) The accelerations a_1 and a_2 may be eliminated by using

$$2a_3 = -(a_1 + a_2) = -(2g - T_A((1/m_1) + (1/m_2))).$$

The tension T_A may be eliminated by using

$$T_A = (1/2)T_C = (1/2)m_3(g - a_3).$$

Combining and solving for a_3 gives

$$a_3 = g\frac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

b) The acceleration of the pulley *B* has the same magnitude as a_3 and is in the opposite direction.

c) $$a_1 = g - \frac{T_A}{m_1} = g - \frac{T_C}{2m_1} = g - \frac{m_3}{2m_1}(g - a_3).$$

Substituting the above expression for a_3 gives

$$a_1 = g\frac{4m_1m_2 - 3m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

d) A similar analysis (or, interchanging the labels 1 and 2) gives

$$a_2 = g\frac{4m_1m_2 - 3m_1m_3 + m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

e) & f) Once the accelerations are known, the tensions may be found by substitution into the appropriate equation of motion, giving

$$T_A = g\frac{4m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}, \quad \mathrm{T_C} = g\frac{8m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

g) If $m_1 = m_2 = m$ and $m_3 = 2m$, all of the accelerations are zero, $T_C = 2mg$ and $T_A = mg$. All masses and pulleys are in equilibrium, and the tensions are equal to the weights they support, which is what is expected.

5.126: In all cases, the tension in the string will be half of F.

a) $F/2 = 62\ \text{N}$, which is insufficient to raise either block; $a_1 = a_2 = 0$.

b) $F/2 = 62\ \text{N}$. The larger block (of weight 196 N) will not move, so $a_1 = 0$, but the smaller block, of weight 98 N, has a net upward force of 49 N applied to it, and so will accelerate upwards with $a_2 = \frac{49\ \text{N}}{10.0\ \text{kg}} = 4.9\ \text{m/s}^2$.

c) $F/2 = 212\ \text{N}$, so the net upward force on block A is 16 N and that on block B is 114 N, so $a_1 = \frac{16\ \text{N}}{20.0\ \text{kg}} = 0.8\ \text{m/s}^2$ and $a_2 = \frac{114\ \text{N}}{10.0\ \text{kg}} = 11.4\ \text{m/s}^2$.

5.127: Before the horizontal string is cut, the ball is in equilibrium, and the vertical component of the tension force must balance the weight, so $T_A \cos\beta = w$, or $T_A = w/\cos\beta$. At point B, the ball is not in equilibrium; its speed is instantaneously 0, so there is no radial acceleration, and the tension force must balance the radial component of the weight, so $T_B = w\cos\beta$, and the ratio $(T_B/T_A) = \cos^2\beta$.

6.1: a) $(2.40\,\text{N})\,(1.5\,\text{m}) = 3.60\,\text{J}$ b) $(-0.600\,\text{N})(1.50\,\text{m}) = -0.900\,\text{J}$
c) $3.60\,\text{J} - 0.720\,\text{J} = 2.70\,\text{J}$.

6.2: a) "Pulling slowly" can be taken to mean that the bucket rises at constant speed, so the tension in the rope may be taken to be the bucket's weight. In pulling a given length of rope, from Eq. (6.1),

$$W = Fs = mgs = (6.75\,\text{kg})\,(9.80\,\text{m/s}^2)(4.00\,\text{m}) = 264.6\,\text{J}.$$

b) Gravity is directed opposite to the direction of the bucket's motion, so Eq. (6.2) gives the negative of the result of part (a), or $-265\,\text{J}$. c) The net work done on the bucket is zero.

6.3: $(25.0\,\text{N})(12.0\,\text{m}) = 300\,\text{J}$.

6.4: a) The friction force to be overcome is

$$f = \mu_\text{k} n = \mu_\text{k} mg = (0.25)(30.0\,\text{kg})(9.80\,\text{m/s}^2) = 73.5\,\text{N},$$

or 74 N to two figures.

b) From Eq. (6.1), $Fs = (73.5\,\text{N})(4.5\,\text{m}) = 331\,\text{J}$. The work is positive, since the worker is pushing in the same direction as the crate's motion.

c) Since f and s are oppositely directed, Eq. (6.2) gives

$$-fs = -(73.5\,\text{N})(4.5\,\text{m}) = -331\,\text{J}.$$

d) Both the normal force and gravity act perpendicular to the direction of motion, so neither force does work. e) The net work done is zero.

6.5: a) See Exercise 5.37. The needed force is

$$F = \frac{\mu_k mg}{\cos\phi - \mu_k \sin\phi} = \frac{(0.25)(30\,\text{kg})(9.80\,\text{m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2\ \text{N},$$

keeping extra figures. b) $Fs\cos\phi = (99.2\ \text{N})(4.50\,\text{m})\cos 30° = 386.5\,\text{J}$, again keeping an extra figure. c) The normal force is $mg + F\sin\phi$, and so the work done by friction is $-(4.50\,\text{m})(0.25)((30\,\text{kg})(9.80\,\text{m/s}^2) + (99.2\ \text{N})\sin 30°) = -386.5\,\text{J}$. d) Both the normal force and gravity act perpendicular to the direction of motion, so neither force does work. e) The net work done is zero.

6.6: From Eq. (6.2),

$$Fs\cos\phi = (180\ \text{N})(300\,\text{m})\cos 15.0° = 5.22\times 10^4\ \text{J}.$$

6.7: $2Fs\cos\phi = 2(1.80\times10^6\ \text{N})(0.75\times10^3\ \text{m})\cos 14° = 2.62\times10^9\ \text{J}$, or $2.6\times10^9\ \text{J}$ to two places.

6.8: The work you do is:

$$\begin{aligned}\vec{F}\cdot\vec{s} &= ((30\text{N})\hat{i} - (40\text{N})\hat{j})\cdot((-9.0\text{m})\hat{i} - (3.0\text{m})\hat{j})\\ &= (30\ \text{N})(-9.0\ \text{m}) + (-40\ \text{N})(-3.0\ \text{m})\\ &= -270\ \text{N}\cdot\text{m} + 120\ \text{N}\cdot\text{m} = -150\ \text{J}\end{aligned}$$

6.9: a) (i) Tension force is always perpendicular to the displacement and does no work.
(ii) Work done by gravity is $-mg(y_2 - y_1)$. When $y_1 = y_2$, $W_{mg} = 0$.
b) (i) Tension does no work.
(ii) Let l be the length of the string. $W_{mg} = -mg(y_2 - y_1) = -mg(2l) = -25.1\,\text{J}$
The displacement is upward and the gravity force is downward, so it does negative work.

6.10: a) From Eq. (6.6),

$$K = \frac{1}{2}(1600\,\text{kg})\left((50.0\,\text{km/h})\left(\frac{1}{3.6}\frac{\text{m/s}}{\text{km/h}}\right)\right)^2 = 1.54\times10^5\ \text{J}.$$

b) Equation (6.5) gives the explicit dependence of kinetic energy on speed; doubling the speed of any object increases the kinetic energy by a factor of four.

6.11: For the T-Rex, $K = \frac{1}{2}(7000\,\text{kg})((4\,\text{km/hr})\frac{1\,\text{m/s}}{3.6\,\text{km/hr}})^2 = 4.32\times10^3\ \text{J}$. The person's velocity would be $v = \sqrt{2(4.32\times10^3\ \text{J})/70\,\text{kg}} = 11.1\,\text{m/s}$, or about 40 km/h.

6.12: (a) Estimate: $v \approx 1\text{m/s}$ (walking)

$v \approx 2\,\text{m/s}$ (running)

$m \approx 70\,\text{kg}$

Walking: $KE = \frac{1}{2}mv^2 = \frac{1}{2}(70\,\text{kg})(1\,\text{m/s})^2 = 35\,\text{J}$

Running: $KE = \frac{1}{2}(70\,\text{kg})(2\,\text{m/s})^2 = 140\,\text{J}$

(b) Estimate: $v \approx 60\,\text{mph} = 88\,\text{ft/s} \approx 30\,\text{m/s}$

$m \approx 2000\,\text{kg}$

$KE = \frac{1}{2}(2000\,\text{kg})(30\,\text{m/s})^2 = 9\times10^5\ \text{J}$

(c) $KE = W_{\text{gravity}} = mgh$ Estimate $h \approx 2\,\text{m}$

$KE = (1\,\text{kg})(9.8\,\text{m/s}^2)(2\,\text{m}) \approx 20\,\text{J}$

6.13: Let point 1 be at the bottom of the incline and let point 2 be at the skier.

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\ K_2 = 0$

Work is done by gravity and friction, so $W_{\text{tot}} = W_{mg} + W_f$.

$W_{mg} = -mg(y_2 - y_1) = -mgh$

$W_f = -fs = -(\mu_k mg\cos\alpha)(h/\sin\alpha) = -\mu_k mgh/\tan\alpha$

Substituting these expressions into the work-energy theorem and solving for v_0 gives

$v_0 = \sqrt{2gh(1+\mu_k/\tan\alpha)}$

6.14: (a)

$$\begin{aligned}W &= \Delta KE\\ -mgh &= \frac{1}{2}mv_{\mathrm{f}}^2 - \frac{1}{2}mv_0^2\\ v_0 &= \sqrt{v_{\mathrm{f}}^2 + 2gh}\\ &= \sqrt{(25.0\,\mathrm{m/s})^2 + 2(9.80\,\mathrm{m/s^2})(15.0\,\mathrm{m})}\\ &= 30.3\,\mathrm{m/s}\end{aligned}$$

(b)

$$\begin{aligned}W &= \Delta KE\\ -mgh &= \frac{1}{2}mv_{\mathrm{f}}^2 - \frac{1}{2}mv_0^2\\ h &= \frac{v_0^2 - v_{\mathrm{f}}^2}{2\mathrm{g}} = \frac{(30.3\,\mathrm{m/s})^2 - 0^2}{2(9.80\,\mathrm{ms/s^2})}\\ &= 46.8\,\mathrm{m}\end{aligned}$$

6.15: a) parallel to incline: force component $= mg\sin\alpha$, down incline; displacement $= h/\sin\alpha$, down incline

$W_{\|} = (mg\sin\alpha)(h/\sin\alpha) = mgh$ perpendicular to incline: no displacement in this direction, so $W_{\perp} = 0$.

$W_{mg} = W_{\|} + W_{\perp} = mgh$, same as falling height *h*.

b) $W_{\mathrm{tot}} = K_2 - K_1$ gives $mgh = \frac{1}{2}mv^2$ and $v = \sqrt{2gh}$, same as if had been dropped from height *h*. The work done by gravity depends only on the vertical displacement of the object.

When the slope angle is small, there is a small force component in the direction of the displacement but a large displacement in this direction. When the slope angle is large, the force component in the direction of the displacement along the incline is larger but the displacement in this direction is smaller.

c) $h = 15.0\,\mathrm{m}$, so $v = \sqrt{2gh} = 17.1\,\mathrm{s}$

6.16: Doubling the speed increases the kinetic energy, and hence the magnitude of the work done by friction, by a factor of four. With the stopping force given as being independent of speed, the distance must also increase by a factor of four.

6.17: Barring a balk, the initial kinetic energy of the ball is zero, and so

$$W = (1/2)mv^2 = (1/2)(0.145\,\mathrm{kg})(32.0\,\mathrm{m/s})^2 = 74.2\,\mathrm{J}.$$

6.18: As the example explains, the boats have the same kinetic energy K at the finish line, so $(1/2)m_A v_A^2 = (1/2)m_B v_B^2$, or, with $m_B = 2m_A, v_A^2 = 2v_B^2$. a) Solving for the ratio of the speeds, $v_A / v_B = \sqrt{2}$. b) The boats are said to start from rest, so the elapsed time is the distance divided by the average speed. The ratio of the average speeds is the same as the ratio of the final speeds, so the ratio of the elapsed times is $t_B / t_A = v_A / v_B = \sqrt{2}$.

6.19: a) From Eq. (6.5), $K_2 = K_1 / 16$, and from Eq. (6.6), $W = -(15/16)K_1$. b) No; kinetic energies depend on the magnitudes of velocities only.

6.20: From Equations (6.1), (6.5) and (6.6), and solving for F,

$$F = \frac{\Delta K}{s} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{s} = \frac{\frac{1}{2}(8.00\,\text{kg})((6.00\,\text{m/s})^2 - (4.00\,\text{m/s})^2)}{(2.50\,\text{m})} = 32.0\,\text{N}.$$

6.21:

$$s = \frac{\Delta K}{F} = \frac{\frac{1}{2}(0.420\,\text{kg})((6.00\,\text{m/s})^2 - (2.00\,\text{m/s})^2)}{(40.0\,\text{N})} = 16.8\,\text{cm}$$

6.22: a) If there is no work done by friction, the final kinetic energy is the work done by the applied force, and solving for the speed,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(36.0\,\text{N})(1.20\,\text{m})}{(4.30\,\text{kg})}} = 4.48\,\text{m/s}.$$

b) The net work is $Fs - f_\text{k}s = (F - \mu_\text{k}mg)s$, so

$$\begin{aligned} v &= \sqrt{\frac{2(F - \mu_\text{k}mg)s}{m}} \\ &= \sqrt{\frac{2(36.0\,\text{N} - (0.30)(4.30\,\text{kg})(9.80\,\text{m/s}^2))(1.20\,\text{m})}{(4.30\,\text{kg})}} \\ &= 3.61\,\text{m/s}. \end{aligned}$$

(Note that even though the coefficient of friction is known to only two places, the difference of the forces is still known to three places.)

6.23: a) On the way up, gravity is opposed to the direction of motion, and so $W = -mgs = -(0.145\,\text{kg})(9.80\,\text{m/s}^2)(20.0\,\text{m}) = -28.4\,\text{J}$.

b) $$v_2 = \sqrt{v_1^2 + 2\frac{W}{m}} = \sqrt{(25.0\,\text{m/s})^2 + \frac{2(-28.4\,\text{J})}{(0.145\,\text{kg})}} = 15.26\,\text{m/s}.$$

c) No; in the absence of air resistance, the ball will have the same speed on the way down as on the way up. On the way down, gravity will have done both negative and positive work on the ball, but the net work will be the same.

6.24: a) Gravity acts in the same direction as the watermelon's motion, so Eq. (6.1) gives

$$W = Fs = mgs = (4.80\,\text{kg})(9.80\,\text{m/s}^2)(25.0\,\text{m}) = 1176\,\text{J}.$$

b) Since the melon is released from rest, $K_1 = 0$, and Eq. (6.6) gives

$$K = K_2 = W = 1176\,\text{J}.$$

6.25: a) Combining Equations (6.5) and (6.6) and solving for v_2 algebraically,

$$v_2 = \sqrt{v_1^2 + 2\frac{W_{\text{tot}}}{m}} = \sqrt{(4.00\,\text{m/s})^2 + \frac{2(10.0\,\text{N})(3.0\,\text{m})}{(7.00\,\text{kg})}} = 4.96\,\text{m/s}.$$

Keeping extra figures in the intermediate calculations, the acceleration is $a = (10.0\,\text{kg}\cdot\text{m/s}^2)/(7.00\,\text{kg}) = 1.429\,\text{m/s}^2$. From Eq. (2.13), with appropriate change in notation,

$$v_2^2 = v_1^2 + 2as = (4.00\,\text{m/s})^2 + 2(1.429\,\text{m/s}^2)(3.0\,\text{m}),$$

giving the same result.

6.26: The normal force does no work. The work-energy theorem, along with Eq. (6.5), gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2W}{m}} = \sqrt{2gh} = \sqrt{2gL\sin\theta},$$

where $h = L\sin\theta$ is the vertical distance the block has dropped, and θ is the angle the plane makes with the horizontal. Using the given numbers,

$$v = \sqrt{2(9.80\,\text{m/s}^2)(0.75\,\text{m})\sin 36.9^\circ} = 2.97\,\text{m/s}.$$

6.27: a) The friction force is $\mu_k mg$, which is directed against the car's motion, so the net work done is $-\mu_k mgs$. The change in kinetic energy is $\Delta K = -K_1 = -(1/2)mv_0^2$, and so $s = v_0^2 / 2\mu_k g$. b) From the result of part (a), the stopping distance is proportional to the square of the initial speed, and so for an initial speed of 60 km/h, $s = (91.2\,\text{m})(60.0/80.0)^2 = 51.3\,\text{m}$. (This method avoids the intermediate calculation of μ_k, which in this case is about 0.279.)

6.28: The intermediate calculation of the spring constant may be avoided by using Eq. (6.9) to see that the work is proportional to the square of the extension; the work needed to compress the spring 4.00 cm is $(12.0\,\text{J})\left(\frac{4.00\text{ cm}}{3.00\text{ cm}}\right)^2 = 21.3\,\text{J}$.

6.29: a) The magnitude of the force is proportional to the magnitude of the extension or compression;

$$(160\,\text{N})(0.015\,\text{m}/0.050\,\text{m}) = 48\,\text{N}, \qquad (160\,\text{N})(0.020\,\text{m}/0.050\,\text{m}) = 64\,\text{N}.$$

b) There are many equivalent ways to do the necessary algebra. One way is to note that to stretch the spring the original 0.050 m requires $\frac{1}{2}\left(\frac{169\text{ N}}{0.050\text{ m}}\right) = (0.050\,\text{m})^2 = 4\,\text{J}$, so that stretching 0.015 m requires $(4\,\text{J})(0.015/0.050)^2 = 0.360\,\text{J}$ and compressing 0.020 m requires $(4\,\text{J})(0.020/0.050)^2 = 0.64\,\text{J}$. Another is to find the spring constant $k = (160\,\text{N}) \div (0.050\,\text{m}) = 3.20\times10^3\ \text{N/m}$, from which $(1/2)(3.20\times10^3\ \text{N/m})(0.015\,\text{m})^2 = 0.360\,\text{J}$ and $(1/2)(3.20\times10^3\ \text{N/m})(0.020\,\text{m})^2 = 0.64\,\text{J}$.

6.30: The work can be found by finding the area under the graph, being careful of the sign of the force. The area under each triangle is 1/2 $\text{base}\times\text{height}$.

a) $1/2\,(8\,\text{m})(10\,\text{N}) = 40\,\text{J}$.
b) $1/2\,(4\,\text{m})(10\,\text{N}) = +20\,\text{J}$.
c) $1/2\,(12\,\text{m})(10\,\text{N}) = 60\,\text{J}$.

6.31: Use the Work-Energy Theorem and the results of Problem 6.30.

a) $v = \sqrt{\dfrac{(2)(40\,\text{J})}{10\,\text{kg}}} = 2.83\,\text{m/s}$

b) At $x = 12\,\text{m}$, the 40 Joules of kinetic energy will have been increased by 20 J, so $v = \sqrt{\dfrac{(2)(60\,\text{J})}{10\,\text{kg}}} = 3.46\,\text{m/s}$.

6.32: The work you do with your changing force is

$$\int_0^{6.9} F(x)\,dx = \int_0^{6.9} (-20.0\ \text{N})\,dx - \int_0^{6.9} 3.0\,\frac{\text{N}}{\text{m}}\,x\,dx$$

$$= (-20.0\ \text{N})x\,|_0^{6.9} - (3.0\frac{\text{N}}{\text{m}})(x^2/2)\,|_0^{6.9}$$

$$= -138\ \text{N}\cdot\text{m} - 71.4\ \text{N}\cdot\text{m} = -209.4\ \text{J} \quad \text{or} \quad -209\ \text{J}$$

The work is negative because the cow continues to advance as you vainly attempt to push her backward.

6.33: $W_{\text{tot}} = K_2 - K_1$

$$K_1 = \frac{1}{2}mv_0^2, \quad K_2 = 0$$

Work is done by the spring force. $W_{\text{tot}} = -\frac{1}{2}kx^2$, where x is the amount the spring is compressed.

$$-\frac{1}{2}kx^2 = -\frac{1}{2}mv_0^2 \quad \text{and} \quad x = v_0\sqrt{m/k} = 8.5\,\text{cm}$$

6.34: a) The average force is $(80.0\ \text{J})/(0.200\ \text{m}) = 400\ \text{N}$, and the force needed to hold the platform in place is twice this, or 800 N. b) From Eq. (6.9), doubling the distance quadruples the work so an extra 240 J of work must be done. The maximum force is quadrupled, 1600 N.

Both parts may of course be done by solving for the spring constant $k = 2(80.0\ \text{J}) \div (0.200\ \text{m})^2 = 4.00 \times 10^3\ \text{N/m}$, giving the same results.

6.35: a) The static friction force would need to be equal in magnitude to the spring force, $\mu_s mg = kd$ or $\mu_s = \frac{(20.0 \text{ N/m})(0.086 \text{ m})}{(0.100 \text{ kg})(9.80 \text{ m/s}^2)} = 1.76$, which is quite large. (Keeping extra figures in the intermediate calculation for *d* gives a different answer.) b) In Example 6.6, the relation

$$\mu_k mgd + \frac{1}{2}kd^2 = \frac{1}{2}mv_1^2$$

was obtained, and *d* was found in terms of the known initial speed v_1. In this case, the condition on *d* is that the static friction force at maximum extension just balances the spring force, or $kd = \mu_s mg$. Solving for v_1^2 and substituting,

$$\begin{aligned} v_1^2 &= \frac{k}{m}d^2 + 2gd\mu_k d \\ &= \frac{k}{m}\left(\frac{\mu_s mg}{k}\right)^2 + 2\mu_k g\left(\frac{\mu_s mg}{k}\right) \\ &= \frac{mg^2}{k}(\mu_s^2 + 2\mu_s\mu_k) \\ &= \left(\frac{(0.10\,\text{kg})(9.80\,\text{m/s}^2)^2}{(20.0\,\text{N/m})}\right)((0.60)^2 + 2(0.60)(0.47)), \end{aligned}$$

from which $v_1 = 0.67\,\text{m/s}$.

6.36: a) The spring is pushing on the block in its direction of motion, so the work is positive, and equal to the work done in compressing the spring. From either Eq. (6.9) or Eq. (6.10), $W = \frac{1}{2}kx^2 = \frac{1}{2}(200\,\text{N/m})(0.025\,\text{m})^2 = 0.06\,\text{J}$.

b) The work-energy theorem gives

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(0.06\,\text{J})}{(4.0\,\text{kg})}} = 0.18\,\text{m/s}.$$

6.37: The work done in any interval is the area under the curve, easily calculated when the areas are unions of triangles and rectangles. a) The area under the trapezoid is $4.0\,\text{N}\cdot\text{m} = 4.0\,\text{J}$. b) No force is applied in this interval, so the work done is zero. c) The area of the triangle is $1.0\,\text{N}\cdot\text{m} = 1.0\,\text{J}$, and since the curve is *below* the axis $(F_x < 0)$, the work is negative, or $-1.0\,\text{J}$. d) The net work is the sum of the results of parts (a), (b) and (c), 3.0 J. (e) $+1.0\,\text{J} - 2.0\,\text{J} = -1.0\,\text{J}$.

6.38: a) $K = 4.0\,\text{J}$, so $v = \sqrt{2K/m} = \sqrt{2(4.0\,\text{J})/(2.0\,\text{kg})} = 2.00\,\text{m/s}$. b) No work is done between $x = 3.0\,\text{m}$ and $x = 4.0\,\text{m}$, so the speed is the same, 2.00 m/s. c) $K = 3.0\,\text{J}$, so $v = \sqrt{2K/m} = \sqrt{2(3.0\,\text{J})/(2.0\,\text{kg})} = 1.73\,\text{m/s}$.

6.39: a) The spring does positive work on the sled and rider; $(1/2)kx^2 = (1/2)mv^2$, or $v = x\sqrt{k/m} = (0.375\,\text{m})\sqrt{(4000\,\text{N/m})/(70\,\text{kg})} = 2.83\,\text{m/s}$. b) The net work done by the spring is $(1/2)k(x_1^2 - x_2^2)$, so the final speed is

$$v = \sqrt{\frac{k}{m}(x_1^2 - x_2^2)} = \sqrt{\frac{(4000\,\text{N/m}}{(70\,\text{kg})}((0.375\,\text{m})^2 - (0.200\,\text{m})^2)} = 2.40\,\text{m/s}.$$

6.40: a) From Eq. (6.14), with $dl = Rd\phi$,

$$W = \int_{P_1}^{P_2} F\cos\phi\, dl = 2wR\int_0^{\theta_0} \cos\phi\, d\phi = 2wR\sin\theta_0.$$

In an equivalent geometric treatment, when $\vec{F}$ is horizontal, $\vec{F}\cdot d\vec{l} = Fdx$, and the total work is $F = 2w$ times the horizontal distance, in this case (see Fig. 6.20(a)) $R\sin\theta_0$, giving the same result. b) The ratio of the forces is $\frac{2w}{w\tan\theta_0} = 2\cot\theta_0$.

c)
$$\frac{2wR\sin\theta_0}{wR(1-\cos\theta_0)} = 2\frac{\sin\theta_0}{(1-\cos\theta_0)} = 2\cot\frac{\theta_0}{2}.$$

6.41: a) The initial and final (at the maximum distance) kinetic energy is zero, so the positive work done by the spring, $(1/2)kx^2$, must be the opposite of the negative work done by gravity, $-mgL\sin\theta$, or

$$x = \sqrt{\frac{2mgL\sin\theta}{k}} = \sqrt{\frac{2(0.0900\,\text{kg})(9.80\,\text{m/s}^2)(1.80\,\text{m})\sin 40.0°}{(640\,\text{N/m})}} = 5.7\,\text{cm}.$$

At this point the glider is no longer in contact with the spring. b) The intermediate calculation of the initial compression can be avoided by considering that between the point 0.80 m from the launch to the maximum distance, gravity does a negative amount of work given by $-(0.0900\,\text{kg})(9.80\,\text{m/s}^2)(1.80\,\text{m} - 0.80\,\text{m})\sin 40.0° = -0.567\,\text{J}$, and so the kinetic energy of the glider at this point is 0.567 J. At this point the glider is no longer in contact with the spring.

6.42: The initial and final kinetic energies of the brick are both zero, so the net work done on the brick by the spring and gravity is zero, so $(1/2)kd^2 - mgh = 0$, or $d = \sqrt{2mgh/k} = \sqrt{2(1.80\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(3.6\,\mathrm{m})/(450\,\mathrm{N/m})} = 0.53\,\mathrm{m}$. The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero, the spring is at its uncompressed length.

6.43: $\text{Energy} = (\text{power})(\text{time}) = (100\,\mathrm{W})(3600\,\mathrm{s}) = 3.6\times10^5\ \mathrm{J}$

$$K = \frac{1}{2}mv^2 \quad \text{so} \quad v = \sqrt{2K/m} = 100\,\mathrm{s} \quad \text{for} \quad m = 70\,\mathrm{kg}.$$

6.44: Set time to stop:

$$\begin{aligned}
\Sigma F &= ma : \mu_\mathrm{k} mg = ma \\
a &= \mu_\mathrm{k} g = (0.200)(9.80\,\mathrm{m/s^2}) = 1.96\,\mathrm{m/s^2} \\
v &= v_0 + at \\
0 &= 8.00\,\mathrm{m/s} - (1.96\,\mathrm{m/s^2})t \\
t &= 4.08\,\mathrm{s} \\
P &= \frac{KE}{t} = \frac{\frac{1}{2}mv^2}{t} \\
&= \frac{\frac{1}{2}(20.0\,\mathrm{kg})(8.00\,\mathrm{m/s^2})}{4.08\,\mathrm{s}} = 157\,\mathrm{W}
\end{aligned}$$

6.45: The total power is $(165\,\mathrm{N})(9.00\,\mathrm{m/s}) = 1.485\times10^3\ \mathrm{W}$, so the power per rider is 742.5 W, or about 1.0 hp (which is a very large output, and cannot be sustained for long periods).

6.46: a) $$\frac{(1.0\times10^{19}\ \mathrm{J/yr})}{(3.16\times10^7\ \mathrm{s/yr})} = 3.2\times10^{11}\ \mathrm{W}.$$

b) $$\frac{3.2\times10^{11}\ \mathrm{W}}{2.6\times10^8\ \text{folks}} = 1.2\,\text{kW/person}.$$

c) $$\frac{3.2\times10^{11}\ \mathrm{W}}{(0.40)1.0\times10^3\ \mathrm{W/m^2}} = 8.0\times10^8\,\mathrm{m^2} = 800\,\mathrm{km^2}.$$

6.47: The power is $P = F \cdot v$. F is the weight, mg, so $P = (700\ \text{kg})\,(9.8\ \text{m/s}^2)\,(2.5\ \text{m/s}) = 17.15\ \text{kW}.$ So, $17.15\ \text{kW}/75\ \text{kW}. = 0.23,$ or about 23% of the engine power is used in climbing.

6.48: a) The number per minute would be the average power divided by the work (*mgh*) required to lift one box,

$$\frac{(0.50\ \text{hp})\,(746\ \text{W/hp})}{(30\ \text{kg})\,(9.80\ \text{m/s}^2)\,(0.90\ \text{m})} = 1.41\,/\text{s},$$

or $84.6\,/\text{min}.$ b) Similarly,

$$\frac{(100\ \text{W})}{(30\ \text{kg})\,(9.80\ \text{m/s}^2)\,(0.90\ \text{m})} = 0.378\,/\text{s},$$

or $22.7\,/\text{min}.$

6.49: The total mass that can be raised is

$$\frac{(40.0\ \text{hp})\,(746\ \text{W/hp})(16.0\ \text{s})}{(9.80\ \text{m/s}^2)\,(20.0\ \text{m})} = 2436\ \text{kg},$$

so the maximum number of passengers is $\frac{1836\ \text{kg}}{65.0\ \text{kg}} = 28.$

6.50: From any of Equations (6.15), (6.16), (6.18) or (6.19),

$$P = \frac{Wh}{t} = \frac{(3800\ \text{N})\,(2.80\ \text{m})}{(4.00\ \text{s})} = 2.66\times10^3\ \text{W} = 3.57\ \text{hp}.$$

6.51: $F = \dfrac{(0.70)\,P_{\text{ave}}}{v} = \dfrac{(0.70)\,(280{,}000\ \text{hp})(746\ \text{W/hp})}{(65\ \text{km/h})\,((1\ \text{km/h})/(3.6\ \text{m/s}))} = 8.1\times10^6\ \text{N}.$

6.52: Here, Eq. (6.19) is the most direct. Gravity is doing negative work, so the rope must do positive work to lift the skiers. The force $\vec{F}$ is gravity, and $F = Nmg,$ where N is the number of skiers on the rope. The power is then

$$\begin{aligned} P &= (Nmg)\,(v)\cos\phi \\ &= (50)\,(70\ \text{kg})\,(9.80\ \text{m/s}^2)\,(12.0\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\cos(90.0^\circ - 15.0^\circ) \\ &= 2.96\times10^4\ \text{W}. \end{aligned}$$

Note that Eq. (1.18) uses ϕ as the angle between the force and velocity vectors; in this case, the force is vertical, but the angle 15.0° is measured from the horizontal, so $\phi = 90.0^\circ - 15.0^\circ$ is used.

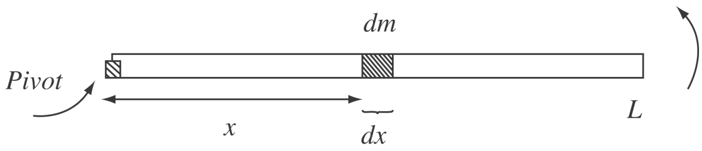
6.53: a) In terms of the acceleration a and the time t since the force was applied, the speed is $v = at$ and the force is ma, so the power is $P = Fv = (ma)(at) = ma^2t$. b) The power at a given time is proportional to the square of the acceleration, tripling the acceleration would mean increasing the power by a factor of nine. c) If the magnitude of the net force is the same, the acceleration will be the same, and the needed power is proportional to the time. At $t = 15.0\ \text{s}$, the needed power is three times that at 5.0 s, or 108 W.

6.54:

$$\begin{aligned}\frac{dK}{dt} &= \frac{d}{dt}\left(\frac{1}{2}mv^2\right)\\ &= mv\frac{dv}{dt}\\ &= mva = mav\\ &= Fv = P.\end{aligned}$$

6.55: Work done in each stroke is $W = Fs$ and $P_{\text{av}} = W/t = 100Fs/t$ $t = 1.00\ \text{s}$, $F = 2mg$ and $s = 0.010\ \text{m}$. $P_{\text{av}} = 0.20\ \text{W}$.

6.56:



Let M = total mass and T = time for one revolution

$$KE = \int \frac{1}{2}(dm)v^2$$

$$dm = \frac{M}{L}dx$$

$$v = \frac{2\pi x}{T}$$

$$KE = \int_0^L \frac{1}{2}\left(\frac{M}{L}dx\right)\left(\frac{2\pi x}{T}\right)^2$$

$$= \frac{1}{2}\left(\frac{M}{L}\right)\left(\frac{4\pi^2}{T^2}\right)\int_0^L x^2 dx$$

$$= \frac{1}{2}\left(\frac{M}{L}\right)\left(\frac{4\pi^2}{T^2}\right)\left(\frac{L^3}{3}\right) = \frac{2}{3}\pi^2 ML^2/T^2$$

5 revolutions in 3 seconds $\rightarrow T = 3/5\,\text{s}$

$$KE = \frac{2}{3}\pi^2(12.0\text{ kg})\,(2.00\text{ m})^2/(3/5\text{ s})^2 = 877\text{ J.}$$

6.57: a) $(140\text{ N})\,(3.80\text{ m}) = 532\text{ J}$ b) $(20.0\text{ kg})\,(9.80\text{ m/s}^2)\,(3.80\text{ m})\,(-\sin 25°) = -315\text{ J}$
c) The normal force does no work.
d)

$$W_f = -f_k s = -\mu_k ns = -\mu_k mgs\cos\theta$$

$$= -(0.30)\,(20.0\text{ kg})\,(9.80\text{ m/s}^2)\,(3.80\text{ m})\cos 25° = -203\text{ J}$$

e) $532\text{ J} - 315\text{ J} - 203\text{ J} = 15\text{ J}$ (14.7 J to three figures).

f) The result of part (e) is the kinetic energy at the top of the ramp, so the speed is $v = \sqrt{2K/m} = \sqrt{2(14.7\text{ J})/(20.0\text{ kg})} = 1.21\text{ m/s}$.

6.58: The work per unit mass is $(W/m) = gh.$

a) The man does work, $(9.8\ \text{N/kg})\,(0.4\ \text{m}) = 3.92\ \text{J/kg}.$

b) $(3.92\ \text{J/kg})/(70\ \text{J/kg}) \times 100 = 5.6\%.$

c) The child does work, $(9.8\ \text{N/kg})\,(0.2\ \text{m}) = 1.96\ \text{J/kg}.$

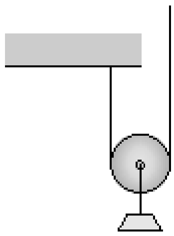
$(1.96\ \text{J/kg})/(70\ \text{J/kg}) \times 100 = 2.8\%.$

d) If both the man and the child can do work at the rate of $70\ \text{J/kg},$ and if the child only needs to use $1.96\ \text{J/kg}$ instead of $3.92\ \text{J/kg},$ the child should be able to do more pull ups.

6.59: a) Moving a distance L along the ramp, $s_{\text{in}} = L,\ s_{\text{out}} = L\sin\alpha,$ so $IMA = \frac{1}{\sin\alpha}.$

b) If $AMA = IMA, (F_{\text{out}}/F_{\text{in}}) = (s_{\text{in}}/s_{\text{out}})$ and so $(F_{\text{out}})\,(s_{\text{out}}) = (F_{\text{in}})\,(s_{\text{in}})$, or $W_{\text{out}} = W_{\text{in}}.$

c)



d)

$$E = \frac{W_{\text{out}}}{W_{\text{in}}} = \frac{(F_{\text{out}})(s_{\text{out}})}{(F_{\text{in}})(s_{\text{in}})} = \frac{F_{\text{out}}/F_{\text{in}}}{s_{\text{in}}/s_{\text{out}}} = \frac{AMA}{IMA}.$$

6.60: a) $m = \dfrac{w}{g} = \dfrac{-W_g/s}{g} = \dfrac{(7.35\times10^3\ \text{J})}{(9.80\ \text{m/s}^2)\,(18.0\ \text{m})} = 41.7\ \text{kg}.$

b) $n = \dfrac{W_n}{s} = \dfrac{8.25\times10^3\ \text{J}}{18.0\ \text{m}} = 458\ \text{N}.$

c) The weight is $mg = \frac{W_g}{s} = 408\ \text{N},$ so the acceleration is the net force divided by the mass, $\dfrac{458\ \text{N} - 408\ \text{N}}{41.7\ \text{kg}} = 1.2\ \text{m/s}^2.$

6.61: a)

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2\pi R}{T}\right)^2 = \frac{1}{2}(86{,}400\ \text{kg})\left(\frac{2\pi(6.66\times10^6\ \text{m})}{(90.1\ \text{min})\,(60\ \text{s/min})}\right)^2 = 2.59\times10^{12}\ \text{J}.$$

b) $(1/2)\,mv^2 = (1/2)\,(86{,}400\ \text{kg})\,((1.00\ \text{m})/(3.00\ \text{s}))^2 = 4.80\times10^3\ \text{J}.$

6.62: a)

$$W_f = -f_k s = -\mu_k mg \cos\theta s$$

$$= -(0.31)\,(5.00\text{ kg})\,(9.80\text{ m/s}^2)\cos 12.0°(1.50\text{ m}) = -22.3\text{ J}$$

(keeping an extra figure) b) $(5.00\text{ kg})\,(9.80\text{ m/s}^2)\sin 12.0°\,(1.50\text{ m}) = 15.3$ J.
c) The normal force does no work. d) $15.3\text{ J} - 22.3\text{ J} = -7.0$ J.
e) $K_2 = K_1 + W = (1/2)\,(5.00\text{ kg})\,(2.2\text{ m/s})^2 - 7.0\text{ J} = 5.1\text{ J}$, and so $v_2 = \sqrt{2(5.1\text{ J})/(5.00\text{ kg})} = 1.4\text{ m/s}$.

6.63: See Problem 6.62: The work done is negative, and is proportional to the distance s that the package slides along the ramp, $W = mg(\sin\theta - \mu_k\cos\theta)s$. Setting this equal to the (negative) change in kinetic energy and solving for s gives

$$s = -\frac{(1/2)mv_1^2}{mg(\sin\theta - \mu_k\cos\theta)} = \frac{v_1^2}{2g(\sin\theta - \mu_k\cos\theta)}$$

$$= \frac{(2.2\text{ m/s})^2}{2(9.80\text{ m/s}^2)(\sin 12° - (0.31)\cos 12°)} = 2.6\text{ m}.$$

As a check of the result of Problem 6.62, $(2.2\text{ m/s})\sqrt{1-(1.5\text{ m})/(2.6\text{ m})} = 1.4\text{ m/s}$.

6.64: a) From Eq. (6.7),

$$W = \int_{x_1}^{x_2} F_x dx = -k\int_{x_1}^{x_2}\frac{dx}{x^2} = -k\left[-\frac{1}{x}\right]_{x_1}^{x_2} = k\left(\frac{1}{x_2} - \frac{1}{x_1}\right).$$

The force is given to be attractive, so $F_x < 0$, and k must be positive. If $x_2 > x_1$, $\frac{1}{x_2} < \frac{1}{x_1}$, and $W < 0$. b) Taking "slowly" to be constant speed, the net force on the object is zero, so the force applied by the hand is opposite F_x, and the work done is negative of that found in part (a), or $k\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$, which is positive if $x_2 > x_1$. c) The answers have the same magnitude but opposite signs; this is to be expected, in that the net work done is zero.

6.65: $F = mg(R_E/r)^2$

$$W = -\int_1^2 F ds = -\int_\infty^{R_E}\left(\frac{mgR_E^2}{r^2}\right)dr = -mgR_E^2(-(1/r)\,|_\infty^{R_E}) = mgR_E$$

$W_{tot} = K_2 - K_1,\ K_1 = 0$

This gives $K_2 = mgR_E = 1.25\times10^{12}$ J

$K_2 = \frac{1}{2}mv_2^2$ so $v_2 = \sqrt{2K_2/m} = 11{,}000\text{ m/s}$

6.66: Let x be the distance past P.

$$\mu_{\mathrm{k}} = 0.100 + Ax$$

when $x = 12.5\,\mathrm{m}$, $\mu_{\mathrm{k}} = 0.600$

$$A = 0.500/12.5\,\mathrm{m} = 0.0400/\mathrm{m}$$

(a)

$$W = \Delta KE : W_{\mathrm{f}} = KE_{\mathrm{f}} - KE_{\mathrm{i}}$$

$$-\int \mu_{\mathrm{k}} mg dx = 0 - \frac{1}{2} m v_{\mathrm{i}}^2$$

$$g \int_0^{x_{\mathrm{f}}} (0.100 + Ax) dx = \frac{1}{2} v_{\mathrm{i}}^2$$

$$g \left[(0.100) x_{\mathrm{f}} + A \frac{x_{\mathrm{f}}^2}{2} \right] = \frac{1}{2} v_{\mathrm{i}}^2$$

$$(9.80\,\mathrm{m/s^2}) \left[(0.100) x_{\mathrm{f}} + (0.0400/\mathrm{m}) \frac{x_{\mathrm{f}}^2}{2} \right] = \frac{1}{2} (4.50\,\mathrm{m/s})^2$$

Solve for $x_{\mathrm{f}} : x_{\mathrm{f}} = 5.11\mathrm{m}$

(b) $\mu_{\mathrm{k}} = 0.100 + (0.0400/\mathrm{m})(5.11\,\mathrm{m}) = 0.304$

(c) $W_{\mathrm{f}} = KE_{\mathrm{f}} - KE_{\mathrm{i}}$

$$-\mu_{\mathrm{k}} mgx = 0 - \frac{1}{2} m v_1^2$$

$$x = v_{\mathrm{i}}^2 / 2\mu_{\mathrm{k}} g = \frac{(4.50\,\mathrm{m/s})^2}{2(0.100)(9.80\,\mathrm{m/s^2})} = 10.3\,\mathrm{m}$$

6.67: a) $\alpha x_a^3 = (4.00\ \mathrm{N/m^3})(1.00\,\mathrm{m})^3 = 4.00\,\mathrm{N}$.

b) $\alpha x_b^3 = (4.00\ \mathrm{N/m^3})(2.00\,\mathrm{m})^3 = 32.0\,\mathrm{N}$. c) Equation 6.7 gives the work needed to move an object against the force; the work done by the force is the negative of this,

$$-\int_{x_1}^{x_2} \alpha x^3 dx = -\frac{\alpha}{4}(x_2^4 - x_1^4).$$

With $x_1 = x_a = 1.00\,\mathrm{m}$ and $x_2 = x_b = 2.00\,\mathrm{m}$, $W = -15.0\,\mathrm{J}$, this work is negative.

6.68: From Eq. (6.7), with $x_1 = 0$,

$$W = \int_0^{x_2} F dx = \int_0^{x_2} (kx - bx^2 + cx^3) dx = \frac{k}{2} x_2^2 - \frac{b}{3} x_2^3 + \frac{c}{4} x_2^4$$
$$= (50.0\ \mathrm{N/m}) x_2^2 - (233\ \mathrm{N/m^2}) x_2^3 + (3000\ \mathrm{N/m^3}) x_2^4$$

a) When $x_2 = 0.050\ \mathrm{m}$, $W = 0.115\ \mathrm{J}$, or 0.12 J to two figures. b) When $x_2 = -0.050\ \mathrm{m}$, $W = 0.173\ \mathrm{J}$, or 0.17 J to two figures. c) It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the $-x$-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

6.69: a) $T = ma_{\mathrm{rad}} = m\frac{v^2}{R} = (0.120\mathrm{kg})\frac{(0.70\ \mathrm{m/s})^2}{(0.40\ \mathrm{m})} = 0.147\ \mathrm{N}$, or 0.15 N to two figures. b) At the later radius and speed, the tension is $(0.120\mathrm{kg})\frac{(2.80\ \mathrm{m/s})^2}{(0.10\ \mathrm{m})} = 9.41\ \mathrm{N}$, or 9.4 N to two figures. c) The surface is frictionless and horizontal, so the net work is the work done by the cord. For a massless and frictionless cord, this is the same as the work done by the person, and is equal to the change in the block's kinetic energy, $K_2 - K_1 = (1/2)m(v_2^2 - v_1^2) = (1/2)(0.120\ \mathrm{kg})((2.80\ \mathrm{m/s})^2 - (0.70\ \mathrm{m/s})^2) = 0.441\ \mathrm{J}$. Note that in this case, the tension cannot be perpendicular to the block's velocity at all times; the cord is in the radial direction, and for the radius to change, the block must have some non-zero component of velocity in the radial direction.

6.70: a) This is similar to Problem 6.64, but here $\alpha > 0$ (the force is repulsive), and $x_2 < x_1$, so the work done is again negative;

$$W = \alpha\left(\frac{1}{x_1} - \frac{1}{x_2}\right) = (2.12\times10^{-26}\ \mathrm{N\cdot m^2}((0.200\ \mathrm{m^{-1}}) - (1.25\times10^{9}\ \mathrm{m^{-1}}))$$
$$= -2.65\times10^{-17}\ \mathrm{J}.$$

Note that x_1 is so large compared to x^2 that the term $\frac{1}{x_1}$ is negligible. Then, using Eq. (6.13)) and solving for v_2,

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(3.00\times10^5\ \mathrm{m/s})^2 + \frac{2(-2.65\times10^{-17}\ \mathrm{J})}{(1.67\times10^{-27}\ \mathrm{kg})}} = 2.41\times10^5\ \mathrm{m/s}.$$

b) With $K_2 = 0, W = -K_1$. Using $W = -\frac{\alpha}{x_2}$,

$$x_2 = \frac{\alpha}{K_1} = \frac{2\alpha}{mv_1^2} = \frac{2(2.12\times10^{-26}\ \mathrm{N\cdot m^2})}{(1.67\times10^{-27}\ \mathrm{kg})(3.00\times10^5\ \mathrm{m/s})^2} = 2.82\times10^{-10}\ \mathrm{m}.$$

c) The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is $3.00\times10^5\ \mathrm{m/s}$.

6.71: The velocity and acceleration as functions of time are

$$v(t) = \frac{dx}{dt} = 2\alpha t + 3\beta t^2, \quad a(t) = 2\alpha + 6\beta t$$

a) $v(t = 4.00\,\text{s}) = 2(0.20\,\text{m/s}^2)(4.00\text{s}) + 3(0.02\,\text{m/s}^3)(4.00\text{s})^2 = 2.56\,\text{m/s}.$

b) $ma = (6.00\,\text{kg})(2(0.20\,\text{m/s}^2) + 6(0.02\,\text{m/s}^3)(4.00\,\text{s}) = 5.28\,\text{N}.$

c) $W = K_2 - K_1 = K_2 = (1/2)(6.00\,\text{kg})(256\,\text{m/s})^2 = 19.7\,\text{J}.$

6.72: In Eq. (6.14), $dl = dx$ and $\phi = 31.0°$ is constant, and so

$$W = \int_{P_1}^{P_2} F \cos\phi dl = \int_{x_1}^{x_2} F \cos\phi dx$$

$$= (5.00\,\text{N/m}^2)\cos 31.0° \int_{1.00\text{ m}}^{1.50\text{ m}} x^2 dx = 3.39\,\text{J}.$$

The final speed of the object is then

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(4.00\,\text{m/s})^2 + \frac{2(3.39\,\text{J})}{(0.250\,\text{kg})}} = 6.57\,\text{m/s}.$$

6.73: a) $K_2 - K_1 = (1/2)m(v_2^2 - v_1^2)$

$$= (1/2)(80.0\,\text{kg})((1.50\,\text{m/s})^2 - (5.00\,\text{m/s})^2) = -910\,\text{J}.$$

b) The work done by gravity is $-mgh = -(80.0\,\text{kg})(9.80\,\text{m/s}^2)(5.20\,\text{m}) = -4.08\times10^3\,\text{J}$, so the work done by the rider is $-910\,\text{J} - (-4.08\times10^3\,\text{J}) = 3.17\times10^3\,\text{J}$.

6.74: a) $W = \int_{x_0}^{\infty} \frac{b}{x^n} dx = \left.\frac{b}{(-n-1))x^{n-1}}\right|_{x_0}^{\infty} = \frac{b}{(n-1)x_0^{n-1}}.$

Note that for this part, for $n > 1$, $x^{1-n} \to 0$ as $x \to \infty$. b) When $0 < n < 1$, the improper integral must be used,

$$W = \lim_{x_2 \to \infty}\left[\frac{b}{(n-1)}(x_2^{n-1} - x_0^{n-1})\right],$$

and because the exponent on the x_2^{n-1} is positive, the limit does not exist, and the integral diverges. This is interpreted as the force F doing an infinite amount of work, even though $F \to 0$ as $x_2 \to \infty$.

6.75: Setting the (negative) work done by the spring to the needed (negative) change in kinetic energy, $\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2$, and solving for the spring constant,

$$k = \frac{mv_0^2}{x^2} = \frac{(1200\,\text{kg})(0.65\,\text{m/s})^2}{(0.070\,\text{m})^2} = 1.03\times10^5\,\text{N/m}.$$

6.76: a) Equating the work done by the spring to the gain in kinetic energy, $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{k}{m}}x_0 = \sqrt{\frac{400\ \text{N/m}}{0.0300\ \text{kg}}}(0.060\ \text{m}) = 6.93\ \text{m/s}.$$

b) W_{tot} must now include friction, so $\frac{1}{2}mv^2 = W_{\text{tot}} = \frac{1}{2}kx_0^2 - fx_0$, where f is the magnitude of the friction force. Then,

$$\begin{aligned} v &= \sqrt{\frac{k}{m}x_0^2 - \frac{2\boldsymbol{f}}{m}x_0} \\ &= \sqrt{\frac{400\ \text{N/m}}{0.0300\ \text{kg}}(0.06\ \text{m})^2 - \frac{2(6.00\ \text{N})}{(0.0300\ \text{kg})}(0.06\ \text{m})} = 4.90\ \text{m/s}. \end{aligned}$$

c) The greatest speed occurs when the acceleration (and the net force) are zero, or $kx = f$, $x = \frac{f}{k} = \frac{6.00\ \text{N}}{400\ \text{N/m}} = 0.0150\ \text{m}$. To find the speed, the net work is $W_{\text{tot}} = \frac{1}{2}k(x_0^2 - x^2) - f(x_0 - x)$, so the maximum speed is

$$\begin{aligned} v_{\max} &= \sqrt{\frac{k}{m}(x_0^2 - x^2) - \frac{2f}{m}(x_0 - x)} \\ &= \sqrt{\frac{400\text{N/m}}{(0.0300\text{kg})}((0.060\text{m})^2 - (0.0150\text{m})^2) - \frac{2(6.00\text{N})}{(0.0300\text{kg})}(0.060\text{m} - 0.0150\ \text{m})} \\ &= 5.20\ \text{m/s}, \end{aligned}$$

which is larger than the result of part (b) but smaller than the result of part (a).

6.77: Denote the initial compression of the spring by x and the distance from the initial position by L. Then, the work done by the spring is $\frac{1}{2}kx^2$ and the work done by friction is $-\mu_k mg(x + L)$; this form takes into account the fact that while the spring is compressed, the frictional force is still present (see Problem 6.76). The initial and final kinetic energies are both zero, so the net work done is zero, and $\frac{1}{2}kx^2 = \mu_k mg(x + L)$. Solving for L,

$$L = \frac{(1/2)kx^2}{\mu_k mg} - x = \frac{(1/2)(250\ \text{N/m})(0.250\text{m})^2}{(0.30)(2.50\ \text{kg})(9.80\ \text{m/s}^2)} - (0.250\text{m}) = 0.813\ \text{m},$$

or 0.81 m to two figures. Thus the book moves $.81\ \text{m} + .25\ \text{m} = 1.06\ \text{m}$, or about 1.1 m.

6.78: The work done by gravity is $W_g = -mgL\sin\theta$ (negative since the cat is moving up), and the work done by the applied force is FL, where F is the magnitude of the applied force. The total work is

$$W_{\text{tot}} = (100\ \text{N})(2.00\ \text{m}) - (7.00\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})\sin 30° = 131.4\ \text{J}.$$

The cat's initial kinetic energy is $\frac{1}{2}mv_1^2 = \frac{1}{2}(7.00\ \text{kg})(2.40\ \text{m/s})^2 = 20.2\ \text{J}$, and

$$v_2 = \sqrt{\frac{2(K_1 + W)}{m}} = \sqrt{\frac{2(20.2\ \text{J} + 131.4\ \text{J})}{(7.00\ \text{kg})}} = 6.58\ \text{m/s}.$$

6.79: In terms of the bumper compression x and the initial speed v_0, the necessary relations are

$$\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2, \quad kx < 5\,mg.$$

Combining to eliminate k and then x, the two inequalties are

$$x > \frac{v^2}{5g} \quad \text{and} \quad k < 25\frac{mg^2}{v^2}.$$

a) Using the given numbers,

$$x > \frac{(20.0\,\mathrm{m/s})^2}{5(9.80\,\mathrm{m/s^2})} = 8.16\,\mathrm{m},$$

$$k < 25\frac{(1700\,\mathrm{kg})(9.80\,\mathrm{m/s^2})^2}{(20.0\,\mathrm{m/s})^2} = 1.02\times10^4\ \mathrm{N/m}.$$

b) A distance of 8 m is not commonly available as space in which to stop a car.

6.80: The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $60.0°$ with the chair's motion, so the total work done is $W_{\mathrm{tot}} = ((600\,\mathrm{N})\cos 30.0° - (85.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2})\cos 60.0°)(2.50\,\mathrm{m}) = 257.8\,\mathrm{J}$, and so the speed at the top of the ramp is

$$v_2 = \sqrt{v_1^2 + \frac{2W_{\mathrm{tot}}}{m}} = \sqrt{(2.00\,\mathrm{m/s})^2 + \frac{2(257.8\,\mathrm{J})}{(85.0\,\mathrm{kg})}} = 3.17\,\mathrm{m/s}.$$

Note that extra figures were kept in the intermediate calculation to avoid roundoff error.

6.81: a) At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy. Therefore, the work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv^2$, or

$$X = \sqrt{\frac{m}{k}}v = \sqrt{\frac{5.00\,\mathrm{kg}}{500\,\mathrm{N/m}}}(6.00\,\mathrm{m/s}) = 0.600\,\mathrm{m}.$$

b) Solving for v in terms of a known X,

$$v = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\,\mathrm{N/m}}{5.00\,\mathrm{kg}}}(0.150\,\mathrm{m}) = 1.50\,\mathrm{m/s}.$$

6.82: The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table). The work done by gravity is (6.00 kg) *gh* and the work done by friction is $-\mu_k(8.00\,\text{kg})\,gh$, so

$$W_{\text{tot}} = (6.00\,\text{kg} - (0.25)(8.00\,\text{kg})\,(9.80\,\text{m/s}^2)\,(1.50\,\text{m}) = 58.8\,\text{J}.$$

This work increases the kinetic energy of both blocks;

$$W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2,$$

so

$$v = \sqrt{\frac{2(58.8\,\text{J})}{(14.00\,\text{kg})}} = 2.90\,\text{m/s}.$$

6.83: See Problem 6.82. Gravity does positive work, while friction does negative work. Setting the net (negative) work equal to the (negative) change in kinetic energy,

$$(m_1 - \mu_k m_2)gh = -\frac{1}{2}(m_1 + m_2)v^2,$$

and solving for μ_k gives

$$\begin{aligned}\mu_k &= \frac{m_1 + (1/2)\,(m_1 + m_2)v^2/gh}{m_2} \\ &= \frac{(6.00\,\text{kg}) + (1/2)\,(14.00\,\text{kg})\,(0.900\,\text{m/s})^2/((9.80\,\text{m/s}^2)\,(2.00\,\text{m}))}{(8.00\,\text{kg})} \\ &= 0.79.\end{aligned}$$

6.84: The arrow will acquire the energy that was used in drawing the bow (*i.e.*, the work done by the archer), which will be the area under the curve that represents the force as a function of distance. One possible way of estimating this work is to approximate the *F vs. x* curve as a parabola which goes to zero at $x = 0$ and $x = x_0$, and has a maximum of F_0 at $x = \frac{x_0}{2}$, so that $F(x) = \frac{4F_0}{x_0^2}x(x_0 - x)$. This may seem like a crude approximation to the figure, but it has the ultimate advantage of being easy to integrate;

$$\int_0^{x_0} F dx = \frac{4F_0}{x_0^2}\int_0^{x_0}(x_0 x - x^2)\,dx = \frac{4F_0}{x_0^2}\left(x_0\frac{x_0^2}{2} - \frac{x_0^3}{3}\right) = \frac{2}{3}F_0 x_0.$$

With $F_0 = 200\,\text{N}$ and $x_0 = 0.75\,\text{m}, W = 100\,\text{J}$. The speed of the arrow is then $\sqrt{\frac{2W}{m}} = \sqrt{\frac{2(100\,\text{J})}{(0.025\,\text{kg})}} = 89\,\text{m/s}$. Other ways of finding the area under the curve in Fig. (6.28) should give similar results.

6.85: $f_k = 0.25\,mg$ so $W_f = W_{\text{tot}} = -(0.25\,mg)s,$ where s is the length of the rough patch.

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2, K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(0.45v_0^2) = 0.2025\left(\frac{1}{2}mv_0^2\right)$

The work-energy relation gives $-(0.25mg)s = (0.2025 - 1)\frac{1}{2}mv_0^2$

The mass divides out and solving gives $s = 1.5\text{ m}$.

6.86: Your friend's average acceleration is

$$a = \frac{v - v_0}{t} = \frac{6.00\text{ m/s}}{3.00\text{ s}} = 2.00\text{ m/s}^2$$

Since there are no other horizontal forces acting, the force you exert on her is given by

$$F_{\text{net}} = ma = (65.0\text{ kg})(2.00\text{ m/s}^2) = 130\text{ N}$$

Her average velocity during your pull is 3.00 m/s, and the distance she travels is thus 9.00 m. The work you do is $Fx = (130\text{ N})(9.00\text{ m}) = 1170\text{ J}$, and the average power is therefore $1170\text{ J}/3.00\text{ s} = 390\text{ W}.$ The work can also be calculated as the change in the kinetic energy.

6.87: a) $(800\text{ kg})(9.80\text{ m/s}^2)(14.0\text{ m}) = 1.098\times10^5\text{ J, or } 1.10\times10^5\text{ J to three figures.}$

b) $(1/2)(800\text{ kg})(18.0\text{ m/s}^2) = 1.30\times10^5\text{ J.}$

c) $\dfrac{1.10\times10^5\text{ J} + 1.30\times10^5\text{ J}}{60\text{ s}} = 3.99\text{ kW.}$

6.88:

$$\begin{aligned} P &= Fv = mav \\ &= m(2\alpha + 6\beta t)(2\alpha t + 3\beta t^2) \\ &= m(4\alpha^2 t + 18\alpha\beta t^2 + 18\beta^2 t^3) \\ &= (0.96\text{ N/s})t + (0.43\text{ N/s}^2)t^2 + (0.043\text{ N/s}^3)t^3. \end{aligned}$$

At $t = 400\text{ s},$ the power output is 13.5 W.

6.89: Let t equal the number of seconds she walks every day. Then,
$(280\text{ J/s})t + (100\text{ J/s})(86400\text{ s} - t) = 1.1\times10^7\text{ J}.$ Solving for t, $t = 13{,}111\text{ s} = 3.6\text{ hours}.$

6.90: a) The hummingbird produces energy at a rate of $0.7\ \text{J/s}$ to $1.75\ \text{J/s}$. At 10 beats/s, the bird must expend between 0.07 J/beat and 0.175 J/beat.

b) The steady output of the athlete is $500\ \text{W}/70\ \text{kg} = 7\ \text{W/kg}$, which is below the 10 W/kg necessary to stay aloft. Though the athlete can expend $1400\ \text{W}/70\ \text{kg} = 20\ \text{W/kg}$ for short periods of time, no human-powered aircraft could stay aloft for very long. Movies of early attempts at human-powered flight bear out this observation.

6.91: From the chain rule, $P = \frac{d}{dt}W = \frac{d}{dt}(mgh) = \frac{dm}{dt}gh,$ for ideal efficiency. Expressing the mass rate in terms of the volume rate and solving gives

$$\frac{(2000\times10^{6}\ \text{W})}{(0.92)(9.80\ \text{m/s}^2)(170\ \text{m})(1000\ \text{kg/m}^3)} = 1.30\times10^{3}\ \frac{\text{m}^3}{\text{s}}.$$

6.92: a) The power P is related to the speed by $Pt = K = \frac{1}{2}mv^2$, so $v = \sqrt{\frac{2Pt}{m}}$.

b) $$a = \frac{dv}{dt} = \frac{d}{dt}\sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2P}{m}}\frac{d}{dt}\sqrt{t} = \sqrt{\frac{2P}{m}}\frac{1}{2\sqrt{t}} = \sqrt{\frac{P}{2mt}}..$$

a) $$x - x_0 = \int v\,dt = \sqrt{\frac{2P}{m}}\int t^{\frac{1}{2}}\,dt = \sqrt{\frac{2P}{m}}\frac{2}{3}t^{\frac{3}{2}} = \sqrt{\frac{8P}{9m}}\frac{2}{3}t^{\frac{3}{2}}.$$

6.93: a) $(7500\times10^{-3}\ \text{kg}^3)(1.05\times10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.63\ \text{m}) = 1.26\times10^{5}\ \text{J}.$

b) $(1.26\times10^{5}\ \text{J})/(86{,}400\ \text{s}) = 1.46\ \text{W}.$

6.94: a) The number of cars is the total power available divided by the power needed per car,

$$\frac{13.4\times10^{6}\ \text{W}}{(2.8\times10^{3}\ \text{N})(27\ \text{m/s})}=177,$$

rounding down to the nearest integer.

b) To accelerate a total mass M at an acceleration a and speed v, the extra power needed is Mav. To climb a hill of angle α, the extra power needed is $Mg\sin\alpha v$. These will be nearly the same if $a\sim g\sin\alpha$; if $g\sin\alpha\sim g\tan\alpha\sim 0.10\ \text{m/s}^2$, the power is about the same as that needed to accelerate at $0.10\ \text{m/s}^2$.

c) $(1.10\times10^{6}\ \text{kg})(9.80\ \text{m/s}^2)(0.010)(27\ \text{m/s})=2.9\ \text{MW}$. d) The power per car needed is that used in part (a), plus that found in part (c) with M being the mass of a single car. The total number of cars is then

$$\frac{13.4\times10^{6}\ \text{W}-2.9\times10^{6}\ \text{W}}{(2.8\times10^{3}\ \text{N}+(8.2\times10^{4}\ \text{kg})(9.80\ \text{m/s}^2)(0.010))(27\ \text{m/s})}=36,$$

rounding to the nearest integer.

6.95: a) $P_0=Fv=(53\times10^{3}\ \text{N})(45\ \text{m/s})=2.4\ \text{MW}.$

b) $P_1=mav=(9.1\times10^{5}\ \text{kg})(1.5\ \text{m/s}^2)(45\ \text{m/s})=61\ \text{MW}.$

c) Approximating $\sin\alpha$, by $\tan\alpha$, and using the component of gravity down the incline as $mg\sin\alpha$,

$P_2=(mg\sin\alpha)v=(9.1\times10^{5}\ \text{kg})(9.80\ \text{m/s}^2)(0.015)(45\ \text{m/s})=6.0\ \text{MW}.$

6.96: a) Along this path, y is constant, and the displacement is parallel to the force, so $W=\alpha y\int xdx=(2.50\ \text{N/m}^2)(3.00\ \text{m})\frac{(2.00\ \text{m})^2}{2}=15.0\ \text{J}.$

b) Since the force has no y-component, no work is done moving in the y-direction.

c) Along this path, y varies with position along the path, given by $y=1.5x$, so $F_x=\alpha(1.5x)x=1.5\alpha x^2$, and

$$W=\int F_x dx=1.5\alpha\int x^2dx=1.5(2.50\ \text{N/m}^2)\frac{(200\ \text{m})^3}{3}=10.0\ \text{J}.$$

6.97: a)

$$
\begin{aligned}
P &= Fv = (F_{\text{roll}} + F_{\text{air}})v \\
&= ((0.0045)(62.0\ \text{kg})(9.80\ \text{m/s}^2) \\
&\quad + (1/2)(1.00)(0.463\ \text{m}^2)(1.2\ \text{kg/m}^3)(12.0\ \text{m/s})^2)(12.0\ \text{m/s}) \\
&= 513\ \text{W}.
\end{aligned}
$$

6.98: a) $F = \dfrac{P}{v} = \dfrac{28.0 \times 10^3\ \text{W}}{(60.0\ \text{km/h})((1\ \text{m/s})/(3.6\ \text{km/h}))} = 1.68 \times 10^3\ \text{N}.$

b) The speed is lowered by a factor of one-half, and the resisting force is lowered by a factor of $(0.65 + 0.35/4)$, and so the power at the lower speed is

$$(28.0\ \text{kW})(0.50)(0.65 + 0.35/4) = 10.3\ \text{kW} = 13.8\ \text{hp}.$$

c) Similarly, at the higher speed,

$$(28.0\ \text{kW})(2.0)(0.65 + 0.35 \times 4) = 114.8\ \text{kW} = 154\ \text{hp}.$$

6.99: a) $$\frac{(8.00\ \text{hp})(746\ \text{W/hp})}{(60.0\ \text{km/h})((1\,\text{m/s})/(3.6\ \text{km/h}))} = 358\ \text{N}.$$

b) The extra power needed is

$$mgv_{\parallel} = (1800\ \text{kg})(9.80\ \text{m/s}^2)\frac{60.6\ \text{km/h}}{3.6\,\frac{\text{km/h}}{\text{m/s}}}\sin(\arctan(1/10)) = 29.3\ \text{kW} = 39.2\ \text{hp},$$

so the total power is 47.2 hp. (Note: If the sine of the angle is approximated by the tangent, the third place will be different.) c) Similarly,

$$mgv_{\parallel} = (1800\ \text{kg})(9.80\ \text{m/s}^2)\frac{60.0\ \text{km/h}}{3.6\,\frac{\text{km/h}}{\text{m/s}}}\sin(\arctan(0.010)) = 2.94\ \text{kW} = 3.94\ \text{hp},$$

This is the rate at which work is done on the car by gravity. The engine must do work on the car at a rate of 4.06 hp. d) In this case, approximating the sine of the slope by the tangent is appropriate, and the grade is

$$\frac{(8.00\ \text{hp})(746\ \text{W/hp})}{(1800\ \text{kg})(9.80\ \text{m/s}^2)(60.0\ \text{km/h})((1\,\text{m/s})/(3.6\ \text{km/h}))} = 0.0203,$$

very close to a 2% grade.

6.100: Use the Work–Energy Theorem, $W = \Delta KE$, and integrate to find the work.

$$\Delta KE = 0 - \frac{1}{2}mv_0^2 \text{ and } W = \int_0^x (-\text{mg} \sin \alpha - \mu \text{mg} \cos \alpha) dx.$$

Then,

$$W = -\text{mg}\int_0^x (\sin \alpha + \text{A}x \cos \alpha) dx, W = -\text{mg}\left[\sin \alpha x + \frac{\text{A}x^2}{2}\cos \alpha\right].$$

Set $W = \Delta KE.$

$$-\frac{1}{2}mv_0^2 = -\text{mg}\left[\sin \alpha x + \frac{\text{A}x^2}{2}\cos \alpha\right].$$

To eliminate x, note that the box comes to a rest when the force of static friction balances the component of the weight directed down the plane. So, $\text{mg} \sin \alpha = \text{A}x \,\text{mg} \cos \alpha$; solve this for x and substitute into the previous equation.

$$x = \frac{\sin \alpha}{\text{A} \cos \alpha}.$$

Then,

$$\frac{1}{2}v_0^2 = +g\left[\sin \alpha \frac{\sin \alpha}{\text{A} \cos \alpha} + \frac{\text{A}\left(\frac{\sin \alpha}{\text{A} \cos \alpha}\right)^2}{2}\cos \alpha\right],$$

and upon canceling factors and collecting terms, $v_0^2 = \frac{3g \sin^2 \alpha}{\text{A} \cos \alpha}$. Or the box will remain stationary whenever $v_0^2 \geq \frac{3g \sin^2 \alpha}{\text{A} \cos \alpha}$.

6.101: a) Denote the position of a piece of the spring by l; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l) = v\frac{l}{L}$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time). The mass of a piece of length dl is $dm = \frac{M}{L}dl,$ and so

$$dK = \frac{1}{2}dmu^2 = \frac{1}{2}\frac{Mv^2}{L^3}l^2 dl,$$

and

$$K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2 dl = \frac{Mv^2}{6}.$$

b) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = \sqrt{(k/m)}x = \sqrt{(3200\ \text{N/m})/(0.053\ \text{kg})}(2.50\times10^{-2}\ \text{m}) = 6.1\ \text{m/s}.$.
c) With the mass of the spring included, the work that the spring does goes into the kinetic energies of both the ball and the spring, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2$. Solving for v,

$$v = \sqrt{\frac{k}{m + M/3}}x = \sqrt{\frac{(3200\ \text{N/m})}{(0.053\ \text{kg}) + (0.243\ \text{kg})/3}}(2.50\times10^{-2}\ \text{m}) = 3.9\ \text{m/s}.$$

d) Algebraically,

$$\frac{1}{2}mv^2 = \frac{(1/2)kx^2}{(1 + M/3m)} = 0.40\ \text{J} \text{ and}$$

$$\frac{1}{6}Mv^2 = \frac{(1/2)kx^2}{(1 + 3m/M)} = 0.60\ \text{J}.$$

6.102: In both cases, a given amount of fuel represents a given amount of work W_0 that the engine does in moving the plane forward against the resisting force. In terms of the range *R* and the (presumed) constant speed *v*,

$$W_0 = RF = R\left(\alpha v^2 + \frac{\beta}{v^2}\right).$$

In terms of the time of flight $T, R = vt,$ so

$$W_0 = vTF = T\left(\alpha v^3 + \frac{\beta}{v}\right).$$

a) Rather than solve for *R* as a function of *v*, differentiate the first of these relations with respect to *v*, setting $\frac{dW_0}{dv} = 0$ to obtain $\frac{dR}{dv}F + R\frac{dF}{dv} = 0.$ For the maximum range, $\frac{dR}{dv} = 0,$ so $\frac{dF}{dv} = 0.$ Performing the differentiation, $\frac{dF}{dv} = 2\alpha v - 2\beta/v^3 = 0,$ which is solved for

$$v = \left(\frac{\beta}{\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \mathrm{N\cdot m^2/s^2}}{0.30\ \mathrm{N\cdot s^2/m^2}}\right)^{1/4} = 32.9\ \mathrm{m/s} = 118\ \mathrm{km/h}.$$

b) Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv) = 0;$ performing the differentiation, $3\alpha v^2 - \beta/v^2 = 0,$ which is solved for

$$v = \left(\frac{\beta}{3\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \mathrm{N\cdot m^2/s^2}}{3(0.30\ \mathrm{N\cdot s^2/m^2}}\right)^{1/4} = 25\ \mathrm{m/s} = 90\ \mathrm{km/h}.$$

6.103: a) The walk will take one-fifth of an hour, 12 min. From the graph, the oxygen consumption rate appears to be about $12\ \mathrm{cm^3/kg\cdot min},$ and so the total energy is

$$(12\ \mathrm{cm^3/kg\cdot min})\,(70\ \mathrm{kg})\,(12\ \mathrm{min})\,(20\ \mathrm{J/cm^3}) = 2.0\times10^5\ \mathrm{J}.$$

b) The run will take 6 min. Using an estimation of the rate from the graph of about $33\ \mathrm{cm^3/kg\cdot min}$ gives an energy consumption of about 2.8×10^5 J. c) The run takes 4 min, and with an estimated rate of about $50\ \mathrm{cm^3/kg\cdot min},$ the energy used is about 2.8×10^5 J. d) Walking is the most efficient way to go. In general, the point where the slope of the line from the origin to the point on the graph is the smallest is the most efficient speed; about 5 km/h.

6.104: From $\vec{F} = m\vec{a}, F_x = ma_x, F_y = ma_y$ and $F_z = ma_z$. The generalization of Eq. (6.11) is then

$$a_x = v_x \frac{dv_x}{dx}, a_y = v_y \frac{dv_y}{dy}, \ a_z = v_z \frac{dv_z}{dz}.$$

The total work is then

$$\begin{aligned}
W_{\text{tot}} &= \int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} F_x dx + F_y dy + F_z dz \\
&= m\left(\int_{x_1}^{x_2} v_x \frac{dv_x}{dx} dx + \int_{y_1}^{y_2} v_y \frac{dv_y}{dy} dy + \int_{z_1}^{z_2} v_z \frac{dv_z}{dz} dz \right) \\
&= m\left(\int_{v_{x1}}^{v_{x2}} v_x dv_x + \int_{v_{y1}}^{v_{y2}} v_y dv_y + \int_{v_{z2}}^{v_{z2}} v_z dv_z \right) \\
&= \frac{1}{2} m(v_{x2}^2 - v_{x1}^2 + v_{y2}^2 - v_{y1}^2 + v_{z2}^2 - v_{z1}^2 \\
&= \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2.
\end{aligned}$$

7.1: From Eq. (7.2),

$$mgy = (800\,\text{kg})\,(9.80\,\text{m/s}^2)\,(440\,\text{m}) = 3.45\times10^6\ \text{J} = 3.45\ \text{MJ}.$$

7.2: a) For constant speed, the net force is zero, so the required force is the sack's weight, $(5.00\,\text{kg})(9.80\,\text{m/s}^2) = 49\ \text{N}$. b) The lifting force acts in the same direction as the sack's motion, so the work is equal to the weight times the distance, $(49.00\ \text{N})\,(15.0\,\text{m}) = 735\ \text{J};$ this work becomes potential energy. Note that the result is independent of the speed, and that an extra figure was kept in part (b) to avoid roundoff error.

7.3: In Eq. (7.7), taking $K_1 = 0$ (as in Example 6.4) and $U_2 = 0,\ K_2 = U_1 + W_{\text{other}}$. Friction does negative work $-fy$, so $K_2 = mgy - fy;$ solving for the speed v_2,

$$v_2 = \sqrt{\frac{2(mg - f)y}{m}} = \sqrt{\frac{2((200\,\text{kg})\,(9.80\,\text{m/s}^2) - 60\ \text{N})\,(3.00\,\text{m})}{(200\,\text{kg})}} = 7.55\,\text{m/s}.$$

7.4: a) The rope makes an angle of $\arcsin\left(\frac{3.0\,\text{m}}{6.0\,\text{m}}\right) = 30^\circ$ with the vertical. The needed horizontal force is then $w\tan\theta = (120\,\text{kg})\,(9.80\,\text{m/s}^2)\tan 30^\circ = 679\ \text{N}$, or 6.8×10^2 N to two figures. b) In moving the bag, the rope does no work, so the worker does an amount of work equal to the change in potential energy, $(120\,\text{kg})\,(9.80\,\text{m/s}^2)\,(6.0\,\text{m})\,(1-\cos 30^\circ) = 0.95\times10^3\ \text{J}$. Note that this is not the product of the result of part (a) and the horizontal displacement; the force needed to keep the bag in equilibrium varies as the angle is changed.

7.5: a) In the absence of air resistance, Eq. (7.5) is applicable. With $y_1 - y_2 = 22.0\,\text{m}$, solving for v_2 gives

$$v_2 = \sqrt{v_1^2 + 2g(y_2 - y_1)} = \sqrt{(12.0\,\text{m/s})^2 + 2(9.80\,\text{m/s}^2)(22.0\,\text{m})} = 24.0\,\text{m/s}.$$

b) The result of part (a), and any application of Eq. (7.5), depends only on the magnitude of the velocities, not the directions, so the speed is again $24.0\,\text{m/s}$. c) The ball thrown upward would be in the air for a longer time and would be slowed more by air resistance.

7.6: a) (Denote the top of the ramp as point 2.) In Eq. (7.7), $K_2 = 0, W_{\text{other}} = -(35\ \text{N}) \times (2.5\ \text{m}) = -87.5\ \text{J}$, and taking $U_1 = 0$ and $U_2 = mgy_2 = (12\ \text{kg})\,(9.80\ \text{m/s}^2)\,(2.5\ \text{m}\sin 30°) = 147\ \text{J}, v_1 = \sqrt{\frac{2(147\ \text{J}+87.5\ \text{J})}{12\ \text{kg}}} = 6.25\ \text{m/s}$, or $6.3\ \text{m/s}$ to two figures. Or, the work done by friction and the change in potential energy are both proportional to the distance the crate moves up the ramp, and so the initial speed is proportional to the square root of the distance up the ramp; $(5.0\ \text{m/s})\sqrt{\frac{2.5\ \text{m}}{1.6\ \text{m}}} = 6.25\ \text{m/s}$.

b) In part a), we calculated W_{other} and U_2. Using Eq. (7.7), $K_2 = \frac{1}{2}(12\ \text{kg})\,(11.0\ \text{m/s})^2 - 87.5\ \text{J} - 147\ \text{J} = 491.5\ \text{J}$

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(491.5\ \text{J})}{(12\ \text{kg})}} = 9.05\ \text{m/s}.$$

7.7: As in Example 7.7, $K_2 = 0,\ U_2 = 94\ \text{J}$, and $U_3 = 0.$ The work done by friction is

$$-(35\ \text{N})\,(1.6\ \text{m}) = -56\ \text{J},\ \text{and so}\ K_3 = 38\ \text{J},\ \text{and}\ v_3 = \sqrt{\frac{2(38\ \text{J})}{12\ \text{Kg}}} = 2.5\ \text{m/s}.$$

7.8: The speed is v and the kinetic energy is $4K$. The work done by friction is proportional to the normal force, and hence the mass, and so each term in Eq. (7.7) is proportional to the total mass of the crate, and the speed at the bottom is the same for any mass. The kinetic energy is proportional to the mass, and for the same speed but four times the mass, the kinetic energy is quadrupled.

7.9: In Eq. (7.7), $K_1 = 0, W_{\text{other}}$ is given as $-0.22\ \text{J}$, and taking $U_2 = 0, K_2 = mgR - 0.22\ \text{J}$, so

$$v_2 = \sqrt{2\left((9.80\ \text{m/s}^2)\,(0.50\ \text{m}) - \frac{0.22\ \text{J}}{0.20\ \text{kg}}\right)} = 2.8\ \text{m/s}.$$

7.10: (a) The flea leaves the ground with an upward velocity of 1.3 m/s and then is in free-fall with acceleration $9.8\,\mathrm{m/s^2}$ downward. The maximum height it reaches is therefore $(v_y^2 - v_{0y}^2)/2(-g) = 9.0\,\mathrm{cm}$. The distance it travels in the first 1.25 ms can be ignored.

(b)

$$\begin{aligned} W = KE &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(210\times10^{-6}\ \mathrm{g})\,(130\,\mathrm{cm/s})^2 \\ &= 1.8\,\mathrm{ergs} = 1.8\times10^{-7}\ \mathrm{J} \end{aligned}$$

7.11: Take $y = 0$ at point A. Let point 1 be A and point 2 be B.

$K_1 + U_1 + W_{\mathrm{other}} = K_2 + U_2$

$U_1 = 0,\ U_2 = mg(2R) = 28{,}224\,\mathrm{J}, W_{\mathrm{other}} = W_f$

$K_1 = \dfrac{1}{2}mv_1^2 = 37{,}500\,\mathrm{J}, K_2 = \tfrac{1}{2}mv_2^2 = 3840\,\mathrm{J}$

The work - energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400\ \mathrm{J}$.

7.12: Tarzan is lower than his original height by a distance $l(\cos 30 - \cos 45)$, so his speed is

$$v = \sqrt{2gl(\cos 30° - \cos 45°)} = 7.9\,\mathrm{m/s},$$

a bit quick for conversation.

7.13: a) The force is applied parallel to the ramp, and hence parallel to the oven's motion, and so $W = Fs = (110\ \mathrm{N})\,(8.0\ \mathrm{m}) = 880\ \ \mathrm{J}$. b) Because the applied force $\vec{\boldsymbol{F}}$ is parallel to the ramp, the normal force is just that needed to balance the component of the weight perpendicular to the ramp, $n = w\cos\alpha$, and so the friction force is $f_{\mathrm{k}} = \mu_{\mathrm{k}} mg\cos\alpha$ and the work done by friction is $W_{\mathrm{f}} = -\mu_{\mathrm{k}} mg\cos\alpha\, s = -(0.25)\,(10.0\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2})\cos 37°(8.0\ \mathrm{m}) = -157\ \ \mathrm{J}$, keeping an extra figure. c) $mgs\sin\alpha = (10.0\ \ \mathrm{kg})(9.80\ \ \mathrm{m/s^2})(8.0\ \ \mathrm{m})\sin 37° = 472\ \mathrm{J}$, again keeping an extra figure. d) $880\ \mathrm{J} - 472\ \mathrm{J} - 157\ \mathrm{J} = 251\ \mathrm{J}$. e) In the direction up the ramp, the net force is

$$\begin{aligned} &F - mg\sin\alpha - \mu_{\mathrm{k}} mg\cos\alpha \\ &\qquad = 110\ \mathrm{N} - (10.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(\sin 37° + (0.25)\cos 37°) \\ &\qquad = 31.46\ \mathrm{N}, \end{aligned}$$

so the acceleration is $(31.46\ \mathrm{N})/10.0\ \mathrm{kg}) = 3.15\ \mathrm{m/s^2}$. The speed after moving up the ramp is $v = \sqrt{2as} = \sqrt{2(3.15\ \mathrm{m/s^2})\,(8.0\ \mathrm{m})} = 7.09\ \mathrm{m/s}$, and the kinetic energy is $(1/2)mv^2 = 252\ \mathrm{J}$. (In the above, numerical results of specific parts may differ in the third place if extra figures are not kept in the intermediate calculations.)

7.14: a) At the top of the swing, when the kinetic energy is zero, the potential energy (with respect to the bottom of the circular arc) is $mgl(1-\cos\theta)$, where l is the length of the string and θ is the angle the string makes with the vertical. At the bottom of the swing, this potential energy has become kinetic energy, so $mgl(1-\cos\theta) = \frac{1}{2}mv^2$, or $v = \sqrt{2gl(1-\cos\theta)} = \sqrt{2(9.80\ \mathrm{m/s^2})\,(0.80\ \mathrm{m})\,(1-\cos 45°)} = 2.1\ \mathrm{m/s}$. b) At $45°$ from the vertical, the speed is zero, and there is no radial acceleration; the tension is equal to the radial component of the weight, or $mg\cos\theta = (0.12\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2})\cos 45° = 0.83\ \mathrm{N}$. c) At the bottom of the circle, the tension is the sum of the weight and the radial acceleration,

$$mg + mv_2^2/l = mg(1 + 2(1-\cos 45°)) = 1.86\ \mathrm{N},$$

or 1.9 N to two figures. Note that this method does not use the intermediate calculation of v.

7.15: Of the many ways to find energy in a spring in terms of the force and the distance, one way (which avoids the intermediate calculation of the spring constant) is to note that the energy is the product of the average force and the distance compressed or extended. a) $(1/2)(800\ \mathrm{N})(0.200\ \mathrm{m}) = 80.0\ \ \mathrm{J}$. b) The potential energy is proportional to the square of the compression or extension; $(80.0\ \mathrm{J})\,(0.050\ \mathrm{m}/0.200\ \mathrm{m})^2 = 5.0\ \mathrm{J}$.

7.16: $U = \frac{1}{2}ky^2$, where y is the vertical distance the spring is stretched when the weight $w = mg$ is suspended. $y = \frac{mg}{k}$, and $k = \frac{F}{x}$, where x and F are the quantities that "calibrate" the spring. Combining,

$$U = \frac{1}{2}\frac{(mg)^2}{F/x} = \frac{1}{2}\frac{((60.0\ \text{kg})\,(9.80\ \text{m/s}^2))^2}{(720\ \text{N}/0.150\ \text{m})} = 36.0\ \text{J}$$

7.17: a) Solving Eq. (7.9) for $x, x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(3.20\ \text{J})}{(1600\ \text{N/m})}} = 0.063\ \text{m}.$

b) Denote the initial height of the book as h and the maximum compression of the spring by x. The final and initial kinetic energies are zero, and the book is initially a height $x + h$ above the point where the spring is maximally compressed. Equating initial and final potential energies, $\frac{1}{2}kx^2 = mg(x + h)$. This is a quadratic in x, the solution to which is

$$\begin{aligned} x &= \frac{mg}{k}\left[1 \pm \sqrt{1 + \frac{2kh}{mg}}\right] \\ &= \frac{(1.20\ \text{kg})(9.80\ \text{m/s}^2)}{(1600\ \text{N/m})}\left[1 \pm \sqrt{1 + \frac{2(1600\ \text{N/m})\,(0.80\ \text{m})}{(1.20\ \text{kg})\,(9.80\ \text{m/s}^2)}}\right] \\ &= 0.116\ \text{m}, -0.101\ \text{m}. \end{aligned}$$

The second (negative) root is not unphysical, but represents an extension rather than a compression of the spring. To two figures, the compression is 0.12 m.

7.18: a) In going from rest in the slingshot's pocket to rest at the maximum height, the potential energy stored in the rubber band is converted to gravitational potential energy; $U = mgy = (10 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\,(22.0\ \text{m}) = 2.16\ \text{J}.$

b) Because gravitational potential energy is proportional to mass, the larger pebble rises only 8.8 m.

c) The lack of air resistance and no deformation of the rubber band are two possible assumptions.

7.19: The initial kinetic energy and the kinetic energy of the brick at its greatest height are both zero. Equating initial and final potential energies, $\frac{1}{2}kx^2 = mgh,$ where h is the greatest height. Solving for h,

$$h = \frac{kx^2}{2mg} = \frac{(1800\ \text{N/m})\,(0.15\ \text{m})^2}{2(1.20\ \text{kg})\,(9.80\ \text{m/s}^2)} = 1.7\ \text{m}.$$

7.20: As in Example 7.8, $K_1 = 0$ and $U_1 = 0.0250\,\text{J}$. For $v_2 = 0.20\ \text{m/s}$, $K_2 = 0.0040\ \text{J}$, so $U_2 = 0.0210\,\text{J} = \frac{1}{2}kx^2$, so $x = \pm\sqrt{\frac{2(0.0210\ \text{J})}{5.00\ \text{N/m}}} = \pm 0.092\ \text{m}$. In the absence of friction, the glider will go through the equilibrium position and pass through $x = -0.092\,\text{m}$ with the same speed, on the opposite side of the equilibrium position.

7.21: a) In this situation, $U_2 = 0$ when $x = 0$, so $K_2 = 0.0250\,\text{J}$ and $v_2 = \sqrt{\frac{2(0.0250\ \text{J})}{0.200\ \text{kg}}} = 0.500\,\text{m/s}$. b) If $v_2 = 2.50\,\text{m/s}$, $K_2 = (1/2)\,(0.200\,\text{kg})(2.50\,\text{m/s})^2 = 0.625\ \text{J} = U_1$, so $x_1 = \sqrt{\frac{2(0.625\ \text{J})}{5.00\ \text{N/m}}} = 0.500\,\text{m}$. Or, because the speed is 5 times that of part (a), the kinetic energy is 25 times that of part (a), and the initial extension is $5 \times 0.100\ \text{m} = 0.500\ \text{m}$.

7.22: a) The work done by friction is
$W_{\text{other}} = -\mu_{\text{k}} mg\Delta x = -(0.05)\,(0.200\,\text{kg})\,(9.80\,\text{m/s}^2)\,(0.020\,\text{m}) = -0.00196\,\text{J}$, so $K_2 = 0.00704\,\text{J}$ and $v_2 = \sqrt{\frac{2(0.00704\ \text{J})}{0.200\ \text{kg}}} = 0.27\,\text{m/s}$. b) In this case $W_{\text{other}} = -0.0098\,\text{J}$, so $K_2 = 0.0250\,\text{J} - 0.0098\,\text{J} = 0.0152\,\text{J}$, and $v_2 = \sqrt{\frac{2(0.0152\ \text{J})}{0.200\ \text{kg}}} = 0.39\,\text{m/s}$.
c) In this case, $K_2 = 0$, $U_2 = 0$, so
$U_1 + W_{\text{other}} = 0 = 0.0250\,\text{J} - \mu_{\text{k}}(0.200\,\text{kg})\,(9.80\,\text{m/s}^2) \times (0.100\,\text{m})$, or $\mu_{\text{k}} = 0.13$.

7.23: a) In this case, $K_1 = 625{,}000\,\text{J}$ as before, $W_{\text{other}} = -17{,}000\,\text{J}$ and

$$\begin{aligned}U_2 &= (1/2)ky_2^2 + mgy_2 \\ &= (1/2)(1.41\times10^5\ \text{N/m})\,(-1.00\,\text{m})^2 + (2000\ \text{kg})\,(9.80\,\text{m/s}^2)\,(-1.00) \\ &= 50{,}900\,\text{J}.\end{aligned}$$

The kinetic energy is then $K_2 = 625{,}000\,\text{J} - 50{,}900\,\text{J} - 17{,}000\,\text{J} = 557{,}100\,\text{J}$, corresponding to a speed $v_2 = 23.6\,\text{m/s}$. b) The elevator is moving down, so the friction force is up (tending to stop the elevator, which is the idea). The net upward force is then $-mg + f - kx = -(2000\,\text{kg})(9.80\ \text{m/s}^2) + 17{,}000\,\text{N} - (1.41\times10^5\ \text{N/m})(-1.00\,\text{m}) = 138{,}40($ for an upward acceleration of $69.2\ \text{m/s}^2$.

7.24: From $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, the relations between *m*, *v*, *k* and *x* are

$$kx^2 = mv^2, \quad kx = 5mg.$$

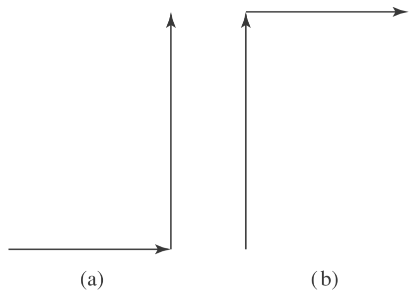
Dividing the first by the second gives $x = \frac{v^2}{5g}$, and substituting this into the second gives $k = 25\frac{mg^2}{v^2}$, so a) & b),

$$x = \frac{(2.50 \text{ m/s})^2}{5(9.80 \text{ m/s}^2)} = 0.128 \text{ m},$$

$$k = 25\frac{(1160 \text{ kg})(9.80 \text{ m/s}^2)^2}{(2.50 \text{ m/s})^2} = 4.46 \times 10^5 \text{ N/m}.$$

7.25: a) Gravity does negative work, $-(0.75\,\text{kg})(9.80 \text{ m/s}^2)(16\,\text{m}) = -118\,\text{J}$. b) Gravity does 118 J of positive work. c) Zero d) Conservative; gravity does no net work on any complete round trip.

7.26: a) & b) $-(0.050\,\text{kg})(9.80 \text{ m/s}^2)(5.0\,\text{m}) = -2.5\,\text{J}$.



c) Gravity is conservative, as the work done to go from one point to another is path-independent.

7.27: a) The displacement is in the *y*-direction, and since $\vec{F}$ has no *y*-component, the work is zero.

b)

$$\int_{P_1}^{P_2} \vec{F} \cdot d\vec{l} = -12\int_{x_1}^{x_2} x^2 dx = -\frac{12 \text{ N/m}^2}{3}(x_2^3 - x_1^3) = -0.104\,\text{J}.$$

c) The negative of the answer to part (b), $0.104\,\text{m}^3$ d) The work is independent of path, and the force is conservative. The corresponding potential energy is $U = \frac{(12 \text{ N/m}^2)x^3}{3} = (4 \text{ N/m}^2)x^3$.

7.28: a) From (0, 0) to (0, L), $x = 0$ and so $\vec{\boldsymbol{F}} = \mathbf{0}$,, and the work is zero. From (0, L) to (L, L), $\vec{\boldsymbol{F}}$ and $d\vec{\boldsymbol{l}}$ are perpendicular, so $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = 0.$ and the net work along this path is zero. b) From (0, 0) to (L, 0), $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = 0.$ From (L, 0) to (L, L), the work is that found in the example, $W_2 = CL^2$, so the total work along the path is CL^2. c) Along the diagonal path, $x = y$, and so $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = Cy\,dy$; integrating from 0 to L gives $\frac{CL^2}{2}$. (It is not a coincidence that this is the average to the answers to parts (a) and (b).) d) The work depends on path, and the field is not conservative.

7.29: a) When the book moves to the left, the friction force is to the right, and the work is $-(1.2\ \text{N})(3.0\ \text{m}) = -3.6\ \text{J}$. b) The friction force is now to the left, and the work is again $-3.6\ \text{J}$. c) $-7.2\ \text{J}$. d) The net work done by friction for the round trip is not zero, and friction is not a conservative force.

7.30: The friction force has magnitude $\mu_k mg = (0.20)(30.0\ \text{kg})(9.80\ \text{m/s}^2) = 58.8\ \text{N}$. a) For each part of the move, friction does $-(58.8\ \text{N})(10.6\ \text{m}) = -623\ \text{J}$, so the total work done by friction is $-1.2\ \text{kN}$. b) $-(58.8\ \text{N})(15.0\ \text{m}) = -882\ \text{N}$.

7.31: The magnitude of the friction force on the book is

$$\mu_k mg = (0.25)(1.5\ \text{kg})(9.80\ \text{m/s}^2) = 3.68\ \text{N}.$$

a) The work done during each part of the motion is the same, and the total work done is $-2(3.68\ \text{N})(8.0\ \text{m}) = -59\ \text{J}$ (rounding to two places). b) The magnitude of the displacement is $\sqrt{2}(8.0\ \text{m})$, so the work done by friction is $-\sqrt{2}(8.0\ \text{m})(3.68\ \text{N}) = -42\ \text{N}$. c) The work is the same both coming and going, and the total work done is the same as in part (a), $-59\ \text{J}$. d) The work required to go from one point to another is not path independent, and the work required for a round trip is not zero, so friction is not a conservative force.

7.32: a) $\frac{1}{2}k(x_1^2 - x_2^2)$ b) $-\frac{1}{2}k(x_1^2 - x_2^2)$. The total work is zero; the spring force is conservative c) From x_1 to x_3, $W = -\frac{1}{2}k(x_3^2 - x_1^2)$. From x_3 to x_2, $W = \frac{1}{2}k(x_2^2 - x_3^2)$. The net work is $-\frac{1}{2}k(x_2^2 - x_1^2)$. This is the same as the result of part (a).

7.33: From Eq. (7.17), the force is

$$F_x = -\frac{dU}{dx} = C_6 \frac{d}{dx}\left(\frac{1}{x^6}\right) = -\frac{6C_6}{x^7}.$$

The minus sign means that the force is attractive.

7.34: From Eq. (7.15), $F_x = -\frac{dU}{dx} = -4\alpha x^3 = -(4.8\ \text{J/m}^4)x^3$, and so

$$F_x(-0.800\ \text{m}) = -(4.8\ \text{J/m}^4)(-0.80\ \text{m})^3 = 2.46\ \text{N}.$$

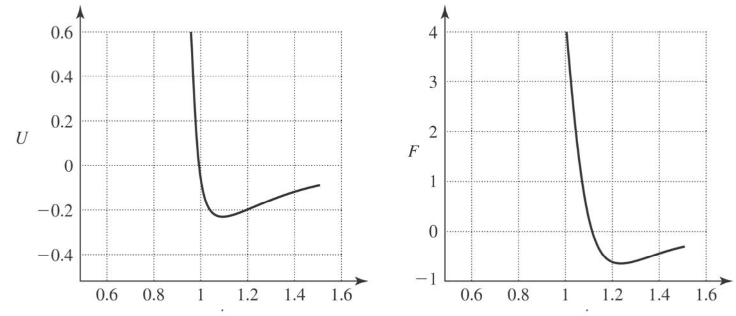
7.35: $\frac{\partial U}{\partial x} = 2kx + k'y$, $\frac{\partial U}{\partial y} = 2ky + k'x$ and $\frac{\partial U}{\partial z} = 0$, so from Eq. (7.19),

$$\vec{\boldsymbol{F}} = -(2kx + k'y)\hat{\boldsymbol{i}} - (2ky + k'x)\hat{\boldsymbol{j}}.$$

7.36: From Eq. (7.19), $\vec{\boldsymbol{F}} = -\frac{\partial U}{\partial x}\hat{\boldsymbol{i}} - \frac{\partial U}{\partial y}\hat{\boldsymbol{j}}$, since U has no z-dependence. $\frac{\partial U}{\partial x} = \frac{-2\alpha}{x^3}$ and $\frac{\partial U}{\partial y} = \frac{-2\alpha}{y^3}$, so

$$\vec{\boldsymbol{F}} = -\alpha\left(\frac{-2}{x^3}\hat{\boldsymbol{i}} + \frac{-2}{y^3}\hat{\boldsymbol{j}}\right).$$

7.37: a) $F_r = -\frac{\partial U}{\partial r} = 12\frac{a}{r^{13}} - 6\frac{b}{r^7}$.



b) Setting $F_r = 0$ and solving for *r* gives $r_{\min} = (2a/b)^{1/6}$. This is the minimum of potential energy, so the equilibrium is stable.

c)

$$U(r_{\min}) = \frac{a}{r_{\min}^{12}} - \frac{b}{r_{\min}^{6}}$$
$$= \frac{a}{((2a/b)^{1/6})^{12}} - \frac{b}{((2a/b)^{1/6})^{6}}$$
$$= \frac{ab^2}{4a^2} - \frac{b^2}{2a} = -\frac{b^2}{4a}.$$

To separate the particles means to remove them to zero potential energy, and requires the negative of this, or $E_0 = b^2/4a$. d) The expressions for E_0 and $r_{\min}$ in terms of *a* and *b* are

$$E_0 = \frac{b^2}{4a} \quad r_{\min}^6 = \frac{2a}{b}.$$

Multiplying the first by the second and solving for *b* gives $b = 2E_0 r_{\min}^6$, and substituting this into the first and solving for *a* gives $a = E_0 r_{\min}^{12}$. Using the given numbers,

$$a = (1.54\times10^{-18}\ \text{J})(1.13\times10^{-10}\ \text{m})^{12} = 6.68\times10^{-138}\ \text{J}\cdot\text{m}^{12}$$
$$b = 2(1.54\times10^{-18}\ \text{J})(1.13\times10^{-10}\ \text{m})^{6} = 6.41\times10^{-78}\ \text{J}\cdot\text{m}^{6}.$$

(Note: the numerical value for *a* might not be within the range of standard calculators, and the powers of ten may have to be handled seperately.)

7.38: a) Considering only forces in the *x*-direction, $F_x = -\frac{dU}{dx}$, and so the force is zero when the slope of the *U vs x* graph is zero, at points *b* and *d*. b) Point *b* is at a potential minimum; to move it away from *b* would require an input of energy, so this point is stable. c) Moving away from point *d* involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so *d* is an unstable point.

7.39: a) At constant speed, the upward force of the three ropes must balance the force, so the tension in each is one-third of the man's weight. The tension in the rope is the force he exerts, or $(70.0\,\text{kg})(9.80\ \text{m/s}^2)/3 = 229\,\text{N}.$ b) The man has risen 1.20 m, and so the increase in his potential energy is $(70.0\,\text{kg})(9.80\ \text{m/s}^2)(1.20\,\text{m}) = 823\,\text{J}.$ In moving up a given distance, the total length of the rope between the pulleys and the platform changes by three times this distance, so the length of rope that passes through the man's hands is $3\times 1.20\,\text{m} = 3.60\,\text{m},$ and $(229\,\text{N})(3.6\,\text{m}) = 824\,\text{J}.$

7.40: First find the acceleration**:**

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(3.00\ \text{m/s})^2}{2(1.20\,\text{m})} = 3.75\ \text{m/s}^2$$

Then, choosing motion in the direction of the more massive block as positive**:**

$$F_{\text{net}} = Mg - mg = (M + m)a = Ma + ma$$

$$M(g - a) = m(g + a)$$

$$\frac{M}{m} = \frac{g + a}{g - a} = \frac{(9.80 + 3.75)\ \text{m/s}^2}{(9.80 - 3.75)\ \text{m/s}^2} = 2.24$$

$$M = 2.24\,\text{m}$$

$$\text{Since}\ M + m = 15.0\ \text{kg}:$$

$$2.24m + m = 15.0\ \text{kg}$$

$$m = 4.63\,\text{kg}$$

$$M = 15.0\,\text{kg} - 4.63\,\text{kg} = 10.4\ \text{kg}$$

7.41: a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$U_1 = U_2 = K_2 = 0$

$W_{\text{other}} = W_f = -\mu_{\text{k}} mgs,$ with $s = 280\,\text{ft} = 85.3\,\text{m}$

The work-energy expression gives $\frac{1}{2} m v_1^2 - \mu_{\text{k}} mgs = 0$

$v_1 = \sqrt{2\mu_{\text{k}} gs} = 22.4\ \text{m/s} = 50\,\text{mph};$ the driver was speeding.

a) 15 mph over speed limit so \$150 ticket.

7.42: a) Equating the potential energy stored in the spring to the block's kinetic energy, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, or

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400\ \text{N/m}}{2.00\ \text{kg}}}(0.220\ \text{m}) = 3.11\ \text{m/s}.$$

b) Using energy methods directly, the initial potential energy of the spring is the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta$, or

$$L = \frac{\frac{1}{2}kx^2}{mg\sin\theta} = \frac{\frac{1}{2}(400\ \text{N/m})(0.220\ \text{m})^2}{(2.00\ \text{kg})(9.80\ \text{m/s}^2)\sin 37.0°} = 0.821\ \text{m}.$$

7.43: The initial and final kinetic energies are both zero, so the work done by the spring is the negative of the work done by friction, or $\frac{1}{2}kx^2 = \mu_k mgl$, where l is the distance the block moves. Solving for μ_k,

$$\mu_k = \frac{(1/2)kx^2}{mgl} = \frac{(1/2)(100\ \text{N/m})(0.20\ \text{m})^2}{(0.50\ \text{kg})(9.80\ \text{m/s}^2)(1.00\ \text{m})} = 0.41.$$

7.44: Work done by friction against the crate brings it to a halt:

$$f_k x = \text{potential energy of compressed spring}$$

$$f_k = \frac{360\ \text{J}}{5.60\ \text{m}} = 64.29\ \text{N}$$

The friction force working over a 2.00-m distance does work $f_k x = (-64.29\ \text{N})(2.00\ \text{m}) = -128.6\ \text{J}$. The kinetic energy of the crate at this point is thus $360\ \text{J} - 128.6\ \text{J} = 231.4\ \text{J}$, and its speed is found from

$$\frac{mv^2}{2} = 231.4\ \text{J}$$

$$v^2 = \frac{2(231.4\ \text{J})}{50.0\ \text{kg}} = 9.256\ \text{m}^2/\text{s}^2$$

$$v = 3.04\ \text{m/s}$$

7.45: a) $mgh = (0.650\ \text{kg})(9.80\ \text{m/s}^2)(2.50\ \text{m}) = 15.9\ \text{J}$

b) The second height is $0.75(2.50\ \text{m}) = 1.875\ \text{m}$, so second $mgh = 11.9\ \text{J}$; loses $15.9\ \text{J} - 11.9\ \text{J} = 4.0\ \text{J}$ on first bounce. This energy is converted to thermal energy.

a) The third height is $0.75(1.875\ \text{m}) = 1.40\ \text{m}$,, so third $mgh = 8.9\ \text{J}$; loses $11.9\ \text{J} - 8.9\ \text{J} = 3.0\ \text{J}$ on second bounce.

7.46: a) $U_A - U_B = mg(h-2R) = \frac{1}{2}mv_A^2$. From previous considerations, the speed at the top must be at least $\sqrt{gR}$. Thus,

$$mg(h-2R) > \frac{1}{2}mgR, \text{ or } h > \frac{5}{2}R.$$

b) $U_A - U_C = (2.50)Rmg = K_C$, so

$$v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 31.3 \text{ m/s}.$$

The radial acceleration is $a_{\text{rad}} = \frac{v_C^2}{R} = 49.0 \text{ m/s}^2$. The tangential direction is down, the normal force at point C is horizontal, there is no friction, so the only downward force is gravity, and $a_{\tan} = g = 9.80 \text{ m/s}^2$.

7.47: a) Use work-energy relation to find the kinetic energy of the wood as it enters the rough bottom: $U_1 = K_2$ gives $K_2 = mgy_1 = 78.4 \text{ J}$.

Now apply work-energy relation to the motion along the rough bottom:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = W_f = -\mu_k mgs$, $K_2 = U_1 = U_2 = 0$; $K_1 = 78.4 \text{ J}$

$78.4 \text{ J} - \mu_k mgs = 0$; solving for s gives $s = 20.0 \text{ m}$.

The wood stops after traveling 20.0 m along the rough bottom.

b) Friction does -78.4 J of work.

7.48: (a)

$$KE_{\text{Bottom}} + W_{\text{f}} = PE_{\text{Top}}$$

$$\frac{1}{2}mv_0^2 - \mu_{\text{k}} mg\cos\theta\, d = mgh$$

$$d = h/\sin\theta$$

$$\frac{1}{2}v_0^2 - \mu_{\text{k}} g\cos\theta \frac{h}{\sin\theta} = gh$$

$$\frac{1}{2}(15\,\text{m/s})^2 - (0.20)(9.8\,\text{m/s}^2)\frac{\cos 40°}{\sin 40°}h = (9.8\,\text{m/s}^2)h$$

$$h = 9.3\,\text{m}$$

(b) Compare maximum static friction force to the weight component down the plane.

$$f_{\text{s}} = \mu_{\text{s}} mg\cos\theta = (0.75)(28\,\text{kg})(9.8\,\text{m/s}^2)\cos 40°$$
$$= 158\,\text{N}$$

$$mg\sin\theta = (28\,\text{kg})(9.8\,\text{m/s}^2)(\sin 40°) = 176\,\text{N} > f_{\text{s}}$$

so the rock will slide down.

(c) Use same procedure as (a), with $h = 9.3\,\text{m}$

$$PE_{\text{Top}} + W_{\text{f}} = KE_{\text{Bottom}}$$

$$mgh - \mu_{\text{k}} mg\cos\theta \frac{h}{\sin\theta} = \frac{1}{2}mv_{\text{B}}^2$$

$$v_{\text{B}} = \sqrt{2gh - 2\mu_{\text{k}} gh\cos\theta/\sin\theta} = 11.8\ \text{m/s}$$

7.49: a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Let point 1 be point A and point 2 be point B. Take $y = 0$ at point B.

$mgy_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$, with $h = 20.0\text{ m}$ and $v_1 = 10.0\text{ m/s}$

$v_2 = \sqrt{v_1^2 + 2gh} = 22.2\text{ m/s}$

b) Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, with point 1 at B and point 2 where the spring has its maximum compression x.

$U_1 = U_2 = K_2 = 0$; $K_1 = \frac{1}{2}mv_1^2$ with $v_1 = 22.2\text{ m/s}$

$W_{\text{other}} = W_f + W_{\text{el}} = -\mu_k mgs - \frac{1}{2}kx^2$, with $s = 100\text{ m} + x$

The work-energy relation gives $K_1 + W_{\text{other}} = 0$.

$\frac{1}{2}mv_1^2 - \mu_k mgs - \frac{1}{2}kx^2 = 0$

Putting in the numerical values gives $x^2 + 29.4x - 750 = 0$. The positive root to this equation is $x = 16.4\text{ m}$.

b) When the spring is compressed $x = 16.4\text{ m}$ the force it exerts on the stone is $F_{\text{el}} = kx = 32.8\text{ N}$. The maximum possible static friction force is

$$\max f_s = \mu_s mg = (0.80)(15.0\text{ kg})(9.80\text{ m/s}^2) = 118\text{ N}.$$

The spring force is less than the maximum possible static friction force so the stone remains at rest.

7.50: First get speed at the top of the hill for the block to clear the pit.

$$y = \frac{1}{2}gt^2$$

$$20\text{ m} = \frac{1}{2}(9.8\text{ m/s}^2)t^2$$

$$t = 2.0\text{ s}$$

$$v_{\text{Top}}t = 40\text{ m} \rightarrow v_{\text{Top}} = \frac{40\text{ m}}{20\text{ s}} = 20\text{ m/s}$$

Energy conservation:

$$KE_{\text{Bottom}} = PE_{\text{Top}} + KE_{\text{Top}}$$

$$\frac{1}{2}mv_{\text{B}}^2 = mgh + \frac{1}{2}mv_{\text{T}}^2$$

$$v_{\text{B}} = \sqrt{v_{\text{T}}^2 + 2gh}$$

$$= \sqrt{(20\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(70\text{ m})}$$

$$= 42\text{ m/s}$$

7.51: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Point 1 is where he steps off the platform and point 2 is where he is stopped by the cord. Let $y = 0$ at point 2. $y_1 = 41.0\,\text{m}$. $W_{\text{other}} = -\frac{1}{2}kx^2$, where $x = 11.0\,\text{m}$ is the amount the cord is stretched at point 2.The cord does negative work.

$K_1 = K_2 = U_2 = 0$, so $mgy_1 - \frac{1}{2}kx^2 = 0$ and $k = 631\,\text{N/m}$.

Now apply $F = kx$ to the test pulls:

$F = kx$ so $x = F/k = 0.602\,\text{m}$.

7.52: For the skier to be moving at no more than $30.0\ \text{m/s}$; his kinetic energy at the bottom of the ramp can be no bigger than

$$\frac{mv^2}{2} = \frac{(85.0\,\text{kg})(30.0\ \text{m/s})^2}{2} = 38{,}250\,\text{J}$$

Friction does $-4000\,\text{J}$ of work on him during his run, which means his combined PE and KE at the top of the ramp must be no more than $38{,}250\,\text{J} + 4000\,\text{J} = 42{,}250\,\text{J}$. His KE at the top is

$$\frac{mv^2}{2} = \frac{(85.0\,\text{kg})(2.0\ \text{m/s})^2}{2} = 170\,\text{J}$$

His PE at the top should thus be no more than $42{,}250\,\text{J} - 170\,\text{J} = 42{,}080\,\text{J}$, which gives a height above the bottom of the ramp of

$$h = \frac{42{,}080\,\text{J}}{mg} = \frac{42{,}080\,\text{J}}{(85.0\,\text{kg})(9.80\ \text{m/s}^2)} = 50.5\,\text{m}.$$

7.53: The net work done during the trip down the barrel is the sum of the energy stored in the spring, the (negative) work done by friction and the (negative) work done by gravity. Using $\frac{1}{2}kx^2 = \frac{1}{2}(F^2/k)$, the performer's kinetic energy at the top of the barrel is

$$K = \frac{1}{2}\frac{(4400\ \text{N})^2}{1100\ \text{N/m}} - (40\ \text{N})(4.0\ \text{m}) - (60\ \text{kg})(9.80\ \text{m/s}^2)(2.5\ \text{m}) = 7.17\times10^3\ \text{J},$$

and his speed is $\sqrt{\frac{2(7.17\times10^3\ \text{J})}{60\ \text{kg}}} = 15.5\ \text{m/s}$.

7.54: To be at equilibrium at the bottom, with the spring compressed a distance x_0, the spring force must balance the component of the weight down the ramp plus the largest value of the static friction, or $kx_0 = w\sin\theta + f$. The work-energy theorem requires that the energy stored in the spring is equal to the sum of the work done by friction, the work done by gravity and the initial kinetic energy, or

$$\frac{1}{2}kx_0^2 = (w\sin\theta - f)L + \frac{1}{2}mv^2,$$

where L is the total length traveled down the ramp and v is the speed at the top of the ramp. With the given parameters, $\frac{1}{2}kx_0^2 = 248\,\text{J}$ and $kx_0 = 1.10\times 10^3\ \text{N}$. Solving for k gives $k = 2440\ \text{N/m}$.

7.55: The potential energy has decreased by $(12.0\,\text{kg})(9.80\ \text{m/s}^2)(2.00\,\text{m}) - (4.0\,\text{kg})\times(9.80\ \text{m/s}^2)(2.00\,\text{m}) = 156.8\,\text{J}$. The kinetic energy of the masses is then $\frac{1}{2}(m_1 + m_2)v^2 = (8.0\,\text{kg})v^2 = 156.8\,\text{J}$, so the common speed is $v = \sqrt{\frac{(156.8\ \text{J})}{8.0\ \text{kg}}} = 4.43\ \text{m/s}$, or $4.4\ \text{m/s}$ to two figures.

7.56: a) The energy stored may be found directly from

$$\frac{1}{2}ky_2^2 = K_1 + W_{\text{other}} - mgy_2 = 625{,}000\text{ J} - 51{,}000\text{ J} - (-58{,}000\text{ J}) = 6.33\times10^5\text{ J}.$$

b) Denote the upward distance from point 2 by h. The kinetic energy at point 2 and at the height h are both zero, so the energy found in part (a) is equal to the negative of the work done by gravity and friction,
$-(mg + f)h = -((2000\text{ kg})(9.80\text{ m/s}^2) + 17{,}000\text{ N})h = (36{,}600\text{ N})h,$ so
$h = \frac{6.33\times10^5\text{ J}}{3.66\times10^4\text{ J}} = 17.3\text{ m}.$ c) The net work done on the elevator between the highest point of the rebound and the point where it next reaches the spring is
$(mg - f)(h - 3.00\text{ m}) = 3.72\times10^4\text{ J}.$ Note that on the way down, friction does negative work. The speed of the elevator is then $\sqrt{\frac{2(3.72\times10^4\text{ J})}{2000\text{ kg}}} = 6.10\text{ m/s}.$ d) When the elevator next comes to rest, the total work done by the spring, friction, and gravity must be the negative of the kinetic energy K_3 found in part (c), or

$$K_3 = 3.72\times10^4\text{ J} = -(mg - f)x_3 + \frac{1}{2}kx_3^2 = -(2{,}600\text{ N})x_3 + (7.03\times10^4\text{ N/m})x_3^2.$$

(In this calculation, the value of k was recalculated to obtain better precision.) This is a quadratic in x_3, the positive solution to which is

$$\begin{aligned} x_3 &= \frac{1}{2(7.03\times10^4\text{ N/m})} \\ &\quad\times\left[2.60\times10^3\text{ N} + \sqrt{(2.60\times10^3\text{ N})^2 + 4(7.03\times10^4\text{ N/m})(3.72\times10^4\text{ J})}\right] \\ &= 0.746\text{ m,} \end{aligned}$$

corresponding to a force of 1.05×10^5 N and a stored energy of 3.91×10^4 J. It should be noted that different ways of rounding the numbers in the intermediate calculations may give different answers.

7.57: The two design conditions are expressed algebraically as $ky = f + mg = 3.66 \times 10^4\ \text{N}$ (the condition that the elevator remains at rest when the spring is compressed a distance y; y will be taken as positive) and $\frac{1}{2}mv^2 + mgy - fy = \frac{1}{2}kx^2$ (the condition that the change in energy is the work $W_{\text{other}} = -fy$). Eliminating y in favor of k by $y = \frac{3.66 \times 10^4\ \text{N}}{k}$ leads to

$$\frac{1}{2}\frac{(3.66 \times 10^4\ \text{N})^2}{k} + \frac{(1.70 \times 10^4\ \text{N})(3.66 \times 10^4\ \text{N})}{k}$$
$$= 62.5 \times 10^4\ \text{J} + \frac{(1.96 \times 10^4\ \text{N})(3.66 \times 10^4\ \text{N})}{k}.$$

This is actually not hard to solve for $k = 919\ \text{N/m}$, and the corresponding x is 39.8 m. This is a very weak spring constant, and would require a space below the operating range of the elevator about four floors deep, which is not reasonable. b) At the lowest point, the spring exerts an upward force of magnitude $f + mg$. Just before the elevator stops, however, the friction force is also directed upward, so the net force is $(f + mg) + f - mg = 2f$, and the upward acceleration is $\frac{2f}{m} = 17.0\ \text{m/s}^2$.

7.58: One mass rises while the other falls, so the net loss of potential energy is

$$(0.5000\ \text{kg} - 0.2000\ \text{kg})(9.80\ \text{m/s}^2)(0.400\ \text{m}) = 1.176\ \text{J}.$$

This is the sum of the kinetic energies of the animals. If the animals are equidistant from the center, they have the same speed, so the kinetic energy of the combination is $\frac{1}{2}m_{\text{tot}}v^2$, and

$$v = \sqrt{\frac{2(1.176\ \text{J})}{(0.7000\ \text{kg})}} = 1.83\ \text{m/s}.$$

7.59: a) The kinetic energy of the potato is the work done by gravity (or the potential energy lost), $\frac{1}{2}mv^2 = mgl$, or $v = \sqrt{2gl} = \sqrt{2(9.80\ \text{m/s}^2)(2.50\ \text{m})} = 7.00\ \text{m/s}$.

b)

$$T - mg = m\frac{v^2}{l} = 2mg,$$

so $T = 3mg = 3(0.100\ \text{kg})(9.80\ \text{m/s}^2) = 2.94\ \text{N}.$

7.60: a) The change in total energy is the work done by the air,

$$(K_2 + U_2) - (K_1 + U_1) = m\left(\frac{1}{2}(v_2^2 - v_1^2) + gy_2\right)$$

$$= (0.145\ \text{kg})\left(\begin{array}{l}(1/2)\,((18.6\ \text{m/s})^2 - (30.0\ \text{m/s})^2 \\ \quad - (40.0\ \text{m/s})^2) + (9.80\ \text{m/s}^2)(53.6\ \text{m})\end{array}\right)$$

$$= -80.0\ \text{J}.$$

b) Similarly,

$$(K_3 + U_3) - (K_2 + U_2) = (0.145\,\text{kg})\left(\begin{array}{l}(1/2)((11.9\ \text{m/s})^2 + (-28.7\ \text{m/s})^2 \\ \quad - (18.6\ \text{m/s})^2) - (9.80\ \text{m/s}^2)(53.6\ \text{m})\end{array}\right)$$

$$= -31.3\ \text{J}.$$

c) The ball is moving slower on the way down, and does not go as far (in the *x*-direction), and so the work done by the air is smaller in magnitude.

7.61: a) For a friction force *f*, the total work done sliding down the pole is $mgd - fd$. This is given as being equal to *mgh*, and solving for *f* gives

$$f = mg\frac{(d-h)}{d} = mg\left(1 - \frac{h}{d}\right).$$

When $h = d,\ f = 0$, as expected, and when $h = 0,\ f = mg$; there is no net force on the fireman. b) $(75\ \text{kg})(9.80\ \text{m/s}^2)(1 - \frac{1.0\ \text{m}}{2.5\ \text{m}}) = 441\ \text{N}$. c) The net work done is $(mg - f)(d - y)$, and this must be equal to $\frac{1}{2}mv^2$. Using the above expression for *f*,

$$\frac{1}{2}mv^2 = (mg - f)(d - y)$$

$$= mg\left(\frac{h}{d}\right)(d - y)$$

$$= mgh\left(1 - \frac{y}{d}\right),$$

from which $v = \sqrt{2gh(1 - y/d)}$. When $y = 0$ $v = \sqrt{2gh}$, which is the original condition. When $y = d$, $v = 0$; the fireman is at the top of the pole.

7.62: a) The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction,

$K_1 = mgh - W_F = (60.0\ \text{kg})(9.8\ \text{N/kg})(65.0\ \text{m}) - 10.500\ \text{J}$, or

$K_1 = 38{,}200\ \text{J} - 10{,}500\ \text{J} = 27{,}720\ \text{J}$. Then $v_1 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(27{,}720\ \text{J})}{60\ \text{kg}}} = 30.4\ \text{m/s}$.

b) $K_2 + K_1 - (W_F + W_A) = 27{,}720\ \text{J} - (\mu_k mgd + f_{\text{air}} d)$, $K_2 = 27{,}720\ \text{J} - [(.2)(588\ \text{N}) \times (82\ \text{m}) + (160\ \text{N})(82\ \text{m})]$, , or $K_2 = 27{,}720\ \text{J} - 22{,}763\ \text{J} = 4957\ \text{J}$. Then,

$$v_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4957\ \text{J})}{60\ \text{kg}}} = 12.85\ \text{m/s} \approx 12.9\ \text{m/s}.$$

c) Use the Work-Energy Theorem to find the force. $W = \Delta KE$,
$F = KE/d = (4957\ \text{J})/(2.5\ \text{m}) = 1983\ \text{N} \approx 2000\ \text{N}$.

7.63: The skier is subject to both gravity and a normal force; it is the normal force that causes her to go in a circle, and when she leaves the hill, the normal force vanishes. The vanishing of the normal force is the condition that determines when she will leave the hill. As the normal force approaches zero, the necessary (inward) radial force is the radial component of gravity, or $mv^2/R = mg\cos\alpha$, where R is the radius of the snowball. The speed is found from conservation of energy; at an angle α, she has descended a vertical distance $R(1-\cos\alpha)$, so $\frac{1}{2}mv^2 = mgR(1-\cos\alpha)$, or $v^2 = 2gR(1-\cos\alpha)$. Using this in the previous relation gives $2(1-\cos\alpha) = \cos\alpha$, or $\alpha = \arccos\left(\frac{2}{3}\right) = 48.2°$. This result does not depend on the skier's mass, the radius of the snowball, or g.

7.64: If the speed of the rock at the top is v_t, then conservation of energy gives the speed v_b from $\frac{1}{2}mv_b^2 = \frac{1}{2}mv_t^2 + mg(2R)$, R being the radius of the circle, and so $v_b^2 = v_t^2 + 4gR$. The tension at the top and bottom are found from $T_t + mg = \frac{mv_t^2}{R}$ and $T_b - mg = \frac{mv_b^2}{R}$, so $T_b - T_t = \frac{m}{R}(v_b^2 - v_t^2) + 2mg = 6mg = 6w$.

7.65: a) The magnitude of the work done by friction is the kinetic energy of the package at point *B*, or $\mu_k mgL = \frac{1}{2} mv_B^2$, or

$$\mu_k = \frac{(1/2)v_B^2}{gL} = \frac{(1/2)(4.80 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(3.00 \text{ m})} = 0.392.$$

b)

$$\begin{aligned} W_{\text{other}} &= K_B - U_A \\ &= \tfrac{1}{2}(0.200 \text{ kg})(4.80 \text{ m/s})^2 - (0.200 \text{ kg})(9.80 \text{ m/s}^2)(1.60 \text{ m}) \\ &= -0.832 \text{ J.} \end{aligned}$$

Equivalently, since $K_A = K_B = 0,\ U_A + W_{AB} + W_{BC} = 0$, or

$$W_{AB} = -U_A - W_{BC} = mg(-(1.60 \text{ m}) - (0.300)(-3.00 \text{ m})) = -0.832 \text{ J.}$$

7.66: Denote the distance the truck moves up the ramp by *x*. $K_1 = \frac{1}{2} mv_0^2$, $U_1 = mgL\sin\alpha$, $K_2 = 0$, $U_2 = mgx\sin\beta$ and $W_{\text{other}} = -\mu_r mgx\cos\beta$. From $W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1)$, and solving for *x*,

$$x = \frac{K_1 + mgL\sin\alpha}{mg(\sin\beta + \mu_r \cos\beta)} = \frac{(v_0^2/2g) + L\sin\alpha}{\sin\beta + \mu_r \cos\beta}.$$

7.67: a) Taking $U(0) = 0$,

$$U(x) = \int_0^x F_x dx = \frac{\alpha}{2}x^2 + \frac{\beta}{3}x^3 = (30.0 \text{ N/m})x^2 + (6.00 \text{ N/m}^2)x^3.$$

b)

$$\begin{aligned} K_2 &= U_1 - U_2 \\ &= ((30.0 \text{ N/m})(1.00 \text{ m})^2 + (6.00 \text{ N/m}^2)(1.00 \text{ m})^3) \\ &\quad - ((30.0 \text{ N/m})(0.50 \text{ m})^2 + (6.00 \text{ N/m}^2)(0.50 \text{ m})^3) \\ &= 27.75 \text{ J,} \end{aligned}$$

and so $v_2 = \sqrt{\frac{2(27.75 \text{ J})}{0.900 \text{ kg}}} = 7.85 \text{ m/s}$.

7.68: The force increases both the gravitational potential energy of the block and the potential energy of the spring. If the block is moved slowly, the kinetic energy can be taken as constant, so the work done by the force is the increase in potential energy, $\Delta U = mga\sin\theta + \frac{1}{2}k(a\theta)^2$.

7.69: With $U_2 = 0,\ K_1 = 0,\ K_2 = \frac{1}{2}mv_2^2 = U_1 = \frac{1}{2}kx^2 + mgh$, and solving for v_2,

$$v_2 = \sqrt{\frac{kx^2}{m} + 2gh} = \sqrt{\frac{(1900\ \text{N/m})(0.045\ \text{m})^2}{(0.150\ \text{kg})} + 2(9.80\ \text{m/s}^2)(1.20\ \text{m})} = 7.01\ \text{m/s}$$

7.70: a) In this problem, use of algebra avoids the intermediate calculation of the spring constant *k*. If the original height is *h* and the maximum compression of the spring is *d*, then $mg(h+d) = \frac{1}{2}kd^2$. The speed needed is when the spring is compressed $\frac{d}{2}$, and from conservation of energy, $mg(h+d/2) - \frac{1}{2}k(d/2)^2 = \frac{1}{2}mv^2$. Substituting for *k* in terms of $h+d$,

$$mg\left(h+\frac{d}{2}\right) - \frac{mg(h+d)}{4} = \frac{1}{2}mv^2,$$

which simplifies to

$$v^2 = 2g\left(\frac{3}{4}h + \frac{1}{4}d\right).$$

Insertion of numerical values gives $v = 6.14\ \text{m/s}$. b) If the spring is compressed a distance *x*, $\frac{1}{2}kx^2 = mgx$, or $x = \frac{2mg}{k}$. Using the expression from part (a) that gives *k* in terms of *h* and *d*,

$$x = (2mg)\frac{d^2}{2mg(h+d)} = \frac{d^2}{h+d} = 0.0210\ \text{m}.$$

7.71: The first condition, that the maximum height above the release point is *h*, is expressed as $\frac{1}{2}kx^2 = mgh$. The magnitude of the acceleration is largest when the spring is compressed to a distance *x*; at this point the net upward force is $kx - mg = ma$, so the second condition is expressed as $x = (m/k)(g+a)$. a) Substituting the second expression into the first gives

$$\frac{1}{2}k\left(\frac{m}{k}\right)^2(g+a)^2 = mgh, \quad \text{or} \quad k = \frac{m(g+a)^2}{2gh}.$$

b) Substituting this into the expression for *x* gives $x = \frac{2gh}{g+a}$.

7.72: Following the hint, the force constant k is found from $w = mg = kd$, or $k = \frac{mg}{d}$. When the fish falls from rest, its gravitational potential energy decreases by mgy; this becomes the potential energy of the spring, which is $\frac{1}{2}ky^2 = \frac{1}{2}\frac{mg}{d}y^2$. Equating these,

$$\frac{1}{2}\frac{mg}{d}y^2 = mgy, \quad \text{or} \quad y = 2d.$$

9.73: a)

$$\begin{aligned}\Delta a_{\text{rad}} &= \omega^2 r - \omega_0^2 r = (\omega^2 - \omega_0^2)r \\ &= [\omega - \omega_0][\omega + \omega_0]r \\ &= \left[\frac{\omega - \omega_0}{t}\right][(\omega + \omega_0)t]r \\ &= [\alpha][2(\theta - \theta_0)r.\end{aligned}$$

b) From the above,

$$\alpha r = \frac{\Delta a_{\text{rad}}}{2\Delta\theta} = \frac{(85.0\ \text{m/s}^2 - 25.0\ \text{m/s}^2)}{2(15.0\ \text{rad})} = 2.00\ \text{m/s}^2.$$

c) Similar to the derivation of part (a),

$$\Delta K = \frac{1}{2}\omega^2 I - \frac{1}{2}\omega_0{}^2 I = \frac{1}{2}[\alpha][2\Delta\theta]I = I\alpha\Delta\theta.$$

d) Using the result of part (c),

$$I = \frac{\Delta K}{\alpha\Delta\theta} = \frac{(45.0\ \text{J} - 20.0\ \text{J})}{((2.00\ \text{m/s}^2)/(0.250\ \text{m}))(15.0\ \text{rad})} = 0.208\ \text{kg}\cdot\text{m}^2.$$

7.74: a) From either energy or force considerations, the speed before the block hits the spring is

$$\begin{aligned} v &= \sqrt{2gL(\sin\theta - \mu_k \cos\theta)} \\ &= \sqrt{2(9.80\ \text{m/s}^2)(4.00\ \text{m})(\sin 53.1° - (0.20)\cos 53.1°)} \\ &= 7.30\ \text{m/s}. \end{aligned}$$

b) This does require energy considerations; the combined work done by gravity and friction is $mg(L+d)(\sin\theta - \mu_k \cos\theta)$, and the potential energy of the spring is $\frac{1}{2}kd^2$, where *d* is the maximum compression of the spring. This is a quadratic in *d*, which can be written as

$$d^2 \frac{k}{2mg(\sin\theta - \mu_k \cos\theta)} - d - L = 0.$$

The factor multiplying d^2 is $4.504\ \text{m}^{-1}$, and use of the quadratic formula gives $d = 1.06\ \text{m}$. c) The easy thing to do here is to recognize that the presence of the spring determines *d*, but at the end of the motion the spring has no potential energy, and the distance below the starting point is determined solely by how much energy has been lost to friction. If the block ends up a distance *y* below the starting point, then the block has moved a distance $L+d$ down the incline and $L+d-y$ up the incline. The magnitude of the friction force is the same in both directions, $\mu_k mg\cos\theta$, and so the work done by friction is $-\mu_k(2L+2d-y)mg\cos\theta$. This must be equal to the change in gravitational potential energy, which is $-mgy\sin\theta$. Equating these and solving for *y* gives

$$y = (L+d)\frac{2\mu_k \cos\theta}{\sin\theta + \mu_k \cos\theta} = (L+d)\frac{2\mu_k}{\tan\theta + \mu_k}.$$

Using the value of *d* found in part (b) and the given values for μ_k and θ gives $y = 1.32\ \text{m}$.

7.75: a) $K_B = W_{\text{other}} - U_B = (20.0\ \text{N})(0.25\ \text{m}) - (1/2)(40.0\ \text{N/m})(.25\ \text{m})^2 = 3.75\ \text{J}$, so $v_B = \sqrt{\frac{2(3.75\ \text{J})}{0.500\ \text{kg}}} = 3.87\ \text{m/s}$, or $3.9\ \text{m/s}$ to two figures. b) At this point (point *C*), $K_C = 0$, and so $U_C = W_{\text{other}}$ and $x_c = -\sqrt{\frac{2(5.00\ \text{J})}{40.0\ \text{N/m}}} = -0.50\ \text{m}$ (the minus sign denotes a displacement to the left in Fig. (7.65)), which is 0.10 m from the wall.

7.76: The kinetic energy K' after moving up the ramp the distance *s* will be the energy initially stored in the spring, plus the (negative) work done by gravity and friction, or

$$K' = \frac{1}{2}kx^2 - mg(\sin\alpha + \mu_k \cos\alpha)s.$$

Minimizing the speed is equivalent to minimizing K', and differentiating the above expression with respect to α and setting $\frac{dK'}{d\alpha} = 0$ gives

$$0 = -mgs(\cos\alpha - \mu_k \sin\alpha),$$

or $\tan\alpha = \frac{1}{\mu_k}$, $\alpha = \arctan\left(\frac{1}{\mu_k}\right)$. Pushing the box straight up $(\alpha = 90°)$ maximizes the vertical displacement *h*, but not $s = h/\sin\alpha$.

7.77: Let $x_1 = 0.18\ \text{m}$, $x_2 = 0.71\ \text{m}$. The spring constants (assumed identical) are then known in terms of the unknown weight *w*, $4kx_1 = w$. The speed of the brother at a given height *h* above the point of maximum compression is then found from

$$\frac{1}{2}(4k)x_2^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 + mgh,$$

or

$$v^2 = \frac{(4k)g}{w}x_2^2 - 2gh = g\left(\frac{x_2^2}{x_1} - 2h\right),$$

so $v = \sqrt{(9.80\ \text{m/s}^2)((0.71\ \text{m})^2/(0.18\ \text{m}) - 2(0.90\ \text{m}))} = 3.13\ \text{m/s}$, or $3.1\ \text{m/s}$ to two figures. b) Setting $v = 0$ and solving for *h*,

$$h = \frac{2kx_2^2}{mg} = \frac{x_2^2}{2x_1} = 1.40\ \text{m},$$

or 1.4 m to two figures. c) No; the distance x_1 will be different, and the ratio $\frac{x_2^2}{x_1} = \frac{(x_1 + 0.53\ \text{m})^2}{x_1} = x_1\left(1 + \frac{0.53\ \text{m}}{x_1}\right)^2$ will be different. Note that on a small planet, with lower *g*, x_1 will be smaller and *h* will be larger.

7.78: a) $a_x = d^2x/dt^2 = -\omega_0^2 x, \quad F_x = ma_x = -m\,\omega_0^2 x$

$a_y = d^2y/dt^2 = -\omega_0^2 y = -\omega_0^2 y, \quad F_y = ma_y = -m\,\omega_0^2 y$

b) $U = -\left[\int F_x dx + \int F_y dy\right] = m\omega_0^2\left[\int xdx + \int ydy\right] = \frac{1}{2}m\omega_0^2(x^2 + y^2)$

c)

$$v_x = dx/dt = -x_0\omega_0 \sin\omega_0 t = -x_0\omega_0(y/y_0)$$
$$v_y = dy/dt = +y_0\omega_0 \cos\omega_0 t = +y_0\omega_0(x/x_0)$$

(i) When $x = x_0$ and $y = 0$, $v_x = 0$ and $v_y = y_0\omega_0$

$$K = \frac{1}{2}m(v_x^2 + v_y^2) = \frac{1}{2}my_0^2\omega_0^2,\ U = \frac{1}{2}\omega_0^2 mx_0^2 \text{ and } E = K + U = \frac{1}{2}m\omega_0^2(x_0^2 + y_0^2)$$

(ii) When $x = 0$ and $y = y_0$, $v_x = -x_0\omega_0$ and $v_y = 0$

$$K = \frac{1}{2}\omega_0^2 mx_0^2,\ U = \frac{1}{2}m\omega_0^2 y_0^2 \text{ and } E = K + U = \frac{1}{2}m\omega_0^2(x_0^2 + y_0^2)$$

Note that the total energy is the same.

7.79: a) The mechanical energy increase of the car is

$K_2 - K_1 = \frac{1}{2}(1500\ \text{kg})(37\ \text{m/s})^2 = 1.027\times10^6\ \text{J}.$

Let α be the number of gallons of gasoline consumed.

$\alpha(1.3\times10^8\ \text{J})(0.15) = 1.027\times10^6\ \text{J}$

$\alpha = 0.053\text{gallons}$

b) $(1.00\ \text{gallons})/\alpha = 19\ \text{accelerations}$

7.80: (a) Stored energy $= mgh = (\rho V)gh = \rho A(1\text{ m})gh$

$$= (1000\text{ kg/m}^3)(3.0\times10^6\text{ m}^2)(1\text{ m})(9.8\,\tfrac{\text{m}}{\text{s}^2})(150\text{ m})$$
$$= 4.4\times10^{12}\text{ J.}$$

(b) 90% of the stored energy is converted to electrical energy, so

$$(0.90)(mgh) = 1000\text{ kW h}$$
$$(0.90)\rho V\,gh = 1000\text{ kW h}$$
$$V = \frac{(1000\text{ kW h})(\frac{3600\text{ s}}{1\text{ h}})}{(0.90)(1000\text{ kg/m}^3)(150\text{ m})(9.8\text{ m/s}^2)}$$
$$= 2.7\times10^3\text{ m}^3$$

Change in level of the lake**:**

$$A\Delta h = V_{\text{water}}$$
$$\Delta h = \frac{V}{A} = \frac{2.7\times10^3\text{ m}^3}{3.0\times10^6\text{ m}^2} = 9.0\times10^{-4}\text{ m}$$

7.81: The potential energy of a horizontal layer of thickness *dy*, area *A*, and height *y* is $dU = (dm)gy$. Let ρ be the density of water.

$dm = \rho dV = \rho A\,dy$, so $dU = \rho Agy\,dy$.

The total potential energy U is

$U = \int_0^h dU = \rho Ag\int_0^h ydy = \frac{1}{2}\rho Ah^2.$

$A = 3.0\times10^6\text{ m}^2$ and $h = 150\text{ m}$, so $U = 3.3\times10^{14}\text{ J} = 9.2\times10^7\text{ kWh}.$

7.82: a) Yes; rather than considering arbitrary paths, consider that

$$\vec{\boldsymbol{F}} = -\left[\frac{\partial}{\partial y}\left(-\frac{Cy^3}{3}\right)\right]\hat{\boldsymbol{j}}.$$

b) No; consider the same path as in Example 7.13 (the field is not the same). For this force, $\vec{\boldsymbol{F}} = \boldsymbol{0}$ along Leg 1, $\vec{\boldsymbol{F}}\cdot d\vec{\boldsymbol{l}} = 0$ along legs 2 and 4, but $\vec{\boldsymbol{F}}\cdot d\vec{\boldsymbol{l}} \neq 0$ along Leg 3.

7.83: a) Along this line, $x = y$, so $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = -\alpha y^3 dy$, and

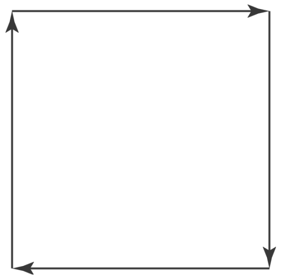
$$\int_{y_1}^{y_2} F_y dy = -\frac{\alpha}{4}(y_2^4 - y_1^4) = -50.6 \text{ J}.$$

b) Along the first leg, $dy = 0$ and so $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = 0$. Along the second leg, $x = 3.00 \text{ m}$, so $F_y = -(7.50 \text{ N/m}^2)y^2$, and

$$\int_{y_1}^{y_2} F_y dy = -(7.5/3 \text{ N/m}^2)(y_2^3 - y_1^3) = -67.5 \text{ J}.$$

c) The work done depends on the path, and the force is not conservative.

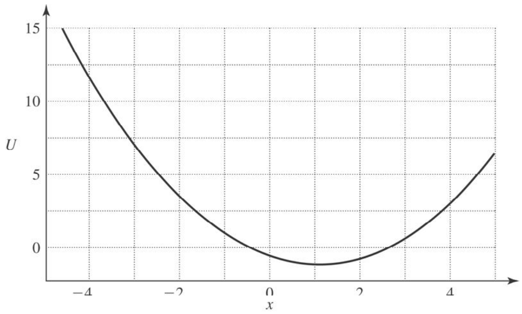
7.84: a)



b) (1): $x = 0$ along this leg, so $\vec{\boldsymbol{F}} = \boldsymbol{0}$ and $W = 0$. (2): Along this leg, $y = 1.50 \text{ m}$, so $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = (3.00 \text{ N/m})x dx$, and $W = (1.50 \text{ N/m})((1.50 \text{ m})^2 - 0) = 3.38 \text{ J}$ (3) $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = 0$, so $W = 0$ (4) $y = 0$, so $\vec{\boldsymbol{F}} = \boldsymbol{0}$ and $W = 0$. The work done in moving around the closed path is 3.38 J. c) The work done in moving around a closed path is not zero, and the force is not conservative.

7.85: a) For the given proposed potential $U(x)$, $-\frac{dU}{dx} = -kx + F$, so this is a possible potential function. For this potential, $U(0) = -F^2/2k$, not zero. Setting the zero of potential is equivalent to adding a constant to the potential; any additive constant will not change the derivative, and will correspond to the same force. b) At equilibrium, the force is zero; solving $-kx + F = 0$ for x gives $x_0 = F/k$. $U(x_0) = -F^2/k$, and this is a minimum of U, and hence a stable point.

c)



d) No; $F_{\text{tot}} = 0$ at only one point, and this is a stable point. e) The extreme values of x correspond to zero velocity, hence zero kinetic energy, so $U(x_\pm) = E$, where $x_\pm$ are the extreme points of the motion. Rather than solve a quadratic, note that $\frac{1}{2}k(x - F/k)^2 - F^2/k$, so $U(x_\pm) = E$ becomes

$$\frac{1}{2}k\left(x_\pm - \frac{F}{k}\right)^2 - F/k = \frac{F^2}{k}$$

$$x_\pm - \frac{F}{k} = \pm 2\frac{F}{k},$$

$$x_+ = 3\frac{F}{k} \quad x_- = -\frac{F}{k}.$$

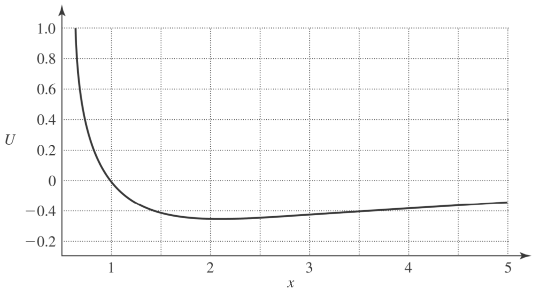
f) The maximum kinetic energy occurs when $U(x)$ is a minimum, the point $x_0 = F/k$ found in part (b). At this point $K = E - U = (F^2/k) - (-F^2/k) = 2F^2/k$, so $v = 2F/\sqrt{mk}$.

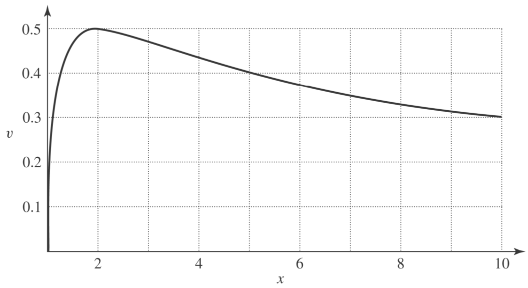
7.86: a) The slope of the U *vs*. x curve is negative at point A, so F_x is positive (Eq. (7.17)). b) The slope of the curve at point B is positive, so the force is negative. c) The kinetic energy is a maximum when the potential energy is a minimum, and that figures to be at around 0.75 m. d) The curve at point C looks pretty close to flat, so the force is zero. e) The object had zero kinetic energy at point A, and in order to reach a point with more potential energy than $U(A)$, the kinetic energy would need to be negative. Kinetic energy is never negative, so the object can never be at any point where the potential energy is larger than $U(A)$. On the graph, that looks to be at about 2.2 m. f) The point of minimum potential (found in part (c)) is a stable point, as is the relative minimum near 1.9 m. g) The only potential maximum, and hence the only point of unstable equilibrium, is at point C.

7.87: a) Eliminating β in favor of α and $x_0 (\beta = \alpha / x_0)$,

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} = \frac{\alpha}{x_0^2}\frac{x_0^2}{x^2} - \frac{\alpha}{x_0 x} = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right)\right].$$

$U(x_0) = \frac{\alpha}{x_0^2}(1-1) = 0$. $U(x)$ is positive for $x < x_0$ and negative for $x > x_0$ (α and β must be taken as positive).





b)

$$v(x) = \sqrt{-\frac{2}{m}U} = \sqrt{\left(\frac{2\alpha}{mx_0^2}\right)\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2\right)}.$$

The proton moves in the positive x-direction, speeding up until it reaches a maximum speed (see part (c)), and then slows down, although it never stops. The minus sign in the square root in the expression for $v(x)$ indicates that the particle will be found only in the region where $U < 0$, that is, $x > x_0$.

c) The maximum speed corresponds to the maximum kinetic energy, and hence the minimum potential energy. This minimum occurs when $\frac{dU}{dx} = 0$, or

$$\frac{dU}{dx} = \frac{\alpha}{x_0}3\left[-2\left(\frac{x_0}{x}\right)^3 + \left(\frac{x_0}{x}\right)^2\right] = 0,$$

which has the solution $x = 2x_0$. $U(2x_0) = -\frac{\alpha}{4x_0^2}$, so $v = \sqrt{\frac{\alpha}{2mx_0^2}}$. d) The maximum speed occurs at a point where $\frac{dU}{dx} = 0$, and from Eq. (7.15), the force at this point is zero. e) $x_1 = 3x_0$, and $U(3x_0) = -\frac{2}{9}\frac{\alpha}{x_0^2}$; $v(x) = \sqrt{\frac{2}{m}(U(x_1) - U(x))} = \sqrt{\frac{2}{m}\left[\left(\frac{-2}{9}\frac{\alpha}{x_0^2}\right) - \frac{\alpha}{x_0^2}\left(\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right)\right]} = \sqrt{\frac{2\alpha}{mx_0^2}\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2 - 2/9\right)}$. The particle is confined to the region where $U(x) < U(x_1)$. The

8.1: a) $(10{,}000\ \text{kg})(12.0\ \text{m/s}) = 1.20\times10^{5}\ \text{kg}\cdot\text{m/s}.$

b) (i) Five times the speed, $60.0\ \text{m/s}$. (ii) $\sqrt{5}\,(12.0\ \text{m/s}) = 26.8\ \text{m/s}.$

8.2: See Exercise 8.3 (a); the iceboats have the same kinetic energy, so the boat with the larger mass has the larger magnitude of momentum by a factor of $\sqrt{(2m)/(m)} = \sqrt{2}.$

8.3: a)

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}.$$

b) From the result of part (a), for the same kinetic energy, $\frac{p_1^2}{m_1} = \frac{p_2^2}{m_2}$, so the larger mass baseball has the greater momentum; $(p_{\text{bird}}/p_{\text{ball}}) = \sqrt{0.040/0.145} = 0.525$. From the result of part (b), for the same momentum $K_1m_1 = K_2m_2$, so $K_1w_1 = K_2w_2$; the woman, with the smaller weight, has the larger kinetic energy. $(K_{\text{man}}/K_{\text{woman}}) = 450/700 = 0.643.$

8.4: From Eq. (8.2),

$$p_x = mv_x = (0.420\ \text{kg})(4.50\ \text{m/s})\cos 20.0° = 1.78\ \text{kg m/s}$$
$$p_y = mv_y = (0.420\ \text{kg})(4.50\ \text{m/s})\sin 20.0° = 0.646\ \text{kg m/s}.$$

8.5: The *y*-component of the total momentum is

$$(0.145\ \text{kg})(1.30\ \text{m/s}) + (0.0570\ \text{kg})(-7.80\ \text{m/s}) = -0.256\ \text{kg}\cdot\text{m/s}.$$

This quantity is negative, so the total momentum of the system is in the $-y$-direction.

8.6: From Eq. (8.2), $p_y = -(0.145\ \text{kg})(7.00\ \text{m/s}) = -1.015\ \text{kg}\cdot\text{m/s}$, and $p_x = (0.045\ \text{kg})(9.00\ \text{m/s}) = 0.405\ \text{kg}\cdot\text{m/s}$, so the total momentum has magnitude

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{(-0.405\ \text{kg}\cdot\text{m/s})^2 + (-1.015\ \text{kg}\cdot\text{m/s})^2} = 1.09\ \text{kg}\cdot\text{m/s},$$

and is at an angle arctan $\left(\frac{-1.015}{+.405}\right) = -68°$, using the value of the arctangent function in the fourth quadrant $(p_x > 0, \text{p}_\text{y} < 0)$.

8.7: $\frac{\Delta p}{\Delta t} = \frac{(0.0450\ \text{kg})(25.0\ \text{m/s})}{2.00\times10^{-3}\ \text{s}} = 563\ \text{N}.$ The weight of the ball is less than half a newton, so the weight is not significant while the ball and club are in contact.

8.8: a) The magnitude of the velocity has changed by

$$(45.0\ \text{m/s}) - (-55.0\ \text{m/s}) = 100.0\ \text{m/s},$$

and so the magnitude of the change of momentum is $(0.145\ \text{kg})\ (100.0\ \text{m/s}) = 14.500\ \text{kg m/s},$ to three figures. This is also the magnitude of the impulse. b) From Eq. (8.8), the magnitude of the average applied force is $\frac{14.500\ \text{kg.m/s}}{2.00\times10^{-3}\text{s}} = 7.25\times10^{3}\ \text{N}.$

8.9: a) Considering the $+x$-components, $p_2 = p_1 + J = (0.16\ \text{kg})(3.00\ \text{m/s}) + (25.0\ \text{N})\times(0.05\ \text{s}) = 1.73\ \text{kg}\cdot\text{m/s}$, and the velocity is $10.8\ \text{m/s}$ in the $+x$-direction. b) $p_2 = 0.48\ \text{kg}\cdot\text{m/s} + (-12.0\ \text{N})(0.05\ \text{s}) = -0.12\ \text{kg}\cdot\text{m/s}$, and the velocity is $+0.75\ \text{m/s}$ in the $-x$-direction.

8.10: a) $\vec{\boldsymbol{F}}\ t = (1.04\times10^{5}\ \text{kg}\cdot\text{m/s})\hat{\boldsymbol{j}}$ **.** b) $(1.04\times10^{5}\ \text{kg}\cdot\text{m/s})\hat{\boldsymbol{j}}$ **.** c) $\frac{(1.04\times10^{5}\ \text{kg.}{}^{\text{m}}\!/_{\text{s}})}{(95{,}000\ \text{kg})}\hat{\boldsymbol{j}} = (1.10\ \text{m/s})\hat{\boldsymbol{j}}.$ d) The initial velocity of the shuttle is not known; the change in the square of the speed is not the square of the change of the speed.

8.11: a) With $t_1 = 0$,

$$J_x = \int_0^{t_2} F_x \, dt = (0.80 \times 10^7 \text{ N/s})t_2^2 - (2.00 \times 10^9 \text{ N/s}^2)t_2^3,$$

which is $18.8 \,\text{kg} \cdot \text{m/s}$, and so the impulse delivered between $t=0$ and $t_2 = 2.50 \times 10^{-3}\text{s}$ is $(18.8 \text{ kg} \cdot \text{m/s})\hat{\boldsymbol{i}}$. b) $J_y = -(0.145 \,\text{kg})\,(9.80 \,\text{m/s}^2)\,(2.50 \times 10^{-3}\text{s})$, and the impulse is $(-3.55 \times 10^{-3} \text{ kg} \cdot \text{m/s})\hat{\boldsymbol{j}}$ c) $\dfrac{J_x}{t_2} = 7.52 \times 10^3 \,\text{N}$, so the average force is $(7.52 \times 10^3 \,\text{N})\hat{\boldsymbol{i}}$.

d) $\vec{\boldsymbol{p}}_2 = \vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{j}}$
$= -(0.145\text{kg})(40.0\hat{\boldsymbol{i}} + 5.0\hat{\boldsymbol{j}})\text{m/s} + (18.8\hat{\boldsymbol{i}} - 3.55 \times 10^{-3}\,\hat{\boldsymbol{j}})$
$= (13.0 \text{ kg.m/s})\hat{\boldsymbol{i}} - (0.73 \,\text{kg.m/s})\hat{\boldsymbol{j}}$.

The velocity is the momentum divided by the mass, or (89.7 m/s) $\hat{\boldsymbol{i}} - (5.0\text{m/s})\hat{\boldsymbol{j}}$.

8.12: The change in the ball's momentum in the *x*-direction (taken to be positive to the right) is
$(0.145 \,\text{kg})\,(-(65.0 \,\text{m/s}) \cos 30^\circ - 50.0\text{m/s}) = -15.41 \,\text{kg} \cdot \text{m/s}$, so the *x*-component of the average force is

$$\frac{-15.41 \,\text{kg} \cdot \text{m/s}}{1.75 \times 10^{-3}\text{s}} = -8.81 \times 10^3 \,\text{N},$$

and the *y*-component of the force is

$$\frac{(0.145\text{kg})(65.0 \,\text{m/s}) \sin 30^\circ}{(1.75 \times 10^{-3}\text{s})} = 2.7 \times 10^3 \,\text{N}.$$

8.13: a) $$J = \int_{t_1}^{t_2} F dt = A(t_2 - t_1) + \frac{B}{3}(t_2^3 - t_1^3),$$

or $J = At_2 + (B/3)t_2^3$ if $\text{t}_1 = 0$. b) $v = \dfrac{p}{m} = \dfrac{J}{m} = \dfrac{A}{m}t_2 + \dfrac{B}{3m}t_2^3$.

8.14: The impluse imparted to the player is opposite in direction but of the same magnitude as that imparted to the puck, so the player's speed is $\frac{(0.16\,\text{kg})\,(20.0\,\text{m/s})}{(75.0\,\text{kg})} = 4.27\,\text{cm/s}$, in the direction opposite to the puck's.

8.15: a) You and the snowball now share the momentum of the snowball when thrown so your speed is $\frac{(0.400\,\text{kg})\,(10.0\,\text{m/s})}{(70.0\,\text{kg}+0.400\,\text{kg})} = 5.68\,\text{cm/s}$. b) The change in the snowball's momentum is $(0.400\,\text{kg})\,(18.0\,\text{m/s}) = 7.20\,\text{kg}\cdot\text{m/s})$, so your speed is $\frac{7.20\,\text{kg}\cdot\text{m/s}}{70.0\,\text{kg}} = 10.3\,\text{cm/s}$.

8.16: a) The final momentum is

$$(0.250\,\text{kg})(-0.120\,\text{m/s}) + (0.350)(0.650\,\text{m/s}) = 0.1975\,\text{kg}\cdot\text{m/s},$$

taking positive directions to the right. a) Before the collision, puck *B* was at rest, so all of the momentum is due to puck *A*'s motion, and

$$v_{A1} = \frac{p}{m_A} = \frac{0.1975\,\text{kg}\cdot\text{m/s}}{0.250\,\text{kg}} = 0.790\,\text{m/s}.$$

b) $$\Delta K = K_2 - K_1 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 - \frac{1}{2}m_A v_{A1}^2$$

$$= \frac{1}{2}(0.250\,\text{kg})\,(-0.120\,\text{m/s})^2 + \frac{1}{2}(0.350\,\text{kg})(0.650\,\text{m/s})^2$$

$$- \frac{1}{2}(0.250\,\text{kg})(-0.7900\,\text{m/s})^2$$

$$= -0.0023\,\text{J}$$

8.17: The change in velocity is the negative of the change in Gretzky's momentum, divided by the defender's mass, or

$$\begin{aligned} v_{B2} &= v_{B1} - \frac{m_A}{m_B}(v_{A2} - v_{A1}) \\ &= -5.00\ \text{m/s} - \frac{756\ \text{N}}{900\ \text{N}}(1.50\,\text{m/s} - 13.0\ \text{m/s}) \\ &= 4.66\ \text{m/s}. \end{aligned}$$

Positive velocities are in Gretzky's original direction of motion, so the defender has changed direction.

b) $$\begin{aligned} K_2 - K_1 &= \frac{1}{2}m_A(v_{A2}^2 - v_{A1}^2) + \frac{1}{2}m_B(v_{B2}^2 - v_{B1}^2) \\ &= \frac{1}{2(9.80\ \text{m/s}^2)}\begin{bmatrix} (756\ \text{N})((1.50\ \text{m/s})^2 - (13.0\ \text{m/s})^2) \\ +(900\ \text{N})((4.66\ \text{m/s})^2 - (-5.00\ \text{m/s})^2) \end{bmatrix} \\ &= -6.58\ \text{kJ}. \end{aligned}$$

8.18: Take the direction of the bullet's motion to be the positive direction. The total momentum of the bullet, rifle, and gas must be zero, so

$$(0.00720\ \text{kg})(601\ \text{m/s} - 1.85\ \text{m/s}) + (2.80\ \text{kg})(-1.85\ \text{m/s}) + p_{\text{gas}} = 0,$$

and $p_{\text{gas}} = 0.866\ \text{kg}\cdot\text{m/s}$. Note that the speed of the bullet is found by subtracting the speed of the rifle from the speed of the bullet relative to the rifle.

8.19: a) See Exercise 8.21; $v_A = \left(\frac{3.00\ \text{kg}}{1.00\ \text{kg}}\right)(0.800\ \text{m/s}) = 3.60\ \text{m/s}$.

b) $(1/2)\,(1.00\ \text{kg})\,(3.60\ \text{m/s})^2 + (1/2)(3.00\ \text{kg})(1.200\ \text{m/s})^2 = 8.64\ \text{J}$.

8.20: In the absence of friction, the horizontal component of the hat-plus-adversary system is conserved, and the recoil speed is

$$\frac{(4.50\ \text{kg})(22.0\ \text{m/s})\cos 36.9°}{(120\ \text{kg})} = 0.66\ \text{m/s}.$$

8.21: a) Taking v_A and v_B to be magnitudes, conservation of momentum is expressed as $m_A v_A = m_B v_B$, so $v_B = \frac{m_A}{m_B} v_A$.

b) $$\frac{K_A}{K_B} = \frac{(1/2)m_A v_A^2}{(1/2)m_B v_B^2} = \frac{m_A v_A^2}{m_B((m_A/m_B)v_A)^2} = \frac{m_B}{m_A}.$$

(This result may be obtained using the result of Exercise 8.3.)

8.22:

$$^{214}\text{Po decay: } {}^{214}\text{Po} \rightarrow {}^{4}\alpha + {}^{210}\text{X}$$

$$Set v_\alpha : KE_\alpha = \frac{1}{2} m_\alpha v_\alpha^2$$

$$v_\alpha = \sqrt{\frac{2KE_\alpha}{m_\alpha}}$$

$$= \sqrt{\frac{2(1.23\times10^{-12}\text{ J})}{6.65\times10^{-27}\text{ kg}}} = 1.92\times10^{7}\text{ m/s}$$

Momentum conservation:

$$0 = m_\alpha v_\alpha - m_\text{x} v_\text{x}$$

$$v_x = \frac{m_\alpha v_\alpha}{m_x} = \frac{m_\alpha v_\alpha}{210 m_\text{p}}$$

$$= \frac{(6.65\times10^{-27}\text{ kg})(1.92\times10^{7}\text{ m/s})}{(210)(1.67\times10^{-27}\text{ kg})}$$

$$= 3.65\times10^{5}\text{ m/s}$$

8.23: Let the $+x$-direction be horizontal, along the direction the rock is thrown. There is no net horizontal force, so P_x is constant. Let object A be you and object B be the rock.

$$0 = -m_A v_A + m_B v_B \cos 35.0°$$

$$v_A = \frac{m_B v_B \cos 35.0°}{m_A} = 2.11\,\text{m/s}$$

8.24: Let Rebecca's original direction of motion be the x-direction. a) From conservation of the x-component of momentum,

$$(45.0\,\text{kg})(13.0\,\text{m/s}) = (45.0\,\text{kg})(8.0\,\text{m/s})\cos 53.1° + (65.0\,\text{kg})v_x,$$

So $v_x = 5.67\,\text{m/s}$. If Rebecca's final motion is taken to have a positive y-component, then

$$v_y = -\frac{(45.0\,\text{kg})(8.0\,\text{m/s})\sin 53.1°}{(65.0\,\text{kg})} = -4.43\,\text{m/s}.$$

Daniel's final speed is

$$\sqrt{v_x^2 + v_y^2} = \sqrt{(5.67\,\text{m/s})^2 + (-4.43\,\text{m/s})^2} = 7.20\,\text{m/s},$$

and his direction is $\arctan\left(\frac{-4.42}{5.67}\right) = -38°$ from the x-axis, which is $91.1°$ from the direction of Rebecca's final motion.

b) $\Delta K = \frac{1}{2}(45.0\,\text{kg})\,(8.0\,\text{m/s})^2 + \frac{1}{2}(65.0\,\text{kg})\,(7.195\,\text{m/s})^2 - \frac{1}{2}(45.0)\,(13.0\,\text{m/s})^2$
$= -680\,\text{J}.$

Note that an extra figure was kept in the intermediate calculation.

8.25: $(m_{\text{Kim}} + m_{\text{Ken}})(3.00\text{ m/s}) = m_{\text{Kim}}(4.00\text{ m/s}) + m_{\text{Ken}}(2.25\text{ m/s})$, so

$$\frac{m_{\text{Kim}}}{m_{\text{Ken}}} = \frac{(3.00\text{ m/s}) - (2.25\text{ m/s})}{(4.00\text{ m/s}) - (3.00\text{ m/s})} = 0.750,$$

and Kim weighs $(0.750)(700\text{ N}) = 525\text{ N}.$

8.26: The original momentum is $(24{,}000\text{ kg})(4.00\text{ m/s}) = 9.60\times10^{4}\text{ kg}\cdot\text{m/s}$, the final mass is $24{,}000\text{ kg} + 3000\text{ kg} = 27{,}000\text{ kg}$, and so the final speed is

$$\frac{9.60\times10^{4}\text{ kg}\cdot\text{m/s}}{2.70\times10^{4}\text{ kg}} = 3.56\text{ m/s}.$$

8.27: Denote the final speeds as v_A and v_B and the initial speed of puck A as v_0, and omit the common mass. Then, the condition for conservation of momentum is

$$v_0 = v_A\cos 30.0^\circ + v_B\cos 45.0^\circ$$
$$0 = v_A\sin 30.0^\circ - v_B\sin 45.0^\circ.$$

The 45.0° angle simplifies the algebra, in that $\sin 45.0^\circ = \cos 45.0^\circ$, and so the v_B terms cancel when the equations are added, giving

$$v_A = \frac{v_0}{\cos 30.0^\circ + \sin 30.0^\circ} = 29.3\text{ m/s}$$

From the second equation, $v_B = \frac{v_A}{\sqrt{2}} = 20.7\text{ m/s}.$ b) Again neglecting the common mass,

$$\frac{K_2}{K_1} = \frac{(1/2)(v_A^2 + v_B^2)}{(1/2)v_0^2} = \frac{(29.3\text{ m/s})^2 + (20.7\text{ m/s})^2}{(40.0\text{ m/s})^2} = 0.804,$$

so 19.6% of the original energy is dissipated.

8.28: a) From $m_1v_1 + m_2v_2 = m_1v + m_2v = (m_1 + m_2)v, v = \frac{m_1v_1+m_2v_2}{m_1+m_2}$. Taking positive velocities to the right, $v_1 = -3.00.\text{m/s}$ and $v_2 = 1.20\text{ m/s}$, so $v = -1.60\text{ m/s}$.

b)
$$\begin{aligned}\Delta K &= \frac{1}{2}(0.500\text{ kg} + 0.250\text{ kg})(-1.60\text{ m/s})^2\\ &\quad - \frac{1}{2}(0.500\text{ kg})(-3.00\text{ m/s})^2 - \frac{1}{2}(0.250\text{ kg})(1.20\text{ m/s})^2\\ &= -1.47\text{ J.}\end{aligned}$$

8.29: For the truck, $M = 6320\text{ kg}$, and $V = 10\text{ m/s}$, for the car, $m = 1050\text{ kg}$ and $v = -15\text{ m/s}$ (the negative sign indicates a westbound direction).

a) Conservation of momentum requires $(M + m)v' = MV + mv$, or

$$v' = \frac{(6320\text{ kg})(10\text{ m/s}) + (1050\text{ kg})(-15\text{ m/s})}{(6320\text{ kg} + 1050\text{ kg})} = 6.4\text{ m/s eastbound.}$$

b) $$V = \frac{-mv}{M} = \frac{-(1050\text{ kg})(-15\text{ m/s})}{6320\text{ kg}} = 2.5\text{ m/s.}$$

c) $\Delta KE = -281\text{ kJ}$ for part (a) and $\Delta KE = -138\text{ kJ}$ for part (b).

8.30: Take north to be the *x*-direction and east to be the *y*-direction (these choices are arbitrary). Then, the final momentum is the same as the intial momentum (for a sufficiently muddy field), and the velocity components are

$$v_x = \frac{(110\text{ kg})(8.8\text{ m/s})}{(195\text{ kg})} = 5.0\text{ m/s}$$
$$v_y = \frac{(85\text{ kg})(7.2\text{ m/s})}{(195\text{ kg})} = 3.1\text{ m/s.}$$

The magnitude of the velocity is then $\sqrt{(5.0\text{ m/s})^2 + (3.1\text{ m/s})^2} = 5.9\text{ m/s}$, at an angle or arctan $\left(\frac{3.1}{5.0}\right) = 32°$ east of north.

8.31: Use conservation of the horizontal component of momentum to find the velocity of the combined object after the collision. Let +*x* be south.

P_x is constant gives

$$(0.250\,\text{kg})(0.200\,\text{m/s}) - (0.150\,\text{kg})(0.600\,\text{s}) = (0.400\,\text{kg})v_{2x}$$

$$v_{2x} = -10.0\,\text{cm/s}\ (v_2 = 10.0\,\text{cm/s, north})$$

$$K_1 = \tfrac{1}{2}(0.250\,\text{kg})(0.200\,\text{s})^2 + \tfrac{1}{2}(0.150\,\text{kg})(0.600\,\text{s})^2 = 0.0320\,\text{J}$$

$$K_2 = \tfrac{1}{2}(0.400\,\text{kg})(0.100\,\text{s})^2 = 0.0020\,\text{J}$$

$$\Delta K = K_2 - K_1 = -0.0300\,\text{J}$$

Kinetic energy is converted to thermal energy due to work done by nonconservative forces during the collision.

8.32: (a) Momentum conservation tells us that both cars have the same change in momentum, but the smaller car has a greater velocity change because it has a smaller mass.

$$M\Delta V = m\Delta v$$

$$\Delta v\,(\text{small car}) = \frac{M}{m}\Delta V\ (\text{large car})$$

$$= \frac{3000\,\text{kg}}{1200\,\text{kg}}\Delta V = 2.5\Delta \text{V}\ (\text{large car})$$

(b) The occupants of the small car experience 2.5 times the velocity change of those in the large car, so they also experience 2.5 times the acceleration. Therefore they feel 2.5 times the force, which causes whiplash and other serious injuries.

8.33: Take east to be the *x*-direction and north to be the *y*-direction (again, these choices are arbitrary). The components of the common velocity after the collision are

$$v_x = \frac{(1400\,\text{kg})\,(-35.0\ \text{km/h})}{(4200\,\text{kg})} = -11.67\ \text{km/h}$$

$$v_\text{y} = \frac{(2800\,\text{kg})\,(-50.0\,\text{km/h})}{(4200\,\text{kg})} = -33.33\ \text{km/h}.$$

The velocity has magnitude $\sqrt{(-11.67\,\text{km/h})^2 + (-33.33\,\text{km/h})^2} = 35.3\ \text{km/h}$ and is at a direction arctan $\left(\frac{-33.33}{-11.67}\right) = 70.7°$ south of west.

8.34: The initial momentum of the car must be the *x*-component of the final momentum as the truck had no intial *x*-component of momentum, so

$$v_{\text{car}} = \frac{p_x}{m_{\text{car}}} = \frac{(m_{\text{car}} + m_{\text{truck}})v\cos\theta}{m_{\text{car}}}$$
$$= \frac{2850\,\text{kg}}{950\,\text{kg}}(16.0\,\text{m/s})\cos(90° - 24°)$$
$$= 19.5\,\text{m}/s.$$

Similarly, $v_{\text{truck}} = \frac{2850}{1900}(16.0\,\text{m/s})\sin 66° = 21.9\,\text{m/s}.$

8.35: The speed of the block immediately after being struck by the bullet may be found from either force or energy considerations. Either way, the distance *s* is related to the speed v_{block} by $v^2 = 2\mu_{\text{k}}gs.$ The speed of the bullet is then

$$v_{\text{bullet}} = \frac{m_{\text{block}} + m_{\text{bullet}}}{m_{\text{bullet}}}\sqrt{2\mu_{\text{k}}gs}$$
$$= \frac{1.205\,\text{kg}}{5.00\times10^{-3}\ \text{kg}}\sqrt{2(0.20)(9.80\,\text{m}/s^2)(0.230\,\text{m})}$$
$$= 229\,\text{m/s},$$

or $2.3\times10^2\ \text{m/s}$ to two places.

8.36: a) The final speed of the bullet-block combination is

$$V = \frac{12.0\times10^{-3}\ \text{kg}}{6.012\,\text{kg}}(380\ \text{m/s}) = 0.758\,\text{m/s}.$$

Energy is conserved after the collision, so $(m+M)gy = \frac{1}{2}(m+M)V^2$, and

$$y = \frac{1}{2}\frac{V^2}{g} = \frac{1}{2}\frac{(0.758\,\text{m/s})^2}{(9.80\,\text{m/s}^2)} = 0.0293\,\text{m} = 2.93\,\text{cm}.$$

b) $K_1 = \frac{1}{2}mv^2 = \frac{1}{2}(12.0\times10^{-3}\ \text{kg})(380\,\text{m/s})^2 = 866\,\text{J}.$

c) From part a), $K_2 = \frac{1}{2}(6.012\,\text{kg})(0.758\,\text{m/s})^2 = 1.73\,\text{J}.$

8.37: Let +y be north and +x be south. Let v_{S1} and v_{A1} be the speeds of Sam and of Abigail before the collision. $m_S = 80.0\ \text{kg}, m_A = 50.0\ \text{kg}, v_{S2} = 6.00\ \text{m/s}, v_{A2} = 9.00\ \text{m/s}$.

P_x is constant gives

$m_S v_{S1} = m_S v_{S2} \cos 37.0° + m_A v_{A2} \cos 23.0°$

$v_{S1} = 9.67\ \text{m/s (Sam)}$

P_y is constant gives

$m_A v_{A1} = m_S v_{S2} \sin 37.0° - m_A v_{A2} \sin 23.0°$

$v_{A1} = 2.26\ \text{m/s (Abigail)}$

b) $K_1 = \frac{1}{2} m_S v_{S1}^2 + \frac{1}{2} m_A v_{A1}^2 = 4101\ \text{J}$

$K_2 = \frac{1}{2} m_S v_{S2}^2 + \frac{1}{2} m_A v_{A2}^2 = 3465\ \text{J}$

$\Delta K = K_2 - K_1 = -640\ \text{J}$

8.38: (a) At maximum compression of the spring, $v_2 = v_{10} = V$. Momentum conservation gives $(2.00\ \text{kg})(2.00\ \text{m/s}) = (12.0\ \text{kg})V$

$$V = 0.333\ \text{m/s}$$

$$\text{Energy conservation}: \frac{1}{2} m_2 v_0^2 = \frac{1}{2}(m_2 + m_{10})V^2 + U_{\text{spr}}$$

$$\frac{1}{2}(2.00\ \text{kg})(2.00\ \text{m/s})^2 = \frac{1}{2}(12.0\ \text{kg})(0.333\ \text{m}/s)^2 + U_{\text{spr}}$$

$$U_{\text{spr}} = 3.33\ \text{J}$$

(b) The collision is elastic and Eqs. (8.24) and (8.25) may be used**:**

$$v_2 = -1.33\ \text{m/s},\ v_{10} = +0.67\ \text{m/s}$$

8.39: In the notation of Example 8.10, with the smaller glider denoted as *A*, conservation of momentum gives $(1.50)v_{A2} + (3.00)v_{B2} = -5.40\ \text{m/s}$. The relative velocity has switched direction, so $v_{A2} - v_{B2} = -3.00\ \text{m/s}$. Multiplying the second of these relations by (3.00) and adding to the first gives $(4.50)v_{A2} = -14.4\ \text{m/s}$, or $v_{A2} = -3.20\ \text{m/s}$, with the minus sign indicating a velocity to the left. This may be substituted into either relation to obtain $v_{B2} = -0.20\ \text{m/s}$; or, multiplying the second relation by (1.50) and subtracting from the first gives $(4.50)v_{B2} = -0.90\ \text{m/s}$, which is the same result.

8.40: a) In the notation of Example 8.10, with the large marble (originally moving to the right) denoted as $A, (3.00)v_{A2} + (1.00)v_{B2} = 0.200\,\text{m/s}$. The relative velocity has switched direction, so $v_{A2} - v_{B2} = -0.600\,\text{m}/s$. Adding these eliminates v_{B2} to give $(4.00)v_{A2} = -0.400\,\text{m/s}$, or $v_{A2} = -0.100\,\text{m/s}$, with the minus sign indicating a final velocity to the left. This may be substituted into either of the two relations to obtain $v_{B2} = 0.500\,\text{m/s}$; or, the second of the above relations may be multiplied by 3.00 and subtracted from the first to give $(4.00)v_{B2} = 2.00\,\text{m/s}$, the same result.

b) $\Delta P_A = -0.009\,\text{kg}\cdot\text{m/s}, \Delta P_B = 0.009\,\text{kg}\cdot\text{m/s}$

c) $\Delta K_A = -4.5\times 10^{-4}, \Delta K_B = 4.5\times 10^{-4}$.

Because the collision is elastic, the numbers have the same magnitude.

8.41: Algebraically, $v_{B2} = \sqrt{20}\,\text{m/s}$. This substitution and the cancellation of common factors and units allow the equations in α and β to be reduced to

$$2 = \cos\alpha + \sqrt{1.8}\cos\beta$$
$$0 = \sin\alpha - \sqrt{1.8}\,\sin\beta.$$

Solving for $\cos\alpha$ and $\sin\,\alpha$, squaring and adding gives

$$\left(2 - \sqrt{1.8}\cos\beta\right)^2 + \left(\sqrt{1.8}\,\sin\beta\right)^2 = 1.$$

Minor algebra leads to $\cos\,\beta = \frac{1.2}{\sqrt{1.8}}$, or $\beta = 26.57°$. Substitution of this result into the first of the above relations gives $\cos\,\alpha = \frac{4}{5}$, and $\alpha = 36.87°$.

8.42: a) Using Eq. (8.24), $\frac{v_A}{V} = \frac{1\,\text{u} - 2\,\text{u}}{1\,\text{u} + 2\,\text{u}} = \frac{1}{3}$. b) The kinetic energy is proportional to the square of the speed, so $\frac{K_A}{K} = \frac{1}{9}$ c) The magnitude of the speed is reduced by a factor of $\frac{1}{3}$ after each collision, so after N collisions, the speed is $\left(\frac{1}{3}\right)^N$ of its original value. To find N, consider

$$\left(\frac{1}{3}\right)^N = \frac{1}{59{,}000} \quad \text{or}$$
$$3^N = 59{,}000$$
$$N\ln(3) = \ln(59{,}000)$$
$$N = \frac{\ln(59{,}000)}{\ln(3)} = 10.$$

to the nearest integer. Of course, using the logarithm in any base gives the same result.

8.43: a) In Eq. (8.24), let $m_A = m$ and $m_B = M$. Solving for M gives

$$M = m\frac{v - v_A}{v + v_A}$$

In this case, $v = 1.50\times10^7\ \text{m/s}$, and $v_A = -1.20\times10^7\ \text{m/s}$, with the minus sign indicating a rebound. Then, $M = m\frac{1.50+1.20}{1.50+)-1.20)} = 9m$. Either Eq. (8.25) may be used to find $v_B = \frac{v}{5} = 3.00\times10^6\ \text{m/s}$, or Eq. (8.23), which gives $v_B = (1.50\times10^7\ \text{m/s}) + (-1.20\times10^7\ \text{m/s})$, the same result.

8.44: From Eq. (8.28),

$$x_{\text{cm}} = \frac{(0.30\ \text{kg})(0.20\ \text{m}) + (0.40\ \text{kg})(0.10\ \text{m}) + (0.20\ \text{kg})(-0.30\ \text{m})}{(0.90\ \text{kg})} = +0.044\ \text{m},$$

$$y_{\text{cm}} = \frac{(0.30\ \text{kg})(0.30\ \text{m}) + (0.40\ \text{kg})(-0.40\ \text{m}) + (0.20\ \text{kg})(0.60\ \text{m})}{(0.90\ \text{kg})} = 0.056\ \text{m}.$$

8.45: Measured from the center of the sun,

$$\frac{(1.99\times10^{30}\ \text{kg})(0) + (1.90\times10^{27}\ \text{kg})(7.78\times10^{11}\ \text{m})}{1.99\times10^{30}\ \text{kg} + 1.90\times10^{27}\ \text{kg}} = 7.42\times10^8\ \text{m}.$$

The center of mass of the system lies outside the sun.

8.46: a) Measured from the rear car, the position of the center of mass is, from Eq. (8.28), $\frac{(1800\ \text{kg})(40.0\ \text{m})}{(1200\ \text{kg} + 1800\ \text{kg})} = 24.0\ \text{m}$, which is 16.0 m behind the leading car.

b) $(1200\ \text{kg})(12.0\ \text{m/s}) + (1800\ \text{kg})(20.0\ \text{m/s}) = 5.04\times10^4\ \text{kg}\cdot\text{m/s}$.

c) From Eq. (8.30),

$$v_{\text{cm}} = \frac{(1200\ \text{kg})(12.0\ \text{m/s}) + (1800\ \text{kg})(20.0\ \text{m/s})}{(1200\ \text{kg} + 1800\ \text{kg})} = 16.8\ \text{m/s}.$$

d) $(1200\ \text{kg} + 1800\ \text{kg})(16.8\ \text{m/s}) = 5.04\times10^4\ \text{kg}\cdot\text{m/s}$.

8.47: a) With $x_1 = 0$ in Eq. (8.28),

$$m_1 = m_2((x_2 / x_{\text{cm}}) - 1) = (0.10\ \text{kg})((8.0\ \text{m})/(2.0\ \text{m}) - 1) = 0.30\ \text{kg}.$$

b) $\vec{\boldsymbol{P}} = M\vec{\boldsymbol{v}}_{cm} = (0.40\ \text{kg})(5.0\ \text{m/s})\,\hat{\boldsymbol{i}} = (2.0\ \text{kg}\cdot\text{m/s})\,\hat{\boldsymbol{i}}$. c) In Eq. (8.32), $\vec{\boldsymbol{v}}_2 = \mathbf{0}$, so $\vec{\boldsymbol{v}}_1 = \vec{\boldsymbol{P}}/(0.30\ \text{kg}) = (6.7\ \text{m/s})\hat{\boldsymbol{i}}$.

8.48: As in Example 8.15, the center of mass remains at rest, so there is zero net momentum, and the magnitudes of the speeds are related by $m_1 v_1 = m_2 v_2$, or $v_2 = (m_1 / m_2) v_1 = (60.0\ \text{kg} / 90.0\ \text{kg})(0.70\ \text{m/s}) = 0.47\ \text{m/s}$.

8.49: See Exercise 8.47(a); with $y_1 = 0$, Eq. (8.28) gives $m_1 = m_2((y_2 / y_{\text{cm}}) - 1) =$ $(0.50\ \text{kg})((6.0\ \text{m})/(2.4\ \text{m}) - 1) = 0.75\ \text{kg}$, so the total mass of the system is 1.25 kg.

b) $\vec{\boldsymbol{a}}_{\text{cm}} = \frac{d}{dt}\vec{\boldsymbol{v}}_{\text{cm}} = (1.50\ \text{m/s}^3)\,t\hat{\boldsymbol{i}}.$

c) $\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}_{\text{cm}} = (1.25\ \text{kg})\,(1.50\ \text{m/s}^3)\,(3.0\ \text{s})\hat{\boldsymbol{i}} = (5.63\ \ \text{N})\hat{\boldsymbol{i}}.$

8.50: $\boldsymbol{p}_z = 0$, so $F_z = 0$. The x-component of force is

$$F_x = \frac{dp_x}{dt} = (-1.50\ \text{N/s})t.$$

$$F_y = \frac{dp_y}{dt} = 0.25\ \text{N}$$

8.51: a) From Eq. (8.38), $F = (1600\ \text{m/s})(0.0500\ \text{kg/s}) = 80.0\ \ \text{N}$. b) The absence of atmosphere would not prevent the rocket from operating. The rocket could be steered by ejecting the fuel in a direction with a component perpendicular to the rocket's velocity, and braked by ejecting in a direction parallel (as opposed to antiparallel) to the rocket's velocity.

8.52: It turns out to be more convenient to do part (b) first; the thrust is the force that accelerates the astronaut and MMU, $F = ma = (70\text{ kg} + 110\text{ kg})(0.029\text{ m/s}^2) = 5.22\ \text{N}$.
a) Solving Eq. (8.38) for $|dm|$,

$$|dm| = \frac{F\,dt}{v_{\text{ex}}} = \frac{(5.22\text{ N})(5.0\text{ s})}{490\text{ m/s}} = 53\,\text{gm}.$$

8.53: Solving for the magnitude of dm in Eq. (8.39),

$$|dm| = \frac{ma}{v_{\text{ex}}}dt = \frac{(6000\text{ kg})(25.0\text{ m/s}^2)}{(2000\text{ m/s})}(1\text{ s}) = 75.0\text{ kg}.$$

8.54: Solving Eq. (8.34) for v_{ex} and taking the magnitude to find the exhaust speed, $v_{ex} = a\frac{m}{dm/dt} = \left(15.0\text{ m/s}^2\right)\ (160\text{ s}) = 2.4\text{ km/s}$. In this form, the quantity $\frac{m}{dm/dt}$ is approximated by $\frac{m}{\Delta m/\Delta t} = \frac{m}{\Delta m}\Delta t = 160$ s.

8.55: a) The average thrust is the impulse divided by the time, so the ratio of the average thrust to the maximum thrust is $\frac{(10.0\text{ N}\cdot\text{s})}{(13.3\text{ N})\,(1.70\text{ s})} = 0.442$. b) Using the average force in Eq. (8.38), $v_{\text{ex}} = \frac{F\,dt}{|dm|} = \frac{10.0\text{ N}\cdot\text{s}}{0.0125\text{ kg}} = 800$ m/s. c) Using the result of part (b) in Eq. (8.40), $v = (800\text{ m/s})\ln(0.0258/0.0133) = 530\text{ m/s}$.

8.56: Solving Eq. (8.4) for the ratio $\frac{m_0}{m}$, with $v_0 = 0$,

$$\frac{m_0}{m} = \exp\left(\frac{v}{v_{\text{ex}}}\right) = \exp\left(\frac{8.00\text{ km/s}}{2.10\text{ km/s}}\right) = 45.1.$$

8.57: Solving Eq. (8.40) for $\frac{m}{m_0}$, the fraction of the original rocket mass that is not fuel,

$$\frac{m}{m_0} = \exp\left(-\frac{v}{v_{\text{ex}}}\right).$$

a) For $v = 1.00\times10^{-3}c = 3.00\times10^5\text{ m/s}, \exp(-(3.00\times10^5\text{ m/s}/(2000\text{m/s})) = 7.2\times10^{-66}$.
b) For $v = 3000\text{ m/s}, \exp(-(3000\text{ m/s})/(2000\text{ m/s})) = 0.22$.

8.58: a) The speed of the ball before and after the collision with the plate are found from the heights. The impulse is the mass times the sum of the speeds,

$$J = m(v_1 + v_2) = m(\sqrt{2gy_1} + \sqrt{2gy_2}) = (0.040\,\text{kg})\sqrt{2(9.80\,\text{m/s}^2}\left(\sqrt{2.00\,\text{m}} + \sqrt{1.60\,\text{m}}\right) = 0.47$$

b) $\frac{J}{\Delta t} = (0.47\ \text{N}\cdot\text{s}/2.00\times10^{-3}\text{s}) = 237\ \text{N}.$

8.59: $\vec{\boldsymbol{p}} = \int \vec{\boldsymbol{F}}\,dt = (\alpha t^3/3)\,\hat{\boldsymbol{j}} + (\beta t + \gamma t^2/2)\hat{\boldsymbol{j}} = (8.33\,\text{N/s}^2\,\text{t}^3)\hat{\boldsymbol{i}} + (30.0\,\text{N}t + 2.5\,\text{N/s}t^2)\,\hat{\boldsymbol{j}}$

After 0.500 s, $\vec{\boldsymbol{p}} = (1.04\,\text{kg}\cdot\text{m/s})\hat{\boldsymbol{i}} + (15.63\,\text{kg}\cdot\text{m/s})\,\hat{\boldsymbol{j}}$, and the velocity is

$$\vec{\boldsymbol{v}} = \vec{\boldsymbol{p}}/m = (0.52\,\text{m/s})\hat{\boldsymbol{i}} + (7.82\,\text{m/s})\,\hat{\boldsymbol{j}}.$$

8.60: a)

$$J_x = F_x t = (-380\ \text{N})\,(3.00\times10^{-3}\text{s}) = -1.14\ \text{N}\cdot\text{s}.$$

$$J_y = F_y t = (110\ \text{N})\,(3.00\times10^{-3}\text{s}) = 0.33\ \text{N}\cdot\text{s}.$$

b)

$$v_{2x} = v_{1x} + J_x/m = (20.0\,\text{m/s}) + \frac{(-1.14\text{N.s})}{(0.560\,\text{N})/(9.80\,\text{m/s}^2)} = 0.05\,\text{m/s}$$

$$v_{2y} = v_{1y} + J_y/m = (-4.0\,\text{m/s}) + \frac{(0.33\,\text{N.s})}{((0.560\,\text{N})/(9.80\,\text{m/s}^2))} = 1.78\,\text{m/s}.$$

8.61: The total momentum of the final combination is the same as the initial momentum; for the speed to be one-fifth of the original speed, the mass must be five times the original mass, or 15 cars.

8.62: The momentum of the convertible must be the south component of the total momentum, so

$$v_{\text{con}} = \frac{(800\,\text{kg}\cdot\text{m/s})\cos 60.0°}{(1500\,\text{kg})} = 2.67\,\text{m/s}.$$

Similarly, the speed of the station wagon is

$$v_{\text{sw}} = \frac{(800\,\text{kg}\cdot\text{m/s})\sin 60.0°}{(2000\,\text{kg})} = 3.46\,\text{m/s}.$$

8.63: The total momentum must be zero, and the velocity vectors must be three vectors of the same magnitude that sum to zero, and hence must form the sides of an equilateral triangle. One puck will move $60°$ north of east and the other will move $60°$ south of east.

8.64: a) $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{tot} v_x$, therefore

$$v_{Cx} = \frac{(0.100\,\text{kg})(0.50\,\text{m/s}) - (0.020\,\text{kg})(-1.50\,\text{m/s}) - (0.030\,\text{kg})(-0.50\,\text{m/s})\cos 60°}{0.050\text{kg}}$$
$$v_{Cx} = 1.75\,\text{m/s}$$

Similarly,

$$v_{Cy} = \frac{(0.100\text{kg})(0\,\text{m/s}) - (0.020\text{kg})(0\,\text{m/s}) - (0.030\,\text{kg})(-0.50\,\text{m/s})\sin 60°}{0.050\,\text{kg}}$$
$$v_{Cy} = 0.26\,\text{m/s}$$

b)

$$\Delta K = \tfrac{1}{2}(0.100\,\text{kg})(0.5\,\text{m/s})^2 - \tfrac{1}{2}(0.020\,\text{kg})(1.50\,\text{m/s})^2 - \tfrac{1}{2}(0.030\,\text{kg})(-0.50\,\text{m/s})^2$$
$$-\tfrac{1}{2}(0.050\,\text{kg}) \times [(1.75\,\text{m/s})^2 + (0.26\,\text{m/s})^2] = -0.092\,\text{J}$$

8.65: a) To throw the mass sideways, a sideways force must be exerted on the mass, and hence a sideways force is exerted on the car. The car is given to remain on track, so some other force (the tracks on the car) act to give a net horizontal force of zero on the car, which continues at $5.00\,\text{m/s}$ east.

b) If the mass is thrown with backward with a speed of $5.00\,\text{m/s}$ relative to the initial motion of the car, the mass is at rest relative to the ground, and has zero momentum. The speed of the car is then $(5.00\,\text{m/s})\frac{(200\,\text{kg})}{(175\,\text{kg})} = 5.71\,\text{m/s},$ and the car is still moving east.

c) The combined momentum of the mass and car must be same before and after the mass hits the car, so the speed is $\frac{(200\,\text{kg})(5.00\,\text{m/s}) + (25.0\,\text{kg})(-6.00\,\text{m/s})}{(225\,\text{kg})} = 3.78\,\text{m/s},$ with the car still moving east.

8.66: The total mass of the car is changing, but the speed of the sand as it leaves the car is the same as the speed of the car, so there is no change in the velocity of either the car or the sand (the sand acquires a downward velocity after it leaves the car, and is stopped on the tracks *after* it leaves the car). Another way of regarding the situation is that v_{ex} in Equations (8.37), (8.38) and (8.39) is zero, and the car does not accelerate. In any event, the speed of the car remains constant at 15.0 m/s. In Exercise 8.24, the rain is given as falling vertically, so its velocity relative to the car as it hits the car is not zero.

8.67: a) The ratio of the kinetic energy of the Nash to that of the Packard is $\frac{m_N v_N^2}{m_P v_P^2} = \frac{(840\ \text{kg})(9\ \text{m/s})^2}{(1620\ \text{kg})(5\ \text{m/s})^2} = 1.68.$ b) The ratio of the momentum of the Nash to that of the Packard is $\frac{m_N v_N}{m_P v_P} = \frac{(840\ \text{kg})(9\ \text{m/s})}{(1620\ \text{kg})(5\ \text{m/s})} = 0.933,$ therefore the Packard has the greater magnitude of momentum. c) The force necessary to stop an object with momentum P in time t is $F = -P/t$. Since the Packard has the greater momentum, it will require the greater force to stop it. The ratio is the same since the time is the same, therefore $F_N / F_P =$ 0.933. d) By the work-kinetic energy theorem, $F = \frac{\Delta k}{d}$. Therefore, since the Nash has the greater kinetic energy, it will require the greater force to stop it in a given distance. Since the distance is the same, the ratio of the forces is the same as that of the kinetic energies, $F_N / F_P =$ 1.68.

8.68: The recoil force is the momentum delivered to each bullet times the rate at which the bullets are fired,

$$F_{\text{ave}} = (7.45 \times 10^{-3}\ \text{kg})\,(293\ \text{m/s})\left(\frac{1000\ \text{bullets/min}}{60\ \text{s/ min}}\right) = 36.4\ \text{N}.$$

8.69: (This problem involves solving a quadratic. The method presented here formulates the answer in terms of the parameters, and avoids intermediate calculations, including that of the spring constant.)

Let the mass of the frame be M and the mass putty be m. Denote the distance that the frame streteches the spring by x_0, the height above the frame from which the putty is dropped as h, and the maximum distance the frame moves from its initial position (with the frame attached) as d.

The collision between the putty and the frame is completely inelastic, and the common speed after the collision is $v_0 = \sqrt{2gh}\,\frac{m}{m+M}$. After the collision, energy is conserved, so that

$$\frac{1}{2}(m+M)v_0^2 + (m+M)gd = \frac{1}{2}k((d+x_0)^2 - x_0^2 \text{, or}$$

$$\frac{1}{2}\frac{m^2}{m+M}(2gh) + (m+M)gd = \frac{1}{2}\frac{mg}{x_0}((d+x_0)^2 - x_0^2,$$

where the above expression for v_0, and $k = mg/x_0$ have been used. In this form, it is seen that a factor of g cancels from all terms. After performing the algebra, the quadratic for d becomes

$$d^2 - d\left(2x_0\frac{m}{M}\right) - 2hx_0\frac{m^2}{m+M} = 0,$$

which has as its positive root

$$d = x_0\left[\left(\frac{m}{M}\right) + \sqrt{\left(\frac{m}{M}\right)^2 + 2\frac{h}{x_0}\left(\frac{m^2}{M(m+M)}\right)}\right].$$

For this situation, $m = 4/3\ M$ and $h/x_0 = 6$, so

$$d = 0.232 \text{ m}.$$

8.70: a) After impact, the block-bullet combination has a total mass of 1.00 kg, and the speed V of the block is found from $\frac{1}{2} M_{\text{total}} V^2 = \frac{1}{2} kX^2$, or $V = \sqrt{\frac{k}{m}} X$. The spring constant k is determined from the calibration; $k = \frac{0.75 \text{ N}}{2.50 \times 10^{-3} \text{ m}} = 300 \text{ N/m}$. Combining,

$$V = \sqrt{\frac{300 \text{ N/m}}{1.00 \text{ kg}}} \left(15.0 \times 10^{-2} \text{ m}\right) = 2.60 \text{ m/s}.$$

b) Although this is not a pendulum, the analysis of the inelastic collision is the same;

$$v = \frac{M_{\text{total}}}{m} V = \frac{1.00 \text{ Kg}}{8.0 \times 10^{-3} \text{ Kg}} (2.60 \text{ m/s}) = 325 \text{ m/s}.$$

8.71: a) Take the original direction of the bullet's motion to be the x-direction, and the direction of recoil to be the y-direction. The components of the stone's velocity after impact are then

$$v_x = \left(\frac{6.00 \times 10^{-3} \text{ Kg}}{0.100 \text{ Kg}}\right) (350 \text{ m/s}) = 21.0 \text{ m/s},$$

$$v_y = -\left(\frac{6.00 \times 10^{-3} \text{ Kg}}{0.100 \text{ Kg}}\right) (250 \text{ m/s}) = 15.0 \text{ m/s},$$

and the stone's speed is $\sqrt{(21.0 \text{ m/s})^2 + (15.0 \text{ m/s})^2} = 25.8 \text{ m/s}$, at an angle of arctan $\left(\frac{15.0}{21.0}\right) = 35.5°$. b) $K_1 = \frac{1}{2}\left(6.00 \times 10^{-3} \text{ kg}\right)(350 \text{ m/s})^2 = 368 \text{ J}$
$K_2 = \frac{1}{2}\left(6.00 \times 10^{-3} \text{ kg}\right)(250 \text{ m/s})^2 + \frac{1}{2}(0.100 \text{ kg})\left(25.8 \text{ m}^2/\text{s}^2\right) = 221 \text{ J}$, so the collision is not perfectly elastic.

8.72: a) The stuntman's speed before the collision is $v_{0s} = \sqrt{2gy} = 9.9\ \text{m/s}$. The speed after the collision is

$$v = \frac{m_s}{m_s + m_v} v_{0s} = \frac{80.0\ \text{kg}}{0.100\ \text{kg}} (9.9\ \text{m/s}) = 5.3\ \text{m/s}.$$

b) Momentum is not conserved during the slide. From the work-energy theorem, the distance x is found from $\frac{1}{2} m_{\text{total}} v^2 = \mu_k m_{\text{total}} gx$, or

$$x = \frac{v^2}{2\mu_{kg}} = \frac{(5.28\ \text{m/s})^2}{2(0.25)(9.80\ \text{m/s}^2)} = 5.7\ \text{m}.$$

Note that an extra figure was needed for V in part (b) to avoid roundoff error.

8.73: Let v be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass m.

Conservation of energy says $\frac{1}{2} mv^2 = mgR$; $v = \sqrt{2gR}$

This is speed v_1 for the collision. Let v_2 be the speed of the combined object just after the collision. Conservation of momentum applied to the collision gives $mv_1 = 2mv_2$ so $v_2 = v_1/2 = \sqrt{gR/2}$

Apply conservation of energy to the motion of the combined object after the collision. Let y_3 be the final height above the bottom of the bowl.

$$\tfrac{1}{2}(2m)v_2^2 = (2m)gy_3$$

$$y_3 = \frac{v_2^2}{2g} = \frac{1}{2g}\left(\frac{gR}{2}\right) = R/4$$

Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.

8.74: Collision: Momentum conservation gives

$$mv_0 = mv_1 + (3m)v_3$$
$$v_0 = v_1 + 3v_3 \qquad (1)$$

Energy Conservation**:**

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(3m)v_3^2 \qquad (2)$$
$$v_0^2 = v_1^2 + 3v_3^2$$

Solve (1) and (2) for $v_3 : v_3 = 2.50 \text{ m/s}$

Energy conservation after collision:

$$\frac{1}{2}(3m)v_3^2 = (3m)gh = (3m)gl(1-\cos\theta)$$

Solve for $\theta : \theta = 68.8°$

8.75: First consider the motion after the collision. The combined object has mass $m_{\text{tot}} = 25.0$ kg. Apply $\Sigma\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the object at the top of the circular loop, where the object has speed v_3.

$$T + mg = m\frac{v_3^2}{R}$$

The minimum speed v_3 for the object not to fall out of the circle is given by setting $T = 0$. This gives $v_3 = \sqrt{Rg},$ where $R = 3.50$ m.

Next, use conservation of energy with point 2 at the bottom of the loop and point 3 at the top of the loop. Take $y = 0$ at the point 2. Only gravity does work, so

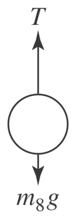
$$K_2 + U_2 = K_3 + U_3$$
$$\tfrac{1}{2}m_{\text{tot}}v_2^2 = \tfrac{1}{2}m_{\text{tot}}v_3^2 + m_{\text{tot}}g(2R)$$

Use $v_3 = \sqrt{Rg}$ and solve for $v_2 : v_2 = \sqrt{5gR} = 13.1 \text{ m/s}$

Now apply conservation of momentum to the collision between the dart and the sphere. Let v_1 be the speed of the dart before the collision.

$$(5.00 \text{ kg})v_1 = (25.0 \text{ kg})(13.1 \text{ m/s})$$
$$v_1 = 65.5 \text{ m/s}$$

8.76: Just after the collision: $\sum F = ma$



$$T - m_8 g = m_8 \frac{v_8^2}{R}$$

$$1600\,\text{N} - (8.00\,\text{kg})(9.80\,\text{m/s}^2) = (8.00\,\text{kg})\frac{v_8^2}{1.35\,\text{m}}$$

$$v_8 = 16.0\,\text{m/s}$$

Energy and momentum are conserved during the elastic collision.

$$\begin{aligned}
m_2 v_0 &= m_2 v_2 + m_8 v_8 \\
(2.00\,\text{kg})v_0 &= (2.00\,\text{kg})v_2 + (8.00\,\text{kg})(16.0\,\text{m/s}) \\
v_0 &= v_2 + 64.0\,\text{m/s} && (1) \\
\frac{1}{2} m_2 v_0^2 &= \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_8 v_8^2 \\
(2.00\,\text{kg})v_0^2 &= (2.00\,\text{kg})v_2^2 + (8.00\,\text{kg})(16.0\,\text{m/s})^2 \\
v_0^2 &= v_2^2 + 1024\,\text{m}^2/\text{s}^2 && (2)
\end{aligned}$$

Solve (1) and (2) for $v_0 : v_0 = 40.0\,\text{m/s}$

8.77: a) The coefficient of friction, from either force or energy consideration, is $\mu_k = v^2/2gs$, where v is the speed of the block after the bullet passes through. The speed of the block is determined from the momentum lost by the bullet, $(4.00 \times 10^{-3}\,\text{kg})(280\,\text{m/s}) = 1.12\,\text{kg} \cdot \text{m/s}$, and so the coefficient of kinetic friction is

$$\mu_k = \frac{((1.12\,\text{kg} \cdot \text{m/s})/(0.80\,\text{kg}))^2}{2(9.80\,\text{m/s}^2)(0.45\text{m})} = 0.22.$$

b) $\frac{1}{2}(4.00 \times 10^{-3}\,\text{kg})((400\,\text{m/s})^2 - (120\,\text{m/s})^2) = 291$ J. c) From the calculation of the momentum in part (a), the block's initial kinetic energy was $\frac{p^2}{2m} = \frac{(1.12\,\text{kg} \cdot \text{m/s})^2}{2(0.80\,\text{kg})} = 0.784\,\text{J}.$

8.78: The speed of the block after the bullet has passed through (but before the block has begun to rise; this assumes a large force applied over a short time, a situation characteristic of bullets) is

$$V = \sqrt{2gy} = \sqrt{2(9.80\ \text{m/s}^2)(0.45\times 10^{-2}\ \text{m})} = 0.297\ \text{m/s}.$$

The final speed v of the bullet is then

$$\begin{aligned} v &= \frac{p}{m} = \frac{mv_0 - MV}{m} = v_0 - \frac{M}{m}V \\ &= 450\ \text{m/s} - \frac{1.00\ \text{kg}}{5.00\times 10^{-3}\ \text{kg}}(0.297\ \text{m/s}) = 390.6\ \text{m/s}, \end{aligned}$$

or 390 m/s to two figures.

8.79: a) Using the notation of Eq. (8.24),

$$\begin{aligned} K_0 - K_2 &= \frac{1}{2}mv^2 - \frac{1}{2}mv_A^2 \\ &= \frac{1}{2}mv^2\left(1 - \left(\frac{m-M}{m+M}\right)^2\right) \\ &= K_0\left(\frac{(m+M)^2 - (m-M)^2}{(m+M)^2}\right) \\ &= K_0\left(\frac{4mM}{(m+M)^2}\right). \end{aligned}$$

b) Of the many ways to do this calculation, the most direct way is to differentiate the expression of part (a) with respect to M and set equal to zero;

$$\begin{aligned} 0 &= (4m\mathrm{K}_0)\frac{d}{dM}\left(\frac{M}{(m+M)^2}\right), \text{ or} \\ 0 &= \frac{1}{(m+M)^2} - \frac{2M}{(m+M)^3} \\ 0 &= (m+M) - 2M \\ m &= M. \end{aligned}$$

c) From Eq.(8.24), with $m_A = m_B = m, v_A = 0;$ the neutron has lost all of its kinetic energy.

8.80: a) From the derivation in Sec. 8.4 of the text we have

$$V_{\mathrm{A}} = \frac{M_{\mathrm{A}} - M_{\mathrm{B}}}{M_{\mathrm{A}} + M_{\mathrm{B}}} V_0 \text{ and } V_{\mathrm{B}} = \frac{2M_{\mathrm{A}}}{M_{\mathrm{A}} + M_{\mathrm{B}}}$$

The ratio of the kinetic energies of the two particles after the collision is

$$\frac{\frac{1}{2} M_{\mathrm{A}} V_{\mathrm{A}}^2}{\frac{1}{2} M_{\mathrm{B}} V_{\mathrm{B}}^2} = \frac{M_{\mathrm{A}}}{M_{\mathrm{B}}} \left(\frac{V_{\mathrm{A}}}{V_{\mathrm{B}}} \right)^2 = \frac{M_{\mathrm{A}}}{M_{\mathrm{B}}} \left(\frac{M_{\mathrm{A}} - M_{\mathrm{B}}}{2M_{\mathrm{A}}} \right)^2 = \frac{(M_{\mathrm{A}} - M_{\mathrm{B}})^2}{4M_{\mathrm{A}} M_{\mathrm{B}}}$$

$$\text{or } KE_{\mathrm{A}} = KE_{\mathrm{B}} \frac{(M_{\mathrm{A}} - M_{\mathrm{B}})^2}{4M_{\mathrm{A}} M_{\mathrm{B}}}$$

b) i) For $M_{\mathrm{A}} = M_{\mathrm{B}}, KE_{\mathrm{A}} = 0;$ i.e., the two objects simply exchange kinetic energies.

ii) For $M_{\mathrm{A}} = 5M_{\mathrm{B}}$,

$$\frac{KE_{\mathrm{A}}}{KE_{\mathrm{B}}} = \frac{(4M_{\mathrm{B}})^2}{4(5M_{\mathrm{B}})(M_{\mathrm{B}})} = \frac{4}{5}$$

i.e., M_{A} gets 4/9 or 44% of the total.

c) We want

$$\frac{KE_{\mathrm{A}}}{KE_{\mathrm{B}}} = 1 = \frac{(M_{\mathrm{A}} - M_{\mathrm{B}})^2}{4M_{\mathrm{A}} M_{\mathrm{B}}} = \frac{M_{\mathrm{A}}^2 - 2M_{\mathrm{A}} M_{\mathrm{B}} + M_{\mathrm{B}}^2}{4M_{\mathrm{A}} M_{\mathrm{B}}}$$

which reduces to

$$M_{\mathrm{A}}^2 - 6M_{\mathrm{A}} M_{\mathrm{B}} + M_{\mathrm{B}}^2 = 0,$$

from which, using the quadratic formula, we get the two possibilities $M_{\mathrm{A}} = 5.83\, M_{\mathrm{B}}$ and $M_{\mathrm{A}} = 0.172\, M_{\mathrm{B}}$.

8.81: a) Apply conservation of energy to the motion of the package from point 1 as it leaves the chute to point 2 just before it lands in the cart. Take y = 0 at point 2, so y_1 = 4.00 m. Only gravity does work, so

$$K_1 + U_1 = K_2 + U_2$$
$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2$$
$$v_2 = \sqrt{v_1^2 + 2gy_1} = 9.35\ \text{m/s}$$

b) In the collision between the package and the cart momentum is conserved in the horizontal direction. (But not in the vertical direction, due to the vertical force the floor exerts on the cart.) Take $+x$ to be to the right. Let A be the package and B be the cart.

P_x is constant gives
$$m_a v_{A1x} + m_B v_{B1x} = (m_A + M_B)v_{2x}$$
$$v_{B1x} = -5.00\ \text{m/s}$$
$v_{A1x} = (3.00\ \text{m/s})\cos 37.0^o$ (The horizontal velocity of the package is constant during its free-fall.)

Solving for v_{2x} gives $v_{2x} = -3.29\ \text{m/s}$. The cart is moving to the left at 3.29 m/s after the package lands in it.

8.82: Even though one of the masses is not known, the analysis of Section (8.4) leading to Eq. (8.26) is still valid, and $v_{\text{red}} = 0.200\ \text{m/s} + 0.050\ \text{m/s} = 0.250\ \text{m/s}$. b) The mass m_{red} may be found from either energy or momentum considerations. From momentum conservation,
$$m_{\text{red}} = \frac{(0.040\ \text{kg})(0.200\ \text{m/s} - 0.050\ \text{m/s})}{(0.250\ \text{m/s})} = 0.024\ \text{kg}.$$
As a check, note that

$$K_1 = \tfrac{1}{2}(0.040\ \text{kg})(0.200\ \text{m/s})^2 = 8.0 \times 10^{-4}\ \text{J, and}$$
$$K_2 = \tfrac{1}{2}(0.040\ \text{kg})(0.050\ \text{m/s})^2 + \tfrac{1}{2}(0.024\ \text{kg})(0.250\ \text{m/s})^2 = 8.0 \times 10^{-4}\ \text{J},$$
so $K_1 = K_2$, as it must for a perfectly elastic collision.

8.83: a) In terms of the primed coordinates,

$$\begin{aligned} v_A^2 &= (\vec{\boldsymbol{v}}'_A + \vec{\boldsymbol{v}}_{\text{cm}}) \cdot (\vec{v}'_A + \vec{v}_{\text{cm}}) \\ &= \vec{\boldsymbol{v}}'_A \cdot \vec{\boldsymbol{v}}'_A + \vec{\boldsymbol{v}}_{\text{cm}} \cdot \vec{\boldsymbol{v}}_{\text{cm}} + 2\vec{\boldsymbol{v}}'_A \cdot \vec{\boldsymbol{v}}_{\text{cm}} \\ &= v_A'^{\,2} + v_{\text{cm}}{}^2 + 2\vec{\boldsymbol{v}}'_A \cdot \vec{\boldsymbol{v}}_{\text{cm}}, \end{aligned}$$

with a similar expression for v_{B}^2. The total kinetic energy is then

$$\begin{aligned} K &= \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \\ &= \frac{1}{2} m_A \left(v_A'^{\,2} + v_{\text{cm}}{}^2 + 2\vec{\boldsymbol{v}}'_A \cdot \vec{\boldsymbol{v}}_{\text{cm}} \right) + \frac{1}{2} m_B \left(v_B'^{\,2} + v_{\text{cm}}{}^2 + 2\vec{\boldsymbol{v}}'_B \cdot \vec{\boldsymbol{v}}_{\text{cm}} \right) \\ &= \frac{1}{2}(m_A + m_B) v_{\text{cm}}^2 + \frac{1}{2}\left(m_A \vec{\boldsymbol{v}}_A'^{\,2} + m_B \vec{\boldsymbol{v}}_B'^{\,2} \right) \\ &\quad + 2\left[m_A \vec{\boldsymbol{v}}'_A . \vec{v}_{\text{cm}} + m_B \vec{\boldsymbol{v}}'_B . \vec{v}_{\text{cm}} \right]. \end{aligned}$$

The last term in brackets can be expressed as

$$2(m_A \vec{\boldsymbol{v}}'_A + m_B\, \vec{\boldsymbol{v}}'_B) \cdot \vec{\boldsymbol{v}}_{\text{cm}},$$

and the term

$$\begin{aligned} m_A \vec{\boldsymbol{v}}'_A + \ m_B \vec{\boldsymbol{v}}'_B &= m_A \vec{\boldsymbol{v}}'_A + m_B \vec{\boldsymbol{v}}'_B - (\, m_A + \mathrm{m}_B) \vec{\boldsymbol{v}}_{\text{cm}} \\ &= \mathbf{0}, \end{aligned}$$

and so the term in square brackets in the expression for the kinetic energy vanishes, showing the desired result. b) In any collision for which other forces may be neglected the velocity of the center of mass does not change, and the $\frac{1}{2} M v_{\text{cm}}^2$ in the kinetic energy will not change. The other terms can be zero (for a perfectly inelastic collision, which is not likely), but never negative, so the minimum possible kinetic energy is $\frac{1}{2} M v_{\text{cm}}^2$.

8.84: a) The relative speed of approach before the collision is the relative speed at which the balls separate after the collision. Before the collision, they are approaching with relative speed $2v$, and so after the collision they are receding with speed $2v$. In the limit that the larger ball has the much larger mass, its speed after the collision will be unchanged (the limit as $m_{\text{A}} >> m_{\text{B}}$ in Eq. (8.24)), and so the small ball will move upward with speed $3v$. b) With three times the speed, the ball will rebound to a height time times greater than the initial height.

8.85: a) If the crate had final speed v, J&J have speed $4.00\ \text{m/s} - v$ relative to the ice, and so $(15.0\ \text{kg})v = (120.0\ \text{kg})(4.00\ \text{m/s} - v)$. Solving for v, $v = \frac{(120.0\ \text{kg})\,(4.00\ \text{m/s})}{(135.0\ \text{kg})} = 3.56\ \text{m/s}$.

b) After Jack jumps, the speed of the crate is $\frac{(75.0\ \text{kg})}{(135.0\ \text{kg})}(4.00\ \text{m/s}) = 2.222\ \text{m/s}$, and the momentum of Jill and the crate is $133.3\ \text{kg}\cdot\text{m/s}$. After Jill jumps, the crate has a speed v and Jill has speed $4.00\ \text{m/s} - v$, and so

$133.3\ \text{kg}\cdot\text{m/s} = (15.0\ \text{kg})v - (45.0\ \text{kg})(4.00\ \text{m/s} - v)$, and solving for v gives $v = 5.22\ \text{m/s}$. c) Repeating the calculation for part (b) with Jill jumping first gives a final speed of $4.67\ \text{m/s}$.

8.86: (a) For momentum to be conserved, the two fragments must depart in opposite directions. We can thus write

$$M_{\mathrm{A}}V_{\mathrm{A}} = -M_{\mathrm{B}}V_{\mathrm{B}}$$

Since $M_{\mathrm{A}} = M - M_{\mathrm{B}}$, we have

$$(M - M_{\mathrm{B}})V_{\mathrm{A}} = -M_{\mathrm{B}}V_{\mathrm{B}}$$

$$\frac{V_{\mathrm{A}}}{V_{\mathrm{B}}} = \frac{-M_{\mathrm{B}}}{M - M_{\mathrm{B}}}$$

Then for the ratio of the kinetic energies

$$\frac{KE_{\mathrm{A}}}{KE_{\mathrm{B}}} = \frac{\frac{1}{2}M_{\mathrm{A}}V_{\mathrm{A}}^2}{\frac{1}{2}M_{\mathrm{B}}V_{\mathrm{B}}^2} = \frac{M_{\mathrm{A}}}{M_{\mathrm{B}}}\frac{M_{\mathrm{B}}^2}{(M - M_{\mathrm{B}})^2} = \frac{M_{\mathrm{B}}}{M_{\mathrm{A}}}$$

The ratio of the KE's is simply the inverse ratio of the masses.

From the two equations

$$KE_{\mathrm{A}} = \frac{M_{\mathrm{B}}}{M_{\mathrm{A}}}KE_{\mathrm{B}} \text{ and } KE_{\mathrm{A}} + KE_{\mathrm{B}} = Q$$

We can solve for KE_{B} to find

$$KE_{\mathrm{A}} = Q - KE_{\mathrm{B}} = Q\left(1 - \frac{M_{\mathrm{A}}}{M_{\mathrm{A}} + M_{\mathrm{B}}}\right)$$

$$KE_{\mathrm{B}} = \frac{Q}{1 + \frac{M_{\mathrm{B}}}{M_{\mathrm{A}}}} = \frac{QM_{\mathrm{A}}}{M_{\mathrm{A}} + M_{\mathrm{B}}}$$

(b) If $M_{\mathrm{B}} = 4M_{\mathrm{A}}$, then M_{A} will get 4 times as much KE as M_{B}, or 80% of Q for M_{A} and 20% for M_{B}.

8.87: Let the proton be moving in the $+x$-direction with speed v_p after the decay. The initial momentum of the neutron is zero, so to conserve momentum the electron must be moving in the $-x$-direction after the collision; let its speed be v_e.

P_x is constant gives $0 = -m_e v_e + m_p v_p$.

$v_e = (m_p/m_e)v - p$

The total kinetic energy after decay is $K_{\text{tot}} = \frac{1}{2}m_e v_e^2 + \frac{1}{2}m_p v_p^2$. Using the momentum equation to replace v_e gives $K_{\text{tot}} = \frac{1}{2}m_p v_p^2\,(1 + m_p/m_e)$.

$$\text{Thus}\frac{K_p}{K_{\text{tot}}} = \frac{1}{1+m_{\text{p}}/m_e} = \frac{1}{1836} = 5.44\times10^{-4} = 0.0544\ \%$$

8.88: The ratios that appear in Eq. (8.42) are $\frac{0.0176}{1.0176}$ and $\frac{1}{1.0176}$, so the kinetic energies are a) $\frac{0.0176}{1.0176}(6.54\times10^{-13}\,\text{J}) = 1.13\times10^{-14}$ J and b) $\frac{1}{1.0176}(6.54\times10^{-13}\,\text{J}) = 6.43\times10^{-13}\,\text{J}$.
Note that the energies do not add to $6.54\times10^{-13}\,\text{J}$ exactly, due to roundoff.

8.89: The "missing momentum" is

$$5.60\times10^{-22}\ \text{kg}\cdot\text{m/s} - (3.50\times10^{-25}\,\text{kg})(1.14\times10^{3}\ \text{m/s}) = 1.61\times10^{-22}\,\text{kg}\cdot\text{m/s}\ .$$

Since the electron has momentum to the right, the neutrino's momentum must be to the left.

8.90: a) For the x - and y - directions, respectively, and m as the common mass of a proton,

$$mv_{A1} = mv_{A2}\cos\alpha + mv_2\cos\beta$$
$$0 = mv_{A2}\sin\alpha - mv_{B2}\sin\beta$$

or

$$v_{A1} = v_{A2}\cos\alpha + v_{B2}\cos\beta$$
$$0 = v_{A2}\sin\alpha - v_{B2}\sin\beta.$$

b) After minor algebra,

$$v_{A1}^2 = v_{A2}^2 + v_{B2}^2 + 2v_{A2}v_{B2}(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$
$$= v_{A2}^2 + v_{B2}^2 + 2v_{A2}v_{B2}\cos(\alpha+\beta).$$

c) For a perfectly elastic collision,

$$\frac{1}{2}mv_{A1}^2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2 \text{ or } v_{A1}^2 = v_{A2}^2 + v_{B2}^2.$$

Substitution into the above result gives $\cos(\alpha+\beta) = 0$. d) The only positive angle with zero cosine is $\frac{\pi}{2}(90°)$.

8.91: See Problem 8.90. Puck B moves at an angle $65.0°$(i.e.$90° - 25° = 65°$) from the original direction of puck A's motion, and from conservation of momentum in the y-direction, $v_{B2} = 0.466v_{A2}$. Substituting this into the expression for conservation of momentum in the x – direction, $v_{A2} = v_{A1}/(\cos 25.0° + 0.466\cos 65°) = 13.6\text{ m/s}$, and so $v_{B2} = 6.34$ m/s.

As an alternative, a coordinate system may be used with axes along the final directions of motion (from Problem 8.90, these directions are known to be perpendicular). The initial direction of the puck's motion is $25.0°$ from the final direction, so $v_{A2} = v_{A1}\cos 25.0°$ and $v_{B2} = v_{A1}\cos 65.0$, giving the same results.

8.92: Since mass is proportional to weight, the given weights may be used in determining velocities from conservation of momentum. Taking the positive direction to the left,

$$v = \frac{(800\ \text{N})(5.00\ \text{m/s})\cos 30.0^\circ - (600\ \text{N})(7.00\ \text{m/s})\cos 36.9^\circ}{1000\ \text{N}} = 0.105\ \text{m/s}$$

8.93: a) From symmetry, the center of mass is on the vertical axis, a distance $(L/2)\cos(\alpha/2)$ from the apex. b) The center of mass is on the (vertical) axis of symmetry, a distance $2(L/2)/3 = L/3$ from the center of the bottom of the $\sqcup$. c) Using the wire frame as a coordinate system, the coordinates of the center of mass are equal, and each is equal to $(L/2)/2 = L/4$. The distance of this point from the corner is $\left(1/\sqrt{8}\right)L = (0.353)L.$ This may also be found from consideration of the situation of part (a), with $\alpha = 45^\circ$ d) By symmetry, the center of mass is in the center of the equilateral triangle, a distance $(L/2)(\tan 60^\circ) = L/\sqrt{12} = (0.289)L$ above the center of the base.

8.94: The trick here is to notice that the final configuration is the same as if the canoe (assumed symmetrical) has been rotated about its center of mass. Intially, the center of mass is a distance $\frac{(45.0\ \text{kg})(1.5\ \text{m})}{(105\ \text{kg})} = 0.643\ \text{m}$ from the center of the canoe, so in rotating about this point the center of the canoe would move $2 \times 0.643\ \text{m} = 1.29\ \text{m}.$

8.95: Neglecting friction, the total momentum is zero, and your speed will be one-fifth of the slab's speed, or 0.40 m/s.

8.96: The trick here is to realize that the center of mass will continue to move in the original parabolic trajectory, "landing" at the position of the original range of the projectile. Since the explosion takes place at the highest point of the trajectory, and one fragment is given to have zero speed after the explosion, neither fragment has a vertical component of velocity immediately after the explosion, and the second fragment has *twice* the velocity the projectile had before the explosion. a) The fragments land at positions symmetric about the original target point. Since one lands at $\frac{1}{2}R,$ the other lands at

$$\frac{3}{2}R = \frac{3}{2}\frac{v_0^2}{g}\sin 2\alpha_0 = \frac{3}{2}\frac{(80\,\mathrm{m/s})^2}{(9.80\,\mathrm{m/s^2})}\sin 120^\circ = 848\,\mathrm{m}.$$

b) In terms of the mass m of the original fragment and the speed v before the explosion, $K_1 = \frac{1}{2}mv^2$and $K_2 = \frac{1}{2}\frac{m}{2}(2v)^2 = mv^2$,so $\Delta K = mv^2 - \frac{1}{2}mv^2 = \frac{1}{2}mv^2.$ The speed v is related to v_0 by $v = v_0\cos\alpha_0$, so

$$\Delta K = \frac{1}{2}mv_0^2\cos^2\alpha_0 = \frac{1}{2}(20.0\text{ kg})(80\text{ m/s})\cos\,60.0^\circ)^2 = 1.60\times10^4\text{ J}.$$

8.97: Apply conservation of energy to the explosion. Just before the explosion the sheel is at its maximum height and has zero kinetic energy. Let A be the piece with mass 1.40 kg and B be the piece with mass 0.28 kg. Let v_A and v_B be the speeds of the two pieces immediately after the collision.

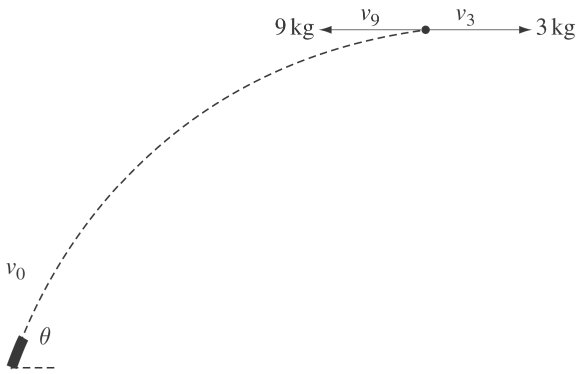
$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = 860\ \text{J}$$

Since the two fragments reach the ground at the same time, their velocitues just after the explosion must be horizontal. The initial momentum of the shell before the explosion is zero, so after the explosion the pieces must be moving in opposite horizontal directions and have equal magnitude of momentum**:** $m_A v_A = m_B v_B$.

Use this to eliminate v_A in the first equation and solve for v_B :
$\tfrac{1}{2}m_B v_B^2 (1 + m_B / m_A) = 860\ \text{J}$ and $v_B = 71.6\ \text{m/s}$.
Then $v_A = (m_B / m_A) v_B = 14.3\ \text{m/s}$.

b) Use the vertical motion from the maximum height to the ground to find the time it takes the pieces to fall to the ground after the explosion. Take $+y$ downward.
$v_{0y} = 0, a_y = +9.80\ \text{m/s}^2, y - y_0 = 80.0\ \text{m}, t = ?$
$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $\text{t} = 4.04\ \text{s}$.
During this time the horizontal distance each piece moves is
$x_A = v_A t = 57.8\ \text{m}$ and $x_B = v_B t = 289.1\ \text{m}$.
They move in opposite directions, so they are $x_A + x_B = 347\ \text{m}$ apart when they land.

8.98: The two fragments are 3.00 kg and 9.00 kg. Time to reach maximum height = time to fall back to the ground.



$$v_9 = v_0 \sin\theta - gt$$

$$0 = (150\,\text{m/s})\sin 55.0^\circ - 9.8\,\text{m/s}^2\, t$$

$$t = 12.5\,\text{s}.$$

The heavier fragment travels back to its starting point, so it reversed its velocity. $v_x = v_0 \cos\theta = (150\,\text{m/s})\cos 55^\circ = 86.0\ \text{m/s}$ to the left after the explosion; this is v_9. Now get v_3 using momentum conversation.

$$Mv_0 = m_3 v_3 + m_9 v_9$$

$$(12\,\text{kg})(86.0\,\text{m/s}) = (3.00\,\text{kg})v_3 + (9.00\text{kg})(-86.0\,\text{m/s})$$

$$v_3 = 602\,\text{m/s}$$

$x_3 = x_{\text{Before explosion}} + x_{\text{After explosion}} = (86.0\ \text{m/s})(12.5\,\text{s}) + (602\,\text{m/s})(12.5)$

$x_3 = 8600\text{m from where it was launched}$

Energy released = Energy after explosion – Energy before explosion

$$= \frac{1}{2}\text{m}_3 v_3^2 + \frac{1}{2}\text{m}_9 v_9^2 - \frac{1}{2}(m_3 + m_9)v_9^2$$

$$= \frac{1}{2}(3.00\,\text{kg})(602\ \text{m/s})^2 + \frac{1}{2}(9.00\,\text{kg})(86.0\ \text{m/s})^2$$

$$-\frac{1}{2}(12.0\,\text{kg})(86.0\ \text{m/s})^2$$

$$= 5.33\times 10^5\ \text{J}$$

8.99: The information is not sufficient to use conservation of energy. Denote the emitted neutron that moves in the $+y$-direction by the subscript 1 and the emitted neutron that moves in the –*y*-direction by the subscript 2. Using conservation of momentum in the *x*- and *y*-directions, neglecting the common factor of the mass of neutron,

$$v_0 = (2v_0/3)\cos 10° + v_1 \cos 45° + v_2 \cos 30°$$
$$0 = (2v_0/3)\sin 10° + v_1 \sin 45° - v_2 \sin 30°.$$

With $\sin 45° = \cos 45°$, these two relations may be subtracted to eliminate $v_{1,}$ and rearrangement gives

$$v_0(1 - (2/3)\cos 10° + (2/3)\sin 10°) = v_2(\cos 30° + \sin 30°),$$

from which $v_2 = 1.01 \times 10^3\ \text{m/s or}\ 1.0 \times 10^3\ \text{m/s}$ to two figures. Substitution of this into either of the momentum relations gives $v_1 = 221\ \text{m/s}$. All that is known is that there is no z-component of momentum, and so only the ratio of the speeds can be determined. The ratio is the inverse of the ratio of the masses, so $v_{\text{Kr}} = (1.5)v_{\text{Ba}}$.

8.100: a) With block *B* initially at rest, $v_{\text{cm}} = \frac{m_A}{m_A + m_B} v_{A1}$. b) Since there is no net external force, the center of mass moves with constant velocity, and so a frame that moves with the center of mass is an inertial reference frame. c) The velocities have only x-components, and the x-components are

$u_{A1} = v_{A1} - v_{\text{cm}} = \frac{m_B}{m_A + m_B} v_{A1}, u_{B1} = -v_{\text{cm}} = -\frac{m_A}{m_A + m_B} u_{A1}$. Then, $P_{\text{cm}} = m_A u_{A1} + m_B u_{B1} = 0$.

d) Since there is zero momentum in the center-of-mass frame before the collision, there can be no momentum after the collision; the momentum of each block after the collision must be reversed in direction. The only way to conserve kinetic energy is if the

momentum of each has the same magnitude so in the center-of-mass frame, the blocks change direction but have the same speeds. Symbolically, $u_{A2} = -u_{A1}, u_{B2} = -u_{B1}$. e) The velocities all have only x–components; these components are

$u_{A1} = \frac{0.200}{0.600} 6.00\ \text{m/s} = 2.00\ \text{m/s}, u_{B1} = -\frac{0.400}{0.600} 6.00\ \text{m/s} = -4.00\ \text{m/s}, u_{A2} = -2.00\ \text{m/s}, u_{B2} = 4.$ and $v_{A2} = +2.00\ \text{m/s}, v_{B2} = 8.00\ \text{m/s}.$ and Equation (8.24) predicts $v_{A2} = +\frac{1}{3} v_{A1}$ and Eq. (8.25) predicts $v_{B2} = \frac{4}{3} u_{A1}$, which are in agreement with the above.

8.101: a) If the objects stick together, their relative speed is zero and $\in = 0$. b) From Eq. (8.27), the relative speeds are the same, and $\in = 1$. c) Neglecting air resistance, the speeds before and after the collision are $\sqrt{2gh}$ and $\sqrt{2gH_1}$, and $\in = \frac{\sqrt{2gH_1}}{\sqrt{2gh}} = \sqrt{H_1/h}$. d) From part (c), $H_1 = \in^2 h = (0.85)^2\,(1.2\text{ m}) = 0.87\text{ m}$. e) $H_{k+1} = H_k \in^2$, and by induction $H_n = \in^{2n} h$. f) $(1.2\text{ m})(0.85)^{16} = 8.9\text{ cm}$.

8.102: a) The decrease in potential energy $(-\Delta < 0)$ means that the kinetic energy increases. In the center of mass frame of two hydrogen atoms, the net momentum is necessarily zero and after the atoms combine and have a common velocity, that velocity must have zero magnitude, a situation precluded by the necessarily positive kinetic energy. b) The initial momentum is zero before the collision, and must be zero after the collision. Denote the common initial speed as v_0, the final speed of the hydrogen atom as v, the final speed of the hydrogen molecule as V, the common mass of the hydrogen atoms as m and the mass of the hydrogen molecules as $2m$. After the collision, the two particles must be moving in opposite directions, and so to conserve momentum, $v = 2V$. From conservation of energy,

$$\frac{1}{2}(2m)V^2 - \Delta + \frac{1}{2}mv^2 = 3\tfrac{1}{2}mv_0^2$$
$$mV^2 - \Delta + 2mV^2 = \tfrac{3}{2}mv_0^2$$
$$V^2 = \frac{v_0^2}{2} + \frac{\Delta}{3m},$$

from which $V = 1.203 \times 10^4$ m/s, or 1.20×10^4 m/s to two figures and the hydrogen atom speed is $v = 2.41 \times 10^4$ m/s.

8.103: a) The wagon, after coming down the hill, will have speed $\sqrt{2gL\sin\alpha}$ = 10 m/s. After the "collision", the speed is $\left(\frac{300\text{ kg}}{435\text{ kg}}\right)(10\text{ m/s}) = 6.9\text{ m/s}$, and in the 5.0 s, the wagon will not reach the edge. b) The "collision" is completely inelastic, and kinetic energy is not conserved. The change in kinetic energy is
$\tfrac{1}{2}(435\text{ kg})(6.9\text{ m/s})^2 - \tfrac{1}{2}(300\text{ kg})(10\text{ m/s})^2 = -4769\text{ J}$, so about 4800 J is lost.

8.104: a) Including the extra force, Eq. (8.37) becomes

$$m\frac{dv}{dt} = -v_{\text{ex}}\frac{dm}{dt} - mg,$$

where the positive direction is taken upwards (usually a sign of good planning). b) Diving by a factor of the mass m,

$$a = \frac{dv}{dt} = -\frac{v_{\text{ex}}}{m}\frac{dm}{dt} - g.$$

c) $20\ \text{m/s}^2 - 9.80\ \text{m/s}^2 = 10.2\ \text{m/s}^2$. d) $3327\ \text{m/s} - (9.80\ \text{m/s}^2)(90) = 2.45\ \text{km/s}$, which is about three-fourths the speed found in Example 8.17.

8.105: a) From Eq. (8.40), $v = v_{\text{ex}}\ln\left(\frac{13{,}000\ \text{kg}}{3{,}300\ \text{kg}}\right) = (1.37)v_{\text{ex}}$.

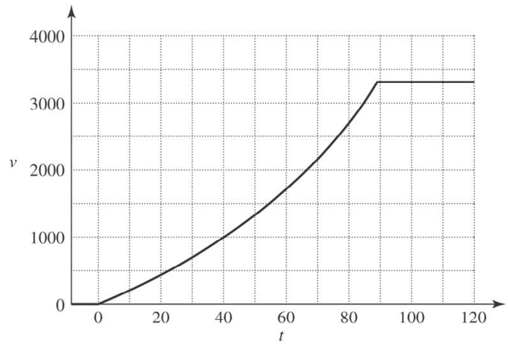
b) $v_{\text{ex}}\ln(13{,}000/4{,}000) = (1.18)v_{\text{ex}}$.

c) $(1.18)v_{\text{ex}} + v_{\text{ex}}\text{In}(1000/300) = (2.38)v_{\text{ex}}$. d) Setting the result of part (c) equal to 7.00 km/s and solving for v_{ex} gives $v_{\text{ex}} = 2.94\ \text{km/s}$.

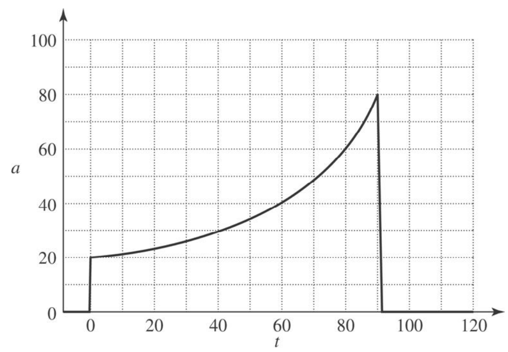
8.106: a) There are two contribution to $F_{\text{net}}, F_{\text{net}} = v_{\text{ex}}\,|\,dm/dt\,| - v\,|\,dm/dt\,|$, or $F_{\text{net}} = (v_{\text{ex}} - v)|dm/dt|$.

b) $F_{\text{net}}/|\,dm/dt\,| = (1300\ \text{N})/(150\ \text{kg/s}) = 8.66\ \text{m/s} = 31$ km/h. This equal to $v_{\text{ex}} - v$.

8.107: a) For $t < 0$ the rocket is at rest. For $0 \le t \le 90$ s, Eq. (8.40) is valid, and $v(t) = (2400\,\text{m/s})\ln(1/(1-(t/120\,\text{s})))$. At $t = 90$ s, this speed is 3.33 km/s, and this is also the speed for $t > 90$ s.



b) The acceleration is zero for $t < 0$ and $t > 90$ s. For $0 \le t \le 90$ s, Eq. (8.39) gives, with $\frac{dm}{dt} = -m_0/120\,\text{s}$, $a = \frac{20\,\text{m/s}^2}{(1-(t/120\,\text{s}))}$.



c) The maximum acceleration occurs at the latest time of firing, $t = 90$ s, at which time the acceleration is, from the result of part (a), $\frac{20\,\text{m/s}^2}{(1-90/120)} = 80\,\text{m/s}^2$, and so the astronaut is subject to a force of 6.0 kN, about eight times her weight on earth.

8.108: The impulse applied to the cake is $J = \mu_{k1}mgt = mv$, where m is the mass of the cake and v is its speed after the impulse is applied. The distance d that the cake moves during this time is then $d = \frac{1}{2}\mu_{k1}gt^2$. While sliding on the table, the cake must lose its kinetic energy to friction, or $\mu_{k2}mg(r-d) = \frac{1}{2}mv^2$. Simplification and substitution for v gives $r - d = \frac{1}{2}g\frac{\mu_{k1}^2}{\mu_{k2}}t^2$, substituting for d in terms of t^2 gives

$$r = \frac{1}{2}gt^2\left(\mu_{k1} + \frac{\mu_{k1}^2}{\mu_{k2}}\right) = \frac{1}{2}gt^2\frac{\mu_{k1}}{\mu_{k2}}(\mu_{k1} + \mu_{k2}),$$

which gives t = 0.59 s.

8.109: a) Noting than $dm = \frac{M}{L}dx$ avoids the intermediate variable ρ. Then,

$$x_{cm} = \frac{1}{M}\int_0^L x\frac{M}{L}dx = \frac{L}{2}.$$

b) In this case, the mass M may be found in terms of ρ and L, specifically by using $dm = \rho Adx = \alpha Adx$ to find that $M = \alpha A\int xdx = \alpha AL^2/2$. Then,

$$x_{cm} = \frac{2}{\alpha AL^2}\int_0^L \alpha Ax^2dx = \frac{2}{\alpha AL^2}\frac{L^3}{3} = \frac{2L}{3}.$$

8.110: By symmetry, $x_{cm} = 0$. Using plane polar coordinates leads to an easier integration, and using the Theorem of Pappus $\left(2\pi y_{cm}\left(\frac{\pi a^2}{2}\right) = \frac{4}{3}\pi a^3\right)$ is easiest of all, but the method of Problem 8.109 involves Cartesian coordinates.

For the x-coordinate, $dm = \rho t\sqrt{a^2 - x^2}\,dx$, which is an even function of x, so $\int x\,dx = 0$. For the y-coordinate, $dm = \rho t2\sqrt{a^2 - y^2}\,dy$, and the range of integration is from 0 to a, so

$$y_{cm} = \frac{2\rho t}{M}\int_0^a y\sqrt{a^2 - y^2},dy.$$

Making the substitutions $M = \frac{1}{2}\rho\pi a^2t$, $u = a^2 - y^2$, $du = -2y$, and

$$y_{cm} = \frac{-2}{\pi a^2}\int_{a^2}^{o} u^{\frac{1}{2}}du = \frac{-4}{3\pi a^2}\left[u^{\frac{3}{2}}\right]_{a^2}^{0} = \frac{4a}{3\pi}.$$

8.111: a) The tension in the rope at the point where it is suspended from the table is $T=(\lambda x)g$, where x is the length of rope over the edge, hanging vertically. In raising the rope a distance $-dx$, the work done is $(\lambda g)\,x\,(-dx)(dx \text{ is negative})$. The total work done is then

$$-\int_{l/4}^{0}(\lambda g)\,x\,dx=(\lambda g)\frac{x^2}{2}\Big|_0^{l/4}=\frac{\lambda g l^2}{32}.$$

b) The center of mass of the hanging piece is initially a distance $l/8$ below the top of the table, and the hanging weight is $(\lambda g)(l/4)$, so the work required to raise the rope is $(\lambda g)(l/4)(l/8)=\lambda g l^2/32,$ as before.

8.112: a) For constant acceleration a, the downward velocity is $v=at$ and the distance x that the drop has fallen is $x=\frac{1}{2}at^2$. Substitution into the differential equation gives

$$\frac{1}{2}at^2g=\frac{1}{2}at^2a+(at)^2=\frac{3}{2}a^2t^2,$$

the non-zero solution of which is $a=\frac{g}{3}$.

b) $$\frac{1}{2}at^2=\frac{1}{2}\left(\frac{9.80\ \text{m/s}^2}{3}\right)(3.00\ \text{s})^2=14.7\ \text{m}.$$

c) $kx=(2.00\ \text{g/m})(14.7\ \text{m})=29.4\ \text{g}.$

9.1: a) $$\frac{1.50\ \text{m}}{2.50\ \text{m}} = 0.60\ \text{rad} = 34.4°.$$

b) $$\frac{(14.0\ \text{cm})}{(128°)(\pi\ \text{rad}/180°)} = 6.27\ \text{cm}.$$

c) $$(1.50\ \text{m})(0.70\ \text{rad}) = 1.05\ \text{m}.$$

9.2: a) $$\left(1900\frac{\text{rev}}{\text{min}}\right)\times\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 199\ \text{rad/s}.$$

b) $$(35° \times \pi\ \text{rad}/180°)/(199\ \text{rad/s}) = 3.07\times 10^{-3}\ \text{s}.$$

9.3: a) $\alpha_z = \dfrac{d\omega_z}{dt} = (12.0\ \text{rad/s}^3)\,t$, so at $t = 3.5\ \text{s}$, $\alpha = 42\ \text{rad/s}^2$. The angular acceleration is proportional to the time, so the average angular acceleration between any two times is the arithmetic average of the angular accelerations. b) $\omega_z = (6.0\ \text{rad/s}^3)t^2$, so at $t = 3.5\ \text{s}$, $\omega_z = 73.5\ \text{rad/s}$. The angular velocity is not linear function of time, so the average angular velocity is not the arithmetic average or the angular velocity at the midpoint of the interval.

9.4: a) $$\alpha_z(\text{t}) = \tfrac{d\omega_z}{dt} = -2\beta t = (-1.60\ \text{rad/s}^3)t.$$

b) $$\alpha_z(3.0\ \text{s}) = (-1.60\ \text{rad/s}^3)(3.0\ \text{s}) = -4.80\ \text{rad/s}^2.$$

$$\alpha_{\text{av}-z} = \frac{\omega(3.0\ \text{s}) - \omega(0)}{3.0\ \text{s}} = \frac{-2.20\ \text{rad/s} - 5.00\ \text{rad/s}}{3.0\ \text{s}} = -2.40\ \text{rad/s}^2,$$

which is half as large (in magnitude) as the acceleration at $t = 3.0\ \text{s}$.

9.5: a) $\omega_z = \gamma + 3\beta t^2 = (0.400\ \text{rad/s}) + (0.036\ \text{rad/s}^3)t^2$ b) At $t = 0$, $\omega_z = \gamma = 0.400\ \text{rad/s}$. c) At $t = 5.00\ \text{s}$, $\omega_z = 1.3\ \text{rad/s}$, $\theta = 3.50\ \text{rad}$, so $\omega_{\text{av}-z} = \frac{3.50\ \text{rad}}{5.00\ \text{s}} = 0.70\ \text{rad/s}$. The acceleration is not constant, but increasing, so the angular velocity is larger than the average angular velocity.

9.6: $\omega_z = (250\ \text{rad/s}) - (40.0\ \text{rad/s}^2)t - (4.50\ \text{rad/s}^3)t^2$, $\alpha_z = -(40.0\ \text{rad/s}^2) - (9.00\ \text{rad/s}^3)t$. a) Setting $\omega_z = 0$ results in a quadratic in t; the only positive time at which $\omega_z = 0$ is $t = 4.23\text{s}$. b) At $t = 4.23\ \text{s}$, $\alpha_z = -78.1\ \text{rad/s}^2$.
c) At $t = 4.23\ \text{s}$, $\theta = 586\ \text{rad} = 93.3\ \text{rev}$.
d) At $t = 0$, $\omega_z = 250\ \text{rad/s}$. e) $\omega_{\text{av}-z} = \frac{586\ \text{rad}}{4.23\ \text{s}} = 138\ \text{rad/s}$.

9.7: a) $\omega_z = \frac{d\theta}{dt} = 2bt - 3ct^2$ and $\alpha_z = \frac{dw_z}{dt} = 2b - 6ct$. b) Setting $\alpha_z = 0, t = \frac{b}{3c}$.

9.8: (a) The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) The angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{8.00\ \text{rad/s} - (-6.00\ \text{rad/s})}{7.00\ \text{s}} = 2.00\ \text{rad/s}^2$$

Thus it takes 3.00 seconds for the wheel to stop $(\omega_z = 0)$. During this time its speed is decreasing. For the next 4.00 s its speed is increasing from $0\ \text{rad/s}$ to $+8.00\ \text{rad/s}$.

(c) We have

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ &= 0 + (-6.00\ \text{rad/s})(7.00\ \text{s}) + \tfrac{1}{2}(2.00\ \text{rad/s}^2)(7.00\ \text{s})^2 \\ &= -42.0\ \text{rad} + 49.0\ \text{rad} = +7.00\ \text{rad}.\end{aligned}$$

Alternatively, the average angular velocity is

$$\frac{-6.00\ \text{rad/s} + 8.00\ \text{rad/s}}{2} = 1.00\ \text{rad/s}$$

Which leads to displacement of 7.00 rad after 7.00 s.

9.9: a) $\omega - \theta_0 = 200\ \text{rev}$, $\omega_0 = 500\ \text{rev/min} = 8.333\ \text{rev/s}$, $t = 30.0\ \text{s}$, $\omega = ?$

$$\theta - \theta_0 = \left(\frac{\omega_0 + \omega}{2}\right)t \text{ gives } \omega = 5.00\ \text{rev/s} = 300\ \text{rpm}$$

b) Use the information in part (a) to find α:

$\omega = \omega_0 + \alpha t$ gives $\alpha = -0.1111\ \text{rev/s}^2$

Then $\omega = 0$, $\alpha = -0.1111\ \text{rev/s}^2$, $\omega_0 = 8.333\ \text{rev/s}$, $t = ?$

$\omega = \omega_0 + \alpha t$ gives $t = 75.0$ and

$$\theta - \theta_0 = \left(\frac{\omega_0 + \omega}{2}\right)t \text{ gives } \theta - \theta_0 = 312\ \text{rev}$$

9.10: a) $\omega_z = \omega_{0z} + \alpha_z t = 1.50 \text{ rad/s} + (0.300 \text{ rad/s}^2)(2.50 \text{ s}) = 2.25 \text{ rad/s}.$

b) $\theta = \omega_{0z} t + 1/2\, \alpha_z t^2 = (1.50 \text{ rad/s})(2.50 \text{ s}) + \frac{1}{2}(0.300 \text{ rad/s}^2)(2.50 \text{ s})^2 = 4.69 \text{ rad}.$

9.11: a) $$\frac{(200 \text{ rev/min} - 500 \text{ rev/min}) \times \left(\frac{1 \text{ min}}{60 \text{ s}}\right)}{(4.00 \text{ s})} = -1.25 \frac{\text{rev}}{\text{s}^2}.$$

The number of revolutions is the average angular velocity, 350 rev/min, times the time interval of 0.067 min, or 23.33 rev. b) The angular velocity will decrease by another 200 rev/min in a time $\frac{200 \text{ rev/min}}{60 \text{ s/min}} \cdot \frac{1}{1.25 \text{ rev/s}^2} = 2.67 \text{ s}.$

9.12: a) Solving Eq. (9.7) for *t* gives $t = \frac{\omega_z - \omega_{0z}}{\alpha_z}$.

Rewriting Eq. (9.11) as $\theta - \theta_0 = t(\omega_{0z} + \frac{1}{2}\alpha_z t)$ and substituting for *t* gives

$$\begin{aligned}\theta - \theta_0 &= \left(\frac{\omega_z - \omega_{0z}}{\alpha_z}\right)\left(\omega_{0z} + \frac{1}{2}(\omega_z - \omega_{0z})\right) \\ &= \frac{1}{\alpha}(\omega_z - \omega_{0z})\left(\frac{\omega_z + \omega_{0z}}{2}\right) \\ &= \frac{1}{2\alpha}(\omega_z^2 - \omega^2{}_{0z}),\end{aligned}$$

which when rearranged gives Eq. (9.12).

b) $\alpha_z = (1/2)(1/\Delta\theta)(\omega_z^2 - \omega_{0z}^2) = (1/2)(1/(7.00 \text{ rad}))((16.0 \text{ rad/s})^2 - (12.0 \text{ rad/s})^2) =$ 8 rad/s^2.

9.13: a) From Eq. (9.7), with $\omega_{0z} = 0, t = \frac{\omega_z}{\alpha_z} = \frac{36.0 \text{ rad/s}}{1.50 \text{ rad/s}^2} = 24.0 \text{ s}.$

b) From Eq. (9.12), with $\omega_{0z} = 0,\ \theta - \theta_0 = \frac{(36.0 \text{ rad/s})^2}{2(1.50 \text{ rad})/\text{s}^2} = 432 \text{ rad} = 68.8 \text{ rev}.$

9.14: a) The average angular velocity is $\frac{162 \text{ rad}}{4.00 \text{ s}} = 40.5 \text{ rad/s},$ and so the initial angular velocity is $2\omega_{\text{av}-z} - \omega_{2z} = \omega_{0z}, \omega_{0z} = -27 \text{ rad/s}.$

b) $$\alpha_z = \frac{\Delta\omega_z}{\Delta t} = \frac{108 \text{rad/s} - (-27 \text{rad/s})}{4.00 \text{s}} = 33.8 \text{rad/s}^2.$$

9.15: From Eq. (9.11),

$$\omega_{0z} = \frac{\theta - \theta_0}{t} - \frac{\alpha_z t}{2} = \frac{60.0\,\text{rad}}{4.00\,\text{s}} - \frac{(2.25\,\text{rad/s}^2)(4.00\,\text{s})}{2} = 10.5\,\text{rad/s}.$$

9.16: From Eq. (9.7), with $\omega_{0z} = 0, \alpha_z = \frac{\omega_z}{t} = \frac{140\,\text{rad/s}}{6.00\,\text{s}} = 23.33\,\text{rad/s}^2$. The angle is most easily found from $\theta = \omega_{\text{av}-z} t = (70\,\text{rad/s})(6.00\,\text{s}) = 420\,\text{rad}$.

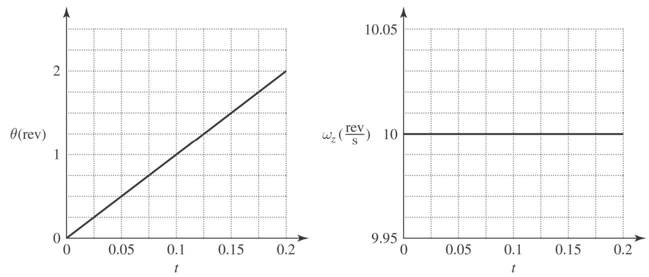
9.17: From Eq. (9.12), with $\omega_z = 0,$ the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.0 rev.

9.18: The following table gives the revolutions and the angle θ through which the wheel has rotated for each instant in time and each of the three situations:

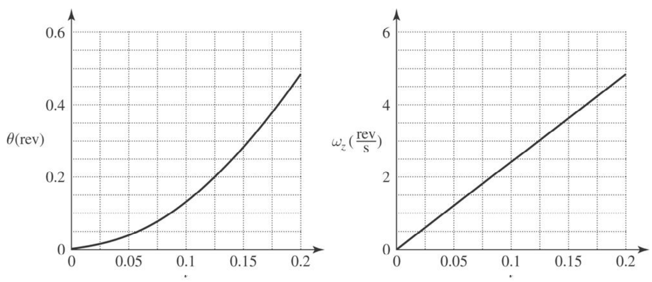
	(a)		(b)		(c)	
t	rev's	θ	rev's	θ	rev's	θ
0.05	0.50	180	0.03	11.3	0.44	158
0.10	1.00	360	0.13	45	0.75	270
0.15	1.50	540	0.28	101	0.94	338
0.20	2.00	720	0.50	180	1.00	360

The θ and ω_z graphs are as follows:

a)



b)



c)

9.19: a) Before the circuit breaker trips, the angle through which the wheel turned was $(24.0\,\text{rad/s})\,(2.00\,\text{s}) + (30.0\,\text{rad/s}^2)\,(2.00\,\text{s})^2/2 = 108\,\text{rad}$, so the total angle is $108\,\text{rad} + 432\,\text{rad} = 540\,\text{rad}$. b) The angular velocity when the circuit breaker trips is $(24.0\,\text{rad/s}) + (30.0\,\text{rad/s}^2)(2.00\,\text{s}) = 84\,\text{rad/s}$, so the average angular velocity while the wheel is slowing is $42.0\,\text{rad/s}$, and the time to slow to a stop is $\frac{432\,\text{rad}}{42.0\,\text{rad/s}} = 10.3\,\text{s}$, so the time when the wheel stops is $12.3\,\text{s}$. c) Of the many ways to find the angular acceleration, the most direct is to use the intermediate calculation of part (b) to find that while slowing down $\Delta\omega_z = -84\,\text{rad/s}$ so $\alpha_z = \frac{-84\,\text{rad/s}}{10.3\,\text{s}} = -8.17\,\text{rad/s}^2$.

9.20: a) Equation (9.7) is solved for $\omega_{0z} = \omega_z - \alpha_z t$, which gives $\omega_{z-\text{ave}} = \omega_z - \frac{\alpha_z}{2}t$, or $\theta - \theta_0 = \omega_z t - \frac{1}{2}\alpha_z t^2$. b) $2\left(\frac{\omega_z}{t} - \frac{\Delta\theta}{t^2}\right) = -0.125\,\text{rad/s}^2$. c) $\omega_z - \alpha_z t = 5.5\,\text{rad/s}$.

9.21: The horizontal component of velocity is $r\omega$, so the magnitude of the velocity is

a) 47.1 m/s

b) $\sqrt{\left((5.0\,\text{m})(90\,\text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2 + (4.0\,\text{m/s})^2} = 47.3\,\text{m/s}.$

9.22: a) $\frac{1.25\,\text{m/s}}{25.0\times10^{-3}\,\text{m}} = 50.0\ \text{rad/s}$, $\frac{1.25\,\text{m}}{58.0\times10^{-3}} = 21.55\ \text{rad/s}$, or $21.6\ \text{rad/s}$ to three figures.

b) $(1.25\ \text{m/s})\,(74.0\ \text{min})\,(60\ \text{s/min}) = 5.55\ \text{km}$.

c) $\alpha_z = \frac{50.0\,\text{rad/s} - 21.55\,\text{rad/s}}{(74.0\,\text{min})\,(60\,\text{s/min})} = 6.41\times10^{-3}\ \text{rad/s}^2$.

9.23: a) $\omega^2 r = (6.00\,\text{rad/s})^2\ (0.500\,\text{m}) = 18\ \text{m/s}^2$.

b) $v = \omega r = (6.00\,\text{rad/s})\ (0.500\,\text{m}) = 3.00\,\text{m/s}$, and $\frac{v^2}{r} = \frac{(3.00\,\text{m/s})^2}{(0.500\,\text{m})} = 18\,\text{m/s}^2$.

9.24: From $a_{\text{rad}} = \omega^2 r,$

$$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{400{,}000 \times 9.80\ \text{m/s}^2}{2.50 \times 10^{-2}\ \text{m}}} = 1.25 \times 10^4\ \ \text{rad/s},$$

which is $(1.25 \times 10^4\ \text{rad/s})\left(\frac{1\,\text{rev}/2\pi\ \text{rad}}{1\,\text{min}/60\,\text{s}}\right) = 1.20 \times 10^5\ \ \text{rev/min}.$

9.25: a) $a_{\text{rad}} = 0, a_{\tan} = \alpha r = \left(0.600\,\text{rad/s}^2\right)\left(0.300\,\text{m}\right) = 0.180\,\text{m/s}^2$ and so $a = 0.180\,\text{m/s}^2.$
b) $\theta = \frac{\pi}{3}$ rad, so $a_{\text{rad}} = \omega^2 r = 2\left(0.600\,\text{rad/s}^2\right)\left(\pi/3\,\text{rad}\right)\left(0.300\,\text{m}\right) = 0.377\,\text{m/s}^2.$

The tangential acceleration is still $0.180\,\text{m/s}^2,$ and so on

$$a = \sqrt{\left(0.180\,\text{m/s}^2\right)^2 + \left(0.377\,\text{m/s}^2\right)^2} = 0.418\,\text{m/s}^2.$$

c) For an angle of 120°, $a_{\text{rad}} = 0.754\,\text{m/s}^2$, and $a = 0.775\,\text{m/s}^2$, since $a_{\tan}$ is still $0.180\,\text{m/s}^2$

9.26: a) $\omega_z = \omega_{0z} + \alpha_z t = 0.250\,\text{rev/s} + \left(0.900\,\text{rev/s}^2\right)\left(0.200\,\text{s}\right) = 0.430\,\text{rev/s}$
(note that since ω_{0z} and α_z are given in terms of revolutions, it's not necessary to convert to radians). b) $\omega_{\text{av}-z}\Delta t = (0.340\,\text{rev/s})\,(0.2\text{s}) = 0.068\,\text{rev}$. c) Here, the conversion to radians must be made to use Eq. (9.13), and

$$v = r\omega = \left(\frac{0.750\,\text{m}}{2}\right)\left(0.430\,\text{rev/s} \times 2\pi\ \ \text{rad/rev}\right) = 1.01\,\text{m/s}.$$

d) Combining equations (9.14) and (9.15),

$$a = \sqrt{a^2{}_{\text{rad}} + a^2{}_{\tan}} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}$$
$$= [((0.430\,\text{rev/s} \times 2\pi\,\text{rad/rev})^2(0.375\,\text{m}))^2 + ((0.900\ \text{rev/s}^2 \times 2\pi\,\text{rad/rev})(0.375\,\text{m}))^2\Big]^{\frac{1}{2}}$$
$$= 3.46\,\text{m/s}^2.$$

9.27:
$$r = \frac{a_{\text{rad}}}{\omega^2} = \frac{(3000)(9.80\,\text{m/s}^2)}{\left((5000\,\text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2} = 10.7\,\text{cm},$$

so the diameter is more than 12.7 cm, contrary to the claim.

9.28: a) Combining Equations (9.13) and (9.15),

$$a_{\text{rad}} = \omega^2 r = \omega^2 \left(\frac{v}{\omega}\right) = \omega v.$$

b) From the result of part (a), $\omega = \frac{a_{\text{rad}}}{v} = \frac{0.500\ \text{m/s}}{2.00\ \text{m/s}} = 0.250\ \text{rad/s}$.

9.29: a) $\omega r = (1250\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\left(\frac{12.7\times10^{-3}\ \text{m}}{2}\right) = 0.831\ \text{m/s}$.

b) $\frac{v^2}{r} = \frac{(0.831\ \text{m/s})^2}{(12.7\times10^{-3}\ \text{m})/2} = 109\ \text{m/s}^2$.

9.30: a) $\alpha = \frac{a_{\tan}}{r} = \frac{-10.0\ \text{m/s}^2}{0.200\ \text{m}} = -50.0\ \text{rad/s}^2$ b) At $t = 3.00\ \text{s}$, $v = 50.0\ \text{m/s}$ and $\omega = \frac{v}{r} = \frac{50.0\ \text{m/s}}{0.200} = 250\ \text{rad/s}$ and at $t = 0$, $v = 50.0\ \text{m/s} + (-10.0\ \text{m/s}^2)$ $(0 - 3.00\ \text{s}) = 80.0\ \text{m/s}$, so $\omega = 400\ \text{rad/s}$. c) $\omega_{\text{ave}} t = (325\ \text{rad/s})(3.00\ \text{s})$ $= 975\ \text{rad} = 155\ \text{rev}$. d) $v = \sqrt{a_{\text{rad}} r} = \sqrt{(9.80\ \text{m/s}^2)(0.200\ \text{m})} = 1.40\ \text{m/s}$. This speed will be reached at time $\frac{50.0\ \text{m/s} - 1.40\ \text{m/s}}{10.0\ \text{m/s}} = 4.86\ \text{s}$ after $t = 3.00\ \text{s}$, or at $t = 7.86\ \text{s}$. (There are many equivalent ways to do this calculation.)

9.31: (a) For a given radius and mass, the force is proportional to the square of the angular velocity; $\left(\frac{640\ \text{rev/min}}{423\ \text{rev/min}}\right)^2 = 2.29$ (note that conversion to rad/s is not necessary for this part). b) For a given radius, the tangential speed is proportional to the angular velocity; $\frac{640}{423} = 1.51$ (again conversion of the units of angular speed is not necessary).

c) $(640\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\left(\frac{0.470\ \text{m}}{2}\right) = 15.75\ \text{m/s}$, or $15.7\ \text{m/s}$ to three figures, and $a_{\text{rad}} = \frac{v^2}{r} = \frac{(15.75\ \text{m/s})^2}{(0.470\ \text{m/2})} = 1.06\times10^3\ \text{m/s}^2 = 108\, g$.

9.32: (a)

$$v_{\text{T}} = R\omega$$

$$2.00\ \text{cm/s} = R\left(\frac{7.5\ rev}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ rad}{1\ \text{rev}}\right)$$

$$R = 2.55\ \text{cm}$$

$$D = 2R = 5.09\ \text{cm}$$

b)

$$a_{\text{T}} = R\alpha$$

$$\alpha = \frac{a_{\text{T}}}{R} = \frac{0.400\ \text{m/s}^2}{0.0255\ \text{m}} = 15.7\ \text{rad/s}^2$$

9.33: The angular velocity of the rear wheel is $\omega_{\mathrm{r}} = \frac{v_{\mathrm{r}}}{r} = \frac{5.00\ \mathrm{m/s}}{0.330\ \mathrm{m}} = 15.15\ \mathrm{rad/s}.$

The angular velocity of the front wheel is $\omega_{\mathrm{f}} = 0.600\ \mathrm{rev/s} = 3.77\ \mathrm{rad/s}$

Points on the chain all move at the same speed, so $r_{\mathrm{r}}\omega_{\mathrm{r}} = r_{\mathrm{f}}\omega_{\mathrm{f}}$

$r_{\mathrm{r}} = r_{\mathrm{r}}(\omega_{\mathrm{f}}/\omega_{\mathrm{r}}) = 2.99\ \mathrm{cm}$

9.34: The distances of the masses from the axis are $\frac{L}{4}, \frac{L}{4}$ and $\frac{3L}{4}$, and so from Eq. (9.16), the moment of inertia is

$$I = m\left(\frac{L}{4}\right)^2 + m\left(\frac{L}{4}\right)^2 + m\left(\frac{3L}{4}\right)^2 = \frac{11}{16}mL^2.$$

9.35: The moment of inertia of the cylinder is $M\frac{L^2}{12}$ and that of each cap is $m\frac{L^2}{4}$, so the moment of inertia of the combination is $\left(\frac{M}{12} + \frac{m}{2}\right)L^2$.

9.36: Since the rod is 500 times as long as it is wide, it can be considered slender.

a) From Table $(9.2(\mathrm{a}))$,

$$I = \frac{1}{12}ML^2 = \frac{1}{12}(0.042\ \mathrm{kg})(1.50\ \mathrm{m})^2 = 7.88\times10^{-3}\,\mathrm{kg}\cdot\mathrm{m}^2.$$

b) From Table $(9.2(\mathrm{b}))$,

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(0.042\ \mathrm{kg})(1.50\ \mathrm{m})^2 = 3.15\times10^{-2}\ \mathrm{kg}\cdot\mathrm{m}^2.$$

c) For this slender rod, the moment of inertia about the axis is obtained by considering it as a solid cylinder, and from Table $(9.2(\mathrm{f}))$,

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.042\ \mathrm{kg})\,(1.5\times10^{-3}\ \mathrm{m})^2 = 4.73\times10^{-8}\ \mathrm{kg}\cdot\mathrm{m}^2.$$

9.37: a) For each mass, the square of the distance from the axis is $2(0.200\ \text{m})^2 = 8.00\times10^{-2}\ \text{m}^2$, and the moment of inertia is $4(0.200\ \text{kg})\,(0.800\times10^{-2}\ \text{m}^2) = 6.40\times10^{-2}\ \text{kg}\cdot\text{m}^2$. b) Each sphere is $0.200\ \text{m}$ from the axis, so the moment of inertia is $4(0.200\ \text{kg})(0.200\ \text{m})^2 = 3.20\times10^{-2}\ \text{kg}\cdot\text{m}^2$.

a) The two masses through which the axis passes do not contribute to the moment of inertia. $I = 2(0.2\ \text{kg})\left(0.2\sqrt{2}\ \text{m}\right)^2 = 0.032\ \text{kg}\cdot\text{m}^2$.

9.38: (a) $I = I_{\text{bar}} + I_{\text{balls}} = \frac{1}{12}M_{\text{bar}}L^2 + 2m_{\text{balls}}\left(\frac{L}{2}\right)^2$

$$= \frac{1}{12}(4.00\ \text{kg})(2.00\ \text{m})^2 + 2(0.500\ \text{kg})(1.00\ \text{m})^2 = 2.33\ \text{kg}\cdot\text{m}^2$$

(b) $I = \frac{1}{3}m_{\text{bar}}L^2 + m_{\text{ball}}L^2$

$$= \frac{1}{3}(4.00\ \text{kg})(2.00\ \text{m})^2 + (0.500\ \text{kg})(2.00\ \text{m})^2 = 7.33\ \text{kg}\cdot\text{m}^2$$

c) $I = 0$ because all masses are on the axis

(d) $I = m_{\text{bar}}d^2 + 2m_{\text{ball}}d^2 = M_{\text{Total}}d^2$

$$= (5.00\ \text{kg})(0.500\ \text{m})^2 = 1.25\ \text{kg}\cdot\text{m}^2$$

9.39: $I = I_{\text{d}} + I_{\text{r}}$ (d = disk, r = ring)

disk: $m_{\text{d}} = (3.00\ \text{g/cm}^3)\pi r_{\text{d}}^2 = 23.56\ \text{kg}$

$$I_{\text{d}} = \frac{1}{2}m_{\text{d}}r_{\text{d}}^2 = 2.945\ \text{kg}\cdot\text{m}^2$$

ring: $m_{\text{r}} = (2.00\ \text{g/cm}^3)\,\pi\,(r_2^2 - r_1^2) = 15.08\ \text{kg}$ $\quad (r_1 = 50.0\ \text{cm}, r_2 = 70.0\ \text{cm})$

$$I_{\text{r}} = \frac{1}{2}m_{\text{r}}(r_1^2 + r_2^2) = 5.580\ \text{kg}\cdot\text{m}^2$$

$$I = I_{\text{d}} + I_{\text{r}} = 8.52\ \text{kg}\cdot\text{m}^2$$

9.40: a) In the expression of Eq. (9.16), each term will have the mass multiplied by f^3 and the distance multiplied by f, and so the moment of inertia is multiplied by $f^3(f)^2 = f^5$. b) $(2.5)(48)^5 = 6.37\times10^8$.

9.41: Each of the eight spokes may be treated as a slender rod about an axis through an end, so the moment of inertia of the combination is

$$\begin{aligned} I &= m_{\text{rim}}R^2 + 8\left(\frac{m_{\text{spoke}}}{3}\right)R^2 \\ &= \left[(1.40\,\text{kg}) + \frac{8}{3}(0.20\,\text{kg})\right](0.300\,\text{m})^2 \\ &= 0.193\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

9.42: a) From Eq. (9.17), with *I* from Table (9.2(a)),

$$K = \frac{1}{2}\frac{1}{12}mL^2\omega^2 = \frac{1}{24}(117\,\text{kg})(2.08\,\text{m})^2(2400\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad/rev}}{60\,\text{s/min}})^2 = 1.3\times10^6\text{ J}.$$

b) From $mgy = K$,

$$y = \frac{K}{mg} = \frac{\left(1.3\times10^6\,\text{J}\right)}{(117\,\text{kg})(9.80\,\text{m/s}^2)} = 1.16\times10^3\ \text{m} = 1.16\,\text{km}.$$

9.43: a) The units of moment of inertia are $[\text{kg}][\text{m}^2]$ and the units of ω are equivalent to $[\text{s}^{-1}]$ and so the product $\frac{1}{2}I\omega^2$ has units equivalent to $[\text{kg}\cdot\text{m}\cdot\text{s}^{-2}] = [\text{kg}\cdot(\text{m/s})^2]$, which are the units of Joules. A radian is a ratio of distances and is therefore unitless.

b) $K = \pi^2 I\omega^2/1800$, when is in rev/min.

9.44: Solving Eq. (9.17) for *I*,

$$I = \frac{2K}{\omega^2} = \frac{2(0.025\text{J})}{(45\,\text{rev/min}\times\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}})^2} = 2.25\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

9.45: From Eq. (9.17), $K_2 - K = \frac{1}{2} I(\omega_2^2 - \omega_1^2)$, and solving for I,

$$\begin{aligned} I &= 2\frac{(K_2 - K_1)}{(\omega_2^2 - \omega_1^2)} \\ &= 2\frac{(-500\,\text{J})}{((520\,\text{rev/min})^2 - (650\,\text{rev/min})^2)\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)^2} \\ &= 0.600\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

9.46: The work done on the cylinder is PL, where L is the length of the rope. Combining Equations (9.17), (9.13) and the expression for I from Table (9.2(g)),

$$PL = \frac{1}{2}\frac{\omega}{g}v^2, \text{ or } P = \frac{1}{2}\frac{\omega}{g}\frac{v^2}{L} = \frac{(40.0\text{ N})(6.00\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)(5.00\,\text{m})} = 14.7\text{ N}.$$

9.47: Expressing ω in terms of α_{rad}, $\omega^2 = \frac{a_{\text{rad}}}{R}$. Combining with $I = \frac{1}{2}MR^2$, Eq. (9.17) becomes $K = \frac{1}{2}\frac{1}{2}MRa_{\text{rad}} = \frac{(70.0\text{ kg})(1.20\text{ m})(3500\,\text{m/s})^2}{4} = 7.35\times10^4\text{ J}.$

9.48: a) With $I = MR^2$, with expression for v is

$$v = \sqrt{\frac{2gh}{1 + M/m}}.$$

b) This expression is smaller than that for the solid cylinder; more of the cylinder's mass is concentrated at its edge, so for a given speed, the kinetic energy of the cylinder is larger. A larger fraction of the potential energy is converted to the kinetic energy of the cylinder, and so less is available for the falling mass.

9.49: a) $\omega = \frac{2\pi}{T}$, so Eq. (9.17) becomes $K = 2\pi^2 I/T^2$.

b) Differentiating the expression found in part (a) with respect to T,

$$\frac{dK}{dt} = (-4\pi^2 I/T^3)\tfrac{dT}{dt}.$$

c) $2\pi^2 (8.0\ \text{kg}\cdot\text{m}^2)/(1.5\ \text{s})^2 = 70.2\ \text{J}$, or 70 to two figures.

d) $(-4\pi^2(8.0\ \text{kg}\cdot\text{m}^2)/(1.5\ \text{s})^3)(0.0060) = -0.56\ \text{W}$.

9.50: The center of mass has fallen half of the length of the rope, so the change in gravitational potential energy is

$$-\frac{1}{2}mgL = -\frac{1}{2}(3.00\ \text{kg})(9.80\ \text{m/s}^2)(10.0\ \text{m}) = -147\ \text{J}.$$

9.51: $(120\ \text{kg})(9.80\ \text{m/s}^2)(0.700\ \text{m}) = 823\ \text{J}$.

9.52: In Eq; (9.19), $I_{\text{cm}} = MR^2$ and $d = R^2$, so $I_P = 2MR^2$.

9.53: $\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$, so $d^2 = \frac{4}{15}R^2$, and the axis comes nearest to the center of the sphere at a distance $d = (2/\sqrt{15})R = (0.516)R$.

9.54: Using the parallel-axis theorem to find the moment of inertia of a thin rod about an axis through its end and perpendicular to the rod,

$$I_p = I_{\text{cm}} + Md^2 = \frac{M}{12}L^2 + M\left(\frac{L}{2}\right)^2 = \frac{M}{3}L^2.$$

9.55: $Ip = I_{\text{cm}} + md^2$, so $I = \frac{1}{12}M\left(a^2+b^2\right) + M\left((\tfrac{a}{2})^2 + (\tfrac{b}{2})^2\right)$, which gives

$$I = \frac{1}{12}M\left(a^2+b^2\right) + \frac{1}{4}\text{M}\left(a^2+b^2\right), \text{ or } I = \frac{1}{3}M\left(a^2+b^2\right).$$

9.56: a) $I = \frac{1}{12}Ma^2$ b) $I = \frac{1}{12}Mb^2$

9.57: In Eq. (9.19), $I_{cm} = \frac{M}{12}L^2$ *and* $d = (L/2 - h)$, so

$$\begin{aligned} Ip &= M\left[\frac{1}{12}L^2 + \left(\frac{L}{2} - h\right)^2\right] \\ &= M\left[\frac{1}{12}L^2 + \frac{1}{4}L^2 - Lh + h^2\right] \\ &= M\left[\frac{1}{3}L^2 - Lh + h^2\right], \end{aligned}$$

which is the same as found in Example 9.12.

9.58: The analysis is identical to that of Example 9.13, with the lower limit in the integral being zero and the upper limit being *R*, and the mass $M = \pi L\rho R^2$. The result is $I = \frac{1}{2}MR^2$, as given in Table $(9.2(\mathrm{f}))$.

9.59: With $dm = \frac{M}{L}dx$

$$I = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L}\frac{x^3}{3}\bigg|_0^L = \frac{M}{3}L^2.$$

9.60: For this case, $dm = \gamma\ dx.$

a) $$M = \int dm = \int_0^L \gamma x\ dx = \gamma \frac{x^2}{2}\Bigg|_0^L = \frac{\gamma L^2}{2}$$

b) $$I = \int_0^L x^2(\gamma x)dx = \gamma \frac{x^4}{4}\Bigg|_0^L = \tfrac{\gamma L^4}{4} = \tfrac{M}{2}L^2.$$

This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater further away from the axis than nearer the axis.

c) $$\begin{aligned} I &= \int_0^L (L-x)^2 \gamma x dx \\ &= \gamma \int_0^L (L^2x - 2Lx^2 + x^3)dx \\ &= \gamma \left(L^2 \frac{x^2}{2} - 2L\frac{x^3}{3} + \frac{x^4}{4} \right)\Bigg|_0^L \\ &= \gamma \frac{L^4}{12} \\ &= \frac{M}{6}L^2. \end{aligned}$$

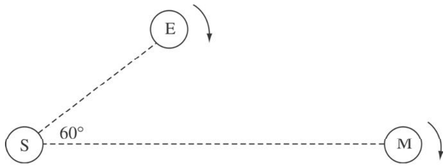
This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

9.61: a) For a clockwise rotation, $\vec{\boldsymbol{\omega}}$ will be out of the page. b) The upward direction crossed into the radial direction is, by the right-hand rule, counterclockwise. $\vec{\omega}$ and $\vec{r}$ are perpendicular, so the magnitude of $\vec{\omega} \times \vec{r}$ is $\omega r = v$. c) Geometrically, $\vec{\omega}$ is perpendicular to $\vec{\boldsymbol{v}}$, and so $\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{v}}$ has magnitude $\omega v = a_{\text{rad,}}$ and from the right-hand rule, the upward direction crossed into the counterclockwise direction is inward, the direction of $\vec{\boldsymbol{a}}_{\text{rad}}$. Algebraically,

$$\begin{aligned} \vec{\boldsymbol{a}}_{\text{rad}} &= \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{v}} = \vec{\boldsymbol{\omega}} \times (\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}) \\ &= \vec{\boldsymbol{\omega}}(\vec{\boldsymbol{\omega}} \cdot \vec{\boldsymbol{r}}) - \vec{\boldsymbol{r}}(\vec{\boldsymbol{\omega}} \cdot \vec{\boldsymbol{\omega}}) \\ &= -\omega^2 \vec{\boldsymbol{r}}, \end{aligned}$$

where the fact that $\vec{\boldsymbol{\omega}}$ and $\vec{\boldsymbol{r}}$ are perpendicular has been used to eliminate their dot product.

9.62:



For planetary alignment, earth must go through $60°$ more than Mars:

$$\theta_E = \theta_M + 60°$$
$$w_E t = \omega_M t + 60°$$
$$t = \frac{60°}{\omega_E - \omega_M}$$
$$w_E = \frac{360°}{1\text{yr}} \text{ and } w_M = \frac{360°}{1.9\text{yr}}$$
$$t = \frac{60°}{\frac{360°}{1\text{yr}} - \frac{360°}{1.9\text{yr}}} = 0.352yr\left(\frac{365\text{d}}{1\text{yr}}\right) = 128\,\text{d}$$

9.63: a) $v = 60\,\text{mph} = 26.82\ \text{m/s}$

$r = 12\,\text{in.} = 0.3048\text{m}$

$\omega = \frac{v}{r} = 88.0\,\text{rad/s} = 14.0\ \text{rev/s} = 840\,\text{rpm}$

b) same ω as in part (a) since speedometer reads same

$r = 15\,\text{in.} = 0.381\,\text{m}$

$v = r\omega = (0.381\text{m})(88.0\,\text{rad/s}) = 33.5\,\text{m/s} = 75\,\text{mph}$

c) $v = 50\,\text{mph} = 22.35\,\text{m/s}$

$r = 10\,\text{in.} = 0.254\,\text{m}$

$\omega = \frac{v}{r} = 88.0\ \text{rad/s}$; .this is the same as for 60 mph with correct tires, so speedometer read 60 mph.

9.64: a) For constant angular acceleration $\theta = \frac{\omega^2}{2\alpha}$, and so $a_{\text{rad}} = \omega^2 r = 2\alpha\theta r.$

b) Denoting the angle that the acceleration vector makes with the radial direction as β, and using Equations (9.14) and (9.15),

$$\tan\beta = \frac{a_{\tan}}{a_{\text{rad}}} = \frac{\alpha\,\text{r}}{\omega^2 r} = \frac{\alpha\,\text{r}}{2\,\alpha\theta\,r} = \frac{1}{2\theta},$$

so $\theta = \frac{1}{2\tan\beta} = \frac{1}{2\tan 36.9°} = 0.666\,\text{rad}.$

9.65: a) $\omega_z = \dfrac{d\theta}{dt} = 2\gamma t - 3\beta t^2 = (6.40\ \text{rad/s}^2)t - (1.50\ \text{rad/s}^3)t^2.$

b) $$\alpha_z = \frac{dw_z}{dt} = 2\gamma - 6\beta\, t = (6.40\ \text{rad/s}^2) - (3.00\ \text{rad/s}^3)t.$$

c) An extreme of angular velocity occurs when $\alpha_z = 0$, which occurs at $t = \frac{\gamma}{3\beta} = \frac{3.20\ \text{rad/s}^2}{1.50\ \text{rad/s}^3} = 2.13$ s, and at this time

$$\omega_z = (2\gamma)(\gamma/3\beta) - (3\beta)(\gamma/3\beta)^2 = \gamma^2/3\beta = \frac{(3.20\ \text{rad/s}^2)^2}{3(0.500\ \text{rad/s}^3)} = 6.83\text{rad/s}.$$

9.66: a) By successively integrating Equations (9.5) and (9.3),

$$\omega_z = \gamma t - \frac{\beta}{2}t^2 = (1.80\ \text{rad/s}^2)t - (0.125\ \text{rad/s}^3)t^2,$$

$$\theta = \frac{\gamma}{2}t^2 - \frac{\beta}{6}t^3 = (0.90\ \text{rad/s}^2)t^2 - (0.042\ \text{rad/s}^3)t^3.$$

b) The maximum positive angular velocity occurs when $\alpha_z = 0,\ t = \dfrac{\gamma}{\beta},$ the angular velocity at this time is

$$\omega_z = \gamma\left(\frac{\gamma}{\beta}\right) - \frac{\beta}{2}\left(\frac{\gamma}{\beta}\right)^2 = \frac{1}{2}\frac{\gamma^2}{\beta} = \frac{1}{2}\frac{(1.80\ \text{rad/s}^2)^2}{(0.25\ \text{rad/s}^3)} = 6.48\ \text{ rad/s}.$$

The maximum angular displacement occurs when $\omega_z = 0,$ at time $t = \dfrac{2\gamma}{\beta}$ ($t = 0$ is an inflection point, and $\theta(0)$ is not a maximum) and the angular displacement at this time is

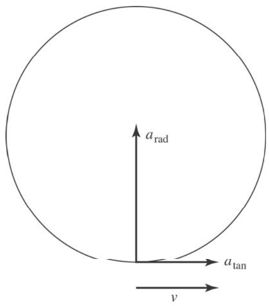
$$\theta = \frac{\gamma}{2}\left(\frac{2\gamma}{\beta}\right)^2 - \frac{\beta}{6}\left(\frac{2\gamma}{\beta}\right)^3 = \frac{2}{3}\frac{\gamma^3}{\beta^2} = \frac{2}{3}\frac{(1.80\ \text{rad/s}^2)^3}{(0.25\ \text{rad/s}^3)^2} = 62.2\ \text{rad}.$$

9.67: a) The scale factor is 20.0, so the actual speed of the car would be $35\ \text{km/h} = 9.72\ \text{m}$
b) $(1/2)mv^2 = 8.51\,\text{J}.$ c) $\omega = \sqrt{\frac{2K}{I}} = 652\ \text{rad/s}.$

9.68: a) $\alpha = \frac{a_{\tan}}{r} = \frac{3.00\ \text{m/s}^2}{60.0\ \text{m}} = 0.050\ \text{rad/s}^2$. b) $\alpha t = (0.05\ \text{rad/s}^2)(6.00s) = 0.300\ \text{rad/s}$.

c) $a_{\text{rad}} = \omega^2 r = (0.300\ \text{rad/s})^2\ (60.0\ \text{m}) = 5.40\ \text{m/s}^2$.

d)



e) $a = \sqrt{a^2{}_{\text{rad}} + a^2{}_{\tan}} = \sqrt{(5.40\ \text{m/s}^2)^2 + (3.00\ \text{m/s}^2)^2} = 6.18\ \text{m/s}^2$,
and the magnitude of the force is $F = ma = (1240\ \text{kg})(6.18\ \text{m/s}^2) = 7.66\ \text{kN}$.

f) arctan $\left(\frac{a_{\text{rad}}}{a_{\tan}}\right) = \arctan\left(\frac{5.40}{3.00}\right) = 60.9°$.

9.69: a) Expressing angular frequencies in units of revolutions per minute may be accomodated by changing the units of the dynamic quantities; specifically,

$$\begin{aligned}\omega_2 &= \sqrt{\omega_1^2 + \frac{2W}{I}} \\ &= \sqrt{(300\ \text{rev/min})^2 + \left(\frac{2(-4000\ \text{J})}{16.0\ \text{kg}\cdot\text{m}^2}\right)\Big/\left(\frac{\pi}{30}\ \frac{\text{rad/s}}{\text{rev/min}}\right)^2} \\ &= 211\text{rev/min}.\end{aligned}$$

b) At the initial speed, the 4000 J will be recovered; if this is to be done is 5.00 s, the power must be $\frac{4000\ \text{J}}{5.00\ \text{s}} = 800\ \text{W}$.

9.70: a) The angular acceleration will be zero when the speed is a maximum, which is at the bottom of the circle. The speed, from energy considerations, is $v = \sqrt{2gh} = \sqrt{2gR(1-\cos\beta)}$, where β is the angle from the vertical at release, and

$$\omega = \frac{v}{R} = \sqrt{\frac{2g}{R}(1-\cos\beta)} = \sqrt{\frac{2(9.80\ \text{m/s}^2)}{(2.50\ \text{m})}(1-\cos 36.9°)} = 1.25\ \text{rad/s}.$$

b) α will again be 0 when the meatball again passes through the lowest point.

c) a_{rad} is directed toward the center, and $a_{\text{rad}} = \omega^2 R,\ a_{\text{rad}} = (1.25\ \text{rad/s}^2)(2.50\ \text{m}) = 3.93$

d) $a_{\text{rad}} = \omega^2 R = (2g/R)(1-\cos\beta)R = (2g)(1-\cos\beta)$, independent of R.

9.71: a) $(60.0\ \text{rev/s})(2\pi\ \text{rad/rev})(0.45\times10^{-2}\ \text{m}) = 1.696\ \text{m/s}$.

b) $\omega = \frac{v}{r} = \frac{1.696\ \text{m/s}}{2.00\times10^{-2}\ \text{m}} = 84.8\ \text{rad/s}$.

9.72: The second pulley, with half the diameter of the first, must have twice the angular velocity, and this is the angular velocity of the saw blade.

a) $$(2(3450\ \text{rev/min}))\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\left(\frac{0.208\ \text{m}}{2}\right) = 75.1\ \text{m/s}.$$

b) $$a_{\text{rad}} = \omega^2 r = \left(2(3450\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2\left(\frac{0.208\ \text{m}}{2}\right) = 5.43\times10^4\ \text{m/s}^2,$$

so the force holding sawdust on the blade would have to be about 5500 times as strong as gravity.

9.73: a)

$$\Delta a_{\text{rad}} = \omega^2 r - \omega_0^2 r = (\omega^2 - \omega_0^2) r$$

$$= [\omega - \omega_0][\omega + \omega_0] r$$

$$= \left[\frac{\omega - \omega_0}{t}\right][(\omega + \omega_0) t] r$$

$$= [\alpha][2(\theta - \theta_0) r.$$

b) From the above,

$$\alpha r = \frac{\Delta a_{\text{rad}}}{2\Delta\theta} = \frac{(85.0\ \text{m/s}^2 - 25.0\ \text{m/s}^2)}{2(15.0\ \text{rad})} = 2.00\ \text{m/s}^2.$$

c) Similar to the derivation of part (a),

$$\Delta K = \frac{1}{2}\omega^2 I - \frac{1}{2}\omega_0{}^2 I = \frac{1}{2}[\alpha][2\Delta\theta] I = I\alpha\Delta\theta.$$

d) Using the result of part (c),

$$I = \frac{\Delta K}{\alpha\Delta\theta} = \frac{(45.0\ \text{J} - 20.0\ \text{J})}{((2.00\ \text{m/s}^2)/(0.250\ \text{m}))(15.0\ \text{rad})} = 0.208\ \text{kg}\cdot\text{m}^2.$$

9.74: $I = I_{\text{wood}} + I_{\text{lead}}$

$$= \frac{2}{5} m_{\text{w}} R^2 + \frac{2}{3} m_{\text{L}} R^2$$

$$m_{\text{w}} = \rho_{\text{w}} V_{\text{w}} = \rho_{\text{w}} \frac{4}{3}\pi R^3$$

$$m_{\text{L}} = \sigma_{\text{L}} A_{\text{L}} = \sigma_{\text{L}} 4\pi R^2$$

$$I = \frac{2}{5}\left(\rho_{\text{w}} \frac{4}{3}\pi R^3\right) R^2 + \frac{2}{3}(\sigma_{\text{L}} 4\pi R^2) R^2$$

$$= \frac{8}{3}\pi R^4 \left(\frac{\rho_{\text{w}} R}{5} + \sigma_{\text{L}}\right)$$

$$= \frac{8\pi}{3}(0.20\ \text{m})^4 \left[\frac{(800\ \text{kg/m}^3)(0.20\ \text{m})}{5} + 20\ \text{kg/m}^2\right]$$

$$= 0.70\ \text{kgm}^2$$

9.75: I approximate my body as a vertical cylinder with mass 80 kg, length 1.7 m, and diameter 0.30 m (radius 0.15 m)

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(80\,\text{kg})\,(0.15\,\text{m})^2 = 0.9\,\text{kg}\cdot\text{m}^2$$

9.76: Treat the V like two thin 0.160 kg bars, each 25 cm long.

$$\text{I} = 2\left(\frac{1}{3}mL^2\right) = 2\left(\frac{1}{3}\right)(0.160\,\text{kg})(0.250\,\text{m})^2$$
$$= 6.67\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

9.77: a) $\omega = 90.0\,\text{rpm} = 9.425\,\text{rad/s}$

$$K = \frac{1}{2}I\omega^2 \text{ so } I = \frac{2K}{\omega^2} = \frac{2(10.0\times10^6\ J)}{(9.425\,\text{rad/s})^2} = 2.252\times10^5\ \text{kg}\cdot\text{m}^2$$

$m = \rho V = \rho\pi R^2 t(\rho = 7800\,\text{kg/m}^3$ is the density of iron and t=0.100 m is the thickness of the flywheel)

$$I = \frac{1}{2}mR^2 = \frac{1}{2}\rho\pi t R^4$$

$$R = (2I/\rho\pi t)^{1/4} = 3.68\,\text{m};\ \text{diameter} = 7.36\,\text{m}$$

b) $a_c = R\omega^2 = 327\,\text{m/s}^2$

9.78: Quantitatively, from Table (9.2), $I_A = \frac{1}{2}mR^2, I_B = mR^2$ and $I_C = \frac{2}{3}mR^2\cdot$ a) Object A has the smallest moment of inertia because, of the three objects, its mass is the most concentrated near its axis. b) Conversely, object B's mass is concentrated and farthest from its axis. c) Because $I_{\text{sphere}} = 2/5\,mR^2$, the sphere would replace the disk as having the smallest moment of inertia.

9.79: a) See Exercise 9.50.

$$K = \frac{2\pi^2 I}{T^2} = \frac{2\pi^2 (0.3308)(5.97\times10^{24}\ \text{kg})(6.38\times10^6\,\text{m})^2}{(86{,}164\,\text{s})^2} = 2.14\times10^{29}\ \text{J}.$$

b) $$\frac{1}{2}M\left(\frac{2\pi R}{T}\right)^2 = \frac{2\pi^2(5.97\times10^{24}\ \text{kg})(1.50\times10^{11}\ \text{m})^2}{(3.156\times10^7\ \text{s})^2} = 2.66\times10^{33}\ \text{J}.$$

c) Since the Earth's moment on inertia is less than that of a uniform sphere, more of the Earth's mass must be concentrated near its center.

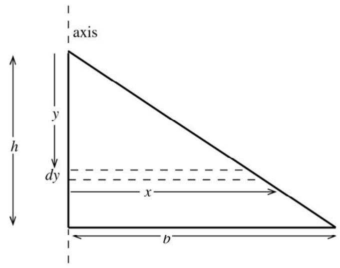
9.80: Using energy considerations, the system gains as kinetic energy the lost potential energy, *mgR*. The kinetic energy is

$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega R)^2 = \frac{1}{2}(I + mR^2)\ \omega^2 \cdot$$

Using $I = \frac{1}{2}mR^2$ and solving for ω,

$$\omega^2 = \frac{4}{3}\frac{g}{R}, \quad \text{and} \quad \omega = \sqrt{\frac{4}{3}\frac{g}{R}}.$$

9.81: a)



Consider a small strip of width *dy* and a distance y below the top of the triangle. The length of the strip is $x = (y/h)b.$

The strip has area $x\,dy$ and the area of the sign is $\frac{1}{2}bh,$ so the mass of the strip is

$$dm = M\left(\frac{xdy}{\frac{1}{2}bh}\right) = M\left(\frac{yb}{h}\right)\left(\frac{2\,dy}{bh}\right) = \left(\frac{2M}{h^2}\right)y\,dy$$

$$dI = \tfrac{1}{3}(dm)x^2 = \frac{2Mb^2}{3h^4}y^3\,dy$$

$$I = \int_0^h dI = \frac{2Mb^2}{3h^4}\int_0^h y^3 dy = \frac{2Mb^2}{3h^4}\left(\frac{1}{4}y^4\,\Big|_0^h\right) = \frac{1}{6}Mb^2$$

b) $I = \tfrac{1}{6}Mb^2 = 2.304\,\mathrm{kg\cdot m^2}$

$\omega = 2.00\,\mathrm{rev/s} = 4.00\pi\,\mathrm{rad/s}$

$K = \tfrac{1}{2}I\omega^2 = 182\,\mathrm{J}$

9.82: (a) The kinetic energy of the falling mass after 2.00 m is $KE = \frac{1}{2}mv^2 = \frac{1}{2}(8.00\ \text{kg})(5.00\ \text{m/s})^2 = 100\ \text{J}.$ The change in its potential energy while falling is $mgh = (8.00\ \text{kg})(9.8\ \text{m/s}^2)(2.00\ \text{m}) = 156.8\ \text{J}$

The wheel must have the "missing" 56.8 J in the form of rotational KE. Since its outer rim is moving at the same speed as the falling mass, $5.00\ \text{m/s}$:

$$v = r\omega$$

$$\omega = \frac{v}{r} = \frac{5.00\text{m/s}}{0.370\text{m}} = 13.51\ \text{rad/s}$$

$KE = \dfrac{1}{2}I\omega^2$; therefore

$$I = \frac{2KE}{\omega^2} = \frac{2(56.8\ \text{J})}{(13.51\ \text{rad/s})^2} = 0.6224\ \text{kg}\cdot\text{m}^2 \text{ or } 0.622\ \text{kg}\cdot\text{m}^2$$

(b) The wheel's mass is $280\ \text{N}/9.8\ \text{m/s}^2$ = 28.6 kg. The wheel with the largest possible moment of inertia would have all this mass concentrated in its rim. Its moment of inertia would be

$$I = MR^2 = (28.6\text{kg})(0.370\text{m})^2 = 3.92\ \text{kg}\cdot\text{m}^2$$

The boss's wheel is physically impossible.

9.83: a) $(0.160\ \text{kg})(-0.500\ \text{m})(9.80\ \text{m/s}^2) = -0.784\ \text{J}.$ b) The kinetic energy of the stick is 0.784 J, and so the angular velocity is

$$\omega = \sqrt{\frac{2k}{I}} = \sqrt{\frac{2k}{ML^2/3}} = \sqrt{\frac{2(0.784\ \text{J})}{(0.160\ \text{kg})(1.00\ \text{m})^2/3}} = 5.42\ \text{rad/s}.$$

This result may also be found by using the algebraic form for the kinetic energy, $K = MgL/2$, from which $\omega = \sqrt{3g/L},$ giving the same result. Note that ω is independent of the mass.

c) $v = \omega L = (5.42\ \text{rad/s})(1.00\ \text{m}) = 5.42\ \text{m/s}$

d) $\sqrt{2gL} = 4.43\ \text{m/s}$; This is $\sqrt{2/3}$ of the result of part (c).

9.84: Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle).When the rope has unwound, its center of mass is a distance πR below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $\omega_0 R,$ and when the rope has unwound, and the cylinder has angular speed $\omega,$ the speed of the rope is ωR (the upper end of the rope has the same tangential speed at the edge of the cylinder). From conservation of energy, using $I = (1/2)MR^2$ for a uniform cylinder,

$$\left(\frac{M}{4} + \frac{m}{2}\right) R^2 \omega_0^2 = \left(\frac{M}{4} + \frac{m}{2}\right) R^2 \omega^2 - mg\pi R.$$

Solving for ω gives

$$\omega = \sqrt{\omega_0^2 + \frac{(4\pi mg/R)}{(M + 2m)}},$$

and the speed of any part of the rope is $v = \omega R.$

9.85: In descending a distance d, gravity has done work $m_B gd$ and friction has done work $-\mu_K m_A gd,$ and so the total kinetic energy of the system is $gd(m_B - \mu_K m_A).$ In terms of the speed v of the blocks, the kinetic energy is

$$K = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(m_A + m_B + I/R^2)v^2,$$

where $\omega = v/R,$ and condition that the rope not slip, have been used. Setting the kinetic energy equal to the work done and solving for the speed v,

$$v = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{(m_A + m_B + I/R^2)}}.$$

9.86: The gravitational potential energy which has become kinetic energy is $K = (4.00\ \text{kg} - 2.00\ \text{kg})(9.80\ \text{m/s}^2)(5.00\ \text{m}) = 98.0\ \text{J}.$ In terms of the common speed v of the blocks, the kinetic energy of the system is

$$K = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$
$$= v^2\,\frac{1}{2}\left(4.00\,\text{kg} + 2.00\,\text{kg} + \frac{(0.480\,\text{kg}\cdot\text{m}^2)}{(0.160\,\text{m})^2}\right) = v^2(12.4\,\text{kg}).$$

Solving for v gives $v = \sqrt{\frac{98.0\,\text{J}}{12.4\,\text{kg}}} = 2.81\,\text{m/s}.$

9.87: The moment of inertia of the hoop about the nail is $2MR^2$ (see Exercise 9.52), and the initial potential energy with respect to the center of the loop when its center is directly below the nail is $gR\ (1-\cos\beta)$. From the work-energy theorem,

$$K = \frac{1}{2}I\omega^2 = M\omega^2 R^2 = MgR(1-\ \cos\beta),$$

from which $\omega = \sqrt{(g/R)(1-\cos\beta)}.$

9.88: a) $K = \dfrac{1}{2}I\omega^2$

$$= \frac{1}{2}\left(\frac{1}{2}(1000\,\text{kg})(0.90\,\text{m})^2\right)\left(3000\,\text{rev/min}\times\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}}\right)^2$$

$$= 2.00\times10^7\ \text{J}.$$

b) $$\frac{K}{P_{\text{ave}}} = \frac{2.00\times10^7\,\text{J}}{1.86\times10^4\ \text{W}} = 1075\ \text{s},$$

which is about 18 min.

9.89: a) $$\frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2 = \frac{1}{2}((0.80\,\text{kg})(2.50\times10^{-2}\,\text{m})^2 + (1.60\,\text{kg})(5.00\times10^{-2}\,\text{m})^2)$$

$$= 2.25\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

b) See Example 9.9. In this case, $\omega = v/R_1$, and so the expression for v becomes

$$v = \sqrt{\frac{2gh}{1+(I/mR^2)}}$$

$$= \sqrt{\frac{2(9.80\,\text{m/s}^2)(2.00\,\text{m})}{(1+((2.25\times10^{-3}\,\text{kg}\cdot\text{m}^2)/(1.50\,\text{kg})(0.025\,\text{m})^2))}} = 3.40\,\text{m/s}.$$

c) The same calculation, with R_2 instead of R_1 gives $v = 4.95\,\text{m/s}$. This does make sense, because for a given total energy, the disk combination will have a larger fraction of the kinetic energy with the string of the larger radius, and with this larger fraction, the disk combination must be moving faster.

9.90: a) In the case that no energy is lost, the rebound height h' is related to the speed v by $h' = \frac{v^2}{2g}$, and with the form for h given in Example 9.9, $h' = \frac{h}{1+M/2m}$. b) Considering the system as a whole, some of the initial potential energy of the mass went into the kinetic energy of the cylinder. Considering the mass alone, the tension in the string did work on the mass, so its total energy is not conserved.

9.91: We can use $K(\text{cylinder}) = 250\,\text{J}$ to find ω for the cylinder and v for the mass.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(10.0\,\text{kg})(0.150\,\text{m})^2 = 0.1125\,\text{kg}\cdot\text{m}^2$$

$$K = \tfrac{1}{2}I\omega^2 \text{ so } \omega = \sqrt{2K/I} = 66.67\,\text{rad/s}$$

$$v = R\omega = 10.0\,\text{m/s}$$

Use conservation of energy $K_1 + U_1 = K_2 + U_2$. Take $y = 0$ at lowest point of the mass, so $y_2 = 0$ and $y_1 = h$, the distance the mass descends. $K_1 = U_2 = 0$ so $U_1 = K_2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where $m = 12.0\,\text{kg}$

For the cylinder, $I = \frac{1}{2}MR^2$ and $\omega = v/R$, so $\frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$.

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{4}Mv^2$$

$$h = \frac{v^2}{2g}\left(1 + \frac{M}{2m}\right) = 7.23\,\text{m}$$

9.92: Energy conservation: Loss of PE of box equals gain in KE of system.

$$m_{\text{box}}gh = \frac{1}{2}m_{\text{box}}v_{\text{box}}^2 + \frac{1}{2}I_{\text{pulley}}\omega_{\text{pulley}}^2 + \frac{1}{2}I_{\text{cylinder}}\omega_{\text{cylinder}}^2$$

$$\omega_{\text{pulley}} = \frac{v_{\text{Box}}}{r_{\text{p}}} \text{ and } \omega_{\text{cylinder}} = \frac{v_{\text{Box}}}{r_{\text{cylinder}}}$$

$$m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{2}\left(\frac{1}{2}m_{\text{P}}r_p^2\right)\left(\frac{v_B}{r_p}\right)^2 + \frac{1}{2}\left(\frac{1}{2}m_{\text{C}}r_{\text{C}}^2\right)\left(\frac{v_{\text{B}}}{r_{\text{C}}}\right)^2$$

$$m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{4}m_{\text{P}}v_{\text{B}}^2 + \frac{1}{4}m_{\text{C}}v_{\text{B}}^2$$

$$v_{\text{B}} = \sqrt{\frac{m_{\text{B}}gh}{\frac{1}{2}m_{\text{B}} + \frac{1}{4}m_{\text{p}} + \frac{1}{4}m_{\text{C}}}}$$

$$= \sqrt{\frac{(3.00\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m})}{1.50\ \text{kg} + \frac{1}{4}(7.00\ \text{kg})}} = 3.68\ \text{m/s}$$

9.93: a) The initial moment of inertia is $I_0 = \frac{1}{2}MR^2$. The piece punched has a mass of $\frac{M}{16}$ and a moment of inertia with respect to the axis of the original disk of

$$\frac{M}{16}\left[\frac{1}{2}\left(\frac{R}{4}\right)^2 + \left(\frac{R}{2}\right)^2\right] = \frac{9}{512}MR^2.$$

The moment of inertia of the remaining piece is then

$$I = \frac{1}{2}MR^2 - \frac{9}{512}MR^2 = \frac{247}{512}MR^2.$$

b) $I = \frac{1}{2}MR^2 + M(R/2)^2 - \frac{1}{2}(M/16)(R/4)^2 = \frac{383}{512}MR^2.$

9.94: a) From the parallel-axis theorem, the moment of inertia is

$I_P = (2/5)MR^2 + ML^2$, and

$$\frac{I_P}{ML^2} = \left(1 + \left(\frac{2}{5}\right)\left(\frac{R}{L}\right)^2\right).$$

If $R = (0.05)L$, the difference is $(2/5)(0.05)^2 = 0.001$. b) $(I_{\text{rod}}/ML^2) = (m_{\text{rod}}/3M)$, which is 0.33% when $m_{\text{rod}} = (0.01)M$.

9.95: a) With respect to O, each element $r_i^{\,2}$ in Eq. (9.17) is $x_i^{\,2} + y_i^{\,2}$, and so

$$I_O = \sum_i m_i r_i^{\,2} = \sum_i m_i (x_i^{\,2} + y_i^{\,2}) = \sum_i m_i x_i^{\,2} + \sum_i m_i y_i^{\,2} = I_x + I_y.$$

b) Two perpendicular axes, both perpendicular to the washer's axis, will have the same moment of inertia about those axes, and the perpendicular-axis theorem predicts that they will sum to the moment of inertia about the washer axis, which is $\frac{M}{2}(R_1^{\,2} + R_2^{\,2})$, and so $I_x = I_y = \frac{M}{4}(R_1^{\,2} + R_2^{\,2})$.

c) From Table (9.2), $I = \frac{1}{12}m(L^{\,2} + L^2) = \frac{1}{6}mL^2$.
Since $I_0 = I_x + I_y$, and $I_x = I_y$, both I_x and I_y must be $\frac{1}{12}\, mL^2$.

9.96: Each side has length a and mass $\frac{M}{4}$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is $\frac{1}{12}\frac{M}{4}a^2 = \frac{Ma^2}{48}$. The moment of inertia of each side about the axis through the center of the square is, from the perpendicular axis theorem, $\frac{Ma^2}{48} + \frac{M}{4}\left(\frac{a}{2}\right)^2 = \frac{Ma^2}{12}$. The total moment of inertia is the sum of the contributions from the four sides, or $4 \times \frac{Ma^2}{12} = \frac{Ma^2}{3}$.

9.97: Introduce the auxiliary variable L, the length of the cylinder, and consider thin cylindrical shells of thickness dr and radius r; the cross-sectional area of such a shell is $2\pi\, r\, dr,$ and the mass of shell is $dm = 2\pi\, rL\rho\, dr = 2\pi\, \alpha\, Lr^2 dr.$ The total mass of the cylinder is then

$$M = \int dm = 2\pi\, L\alpha \int_0^R r^2 dr = 2\pi\, L\alpha \frac{R^3}{3}$$

and the moment of inertia is

$$I = \int r^2 dm = 2\pi\, L\alpha \int_o^R r^4 dr = 2\pi\, L\alpha \frac{R^5}{5} = \frac{3}{5} MR^2.$$

b) This is less than the moment of inertia if all the mass were concentrated at the edge, as with a thin shell with $I = MR^2,$ and is greater than that for a uniform cylinder with $I = \frac{1}{2} MR^2,$ as expected.

9.98: a) From Exercise 9.49, the rate of energy loss is $\frac{4\pi^2 I}{T^3} \frac{dT}{dt}$; solving for the moment of inertia I in terms of the power P,

$$I = \frac{PT^3}{4\pi} \frac{1}{dT/dt} = \frac{(5 \times 10^{31}\ \text{W})(0.0331\text{s})^3}{4\pi^2} \frac{1\,\text{s}}{4.22 \times 10^{-13}\ \text{s}} = 1.09 \times 10^{38}\ \text{kg} \cdot \text{m}^2.$$

b) $R = \sqrt{\dfrac{5I}{2M}} = \sqrt{\dfrac{5(1.08 \times 10^{38}\ \text{kg} \cdot \text{m}^2)}{2(1.4)(1.99 \times 10^{30}\ \text{kg})}} = 9.9 \times 10^3\ \text{m},\ \text{about}\ 10\ \text{km}.$

c) $$\frac{2\pi R}{T} = \frac{2\pi(9.9 \times 10^3 \text{m})}{(0.0331\text{s})} = 1.9 \times 10^6\ \text{m/s} = 6.3 \times 10^{-3}\ \text{c}.$$

d) $$\frac{M}{V} = \frac{M}{(4\pi/3)R^3} = 6.9 \times 10^{17}\ \text{kg/m}^3,$$

which is much higher than the density of ordinary rock by 14 orders of magnitude, and is comparable to nuclear mass densities.

9.99: a) Following the hint, the moment of inertia of a uniform sphere in terms of the mass density is $I = \frac{2}{5}MR^2 = \frac{8\pi}{15}\rho R^5,$ and so the difference in the moments of inertia of two spheres with the same density ρ but different radii R_2 and R_1 is $I = \rho(8\pi/15)(R_2^5 - R_1^5)$.

b) A rather tedious calculation, summing the product of the densities times the difference in the cubes of the radii that bound the regions and multiplying by $4\pi/3$, gives $M = 5.97 \times 10^{24}$ kg. c) A similar calculation, summing the product of the densities times the difference in the fifth powers of the radii that bound the regions and multiplying by $8\pi/15$, gives $I = 8.02 \times 10^{22}\ \text{kg} \cdot \text{m}^2 = 0.334MR^2$.

9.100: Following the procedure used in Example 9.14 (and using z as the coordinate along the vertical axis) $r(z) = z\frac{R}{h}, dm = \pi\rho\frac{R^2}{h^2}z^2\, dz$ and $dI = \frac{\pi\rho}{2}\frac{R^4}{h^4}z^4\, dz.$ Then,

$$I = \int dI = \frac{\pi\rho}{2}\frac{R^4}{h}\int_0^h z^4\, dz = \frac{\pi\rho}{10}\frac{R^4}{h^4}\left[z^5\right]_0^h = \frac{1}{10}\pi\rho R^4 h.$$

The volume of a right circular cone is $V = \frac{1}{3}\pi R^2 h,$ the mass is $\frac{1}{3}\pi R^2 h$and so

$$I = \frac{3}{10}\left(\frac{\pi\rho R^2 h}{3}\right)R^2 = \frac{3}{10}MR^2.$$

9.101: a) $ds = r\,d\theta = r_0 d\theta + \beta\theta\, d\theta$, so $s(\theta) = r_0\theta + \frac{\beta}{2}\theta^2$. b) Setting $s = vt = r_0\theta + \frac{\beta}{2}\theta^2$ gives a quadratic in θ. The positive solution is

$$\theta(t) = \frac{1}{\beta} = \left[\sqrt{r_0^{\;2} + 2\beta vt} - r_0\right].$$

(The negative solution would be going backwards, to values of r smaller than r_0 .)

c) Differentiating,

$$\omega_z(t) = \frac{d\theta}{dt} = \frac{v}{\sqrt{r_0^2 + 2\beta vt}},$$

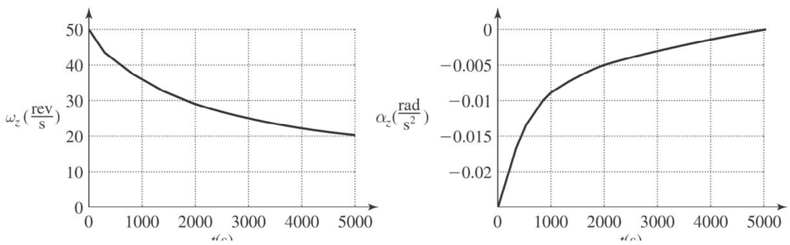
$$\alpha_z = \frac{d\omega}{dt} = -\frac{\beta v^2}{\left(r_0^2 = 2\beta vt\right)^{3/2}}.$$

The angular acceleration α_z is not constant. d) $r_0 = 25.0\text{ mm}$; It is crucial that θ is measured in radians, so $\beta = (1.55\ \mu\text{m/rev})(1\text{ rev}/2\pi\text{ rad}) = 0.247\ \mu\text{m/rad}.$ The total angle turned in 74.0 min = 4440 s is

$$\theta = \frac{1}{2.47\times10^{-7}\,\text{m/rad}}\left[\sqrt{\begin{matrix}2(2.47\times10^{-7}\,\text{m/rad})(1.25\,\text{m/s})(4440\,\text{s})\\ +(25.0\times10^{-3}\,\text{m})^2\end{matrix}} - 25.0\times10^{-3}\text{ m}\right]$$
$$=1.337\times10^{5}\text{ rad}$$

which is 2.13 × 10^4 rev.

e)



10.1: Equation (10.2) or Eq. (10.3) is used for all parts.

a) $(4.00\,\text{m})(10.0\,\text{N})\sin 90° = 40.00\,\text{N}\cdot\text{m}$, out of the page.

b) $(4.00\,\text{m})(10.0\,\text{N})\sin 120° = 34.6\,\text{N}\cdot\text{m}$, out of the page.

c) $(4.00\,\text{m})(10.0\,\text{N})\sin 30° = 20.0\,\text{N}\cdot\text{m}$, out of the page.

d) $(2.00\,\text{m})(10.00\,\text{N})\sin 60° = 17.3\,\text{N}\cdot\text{m}$, into the page.

e) The force is applied at the origin, so $\tau = 0$.

f) $(4.00\,\text{m})(10.0\,\text{N})\sin 180° = 0$.

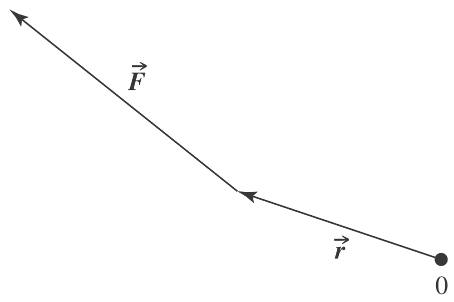
10.2:
$$\tau_1 = -(8.00\,\text{N})(5.00\,\text{m}) = -40.0\,\text{N}\cdot\text{m},$$
$$\tau_2 = (12.0\,\text{N})(2.00\,\text{m})\sin 30° = 12.0\,\text{N}\cdot\text{m},$$

where positive torques are taken counterclockwise, so the net torque is $-28.0\,\text{N}\cdot\text{m}$, with the minus sign indicating a clockwise torque, or a torque into the page.

10.3: Taking positive torques to be counterclockwise (out of the page), $\tau_1 = -(0.090\,\text{m})\times(180.0\,\text{N}) = -1.62\,\text{N}\cdot\text{m}$, $\tau_2 = (0.09\,\text{m})(26.0\,\text{N}) = 2.34\,\text{N}\cdot\text{m}$, $\tau_3 = \left(\sqrt{2}\right)(0.090\,\text{m})\,(14.0\,\text{N}) = 1.78\,\text{N}\cdot\text{m}$, so the net torque is $2.50\,\text{N}\cdot\text{m}$, with the direction counterclockwise (out of the page). Note that for τ_3 the applied force is perpendicular to the lever arm.

10.4:
$$\tau_1 + \tau_2 = -F_1R + F_2R = (F_2 - F_1)R$$
$$= (5.30\,\text{N} - 7.50\,\text{N})(0.330\,\text{m}) = -0.726\,\text{N}\cdot\text{m}.$$

10.5: a)



b) Into the plane of the page.

c) $\vec{\boldsymbol{r}}\times\vec{\boldsymbol{F}} = [(-0.450\,\text{m})\,\hat{\boldsymbol{i}} + (0.150\,\text{m})\,\hat{\boldsymbol{j}}]\times[(-5.00\,\text{n})\hat{\boldsymbol{i}} + (4.00\,\text{N})\hat{\boldsymbol{i}}]$

$= (-0.450\,\text{m})\,(4.00\,\text{N}) - (0.150\,\text{m})(-5.00\,\text{N})\hat{\boldsymbol{k}}$

$= (-1.05\,\text{N}\cdot\text{m})\hat{\boldsymbol{k}}$

10.6: (a)

$$\tau_A = (50\ \text{N})(\sin 60°)(0.2\ \text{m}) = 8.7\ \text{N}\cdot\text{m, CCW}$$
$$\tau_B = 0$$
$$\tau_C = (50\ \text{N})(\sin 30°)(0.2\ \text{m}) = 5\ \text{N}\cdot\text{m, CW}$$
$$\tau_D = (50\ \text{N})(0.2\ \text{m}) = 10\ \text{N}\cdot\text{m, CW}$$

(b) $\sum\tau = 8.7\ \text{N}\cdot\text{m} - 5\ \text{N}\cdot\text{m} - 10\ \text{N}\cdot\text{m}$
$= -6.3\ \text{N}\cdot\text{m, CW}$

10.7: $I = \frac{2}{3}MR^2 + 2mR^2$, where $M = 8.40$ kg, $m = 2.00$ kg

$I = 0.600\ \text{kg}\cdot\text{m}^2$

$\omega_0 = 75.0\ \text{rpm} = 7.854\ \text{rad/s};\ \omega = 50.0\ \text{rpm} = 5.236\ \text{rad/s};\ t = 30.0\ \text{s},\ \alpha = ?$

$\omega = \omega_0 + \alpha t$ gives $\alpha = -0.08726\ \text{rad/s}^2$;

$\Sigma\tau = I\alpha,\ \ \tau_f = I\alpha = -0.0524\ \text{N}\cdot\text{m}$

10.8: a) $\tau = I\alpha = I\dfrac{\Delta\omega}{\Delta t} = \left(2.50\ \text{kg}\cdot\text{m}^2\right)\dfrac{\left(400\ \text{rev/min}\times\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}}\right)}{(8.00\ \text{s})} = 13.1\ \text{N}\cdot\text{m}.$

b) $\dfrac{1}{2}I\omega^2 = \dfrac{1}{2}(2.50\ \text{kg}\cdot\text{m}^2)\left(400\ \text{rev/min}\times\dfrac{2\pi}{60}\dfrac{\text{rad/s}}{\text{rev/min}}\right)^2 = 2.19\times10^3\ \text{J}.$

10.9: $v = \sqrt{2as} = \sqrt{2\left(0.36\ \text{m/s}^2\right)(2.0\ \text{m})} = 1.2\ \text{m/s}$, the same as that found in Example 9-8.

10.10:

$$\alpha = \frac{\tau}{I} = \frac{FR}{I} = \frac{(40.0\ \text{N})(0.250\ \text{m})}{\left(5.0\ \text{kg}\cdot\text{m}^2\right)} = 2.00\ \text{rad/s}^2.$$

10.11: a)

$$n = Mg + T = g\left[M + \frac{m}{1+2m/M}\right] = g\left[\frac{M+3m}{1+2m/M}\right]$$

b) This is less than the total weight; the suspended mass is accelerating down, so the tension is less than *mg*. c) As long as the cable remains taut, the velocity of the mass does not affect the acceleration, and the tension and normal force are unchanged.

10.12: a) The cylinder does not move, so the net force must be zero. The cable exerts a horizontal force to the right, and gravity exerts a downward force, so the normal force must exert a force up and to the left, as shown in Fig. (10.9).

b) $n = \sqrt{(9.0\ \text{N})^2 + ((50\ \text{kg})(9.80\ \text{m/s}^2))^2}$ 490 N, at an angle of arctan $\left(\frac{9.0}{490}\right) = 1.1°$ from the vertical (the weight is much larger than the applied force F).

10.13: $\mu_\text{k} = \dfrac{f}{n} = \dfrac{\tau/R}{n} = \dfrac{I\alpha}{Rn} = \dfrac{MR(\omega_0/t)}{2n}$

$$= \frac{(50.0\ \text{kg})(0.260\ \text{m})(850\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)}{2(7.50\ \text{s})(160\ \text{N})} = 0.482.$$

10.14: (a) Falling stone: $g = \frac{1}{2}at^2$

$$12.6\ \text{m} = \tfrac{1}{2}a(3.00\ \text{s})^2$$

$$a = 2.80\ \text{m/s}^2$$

$$\text{Stone}: \Sigma F = ma : mg - T = ma (1)$$

$$\text{Pulley}: \Sigma \tau = I\alpha : TR = \tfrac{1}{2}MR^2\alpha = \tfrac{1}{2}MR^2\left(\tfrac{a}{\text{R}}\right)$$

$$T = \tfrac{1}{2}Ma (2)$$

Solve (1) and (2):

$$M = \frac{M}{2}\left(\frac{a}{g-a}\right) = \left(\frac{10.0\ \text{kg}}{2}\right)\left(\frac{2.80\ \text{m/s}^2}{9.80\ \text{m/s}^2 - 2.80\ \text{m/s}^2}\right)$$

$$M = 2.00\ \text{kg}$$

(b)From (2):

$$T = \frac{1}{2}Ma = \frac{1}{2}(10.0\ \text{kg})(2.80\ \text{m/s}^2)$$

$$T = 14.0\ \text{N}$$

10.15: $I = \frac{1}{2}mR^2 = \frac{1}{2}(8.25\,\text{kg})(0.0750\,\text{m})^2 = 0.02320\,\text{kg}\cdot\text{m}^2$

$\omega_0 = 220\,\text{rpm} = 23.04\,\text{rad/s};\ \omega = 0;\ \theta - \theta_0 = 5.25\,\text{rev} = 33.0\,\text{rad},\ \alpha = ?$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ gives $\alpha = -8.046\,\text{rad/s}^2$

$\sum \tau = I\alpha$

$\sum \tau = \tau_f = -f_\text{k}R = -\mu_\text{k}nR$

$$-\mu_\text{k}nR = I\alpha \text{ so } n = \frac{I\alpha}{\mu_\text{k}R} = 7.47\,\text{N}$$

10.16: This is the same situtation as in Example 10.3. a) $T = mg/(1+2m/M) = 42.0\,\text{N}$. b) $v = \sqrt{2\,gh/(1+M/2m} = 11.8\,\text{m/s}$. c) There are many ways to find the time of fall. Rather than make the intermediate calculation of the acceleration, the time is the distance divided by the average speed, or $h/(v/2) = 1.69\,\text{s}$. d) The normal force in Fig. (10.10(b)) is the sum of the tension found in part (a) and the weight of the windlass, a total 159.6 N (keeping extra figures in part (a)).

10.17: See Example 10.4. In this case, the moment of inertia I is unknown, so $a_1 = (m_2g)/(m_1 + m_2 + (I/R^2))$. a) $a_1 = 2(1.20\,\text{m})/(0.80\,\text{s})^2 = 3.75\,\text{m/s}^2$, so $T_1\ m_1a_1 = 7.50\,\text{N}$ and $T_2 = m_2(g - a_1) = 18.2\,\text{N}$.

b) The torque on the pulley is $(T_2 - T_1)R = 0.803\,\text{N}\cdot\text{m}$, and the angular acceleration is $\alpha = a_1/R = 50\,\text{rad/s}^2$, so $I = \tau/\alpha = 0.016\,\text{kg}\cdot\text{m}^2$.

10.18:
$$\alpha = \frac{\tau}{I} = \frac{Fl}{\frac{1}{3}Ml^2} = \frac{3F}{Ml}.$$

10.19: The acceleration of the mass is related to the tension by $Ma_\text{cm} = Mg - T$, and the angular acceleration is related to the torque by $I\alpha = \tau = TR$, or $a_\text{cm} = T/M$, where $\alpha = a_\text{cm}/RSt$ and $I = MR^2$ have been used.
a) Solving these for T gives $T = Mg/2 = 0.882\,\text{N}$. b) Substituting the expression for T into either of the above relations gives $a_\text{cm} = g/2$, from which $t = \sqrt{2h/a_\text{cm}} = \sqrt{4h/g} = 0.553\,\text{s}$. c) $\omega = v_\text{cm}/R = a_\text{cm}t/R = 33.9\,\text{rad/s}$.

10.20: See Example 10.6 and Exercise 10.21. In this case, $K_2 = Mv_\text{cm}^2$ and $v_\text{cm} = \sqrt{gh}$, $\omega = v_\text{cm}/R = 33.9\,\text{rad/s}$.

10.21: From Eq. (10.11), the fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{\text{cm}}\omega^2}{(1/2)Mv_{\text{cm}}^2+(1/2)I_{\text{cm}}\omega^2}=\frac{1}{1+(M/I_{\text{cm}})/v_{\text{cm}}^2/\omega^2}=\frac{1}{1+\frac{MR^2}{I_{\text{cm}}}},$$

where $v_{\text{cm}}=R\omega$ for an object that is rolling without slipping has been used.

a) $I_{\text{cm}}=(1/2)MR^2$, so the above ratio is $1/3$. b) $\mathrm{I}=(2/5)\mathrm{MR}^2$, so the above ratio is $2/7$. c) $I=2/3MR^2$, so the ratio is $2/5$. d) $I=5/8\,\mathrm{MR}^2$, so the ratio is $5/13$.

10.22: a) The acceleration down the slope is $a=g\sin\theta-\frac{f}{M}$, the torque about the center of the shell is

$$\tau=Rf=I\alpha=I\frac{a}{R}=\frac{2}{3}MR^2\frac{a}{R}=\frac{2}{3}MRa,$$

so $\frac{f}{M}=\frac{2}{3}a$. Solving these relations a for f and simultaneously gives $\frac{5}{3}a=g\sin\theta$, or

$$a=\frac{3}{5}g\sin\theta=\frac{3}{5}(9.80\,\text{m/s}^2)\sin 38.0°=3.62\,\text{m/s}^2,$$

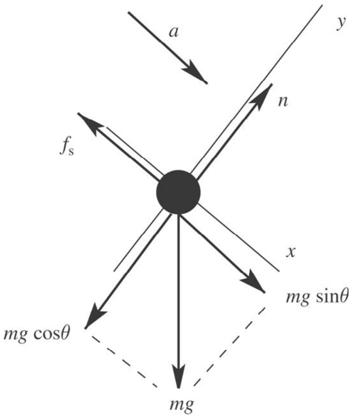
$$f=\frac{2}{3}Ma=\frac{2}{3}(2.00\,\text{kg})(3.62\,\text{m/s}^2)=4.83\text{ N}.$$

The normal force is $Mg\cos\theta$, and since $f\le\mu_{\text{s}}n$,

$$\mu_{\text{s}}\ge\frac{f}{n}=\frac{\frac{2}{3}Ma}{Mg\cos\theta}=\frac{2}{3}\frac{a}{g\cos\theta}=\frac{2}{3}\frac{\frac{3}{5}g\sin\theta}{g\cos\theta}=\frac{2}{5}\tan\theta=0.313.$$

b) $a=3.62\,\text{m/s}^2$ since it does not depend on the mass. The frictional force, however, is twice as large, 9.65 N, since it does depend on the mass. The minimum value of μ_{s} also does not change.

10.23:



$n = mg\cos\alpha$

$mg\sin\theta - \mu_s mg\cos\theta = ma$

$g(\sin\theta - \mu_s\cos\theta) = a$ (eq.1)

n and mg act at the center of the ball and provide no torque.

$\sum\tau = \tau_f = \mu_s mg\cos\theta R; I = \frac{2}{5}mR^2$

$\sum\tau = I\alpha$ gives $\mu_s mg\cos\theta = \frac{2}{5}mR^2\alpha$

No slipping means $\alpha = a/R$, so $\mu_s g\cos\theta = \frac{2}{5}a$ (eq.2)

We have two equations in the two unknowns a and μ_s. Solving gives

$a = \frac{5}{7}g\sin\theta$ and $\mu_s = \frac{2}{7}\tan\theta = \frac{2}{7}\tan 65.0° = 0.613$

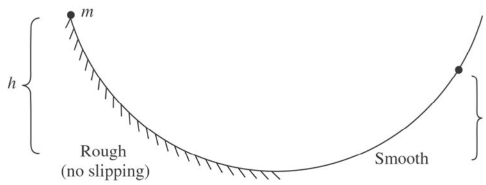
b) Repeat the calculation of part (a), but now $I = \frac{2}{3}mR^2$.

$a = \frac{3}{5}g\sin\theta$ and $\mu_s = \frac{2}{5}\tan\theta = \frac{2}{5}\tan 65.0° = 0.858$

The value of μ_s calculated in part (a) is not large enough to prevent slipping for the hollow ball.

c) There is no slipping at the point of contact.

10.24:



$v_{\text{cm}} = R\omega$ for no slipping

a) Get v at bottom:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$v = \sqrt{\frac{10}{7}gh}$$

Now use energy conservation. Rotational *KE* does not change

$$\frac{1}{2}mv^2 + KE_{\text{Rot}} = mgh' + KE_{\text{Rot}}$$

$$h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$$

(b) $mgh = mgh' \rightarrow h' = h$ With friction on both halves, all the *PE* gets converted back to *PE*. With one smooth side, some of the *PE* remains as rotational *KE*.

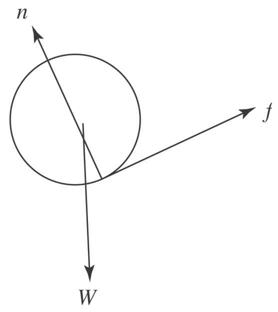
10.25: $wh - W_f = K_1 = (1/2)I_{\text{cm}}w^2{}_0 + \tfrac{1}{2}mv^2{}_{\text{cm}}$

Solving for h with $v_{\text{cm}} = Rw$

$$h = \frac{\frac{1}{2}\left(\frac{w}{9.80\ \text{m/s}^2}\right)[(0.800)(0.600\ \text{m})^2(25.0\ \text{rad/s})^2 + (0.600\ \text{m})^2(25.0\ \text{rad/s})^2]}{w}$$

$$-\frac{3500\ \text{J}}{392\ \text{N}} = 11.7\ \text{m}.$$

10.26: a)



The angular speed of the ball must decrease, and so the torque is provided by a friction force that acts up the hill.

b) The friction force results in an angular acceleration, related by $I\alpha = fR$. The equation of motion is $mg \sin\beta - f = ma_{\text{cm}},$ and the acceleration and angular acceleration are related by $a_{\text{cm}} = R\alpha$ (note that positive acceleration is taken to be *down* the incline, and relation between a_{cm} and α is correct for a friction force directed uphill). Combining,

$$mg \sin\beta = ma\left(1+\frac{I}{mR^2}\right) = ma(7/5),$$

from which $a_{\text{cm}} = (5/7)g \sin\beta$. c) From either of the above relations between if f and a_{cm},

$$f = \frac{2}{5}ma_{\text{cm}} = \frac{2}{7}mg \sin\beta \leq \mu_{\text{s}}n = \mu_{\text{s}}mg \cos\beta,$$

from which $\mu_{\text{s}} \geq (2/7)\tan\beta$.

10.27: a) $\omega = \alpha\Delta t = (FR/I)\Delta t = ((18.0\ \text{N})(2.40\ \text{m})/(2100\ \text{kg}\cdot\text{m}^2))(15.0\ \text{s}) = 0.3086\ \text{rad/s}$, or 0.309 rad/s to three figures.

b) $W = K_2 = (1/2)I\omega^2 = (1/2)\times\ (2.00\ \text{kg}\cdot\text{m}^2)(0.3086\ \text{rad/s})^2 = 100\ \text{J}$.

c) From either $P = \tau\omega_{\text{ave}}$ or $P = W/\Delta t$, $P = 6.67\ \text{W}$.

10.28: a) $$\tau = \frac{P}{\omega} = \frac{(175\ \text{hp})(746\ \text{W/hp})}{(2400\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)} = 519\ \text{N}\cdot\text{m}.$$

b) $W = \tau\Delta\theta = (519\ \text{N}\cdot\text{m})(2\pi) = 3261\text{J}.$

10.29: a) $$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t}$$

$$= \frac{\left((1/2)(1.50\ \text{kg})(0.100\ \text{m})^2\right)(1200\ \text{rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)}{2.5\ \text{s}}$$

$$= 0.377\ \text{N}\cdot\text{m}.$$

b) $$\omega_{\text{ave}}\Delta t = \frac{(600\ \text{rev/min})(2.5\ \text{s})}{60\ \text{s/min}} 25.0\ \text{rev} = 157\ \text{rad}.$$

c) $$\tau\Delta\theta = 59.2\ \text{J}.$$

d) $$K = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\left((1/2)(1.5\ \text{kg})(0.100\ \text{m})^2\right)\left((1200\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2$$

$$= 59.2\ \text{J},$$

the same as in part (c).

10.30: From Eq. (10.26), the power output is

$$P = \tau\omega = (4.30\ \text{N}\cdot\text{m})\left(4800\ \text{rev/min} \times \frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}}\right) = 2161\ \text{W},$$

which is 2.9 hp.

10.31: a) With no load, the only torque to be overcome is friction in the bearings (neglecting air friction), and the bearing radius is small compared to the blade radius, so any frictional torque could be neglected.

b) $$F = \frac{\tau}{R} = \frac{P/\omega}{R} = \frac{(1.9\ \text{hp})(746\ \text{W/hp})}{(2400\ \text{rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)(0.086\ \text{m})} = 65.6\ \text{N}.$$

10.32: $I = \frac{1}{2}mL^2 = \frac{1}{2}(117\ \text{kg})(2.08\ \text{m})^2 = 42.2\ \text{kg}\cdot\text{m}^2$

a) $$\alpha = \frac{\tau}{I} = \frac{1950\ \text{N}\cdot\text{m}}{42.2\ \text{kg}\cdot\text{m}^2} = 46.2\ \text{rad/s}^2.$$

b) $\omega = \sqrt{2\alpha\theta} = \sqrt{2(46.2\ \text{rad/s}^2)(5.0\ \text{rev}\times 2\pi\ \text{rev})} = 53.9\ \text{rad/s}.$

c) From either $W = K = \dfrac{1}{2}\omega^2$ or Eq. (10.24),

$$W = \tau\theta = (1950\ \text{N.m})(5.00\ \text{rev}\times 2\pi\ \text{rad/rev}) = 6.13\times 10^4\ \ \text{J}.$$

d), e) The time may be found from the angular acceleration and the total angle, but the instantaneous power is also found from $P = \tau\omega = 105\ \text{kW}(141\ \text{hp})$. The average power is half of this, or $52.6\ \text{kW}$.

10.33: a) $\tau = P/\omega = (150\times 10^3\ \text{W}) \Big/ \left((400\ \text{rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\right) = 358\ \text{N}\cdot\text{m}.$

b) If the tension in the rope is $F, F = w$ and so $w = \tau/R = 1.79\times 10^3\ \ \text{N}.$

c) Assuming ideal efficiency, the rate at which the weight gains potential energy is the power output of the motor, or $wv = P$, so $v = P/w = 83.8\ \text{m/s}.$ Equivalently, $v = \omega R.$

10.34: As a point, the woman's moment of inertia with respect to the disk axis is mR^2, and so the total angular momentum is

$$\begin{aligned} L = L_{\text{disk}} + L_{\text{woman}} &= (I_{\text{disk}} + I_{\text{woman}})\omega = \left(\frac{1}{2}M + m\right)R^2\omega \\ &= \left(\frac{1}{2}110\ \text{kg} + 50.0\ \text{kg}\right)(4.00\ \text{m})^2(0.500\ \text{rev/s}\times 2\pi\ \text{rad/rev}) \\ &= 5.28\times 10^3\ \text{kg}\cdot\text{m}^2/\text{s}. \end{aligned}$$

10.35: a) $mvr\sin\varphi = 115\ \text{kg}\cdot\text{m}^2/\text{s},$ with a direction from the right hand rule of into the page.

b) $dL/dt = \tau = (2\ \text{kg})(9.8\ \text{N/kg})\cdot(8\ \text{m})\cdot\sin(90^\circ - 36.9^\circ) = 125\ \text{N}\cdot\text{m} = 125\ \text{kg}\cdot\text{m}^2/\text{s}^2$, out of the page.

10.36: For both parts, $L = I\omega$. Also, $\omega = v/r$, so $L = I(v/r)$.

a) $L = (mr^2)(v/r) = mvr$

$L = (5.97\times10^{24}\ \text{kg})(2.98\times10^4\ \text{m/s})(1.50\times10^{11}\ \text{m}) = 2.67\times10^{40}\ \text{kg}\cdot\text{m}^2/\text{s}$

b) $L = (2/5mr^2)(\omega)$

$L = (2/5)(5.97\times10^{24}\ \text{kg})(6.38\times10^6\ \text{m})^2(2\pi\ \text{rad}/(24.0\ \text{hr}\times3600\ \text{s/hr}))$

$= 7.07\times10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}$

10.37: The period of a second hand is one minute, so the angular momentum is

$$L = I\omega = \frac{M}{3}l^2\frac{2\pi}{T}$$
$$= \left(\frac{6.0\times10^{-3}\ \text{kg}}{3}\right)(15.0\times10^{-2}\ \text{m})^2\frac{2\pi}{60\ \text{s}} = 4.71\times10^{-6}\ \text{kg}\cdot\text{m}^2/\text{s}.$$

10.38: The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi\ \text{rad}}{(30\ \text{d})(86{,}400\ \text{s/d}}\right)\left(\frac{7.0\times10^5\ \text{km}}{16\ \text{km}}\right)^2 = 4.6\times10^3\ \text{rad/s}.$$

10.39: a) The net force is due to the tension in the rope, which always acts in the radial direction, so the angular momentum with respect to the hole is constant.

b) $L_1 = m\omega_1 r^2{}_1, L_2 = m\omega_2 r_2{}^2$, and with $L_1 = L_2, \omega_2 = \omega_1(r_1/r_2)^2 = 7.00\ \text{rad}/s$.

c) $\Delta K = (1/2)m((\omega_2 r_2)^2 - (\omega_1 r_1)^2) = 1.03\times10^{-2}\ \text{J}$.

d) No other force does work, so $1.03\times10^{-2}\ \text{J}$ of work were done in pulling the cord.

10.40: The skater's initial moment of inertia is

$$I_1 = (0.400\,\text{kg}\cdot\text{m}^2) + \frac{1}{2}(8.00\,\text{kg})(1.80\,\text{m})^2 = 2.56\,\text{kg}\cdot\text{m}^2,$$

and her final moment of inertia is

$$I_2 = (0.400\,\text{kg}\cdot\text{m}^2) + (8.00\,\text{kg})(25\times10^{-2}\,\text{m}) = 0.9\,\text{kg}\cdot\text{m}^2.$$

Then from Eq. (10.33),

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = (0.40\,\text{rev/s})\frac{2.56\,\text{kg}\cdot\text{m}^2}{0.9\,\text{kg}\cdot\text{m}^2} = 1.14\,\text{rev/s}.$$

Note that conversion from rev/s to rad/s is not necessary.

10.41: If she had tucked, she would have made $(2)(3.6\,\text{kg}\cdot\text{m}^2)/18\,\text{kg}\cdot\text{m}^2) = 0.40$ rev in the last 1.0 s, so she would have made $(0.40\,\text{rev})(1.5/1.0) = 0.60$ rev in the total 1.5 s.

10.42: Let

$$I_1 = I_0 = 1200\text{kg}\cdot\text{m}^2,$$
$$I_2 = I_0 + mR^2 = 1200\text{kg}\cdot\text{m}^2 + (40.0\text{kg})(2.00\,\text{m})^2 = 1360\text{kg}\cdot\text{m}^2.$$

Then, from Eq. (10.33),

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \left(\frac{2\pi\ \text{rad}}{6.00\,\text{s}}\right)\frac{1200\,\text{kg.m}^2}{1360\,\text{kg.m}^2} = 0.924\,\text{rad/s}.$$

10.43: a) From conservation of angular momentum,

$$\omega_2 = \omega_1 \frac{I_1}{I_0 + mR^2} = \omega_1 \frac{(1/2)MR^2}{(1/2)MR^2 + mR^2} = \omega_1 \frac{1}{1 + 2m/M}$$

$$= \frac{3.0\ \text{rad/s}}{1 + 2(70)/120} = 1.385\ \text{rad/s}$$

or $1.39\ \text{rad/s}$ to three figures

b) $K_1 = (1/2)(1/2)(120\ \text{kg})(2.00\ \text{m})^2 (3.00\ \text{rad/s})^2 = 1.80\ \text{kJ}$, and $K_2 = (1/2)\left(I_0 + (70\ \text{kg})(2.00\ \text{m})^2\right)\omega_2^2 = 499\ \text{J}$. In changing the parachutist's horizontal component of velocity and slowing down the turntable, friction does negative work.

10.44: Let the width of the door be l;

$$\omega = \frac{L}{I} = \frac{mv(l/2)}{(1/3)Ml^2 + m(l/2)^2}$$

$$= \frac{(0.500\ \text{kg})(12.0\ \text{m/s})(0.500\ \text{m})}{(1/3)(40.0\ \text{kg})(1.00\ \text{m})^2 + (0.500\ \text{kg})(0.500\ \text{m})^2} = 0.223\ \text{rad/s}.$$

Ignoring the mass of the mud in the denominator of the above expression gives $\omega = 0.225\ \text{rad/s}$, so the mass of the mud in the moment of inertia does affect the third significant figure.

10.45: Apply conservation of angular momentum $\vec{L}$, with the axis at the nail. Let object A be the bug and object B be the bar.

Initially, all objects are at rest and $L_1 = 0$.

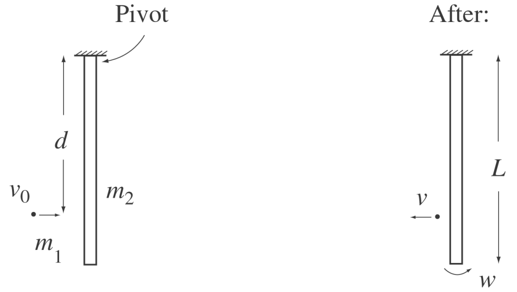
Just after the bug jumps, it has angular momentum in one direction of rotation and the bar is rotating with angular velocity ω_B in the opposite direction.

$L_2 = m_A v_A r - I_B \omega_B$ where $r = 1.00\ \text{m}$ and $I_B = \frac{1}{3} m_B r^2$

$L_1 = L_2$ gives $m_A v_A r = \frac{1}{3} m_B r^2 \omega_B$

$$\omega_B = \frac{3 m_A v_A}{m_B r} = 0.120\ \text{rad/s}$$

10.46:



(a) Conservation of angular momentum:

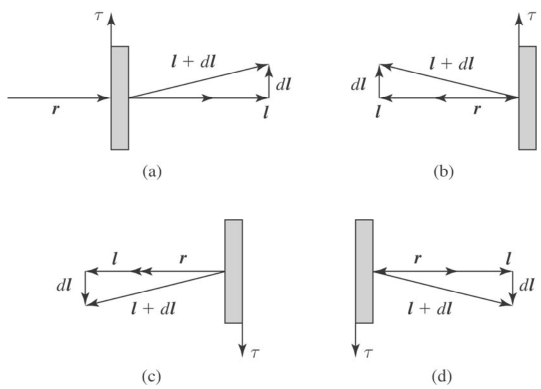
$$m_1 v_0 d = -m_1 v d + \tfrac{1}{3} m_2 L^2 \omega$$

$$(3.00\,\text{kg})(10.0\,\text{m/s})(1.50\,\text{m}) = -(3.00\,\text{kg})(6.00\,\text{m/s})(1.50\,\text{m}) + \frac{1}{3}\left(\frac{90.0\,\text{N}}{9.80\,\text{m/s}^2}\right)(2.00\,\text{m})^2 \omega$$

$$\omega = 5.88\,\text{rad/s}$$

(b) There are no unbalanced torques about the pivot, so angular momentum is conserved. But the pivot exerts an unbalanced horizontal external force on the system, so the linear momentum is not conserved.

10.47:



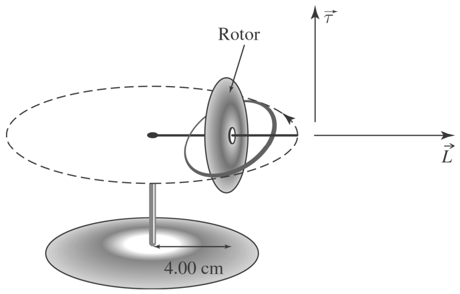
10.48: a) Since the gyroscope is precessing in a horizontal plane, there can be no net vertical force on the gyroscope, so the force that the pivot exerts must be equal in magnitude to the weight of the gyroscope,
$F = \omega = mg = (0.165\,\text{kg})(9.80\,\text{m/s}^2) = 1.617\,\text{N},\ 1.62\,\text{N}$ to three figures.

b) Solving Eq. (10.36) for ω,

$$\omega = \frac{\omega R}{I\Omega} = \frac{(1.617\,\text{N})(4.00\times10^{-2}\,\text{m})}{(1.20\times10^{-4}\,\text{kg}\cdot\text{m}^2)(2\pi)\,\text{rad/2.20\,s}} = 188.7\,\text{rad/s},$$

which is $1.80\times10^3\,\text{rev/min}$. Note that in this and similar situations, since Ω appears in the denominator of the expression for ω, the conversion from rev/s and back to rev/min *must* be made.

c)



10.49: a) $\dfrac{K}{P} = \dfrac{(1/2)((1/2)MR^2)\omega^2}{P}$

$$= \frac{(1/2)((1/2)(60{,}000\,\text{kg})(2.00\,\text{m})^2)((500\,\text{rev/min})(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}))^2}{7.46\times10^4\,\text{W}}$$

$$= 2.21\times10^3\,\text{s},$$

or 36.8 min.

b) $\tau = I\Omega\omega$

$$= (1/2)(60{,}000\,\text{kg})(2.00\,\text{m})^2(500\,\text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)(1.00°/\text{s})\left(\frac{2\pi\,\text{rad}}{360°}\right)$$

$$= 1.10\times10^5\,\text{N}\cdot\text{m}.$$

10.50: Using Eq. (10.36) for all parts, a) halved b) doubled (assuming that the added weight is distributed in such a way that r and I are not changed) c) halved (assuming that w and r are not changed) d) doubled e) unchanged.

10.51: a) Solving Eq. (10.36) for $\tau, \tau = I\omega\,\Omega = (2/5)MR^2\omega\,\Omega$. Using $\omega = \frac{2\pi\ \text{rad}}{86{,}400\ \text{s}}$ and $\Omega = \frac{2\pi}{(26{,}000\ \text{y})(3.175\times10^7\ \text{s/y})}$ and the mass and radius of the earth from Appendix F, $\tau \sim 5.4\times10^{22}\ \text{N}\cdot\text{m}.$

10.52: a) The net torque must be

$$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = (1.86\ \text{kg}\cdot\text{m}^2)\frac{\left(120\ \text{rev/min}\times\dfrac{2\pi}{60}\dfrac{\text{rad/s}}{\text{rev/min}}\right)}{(9.00\ \text{s})} = 2.60\ \text{N}\cdot\text{m}.$$

This torque must be the sum of the applied force FR and the opposing frictional torques τ_f at the axle and $fr = \mu_\text{k}nr$ due to the knife. Combining,

$$\begin{aligned} F &= \frac{1}{R}(\tau + \tau_\text{f} + \mu_\text{k}nr) \\ &= \frac{1}{0.500\ \text{m}}\big((2.60\ \text{N}\cdot\text{m}) + (6.50\ \text{N}\cdot\text{m}) + (0.60)(160\ \text{N})(0.260\ \text{m})\big) \\ &= 68.1\ \text{N}. \end{aligned}$$

b) To maintain a constant angular velocity, the net torque τ is zero, and the force F' is $F' = \frac{1}{0.500\ \text{m}}(6.50\ \text{N}\cdot\text{m} + 24.96\ \text{N}\cdot\text{m}) = 62.9\ \text{N}.$ c) The time t needed to come to a stop is found by taking the magnitudes in Eq. (10.27), with $\tau = \tau_\text{f}$ constant;

$$t = \frac{L}{\tau_\text{f}} = \frac{\omega I}{\tau_\text{f}} = \frac{\left(120\ \text{rev/min}\times\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}}\right)\left(1.86\ \text{kg}\cdot\text{m}^2\right)}{\left(6.50\ \text{N}\cdot\text{m}\right)} = 3.6\ \text{s}.$$

Note that this time can also be found as $t = \left(9.00\ \text{s}\right)\frac{2.60\text{N}\cdot\text{m}}{6.50\text{N}\cdot\text{m}}$.

10.53: a) $I = \dfrac{\tau}{\alpha} = \dfrac{\tau\Delta t}{\Delta\omega} = \dfrac{\left(5.0\ \text{N}\cdot\text{m}\right)\left(2.0\ \text{s}\right)}{\left(100\ \text{rev/min}\right)\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)} = 0.955\ \text{kg}\cdot\text{m}^2.$

b) Rather than use the result of part (a), the magnitude of the torque is proportional to α and hence inversely proportional to $|\Delta t|$; equivalently, the magnitude of the change in angular momentum is the same and so the magnitude of the torque is again proportional to $1/|\Delta t|$. Either way, $\tau_\text{f} = \left(5.0\ \text{N}\cdot\text{m}\right)\dfrac{2\ \text{s}}{125\ \text{s}} = 0.080\ \text{N}\cdot\text{m}.$

c) $\omega_\text{ave}\Delta t = \left(50.0\ \text{rev/min}\right)\left(125\ \text{s}\right)\left(1\ \text{min}/60\ \text{s}\right) = 104.2\ \text{rev}.$

10.54: a) The moment of inertia is not given, so the angular acceleration must be found from kinematics;

$$\alpha = \frac{2\theta}{t^2} = \frac{2s}{rt^2} = \frac{2(5.00\,\text{m})}{(0.30\,\text{m})(2.00\,\text{s})^2} = 8.33\ \text{rad/s}^2.$$

b) $\alpha t = (8.33\,\text{rad/s}^2)(2.00\,\text{s}) = 16.67\,\text{rad/s}.$

c) The work done by the rope on the flywheel will be the final kinetic energy; $K = W = Fs = (40.0\,\text{N})(5.0\,\text{m}) = 200\,\text{J}.$

d) $$I = \frac{2K}{\omega^2} = \frac{2(200\,\text{J})}{(16.67\,\text{rad/s})^2} = 1.44\,\text{kg}\cdot\text{m}^2.$$

10.55: a) $P = \tau\omega = \tau\alpha t = \tau\left(\frac{\tau}{I}\right)t = \tau^2\left(\frac{t}{I}\right).$

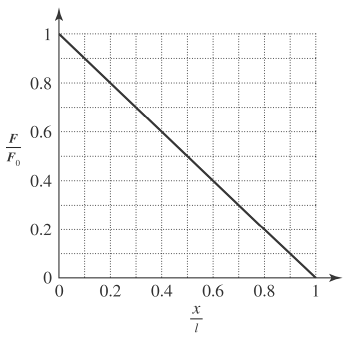
b) From the result of part (a), the power is $(500\,\text{W})\left(\frac{60.0}{20.0}\right)^2 = 4.50\,\text{kW}.$

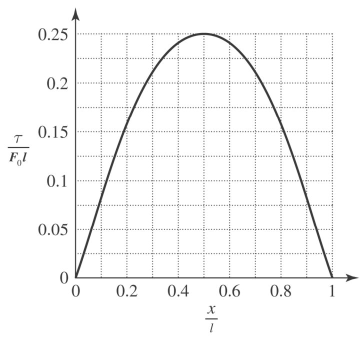
c) $P = \tau\omega = \tau\sqrt{2\alpha\theta} = \tau\sqrt{2(\tau/I)\theta} = \tau^{3/2}\sqrt{2\theta/I}.$

d) From the result of part (c), the power is $(500\,\text{W})\left(\frac{6.00}{20.00}\right)^{3/2} = 2.6\,\text{kW}.$ e) No; the power is proportional to the time t or proportional to the square root of the angle.

10.56: a) From the right-hand rule, the direction of the torque is $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}},$ the $+z$ direction.

b), c)





d) The magnitude of the torque is $F_0(x - x^2/l),$ which has it maximum at $l/2.$ The torque at $x = l/2$ is $F_0\, l/4.$

10.57: $$t^2 = \frac{2\theta}{\alpha} = \frac{2\theta}{(\tau/I)} = \frac{2\theta\, I}{\tau}.$$

The angle in radiants is $\pi/2$, the moment of inertia is

$$(1/3)((750\ \text{N})/(9.80\ \text{m}/\text{s}^2)(1.25\ \text{m}))^3 = 39.9\ \text{kg}\cdot\text{m}^2$$

and the torque is $(220\ \text{N})(1.25\ \text{m}) = 275\ \text{N}\cdot\text{m}$. Using these in the above expression gives $t^2 = 0.455\ \text{s}^2$, so $t = 0.675\ \text{s}$.

10.58: a) From geometric consideration, the lever arm and the sine of the angle between $\vec{F}$ and $\vec{r}$ are both maximum if the string is attached at the end of the rod. b) In terms of the distance x where the string is attached, the magnitude of the torque is $Fxh/\sqrt{x^2+h^2}$. This function attains its maximum at the boundary, where $x = h$, so the string should be attached at the right end of the rod. c) As a function of x, l and h, the torque has magnitude

$$\tau = F\frac{xh}{\sqrt{(x-l/2)^2+h^2}}.$$

This form shows that there are two aspects to increasing the torque; maximizing the lever arm l and maximizing sin ϕ. Differentiating τ with respect to x and setting equal to zero gives $x_{\text{max}} = (l/2)(1+(2h/l)^2)$. This will be the point at which to attach the string unless $2h > l$, in which case the string should be attached at the furthest point to the right, $x = l$.

10.59: a) A distance $L/4$ from the end with the clay.

b) In this case $I = (4/3)ML^2$ and the gravitational torque is $(3L/4)(2Mg)\sin\theta = (3Mg\,L/2)\sin\theta$, so $\alpha = (9g/8L)\sin\theta$.

c) In this case $I = (1/3)ML^2$ and the gravitational torque is $(L/4)(2Mg)\sin\theta = (Mg\,L/2)\sin\theta$, so $\alpha = (3g/2L)\sin\theta$. This is greater than in part (b).

d) The greater the angular acceleration of the upper end of the cue, the faster you would have to react to overcome deviations from the vertical.

10.60: In Fig. (10.22) and Eq. (10.22), with the angle θ measured from the vertical, $\sin\theta = \cos\theta$ in Eq. (10.2). The torque is then $\tau = FR\cos\theta$.

a) $$W = \int_0^{\pi/2} FR\cos\theta \, d\,\theta = FR$$

b) In Eq. (6.14), *dl* is the horizontal distance the point moves, and so $W = F\int dl = FR$, the same as part (a). c) From $K_2 = W = (MR^2/4)\omega^2, \omega = \sqrt{4F/MR}$. d) The torque, and hence the angular acceleration, is greatest when $\theta = 0$, at which point $\alpha = (\tau/I) = 2F/MR$, and so the maximum tangential acceleration is $2F/M$. e) Using the value for ω found in part (c), $a_{\text{rad}} = \omega^2 R = 4F/M$.

10.61: The tension in the rope must be $m(g + a) = 530\ \text{N}$. The angular acceleration of the cylinder is $a/R = 3.2\ \text{rad/s}^2$, and so the net torque on the cylinder must be 9.28 $\text{N}\cdot\text{m}$. Thus, the torque supplied by the crank is $(530\ \text{N})(0.25\ \text{m}) + (9.28\ \text{N}\cdot\text{m}) = 141.8\ \text{N}\cdot\text{m}$, and the force applied to the crank handle is $\frac{141.8\ \text{N}\cdot\text{m}}{0.12\ \text{m}} = 1.2\ \text{kN}$ to two figures.

10.62: At the point of contact, the wall exerts a friction force *f* directed downward and a normal force *n* directed to the right. This is a situation where the net force on the roll is zero, but the net torque is *not* zero, so balancing torques would not be correct. Balancing vertical forces, $F_{\text{rod}}\cos\theta = f + w + F$, and balacing horizontal forces $F_{\text{rod}}\sin\theta = n.$ With $f = \mu_k n$, these equations become

$$F_{\text{rod}}\cos\theta = \mu_k n + F + w,$$
$$F_{\text{rod}}\sin\theta = n.$$

(a) Eliminating *n* and solving for F_{rod} gives

$$F_{\text{rod}} = \frac{\omega + F}{\cos\theta - \mu_k \sin\theta} = \frac{(16.0\ \text{kg})(9.80\ \text{m/s}^2) + (40.0\ \text{N})}{\cos 30° - (0.25)\sin 30°} = 266\ \text{N}.$$

b) With respect to the center of the roll, the rod and the normal force exert zero torque. The magnitude of the net torque is $(F - f)R$, and $f = \mu_k n$ may be found insertion of the value found for F_{rod} into either of the above relations; *i.e.*, $f = \mu_k F_{\text{rod}}\sin\theta = 33.2\ \text{N}$. Then,

$$\alpha = \frac{\tau}{I} = \frac{(40.0\ \text{N} - 31.54\ \text{N})(18.0\times 10^{-2}\ \text{m})}{(0.260\ \text{kg}\cdot\text{m}^2)} = 4.71\ \text{rad/s}^2.$$

10.63: The net torque on the pulley is TR, where T is the tension in the string, and $\alpha = TR/I$. The net force on the block down the ramp is $mg(\sin\beta - \mu_k \cos\beta) - T = ma$. The acceleration of the block and the angular acceleration of the pulley are related by $a = \alpha R$.

a) Multiplying the first of these relations by I/R and eliminating α in terms of a, and then adding to the second to eliminate T gives

$$a = mg\frac{(\sin\beta - \mu_k\cos\beta)}{m + I/R^2} = \frac{g(\sin\beta - \mu_k\cos\beta)}{(1 + I/mR^2)},$$

and substitution of numerical values given $1.12\ \text{m/s}^2$. b) Substitution of this result into either of the above expressions involving the tension gives $T = 14.0$ N.

10.64: For a tension T in the string, $mg - T = ma$ and $TR = I\alpha = I\frac{a}{R}$. Eliminating T and solving for a gives

$$a = g\frac{m}{m + I/R^2} = \frac{g}{1 + I/mR^2},$$

where m is the mass of the hanging weight, I is the moment of inertia of the disk combination $(I = 2.25\times10^{-3}\ \text{kg}\cdot\text{m}^2 \text{ from Problem 9.89})$ and R is the radius of the disk to which the string is attached.

a) With $m = 1.50$ kg, $R = 2.50\times10^{-2}\,\text{m}, a = 2.88\,\text{m/s}^2$.

b) With $m = 1.50$ kg, $R = 5.00\times10^{-2}\,\text{m}, a = 6.13\,\text{m/s}^2$.

The acceleration is larger in case (b); with the string attached to the larger disk, the tension in the string is capable of applying a larger torque.

10.65: Taking the torque about the center of the roller, the net torque is $fR = \alpha I$, $I = MR^2$ for a hollow cylinder, and with $\alpha = a/R$, $f = Ma$ (note that this is a relation between magnitudes; the vectors $\vec{\boldsymbol{f}}$ and $\vec{\boldsymbol{a}}$ are in opposite directions). The net force is $F - f = Ma$, from which $F = 2Ma$ and so $a = F/2M$ and $f = F/2$.

10.66: The accelerations of blocks A and B will have the same magnitude a. Since the cord does not slip, the angular acceleration of the pulley will be $\alpha = \frac{a}{R}$. Denoting the tensions in the cord as T_A and T_B, the equations of motion are

$$m_A g - T_A = m_A a$$
$$T_B - m_B g = m_B a$$
$$T_A - T_B = \frac{I}{R^2} a,$$

where the last equation is obtained by dividing $\tau = I\alpha$ by R and substituting for α in terms of a.

Adding the three equations eliminates both tensions, with the result that

$$a = g \frac{m_A - m_B}{m_A + m_B + I/R^2}$$

Then,

$$\alpha = \frac{a}{R} = g \frac{m_A - m_B}{m_A R + m_B R + I/R}.$$

The tensions are then found from

$$T_A = m_A(g - a) = g \frac{2m_A m_B + m_A I/R^2}{m_A + m_B + I/R^2}$$
$$T_B = m_B(g + a) = g \frac{2m_B m_A + m_B I/R^2}{m_A + m_B + I/R^2}.$$

As a check, it can be shown that $(T_A - T_B)R = I\alpha$.

10.67: For the disk, $K = (3/4)Mv^2$ (see Example 10.6). From the work-energy theorem, $K_1 = MgL\sin\beta$, from which

$$L = \frac{3v^2}{4g\sin\beta} = \frac{3(2.50\ \text{m/s})^2}{4(9.80\ \text{m/s}^2)\sin 30.0°} = 0.957\ \text{m}.$$

This same result may be obtained by an extension of the result of Exercise 10.26; for the disk, the acceleration is $(2/3)g\sin\beta$, leading to the same result.

b) Both the translational and rotational kinetic energy depend on the mass which cancels the mass dependence of the gravitational potential energy. Also, the moment of inertia is proportional to the square of the radius, which cancels the inverse dependence of the angular speed on the radius.

10.68: The tension is related to the acceleration of the yo-yo by $(2m)g - T = (2m)a$, and to the angular acceleration by $Tb = I\alpha = I\frac{a}{b}$. Dividing the second equation by b and adding to the first to eliminate T yields

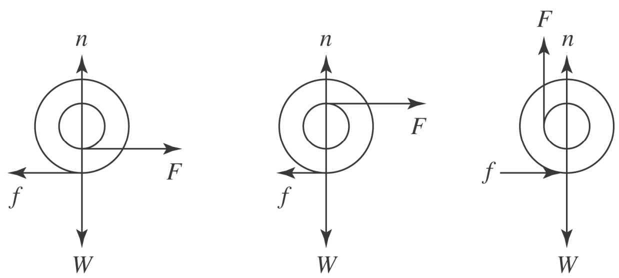
$$a = g\frac{2m}{(2m + I/b^2)} = g\frac{2}{2 + (R/b)^2}, \quad \alpha = g\frac{2}{2b + R^2/b},$$

where $I = 2\frac{1}{2}mR^2 = mR^2$ has been used for the moment of inertia of the yo-yo. The tension is found by substitution into either of the two equations; *e.g.*,

$$T = (2m)(g - a) = (2mg)\left(1 - \frac{2}{2 + (R/b)^2}\right) = 2mg\frac{(R/b)^2}{2 + (R/b)^2} = \frac{2mg}{(2(b/R)^2 + 1)}.$$

10.69: a) The distance the marble has fallen is $y = h - (2R - r) = h + r - 2R.$ The radius of the path of the center of mass of the marble is $R - r,$ so the condition that the ball stay on the track is $v^2 = g(R - r).$ The speed is determined from the work-energy theorem, $mgy = (1/2)mv^2 + (1/2)I\omega^2.$ At this point, it is crucial to know that even for the curved track, $\omega = v/r;$ this may be seen by considering the time T to move around the circle of radius $R - r$ at constant speed V is obtained from $2\pi\,(R - r) = Vt,$ during which time the marble rotates by an angle $2\pi\left(\frac{R}{r} - 1\right) = \omega\,T,$ from which $\omega = V/r.$ The work-energy theorem then states $mgy = (7/10)mv^2,$ and combining, canceling the factors of m and g leads to $(7/10)(R - r) = h + r - 2R,$ and solving for h gives $h = (27/10)R - (17/10)r.$ b) In the absence of friction, $mgy = (1/2)mv^2,$ and substitution of the expressions for y and v^2 in terms of the other parameters gives $(1/2)(R - r) = h - r - 2R,$ which is solved for $h = (5/2)R - (3/2)r.$

10.70: In the first case, $\vec{\boldsymbol{F}}$ and the friction force act in opposite directions, and the friction force causes a larger torque to tend to rotate the yo-yo to the right. The net force to the right is the difference $F - f,$ so the net force is to the right while the net torque causes a clockwise rotation. For the second case, both the torque and the friction force tend to turn the yo-yo clockwise, and the yo-yo moves to the right. In the third case, friction tends to move the yo-yo to the right, and since the applied force is vertical, the yo-yo moves to the right.



10.71: a) Because there is no vertical motion, the tension is just the weight of the hoop: $T = Mg = (0.180\ \text{kg})(9.8\ \text{N/kg}) = 1.76\ \text{N}$ b) Use $\tau = I\alpha$ to find $\alpha.$ The torque is $RT,$ so $\alpha = RT/I = RT/MR^2 = T/MR = Mg/MR,$

so $\alpha = g/R = (9.8\ \text{m/s}^2)/(0.08\ \text{m}) = 122.5\ \text{rad/s}^2$

c) $a = R\alpha = 9.8\ \text{m/s}^2$

d) T would be unchanged because the mass M is the same, α and a would be twice as great because I is now $\frac{1}{2}MR^2.$

10.72: (a) $\Sigma\tau = I\alpha$ and $a_{\mathrm{T}} = R\alpha$

$$PR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2\left(\frac{a_{\mathrm{T}}}{R}\right)$$

$$a_{\mathrm{T}} = \frac{2P}{M} = \frac{200\ \mathrm{N}}{4.00\ \mathrm{kg}} = 50\ \mathrm{m/s^2}$$

Distance the cable moves: $x = \frac{1}{2}at^2$

$$50\ \mathrm{m} = \frac{1}{2}\left(50\ \mathrm{m/s^2}\right)t^2 \rightarrow t = 1.41\ \mathrm{s}.$$

$$v = v_0 + at = 0 + \left(50\ \mathrm{m/s^2}\right)\left(1.41\ \mathrm{s}\right) = 70.5\ \mathrm{m/s}$$

(b) For a hoop, $I = MR^2$, which is twice as large as before, so α and a_{T} would be half as large. Therefore the time would be longer. For the speed, $v^2 = v_0^2 + 2ax$, in which x is the same, so v would be smaller since a is smaller

10.73: Find the speed v the marble needs at the edge of the pit to make it to the level ground on the other side. The marble must travel 36 m horizontally while falling vertically 20 m.

Use the vertical motion to find the time. Take + y to be downward.

$v_{0y} = 0,\ a_y = 9.80\ \mathrm{m/s^2},\ y - y_0 = 20\ \mathrm{m},\ t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $t = 2.02\ \mathrm{s}$

Then $x - x_0 = v_{0x}t$ gives $v_{0x} = 17.82\ \mathrm{m/s}$.

Use conservation of energy, where point 1 is at the starting point and point 2 is at the edge of the pit, where $v = 17.82\ \mathrm{m/s}$. Take $y = 0$ at point 2, so $y_2 = 0$ and $y_1 = h$.

$K_1 + U_1 = K_2 + U_2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rolling without slipping means $\omega = v/r$. $I = \frac{2}{5}mr^2$, so $\frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$

$mgh = \frac{7}{10}mv^2$

$$h = \frac{7v^2}{10g} = \frac{7(17.82\ \mathrm{m/s})}{10(9.80\ \mathrm{m/s^2})} = 23\ \mathrm{m}$$

b) $\frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$, Independent of r.

c) All is the same, except there is no rotational kinetic energy term in $K: K = \frac{1}{2}mv^2$

$mgh = \frac{1}{2}mv^2$

$$h = \frac{v^2}{2g} = 16\ \mathrm{m},\ 0.7\ \text{times smaller than the answer in part (a).}$$

10.74: Break into 2 parts, the rough and smooth sections.

$Rough: mgh_1 = \frac{1}{2}mv_2 + \frac{1}{2}I\omega^2$

$$mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$v^2 = \frac{10}{7}gh_1$$

Smooth: Rotational *KE* does not change.

$$mgh_2 + \frac{1}{2}mv^2 + KE_{\text{Rot}} = \frac{1}{2}mv_{\text{Bottom}}^2 + KE_{\text{Rot}}$$

$$gh_2 + \frac{1}{2}\left(\frac{10}{7}gh_1\right) = \frac{1}{2}v_{\text{B}}^2$$

$$v_{\text{B}} = \sqrt{\frac{10}{7}gh_1 + 2gh_2}$$

$$= \sqrt{\frac{10}{7}(9.80\ \text{m/s}^2)(25\ \ \text{m}) + 2(9.80\ \text{m/s}^2)(25\ \text{m})}$$

$$= 29.0\ \text{m/s}$$

10.75: a) Use conservation of energy to find the speed v_2 of the ball just before it leaves the top of the cliff. Let point 1 be at the bottom of the hill and point 2 be at the top of the hill. Take $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = 28.0\,\text{m}$.

$$K_1 + U_1 = K_2 + U_2$$

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}I\omega_1^2 = mgy_2 + \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}I\omega_2^2$$

Rolling without slipping means $\omega = v/r$ and $\tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)(v/r)^2 = \tfrac{1}{5}mv^2$

$$\tfrac{7}{10}mv_1^2 = mgy_2 + \tfrac{7}{10}mv_2^2$$

$$v_2 = \sqrt{v_1^2 - \tfrac{10}{7}gy_2} = 15.26\,\text{m/s}$$

Consider the projectile motion of the ball, from just after it leaves the top of the cliff until just before it lands. Take $+y$ to be downward.

Use the vertical motion to find the time in the air:

$v_{0y} = 0,\ a_y = 9.80\ \text{m/s}^2,\ y - y_0 = 28.0\ \text{m},\ t = ?$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = 2.39\,\text{s}$

During this time the ball travels horizontally $x - x_0 = v_{0x}t = (15.26\,\text{m/s})(2.39\,\text{s}) = 36.5\,\text{m}$

Just before it lands, $v_y = v_{0y} + a_y t = 23.4\,\text{s}$ and $v_x = v_{0x} = 15.3\,\text{s}$ $v = \sqrt{v_x^2 + v_y^2} = 28.0\ \text{m/s}$

b) At the bottom of the hill, $\omega = v/r = (25.0\,\text{m/s})r.$ The rotation rate doesn't change while the ball is in the air, after it leaves the top of the cliff, so just before it lands $\omega = (15.3\,\text{s})r.$ The total kinetic energy is the same at the bottom of the hill and just before it lands, but just before it lands less of this energy is rotational kinetic energy, so the translational kinetic energy is greater.

10.76: (a) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ (1)

$$I = I_{\text{rim}} + I_{\text{spokes}} = M_{\text{r}}R^2 + 6\left(\frac{1}{3}m_{\text{s}}R^2\right)$$

Uniform density means: $m_{\text{r}} = \lambda 2\pi R$ and $m_{\text{s}} = \lambda R.$ No slipping means that $\omega = v/R.$
Also, $m = m_{\text{r}} + m_{\text{s}} = 2\pi R\lambda + 6R\lambda = 2R\lambda(\pi + 3)$ substituting into (1) gives

$$2R\lambda(\pi+3)gh = \frac{1}{2}(2R\lambda)(\pi+3)(R\omega)^2 + \frac{1}{2}\left[2\pi R\lambda R^2 + 6\left(\frac{1}{3}\pi RR^2\right)\right]\omega^2$$

$$\omega = \sqrt{\frac{(\pi+3)gh}{R^2(\pi+2)}} = \sqrt{\frac{(\pi+3)(9.80\text{ m/s}^2)(58.0\text{ m})}{(0.210\text{ m})^2(\pi+2)}} = 124\text{ rad/s}$$

and $v = R\omega = 26.0\text{ m/s}$

(b) Doubling the density would have no effect because it does not appear in the answer. $\omega \,\alpha\, \frac{1}{R}$, so doubling the diameter would double the radius which would reduce ω by half, but $v = R\omega$ would be unchanged.

10.77: a) The front wheel is turning at $\omega = 1.00\text{ rev/s} = 2\pi\text{ rad/s}.$
$\upsilon = r\omega = (0.330\text{ m})(2\pi\text{ rad/s}) = 2.07\text{ s}$

b) $\omega = v/r = (2.07\text{ m/s})/(0.655\text{ m} = 3.16\text{ rad/s} = 0.503\text{ rev/s}$

c) $\omega = v/r = (2.07\text{ m/s})/(0.220\text{ m}) = 9.41\text{ rad/s} = 1.50\text{ rev/s}$

10.78: a) The kinetic energy of the ball when it leaves the tract (when it is still rolling without slipping) is $(7/10)mv^2$ and this must be the work done by gravity, W = mgh, so $v = \sqrt{10gh/7}.$ The ball is in the air for a time $t = \sqrt{2y/g},$ so $x = vt = \sqrt{20hy/7}.$

b) The answer does not depend on g, so the result should be the same on the moon.

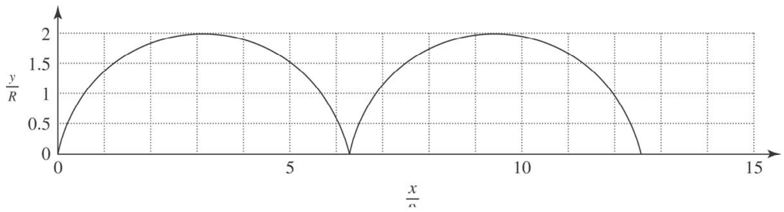
c) The presence of rolling friction would decrease the distance.

d) For the dollar coin, modeled as a uniform disc, $K = (3/4)mv^2$, and so $x = \sqrt{8hy/3}.$

10.79: a) $v=\sqrt{\dfrac{10K}{7m}}=\sqrt{\dfrac{(10)(0.800)(1/2)(400\ \text{N/m})(0.15\ \text{m})^2}{7(0.0590\ \text{kg})}}=9.34\ \text{m/s}.$

b) Twice the speed found in part (a), 18.7 m/s. c) If the ball is rolling without slipping, the speed of a point at the bottom of the ball is zero. d) Rather than use the intermediate calculation of the speed, the fraction of the initial energy that was converted to gravitational potential energy is $(0.800)(0.900)$, so $(0.720)(1/2)kx^2=mgh$ and solving for h gives 5.60 m.

10.80: a)



b) R is the radius of the wheel (y varies from 0 to $2R$) and T is the period of the wheel's rotation.

c) Differentiating,

$$v_x=\frac{2\pi R}{T}\left[1-\cos\left(\frac{2\pi t}{T}\right)\right]\qquad a_x=\left(\frac{2\pi}{T}\right)^2R\sin\left(\frac{2\pi t}{T}\right)$$

$$v_y=\frac{2\pi R}{T}\sin\left(\frac{2\pi t}{T}\right)\qquad a_y=\left(\frac{2\pi}{T}\right)^2R\cos\left(\frac{2\pi t}{T}\right).$$

d) $v_x=v_y=0$ when $\left(\frac{2\pi t}{T}\right)=2\pi$ or any multiple of 2π, so the times are integer multiples of the period T. The acceleration components at these times are $a_x=0, a_y=\dfrac{4\pi^2R}{T^2}.$

e) $\sqrt{a_x^2+a_y^2}=\left(\dfrac{2\pi}{T}\right)^2R\sqrt{\cos^2\left(\dfrac{2\pi t}{T}\right)+\sin^2\left(\dfrac{2\pi t}{T}\right)}=\dfrac{4\pi^2R}{T^2},$

independent of time. This is the magnitude of the radial acceleration for a point moving on a circle of radius R with constant angular velocity $\frac{2\pi}{T}$. For motion that consists of this circular motion superimposed on motion with constant velocity $(\vec{a}=0)$, the acceleration due to the circular motion will be the total acceleration.

10.81: For rolling without slipping, the kinetic energy is $(1/2)(m+I/R^2)v^2 = (5/6)mv^2$; initially, this is 32.0 J and at the return to the bottom it is 8.0 J. Friction has done $-24.0\ \text{J}$ of work, $-12.0\ \text{J}$ each going up and down. The potential energy at the highest point was 20.0 J, so the height above the ground was $\frac{20.0\ \text{J}}{(0.600\ \text{kg})(9.80\ \text{m/s}^2)} = 3.40\ \text{m}$.

10.82: Differentiating , and obtaining the answer to part (b),

$$\omega = \frac{d\theta}{dt} = 3bt^2 = 3b\left(\frac{\theta}{b}\right)^{2/3} = 3b^{1/3}\theta^{2/3},$$

$$\alpha = -\frac{d\omega}{dt} = 6bt = 6b\left(\frac{\theta}{b}\right)^{1/3} = 6b^{2/3}\theta^{1/3}.$$

a) $$W = \int I_{\text{cm}}\alpha\, d\theta = 6b^{2/3} I_{\text{cm}} \int \theta^{1/3} d\theta = \frac{9}{2} I_{\text{cm}} b^{2/3}\theta^{4/3}.$$

c) The kinetic energy is

$$K = \frac{1}{2} I_{\text{cm}}\omega^2 = \frac{9}{2} I_{\text{cm}} b^{2/3}\theta^{4/3},$$

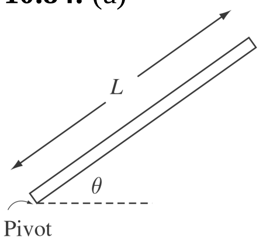
in agreement with Eq. (10.25); the total work done is the change in kinetic energy.

10.83: Doing this problem using kinematics involves four unknowns (six, counting the two angular accelerations), while using energy considerations simplifies the calculations greatly. If the block and the cylinder both have speed *v*, the pulley has angular velocity *v/R* and the cylinder has angular velocity *v/2R*, the total kinetic energy is

$$K = \frac{1}{2}\left[Mv^2 + \frac{M(2R)^2}{2}(v/2R)^2 + \frac{MR^2}{2}(v/R)^2 + Mv^2\right] = \frac{3}{2}Mv^2.$$

This kinetic energy must be the work done by gravity; if the hanging mass descends a distance y, $K = Mgy$, or $v^2 = (2/3)gy$. For constant acceleration, $v^2 = 2ay$, and comparison of the two expressions gives $a = g/3$.

10.84: (a)



$$\Sigma\tau = I\alpha$$

$$mg\frac{1}{2}\cos\theta = \frac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3}{2}g\frac{\cos\theta}{\mathrm{L}} = \left(\frac{3}{2}\right)\frac{(9.80\ \mathrm{m/s^2})\cos 60°}{8.00\ \mathrm{m}} = 0.92\ \mathrm{rad/s^2}$$

(b) As the bridge lowers, θ changes, so α is not constant. Therefore Eq. (9.17) is not valid.

(c) Conservation of energy:

$$PE_{\mathrm{i}} = KE_{\mathrm{f}} \rightarrow mgh = \frac{1}{2}I\omega^2$$

$$mg\tfrac{L}{2}\sin\theta = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$\omega = \sqrt{\frac{3g\sin\theta}{L}}$$

$$= \sqrt{\frac{3(9.8\ \mathrm{m/s^2})\sin 60°}{8.00\ \mathrm{m}}} = 1.78\ \mathrm{rad/s}$$

10.85: The speed of the ball just before it hits the bar is $v = \sqrt{2gy} = 15.34\ \mathrm{m/s}$.

Use conservation of angular momentum to find the angular velocity ω of the bar just after the collision. Take the axis at the center of the bar.

$L_1 = mvr = (5.00\ \mathrm{kg})(15.34\mathrm{m/s})(2.00\ \mathrm{m}) = 153.4\ \mathrm{kg\cdot m^2}$

Immediately after the collsion the bar and both balls are rotating together.

$L_2 = I_{\mathrm{tot}}\omega$

$I_{\mathrm{tot}} = \frac{1}{12}Ml^2 + 2mr^2 = \frac{1}{12}(8.00\ \mathrm{kg})(4.00\ \mathrm{m})^2 + 2(5.00\ \mathrm{kg})(2.00\ \mathrm{m})^2 = 50.67\ \mathrm{kg\cdot m^2}$

$L_2 = L_1 = 153.4\ \mathrm{kg\cdot m^2}$

$\omega = L_2/I_{\mathrm{tot}} = 3.027\ \mathrm{rad/s}$

Just after the collision the second ball has linear speed

$v = rw = (2.00\ \mathrm{m})(3.027\mathrm{rad/s}) = 6.055\mathrm{m/s}$ and is moving upward.

$\frac{1}{2}mv^2 = mgy$ gives $\mathrm{y} = 1.87\ \mathrm{m}$ for the height the second ball goes.

10.86: a) The rings and the rod exert forces on each other, but there is no net force or torque on the system, and so the angular momentum will be constant. As the rings slide toward the ends, the moment of inertia changes, and the final angular velocity is given by Eq. (10.33),

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \left[\frac{\frac{1}{12} ML^2 + 2mr_1^2}{\frac{1}{12} ML^2 + 2mr_2^2} \right] = \omega_1 \frac{5.00 \times 10^{-4}\ \text{kg} \cdot \text{m}^2}{2.00 \times 10^{-3}\ \text{kg} \cdot \text{m}^2} = \frac{\omega_1}{4},$$

ans so $\omega_2 = 7.5\ \text{rev/min}$. Note that conversion from rev/min to rad/s is not necessary.

b) The forces and torques that the rings and the rod exert on each other will vanish, but the common angular velocity will be the same, 7.5 rev/min.

10.87: The intial angular momentum of the bullet is $(m/4)(v)(L/2)$, and the final moment of intertia of the rod and bullet is $(m/3)L^2 + (m/4)(L/2)^2 = (19/48)mL^2$. Setting the initial angular moment equal to ωI and solving for ω gives $\omega = \dfrac{mvL/8}{(19/48)mL^2} = \dfrac{6}{19} v/L.$

b) $$\frac{(1/2)I\omega^2}{(1/2)(m/4)v^2} = \frac{(19/48)mL^2((6/19)(v/L))^2}{(m/4)v^2} = \frac{3}{19}.$$

10.88: Assuming the blow to be concentrated at a point (or using a suitably chosen "average" point) at a distance r from the hinge, $\Sigma\tau_{\text{ave}} = rF_{\text{ave}}$, and $\Delta L = rF_{\text{ave}} \Delta t = rJ$. The angular velocity ω is then

$$\omega = \frac{\Delta L}{I} = \frac{rF_{\text{ave}} \Delta t}{I} = \frac{(l/2)F_{\text{ave}} \Delta t}{\frac{1}{3} ml^2} = \frac{3}{2} \frac{F_{\text{ave}} \Delta t}{ml},$$

Where l is the width of the door. Substitution of the given numeral values gives $\omega = 0.514\ \text{rad/s}$.

10.89: a) The initial angular momentum is $L = mv(l/2)$ and the final moment of inertia is $I = I_0 + m(l/2)^2$, so

$$\omega = \frac{mv(l/2)}{(M/3)l^2 + m(l/2)^2} = 5.46\ \text{rad/s}.$$

b) $(M + m)gh = (1/2)\omega^2 I$, and after solving for h and substitution of numerical values, $h = 3.16 \times 10^{-2}\ \text{m}$. c) Rather than recalculate the needed value of ω, note that ω will be proportional to v and hence h will be proportional to v^2; for the board to swing all the way over, $h = 0.250\ \text{m}$. and so $v = (360\ \text{m/s})\sqrt{\frac{0.250\ \text{m}}{0.0316\ \text{m}}} = 1012\ \text{m/s}$.

10.90: Angular momentum is conserved, so $I_0\omega_0 = I_2\omega_2$, or, using the fact that for a common mass the moment of inertia is proportional to the square of the radius, $R_0^2\omega_0 = R_2^2\omega_2$, or $R_0^2\omega_0 = (R_0 + \Delta R)^2(\omega_0 + \Delta\omega) \sim R_0^2\omega_0 + 2R_0\Delta R\omega_0 + R_0^2\Delta\omega$, where the terms in $\Delta R\Delta\omega$ and $\Delta\omega^2$ have been omitted. Canceling the $R_0^2\omega_0$ term gives

$$\Delta R = -\frac{R_0}{2}\frac{\Delta\omega}{\omega_0} = -1.1\,\text{cm}.$$

10.91: The initial angular momentum is $L_1 = \omega_0 I_A$ and the initial kinetic energy is $K_1 = I_A\omega_0^2/2.$ The final total moment of inertia is $4I_A$, so the final angular velocity is $(1/4)\omega_0$ and the final kinetic energy is $(1/2)4I_A(\omega_0/4)^2 = (1/4)K_1$. (This result may be obtained more directly from $K = L^2/I$. Thus, $\Delta K = -(3/4)K_1$ and $\text{K}_1 = -(4/3)(-2400\,\text{J})$ $= 3200\,\text{J}$.

10.92: The tension is related to the block's mass and speed, and the radius of the circle, by $T = m\dfrac{v^2}{r}$. The block's angular momentum with respect to the hole is $L = mvr$, so in terms of the angular momentum,

$$T = mv^2\frac{1}{r} = \frac{m^2v^2}{m}\frac{r^2}{r^3} = \frac{(mvr)^2}{mr^3} = \frac{L^2}{mr^3}.$$

The radius at which the string breaks can be related to the initial angular momentum by

$$r^3 = \frac{L^2}{mT_{\text{max}}} = \frac{(mv_1r_1)^2}{mT_{\text{max}}} = \frac{((0.250\,\text{kg})(4.00\,\text{m/s})(0.800\,\text{m}))^2}{(0.250\,\text{kg})(30.0\,\text{N})},$$

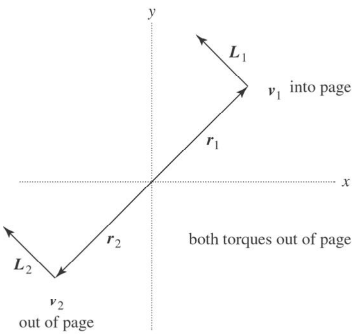
from which $r = 0.440\,\text{m}$.

10.93: The train's speed relative to the earth is $0.600\,\text{m/s} + \omega\,(0.475\,\text{m})$, so the total angular momentum is

$$((0.600\,\text{m/s}) + \omega(0.475\,\text{m}))(1.20\,\text{kg})(0.475\,\text{m}) + \omega(1/2)(7.00\,\text{kg})\left(\frac{1.00\,\text{m}}{2}\right)^2 = 0,$$

from which $\omega = -0.298\,\text{rad/s}$, with the minus sign indicating that the turntable moves clockwise, as expected.

10.94: a), g)



b) Using the vector product form for the angular momentum, $\vec{\boldsymbol{v}}_1 = -\vec{\boldsymbol{v}}_2$ and $\vec{\boldsymbol{r}}_1 = -\vec{\boldsymbol{r}}_2$, so

$$m\vec{\boldsymbol{r}}_2 \times \vec{\boldsymbol{v}}_2 = m\vec{\boldsymbol{r}}_1 \times \vec{\boldsymbol{v}}_1,$$

so the angular momenta are the same. c) Let $\vec{\boldsymbol{\omega}} = \omega\hat{\boldsymbol{j}}$. Then,

$$\vec{\boldsymbol{v}}_1 = \vec{\omega} \times \vec{r}_1 = \omega\left(z\hat{\boldsymbol{i}} - x\hat{\boldsymbol{k}}\right), \text{ and}$$

$$\vec{\boldsymbol{L}}_1 = m\vec{r}_1 \times \vec{\boldsymbol{v}}_1 = m\omega\left((-xR)\hat{\boldsymbol{i}} + \left(x^2 + y^2\right)\hat{\boldsymbol{j}} + (xR)\hat{\boldsymbol{k}}\right).$$

With $x^2 + y^2 = R^2$, the magnitude of $\vec{\boldsymbol{L}}$ is $2m\omega R^2$, and $\vec{\boldsymbol{L}}_1 \cdot \vec{\boldsymbol{\omega}} = m\omega^2 R^2$, and so $\cos\theta = \frac{m\omega^2 R^2}{(2m\omega R^2)(\omega)} = \frac{1}{2}$, and $\theta = \frac{\pi}{6}$. This is true for $\vec{L}_2$ as well, so the total angular momentum makes an angle of $\frac{\pi}{6}$ with the +y-axis. d) From the intermediate calculation of part (c), $L_{y1} = m\omega R^2 = mvR,$ so the total y-component of angular momentum is $L_y = 2mvR.$ e) L_y is constant, so the net y-component of torque is zero. f) Each particle moves in a circle of radius R with speed v, and so is subject to an inward force of magnitude $mv^2/R.$ The lever arm of this force is R, so the torque on each has magnitude $mv^2.$ These forces are directed in opposite directions for the two particles, and the position vectors are opposite each other, so the torques have the same magnitude and direction, and the net torque has magnitude $2mv^2$.

10.95: a) The initial angular momentum with respect to the pivot is mvr, and the final total moment of inertia is $I + mr^2$, so the final angular velocity is $\omega = mvr/\left(mr^2 + I\right)$.

b) The kinetic energy after the collision is

$$K = \frac{1}{2}\omega^2\left(mr^2 + I\right) = (M + m)gh, \text{ or}$$

$$\omega = \sqrt{\frac{2(M + m)gh}{\left(mr^2 + I\right)}}.$$

c) Substitution of $I = Mr^2$ into either of the result of part (a) gives $\omega = \left(\frac{m}{m + M}\right)(v/r)$, and into the result of part (b), $\omega = \sqrt{2gh}(1/r)$, which are consistent with the forms for v.

10.96: The initial angular momentum is $I\omega_1 - mRv_1$, with the minus sign indicating that runner's motion is opposite the motion of the part of the turntable under his feet. The final angular momentum is $\omega_2(\mathrm{I} + mR^2)$, so

$$\begin{aligned}\omega_2 &= \frac{I\omega_1 - mRv_1}{I + mR^2} \\ &= \frac{(80\ \text{kg}\cdot\text{m}^2)(0.200\ \text{rad/s}) - (55.0\ \text{kg})(3.00\ \text{m})(2.8\ \text{m/s})}{(80\ \text{kg}\cdot\text{m}^2) + (55.0\ \text{kg})(3.00\ \text{m})^2} \\ &= -0.776\ \text{rad/s},\end{aligned}$$

where the minus sign indicates that the turntable has reversed its direction of motion *(i.e.,* the man had the larger magnitude of angular momentum initially).

10.97: From Eq. (10.36),

$$\Omega = \frac{\omega r}{\mathrm{I}\omega} = \frac{(50.0\ \text{kg})(9.80\ \text{m/s}^2)(0.040\ \text{m})}{(0.085\ \text{kg}\cdot\text{m}^2)((6.0\ \text{m/s})/(0.33\ \text{m}))} = 12.7\ \text{rad/s},$$

or 13 rad/s to two figures, which is quite large.

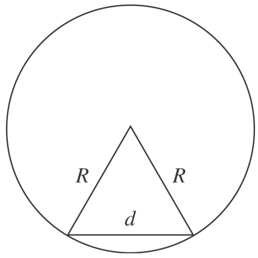
10.98: The velocity of the center of mass will change by $\Delta v_{\text{cm}} = \frac{J}{m}$, and the angular velocity will change by $\Delta\omega = \frac{J(x - x_{\text{cm}})}{I}\cdot$ The change is velocity of the end of the bat will then be $\Delta v_{\text{end}} = \Delta v_{\text{cm}} - \Delta\omega x_{\text{cm}} = \dfrac{J}{m} - \dfrac{J(x - x_{cm})x_{\text{cm}}}{I}\cdot$ Setting $\Delta v_{\text{end}} = 0$ allows cancellation of J, and gives $I = (x - x_{\text{cm}})x_{\text{cm}}m,$ which when solved for x is

$$x = \frac{I}{x_{\text{cm}}m} + x_{\text{cm}} = \frac{(5.30\times10^{-2}\,\text{kg}\cdot\text{m}^2)}{(0.600\,\text{m})(0.800\,\text{kg})} + (0.600\,\text{m}) = 0.710\,\text{m}.$$

10.99: In Fig. (10.34(a)), if the vector $\vec{\boldsymbol{r}}$, and hence the vector $\vec{\boldsymbol{L}}$ are not horizontal but make an angle β with the horizontal, the torque will still be horizontal (the torque must be perpendicular to the vertical weight). The magnitude of the torque will be $\omega\, r \cos\beta$, and this torque will change the direction of the horizontal component of the angular momentum, which has magnitude $L\cos\beta$. Thus, the situation of Fig. (10.36) is reproduced, but with $\vec{\boldsymbol{L}}_{\text{horiz}}$ instead of $\vec{\boldsymbol{L}}$. Then, the expression found in Eq. (10.36) becomes

$$\Omega = \frac{d\phi}{dt} = \frac{\left|d\vec{\boldsymbol{L}}\right| \big/ \left|\vec{\boldsymbol{L}}_{\text{horiz}}\right|}{dt} = \frac{\tau}{\left|\vec{\boldsymbol{L}}_{\text{horiz}}\right|} = \frac{mgr\cos\beta}{L\cos\beta} = \frac{\omega r}{\mathrm{I}\omega}\cdot$$

10.100: a)



The distance from the center of the ball to the midpoint of the line joining the points where the ball is in contact with the rails is $\sqrt{R^2-(d/2)^2}$, so $v_{\text{cm}}=\omega\sqrt{R^2-d^2/4}.$ when $d=0,$ this reduces to $v_{cm}=\omega R,$ the same as rolling on a flat surface. When $d=2R,$ the rolling radius approaches zero, and $v_{cm}\to 0$ for any ω.

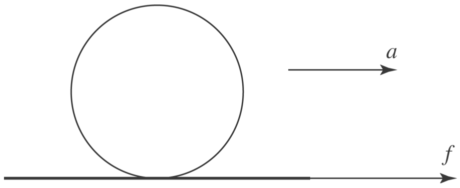
b)
$$K=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

c)
$$=\frac{1}{2}\left[mv_{\text{cm}}^2+(2/5)mR^2\left(\frac{v_{\text{cm}}}{\sqrt{R^2-(d^2/4)}}\right)^2\right]$$

$$=\frac{mv_{\text{cm}}^2}{10}\left[5+\frac{2}{(1-d^2/4R^2)}\right].$$

Setting this equal to mgh and solving for v_{cm} gives the desired result. c) The denominator in the square root in the expression for v_{cm} is larger than for the case $d=0,$ so v_{cm} is smaller. For a given speed, ω is large than the $d=0$ case, so a larger fraction of the kinetic energy is rotational, and the translational kinetic energy, and hence v_{cm}, is smaller. d) Setting the expression in part (b) equal to 0.95 of that of the $d=0$ case and solving for the ratio d/R gives $d/R=1.05.$ Setting the ratio equal to 0.995 gives $d/R=0.37.$

10.101: a)



The friction force is $f = \mu_k n = \mu_k Mg$, so $a = \mu_k g$. The magnitude of the angular acceleration is $\frac{fR}{I} = \frac{\mu_k MgR}{(1/2)MR^2} = \frac{2\mu_k g}{R}$. b) Setting $v = at = \omega R = (\omega_0 - \omega t)R$ and solving for t gives

$$t = \frac{R\omega_0}{a + R\alpha} = \frac{R\omega_0}{\mu_k g + 2\mu_k g} = \frac{R\omega_0}{3\mu_k g},$$

and

$$d = \frac{1}{2}at^2 = \frac{1}{2}(\mu_k g)\left(\frac{R\omega_0}{3\mu_k g}\right)^2 = \frac{R^2\omega_0^2}{18\mu_k g}.$$

c) The final kinetic energy is $(3/4)Mv^2 = (3/4)M(\mathrm{at})^2$, so the change in kinetic energy is

$$\frac{3}{4}M\left(\mu_k g\frac{R\omega_0}{3\mu_k g}\right)^2 - \frac{1}{4}MR^2\omega_0^2 = \frac{1}{6}MR^2\omega_0^2.$$

10.102: Denoting the upward forces that the hands exert as F_L and F_R, the conditions that F_L and F_R must satisfy are

$$F_L + F_R = w$$
$$F_L - F_R = \Omega \frac{I\omega}{r},$$

where the second equation is $\tau = \Omega L,$ divided by r. These two equations can be solved for the forces by first adding and then subtracting, yielding

$$F_L = \frac{1}{2}\left(\omega + \Omega \frac{I\omega}{r}\right)$$
$$F_R = \frac{1}{2}\left(\omega - \Omega \frac{I\omega}{r}\right).$$

Using the values $\omega = mg = (8.00\,\text{kg})(9.80\,\text{m/s}^2) = 78.4\,\text{N}$ and

$$\frac{I\omega}{r} = \frac{(8.00\,\text{kg})(0.325\,\text{m})^2(5.00\,\text{rev/s} \times 2\pi\,\text{rad/rev})}{(0.200\,\text{m})} = 132.7\,\text{kg}\cdot\text{m/s}$$

gives

$$F_L = 39.2\,\text{N} + \Omega(66.4\,\text{N}\cdot\text{s}),\ F_R = 39.2\,\text{N} - \Omega(66.4\,\text{N}\cdot\text{s}).$$

a) $\Omega = 0, F_L = F_R = 39.2\,\text{N}.$

b) $\Omega = 0.05\,\text{rev/s} = 0.314\,\text{rad/s}, F_L = 60.0\,\text{N}, F_R = 18.4\,\text{N}.$

c) $\Omega = 0.3\,\text{rev/s} = 1.89\,\text{rad/s}, F_L = 165\,\text{N}, F_R = -86.2\,\text{N},$
with the minus sign indicating a downward force.

d) $F_R = 0$ gives $\Omega = \frac{39.2\,\text{N}}{66.4\,\text{N}\cdot\text{s}} = 0.575\,\text{rad/s}$, which is $0.0916\,\text{rev/s}$.

10.103: a) See Problem 10.92; $T = mv_1^2 r_1^2 / r^3$. b) $\vec{T}$ and $d\vec{r}$ are always antiparallel, so

$$W = -\int_{r1}^{r2} T\,dr = mv_1^2 r_1^2 \int_{r2}^{r1} \frac{dr}{r^3} = \frac{mv_1^2}{2} r_1^2 \left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right].$$

c) $v_2 = v_1(r_1/r_2)$, so

$$\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{mv_1^2}{2}\left[\left(\frac{r_1}{r_2}\right)^2 - 1\right],$$

which is the same as the work found in part (b).

11.1: Take the origin to be at the center of the small ball; then,

$$x_{\text{cm}} = \frac{(1.00\,\text{kg})(0) + (2.00\,\text{kg})(0.580\,\text{m})}{3.00\,\text{kg}} = 0.387\,\text{m}$$

from the center of the small ball.

11.2: The calculation of Exercise 11.1 becomes

$$x_{\text{cm}} = \frac{(1.00\,\text{kg})(0) + (1.50\,\text{kg})(0.280\,\text{m}) + (2.00\,\text{kg})(0.580\,\text{m})}{4.50\,\text{kg}} = 0.351\,\text{m}$$

This result is smaller than the one obtained in Exercise 11.1.

11.3: In the notation of Example 11.1, take the origin to be the point S, and let the child's distance from this point be x. Then,

$$s_{\text{cm}} = \frac{M(-D/2) + mx}{M + m} = 0, \quad x = \frac{MD}{2m} = 1.125\,\text{m},$$

which is $(L/2 - D/2)/2$, halfway between the point S and the end of the plank.

11.4: a) The force is applied at the center of mass, so the applied force must have the same magnitude as the weight of the door, or $300\ \text{N}$. In this case, the hinge exerts no force.

b) With respect to the hinge, the moment arm of the applied force is twice the distance to the center of mass, so the force has half the magnitude of the weight, or $150\ \text{N}$. The hinge supplies an upward force of $300\ \text{N} - 150\ \text{N} = 150\ \text{N}$.

11.5: $F(8.0\,\text{m})\sin 40° = (2800\,\text{N})(10.0\,\text{m})$, so $F = 5.45\,\text{kN}$, keeping an extra figure.

11.6: The other person lifts with a force of $160\,\text{N} - 60\,\text{N} = 100\,\text{N}$. Taking torques about the point where the 60-N force is applied,

$$(100\,\text{N})x = (160\ \text{N})(1.50\,\text{m}), \text{ or } x = (1.50\,\text{m})\left(\frac{160\ \text{N}}{100\ \text{N}}\right) = 2.40\,\text{m}.$$

11.7: If the board is taken to be massless, the weight of the motor is the sum of the applied forces, $1000\,\text{N}$. The motor is a distance $\frac{(2.00\ \text{m})(600\ \text{N})}{(1000\ \text{N})} = 1.200\,\text{m}$ from the end where the 400-N force is applied.

11.8: The weight of the motor is $400\,\text{N} + 600\,\text{N} - 200\,\text{N} = 800\,\text{N}.$ Of the myriad ways to do this problem, a sneaky way is to say that the lifters each exert 100 N to the lift the board, leaving $500\,\text{N}$ and $300\,\text{N}$ to the lift the motor. Then, the distance of the motor from the end where the 600-N force is applied is $\frac{(2.00\text{ m})(300\text{ N})}{(800\text{ N})} = 0.75\text{ m}$ .The center of gravity is located at $\frac{(200\text{ N})(1.0\text{ m})+(800\text{ N})(0.75\text{ m})}{(1000\text{ N})} = 0.80\,\text{m}$ from the end where the $600\,\text{N}$ force is applied.

11.9: The torque due to T_x is $-T_x h = -\frac{Lw}{D}\cot\theta\, h,$ and the torque due to T_y is $T_y D = Lw$. The sum of these torques is $Lw(1-\frac{h}{D}\cot\theta).$ From Figure (11.9(b)), $h = D\tan\theta,$ so the net torque due to the tension in the tendon is zero.

11.10: a) Since the wall is frictionless, the only vertical forces are the weights of the man and the ladder, and the normal force. For the vertical forces to balance, $n_2 = w_1 + w_{\text{m}} = 160\text{ N} + 740\text{ N} = 900\text{ N},$ and the maximum frictional forces is $\mu_{\text{s}} n_2 = (0.40)(900\,\text{N}) = 360\,\text{N}$ (see Figure 11.7(b)). b) Note that the ladder makes contact with the wall at a height of 4.0 m above the ground. Balancing torques about the point of contact with the ground,

$$(4.0\,\text{m})n_1 = (1.5\,\text{m})(160\,\text{N}) + (1.0\,\text{m})(3/5))(740\,\text{N}) = 684\,\text{N}\cdot\text{m},$$

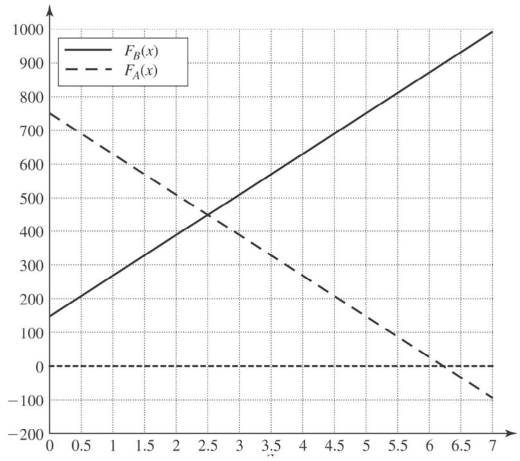
so $n_1 = 171.0\,\text{N},$ keeping extra figures. This horizontal force about must be balanced by the frictional force, which must then be 170 N to two figures. c) Setting the frictional force, and hence n_1, equal to the maximum of 360 N and solving for the distance *x* along the ladder,

$$(4.0\,\text{m})(360\,\text{N}) = (1.50\,\text{m})(160\,\text{N}) + x(3/5)(740\,\text{N}),$$

so x = 2.70 m, or 2.7 m to two figures.

11.11: Take torques about the left end of the board in Figure (11.21). a) The force F at the support point is found from $F(1.00\,\text{m}) = +(280\,\text{N})(1.50\,\text{m}) + (500\,\text{N})(3.00\,\text{m}),$ or $F = 1920\,\text{N}.$ b) The net force must be zero, so the force at the left end is $(1920\,\text{N}) - (500\,\text{N}) - (280\,\text{N}) = 1140\,\text{N},$ downward.

11.12: a)



b) $x = 6.25\,\text{m}$ when $F_A = 0$, which is 1.25 m beyond point B. c) Take torques about the right end. When the beam is just balanced, $F_A = 0$, so $F_B = 900\,\text{N}$. The distance that point B must be from the right end is then $\frac{(300\text{ N})(4.50\text{ m})}{(900\text{ N})} = 1.50\,\text{m}$.

11.13: In both cases, the tension in the vertical cable is the weight ω. a) Denote the length of the horizontal part of the cable by L. Taking torques about the pivot point, $TL\tan 30.0° = wL + w(L/2)$, from which $T = 2.60w$. The pivot exerts an upward vertical force of $2w$ and a horizontal force of $2.60w$, so the magnitude of this force is $3.28w$, directed $37.6°$ from the horizontal. b) Denote the length of the strut by L, and note that the angle between the diagonal part of the cable and the strut is $15.0°$. Taking torques about the pivot point, $TL\sin 15.0° = wL\sin 45.0° + (w/2)L\sin 45°$, so $T = 4.10w$. The horizontal force exerted by the pivot on the strut is then $T\cos 30.0° = 3.55\omega$ and the vertical force is $(2w) + T\sin 30° = 4.05w$, for a magnitude of $5.38w$, directed $48.8°$.

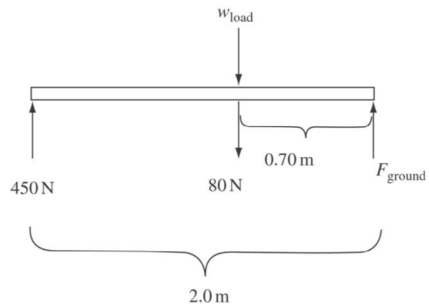
11.14: a) Taking torques about the pivot, and using the 3-4-5 geometry,

$$(4.00\text{ m})(3/5)T = (4.00\text{ m})(300\text{ N}) + (2.00\text{ m})(150\text{ N}),$$

so $T = 625\text{ N}$. b) The horizontal force must balance the horizontal component of the force exerted by the rope, or $T(4/5) = 500\text{ N}$. The vertical force is $300\text{ N} + 150\text{ N} - T(3/5) = 75\text{ N}$, upwards.

11.15: To find the horizontal force that one hinge exerts, take the torques about the other hinge; then, the vertical forces that the hinges exert have no torque. The horizontal force is found from $F_H(1.00\,\text{m}) = (280\,\text{N})(0.50\,\text{m})$, from which $F_H = 140\,\text{N}$. The top hinge exerts a force away from the door, and the bottom hinge exerts a force toward the door. Note that the magnitudes of the forces must be the same, since they are the only horizontal forces.

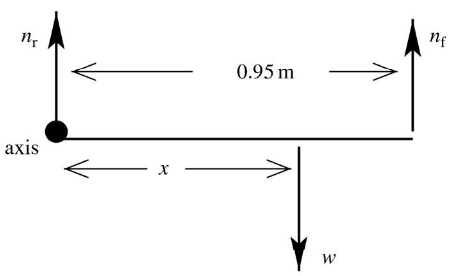
11.16: (a) Free body diagram of wheelbarrow:



$$\Sigma\tau_{wheel} = 0$$
$$-(450\text{ N})(2.0\text{ m}) + (80\text{ N})(0.70\text{ m}) + W_L(0.70\text{ m}) = 0$$
$$W_L = 1200\text{ N}$$

(b) From the ground.

11.17: Consider the forces on Clea.



$n_r = 89\text{ N},\ n_f = 157\text{ N}$

$n_r + n_f = w$ so $w = 246\text{ N}$

11.18: a) Denote the length of the boom by L, and take torques about the pivot point. The tension in the guy wire is found from

$$TL\sin 60° = (5000\ \text{N})\, L\cos 60.0° + (2600\ \text{N})(0.35\, L)\cos 60.0°,$$

so $T = 3.14\ \text{kN}$. The vertical force exerted on the boom by the pivot is the sum of the weights, 7.06 kN and the horizontal force is the tension, 3.14 kN. b) No; $\tan\left(\frac{\text{F}_\text{v}}{\text{F}_\text{H}}\right) \neq 0.$

11.19: To find the tension T_L in the left rope, take torques about the point where the rope at the right is connected to the bar. Then,
$T_\text{L}\ (3.00\ \text{m})\sin 150° = (240\ \text{N})(1.50\ \text{m}) + (90\ \text{N})(0.50\ \text{m})$, so $T_\text{L} = 270\ \text{N}$. The vertical component of the force that the rope at the end exerts must be
$(330\ \text{N}) - (270\ \text{N})\sin 150° = 195\ \text{N}$, and the horizontal component of the force is
$-(270\ \text{N})\cos 150°$, so the tension is the rope at the right is $T_\text{R} = 304\ \text{N}$. and $\theta = 39.9°$.

11.20: The cable is given as perpendicular to the beam, so the tension is found by taking torques about the pivot point;
$T(3.00\ \text{m}) = (1.00\ \text{kN})(2.00\ \text{m})\cos 25.0° + (5.00\ \text{kN})(4.50\ \text{m})\ \cos 25.0°$, or $T = 7.40\ \text{kN}$. The vertical component of the force exerted on the beam by the pivot is the net weight minus the upward component of T, $6.00\ \text{kN} - T\cos 25.0° = 0.17\ \text{kN}$. The horizontal force is $T\sin 25.0° = 3.13\ \text{kN}$.

11.21: a) $F_1(3.00\ \text{m}) - F_2(3.00\ \text{m} + l) = (8.00\ \text{N})(-l)$. This is given to have a magnitude of $6.40\ \text{N.m}$, so $l = 0.80\text{m}$. b) The net torque is clockwise, either by considering the figure or noting the torque found in part (a) was negative. c) About the point of contact of $\vec{\boldsymbol{F}}_2$, the torque due to $\vec{\boldsymbol{F}}_1$ is $-F_1 l$, and setting the magnitude of this torque to $6.40\ \text{N}\cdot\text{m}$ gives $l = 0.80\ \text{m}$, and the direction is again clockwise.

11.22: From Eq. (11.10),

$$Y = F\frac{l_0}{\Delta l A} = F\frac{(0.200\ \text{m})}{(3.0\times 10^{-2}\ \text{m})(50.0\times 10^{-4}\ \text{m}^2)} = F(1333\ \text{m}^{-2}).$$

Then, $F = 25.0\ \text{N}$ corresponds to a Young's modulus of $3.3\times 10^4\ \text{Pa}$, and $F = 500\ \text{N}$ corresponds to a Young's modulus of $6.7\times 10^5\ \text{Pa}$.

11.23: $$A = \frac{Fl_0}{Y\Delta l} = \frac{(400\,\text{N})(2.00\,\text{m})}{(20\times10^{10}\ \text{Pa})(0.25\times10^{-2}\ \text{m})} = 1.60\times10^{-6}\ \text{m}^2,$$

and so $d = \sqrt{4A/\pi} = 1.43\times10^{-3}$ m, or 1.4 mm to two figures.

11.24: a) The strain, from Eq. (11.12), is $\frac{\Delta l}{l_0} = \frac{F}{YA}$. For steel, using Y from Table (11.1) and $A = \pi\frac{d^2}{4} = 1.77\times10^{-4}\ \text{m}^2$,

$$\frac{\Delta l}{l_0} = \frac{(4000\,\text{N})}{(2.0\times10^{11}\ \text{Pa})(1.77\times10^{-4}\ \text{m}^2)} = 1.1\times10^{-4}.$$

Similarly, the strain for copper $(Y = 1.10\times10^{11}\text{Pa})$ is 2.1×10^{-4}. b) Steel: $(1.1\times10^{-4})\times(0.750\,\text{m}) = 8.3\times10^{-5}\text{m}$. Copper: $(2.1\times10^{-4})(0.750\,\text{m}) = 1.6\times10^{-4}\text{m}\cdot$

11.25: From Eq. (11.10),

$$Y = \frac{(5000\,\text{N})(4.00\,\text{m})}{(0.50\times10^{-4}\text{m}^2)(0.20\times10^{-2}\ \text{m})} = 2.0\times10^{11}\ \text{Pa}.$$

11.26: From Eq. (11.10),

$$Y = \frac{(65.0\,\text{kg})(9.80\,\text{m/s}^2)(45.0\,\text{m})}{(\pi(3.5\times10^{-3}\ \text{m})^2)(1.10\,\text{m})} = 6.8\times10^{8}\ \text{Pa}.$$

11.27: a) The top wire is subject to a tension of $(16.0\,\text{kg})(9.80\ \text{m/s}^2) = 157\,\text{N}$ and hence a tensile strain of $\frac{(157\ \text{N})}{(20\times10^{10}\ \text{Pa})(2.5\times10^{-7}\ \text{m}^2)} = 3.14\times10^{-3}$, or 3.1×10^{-3} to two figures. The bottom wire is subject to a tension of 98.0 N, and a tensile strain of 1.96×10^{-3}, or 2.0×10^{-3} to two figures. b) $(3.14\times10^{-3})(0.500\,\text{m}) = 1.57\,\text{mm}$, $(1.96\times10^{-3})(0.500\,\text{m}) = 0.98\,\text{mm}$.

11.28: a) $\frac{(8000\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(12.5\times10^{-2}\ \text{m})^2} = 1.6\times10^{6}\ \text{Pa}$. b) $\frac{1.6\times10^{6}\ \text{Pa}}{2.0\times10^{10}\ \text{Pa}} = 0.8\times10^{-5}$. c) $(0.8\times10^{-5})\times(2.50\ \text{m}) = 2\times10^{-5}\ \text{m}$.

11.29: $(2.8-1)(1.013\times10^{5}\ \text{Pa})(50.0\,\text{m}^2) = 9.1\times10^{6}\ \text{N}.$

11.30: a) The volume would increase slightly. b) The volume change would be twice as great. c) The volume is inversely proportional to the bulk modulus for a given pressure change, so the volume change of the lead ingot would be four times that of the gold.

11.31: a) $\frac{250\ \text{N}}{0.75\times10^{-4}\ \text{m}^2} = 3.33\times10^6\ \text{Pa}$. b) $(3.33\times10^6\ \text{Pa})(2)(200\times10^{-4}\ \text{m}^2) = 133\,\text{kN}$.

11.32: a) Solving Eq. (11.14) for the volume change,

$$\begin{aligned}\Delta V &= -kV\Delta P\\ &= -(45.8\times10^{-11}\ \text{Pa}^{-1})(1.00\,\text{m}^3)(1.16\times10^8\ \text{Pa} - 1.0\times10^5\ \text{Pa})\\ &= -0.0531\,\text{m}^3.\end{aligned}$$

b) The mass of this amount of water not changed, but its volume has decreased to $1.000\,\text{m}^3 - 0.053\,\text{m}^3 = 0.947\,\text{m}^3$, and the density is now $\frac{1.03\times10^3\ \text{kg}}{0.947\ \text{m}^3} = 1.09\times10^3\ \text{kg/m}^3$.

11.33: $$B = \frac{(600\,\text{cm}^3)(3.6\times10^6\ \text{Pa})}{(0.45\,\text{cm}^3)} = 4.8\times10^9\,\text{Pa},\quad k = \frac{1}{B} = 2.1\times10^{-10}\ \text{Pa}^{-1}.$$

11.34: a) Using Equation (11.17),

$$\text{Shear strain } = \frac{F_{\parallel}}{AS} = \frac{(9\times10^5\ \text{N})}{[(.10\,\text{m})(.005\,\text{m})][7.5\times10^{10}\ \text{Pa}]} = 2.4\times10^{-2}.$$

b) Using Equation (11.16), $x = \text{ Shear stain } \cdot h = (.024)(.1\,\text{m}) = 2.4\times10^{-3}\ \text{m}$.

11.35: The area A in Eq. (11.17) has increased by a factor of 9, so the shear strain for the larger object would be $1/9$ that of the smaller.

11.36: Each rivet bears one-quarter of the force, so

$$\text{Shear stress } = \frac{F_{\parallel}}{A} = \frac{\frac{1}{4}(1.20\times10^4\ \text{N})}{\pi(.125\times10^{-2}\ \text{m})^2} = 6.11\times10^8\ \text{Pa}.$$

11.37: $\frac{F}{A}=\frac{(90.8\ \text{N})}{\pi(0.92\times10^{-3}\ \text{m})^2}=3.41\times10^7\ \text{Pa}$, or $3.4\times10^7\ \text{Pa}$ to two figures.

11.38: a) $(1.6\times10^{-3})(20\times10^{10}\ \text{Pa})(5\times10^{-6}\ \text{m}^2)=1.60\times10^3\ \text{N}$. b) If this were the case, the wire would stretch 6.4 mm.

c) $(6.5\times10^{-3})(20\times10^{10}\ \text{Pa})(5\times10^{-6}\ \text{m}^2)=6.5\times10^3\ \text{N}$.

11.39: $a=\dfrac{F_{\text{tot}}}{m}=\dfrac{(2.40\times10^8\ \text{Pa})(3.00\times10^{-4}\ \text{m}^2)/3}{(1200\ \text{kg})}-9.80\ \text{m/s}^2=10.2\ \text{m/s}^2.$

11.40: $A=\frac{350\ \text{N}}{4.7\times10^8\ \text{Pa}}=7.45\times10^{-7}\ \text{m}^2$, so $d=\sqrt{4A/\pi}=0.97\ \text{mm}$.

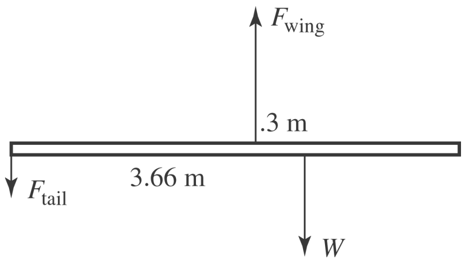
11.41: a) Take torques about the rear wheel, so that $f\omega d=\omega x_{\text{cm}}$, or $x_{\text{cm}}=fd$.
b) $(0.53)(2.46\ \text{m})=1.30\ \text{m}$ to three figures.

11.42: If Lancelot were at the end of the bridge, the tension in the cable would be (from taking torques about the hinge of the bridge) obtained from

$T(12.0\ \text{N})=(600\ \text{kg})(9.80\ \text{m/s}^2)(12.0\ \text{m})+(200\ \text{kg})(9.80\ \text{m/s}^2)(6.0\ \text{m})$,
so $T=6860\ \text{N}$. This exceeds the maximum tension that the cable can have, so Lancelot is going into the drink. To find the distance x Lancelot can ride, replace the 12.0 m multiplying Lancelot's weight by x and the tension T by $T_{\text{max}}=5.80\times10^3\ \text{N}$ and solve for x;

$$x=\frac{(5.80\times10^3\ \text{N})(12.0\ \text{m})-(200\ \text{kg})(9.80\ \text{m/s}^2)(6.0\ \text{m})}{(600\ \text{kg})(9.80\ \text{m/s}^2)}=9.84\ \text{m}.$$

11.43: For the airplane to remain in level flight, both $\sum F = 0$ and $\sum \tau = 0$.



Taking the clockwise direction as positive, and taking torques about the center of mass,

$$\text{Forces:} - F_{\text{tail}} - W + F_{\text{wing}} = 0$$

$$\text{Torques:} \; -(3.66\,\text{m})F_{\text{tail}} + (.3\,\text{m})F_{\text{wing}} = 0$$

A shortcut method is to write a second torque equation for torques about the tail, and solve for the F_{wing} : $-(3.66\,\text{m})(6700\,\text{N}) + (3.36\,\text{m})F_{\text{wing}} = 0.$ This gives $F_{\text{wing}} = 7300\ \text{N(up)}$, and $F_{\text{tail}} = 6700\,\text{N} - 7300\,\text{N} = -600\ \text{N(down)}$.

Note that the rear stabilizer provides a *downward* force, does not hold up the tail of the aircraft, but serves to counter the torque produced by the wing. Thus balance, along with weight, is a crucial factor in airplane loading.

11.44: The simplest way to do this is to consider the *changes* in the forces due to the extra weight of the box. Taking torques about the rear axle, the force on the front wheels is decreased by $3600\,\text{N}\frac{1.00\text{ m}}{3.00\text{ m}} = 1200\,\text{N},$ so the net force on the front wheels is $10{,}780\,\text{N} - 1200\,\text{N} = 9.58\times10^{3}\ \text{N}$ to three figures. The weight added to the rear wheels is then $3600\,\text{N} + 1200\,\text{N} = 4800\,\text{N},$ so the net force on the rear wheels is $8820\,\text{N} + 4800\,\text{N} = 1.36\times10^{4}\ \text{N},$ again to three figures.

b) Now we want a shift of $10{,}780\,\text{N}$ away from the front axle. Therefore, $W\frac{1.00\text{ m}}{3.00\text{ m}} = 10{,}780\ \text{N}$ and so $w = 32{,}340\,\text{N}.$

11.45: Take torques about the pivot point, which is 2.20 m from Karen and 1.65 m from Elwood. Then $w_{\text{Elwood}}(1.65\,\text{m}) = (420\,\text{N})(2.20\,\text{m}) + (240\,\text{N})(0.20\,\text{m}),$ so Elwood weighs 589 N. b) Equilibrium is neutral.

11.46: a) Denote the weight per unit length as α, so $w_1 = \alpha(10.0\,\text{cm})$, $w_2 = \alpha(8.0\,\text{cm})$, and $w_3 = \alpha l$.
The center of gravity is a distance x_{cm} to the right of point O where

$$x_{\text{cm}} = \frac{w_1(5.0\,\text{cm}) + w_2(9.5\,\text{cm}) + w_3(10.0\,\text{cm} - l/2)}{w_1 + w_2 + w_3}$$

$$= \frac{(10.0\,\text{cm})(5.0\,\text{cm}) + (8.0\,\text{cm})(9.5\,\text{cm}) + l(10.0\,\text{cm} - l/2)}{(10.0\,\text{cm}) + (8.0\,\text{cm}) + l}.$$

Setting $x_{\text{cm}} = 0$ gives a quadratic in l, which has as its positive root $l = 28.8\,\text{cm}$.

b) Changing the material from steel to copper would have no effect on the length l since the weight of each piece would change by the same amount.

11.47: Let $\vec{r}_i' = \vec{r}_i - \vec{R}$, where $\vec{R}$ is the vector from the point O to the point P. The torque for each force with respect to point P is then $\vec{\tau}_i' = \vec{r}_i' \times \vec{F}_i$, and so the net torque is

$$\begin{aligned} \sum \vec{\tau}_i &= \sum\left(\vec{r}_i - \vec{R}\right) \times \vec{F}_i \\ &= \sum \vec{r}_i \times \vec{F}_i - \sum \vec{R} \times \vec{F}_i \\ &= \sum \vec{r}_i \times \vec{F}_i - \vec{R} \times \sum \vec{F}_i. \end{aligned}$$

In the last expression, the first term is the sum of the torques about point O, and the second term is given to be zero, so the net torques are the same.

11.48: From the figure (and from common sense), the force $\vec{F}_1$ is directed along the length of the nail, and so has a moment arm of (0.0800 m) $\sin 60°$. The moment arm of $\vec{F}_2$ is 0.300 m, so

$$F_2 = F_1 \frac{(0.0800\,\text{m})\ \sin 60°}{(0.300\,\text{m})} = (500\,\text{N})(0.231) = 116\,\text{N}.$$

11.49: The horizontal component of the force exerted on the bar by the hinge must balance the applied force $\vec{F}$, and so has magnitude 120.0 N and is to the left. Taking torques about point A, $(120.0\ \text{N})(4.00\ \text{m}) + F_{\text{V}}(3.00\ \text{m})$, so the vertical component is $-160\ \text{N}$, with the minus sign indicating a downward component, exerting a torque in a direction opposite that of the horizontal component. The force exerted by the bar on the hinge is equal in magnitude and opposite in direction to the force exerted by the hinge on the bar.

11.50: a) The tension in the string is $w_2 = 50\ \text{N}$, and the horizontal force on the bar must balance the horizontal component of the force that the string exerts on the bar, and is equal to $(50\ \text{N})\sin 37° = 30\ \text{N}$, to the left in the figure. The vertical force must be $(50\ \text{N})\cos 37° + 10\ \text{N} = 50\ \text{N}$, up. b) $\arctan\left(\dfrac{50\ \text{N}}{30\ \text{N}}\right) = 59°$. c) $\sqrt{(30\ \text{N})^2 + (50\ \text{N})^2} = 58\ \text{N}$.

d) Taking torques about (and measuring the distance from) the left end, $(50\ \text{N})x = (40\ \text{N})(5.0\ \text{m})$, so $x = 4.0\ \text{m}$, where only the vertical components of the forces exert torques.

11.51: a) Take torques about her hind feet. Her fore feet are 0.72 m from her hind feet, and so her fore feet together exert a force of $\frac{(190\ \text{N})(0.28\ \text{m})}{(0.72\ \text{m})} = 73.9\ \text{N}$, so each foot exerts a force of 36.9 N, keeping an extra figure. Each hind foot then exerts a force of 58.1 N. b) Again taking torques about the hind feet, the force exerted by the fore feet is $\frac{(190\ \text{N})(0.28\ \text{m}) + (25\ \text{N})(0.09\ \text{m})}{0.72\ \text{m}} = 105.1\ \text{N}$, so each fore foot exerts a force of 52.6 N and each hind foot exerts a force of 54.9 N.

11.52: a) Finding torques about the hinge, and using L as the length of the bridge and w_{T} and w_{B} for the weights of the truck and the raised section of the bridge,

$$TL\sin 70° = w_{\text{T}}\left(\tfrac{3}{4}L\right)\cos 30° + w_{\text{B}}\left(\tfrac{1}{2}\text{L}\right)\cos 30°, \text{ so}$$

$$T = \frac{\left(\frac{3}{4}m_{\text{T}} + \frac{1}{2}m_{\text{B}}\right)(9.80\ \text{m/s}^2)\cos 30°}{\sin 70°} = 2.57\times10^5\ \text{N}.$$

b) Horizontal: $T\cos(70° - 30°) = 1.97\times10^5\ \text{N}$. Vertical: $w_{\text{T}} + w_{\text{B}} - T\sin 40°$
$= 2.46\times10^5\ \text{N}$.

11.53: a) Take the torque exerted by $\vec{F}_2$ to be positive; the net torque is then $-F_1(x)\sin\phi + F_2(x+l)\sin\phi = Fl\sin\phi,$ where F is the common magnitude of the forces.
b) $\tau_1 = -(14.0\text{ N})(3.0\text{ m})\sin 37° = -25.3\text{ N}\cdot\text{m},$ keeping an extra figure, and $\tau_2 = (14.0\text{ N})(4.5\text{ m})\sin 37° = 37.9\text{ N}\cdot\text{m},$ and the net torque is $12.6\text{ N}\cdot\text{m}.$ About point P, $\tau_1 = (14.0\text{ N})(3.0\text{ m})(\sin 37°) = 25.3\text{ N}\cdot\text{m},$ and $\tau_2 = (-14.0\text{ N})(1.5\text{ m})(\sin 37°) = -12.6\text{ N}\cdot\text{m},$ and the net torque is $12.6\text{ N}\cdot\text{m}.$ The result of part (a) predicts $(14.0\text{ N})(1.5\text{ m})\sin 37°,$ the same result.

11.54: a) Take torques about the pivot. The force that the ground exerts on the ladder is given to be vertical, and $F_V(6.0\text{ m})\sin\theta = (250\text{ N})(4.0\text{ m})\sin\theta$ $+ (750\text{ N})(1.50\text{ m})\sin\theta,$ so $F_V = 354\text{ N}.$ b) There are no other horizontal forces on the ladder, so the horizontal pivot force is zero. The vertical force that the pivot exerts on the ladder must be $(750\text{ N}) + (250\text{ N}) - (354\text{ N}) = 646\text{ N},$ up, so the ladder exerts a downward force of 646 N on the pivot. c) The results in parts (a) and (b) are independent of $\theta.$

11.55: a) $V = mg + w$ and $H = T.$ To find the tension, take torques about the pivot point. Then, denoting the length of the strut by $L,$

$$T\left(\frac{2}{3}L\right)\sin\theta = w\left(\frac{2}{3}L\right)\cos\theta + mg\left(\frac{L}{6}\right)\cos\theta,\text{ or}$$

$$T = \left(w + \frac{mg}{4}\right)\cot\theta.$$

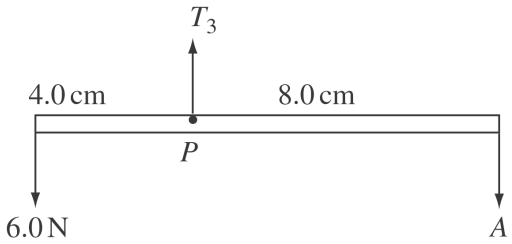
b) Solving the above for w, and using the maximum tension for T,

$$w = T\tan\theta - \frac{mg}{4} = (700\text{ N})\tan 55.0° - (5.0\text{ kg})(9.80\ \text{m/s}^2) = 951\text{ N}.$$

c) Solving the expression obtained in part (a) for $\tan\theta$ and letting $\omega \to 0,\ \tan\theta = \frac{mg}{4T} = 0.700,$ so $\theta = 4.00°.$

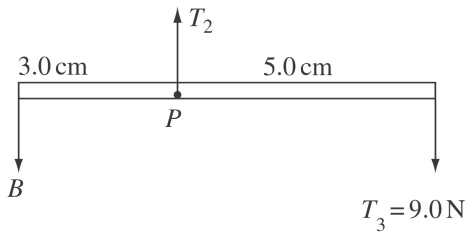
11.56: (a) and (b)

Lower rod:



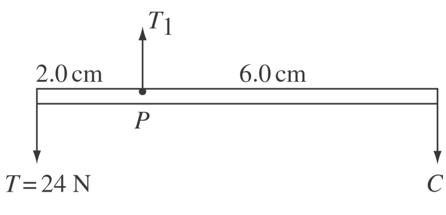
$$\Sigma\tau_p = 0 : (6.0\text{ N})(4.0\text{ cm}) = A(8.0\text{ cm})$$
$$A = 3.0\text{ N}$$
$$\Sigma F = 0 : T_3 = 6.0\text{ N} + A = 6.0\text{ N} + 3.0\text{ N} = 9.0\text{ N}$$

Middle rod:



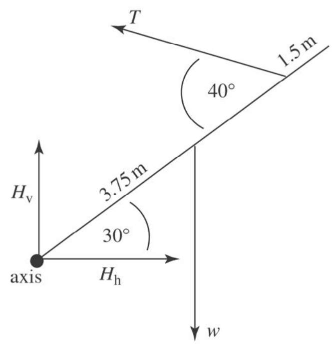
$$\Sigma\tau_p = 0 : B(3.0\text{ cm}) = (9.0\text{ N})(5.0\text{ cm})$$
$$B = 15\text{ N}$$
$$\Sigma F = 0 : T_2 = B + T_3 = 15\text{ N} + 9.0\text{ N} = 24\text{ N}$$

Upper rod:



$$\Sigma\tau_p = 0 : (24\text{ N})(2.0\text{ cm}) = C(6.0\text{ cm})$$
$$C = 8.0\text{ N}$$
$$\Sigma F = 0 : T_1 = T_2 + C = 24\text{ N} + 8.0\text{ N} = 32\text{ N}$$

11.57:

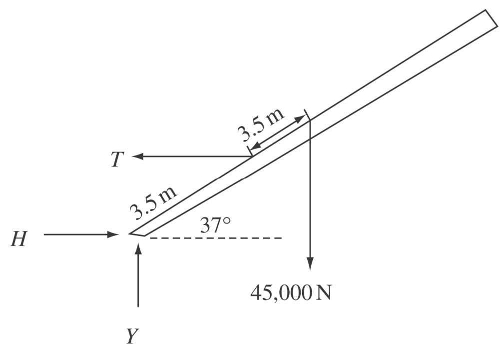


$\Sigma\tau = 0$, axis at hinge

$T(6.0\text{ m})(\sin 40°) - w(3.75\text{ m})(\cos 30°) = 0$

$$T = 760\text{ N}$$

11.58: (a)



$$\Sigma\tau_{\text{Hinge}} = 0$$

$$T(3.5\text{ m})\sin 37° = (45{,}000\text{ N})(7.0\text{ m})\cos 37°$$

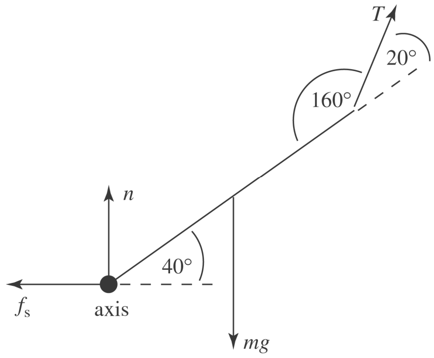
$$T = 120{,}000\text{ N}$$

(b) $\sum F_x = 0 : H = T = 120{,}000\text{ N}$

$\sum F_x = 0 : V = 45{,}000\text{ N}$

The resultant force exerted by the hinge has magnitude 1.28×10^5 N and direction $20.6°$ above the horizontal.

11.59:



a) $\sum \tau = 0$, axis at lower end of beam
Let the length of the beam be L.

$$T(\sin 20°)L = -mg\left(\frac{L}{2}\right)\cos 40° = 0$$

$$T = \frac{\frac{1}{2}mg\cos 40°}{\sin 20°} = 2700\ \text{N}$$

b) Take $+y$ upward.

$\sum F_y = 0$ gives $n - w + T\sin 60° = 0$ so $n = 73.6\ \text{N}$
$\sum F_x = 0$ gives $f_s = T\cos 60° = 1372\ \text{N}$
$f_s = \mu_s n,\ \mu_s = \dfrac{f_s}{n} = \dfrac{1372\ \text{N}}{73.6\ \text{N}} = 19$

The floor must be very rough for the beam not to slip.

11.60: a) The center of mass of the beam is 1.0 m from the suspension point. Taking torques about the suspension point,

$$w(4.00\ \text{m}) + (140.0\ \text{N})(1.00\ \text{m}) = (100\ \text{N})(2.00\ \text{m})$$

(note that the common factor of $\sin\ 30°$ has been factored out), from which $w = 15.0\ \text{N}$.

b) In this case, a common factor of $\sin\ 45°$ would be factored out, and the result would be the same.

11.61: a) Taking torques about the hinged end of the pole $(200\ \text{N})(2.50\ \text{m}) + (600\ \text{N}) \times (5.00\ \text{m}) - T_y(5.00\ \text{m}) = 0$. Therefore the *y*-component of the tension is $T_y = 700\ \text{N}$. The *x*-component of the tension is then $T_x = \sqrt{(1000\ \text{N})^2 - (700\ \text{N})^2} = 714\ \text{N}$. The height above the pole that the wire must be attached is $(5.00\ \text{m})\frac{700}{714} = 4.90\ \text{m}$. b) The *y*-component of the tension remains 700 N and the *x*-component becomes $(714\ \text{N})\frac{4.90\ \text{m}}{4.40\ \text{m}} = 795\ \text{N}$, leading to a total tension of $\sqrt{(795\ \text{N})^2 + (700\ \text{N})^2} = 1059\ \text{N}$, an increase of 59 N.

11.62: *A* and *B* are straightforward, the tensions being the weights suspended; $T_A = (0.0360\ \text{kg})(9.80\ \text{m/s}^2) = 0.353\ \text{N}, T_B = (0.0240\ \text{kg} + 0.0360\ \text{kg})(9.80\ \text{m/s}^2) = 0.588\ \text{N}$ To find T_C and T_D, a trick making use of the right angle where the strings join is available; use a coordinate system with axes parallel to the strings. Then, $T_C = T_B \cos 36.9° = 0.470\ \text{N}, T_D = T_B \cos 53.1° = 0.353\ \text{N}$, To find T_E, take torques about the point where string *F* is attached;

$$\begin{aligned} T_E(1.000\ \text{m}) &= T_D \sin 36.9°(0.800\ \text{m}) + T_C \sin 53.1°(0.200\ \text{m}) \\ &\quad + (0.120\ \text{kg})(9.80\ \text{m/s}^2)(0.500\ \text{m}) \\ &= 0.833\ \text{N}\cdot\text{m}, \end{aligned}$$

so $T_E = 0.833\ \text{N}. T_F$ may be found similarly, or from the fact that $T_E + T_F$ must be the total weight of the ornament. $(0.180\ \text{kg})(9.80\ \text{m/s}^2) = 1.76\ \text{N}$, from which $T_F = 0.931\ \text{N}$.

11.63: a) The force will be vertical, and must support the weight of the sign, and is 300 N. Similarly, the torque must be that which balances the torque due to the sign's weight about the pivot, $(300\ \text{N})(0.75\ \text{m}) = 225\ \text{N}\cdot\text{m}$. b) The torque due to the wire must balance the torque due to the weight, again taking torques about the pivot. The minimum tension occurs when the wire is perpendicular to the lever arm, from one corner of the sign to the other. Thus, $T\sqrt{(1.50\ \text{m})^2 + (0.80\ \text{m})^2} = 225\ \text{N}\cdot\text{m}$, or $T = 132\ \text{N}$. The angle that the wire makes with the horizontal is $90° - \arctan\left(\frac{0.80}{1.50}\right) = 62.0°$. Thus, the vertical component of the force that the pivot exerts is $(300\ \text{N}) - (132\ \text{N}) \sin 62.0° = 183\ \text{N}$ and the horizontal force is $(132\ \text{N})\cos 62.0° = 62\ \text{N}$, for a magnitude of 193 N and an angle of 71° above the horizontal.

11.64: a) $\Delta w = -\sigma\,(\Delta l/l)\,w_0 = -(0.23)(9.0\times10^{-4})\sqrt{4(0.30\times10^{-4}\ \text{m}^2)/\pi} = 1.3\ \mu\text{m}.$

b)

$$F_{\perp} = AY\frac{\Delta l}{l} = AY\frac{1}{\sigma}\frac{\Delta w}{w}$$
$$= \frac{(2.1\times10^{11}\ \text{Pa})\,(\pi\,(2.0\times10^{-2}\ \text{m})^2)}{0.42}\,\frac{0.10\times10^{-3}\ \text{m}}{2.0\times10^{-2}\ \text{m}} = 3.1\times10^{6}\ \text{N},$$

where the Young's modulus for nickel has been used.

11.65: a) The tension in the horizontal part of the wire will be 240 N. Taking torques about the center of the disk, $(240\ \text{N})(0.250\ \text{m}) - w(1.00\ \text{m})) = 0$, or w = 60 N.
b) Balancing torques about the center of the disk in this case,
$(240\ \text{N})\,(0.250\ \text{m}) - ((60\ \text{N})(1.00\ \text{m}) + (20\ \text{N})(2.00\ \text{m}))\cos\theta = 0,$ so $\theta = 53.1°$.

11.66: a) Taking torques about the right end of the stick, the friction force is half the weight of the stick, $f = \frac{w}{2}\cdot$ Taking torques about the point where the cord is attached to the wall (the tension in the cord and the friction force exert no torque about this point),and noting that the moment arm of the normal force is
$l\tan\theta, n\tan\theta = \frac{w}{2}\cdot$ Then, $\frac{f}{n} =$ $\tan\theta < 0.40$, so $\theta <$ arctan (0.40) = 22°.

b) Taking torques as in part (a), and denoting the length of the meter stick as l,

$$fl = w\frac{l}{2} + w(l-x) \text{ and } nl\tan\theta = w\frac{l}{2} + wx.$$

In terms of the coefficient of friction μ_s,

$$\mu_s > \frac{f}{n} = \frac{\frac{l}{2} + (l-x)}{\frac{l}{2} + x}\tan\theta = \frac{3l-2x}{l+2x}\tan\theta.$$

Solving for *x*,

$$x > \frac{l}{2}\,\frac{3\tan\theta - \mu_s}{\mu_s + \tan\theta} = 30.2\ \text{cm}.$$

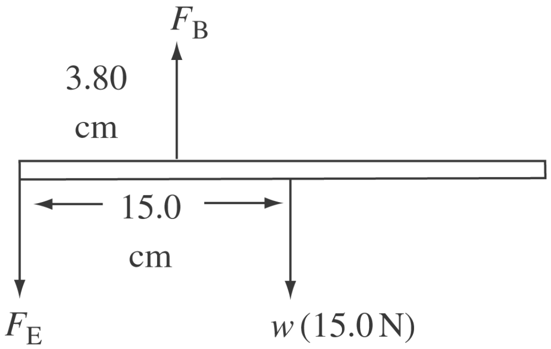
c) In the above expression, setting $x = 10$ cm and solving for μ_s gives

$$\mu_s > \frac{(3 - 20/l)\tan\theta}{1 + 20/l} = 0.625.$$

11.67: Consider torques around the point where the person on the bottom is lifting. The center of mass is displaced horizontally by a distance $(0.625\,\text{m} - 0.25\,\text{m})\sin 45°$ and the horizontal distance to the point where the upper person is lifting is $(1.25\,\text{m})\sin 45°$, and so the upper lifts with a force of $w\frac{0.375\sin 45°}{1.25\sin 45°} = (0.300)w = 588\ \text{N}.$ The person on the bottom lifts with a force that is the difference between this force and the weight, 1.37 kN. The person above is lifting less.

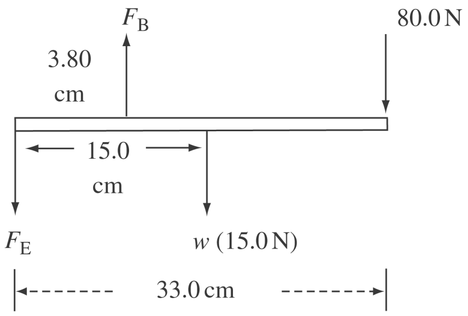
11.68:

(a)



$$\Sigma\tau_{\text{Elbow}} = 0$$
$$F_B(3.80 \text{ cm}) = (15.0 \text{ N})(15.0 \text{ cm})$$
$$F_B = 59.2 \text{ N}$$

(b)



$$\Sigma\tau_E = 0$$
$$F_B(3.80 \text{ cm}) = (15.0 \text{ N})(15.0 \text{ cm}) + (80.0 \text{ N})(33.0 \text{ cm})$$
$$F_B = 754 \text{ N}$$

11.69: a) The force diagram is given in Fig. 11.9.

$$\Sigma\tau = 0, \text{ axis at elbow}$$

$$wL - (T \sin\theta)D = 0$$

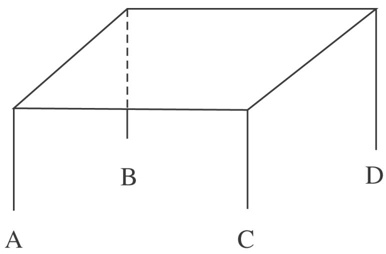
$$\sin\theta = \frac{h}{\sqrt{h^2 + D^2}} \text{ so } w = T\frac{hD}{L\sqrt{h^2 + D^2}}$$

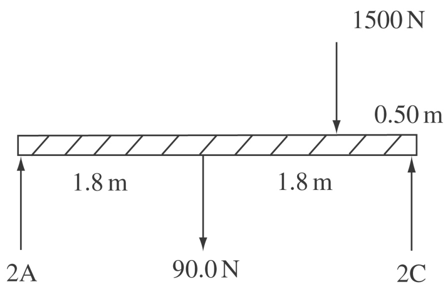
$$w_{max} = T_{max}\frac{hD}{L\sqrt{h^2 + D^2}}$$

b) $\dfrac{dw_{max}}{dD} = \dfrac{T_{max}h}{L\sqrt{h^2 + D^2}}\left(1 - \dfrac{D^2}{h^2 + D^2}\right)$; the derivative is positive

c) The result of part (b) shows that w_{max} increases when D increases.

11.70:





By symmetry, $A=B$ and $C=D$. Redraw the table as viewed from the AC side.

$$\Sigma\tau \text{ (about right end)} = 0:$$

$$2A(3.6 \text{ m}) = (90.0 \text{ N})(1.8 \text{ m}) + (1500 \text{ N})(0.50 \text{ m})$$

$$A = 130 \text{ N} = B$$

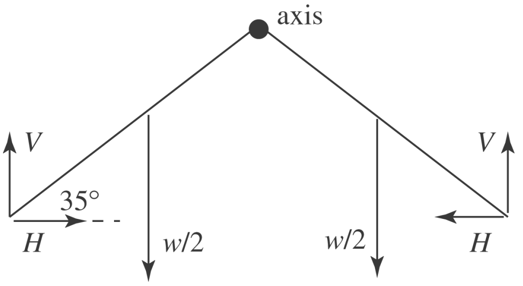
$$\Sigma F = 0: A + B + C + D = 1590 \text{ N}$$

Use $A = B = 130$ N and $C = D$

$$C = D = 670 \text{ N}$$

By Newton's third law of motion, the forces A, B, C, and D on the table are the same as the forces the table exerts on the floor.

11.71: a) Consider the forces on the roof



V and H are the vertical and horizontal forces each wall exerts on the roof.
$w = 20{,}000$ N is the total weight of the roof.

$2V = w$ so $V = w/2$

Apply $\Sigma\tau = 0$ to one half of the roof, with the axis along the line where the two halves join. Let each half have length L.

$(w/2)(L/2)(\cos 35.0°) + HL\sin 35.0° - VL\cos 35.0° = 0$

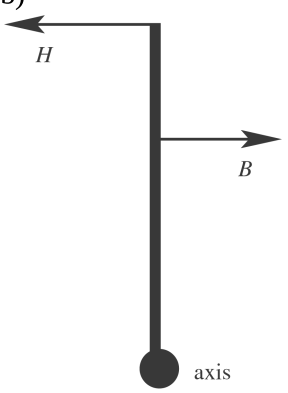
L divides out, and use $V = w/2$

$H\sin 35.0° = \frac{1}{4}w\cos 35.0°$

$$H = \frac{w}{4\tan 35.0°} = 7140\ \text{N}$$

By Newton's 3rd law, the roof exerts a horizontal, outward force on the wall. For torque about an axis at the lower end of the wall, at the ground, this force has a larger moment arm and hence larger torque the taller the walls.

b)



Consider the torques on one of the walls.

11.72: a) Take torques about the upper corner of the curb. The force $\vec{F}$ acts at a perpendicular distance $R-h$ and the weight acts at a perpendicular distance $\sqrt{R^2-(R-h)^2}=\sqrt{2Rh-h^2}$. Setting the torques equal for the minimum necessary force,

$$F=mg\frac{\sqrt{2Rh-h^2}}{R-h}.$$

b) The torque due to gravity is the same, but the force $\vec{F}$ acts at a perpendicular distance $2R-h$, so the minimum force is $(mg)\sqrt{2Rh-hv}$ /2*R*–*h*. c) Less force is required when the force is applied at the top of the wheel.

11.73: a) There are several ways to find the tension. Taking torques about point *B* (the force of the hinge at *A* is given as being vertical, and exerts no torque about *B*), the tension acts at distance $r=\sqrt{(4.00\,\mathrm{m})^2+(2.00\,\mathrm{m})^2}=4.47$ m and at an angle of $\phi=30°+\arctan\left(\frac{2.00}{4.00}\right)=56.6°$. Setting

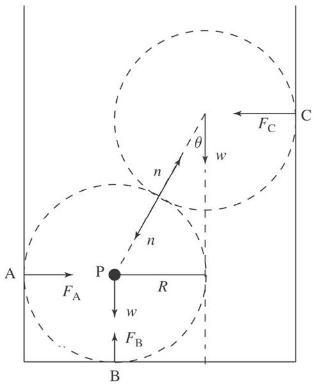
$Tr\sin\varphi=(500\,\mathrm{N})(2.00\,\mathrm{m})$ and solving for T gives $T=268\,\mathrm{N}$. b) The hinge at *A* is given as exerting no horizontal force, so taking torques about point D, the lever arm for the vertical force at point B is $(2.00\,\mathrm{m})+(4.00\,\mathrm{m})\tan 30.0°=4.31\,\mathrm{m}$, so the horizontal force at B is $\frac{(500\,\mathrm{N})(2.00\,\mathrm{m})}{4.31\,\mathrm{m}}=232\,\mathrm{N}$. Using the result of part (a),

however, $(268\text{ N})\cos 30.0°=232\text{ N}$ In fact, finding the horizontal force at B first simplifies the calculation of the tension slightly. c) $(500\text{ N})-(268\text{ N})\sin 30.0°=366\text{ N}$. Equivalently, the result of part (b) could be used, taking torques about point C, to get the same result.

11.74: a) The center of gravity of top block can be as far out as the edge of the lower block. The center of gravity of this combination is then $3L/4$ from the right edge of the upper block, so the overhang is $3L/4.$

b) Take the two-block combination from part (a), and place it on the third block such that the overhang of $3L/4$ is from the right edge of the third block; that is, the center of gravity of the first two blocks is above the right edge of the third block. The center of mass of the three-block combination, measured from the right end of the bottom block, is $-L/6$ and so the largest possible overhang is $(3L/4)+(L/6)=11L/12.$
Similarly, placing this three-block combination with its center of gravity over the right edge of the fourth block allows an extra overhang of $L/8,$ for a total of $25L/24.$ c) As the result of part (b) shows, with only four blocks, the overhang can be larger than the length of a single block.

11.75: a)



$$F_B = 2w = 1.47 \text{ N}$$

$$\sin\theta = R/2R \text{ so } \theta = 30°$$

$$\tau = 0, \text{ axis at } P$$

$$F_C(2R\cos\theta) - wR = 0$$

$$F_C = \frac{mg}{2\cos 30°} = 0.424 \text{ N}$$

$$F_A = F_C = 0.424 \text{ N}$$

b) Consider the forces on the bottom marble. The horizontal forces must sum to zero, so

$$F_A = n\sin\theta$$

$$n = \frac{F_A}{\sin 30°} = 0.848 \text{ N}$$

Could use instead that the vertical forces sum to zero

$$F_B - mg - n\cos\theta = 0$$

$$n = \frac{F_B - mg}{\cos 30°} = 0.848 \text{ N, which checks.}$$

11.76: (a) Writing an equation for the torque on the right-hand beam, using the hinge as an axis and taking counterclockwise rotation as positive:

$$F_{\text{wire}} L \sin\frac{\theta}{2} - F_c \frac{L}{2}\cos\frac{\theta}{2} - w\frac{L}{2}\sin\frac{\theta}{2} = 0$$

where θ is the angle between the beams, F_{c} is the force exerted by the cross bar, and w is the weight of one beam. The length drops out, and all other quantities except F_{c} are known, so

$$F_{\text{c}} = \frac{F_{\text{wire}} \sin\frac{\theta}{2} - \frac{1}{2} w \sin\frac{\theta}{2}}{\frac{1}{2}\cos\frac{\theta}{2}} = (2F_{\text{wire}} - w)\tan\frac{\theta}{2}$$

Therefore

$$F = 260 \tan\frac{53°}{2} = 130\ \text{N}$$

b) The cross bar is under compression, as can be seen by imagining the behavior of the two beams if the cross bar were removed. It is the cross bar that holds them apart.

c) The upward pull of the wire on each beam is balanced by the downward pull of gravity, due to the symmentry of the arrangement. The hinge therefore exerts no vertical force. It must, however, balance the outward push of the cross bar: 130 N horizontally to the left for the right-hand beam and 130 N to the right for the left-hand beam. Again, it’s instructive to visualize what the beams would do if the hinge were removed.

11.77: a) The angle at which the bale would slip is that for which $f = \mu_{\text{s}} N = \mu_{\text{s}} w \cos\beta = w \sin\beta$, or $\beta = \arctan(\mu_{\text{s}}) = 31.0°$. The angle at which the bale would tip is that for which the center of gravity is over the lower contact point, or $\arctan(\frac{0.25\ \text{m})}{0.50\ \text{m})} = 26.6°$, or $27°$ to two figures. The bale tips before it slips. b) The angle for tipping is unchanged, but the angle for slipping is $\arctan(0.40) = 21.8°$, or $22°$ to two figures. The bale now slips before it tips.

11.78: a) $F = f = \mu_{\text{k}} N = \mu_{\text{k}} mg = (0.35)(30.0\ \text{kg})(9.80\ \text{m/s}^2) = 103\ \text{N}$

b) With respect to the forward edge of the bale, the lever arm of the weight is $\frac{0.250\ \text{m}}{2} = 0.125\,\text{m}$ and the lever arm h of the applied force is then h $= (0.125\,\text{m})\frac{mg}{F} = (0.125\,\text{m})\frac{1}{\mu_{\text{k}}} = \frac{0.125\ \text{m}}{0.35} = 0.36\,\text{m}.$

11.79: a) Take torques about the point where wheel B is in contact with the track. With respect to this point, the weight exerts a counterclockwise torque and the applied force and the force of wheel A both exert clockwise torques. Balancing torques, $F_A(2.00\,\text{m}) + (F)(1.60\,\text{m}) = (950\,\text{N})(1.00\,\text{m})$. Using $F = \mu_k w = 494\,\text{N}, F_A = 80\,\text{N}, \text{and } F_B = w - F_A = 870\,\text{N}.$ b) Again taking torques about the point where wheel B is in contact with the tract, and using $F = 494\,\text{N}$ as in part (a), $(494\,\text{N})\,h = (950\,\text{N})(1.00\text{N})$, so $h = 1.92\,\text{m}$.

11.80: a) The torque exerted by the cable about the left end is $TL\sin\theta$. For any angle $\theta, \sin(180° - \theta) = \sin\theta,$ so the tension T will be the same for either angle. The horizontal component of the force that the pivot exerts on the boom will be $T\cos\theta$ or $T\cos(180° - \theta) = -T\cos\theta$. b) From the result of part (a), $T\ \alpha\ \frac{1}{\sin\theta}$, and this becomes infinite as $\theta \to 0$ or $\to 180°.$ Also, c), the tension is a minimum when $\sin\theta$ is a maximum, or $\theta = 90°,$ a vertical string. d) There are no other horizontal forces, so for the boom to be in equilibrium, the pivot exerts zero horizontal force on the boom.

11.81: a) Taking torques about the contact point on the ground, $T(7.0\text{ m})\sin\theta = w(4.5\text{ m})\sin\theta$, so $T = (0.64)w = 3664$ N. The ground exerts a vertical force on the pole, of magnitude $w - T = 2052$ N. b) The factor of $\sin\theta$ appears in both terms of the equation representing the balancing of torques, and cancels.

11.82: a) Identifying x with Δl in Eq. (11.10), $k = YA/l_0.$
b) $(1/2)kx^2 = YAx^2/2l_0.$

11.83: a) At the bottom of the path the wire exerts a force equal in magnitude to the centripetal acceleration plus the weight,

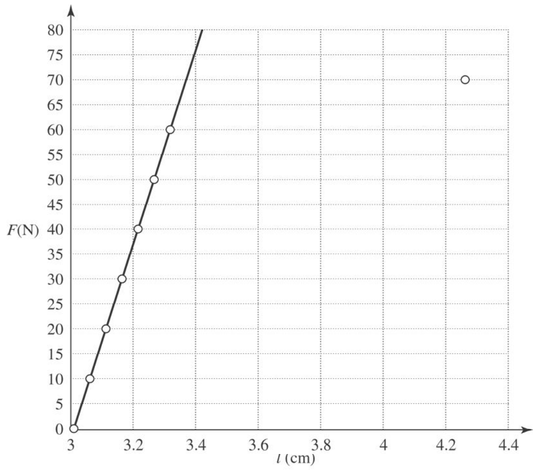
$$F = m(((2.00\text{ rev/s})(2\pi\text{ rad/rev}))^2(0.50\,\text{m}) + 9.80\text{ m/s}^2) = 1.07\times10^3\text{ N}.$$

From Eq. (11.10), the elongation is

$$\frac{(1.07\times10^3\text{ N})(0.50\,\text{m})}{(0.7\times10^{11}\text{ Pa})(0.014\times10^{-4}\text{ m}^2)} = 5.5\,\text{mm}.$$

b) Using the same equations, at the top the force is 830 N, and the elongation is 0.0042 m.

11.84: a)



b) The ratio of the added force to the elongation, found from taking the slope of the graph, doing a least-squares fit to the linear part of the data, or from a casual glance at the data gives $\frac{F}{\Delta l} = 2.00\times 10^4\ \text{N/m}$. From Eq. (11.10),

$$Y = \frac{F}{\Delta l}\frac{l_0}{A} = (2.00\times 10^4\ \text{N/m})\frac{(3.50\ \text{m})}{(\pi(0.35\times 10^{-3}\ \text{m})^2)} = 1.8\times 10^{11}\ \text{Pa}.$$

c) The total force at the proportional limit is $20.0\ \text{N} + 60\ \text{N} = 80\ \text{N}$, and the stress at this limit is $\frac{(80\ \text{N})}{\pi(0.35\times 10^{-3}\ \text{m})^2} = 2.1\times 10^8\ \text{Pa}$.

11.85: a) For the same stress, the tension in wire *B* must be two times in wire *A*, and so the weight must be suspended at a distance $(2/3)(1.05\ \text{m}) = 0.70\ \text{m}$ from wire *A*..

b) The product YA for wire *B* is $(4/3)$ that of wire *B*, so for the same strain, the tension in wire *B* must be $(4/3)$ that in wire *A*, and the weight must be 0.45 m from wire *B*.

11.86: a) Solving Eq. (11.10) for Δl and using the weight for *F*,

$$\Delta l = \frac{Fl_0}{YA} = \frac{(1900\ \text{N})(15.0\ \text{m})}{(2.0\times 10^{11}\ \text{Pa})(8.00\times 10^{-4}\ \text{m}^2)} = 1.8\times 10^{-4}\ \text{m}.$$

b) From Example 5.21, the force that each car exerts on the cable is $F = m\omega^2 l_0 = \frac{w}{g} w^2 l_0$, and so

$$\Delta l = \frac{Fl_0}{YA} = \frac{w\omega^2 l_0^2}{gYA} = \frac{(1900\ \text{N})(0.84\ \text{rad/s})^2(15.0\ \text{m})^2}{(9.80\ \text{m/s}^2)(2.0\times 10^{11}\ \text{Pa})(8.00\times 10^{-4}\ \text{m}^2)} = 1.9\times 10^{-4}\ \text{m}.$$

11.87: Use subscripts 1 to denote the copper and 2 to denote the steel. a) From Eq. (11.10), with $\Delta l_1 = \Delta l_2$ and $F_1 = F_2$,

$$L_2 = L_1\left(\frac{A_2Y_2}{A_1Y_1}\right) = (1.40\,\text{m})\left(\frac{(1.00\,\text{cm}^2)(21\times10^{10}\text{ Pa})}{(2.00\,\text{cm}^2)(9\times10^{10}\text{ Pa})}\right) = 1.63\,\text{m}.$$

b) For nickel, $\frac{F}{A_1} = 4.00\times10^8$ Pa and for brass, $\frac{F}{A_2} = 2.00\times10^8$ Pa. c) For nickel, $\frac{4.00\times10^8\text{ Pa}}{21\times10^{10}\text{ Pa}} = 1.9\times10^{-3}$ and for brass, $\frac{2.00\times10^8\text{ Pa}}{9\times10^{10}\text{ Pa}} = 2.2\times10^{-3}$.

11.88: a) $F_{\text{max}} = YA\left(\frac{\Delta l}{l_0}\right)_{\text{max}} = (1.4\times10^{10}\text{ Pa})(3.0\times10^{-4}\text{ m}^2)(0.010) = 4.2\times10^4\text{ N}.$

b) Neglect the mass of the shins (actually the lower legs and feet) compared to the rest of the body. This allows the approximation that the compressive stress in the shin bones is uniform. The maximum height will be that for which the force exerted on each lower leg by the ground is F_{max} found in part (a), minus the person's weight. The impulse that the ground exerts is $J = (4.2\times10^4\text{ N} - (70\text{ kg})(9.80\text{ m/s}^2))(0.030\text{ s}) = 1.2\times10^3\text{ kg}\cdot\text{m/s}$. The speed at the ground is $\sqrt{2gh}$, so $2J = m\sqrt{2gh}$ and solving for h,

$$h = \frac{1}{2g}\left(\frac{2J}{m}\right)^2 = 64\,\text{m},$$

but this is not recommended.

11.89: a) Two times as much, 0.36 mm, b) One-fourth (which is $(1/2)^2$) as much, 0.045 mm.c) The Young's modulus for copper is approximately one-half that for steel, so the wire would stretch about twice as much. $(0.18\,\text{mm})\frac{20\times10^{10}\text{ Pa}}{11\times10^{10}\text{ Pa}} = 0.33\,\text{mm}.$

11.90: Solving Eq. (11.14) for ΔV,

$$\begin{aligned}\Delta V &= -kV_0\Delta P = -kV_0\frac{mg}{A}\\ &= -(110\times10^{-11}\text{ Pa}^{-1})(250\text{ L})\frac{(1420\text{ kg})(9.80\text{ m/s}^2)}{\pi(0.150\text{ m})^2}\\ &= -0.0541\text{L}.\end{aligned}$$

The minus sign indicates that this is the volume by which the original hooch has shrunk, and is the extra volume that can be stored.

11.91: The normal component of the force is $F\cos\theta$ and the area (the intersection of the red plane and the bar in Figure (11.52)) is $A/\cos\theta$, so the normal stress is $(F/A)\cos^2\theta$.

b) The tangential component of the force is $F\sin\theta$, so the shear stress is $(\mathrm{F/A})\sin\theta\cos\theta$.

c) $\cos^2\theta$ is a maximum when $\cos\theta = 1$, or $\theta = 0$. d) The shear stress can be expressed as $(F/2A)\sin(2\theta)$, which is maximized when $\sin(2\theta) = 1$, or $\theta = \frac{90°}{2} = 45°$. Differentiation of the original expression with respect to θ and setting the derivative equal to zero gives the same result.

11.92: a) Taking torques about the pivot, the tension T in the cable is related to the weight by $T\sin\theta\, l_0 = mgl_0/2$, so $T = \frac{mg}{2\sin\theta}$. The horizontal component of the force that the cable exerts on the rod, and hence the horizontal component of the force that the pivot exerts on the rod, is $\frac{mg}{2}\cot\theta$ and the stress is $\frac{mg}{2A}\cot\theta$.

b)

$$\Delta l = \frac{l_0 F}{AY} = \frac{mgl_0\cot\theta}{2AY}.$$

c) In terms of the density and length, $(m/A) = \rho l_0$, so the stress is $(\rho l_0 g/2)\cot\theta$ and the change in length is $(\rho l_0^2 g/2Y)\cot\theta$. d) Using the numerical values, the stress is 1.4×10^5 Pa and the change in length is 2.2×10^{-6} m. e) The stress is proportional to the length and the change in length is proportional to the square of the length, and so the quantities change by factors of 2 and 4.

11.93: a) Taking torques about the left edge of the left leg, the bookcase would tip when $F = \frac{(1500\text{ N})(0.90\text{ m})}{(1.80\text{ m})} = 750\,\text{N}$, and would slip when $F = (\mu_s)(1500\,\text{N}) = 600\,\text{N}$, so the bookcase slides before tipping. b) If F is vertical, there will be no net horizontal force and the bookcase could not slide. Again taking torques about the left edge of the left leg, the force necessary to tip the case is $\frac{(1500\text{ N})(0.90\text{ m})}{(0.10\text{ m})} = 13.5\,\text{kN}$.

c) To slide, the friction force is $f = \mu_s(w + F\cos\theta)$, and setting this equal to $F\sin\theta$ and solving for F gives

$$F = \frac{\mu_s w}{\sin\theta - \mu_s\cos\theta}.$$

To tip, the condition is that the normal force exerted by the right leg is zero, and taking torques about the left edge of the left leg,
$F\sin\theta\,(1.80\,\text{m}) + F\cos\theta(0.10\,\text{m}) = w(0.90\,\text{m})$, and solving for F gives

$$F = \frac{w}{(1/9)\cos\theta + 2\sin\theta}.$$

Setting the expression equal gives

$$\mu_s((1/9)\cos\theta + 2\sin\theta) = \sin\theta - \mu_s\cos\theta,$$

and solving for θ gives

$$\theta = \arctan\left(\frac{(10/9)\mu_s}{(1\text{-}2\mu_s)}\right) = 66°.$$

11.94: a) Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $\frac{h}{2}$ and the lever arm of both the weight and the normal force is $h\tan\theta$, and so $F\frac{h}{2} = (n-w)h\tan\theta$. Taking torques about the upper point (where the rope is attached to the post), $fh = F\frac{h}{2}$. Using $f \le \mu_s n$ and solving for *F*,

$$F \le 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400\, nN)\left(\frac{1}{0.30} - \frac{1}{\tan 36.9°}\right)^{-1} = 400\, nN,$$

b) The above relations between F, n and f become

$$F\frac{3}{5}h = (n-w)h\tan\theta,\; f = \frac{2}{5}F,$$

and eliminating f and n and solving for F gives

$$F \le w\left(\frac{2/5}{\mu_s} - \frac{3/5}{\tan\theta}\right)^{-1},$$

and substitution of numerical values gives 750 N to two figures. c) If the force is applied a distance *y* above the ground, the above relations become

$$Fy = (n-w)h\tan\theta,\; F(h-y) = fh,$$

which become, on eliminating n and f,

$$w \ge F\left[\frac{\left(1-\frac{y}{h}\right)}{\mu_s} - \frac{\left(\frac{y}{h}\right)}{\tan\theta}\right].$$

As the term in square brackets approaches zero, the necessary force becomes unboundedly large. The limiting value of *y* is found by setting the term in square brackets equal to zero. Solving for *y* gives

$$\frac{y}{h} = \frac{\tan\theta}{\mu_s + \tan\theta} = \frac{\tan 36.9°}{0.30 + \tan 36.9°} = 0.71.$$

11.95: Assume that the center of gravity of the loaded girder is at $L/2$, and that the cable is attached a distance x to the right of the pivot. The sine of the angle between the lever arm and the cable is then $h/\sqrt{h^2+((L/2)-x)^2}$, and the tension is obtained from balancing torques about the pivot;

$$T\left[\frac{hx}{\sqrt{h^2+((L/2)-x)^2}}\right]=wL/2,$$

where w is the total load (the exact value of w and the position of the center of gravity do not matter for the purposes of this problem). The minimum tension will occur when the term in square brackets is a maximum; differentiating and setting the derviative equal to zero gives a maximum, and hence a minimum tension, at $x_{\min}=(h^2/L)+(L/2)$. However, if $x_{\min}>L$, which occurs if $h>L/\sqrt{2}$, the cable must be attached at L, the furthest point to the right.

11.96: The geometry of the 3-4-5 right triangle simplifies some of the intermediate algebra. Denote the forces on the ends of the ladders by F_L and F_R (left and right). The contact forces at the ground will be vertical, since the floor is assumed to be frictionless. a) Taking torques about the right end, $F_L(5.00\,\text{m})=(480\,\text{N})(3.40\,\text{m})+(360\,\text{N})(0.90\,\text{m})$, so $F_L=391\,\text{N}$. F_R may be found in a similar manner, or from $F_R=840\,\text{N}-F_L=449\,\text{N}$. b) The tension in the rope may be found by finding the torque on each ladder, using the point A as the origin. The lever arm of the rope is 1.50 m. For the left ladder, $T(1.50\,\text{m})=F_L(3.20\,\text{m})-(480\,\text{N})(1.60\,\text{m})$, so $T=322.1\,\text{N}$ (322 N to three figures). As a check, using the torques on the right ladder, $T(1.50\,\text{m})=F_R(1.80\,\text{m})-(360\,\text{N})(0.90\,\text{m})$ gives the same result. c) The horizontal component of the force at A must be equal to the tension found in part (b). The vertical force must be equal in magnitude to the difference between the weight of each ladder and the force on the bottom of each ladder, 480 N–391 N = 449 N–360 N = 89 N. The magnitude of the force at A is then

$$\sqrt{(322.1\text{ N})^2+(89\,\text{N})^2}=334\,\text{N}.$$

d) The easiest way to do this is to see that the added load will be distributed at the floor in such a way that $F_L'=F_L+(0.36)(800\,\text{N})=679\,\text{N}$, and $F_R'=F_R+(0.64)(800\,\text{N})=961\,\text{N}$. Using these forces in the form for the tension found in part (b) gives

$$T=\frac{F_L'(3.20\,\text{m})-(480\,\text{N})(1.60\,\text{m})}{(1.50\,\text{m})}=\frac{F_R'(1.80\,\text{m})-(360\,\text{N})(0.90\,\text{m})}{(1.50\,\text{m})}=936.53\,\text{N},$$

which is 937 N to three figures.

11.97: The change in the volume of the oil is $= k_O v_O \Delta p$ and the change in the volume of the sodium is $= k_s v_s \Delta p$. Setting the total volume change equal to Ax (x is positive) and using $\Delta p = F/A$,

$$Ax = (k_O V_O + k_s V_s)(F/A),$$

and solving for k_s gives

$$k_s = \left(\frac{A^2 x}{F} - k_O V_O\right)\frac{1}{V_s}.$$

11.98: a) For constant temperature $(\Delta T = 0)$,

$$\Delta(pV) = (\Delta p)V + p(\Delta V) = 0 \quad \text{and} \quad B = -\frac{(\Delta p)V}{(\Delta V)} = p.$$

b) In this situation,

$$(\Delta p)V^{\gamma} + \gamma p(\Delta V)V^{\gamma-1} = 0, \quad (\Delta p) + \gamma p\frac{\Delta V}{V} = 0,$$

and

$$B = -\frac{(\Delta p)V}{\Delta V} = \gamma p.$$

11.99: a) From Eq.(11.10), $\Delta l = \frac{(4.50\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m})}{(20\times10^{10}\ \text{Pa})(5.00\times10^{-7}\ \text{m}^2)} = 6.62\times10^{-4}\ \text{m}$, or $0.66\ \text{mm}$ to two figures. b) $(4.50\ \text{kg})(9.80\ \text{m/s}^2)(0.0500\times10^{-2}\ \text{m}) = 0.022\ \text{J}$. c) The magnitude F will be vary with distance; the average force is $Y\,A(0.0250\ \text{cm}/l_0) = 16.7\ \text{N}$, and so the work done by the applied force is $(16.7\ \text{N})(0.0500\times10^{-2}\ \text{m}) = 8.35\times10^{-3}\ \text{J}$. d) The wire is initially stretched a distance $6.62\times10^{-4}\ \text{m}$ (the result of part (a)), and so the average elongation during the additional stretching is $9.12\times10^{-4}\ \text{m}$, and the average force the wire exerts is $60.8\ \text{N}$. The work done is negative, and equal to $-(60.8\ \text{N})(0.0500\times10^{-2}\ \text{m}) = -3.04\times10^{-2}\ \text{J}$. e) See problem 11.82. The change in elastic potential energy is

$$\frac{(20\times10^{10}\ \text{Pa})(5.00\times10^{-7}\ \text{m}^2)}{2(1.50\ \text{m})}((11.62\times10^{-4}\ \text{m}^2) - (6.62\times10^{-4}\ \text{m})^2) = 3.04\times10^{-2}\ \text{J},$$

the negative of the result of part (d). (If more figures are kept in the intermediate calculations, the agreement is exact.)

Note: to obtain the numerical results given in this chapter, the following numerical values of certain physical quantities have been used;

$$G = 6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2},\ g = 9.80\ \mathrm{m/s^2}\ \text{and}\ m_\mathrm{E} = 5.97\times10^{24}\ \mathrm{kg}.$$

Use of other tabulated values for these quantities may result in an answer that differs in the third significant figure.

12.1: The ratio will be the product of the ratio of the mass of the sun to the mass of the earth and the square of the ratio of the earth-moon radius to the sun-moon radius. Using the earth-sun radius as an average for the sun-moon radius, the ratio of the forces is

$$\left(\frac{3.84\times10^{8}\ \mathrm{m}}{1.50\times10^{11}\ \mathrm{m}}\right)^2\left(\frac{1.99\times10^{30}\ \mathrm{kg}}{5.97\times10^{24}\ \mathrm{kg}}\right) = 2.18.$$

12.2: Use of Eq. (12.1) gives

$$F_\mathrm{g} = G\frac{m_1m_2}{r^2} = (6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})\frac{(5.97\times10^{24}\ \mathrm{kg})(2150\ \mathrm{kg})}{(7.8\times10^5\ \mathrm{m}+6.38\times10^6\ \mathrm{m})^2} = 1.67\times10^4\ \mathrm{N}.$$

The ratio of this force to the satellite's weight at the surface of the earth is

$$\frac{(1.67\times10^4\ \mathrm{N})}{(2150\ \mathrm{kg})(9.80\ \mathrm{m/s^2})} = 0.79 = 79\%.$$

(This numerical result requires keeping one extra significant figure in the intermediate calculation.) The ratio, which is independent of the satellite mass, can be obtained directly as

$$\frac{Gm_\mathrm{E}\,m/r^2}{mg} = \frac{Gm_\mathrm{E}}{r^2g} = \left(\frac{R_\mathrm{E}}{r}\right)^2,$$

yielding the same result.

12.3:
$$G\frac{(nm_1)(nm_2)}{(nr_{12})^2} = G\frac{m_1m_2}{r_{12}^2} = F_{12}.$$

12.4: The separation of the centers of the spheres is $2R$, so the magnitude of the gravitational attraction is $GM^2/(2R)^2 = GM^2/4R^2$.

12.5: a) Denoting the earth-sun separation as R and the distance from the earth as x, the distance for which the forces balance is obtained from

$$\frac{GM_{\mathrm{S}}m}{(R-x)^2} = \frac{GM_{\mathrm{E}}m}{x^2},$$

which is solved for

$$x = \frac{R}{1+\sqrt{\frac{M_{\mathrm{S}}}{M_{\mathrm{E}}}}} = 2.59\times 10^{8}\ \mathrm{m}.$$

b) The ship could not be at equilibrium for long, in that the point where the forces balance is moving in a circle, and to move in that circle requires some force. The spaceship could continue toward the sun with a good navigator on board.

12.6: a) Taking force components to be positive to the right, use of Eq. (12.1) twice gives

$$F_g = \left(6.673\times 10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)(0.100\ \mathrm{kg})\left[-\frac{(5.00\ \mathrm{kg})}{(0.400\ \mathrm{m})^2}+\frac{(10.0\ \mathrm{kg})}{(0.600\ \mathrm{m})^2}\right],$$
$$= -2.32\times 10^{-11}\ \mathrm{N}$$

with the minus sign indicating a net force to the left.

b) No, the force found in part (a) is the *net* force due to the other two spheres.

12.7: $$\left(6.673\times 10^{-11}\ \mathrm{N.m^2/kg^2}\right)\frac{(70\mathrm{kg})\left(7.35\times 10^{22}\ \mathrm{kg}\right)}{\left(3.78\times 10^{8}\ \mathrm{m}\right)^2} = 2.4\times 10^{-3}\mathrm{N}.$$

12.8: $$\frac{(333{,}000)}{(23{,}500)^2} = 6.03\times 10^{-4}$$

12.9: Denote the earth-sun separation as r_1 and the earth-moon separation as r_2.

a) $$(Gm_{\mathrm{M}})\left[\frac{m_{\mathrm{S}}}{(r_1+r_2)^2}+\frac{m_{\mathrm{E}}}{r_2^2}\right]=6.30\times10^{20}\,\mathrm{N},$$

toward the sun. b)The earth-moon distance is sufficiently small compared to the earth-sun distance ($r_2 << r_2$) that the vector from the earth to the moon can be taken to be perpendicular to the vector from the sun to the moon. The components of the gravitational force are then

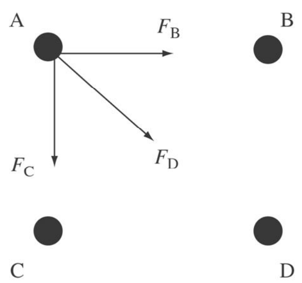
$$\frac{Gm_{\mathrm{M}}m_{\mathrm{S}}}{r_1^2}=4.34\times10^{20}\,\mathrm{N},\ \frac{Gm_{\mathrm{M}}m_{\mathrm{E}}}{r_2^2}=1.99\times10^{20}\,\mathrm{N},$$

and so the force has magnitude $4.77\times10^{20}\,\mathrm{N}$ and is directed $24.6°$ from the direction toward the sun.

c) $$(Gm_{\mathrm{M}})\left[\frac{m_{\mathrm{S}}}{(\mathrm{r}_1-r_2)^2}-\frac{m_{\mathrm{E}}}{r_2^2}\right]=2.37\times10^{20}\,\mathrm{N},$$

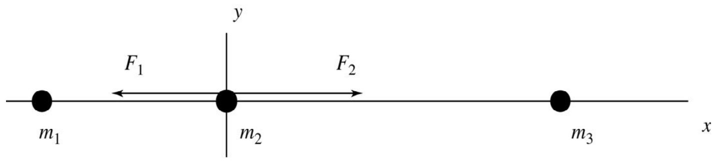
toward the sun.

12.10:



$$\begin{aligned}F_{\mathrm{onA}}&=2F_{\mathrm{B}}\cos45°+\mathrm{F_D}\\&=2\frac{Gm_{\mathrm{A}}m_{\mathrm{B}}\cos45°}{r_{\mathrm{AB}}^2}+\frac{Gm_{\mathrm{A}}m_{\mathrm{D}}}{r_{\mathrm{AD}}^2}\\&=\frac{2(6.67\times10^{-11}\,\mathrm{Nm^2/kg^2})(800\,\mathrm{kg})^2\cos45°}{(0.10\,\mathrm{m})^2}\\&\qquad+\frac{(6.67\times10^{-11}\,\mathrm{Nm^2/kg^2})\,(800\,\mathrm{kg})^2}{(0.10\,\mathrm{m})^2}\\&=8.2\times10^{-3}\,\mathrm{N},\ \text{toward the center of the square}\end{aligned}$$

12.11:



$$m_1 = m_2 = m_3 = 500\,\text{kg}$$
$$r_{12} = 0.10\,\text{m}; r_{23} = 0.40\,\text{m}$$
$$F_1 = G\frac{m_1m_2}{r_{12}^2} = 1.668\times10^{-3}\,\text{N}$$
$$F_3 = G\frac{m_2m_3}{r_{23}^2} = 1.043\times10^{-4}\,\text{N}$$
$$F = F_1 - F_3 = 1.6\times10^{-3}\,\text{N, to the left}$$

12.12: The direction of the force will be toward the larger mass, and the magnitude will be

$$\frac{Gm_2m}{(d/2)^2} - \frac{Gm_1m}{(d/2)^2} = \frac{4Gm(m_2 - m_1)}{d^2}.$$

12.13: For convenience of calculation, recognize that the mass of the small sphere will cancel. The acceleration is then

$$\frac{2G(0.260\,\text{kg})}{(10.0\times10^{-2}\,\text{m})^2}\times\frac{6.0}{10.0} = 2.1\times10^{-9}\,\text{m/s}^2,$$

directed down.

12.14: Equation (12.4) gives

$$g = \frac{\left(6.763\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.5\times10^{22}\,\text{kg}\right)}{\left(1.15\times10^6\,\text{m}\right)^2} = 0.757\,\text{m/s}^2.$$

12.15: To decrease the acceleration due to gravity by one-tenth, the distance from the earth must be increased by a factor of $\sqrt{10}$, and so the distance above the surface of the earth is

$$\left(\sqrt{10}-1\right)\mathrm{R}_{\mathrm{E}} = 1.38\times 10^{7}\,\mathrm{m}.$$

12.16: a) Using $\mathrm{g_E} = 9.80\,\mathrm{m/s^2}$, Eq(12.4) gives

$$\begin{aligned} \mathrm{g_v} &= \frac{Gm_\mathrm{v}}{R_\mathrm{v}^2} = G\left(\frac{m_\mathrm{v}}{m_\mathrm{E}}\right)\mathrm{m_E}\left(\frac{R_\mathrm{E}}{R_\mathrm{v}}\right)^2\left(\frac{1}{R_\mathrm{E}^2}\right) \\ &= \frac{Gm_\mathrm{E}}{R_\mathrm{E}^2}\left(\frac{m_\mathrm{v}}{m_\mathrm{E}}\right)\left(\frac{R_\mathrm{E}}{R_\mathrm{v}}\right)^2 = g_\mathrm{E}(.815)\left(\frac{1}{.949}\right)^2 \\ &= (9.80\,\mathrm{m/s^2})(.905) \\ &= 8.87\,\mathrm{m/s^2}, \end{aligned}$$

where the subscripts v refer to the quantities pertinent to Venus. b) $(8.87\,\mathrm{m/s^2})\,(5.00\,\mathrm{kg}) = 44.3\mathrm{N}$.

12.17: a) See Exercise 12.16;

$$g_{\text{Titania}} = (9.80\,\mathrm{m/s^2})\left(\frac{(8)^2}{1700}\right) = 0.369\,\mathrm{m/s^2}.$$

b) $\frac{\rho_\mathrm{T}}{\rho_\mathrm{E}} = \frac{m_\mathrm{T}}{m_\mathrm{E}}.\frac{r_\mathrm{E}^3}{r_\mathrm{T}^3}$, or rearranging and solving for density, $\rho_T = \rho_\mathrm{E}.\frac{(1/1700)\,m_E}{m_\mathrm{E}}.\frac{r_\mathrm{E}^3}{(1/8 r_\mathrm{E})^3} =$ $(5500\,\mathrm{kg/m^3})\left(\frac{512}{1700}\right) = 1656\,\mathrm{kg/m^3}$, or about 0.39ρE.

12.18: $M = \frac{gR^2}{G} = 2.44\times 10^{21}\,\mathrm{kg}$ and $\rho = \frac{M}{(4\pi/3)R^3} = 1.30\times 10^{3}\,\mathrm{Kg/m^3}$.

12.19: $F = G\dfrac{mm_{\text{E}}}{r^2}$

$r = 600\times10^3\,\text{m} + R_{\text{E}}$ so $F = 610\,\text{N}$

At the surface of the earth, $w = mg = 735\,\text{N}$.

The gravity force is not zero in orbit. The satellite and the astronaut have the same acceleration so the astronaut's apparent weight is zero.

12.20: Get g on the neutron star

$$mg_{\text{ns}} = \frac{GmM_{\text{ns}}}{R^2}$$

$$g_{\text{ns}} = \frac{GM_{\text{ns}}}{R^2}$$

Your weight would be

$$w_{\text{ns}} = mg_{\text{ns}} = \frac{mGM_{\text{ns}}}{R^2}$$

$$= \left(\frac{675\text{N}}{9.8\,\text{m/s}^2}\right)\frac{(6.67\times10^{11}\ \text{Nm}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(10^4\,\text{m})^2}$$

$$= 9.1\times10^{13}\,\text{N}$$

12.21: From eq. (12.1), $\text{G} = Fr^2/m_1m_2$, and from Eq. (12.4), $g = Gm_E/R_{\text{E}}^2$; combining and solving for R_{E},

$$m_{\text{E}} = \frac{gm_1m_2R_{\text{E}}^2}{Fr^2} = 5.98\times10^{24}\ \text{kg}.$$

12.22: a) From Example 12.4 the mass of the lander is 4000 kg. Assuming Phobos to be spherical, its mass in terms of its density ρ and radius R is $(4\pi/3)\rho R^3$, and so the gravitational force is

$$\frac{G(4\pi/3)(4000\,\text{kg})\rho R^3}{R^2} = G(4\pi/3)(4000\,\text{kg})(2000\,\text{kg/m}^3)(12\times10^3\ \text{m}) = 27\ \text{N}.$$

b) The force calculated in part (a) is much less than the force exerted by Mars in Example 12.4.

12.23: $$\sqrt{2GM/R} = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.6\times10^{12}\ \text{kg})/(700\,\text{m})}$$
$$= 0.83\,\text{m/s}.$$

One could certainly walk that fast.

12.24: a) $F = Gm_E m/r^2$ and $|U| = Gm_\text{E}m/r$, so the altitude above the surface of the earth is $\frac{|U|}{F} - R_\text{E} = 9.36\times10^5$ m. b) Either of Eq. (12.1) or Eq. (12.9) can be used with the result of part (a) to find *m*, or noting that $U^2 = G^2 M_E{}^2 m^2/r^2$, $m = U^2/FGM_\text{E}$ $= 2.55\times10^3$ kg.

12.25: The escape speed, from the results of Example 12.5, is $\sqrt{2GM/R}$.

a) $\sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42\times10^{23}\ \text{kg})/(3.40\times10^6\ \text{m})} = 5.02\times10^3\ \text{m/s}$.

b) $\sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.90\times10^{27}\ \text{kg})/(6.91\times10^7\ \text{m})} = 6.06\times10^4\ \text{m/s}$.

c) Both the kinetic energy and the gravitational potential energy are proportional to the mass.

12.26: a) The kinetic energy is $K=\frac{1}{2}mv^2$, or $K=\frac{1}{2}(629\,\text{kg})(3.33\times10^3\,\text{m/s})^2$, or $KE=3.49\times10^9$ J.

b) $$U=-\frac{\text{GM}m}{r}=\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(629\,\text{kg})}{2.87\times10^9\ \text{m}},$$

or $U=-8.73\times10^7$ J.

12.27: a) Eliminating the orbit radius r between Equations (12.12) and (12.14) gives

$$T=\frac{2\pi Gm_E}{v^3}=\frac{2\pi\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\,\text{kg}\right)}{\left(6200\,\text{m/s}\right)^3}$$
$$=1.05\times10^4\,s=175\,\text{min}.$$

b) $$\frac{2\pi v}{T}=3.71\,\text{m/s}^2.$$

12.28: Substitution into Eq. (12.14) gives $T=6.96\times10^3$ s, or 116 minutes.

12.29: Using Eq. (12.12),

$$v=\sqrt{\frac{\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)}{\left(6.38\times10^6\ \text{m}+7.80\times10^5\ \text{m}\right)}}=7.46\times10^3\ \text{m/s}.$$

12.30: Applying Newton's second law to the Earth

$$\sum F = ma;$$

$$\frac{Gm_E m_s}{r^2} = m_E \frac{v^2}{r}$$

$$m_s = \frac{rv^2}{G} \text{ and } v = \frac{2\pi r}{T_{\text{Earth}}}$$

$$m_s = \frac{r\left(\frac{2\pi r}{T_E}\right)^2}{G} = \frac{4\pi^2 r^3}{GT_{\text{E}}^2}$$

$$= \frac{4\pi^2 (1.50\times 10^{11}\ \text{m})^3}{(6.67\times 10^{-11}\ \text{Nm}^2/\text{kg}^2)\,[(365.3d)(\frac{8.64\times 10^4 s}{d})]^{\ 2}}$$

$$= 2.01\times 10^{30}\ \text{kg}$$

12.31: $\sum F = ma_{\text{c}}$ for the baseball.

The net force is the gravity force exerted on the baseball by Deimos, so

$$G\frac{mm_D}{R_D^2} = m\frac{v^2}{R_D}$$

$$v = \sqrt{Gm_D/R_D} = \sqrt{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times 10^{15}\ \text{kg})/(6.0\times 10^3\ \text{m})} = 4.7\,\text{m/s}$$

A world-class sprinter runs 100 m in 10 s so have $v = 10\,\text{m/s}; v = 4.7\,\text{m/s}$ for a thrown baseball is very achieveable.

12.32: Apply Newton's second law to Vulcan.

$$\sum F = ma : \frac{Gm_s m_v}{r^2} = m_v \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$\frac{Gm_s}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_s}}$$

$$= \sqrt{\frac{4\pi^2\left[\frac{2}{3}(5.79\times 10^{10}\,\text{m})\right]^3}{(6.67\times 10^{-11}\,\text{Nm}^2/\text{kg}^2)(1.99\times 10^{30}\,\text{kg})}}$$

$$= 4.14\times 10^6\,\text{s}\left(\frac{1\text{d}}{86{,}400\,\text{s}}\right) = 47.9\text{ days}$$

12.33: a)

$$v = \sqrt{Gm/r}$$

$$= \sqrt{(6.673\times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(0.85\times 1.99\times 10^{30}\,\text{kg})/((1.50\times 10^{11}\,\text{m})(0.11))}$$

$$= 8.27\times 10^4\ \text{m/s}.$$

b) $2\pi r/v = 1.25\times 10^6\,\text{s}$ (about two weeks).

12.34: From either Eq. (12.14) or Eq. (12.19),

$$m_S = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(1.08\times 10^{11}\,\text{m})^3}{(6.673\times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)\,((224.7\,\text{d})(8.64\times 10^4\,\text{s/d}))^2}$$

$$= 1.98\times 10^{30}\,\text{kg}.$$

12.35: a) The result follows directly from Fig. 12.18. b) $(1-0.248)(5.92\times 10^{12}\,\text{m})$ $= 4.45\times 10^{12}\,\text{m}$, $(1+0.010)(4.50\times 10^{12}\,\text{m}) = 4.55\times 10^{12}\,\text{m}$. c) $T = 248\,\text{y}$.

12.36: a) $$r=\sqrt{\frac{Gm_1m_2}{F}}=7.07\times10^{10}\,\mathrm{m}.$$

b) From Eq. (12.19), using the result of part (a),
$$T=\frac{2\pi(7.07\times10^{10}\,\mathrm{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})(1.90\times10^{30}\,\mathrm{kg})}}=1.05\times10^{7}\,\mathrm{s}=121\,\mathrm{days}.$$

c) From Eq. (12.14) the radius is $(8)^{2/3}=$ four times that of the large planet's orbit, or 2.83×10^{11} m.

12.37: a) For a circular orbit, Eq. (12.12) predicts a speed of

$$\sqrt{(6.673\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\,\mathrm{kg})/(43\times10^{9}\,\mathrm{m})}=56\,\mathrm{km/s}.$$

The speed doesn't have this value, so the orbit is not circular. b) The escape speed for any object at this radius is $\sqrt{2}(56\,\mathrm{km/s})=79\,\mathrm{km/s}$, so the spacecraft must be in a closed elliptical orbit.

12.38: a) Divide the rod into differential masses dm at position l, measured from the right end of the rod. Then , $dm = dl \ (M/L)$, and

$$dU = -\frac{Gm\,dm}{l+x} = -\frac{GmM}{L}\frac{dl}{l+x}.$$

Integrating,

$$U = -\frac{GmM}{L}\int_O^L \frac{dl}{l+x} = -\frac{GmM}{L}\ln\left(1+\frac{L}{x}\right).$$

For x >> L, the natural logarithm is $\sim (L/x)$, and $U \to -Gm\,M/x$. b) The x-component of the gravitational force on the sphere is

$$F_x = -\frac{\delta U}{\delta x} = \frac{GmM}{L}\frac{(-L/x^2)}{(1+(L/x))} = -\frac{GmM}{(x^2+Lx)},$$

with the minus sign indicating an attractive force. As x >> L, the denominator in the above expression approaches x^2, and $F_x \to Gm\,M/x^2$, as expected. The derivative may also be taken by expressing

$$\ln\left(1+\frac{L}{x}\right) = \ln(x+L) - \ln x$$

at the cost of a little more algebra.

12.39: a) Refer to the derivation of Eq. (12.26) and Fig. (12.22). In this case, the red ring in Fig. (12.22) has mass M and the common distance s is $\sqrt{x^2 + a^2}$. Then, $U = -GMm/\sqrt{x^2 + a^2}$. b) When $x >> a$, the term in the square root approaches x^2 and $U \to -GMm/x$, as expected.

c)
$$F_x = -\frac{\delta U}{\delta x} = -\frac{GMmx}{(x^2 + a^2)^{3/2,}},$$

with the minus sign indicating an attractive force. d) when $x >> \text{a}$, the term inside the parentheses in the above expression approaches x^2 and $F_x \to -GMmx/(x^2)^{3/2}$ $= -GMm/x^2$, as expected. e) The result of part (a) indicates that $U = \frac{-GMm}{a}$ when $x = 0.$ This makes sense because the mass at the center is a constant distance a from the mass in the ring. The result of part (c) indicates that $F_x = 0$ when $x = 0.$ At the center of the ring, all mass elements that comprise the ring attract the particle toward the respective parts of the ring, and the net force is zero.

12.40: At the equator, the gravitational field and the radial acceleration are parallel, and taking the magnitude of the weight as given in Eq. (12.30) gives

$$w = mg_0 - ma_{\text{rad}}.$$

The difference between the measured weight and the force of gravitational attraction is the term ma_{rad}. The mass m is found by solving the first relation for m, $m = \frac{\omega}{g_0 - a_{rad}}$. Then,

$$ma_{\text{rad}} = w\frac{a_{\text{rad}}}{g_0 - a_{\text{rad}}} = \frac{w}{(g_0/a_{\text{rad}}) - 1}.$$

Using either $g_0 = 9.80\ \text{m/s}^2$ or calculating g_0 from Eq. (12.4) gives $ma_{\text{rad}} = 2.40\ \text{N}.$

12.41: a) $Gm_{\text{N}}m/R^2 = (10.7\ \text{m/s}^2)(5.00\ \text{kg}) = 53.5\ \text{N}$, or $54\ \text{N}$ to two figures.

b) $m(g_0 - a_{\text{rad}}) = (5.00\ \text{kg})\left(10.7\ \ \text{m/s}^2 - \frac{4\pi^2(2.5\times10^7\ \text{m})}{[(16\ \text{h})(3600\ \text{s/h})]^2}\right) = 52.0\ \text{N}.$

12.42: a) $$\frac{GMm}{r^2} = \frac{(R_S c^2/2)}{r^2} = \frac{mc^2 R_S}{2r^2}.$$

b) $$\frac{(5.00\,\text{kg})(3.00\times10^8\ \text{m/s})^2(1.4\times10^{-2}\,\text{m})}{2(3.00\times10^6\ \text{m})^2} = 350\,\text{N}.$$

c) Solving Eq. (12.32) for M,

$$M = \frac{R_S c^2}{2G} = \frac{(14.00\times10^{-3}\ \text{m})(3.00\times10^8\ \text{m/s})^2}{2(6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)} = 9.44\times10^{24}\,\text{kg}.$$

12.43: a) From Eq. (12.12),

$$M = \frac{Rv^2}{G} = \frac{(7.5\,\text{ly})(9.461\times10^{15}\ \text{m/ly})(200\times10^3\,\text{m/s})^2}{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}$$
$$= 4.3\times10^{37}\ \text{kg} = 2.1\times10^7\ M_S.$$

b) It would seem not.

c) $$R_S = \frac{2GM}{c^2} = \frac{2v^2R}{c^2} = 6.32\times10^{10}\ \text{m},$$

which does fit.

12.44: Using the mass of the sun for M in Eq. (12.32) gives

$$R_S = \frac{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(3.00\times10^8\,\text{m/s})^2} = 2.95\,\text{km}.$$

That is, Eq. (12.32) may be rewritten
$$R_S = \frac{2Gm_{\text{sun}}}{c^2}\left(\frac{M}{m_{\text{sun}}}\right) = 2.95\,\text{km}\times\left(\frac{\text{M}}{m_{\text{sun}}}\right).$$

Using 3.0 km instead of 2.95 km is accurate to 1.7%.

12.45: $\dfrac{R_S}{R_E} = \dfrac{2\left(6.67\times10^{-11}\ \text{N m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)}{\left(3\times10^8\ \text{m/s}\right)^2\left(6.38\times10^6\ \text{m}\right)} = 1.4\times10^{-9}.$

12.46: a) From symmetry, the net gravitational force will be in the direction $45°$ from the $+x$-axis (bisecting the x and y axes), with magnitude

$$(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.0150\,\text{kg})\left[\frac{(2.0\,\text{kg})}{(2(0.50\,\text{m})^2)} + 2\frac{(1.0\,\text{kg})}{(0.50\,\text{m})^2}\sin 45°\right]$$
$$= 9.67\times10^{-12}\ \text{N}.$$

b) The initial displacement is so large that the initial potential may be taken to be zero. From the work-energy theorem,

$$\frac{1}{2}mv^2 = Gm\left[\frac{(2.0\,\text{kg})}{\sqrt{2}\,(0.50\,\text{m})} + 2\frac{(1.0\,\text{kg})}{(0.50\,\text{m})}\right].$$

Canceling the factor of m and solving for v, and using the numerical values gives $\upsilon = 3.02\times10^{-5}\ \text{m/s}.$

12.47: The geometry of the 3-4-5 triangle is available to simplify some of the algebra, The components of the gravitational force are

$$F_y = \frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.500\,\text{kg})(80.0\,\text{kg})}{(5.000\,\text{m})^2}\frac{3}{5}$$

$$= 6.406\times10^{-11}\ \text{N}$$

$$F_x = -(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.500\,\text{kg})\left[\frac{(60.0\,\text{kg})}{(4.000\,\text{m})^2} + \frac{(80.0\,\text{kg})}{(5.000\,\text{m})^2}\frac{4}{5}\right]$$

$$= -2.105\times10^{-10}\ \text{N},$$

so the magnitude is $2.20\times10^{-10}\ \text{N}$ and the direction of the net gravitational force is $163°$ counterclockwise from the $+x$-axis. b) A at x = 0, y = 1.39 m.

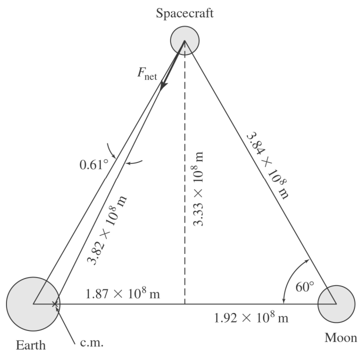
12.48: a) The direction from the origin to the point midway between the two large masses is arctan $\left(\frac{0.100\ \text{m}}{0.200\ \text{m}}\right) = 26.6°$, which is not the angle $(14.6°)$ found in the example.

b) The common lever arm is 0.100 m, and the force on the upper mass is at an angle of $45°$ from the lever arm. The net torque is

$$(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.0100\ \text{kg})(0.500\ \text{kg})\left[\frac{(0.100\ \text{m})\sin 45°}{2(0.200\ \text{m})^2}-\frac{(0.100\ \text{m})}{(0.200\ \text{m})^2}\right]$$
$$=-5.39\times10^{-13}\ \text{N}\cdot\text{m},$$

with the minus sign indicating a clockwise torque. c) There can be no net torque due to gravitational fields with respect to the center of gravity, and so the center of gravity in this case is not at the center of mass.

12.49: a) The simplest way to approach this problem is to find the force between the spacecraft and the center of mass of the earth-moon system, which is 4.67×10^{6} m from the center of the earth.



The distance from the spacecraft to the center of mass of the earth-moon system is 3.82×10^{8} m. Using the Law of Gravitation, the force on the spacecraft is 3.4 N, an angle of $0.61°$ from the earth-spacecraft line. This equilateral triangle arrangement of the earth, moon and spacecraft is a solution of the Lagrange Circular Restricted Three-Body Problem. The spacecraft is at one of the earth-moon system Lagrange points. The Trojan asteriods are found at the corresponding Jovian Lagrange points.

b) The work is $W=-\frac{GMm}{r}=-\frac{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg}+7.35\times10^{22}\ \text{kg})(1250\ \text{kg})}{3.84\times10^{8}\ \text{m}}$, or

$W=-1.31\times10^{9}$ J.

12.50: Denote the 25-kg sphere by a subscript 1 and the 100-kg sphere by a subscript 2. a) Linear momentum is conserved because we are ignoring all other forces, that is, the net external force on the system is zero. Hence, $m_1 v_1 = m_2 v_2.$ This relationship is useful in solving part (b) of this problem. b)From the work-energy theorem,

$$Gm_1 m_2 \left[\frac{1}{r_\mathrm{f}} - \frac{1}{r_\mathrm{i}} \right] = \frac{1}{2}\left(m_1 m_1^2 + m_2 v_2^2 \right)$$

and from conservation of momentum the speeds are related by $m_1 v_1 = m_2 v_2$. Using the conservation of momentum relation to eliminate v_2 in favor of v_1 and simplifying yields

$$v_1^2 = \frac{2Gm_2^2}{m_1 + m_2}\left[\frac{1}{r_\mathrm{f}} - \frac{1}{r_\mathrm{i}} \right],$$

with a similar expression for v_2. Substitution of numerical values gives $v_1 = 1.63\times 10^{-5}\ \mathrm{m/s}, v_2 = 4.08\times 10^{-6}\ \mathrm{m/s}.$ The magnitude of the relative velocity is the sum of the speeds, $2.04\times 10^{-5}\ \mathrm{m/s}.$

c) The distance the centers of the spheres travel $\left(x_1 \text{ and } x_2\right)$ is proportional to their acceleration, and $\frac{x_1}{x_2} = \frac{a_1}{a_2} = \frac{m_2}{m_1}$, or $x_1 = 4x_2$. When the spheres finally make contact, their centers will be a distance of $2R$ apart, or $x_1 + x_2 + 2R = 40\ \mathrm{m}$, or $x_2 + 4x_2 + 2R = 40\ \mathrm{m}$. Thus, $x_2 = 8\ \mathrm{m} - 0.4R$, and $x_1 = 32\ \mathrm{m} - 1.6R$.

12.51: Solving Eq. (12.14) for *r*,

$$\begin{aligned} R^3 &= Gm_\mathrm{E}\left(\frac{T}{2\pi}\right)^2 \\ &= (6.673\times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.97\times 10^{24}\ \mathrm{kg})\left(\frac{(27.3\ \mathrm{d})(86{,}400\ \mathrm{s/d})}{2\pi}\right)^2 \\ &= 5.614\times 10^{25}\ \mathrm{m}^3, \end{aligned}$$

from which $r = 3.83\times 10^8\ \mathrm{m}$.

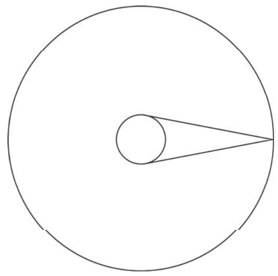
12.52: $g = |\vec{g}| = \frac{(6.673\times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})(20.0\ \mathrm{kg})}{(1.50\ \mathrm{m})^2} = 5.93\times 10^{-10}\ \mathrm{N/kg}$, directed toward the center of the sphere.

12.53: a) From Eq. (12.14),

$$r^3 = Gm_E\left(\frac{T}{2\pi}\right)^2 = (6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})\,(5.97\times10^{24}\ \mathrm{kg})\left(\frac{86{,}164\,\mathrm{s}}{2\pi}\right)^2$$

$$= 7.492\times10^{22}\ \mathrm{m^3},$$

and so $h = r - R_E = 3.58\times10^7\ \mathrm{m}.$ Note that the period to use for the earth's rotation is the siderial day, not the solar day (see Section 12.7). b) For these observers, the satellite is below the horizon.



12.54: Equation 12.14 in the text will give us the planet's mass:

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM_P}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_P}$$

$$M_P = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(5.75\times10^5\ \mathrm{m} + 4.80\times10^6\ \mathrm{m})^3}{(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.8\times10^3\ \mathrm{s})^2}$$

$$= 2.731\times10^{24}\ \mathrm{kg}\text{, or about half earth's mass.}$$

Now we can find the astronaut's weight on the surface (The landing on the north pole removes any need to account for centripetal acceleration):

$$w = \frac{GM_p m_a}{r_p^2} = \frac{(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(2.731\times10^{24}\ \mathrm{kg})(85.6\ \mathrm{kg})}{(4.80\times10^6\ \mathrm{m})^2}$$

$$= 677\ \mathrm{N}$$

12.55: In terms of the density ρ, the ratio M/R is $(4\pi/3)\rho R^2$, and so the escape speed is

$$v = \sqrt{(8\pi/3)(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(2500\ \mathrm{kg/m^3})(150\times10^3\ \mathrm{m})^2} = 177\ \mathrm{m/s}.$$

12.56: a) Following the hint, use as the escape velocity $v = \sqrt{2gh,}$ where h is the height one can jump from the surface of the earth. Equating this to the expression for the escape speed found in Problem 12.55,

$$2gh = \frac{8\pi}{3}\rho G R^2, \text{ or } R^2 = \frac{3}{4\pi}\frac{gh}{\rho G},$$

where $g = 9.80\ \text{m/s}^2$ is for the surface of the earth, not the asteroid. Using $h = 1\ \text{m}$ (variable for different people, of course), $R = 3.7\ \text{km}$. As an alternative, if one's jump speed is known, the analysis of Problem 12.55 shows that for the same density, the escape speed is proportional to the radius, and one's jump speed as a fraction of $60\ \text{m/s}$ gives the largest radius as a fraction of $50\ \text{km}$. b) With $a = v^2/R,\ \rho = \frac{3a}{4\pi GR} = 3.03\times10^3\ \text{kg/m}^3$.

12.57: a) The satellite is revolving west to east, in the same direction the earth is rotating. If the angular speed of the satellite is ω_s and the angular speed of the earth is ω_E, the angular speed ω_{rel} of the satellite relative to you is $\omega_{\text{rel}} = \omega_s - \omega_E$.

$$\omega_{\text{rel}} = (1\ \text{rev})/(12\ \text{h}) = \left(\tfrac{1}{12}\right)\text{rev/h}$$

$$\omega_E = \left(\tfrac{1}{12}\right)\text{rev/h}$$

$$\omega_s = \omega_{\text{rel}} + \omega_E = \left(\tfrac{1}{8}\right)\text{rev/h} = 2.18\times10^{-4}\ \text{rad/s}$$

$$\sum \vec{F} = m\vec{a} \text{ says } G\frac{mm_E}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{Gm_E}{r} \text{ and with } v = r\omega \text{ this gives } r^3 = \frac{Gm_E}{\omega^2};\ r = 2.03\times10^7\ \text{m}$$

This is the radius of the satellite's orbit. Its height h above the surface of the earth is $h = r - R_E = 1.39\times10^7\ \text{m}$.

b) Now the satellite is revolving opposite to the rotation of the earth.

If west to east is positive, then $\omega_{\text{rel}} = \left(-\tfrac{1}{12}\right)\text{rev/h}$

$$\omega_s = \omega_{\text{rel}} + \omega_E = \left(-\tfrac{1}{24}\right)\text{rev/h} = -7.27\times10^{-5}\ \text{rad/s}$$

$$r^3 = \frac{Gm_E}{\omega^2} \text{ gives } r = 4.22\times10^7\ \text{m and } h = 3.59\times10^7\ \text{m}$$

12.58: (a) Get radius of $\text{X}: \frac{1}{4}(2\pi R) = 18{,}850\ \text{km}$

$$\text{R} = 1.20 \times 10^7\ \text{m}$$

Astronant mass: $m = \frac{\omega}{g} = \frac{943\ \text{N}}{9.80\ \text{m/s}^2} = 96.2\ \text{kg}$ Use astronant at north pole to get mass of X:

$$\sum F = ma : \frac{GmM_\text{x}}{R^2} = mg_\text{x}$$

$$M_\text{x} = \frac{mg_\text{x}R^2}{Gm} = \frac{(915\ \text{N})(1.20\times 10^7\ \text{m})^2}{(6.67\times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(96.2\ \text{kg})} = 2.05\times 10^{25}\,\text{kg}$$

Apply Newton's second law to astronant on a scale at the equator of X.

$$\sum F = ma : F_\text{grav} - F_\text{scale} = \frac{mv^2}{R}$$

$$v = \frac{2\pi R}{T} \to F_\text{grav} - F_\text{scale} = \frac{m\left(\frac{2\pi R}{R}\right)^2}{R} = \frac{4\pi^2 mR}{T^2}$$

$$915.0\ \text{N} - 850.0\ \text{N} = \frac{4\pi^2(96.2\ \text{kg})(1.20\times 10^7\ \text{m})}{T^2}$$

$$T = 2.65\times 10^4\,\text{s}\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right) = 7.36\,\text{hr, which is one day}$$

(b) For satellite: $\sum F = ma \to \frac{Gm_\text{s}m_\text{x}}{r^2} = \frac{m_\text{s}v^2}{r}$ where $v = \frac{2\pi r}{T} . \frac{\text{Gm}_\text{x}}{r} = \left(\frac{2\pi r}{T}\right)^2$

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_\text{x}}} = \sqrt{\frac{4\pi^2(1.20\times 10^7\ \text{m} + 2\times 10^6\ \text{m})^3}{(6.67\times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(2.05\times 10^{25}\ \text{kg})}}$$

$$T = 8.90\times 10^3\,\text{s} = 2.47\ \text{hours}$$

12.59: The fractional error is

$$1 - \frac{mgh}{Gmm_\text{E}\left(\frac{1}{R_\text{E}} - \frac{1}{R_\text{E}+h}\right)} = 1 - \frac{g}{Gm_\text{E}}(R_\text{E}+h)(R_\text{E}).$$

At this point, it is advantageous to use the algebraic expression for g as given in Eq. (12.4) instead of numerical values to obtain the fractional difference as $1-(R_\text{E}+h)/R_\text{E} = -h/R_\text{E}$, so if the fractional difference is $-1\%, h = (0.01)R_\text{E} = 6.4\times 10^4$ m.

If the algebraic form for g in terms of the other parameters is not used, and the numerical values from Appendix F are used along with $g = 9.80\ \text{m/s}^2$, $h/R_\text{E} = 8.7\times 10^{-3}$, which is qualitatively the same.

12.60: (a) Get g on Mongo: It takes 4.00 s to reach the maximum height, where $v = 0$ then $v_0 - gt \to 0 = 12.0\,\text{m/s} - g(4.00\,\text{s})$

$$g = 3.00\,\text{m/s}^2$$

Apply Newton's second law to a falling object:

$$\sum F = ma : mg = \frac{GmM}{R^2} \to M = gR^2/G$$

$$2\pi R = C \to R = C/2\pi$$

$$M = gR^2/G = \frac{(3.00\,\text{m/s}^2)\left(\frac{2.00\times10^8\,\text{m}}{2\pi}\right)^2}{6.67\times10^{-11}\ \text{N m}^2/\text{kg}^2} = 4.56\times10^{25}\ \text{kg}$$

b) Apply Newton's second law to the orbiting starship.

$$\Sigma F = ma : \frac{GmM}{r^2} = \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T} \to T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$r = R + 30{,}000\ \text{km} = \frac{C}{2\pi} + 3.0\times10^7\ \text{m}$$

$$T = \sqrt{\frac{4\pi^2\left(\frac{2.00\times10^8\,\text{m}}{2\pi} + 3.0\times10^7\ \text{m}\right)^3}{(6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2)(4.56\times10^{25}\ \text{kg})}}$$

$$= 5.54\times10^4\ \text{s}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 15.4\,\text{h}$$

12.61: At Sacramento, the gravity force on you is $F_1 = G\dfrac{mm_\text{E}}{R_E^2}$.

At the top of Mount Everest, a height of $h = 8800\,\text{m}$ above sea level, the gravity force on you is

$$F_2 = G = \frac{mm_\text{E}}{(R_E + h)^2} = G\frac{mm_\text{E}}{R_\text{E}^2(1 + h/R_\text{E})^2}$$

$$(1 + h/R_E)^{-2} \approx 1 - \frac{2h}{R_\text{E}}, F_2 = F_1\left(\frac{1 - 2h}{R_\text{E}}\right)$$

$$\frac{F_1 - F_2}{F_1} = \frac{2h}{r_\text{E}} = 0.28\,\%$$

12.62: a) The total gravitational potential energy in this model is

$$U = -Gm\left[\frac{m_E}{r} + \frac{m_M}{R_{EM} - r}\right].$$

b) See Exercise 12.5. The point where the net gravitational field vanishes is

$$r = \frac{R_{EM}}{1 + \sqrt{m_M/m_E}} = 3.46 \times 10^8 \text{ m}.$$

Using this value for *r* in the expression in part (a) and the work-energy theorem, including the initial potential energy of $-Gm(m_E/R_E + m_M/(R_{EM} - R_E))$ gives $11.1\,\text{km/s}$. c) The final distance from the earth is not R_M, but the Earth-moon distance minus the radius of the moon, or 3.823×10^8 m. From the work-energy theorem, the rocket impacts the moon with a speed of $2.9\,\text{km/s}$.

12.63: One can solve this problem using energy conservation, units of J/kg for energy, and basic concepts of orbits. $E = K + U$, or $-\frac{GM}{2a} = \frac{1}{2}v^2 - \frac{GM}{r}$, where E, K and U are the energies per unit mass, v is the circular orbital velocity of 1655 m/s at the lunicentric distance of 1.79×10^6 m. The total energy at this distance is -1.37×10^6 J/Kg. When the velocity of the spacecraft is reduced by 20 m/s, the total energy becomes

$$E = \frac{1}{2}(1655\,\text{m/s} - 20\,\text{m/s})^2 - \frac{(6.673 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\ \text{kg})}{(1.79 \times 10^6\ \text{m})},$$

or $E = -1.40 \times 10^6$ J/kg. Since $E = -\frac{GM}{2a}$, we can solve for a, $a = 1.748 \times 10^6$ m, the semi –major axis of the new elliptical orbit. The old distance of 1.79×10^6 m is now the apolune distance, and the perilune can be found from
$a = \frac{r_a + r_p}{2}$, $r_p = 1.706 \times 10^6$ m. Obviously this is less than the radius of the moon, so the spacecraft crashes! At the surface, $U = -\frac{GM}{R_m}$ or, $U = -2.818 \times 10^6$ J/kg.
Since the total energy at the surface is -1.40×10^6 J/kg, the kinetic energy at the surface is 1.415×10^6 J/kg. So, $\frac{1}{2}v^2 = 1.415 \times 10^6$ J/kg, or $v = 1.682 \times 10^3$ m/s $= 6057$ km/h.

12.64: Combining Equations (12.13) and (3.28) and setting $a_{\text{rad}} = 9.80\ \text{m/s}^2$ (so that $\omega = 0$ in Eq. (12.30)),

$$T = 2\pi\sqrt{\frac{R}{a_{\text{rad}}}} = 5.07\times10^3\ \text{s},$$

which is 84.5 min, or about an hour and a half.

12.65: The change in gravitational potential energy is

$$\Delta U = \frac{Gm_{\text{E}}m}{(R_{\text{E}}+h)} - \frac{Gm_{\text{E}}m}{R_{\text{E}}} = -Gm_{\text{E}}m\frac{h}{R_{\text{E}}(R_{\text{R}}+h)},$$

so the speed of the hammer is, from the work-energy theorem,

$$\sqrt{\frac{2Gm_{\text{E}}h}{(R_{\text{E}}+h)R_{\text{E}}}}.$$

12.66: a) The energy the satellite has as it sits on the surface of the Earth is $E_{\text{i}} = \frac{-GmM_{\text{E}}}{R_{\text{E}}}$. The energy it has when it is in orbit at a radius $R \approx R_{\text{E}}$ is $E_{\text{f}} = \frac{-GmM_{\text{E}}}{2R_{\text{E}}}$. The work needed to put it in orbit is the difference between these: $W = E_{\text{f}} - E_{\text{i}} = \frac{GmM_{\text{E}}}{2R_{\text{E}}}$.

b) The total energy of the satellite far away from the Earth is zero, so the additional work needed is $0 - \left(\frac{-GmM_{\text{E}}}{2R_{\text{E}}}\right) = \frac{GmM_{\text{E}}}{2R_{\text{E}}}$.

c) The work needed to put the satellite into orbit was the same as the work needed to put the satellite from orbit to the edge of the universe.

12.67: The escape speed will be

$$v=\sqrt{2G\left[\frac{m_{\mathrm{E}}}{R_{\mathrm{E}}}+\frac{m_{\mathrm{s}}}{R_{\mathrm{ES}}}\right]}=4.35\times10^{4}\ \mathrm{m/s}.$$

a) Making the simplifying assumption that the direction of launch is the direction of the earth's motion in its orbit, the speed relative the earth is

$$v-\frac{2\pi R_{\mathrm{ES}}}{\mathrm{T}}=4.35\times10^{4}\ \mathrm{m/s}-\frac{2\pi(1.50\times10^{11}\ \mathrm{m})}{(3.156\times10^{7}\mathrm{s})}=1.37\times10^{4}\ \mathrm{m/s}.$$

b) The rotational at Cape Canaveral is $\frac{2\pi(6.38\times10^{6}\ \mathrm{m})\cos 28.5°}{86164\ \mathrm{s}}=4.09\times10^{2}\ \mathrm{m/s},$ so the speed relative to the surface of the earth is $1.33\times10^{4}\ \mathrm{m/s}.$ c) In French Guiana, the rotational speed is $4.63\times10^{2}\ \mathrm{m/s},$ so the speed relative to the surface of the earth is $1.32\times10^{4}\ \mathrm{m/s}.$

12.68: a) The SI units of energy are $\mathrm{kg}\cdot\mathrm{m}^{2}/\mathrm{s}^{2}$, so the SI units for ϕ are $\mathrm{m}^{2}/\mathrm{s}^{2}$. Also, it is known from kinetic energy considerations that the dimensions of energy, kinetic or potential, are $\mathrm{mass}\times\mathrm{speed}^{2}$, so the dimensions of gravitational potential must be the same as speed^{2}. b) $\phi=-\frac{U}{m}=\frac{Gm_{\mathrm{E}}}{r}$.

c) $$\Delta\phi=Gm_{\mathrm{E}}\left[\frac{1}{R_{\mathrm{E}}}-\frac{1}{r_{\mathrm{f}}}\right]=3.68\times10^{6}\ \mathrm{J/kg}.$$

d) $m\Delta\phi=5.53\times10^{10}$ J. (An extra figure was kept in the intermediate calculations.)

12.69: a) The period of the asteroid is $T=\frac{2\pi a^{3/2}}{GM}$. Inserting 3×10^{11} m for a gives 2.84 y and 5×10^{11} m gives a period of 6.11 y.

b) If the period is $5.93\ y$, then $a=4.90\times10^{11}\mathrm{m}.$

c) This happens because $0.4=2/5,$ another ratio of integers. So once every 5 orbits of the asteroid and 2 orbits of Jupiter, the asteroid is at its perijove distance. Solving when $T=4.74\ y,\ a=4.22\times10^{11}$ m.

12.70: a) In moving to a lower orbit by whatever means, gravity does positive work, and so the speed does increase. b) From $\frac{v^2}{r} = \frac{Gm_E}{r^2}$, $v = (Gm_E)^{1/2} r^{-1/2}$, so

$$\Delta v = (Gm_E)^{1/2}\left(-\frac{-\Delta r}{2}\right)r^{-3/2} = \left(\frac{\Delta r}{2}\right)\sqrt{\frac{Gm_E}{r^3}}.$$

Note that a positive Δr is given as a decrease in radius. Similarly, the kinetic energy is $K = (1/2)mv^2 = (1/2)Gm_E m/r$, and so $\Delta K = (1/2)(Gm_E m/r^2)\Delta r$, $\Delta U = -(Gm_E m/r^2)$ Δr and $W = \Delta U + \Delta K = -(Gm_E m/2r^2)\Delta r$, is agreement with part (a). c) v $= \sqrt{Gm_E/r} = 7.72\times10^3$ m/s, $\Delta v = (\Delta r/2)\sqrt{Gm_E/r^3} = 28.9$ m/s, $E = -Gm_E m/2r =$ -8.95×10^{10} J (from Eq. (12.15)), $\Delta K = (Gm_E m/2r^2)(\Delta r) = 6.70\times10^8$ J, $\Delta U = -2\Delta K =$ -1.34×10^9 J and $W = -\Delta K = -6.70\times10^8$ J. d)As the term "burns up" suggests, the energy is converted to heat or is dissipated in the collisions of the debris with the grounds.

12.71: a) The stars are separated by the diameter of the circle d =2R, so the gravitational force is $\frac{GM^2}{4R^2}$.

b) The gravitational force found in part (b) is related to the radial acceleration by $F_g = Ma_{rad} = Mv^2/R$ for each star, and substituting the expression for the force from part (a) and solving for v gives $v = \sqrt{GM/4R}$. The period is $T = \frac{2\pi R}{\upsilon} = \sqrt{16\pi^2 R^3/GM} = 4\pi R^{3/2}/\sqrt{GM}$. c) The initial gravitational potential energy is $-GM^2/2R$ and the initial kinetic energy is $2(1/2)Mv^2 = .GM^2/4R$, so the total mechanical energy is $-GM^2/2R$. If the stars have zero speed when they are very far apart, the energy needed to separate them is $GM^2/4R$.

12.72: a) The radii R_1 and R_2 are measured with respect to the center of mass, and so $M_1R_1 = M_2R_2$, and $R_1/R_2 = M_2/M_1$.

b) If the periods were different, the stars would move around the circle with respect to one another, and their separations would not be constant; the orbits would not remain circular. Employing qualitative physical principles, the forces on each star are equal in magnitude, and in terms of the periods, the product of the mass and the radial accelerations are

$$\frac{4\pi^2 M_1R_1}{T_1^2} = \frac{4\pi^2 M_2R_2}{T_2^2}.$$

From the result of part (a), the numerators of these expressions are equal, and so the denominators are equal, and the periods are the same. To find the period in the symmetric from desired, there are many possible routes. An elegant method, using a bit of hindsight, is to use the above expressions to relate the periods to the force $F_g = \frac{GM_1M_2}{(R_1+R_2)^2}$, so that equivalent expressions for the period are

$$M_2T^2 = \frac{4\pi^2 R_1(R_1+R_2)^2}{G}$$

$$M_1T^2 = \frac{4\pi^2 R_2(R_1+R_2)^2}{G}.$$

Adding the expressions gives

$$(M_1+M_2)T^2 = \frac{4\pi^2(R_1+R_2)^3}{G} \text{ or } T = \frac{2\pi(R_1+R_2)^{3/2}}{\sqrt{G(M_1+M_2)}}.$$

c) First we must find the radii of each orbit given the speed and period data. In a circular orbit, $v = \frac{2\pi R}{T}$, or $\mathrm{R} = \frac{vT}{2\pi}$

Thus, $\mathrm{R}_\alpha = \frac{(36\times10^3 \text{ m/s})(137 \text{ d}\times 86{,}400 \text{ s/d})}{2\pi} = 6.78\times10^{10}$ m, and $\mathrm{R}_\beta = \frac{(12\times10^3 \text{ m/s})(137 \text{ d}\times 86{,}400 \text{ s/d})}{2\pi}$ $= 2.26\times10^{10}$ m. Now find the sum of the masses and use $M_\alpha R_\alpha = M_\beta R_\beta$, and the fact that $R_\alpha = 3R_\beta.(M_\alpha + M_\beta) = \frac{4\pi^2 (R_\alpha+R_\beta)^3}{T^2G}$, inserting the values of T, and the radii, $(M_\alpha + M_\beta) = \frac{4\pi^2(6.78\times10^{10}\text{ m}+2.26\times10^{10}\text{ m})^3}{(137\text{ d}\times 86{,}400\text{ s/d})^2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)}.M_\alpha + M_\beta = 3.12\times10^{30}$ kg. Since $M_\beta = M_\alpha R_\alpha/R_\beta = 3M_\alpha, 4M_\alpha = 3.12\times10^{30}$ kg, or $M_\alpha = 7.80\times10^{29}$ kg, and $M_\beta = 2.34\times10^{3}$

d) Let α refer to the star and β refer to the black hole. Use the relationships derived in parts (a) and (b): $R_\beta = (M_\alpha/M_\beta)R_\alpha = (0.67/3.8)R_\alpha = (0.176)R_\alpha, (R_\alpha +$ $\mathrm{R}_\beta) = \sqrt[3]{\frac{(M_\alpha+M_\beta)T^2G}{4\pi^2}}$, and $v = \frac{2\pi R}{T}$. For R_α, inserting the values for M and T and

12.73: From conservation of energy, the speed at the closer distance is

$$v = \sqrt{v_0^2 + 2Gm_s\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} = 6.8 \times 10^4 \text{ m/s}.$$

12.74: Using conservation of energy,

$$\frac{1}{2}m_M v_a^2 - \frac{GM_S m_M}{r_a} = \frac{1}{2}m_M v_p^2 - \frac{GM_S m_M}{r_p}, \text{ or}$$

$$v_p = \sqrt{v_a^2 - 2GM_S\left(\frac{1}{r_a} - \frac{1}{r_p}\right)} = 2.650 \times 10^4 \text{ m/s}.$$

The subscripts a and p denote aphelion and perihelion.

To use conservation of angular momentum, note that at the extremes of distance (periheleion and aphelion), Mars' velocity vector must be perpendicular to its radius vector, and so the magnitude of the angular momentum is $L = mrv$. Since L is constant, the product rv must be a constant, and so

$$v_p = v_a \frac{r_a}{r_p} = (2.198 \times 10^4 \text{ m/s})\frac{(2.492 \times 10^{11} \text{ m})}{(2.067 \times 10^{11} \text{ m})} = 2.650 \times 10^4 \text{ m/s},$$

a confirmation of Kepler's Laws.

12.75: a) The semimajor axis is the average of the perigee and apogee distances, $a = \frac{1}{2}((R_E + h_p) + (R_E + h_a)) = 8.58 \times 10^6$ m. From Eq. (12.19) with the mass of the earth, the period of the orbit is

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_E}} = 7.91 \times 10^3 \text{ s},$$

a little more than two hours. b) See Problem 12.74; $\frac{v_p}{v_a} = \frac{r_a}{r_p} = 1.53$. c) The equation that represents conservation of energy (apart from a common factor of the mass of the spacecraft) is

$$\frac{1}{2}v_p^2 - \frac{Gm_E}{r_p} = \frac{1}{2}v_a^2 - \frac{Gm_E}{r_a} = \frac{1}{2}\left(\frac{r_p}{r_a}\right)^2 v_p^2 - \frac{Gm_E}{r_a},$$

where conservation of angular momentum has been used to eliminate v_a is favor of v_p. Solving for v_p^2 and simplifying,

$$v_p^2 = \frac{2Gm_E r_a}{r_p(r_p + r_a)} = 7.71 \times 10^7 \text{ m}^2/\text{s}^2,$$

from which $v_p = 8.43 \times 10^3$ m/s and $v_a = 5.51 \times 10^3$ m/s. d) The escape speed for a given distance is $v_e = \sqrt{2GM/r}$, and so the difference between escape speed and v_p is, after some algebra,

$$v_e - v_p = \sqrt{\frac{2Gm_E}{r_p}}\left[1 - 1/\sqrt{1 + (r_p/r_a)}\right].$$

Using the given values for the radii gives $v_e - v_p = 2.41 \times 10^3$ m/s. The similar calculation at apogee give $v_e - v_a = 3.26 \times 10^3$ m/s, so it is more efficient to fire the rockets at perigee. Note that in the above, the escape speed v_e is different at the two points, $v_{pe} = 1.09 \times 10^4$ m/s and $v_{ae} = 8.77 \times 10^3$ m/s.

12.76: a) From the value of g at the poles,

$$m_{\mathrm{U}} = \frac{g_{\mathrm{U}} R_{\mathrm{U}}^2}{G} = \frac{\left(11.1\,\mathrm{m/s^2}\right)\left(2.556\times10^{7}\ \mathrm{m}\right)^2}{\left(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)} = 1.09\times10^{26}\ \mathrm{kg}.$$

b) $Gm_{\mathrm{U}}/r^2 = g_{\mathrm{U}}\left(R_{\mathrm{U}}/r\right)^2 = 0.432\,\mathrm{m/s^2}$. c) $Gm_{\mathrm{M}}/R_{\mathrm{M}}^2 = 0.080\,\mathrm{m/s^2}$. d) No; Miranda's gravity is sufficient to retain objects released near its surface.

12.77: Using Eq. (12.15), with the mass M_{m} instead of the mass of the earth, the energy needed is

$$\begin{aligned}
\Delta E &= \frac{Gm_{\mathrm{m}}m}{2}\left[\frac{1}{r_{\mathrm{i}}} - \frac{1}{r_{\mathrm{f}}}\right] \\
&= \frac{\left(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(6.42\times10^{23}\ \mathrm{kg}\right)\left(3000\,\mathrm{kg}\right)}{2} \\
&\quad\times\left[\frac{1}{\left(2.00\times10^{6}\ \mathrm{m} + 3.40\times10^{6}\ \mathrm{m}\right)} - \frac{1}{\left(4.00\times10^{6}\ \mathrm{m} + 3.40\times10^{6}\ \mathrm{m}\right)}\right] \\
&= 3.22\times10^{9}\ \mathrm{J}.
\end{aligned}$$

12.78: a) The semimajor axis is 4×10^{15} m and so the period is

$$\frac{2\pi\left(4\times10^{15}\ \mathrm{m}\right)^{3/2}}{\sqrt{\left(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(1.99\times10^{30}\,\mathrm{kg}\right)}} = 1.38\times10^{14}\ \mathrm{s},$$

which is about 4 million years. b) Using the earth-sun distance as an estimate for the distance of closest approach, $v = \sqrt{2Gm_{\mathrm{S}}/R_{\mathrm{ES}}} = 4\times10^{4}$ m/s. c) $(1/2)mv^2 = Gm_{\mathrm{S}}\,m/R = 10^{24}$ J. This is far larger than the energy of a volcanic eruption and is comparable to the energy of burning the fossil fuel.

12.79: a) From Eq. (12.14) with the mass of the sun,

$$r = \left[\begin{array}{c} \left(6.673 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}\right)\left(1.99 \times 10^{30}\ \mathrm{kg}\right) \\ \times \left(\left(3 \times 10^4\ y\right)\left(3.156 \times 10^7\ \mathrm{s/y}\right)\right)^2 / 4\pi^2 \end{array} \right]^{1/3} = 1.4 \times 10^{14}\ \mathrm{m}.$$

This is about 24 times the orbit radius of Pluto and about $1/250$ of the way to Alpha Centauri.

12.80: Outside the planet it behaves like a point mass, so at the surface:

$$\sum F = ma : \frac{GmM}{R^2} = mg \rightarrow g = GM/R^2$$

Get $M : M = \int dm = \int \rho dV = \int \rho 4\pi r^2 dr.$ The density is $\rho = \rho_0 - br$, where $\rho_0 = 15.0 \times 10^3\ \mathrm{kg/m^3}$ at the center at the surface, $\rho_S = 2.0 \times 10^3\ \mathrm{kg/m^3}$, so $b = \frac{\rho_0 - \rho_s}{R}$

$$M = \int_0^R (\rho_0 - br)\, 4\pi r^2 dr = \frac{4\pi}{3}\rho_0 R^3 - \pi b R^4$$

$$= \frac{4}{3}\pi R^3 \rho_0 - \pi R^4 \left(\frac{\rho_0 - \rho_s}{R}\right) = \pi R^3 \left(\frac{1}{3}\rho_0 + \rho_s\right)$$

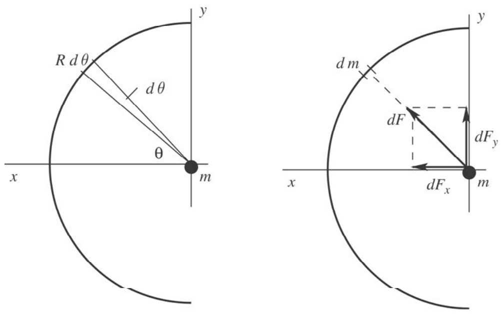
$$g = \frac{GM}{R^2} = \frac{G\pi R^3\left(\frac{1}{3}\rho_0 + \rho_s\right)}{R^2} = \pi R G \left(\frac{1}{3}\rho_0 + \rho_s\right)$$

$$\pi\left(6.38 \times 10^6\ \mathrm{m}\right)\left(6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}\right)\left(\frac{15.0 \times 10^3\ \mathrm{kg/m^3}}{3} + 2.0 \times 10^3\ \mathrm{kg/m^3}\right)$$

$$= 9.36\ \mathrm{m/s^2}$$

12.81: The radius of the semicircle is $R = L/\pi$

Divide the semicircle up into small segments of length $R\,d\theta$



$dM = (M/L)R\,d\theta = (M/\pi)d\theta$

$d\vec{F}$ is the gravity force on m exerted by dM

$\int dF_y = 0$; the y-components from the upper half of the semicircle cancel the y-components from the lower half.

The x-components are all in the $+x$-direction and all add.

$$dF = G\frac{mdM}{R^2}$$

$$dF_x = G\frac{mdM}{R^2}\cos\theta = \frac{Gm\pi m}{L^2}\cos\theta\,d\theta$$

$$F_x = \int_{-\pi/2}^{\pi/2} dF_x = \frac{Gm\pi M}{L2}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{Gm\pi m}{L^2}(2)$$

$$F = \frac{2\pi GmM}{L^2}$$

12.82: The direct calculation of the force that the sphere exerts on the ring is slightly more involved than the calculation of the force that the ring exerts on the ball. These forces are equal in magnitude but opposite in direction, so it will suffice to do the latter calculation. By symmetry, the force on the sphere will be along the axis of the ring in Fig. (12.34), toward the ring. Each mass element dM of the ring exerts a force of magnitude $\frac{GmdM}{a^2+x^2}$ on the sphere, and the x-component of this force is

$$\frac{GmdM}{a^2+x^2}\frac{x}{\sqrt{a^2+x^2}}=\frac{GmdMx}{\left(a^2+x^2\right)^{3/2}}.$$

As $x >> a$ the denominator approaches x^3 and $F \to \frac{GMm}{x^2}$, as expected, and so the force on the sphere is $GmMx/\left(a^2+x^2\right)^{3/2}$, in the $-x$ - direction. The sphere attracts the ring with a force of the same magnitude. (This is an alternative but equivalent way of obtaining the result of parts (c) and (d) of Exercise 12.39.)2

12.83: Divide the rod into differential masses *dm at* position *l*, measured from the right end of the rod. Then, $dm = dl(M/L)$, and the contribution

dF_x from each piece is $dF_x=-\dfrac{GmMdl}{(l+x)^2L}$. Integrating from $l=0$ to $l=L$ gives

$$F=-\frac{GmM}{L}\int_0^L\frac{dl}{(l+x)^2}=\frac{GmM}{L}\left[\frac{1}{x+L}-\frac{1}{x}\right]=-\frac{GmM}{x(x+L)},$$

with the negative sign indicating a force to the left. The magnitude is $F=\frac{GmM}{x(x+L)}$. As $x >>$ L, the denominator approaches x^2 and $F \to \frac{GmM}{x^2}$, as expected. (This is an alternative but equivalent way of obtaining the result of part (b) Exercise 12.39.)

12.84: a) From the result shown in Example 12.10, the force is attractive and its magnitude is proportional to the distance the object is from the center of the earth. Comparison with equations (6.8) and (7.9) show that the gravitational potential energy is given by

$$U(r) = \frac{Gm_E m}{2R_E^3} r^2.$$

This is also given by the integral of F_g from 0 to r with respect to distance. b) From part (a), the initial gravitational potential energy is $\frac{Gm_E m}{1R_E}$. Equating initial potential energy and final kinetic energy (initial kinetic energy and final potential energy are both zero) gives $v^2 = \frac{Gm_E}{R_E}$, so $v = 7.90 \times 10^3$ m/s.

12.85: a) $T = \frac{2\pi r^{3/2}}{\sqrt{GM_{\mathrm{E}}}}$, therefore

$$T + \Delta T = \frac{2\pi}{\sqrt{GM_{\mathrm{E}}}}(r + \Delta r)^{3/2} = \frac{2\pi r^{3/2}}{\sqrt{GM_{\mathrm{E}}}}\left(1 + \frac{\Delta r}{r}\right)^{3/2} \approx \frac{2\pi r^{3/2}}{\sqrt{GM_{\mathrm{E}}}}\left(1 + \frac{3\Delta r}{2r}\right) = T + \frac{3\pi r^{1/2}\Delta r}{\sqrt{GM_{\mathrm{E}}}}.$$

Since $v = \sqrt{\frac{GM_{\mathrm{E}}}{r}}, \Delta T = \frac{3\pi\Delta r}{v}$. $v = \sqrt{GM_{\mathrm{E}}}\, r^{-1/2}$, therefore

$$v - \Delta v = \sqrt{GM_E}\,(r + \Delta r)^{-1/2} = \sqrt{GM_E}\, r^{-1/2}\left(1 + \frac{\Delta r}{r}\right)^{-1/2} \approx \sqrt{GM_{\mathrm{E}}}\, r^{-1/2}\left(1 - \frac{\Delta r}{2r}\right) = v - \frac{\sqrt{GM_E}}{2r^{3/2}}\Delta r.$$

Since $T = \frac{2\pi r^{3/2}}{\sqrt{GM_{\mathrm{E}}}}, \Delta v = \frac{\pi\Delta r}{T}$.

b) Note: Because of the small change in r, several significant figures are needed to see the results. Starting with $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$ (Eq.(12.14)), $T = 2\pi r/v$, and $v = \sqrt{\frac{GM}{r}}$ (Eq.(12.12)) find the velocity and period of the initial orbit:

$$v = \sqrt{\frac{(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.97\times10^{24}\,\mathrm{kg})}{6.776\times10^{6}\,\mathrm{m}}} = 7.672\times10^{3}\ \mathrm{m/s}, \text{and}$$

$T = 2\pi r/v = 5549\,\mathrm{s} = 92.5$ min. We then can use the two derived equations to approximate the

ΔT and $\Delta v, \Delta T = \frac{3\pi\Delta \mathrm{r}}{v}$ and $\Delta v = \frac{\pi\Delta r}{T}$. $\Delta T = \frac{3\pi\,(100\ \mathrm{m})}{7.672\times10^{3}\ \mathrm{m/s}} = 0.1228\,\mathrm{s}$, and Δv

$= \frac{\pi\Delta r}{T} = \frac{\pi(100\ \mathrm{m})}{(5549\,\mathrm{s})} = .05662\mathrm{m/s}$.

Before the cable breaks, the shuttle will have traveled a distance d, d $= \sqrt{(125\,\mathrm{m}^2) - (100\,\mathrm{m}^2)} = 75\,\mathrm{m}$. So, $(75\,\mathrm{m})/(.05662\,\mathrm{m/s}) = 1324.7\,\mathrm{s} = 22$ min. It will take 22 minutes for the cable to break.

c) The ISS is moving faster than the space shuttle, so the total angle it covers in an orbit must be 2π radians more than the angle that the space shuttle covers before they are once again in line. Mathematically, $\frac{vt}{r} - \frac{(v-\Delta v)t}{(r+\Delta r)} = 2\pi$. Using the binomial theorem and neglecting terms of order $\Delta v\Delta r$, $\frac{vt}{r} - \frac{(v-\Delta v)t}{r}\left(1 + \frac{\Delta r}{r}\right)^{-1} \approx t\left(\frac{\Delta v}{r} + \frac{v\Delta r}{r^2}\right) = 2\pi$. Therefore,

$t = \frac{2\pi r}{\left(\Delta v + \frac{v\Delta r}{r}\right)} = \frac{vT}{\frac{\pi\,\Delta r}{T} + \frac{v\Delta r}{r}}$. Since $2\pi r = vT$ and $\Delta r = \frac{v\Delta T}{3\pi}$, $t = \frac{vT}{\frac{\pi}{t}\left(\frac{v\Delta T}{3\pi}\right) + \frac{2\pi}{T}\left(\frac{v\Delta T}{3\pi}\right)} = \frac{T^2}{\Delta T}$, as was to be shown. $t = \frac{T^2}{\Delta T} = \frac{(5549\,\mathrm{s})^2}{(0.1228\,\mathrm{s})} = 2.5\times10^{8}\ \mathrm{s} = 2900\,\mathrm{d} = 7.9\ \mathrm{y}$. It is highly doubtful the shuttle crew would survive the congressional hearings if they miss!

12.86: a) To get from the circular orbit of the earth to the transfer orbit, the spacecraft's energy must increase, and the rockets are fired in the direction opposite that of the motion, that is, in the direction that increases the speed. Once at the orbit of Mars, the energy needs to be increased again, and so the rockets need to be fired in the direction opposite that of the motion. From Fig. (12.37), the semimajor axis of the transfer orbit is the arithmetic average of the orbit radii of the earth and Mars, and so from Eq. (12.19), the energy of spacecraft while in the transfer orbit is intermediate between the energies of the circular orbits. Returning from Mars to the earth, the procedure is reversed, and the rockets are fired against the direction of motion. b) The time will be half the period as given in Eq. (12.19), with the semimajor axis *a* being the average of the orbit radii, $a = 1.89 \times 10^{11}\ \text{m}$, so

$$t = \frac{T}{2} = \frac{\pi (1.89 \times 10^{11}\ \text{m})^{3/2}}{\sqrt{(6.673 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})}} = 2.24 \times 10^7\ \text{s},$$

which is more than $8\frac{1}{2}$ months. c) During this time, Mars will pass through an angle of $(360°)\frac{(2.24 \times 10^7\ \text{s})}{(687\ \text{d})(86{,}400\ \text{s/d})} = 135.9°$, and the spacecraft passes through an angle of $180°$, so the angle between the earth-sun line and the Mars-sun line must be $44.1°$.

12.87: a) There are many ways of approaching this problem; two will be given here.

I) Denote the orbit radius as r and the distance from this radius to either ear as δ. Each ear, of mass m, can be modeled as subject to two forces, the gravitational force from the black hole and the tension force (actually the force from the body tissues), denoted by F. Then, the force equations for the two ears are

$$\frac{GMm}{(r-\delta)^2} - F = m\omega^2(r-\delta)$$
$$\frac{GMm}{(r+\delta)^2} + F = m\omega^2(r+\delta),$$

where ω is the common angular frequency. The first equation reflects the fact that one ear is closer to the black hole, is subject to a larger gravitational force, has a smaller acceleration, and needs the force F to keep it in the circle of radius $r-\delta$. The second equation reflects the fact that the outer ear is further from the black hole and is moving in a circle of larger radius and needs the force F to keep in in the circle of radius $r+\delta$.

Dividing the first equation by $r-\delta$ and the second by $r+\delta$ and equating the resulting expressions eliminates ω, and after a good deal of algebra,

$$F = (3GMm\delta)\frac{(r+\delta)}{(r^2-\delta^2)^2}.$$

At this point it is prudent to neglect δ in the sum and difference, but recognize that F is proportional to δ, and numerically $F = \frac{3GMm\delta}{r^3} = 2.1$ kN. (Using the result of Exercise 12.39 to express the gravitational force in terms of the Schwartzschild radius gives the same result to two figures.)

II) Using the same notation,

$$\frac{GMm}{(r+\delta)^2} - F = m\omega^2(r+\delta),$$

where δ can be of either sign. Replace the product $m\omega^2$ with the value for $\delta = 0$, $m\omega^2 = GMm/r^3$ and solve for

$$F = (GMm)\left[\frac{r+\delta}{r^3} - \frac{1}{(r+\delta)^2}\right] = \frac{GMm}{r^3}\left[r+\delta - r(1+(\delta/r))^{-2}\right].$$

Using the binomial theorem to expand the term in square brackets in powers of δ/r,

$$F = \frac{GMm}{r^3}\left[r+\delta - r(1-2(\delta/r))\right] = \frac{GMm}{r^3}(3\delta),$$

the same result as above.

Method (I) avoids using the binomial theorem or Taylor series expansions; the approximations are made only when numerical values are inserted and higher powers of δ are found to be numerically insignificant.

12.88: As suggested in the problem, divide the disk into rings of radius *r* and thickness *dr*. Each ring has an area $dA = 2\pi r\, dr$ and mass $dM = \frac{M}{\pi a^2} dA = \frac{2M}{a^2} r\, dr$. The magnitude of the force that this small ring exerts on the mass *m* is then $(G\, m\, dM)(x/(r^2 + x^2)^{3/2})$, the expression found in Problem 12.82, with *dM* instead of *M* and the variable *r* instead of *a*. Thus, the contribution *dF* to the force is $dF = \dfrac{2GMmx}{a^2} \dfrac{rdr}{(x^2 + r^2)^{3/2}}$.

The total force *F* is then the integral over the range of *r*;

$$F = \int dF = \frac{2GMmx}{a^2} \int_0^a \frac{r}{(x^2 + r^2)^{3/2}} dr.$$

The integral (either by looking in a table or making the substitution $u = r^2 + a^2$) is

$$\int_0^a \frac{r}{(x^2 + r^2)^{3/2}} dr = \left[\frac{1}{x} - \frac{1}{\sqrt{a^2 + x^2}} \right] = \frac{1}{x} \left[1 - \frac{x}{\sqrt{a^2 + x^2}} \right].$$

Substitution yields the result

$$F = \frac{2GMm}{a^2} \left[1 - \frac{x}{\sqrt{a^2 + x^2}} \right].$$

The second term in brackets can be written as

$$\frac{1}{\sqrt{1 + (a/x)^2}} = (1 + (a/x)^2)^{-1/2} \approx 1 - \frac{1}{2} \left(\frac{a}{x} \right)^2$$

if $x >> a$, where the binomial approximation (or first-order Taylor series expansion) has been used. Substitution of this into the above form gives

$$F \approx \frac{GMm}{x^2},$$

as it should.

12.89: From symmetry, the component of the gravitational force parallel to the rod is zero. To find the perpendicular component, divide the rod into segments of length dx and mass $dm = dx\frac{M}{2L}$, positioned at a distance x from the center of the rod. The magnitude of the gravitational force from each segment is

$$dF = \frac{Gm\,dM}{x^2 + a^2} = \frac{GmM}{2L}\frac{dx}{x^2 + a^2}.$$

The component of dF perpendicular to the rod is $dF\,\frac{a}{\sqrt{x^2+a^2}}$, and so the net gravitational force is

$$F = \int_{-L}^{L} dF = \frac{GmMa}{2L}\int_{-L}^{L}\frac{dx}{(x^2 + a^2)^{3/2}}.$$

The integral can be found in a table, or found by making the substitution $x = a\tan\theta$. Then, $dx = a\sec^2\theta\,d\theta, (x^2 + a^2) = a^2\sec^2\theta,$ and so

$$\int\frac{dx}{(x^2 + a^2)^{3/2}} = \int\frac{a\sec^2\theta\,d\theta}{a^3\sec^3\theta} = \frac{1}{a^2}\int\cos\theta\,d\theta = \frac{1}{a^2}\sin\theta = \frac{x}{a^2\sqrt{x^2 + a^2}},$$

and the definite integral is

$$F = \frac{GmM}{a\sqrt{a^2 + L^2}}.$$

When a >> L, the term in the square root approaches a^2 and $F \to \frac{GmM}{a^2}$, as expected.

13.1: a) $T = \frac{1}{f} = 4.55 \times 10^{-3}\text{s}, \omega = \frac{2\pi}{T} = 2\pi f = 1.38 \times 10^{3}\ \text{rad/s}.$

b) $\frac{1}{4(220\ \text{Hz})} = 1.14 \times 10^{-3}\ \text{s}, \omega = 2\pi f = 5.53 \times 10^{3}\ \text{rad/s}.$

13.2: a) Since the glider is released form rest, its initial displacement (0.120 m) is the amplitude. b) The glider will return to its original position after another 0.80 s, so the period is 1.60 s. c) The frequency is the reciprocal of the period (Eq. (13.2)), $f = \frac{1}{1.60\ \text{s}} = 0.625\ \text{Hz}.$

13.3: The period is $\frac{0.50\ \text{s}}{440} = 1.14 \times 10^{-3}\ \text{s}$ and the angular frequency is $\omega = \frac{2\pi}{T} = 5.53 \times 10^{3}\ \text{rad/s}.$

13.4: (a) From the graph of its motion, the object completes one full cycle in 2.0 s; its period is thus 2.0 s and its frequency $= 1/\text{period} = 0.5\ \text{s}^{-1}$. (b) The displacement varies from $-0.20\ \text{m}$ to $+0.20\ \text{m}$, so the amplitude is 0.20 m. (c) 2.0 s (see part a)

13.5: This displacement is $\frac{1}{4}$ of a period.

$T = 1/f = 0.200\ \text{s}, \text{so}\ t = 0.0500\ \text{s}.$

13.6: The period will be twice the time given as being between the times at which the glider is at the equilibrium position (see Fig. (13.8));

$$k = \omega^2 m = \left(\frac{2\pi}{T}\right)^2 m = \left(\frac{2\pi}{2(2.60\ \text{s})}\right)^2 (0.200\ \text{kg}) = 0.292\ \text{N/m}.$$

13.7: a) $T = \frac{1}{f} = 0.167\ \text{s}.$ b) $\omega = 2\pi f = 37.7\ \text{rad/s}.$ c) $m = \frac{k}{\omega^2} = 0.084\ \text{kg}.$

13.8: Solving Eq. (13.12) for k,

$$k = m\left(\frac{2\pi}{T}\right)^2 = (0.600\ \text{kg})\left(\frac{2\pi}{0.150\ \text{s}}\right)^2 = 1.05 \times 10^{3}\ \text{N/m}.$$

13.9: From Eq. (13.12) and Eq. (13.10), $T = 2\pi\sqrt{\frac{0.500\ \text{kg}}{140\ \text{N/m}}} = 0.375\ \text{s},\ f = \frac{1}{T} = 2.66\ \text{Hz},$ $\omega = 2\pi f = 16.7\ \text{rad/s}.$

13.10: a) $a_x = \frac{d^2x}{dt^2} = -\omega^2 A \sin(\omega t + \beta) = -\omega^2 x,$ so $x(t)$ is a solution to Eq. (13.4) if $\omega^2 = \frac{k}{m}$. b) $a = 2A\omega$ a constant, so Eq. (13.4) is not satisfied. c) $v_x = \frac{dx}{dt} = i\omega^{i(\omega t + \beta)},$ $a_x = \frac{dv_x}{dt} = (i\omega)^2 A e^{i(\omega t + \beta)} = -\omega^2 x,$ so $x(t)$ is a solution to Eq. (13.4) if $\omega^2 = k/m \cdot$

13.11: a) $x = (3.0\ \text{mm})\cos((2\pi)(440\ \text{Hz})t)$ b) $(3.0\times10^{-3}\ \text{m})(2\pi)(440\ \text{Hz}) = 8.29\ \text{m/s}$, $(3.0\ \text{mm})(2\pi)^2(440\ \text{Hz})^2 = 2.29\times10^4\ \text{m/s}^2$. c) $j(t) = (6.34\times10^7\ \text{m/s}^3)\sin((2\pi)(440\ \text{Hz})t)$, $j_{\max} = 6.34\times10^7\ \text{m/s}^3$.

13.12: a) From Eq. (13.19), $A = \left|\frac{v_0}{\omega}\right| = \left|\frac{v_0}{\sqrt{k/m}}\right| = 0.98\ \text{m}$. b) Equation (13.18) is indeterminant, but from Eq. (13.14), $\phi = \pm\frac{\pi}{2}$, and from Eq. (13.17), sin $\phi > 0$, so $\phi = +\frac{\pi}{2}$. c) $\cos(\omega t + (\pi/2)) = -\sin\omega t$, so $x = (-0.98\ \text{m})\sin((12.2\ \text{rad/s})t))$.

13.13: With the same value for ω, Eq. (13.19) gives $A = 0.383\ \text{m}$ and Eq. (13.18) gives and $x = (0.383\ \text{m})\cos\left((12.2\ \text{rad/s})t + 1.02\ \text{rad}\right)$.

$$\phi = \arctan\left(-\frac{(-4.00\ \text{m/s})}{(0.200\ \text{m})\sqrt{300\ \text{N/m}/2.00\ \text{kg}}}\right) = 1.02\ \text{rad} = 58.5°,$$

and $x = (0.383\ \text{m})\cos((12.2\ \text{rad/s})t + 1.02\ \text{rad})$.

13.14: For SHM, $a_x = -\omega^2 x = -(2\pi f)^2 x = -\left(2\pi(2.5\ \text{Hz})\right)^2(1.1\times10^{-2}\ \text{m}) = -2.71\ \text{m/s}^2$.
b) From Eq. (13.19) the amplitude is 1.46 cm, and from Eq. (13.18) the phase angle is 0.715 rad. The angular frequency is $2\pi f = 15.7\ \text{rad/s}$, so

$$x = (1.46\ \text{cm})\cos((15.7\ \text{rad/s})t + 0.715\ \text{rad})$$
$$v_x = (-22.9\ \text{cm/s})\sin((15.7\ \text{rad/s})t + 0.715\ \text{rad})$$
$$a_x = (-359\ \text{cm/s}^2)\cos((15.7\ \text{rad/s})t + 0.715\ \text{rad}).$$

13.15: The equation describing the motion is $x = A\sin\omega t$; this is best found from either inspection or from Eq. (13.14) (Eq. (13.18) involves an infinite argument of the arctangent). Even so, x is determined only up to the sign, but that does not affect the result of this exercise. The distance from the equilibrium position is $A\sin\left(2\pi(t/T)\right) = (0.600\ \text{m})\sin(4\pi/5) = 0.353\ \text{m}$.

13.16: Empty chair: $T = 2\pi\sqrt{\frac{m}{k}}$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(42.5\,\text{kg})}{(1.30\,\text{s})^2} = 993\,\text{N/m}$$

With person in chair:

$$T = 2\pi\sqrt{m/k}$$

$$m = \frac{T^2 k}{4\pi^2} = \frac{(2.54\,\text{s})^2(993\,\text{N/m})}{4\pi^2} = 162\,\text{kg}$$

$$m_{\text{person}} = 162\,\text{kg} - 42.5\,\text{kg} = 120\,\text{kg}$$

13.17: $T = 2\pi\sqrt{m/k}, m = 0.400\,\text{kg}$

Use $a_x = -2.70\,\text{m/s}^2$ to calculate k:

$$-kx = ma_x \text{ gives } k = -\frac{ma_x}{x} = -\frac{(0.400\,\text{kg})(-2.70\,\text{m/s}^2)}{0.300\,\text{m}} = +3.60\,\text{N/m}$$

$$T = 2\pi\sqrt{m/k} = 2.09\,\text{s}$$

13.18: We have $v_x(t) = (3.60\,\text{cm/s})\sin((4.71\,\text{s}^{-1})\,t - \pi/2)$. Comparing this to the general form of the velocity for SHM:

$$-\omega A = 3.60\,\text{cm/s}$$

$$\omega = 4.71\,\text{s}^{-1}$$

$$\phi = -\pi/2$$

(a) $$\text{T} = 2\pi/\omega = 2\pi/4.71\,\text{s}^{-1} = 1.33\,\text{s}$$

(b) $$A = \frac{3.60\,\text{cm/s}}{\omega} = \frac{3.60\,\text{cm/s}}{4.71\,\text{s}^{-1}} = 0.764\,\text{cm}$$

(c) $$a_{\max} = \omega^2 A = (4.71\,\text{s}^{-1})^2(0.764\,\text{cm}) = 16.9\,\text{cm/s}^2$$

13.19: a) $x(t) = (7.40\,\text{cm})\cos((4.16\,\text{rad/s})t - 2.42\,\text{rad})$

When $t = T$, $(4.16\,\text{rad/s})T = 2\pi$ so $T = 1.51\,\text{s}$

b) $T = 2\pi\sqrt{m/k}$ so $k = m(2\pi/T)^2 = 26.0\,\text{N/m}$

c) $A = 7.40\,\text{cm} = 0.0740\,\text{m}$

$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ gives $v_{\max} = A\sqrt{k/m} = 0.308\,\text{m/s}$

d) $F = -kx$ so $F_{\max} = kA = 1.92\,\text{N}$

e) $x(t)$ evaluated at $t = 1.00\,\text{s}$ gives $x = -0.0125\,\text{m}$

$v = \pm\sqrt{k/m}\sqrt{A^2 - x^2} = \pm\sqrt{26.0/1.50}\sqrt{(0.0740)^2 - (0.0125)^2}\ \text{m/s} = \pm 0.303\,\text{m/s}$

Speed is $0.303\,\text{m/s}$.

$a = -kx/m = -(26.0/1.50)(-0.0125)\,\text{m/s}^2 = +0.216\,\text{m/s}^2$

13.20: See Exercise 13.15;

$$t = (\arccos(-1.5/6))(0.3/(2\pi)) = 0.0871\,\text{s}.$$

13.21: a) Dividing Eq. (13.17) by ω,

$$x_0 = A\cos\phi,\ \frac{v_0}{\omega} = -A\sin\phi.$$

Squaring and adding,

$$x_0^2 + \frac{v_0^2}{\omega^2} = A^2,$$

which is the same as Eq. (13.19). b) At time $t = 0$, Eq. (13.21) becomes

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}\frac{k}{\omega^2}v_0^2 + \frac{1}{2}kx_0^2,$$

where $m = k\omega^2$ (Eq. (13.10)) has been used. Dividing by $k/2$ gives Eq. (13.19).

13.22: a) $v_{\max} = (2\pi f)A = (2\pi(392\,\text{Hz}))(0.60\times10^{-3}\,\text{m}) = 1.48\,\text{m/s}.$

b) $K_{\max} = \dfrac{1}{2}m(V_{\max})^2 = \dfrac{1}{2}(2.7\times10^{-5}\,\text{kg})(1.48\,\text{m/s})^2 = 2.96\times10^{-5}\,\text{J}.$

13.23: a) Setting $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$ in Eq. (13.21) and solving for x gives $x = \pm\frac{A}{\sqrt{2}}$.

Eliminating x in favor of v with the same relation gives $v_x = \pm\sqrt{kA^2/2m} = \pm\frac{\omega A}{\sqrt{2}}$. b) This happens four times each cycle, corresponding the four possible combinations of + and – in the results of part (a). The time between the occurrences is one-fourth of a period or $T/4 = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega}$. c) $U = \frac{1}{4}E, K = \frac{3}{4}E$ $\left(U = \frac{kA^2}{8}, K = \frac{3kA^2}{8}\right)$

13.24: a) From Eq. (13.23),

$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{450\ \text{N/m}}{0.500\,\text{kg}}}(0.040\,\text{m}) = 1.20\,\text{m/s}.$$

b) From Eq. (13.22),

$$v = \sqrt{\frac{450\,\text{N}}{0.500\,\text{kg}}}\sqrt{(0.040\,\text{m})^2 - (-0.015\,\text{m})^2} = 1.11\,\text{m/s}.$$

c) The extremes of acceleration occur at the extremes of motion, when $x = \pm A$, and

$$a_{\max} = \frac{kA}{m} = \frac{(450\,\text{N/m})(0.040\,\text{m})}{(0.500\,\text{kg})} = 36\,\text{m/s}^2$$

d) From Eq. (13.4), $a_x = -\frac{(450\ \text{N/m})(-0.015\ \text{m})}{(0.500\ \text{kg})} = 13.5\,\text{m/s}^2$.

e) From Eq. (13.31), $E = \frac{1}{2}(450\,\text{N/m})(0.040\,\text{m})^2 = 0.36\ \text{J}$.

13.25: a) $a_{\max} = \omega^2 A = (2\pi f)^2 A = \big(2\pi(0.85\,\text{Hz})\big)^2(18.0\times10^{-2}\ \text{m}) = 5.13\,\text{m/s}^2$. $v_{\max} = \omega A = 2\pi f A = 0.961\,\text{m/s}$. b) $a_x = -(2\pi f)^2 x = -2.57\,\text{m/s}^2$,

$$\begin{aligned} v &= (2\pi f)\sqrt{A^2 - x^2} \\ &= \big(2\pi(0.85\,\text{Hz})\big)\sqrt{(18.0\times10^{-2}\ \text{m})^2 - (9.0\times10^{-2}\ \text{m})^2} = 0.833\,\text{m/s}. \end{aligned}$$

c) The fraction of one period is $(1/2\pi)\arcsin\,(12.0/18.0)$, and so the time is $(T/2\pi)\times\arcsin\,(12.0/18.0) = 1.37\times10^{-1}$ s. Note that this is also $\arcsin\,(x/A)/\omega$.

d) The conservation of energy equation can be written $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. We are given amplitude, frequency in Hz, and various values of x. We could calculate velocity from this information if we use the relationship $k/m = \omega^2 = 4\pi^2 f^2$ and rewrite the conservation equation as $\frac{1}{2}A^2 = \frac{1}{2}\frac{v^2}{4\pi^2 f^2} + \frac{1}{2}x^2$. Using energy principles is generally a good approach when we are dealing with velocities and positions as opposed to accelerations and time when using dynamics is often easier.

13.26: In the example, $A_2 = A_1\sqrt{\frac{M}{M+m}}$ and now we want $A_2 = \frac{1}{2}A_1$. So $\frac{1}{2} = \sqrt{\frac{M}{M+m}}$, or $m = 3M$. For the energy, $E_2 = \frac{1}{2}kA_2^2$, but since $A_2 = \frac{1}{2}A_1$, $E_2 = \frac{1}{4}E_1$, or $\frac{3}{4}E_1$ is lost to heat.

13.27: a) $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = 0.0284 \text{ J}$.

b) $\sqrt{x_0^2 + \dfrac{v_0^2}{\omega^2}} = \sqrt{(0.012\,\text{m})^2 + \dfrac{(0.300\,\text{m/s})^2}{(300\,\text{N/m})/(0.150\,\text{kg})}} = 0.014\,\text{m}.$

c) $\omega A = \sqrt{k/m}A = 0.615\,\text{m/s}\cdot$

13.28: At the time in question we have

$$x = A\cos(\omega t + \phi) = 0.600\,\text{m}$$
$$v = -\omega A\sin(\omega t + \phi) = 2.20\,\text{m/s}$$
$$a = -\omega^2 A\cos(\omega t + \phi) = -8.40\,\text{m/s}^2$$

Using the displacement and acceleration equations:

$$-\omega^2 A\cos(\omega t + \phi) = -\omega^2(0.600\,\text{m}) = -8.40\,\text{m/s}^2$$

$\omega^2 = 14.0$ and $\omega = 3.742\,\text{s}^{-1}$ To find A, multiply the velocity equation by ω:

$$-\omega^2 A\sin(\omega t + \phi) = (3.742\,\text{s}^{-1})(2.20\,\text{m/s}) = 8.232\,\text{m/s}^2$$

Next square both this new equation and the acceleration equation and add them:

$$\omega^4 A^2 \sin^2(\omega t + \phi) + \omega^4 A^2 \cos^2(\omega t + \phi) = (8.232\,\text{m/s}^2)^2 + (-8.40\,\text{m/s}^2)^2$$
$$= \omega^4 A^2 \sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)$$
$$\omega^4 A^2 = 67.77\,\text{m}^2/\text{s}^4 + 70.56\,\text{m}^2/\text{s}^4 = 138.3\,\text{m}^2/\text{s}^4$$
$$A^2 = \frac{138.3\,\text{m}^2/\text{s}^4}{\omega^4} = \frac{138.3\,\text{m}^2/\text{s}^4}{(3.742\,\text{s}^{-1})^4} = 0.7054\,\text{m}^2$$
$$A = 0.840\,\text{m}$$

The object will therefore travel $0.840\,\text{m} - 0.600\,\text{m} = 0.240\,\text{m}$ to the right before stopping at its maximum amplitude.

13.29: $v_{\text{max}} = A\sqrt{k/m}$

Use T to find k/m:

$T = 2\pi\sqrt{m/k}$ so $k/m = (2\pi/T)^2 = 158\,\text{s}^{-2}$

Use a_{max} to find A:

$a_{\text{max}} = kA/m$ so $A = a_{\text{max}}/(k/m) = 0.0405\,\text{m}.$

Then $v_{\text{max}} = A\sqrt{k/m} = 0.509\,\text{m/s}$

13.30: Using $k = \frac{F_0}{L_0}$ from the calibration data,

$$m = \frac{(F_0/L_0)}{(2\pi f)^2} = \frac{(200\ \text{N})/(1.25\times 10^{-1}\ \text{m})}{(2\pi(2.60\ \text{Hz}))^2} = 6.00\ \text{kg}.$$

13.31: a) $k = \dfrac{mg}{\Delta l} = \dfrac{(650\ \text{kg})\,(9.80\ \text{m/s}^2)}{(0.120\ \text{m})} = 531\times 10^3\ \text{N/m}.$

b) $T = 2\pi = \sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{\Delta l}{g}} = 2\pi\sqrt{\dfrac{0.120\ \text{m}}{9.80\ \text{m/s}^2}} = 0.695\ \text{s}.$

13.32: a) At the top of the motion, the spring is unstretched and so has no potential energy, the cat is not moving and so has no kinetic energy, and the gravitational potential energy relative to the bottom is $2mgA = 2(4.00\ \text{kg})(9.80\ \text{m/s}^2)\times(0.050\ \text{m}) = 3.92\ \text{J}$. This is the total energy, and is the same total for each part.

b) $U_{\text{grav}} = 0, K = 0, \text{so}\, U_{\text{spring}} = 3.92\ \text{J}$.

c) At equilibrium the spring is stretched half as much as it was for part (a), and so $U_{\text{spring}} = \frac{1}{4}(3.92\ \text{J}) = 0.98\ \text{J}, U_{\text{grav}} = \frac{1}{2}(3.92\ \text{J}) = 1.96\ \text{J}, \text{and so}\ K = 0.98\ \text{J}$.

13.33: The elongation is the weight divided by the spring constant,

$$\Delta l = \frac{w}{k} = \frac{mg}{\omega^2 m} = \frac{gT^2}{4\pi^2} = 3.97\ \text{cm}.$$

13.34: See Exercise 9.40. a) The mass would decrease by a factor of $(1/3)^3 = 1/27$ and so the moment of inertia would decrease by a factor of $(1/27)(1/3)^2 = (1/243)$, and for the same spring constant, the frequency and angular frequency would increase by a factor of $\sqrt{243} = 15.6$. b) The torsion constant would need to be *decreased* by a factor of 243, or changed by a factor of 0.00412 (approximately).

13.35: a) With the approximations given, $I = mR^2 = 2.72\times 10^{-8}\ \text{kg}\cdot\text{m}^2$, or $2.7\times 10^{-8}\ \text{kg}\cdot\text{m}^2$ to two figures.

b) $\kappa = (2\pi f)^2 I = (2\pi 2\ \text{Hz})^2(2.72\times 10^{-8}\ \text{kg}\cdot\text{m}^2) = 4.3\times 10^{-6}\ \text{N}\cdot\text{m/rad}$.

13.36: Solving Eq. (13.24) for κ in terms of the period,

$$\begin{aligned}\kappa &= \left(\frac{2\pi}{T}\right)^2 I \\ &= \left(\frac{2\pi}{1.00\ \text{s}}\right)^2((1/2)(2.00\times 10^{-3}\ \text{kg})(2.20\times 10^{-2}\ \text{m})^2) \\ &= 1.91\times 10^{-5}\ \text{N}\cdot\text{m/rad}.\end{aligned}$$

13.37:

$$I = \frac{\kappa}{(2\pi f)^2} = \frac{0.450\ \text{N}\cdot\text{m/rad}}{\left(2\pi(125)/(265\,\text{s})\right)^2} = 0.0152\ \text{kg}\cdot\text{m}^2.$$

13.38: The equation $\theta = \Theta\cos(\omega t + \varphi)$ describes angular SHM. In this problem, $\varphi = 0$.

a) $\frac{d\theta}{dt} = -\omega\,\Theta\sin(\omega\,t)$ and $\frac{d^2\theta}{dt^2} = -\omega^2\,\Theta\cos(\omega\,t)$.

b) When the angular displacement is $\Theta, \Theta = \Theta\cos(\omega\,t)$, and this occurs at $t = 0$, so

$$\frac{d\theta}{dt} = 0 \text{ since } \sin(0) = 0, \text{ and } \frac{d^2\theta}{dt^2} = -\omega^2\Theta, \text{ since } \cos(0) = 1.$$

When the angular displacement is $\Theta/2, \frac{\Theta}{2} = \Theta\cos(\omega\,t)$, or $\frac{1}{2} = \cos(\omega\,t)$.

$$\frac{d\theta}{dt} = \frac{-\omega\Theta\sqrt{3}}{2} \text{ since } \sin(\omega\,t) = \frac{\sqrt{3}}{2}, \text{ and } \frac{d^2\theta}{dt^2} = \frac{-\omega^2\Theta}{2}, \text{ since } \cos(\omega\,t) = 1/2.$$

This corresponds to a displacement of $60°$.

13.39: Using the same procedure used to obtain Eq. (13.29), the potential may be expressed as

$$U = U_0[(1 + x/R_0)^{-12} - 2(1 + x/R_0)^{-6}].$$

Note that at $r = R_0, U = -U_0$. Using the appropriate forms of the binomial theorem for $|\,x/R_0\,| << 1$,

$$U \approx U_0\left[\begin{array}{l}\left(1 - 12(x/R_0) + \dfrac{(-12)(-13)}{2}(x/R_0)^2\right) \\ -2\left(1 - 6(x/R_0) + \dfrac{(-6)(-7)}{2}(x/R_0)^2\right)\end{array}\right]$$

$$= U_0\left[-1 + \frac{36}{R_0^2}x^2\right]$$

$$= \frac{1}{2}kx^2 - U_0.$$

where $k = 72U_0/R^2$ has been used. Note that terms in u^2 from Eq. (13.28) must be kept ; the fact that the first-order terms vanish is another indication that R_0 is an extreme (in this case a minimum) of U.

13.40: $f = \frac{1}{2\pi}\sqrt{\frac{k}{(m/2)}} = \frac{1}{2\pi}\sqrt{\frac{2(580\ \text{N/m})}{(1.008)(1.66\times10^{-27}\ \text{kg})}} = 1.33\times10^{14}\ \text{Hz}.$

13.41: $T = 2\pi\sqrt{L/g}$, so for a different acceleration due to gravity g',

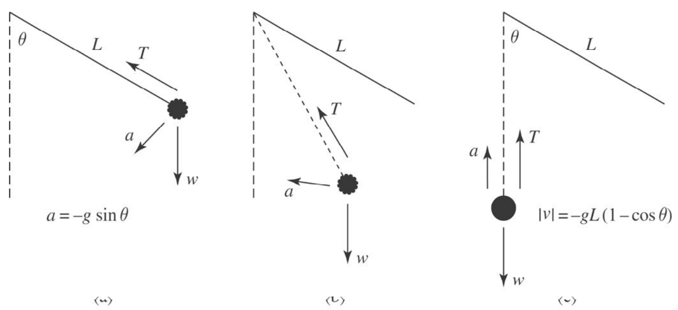
$$T' = T\sqrt{g/g'} = (1.60\,\text{s})\sqrt{9.80\ \text{m/s}^2/3.71\,\text{m/s}^2} = 2.60\,\text{s}.$$

13.42: a) To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\frac{T}{4}=\frac{\pi}{2}\sqrt{\frac{L}{g}}=0.25\,\text{s}$.

b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

13.43: Besides approximating the pendulum motion as SHM, assume that the angle is sufficiently small that the length of the spring does not change while swinging in the arc. Denote the angular frequency of the vertical motion as $\omega_0=\sqrt{\frac{k}{m}}=\sqrt{\frac{kg}{\omega}}$ and $\omega'=\sqrt{\frac{g}{L}}$ $=\frac{1}{2}\omega_0=\sqrt{\frac{kg}{4w}}$, which is solved for $L=4w/k$. But L is the length of the stretched spring; the unstretched length is $L_0=L-w/k=3w/k=3(1.00\text{ N})/(1.50\text{ N/m})=2.00\text{ m}$.

13.44:



13.45: The period of the pendulum is $T=(136\text{ s})/100=1.36\text{ s}$. Then,

$$g=\frac{4\pi^2L}{T^2}=\frac{4\pi^2(.5\text{ m})}{(1.36\text{ s})^2}=10.67\text{ m/s}^2.$$

13.46: From the parallel axis theorem, the moment of inertia of the hoop about the nail is $I=MR^2+MR^2=2MR^2$, so $T=2\pi\sqrt{2R/g}$, with $d=R$ in Eq.(13.39). Solving for R, $R=gT^2/8\pi^2=0.496\text{ m}$.

13.47: For the situation described, $I=mL^2$ and $d=L$ in Eq. (13.39); canceling the factor of m and one factor of L in the square root gives Eq. (13.34).

13.48: a) Solving Eq. (13.39) for *I*,

$$I = \left(\frac{T}{2\pi}\right)^2 mgd = \left(\frac{0.940\ \text{s}}{2\pi}\right)^2 (1.80\ \text{kg})(9.80\ \text{m/s}^2)(0.250\ \text{m}) = 0.0987\ \text{kg}\cdot\text{m}^2.$$

b) The small-angle approximation will not give three-figure accuracy for $\Theta = 0.400$ rad. From energy considerations,

$$mgd(1-\cos\Theta) = \frac{1}{2} I\Omega_{\text{max}}^2.$$

Expressing Ω_{max} in terms of the period of small-angle oscillations, this becomes

$$\Omega_{\text{max}} = \sqrt{2\left(\frac{2\pi}{T}\right)^2 (1-\cos\Theta)} = \sqrt{2\left(\frac{2\pi}{0.940\ \text{s}}\right)^2 (1-\cos(0.40\ \text{rad}))} = 2.66\ \text{rad/s}.$$

13.49: Using the given expression for *I* in Eq. (13.39), with *d*=*R* (and of course *m*=*M)*, $T = 2\pi\sqrt{5R/3g} = 0.58\ \text{s}$.

13.50: From Eq. (13.39),

$$I = mgd\left(\frac{T}{2\pi}\right)^2 = (1.80\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m})\left(\frac{120\ \text{s}/100}{2\pi}\right)^2 = 0.129\ \text{kg.m}^2.$$

13.51: a) From Eq. (13.43),

$$\omega' = \sqrt{\frac{(2.50\ \text{N/m})}{(0.300\ \text{kg})} - \frac{(0.90\ \text{kg/s})^2}{4(0.300\ \text{kg})^2}} = 2.47\ \text{rad/s, so}\ f' = \frac{\omega'}{2\pi} = 0.393\ \text{Hz}.$$

b) $b = 2\sqrt{km} = 2\sqrt{(2.50\ \text{N/m})(0.300\ \text{kg})} = 1.73\ \text{kg/s}.$

13.52: From Eq. (13.42) $A_2 = A_1 \exp\left(-\frac{b}{2m}t\right)$. Solving for *b*,

$$b = \frac{2m}{t}\ln\left(\frac{A_1}{A_2}\right) = \frac{2(0.050\ \text{kg})}{(5.00\ \text{s})}\ln\left(\frac{0.300\ \text{m}}{0.100\ \text{m}}\right) = 0.0220\ \text{kg/s}.$$

As a check, note that the oscillation frequency is the same as the undamped frequency to $4.8\times10^{-3}\%$, so Eq. (13.42) is valid.

13.53: a) With $\phi = 0, x(0) = A$.

b) $$v_x = \frac{dx}{dt} = Ae^{-(b/2m)t}\left[-\frac{b}{2m}\cos\omega' t - \omega' \sin\omega' t\right],$$

and at $t = 0, v = -Ab/2m$; the graph of x versus t near $t = 0$ slopes down.

c) $$a_x = \frac{dv_x}{dt} = Ae^{-(b/2m)t}\left[\left(\frac{b^2}{4m^2} - \omega'^2\right)\cos\omega' t + \frac{\omega' b}{2m}\sin\omega' t\right],$$

and at $t = 0$,

$$a_x = A\left(\frac{b^2}{4m^2} - \omega'^2\right) = A\left(\frac{b^2}{2m^2} - \frac{k}{m}\right).$$

(Note that this is $(-bv_0 - kx_0)/m$.) This will be negative if $b < \sqrt{2km}$, zero if $b = \sqrt{2km}$ and positive if $b > \sqrt{2km}$. The graph in the three cases will be curved down, not curved, or curved up, respectively.

13.54: At resonance, Eq. (13.46) reduces to $A = F_{\mathrm{max}}/b\omega_{\mathrm{d}}$. a) $\frac{A_1}{3}$. b) $2A_1$. Note that the resonance frequency is independent of the value of b (see Fig. (13.27)).

13.55: a) The damping constant has the same units as force divided by speed, or $[\mathrm{kg\cdot m/s^2}]/[\mathrm{m/s}] = [\mathrm{kg/s}]$. b)The units of $\sqrt{km}$ are the same as $[[\mathrm{kg/s^2}][\mathrm{kg}]]^{1/2} = [\mathrm{kg/s}]$, the same as those for b. c) $\omega_{\mathrm{d}}^2 = k/m$. (i) $b\omega_{\mathrm{d}} = 0.2\,k$, so $A = F_{\mathrm{max}}/(0.2k) = 5F_{\mathrm{max}}/k$. (ii) $b\omega_{\mathrm{d}} = 0.4k$, so $A = F_{\mathrm{max}}/(0.4\,k) = 2.5F_{\mathrm{max}}/k$, as shown in Fig.(13.27).

13.56: The resonant frequency is

$$\sqrt{k/m} = \sqrt{(2.1\times10^6\ \mathrm{N/m})/108\ \mathrm{kg})} = 139\ \mathrm{rad/s} = 22.2\ \mathrm{Hz},$$

and this package does not meet the criterion.

13.57: a)

$$a = A\omega^2 = \left(\frac{0.100\,\text{m}}{2}\right)\left((3500\,\text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2 = 6.72\times10^3\ \text{m/s}^2.$$

b) $ma = 3.02\times10^3$ N. c) $\omega A = (3500\,\text{rev/min})(.05\ \text{m})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right) = 18.3\ \text{m/s}.$

$K = \frac{1}{2}mv^2 = (\frac{1}{2})(.45\,\text{kg})(18.3\,\text{m/s})^2 = 75.6\,\text{J}.$ d) At the midpoint of the stroke, cos($\omega\, t$)=0 and so $\omega t = \pi/2$, thus $t = \pi/2\omega$. $\omega = (3500\ \text{rev/min})(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}) = \frac{350\pi}{3}$ rad/s, so $t = \frac{3}{2(350)}$ s. Then $P = \Delta K/\Delta t$, or $P = 75.6\ \text{J}/(\frac{3}{2(350)}\ \text{s}) = 1.76\times10^4\,\text{W}$.

e) If the frequency doubles, the acceleration and hence the needed force will quadruple (12.1×10^3 N). The maximum speed increases by a factor of 2 since $v\ \alpha\ \omega$, so the speed will be 36.7 m/s. Because the kinetic energy depends on the square of the velocity, the kinetic energy will increase by a factor of four (302 J). But, because the time to reach the midpoint is halved, due to the doubled velocity, the power increases by a factor of eight (141 kW).

13.58: Denote the mass of the passengers by *m* and the (unknown) mass of the car by *M*. The spring cosntant is then $k = mg/\Delta l$. The period of oscillation of the empty car is $T_\text{E} = 2\pi\sqrt{M/k}$ and the period of the loaded car is

$$T_\text{L} = 2\pi\sqrt{\frac{M+m}{k}} = \sqrt{T_\text{E}^2 + (2\pi)^2\frac{\Delta l}{g}},\ \text{so}$$

$$T_\text{E} = \sqrt{T_\text{L}^2 - (2\pi)^2\frac{\Delta l}{g}} = 1.003\,\text{s}.$$

13.59: a) For SHM, the period, frequency and angular frequency are independent of amplitude, and are not changed. b) From Eq. (13.31), the energy is decreased by a factor of $\frac{1}{4}$. c) From Eq. (13.23), the maximum speed is decreased by a factor of $\frac{1}{2}$ d) Initially, the speed at $A_1/4$ was $\frac{\sqrt{15}}{4}\omega A_1$; after the amplitude is reduced, the speed is $\omega\sqrt{(A_1/2)^2 - (A_1/4)^2} = \frac{\sqrt{3}}{4}\omega A_1$, so the speed is decreased by a factor of $\frac{1}{\sqrt{5}}$ (this result is valid at $x = -A_1/4$ as well). e) The potential energy depends on position and is unchanged. From the result of part (d), the kinetic energy is decreased by a factor of $\frac{1}{5}$.

13.60: This distance L is $L = mg/k$; the period of the oscillatory motion is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{g}},$$

which is the period of oscillation of a simple pendulum of lentgh *L*.

13.61: a) Rewriting Eq. (13.22) in terms of the period and solving,

$$T = \frac{2\pi\sqrt{A^2 - x^2}}{v} = 1.68\,\text{s}.$$

b) Using the result of part (a),

$$x = \sqrt{A^2 - \left(\frac{vT}{2\pi}\right)^2} = 0.0904\,\text{m}.$$

c) If the block is just on the verge of slipping, the friction force is its maximum, $f = \mu_s n = \mu_s mg$. Setting this equal to $ma = mA(2\pi/T)^2$ gives $\mu_s = A(2\pi/T)^2/g = 0.143$.

13.62: a) The normal force on the cowboy must always be upward if he is not holding on. He leaves the saddle when the normal force goes to zero (that is, when he is no longer in contact with the saddle, and the contact force vanishes). At this point the cowboy is in free fall, and so his acceleration is $-g$; this must have been the acceleration just before he left contact with the saddle, and so this is also the saddle's acceleration.
b) $x = +a/(2\pi f)^2 = +(9.80\,\text{m/s}^2)/2\pi(1.50\,\text{Hz}))^2 = 0.110\,\text{m}$. c) The cowboy's speed will be the saddle's speed, $v = (2\pi f)\sqrt{A^2 - x^2} = 2.11\,\text{m/s}$. d) Taking $t = 0$ at the time when the cowboy leaves, the position of the saddle as a function of time is given by Eq. (13.13), with $\cos\phi = -\frac{g}{\omega^2 A}$; this is checked by setting $t = 0$ and finding that $x = \frac{g}{\omega^2} = -\frac{a}{\omega^2}$. The cowboy's position is $x_c = x_0 + v_0 t - (g/2)t^2$. Finding the time at which the cowboy and the saddle are again in contact involves a transcendental equation which must be solved numerically; specifically,

$$(0.110\,\text{m}) + (2.11\,\text{m/s})t - (4.90\,\text{m/s}^2)t^2 = (0.25\,\text{m})\cos((9.42\,\text{rad/s})t - 1.11\,\text{rad}),$$

which has as its least non-zero solution $t = 0.538\,\text{s}$. e) The speed of the saddle is $(-2.36\ \text{m/s})\sin(\omega t + \phi) = 1.72\,\text{m/s}$, and the cowboy's speed is $(2.11\ \text{m/s}) - (9.80\,\text{m/s}^2) \times (0.538\,\text{s}) = -3.16\,\text{m/s}$, giving a relative speed of $4.87\,\text{m/s}$ (extra figures were kept in the intermediate calculations).

13.63: The maximum acceleration of both blocks, assuming that the top block does not slip, is $a_{\text{max}} = kA/(m + M)$, and so the maximum force on the top block is $\left(\frac{m}{m+M}\right)kA = \mu_s mg$, and so the maximum amplitude is $A_{\text{max}} = \mu_s (m + M)g/k$.

13.64: (a) Momentum conservation during the collision: $mv_0 = (2m)V$

$$V = \frac{1}{2}v_0 = \frac{1}{2}(2.00\ \text{m/s}) = 1.00\ \text{m/s}$$

Energy conservation after the collision:

$$\frac{1}{2}MV^2 = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{MV^2}{k}} = \sqrt{\frac{(20.0\ \text{kg})(1.00\ \text{m/s})^2}{80.0\ \text{N/m}}} = 0.500\ \text{m (amplitude)}$$

$$\omega = 2\pi f = \sqrt{k/M}$$

$$f = \frac{1}{2\pi}\sqrt{k/M} = \frac{1}{2\pi}\sqrt{\frac{80.0\ \text{N/m}}{20.0\ \text{kg}}} = 0.318\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{0.318\ \text{Hz}} = 3.14\ \text{s}$$

(b) It takes $1/2$ period to first return: $\frac{1}{2}(3.14\ \text{s}) = 1.57\ \text{s}$

13.65: a) $m \to m/2$

Splits at $x = 0$ where energy is all kinetic energy, $E = \frac{1}{2}mv^2$, so $E \to E/2$

k stays same

$E = \frac{1}{2}kA^2$ so $A = \sqrt{2E/k}$

Then $E \to E/2$ means $A \to A/\sqrt{2}$

$T = 2\pi\sqrt{m/k}$ so $m \to m/2$ means $T \to T/\sqrt{2}$

b) $m \to m/2$

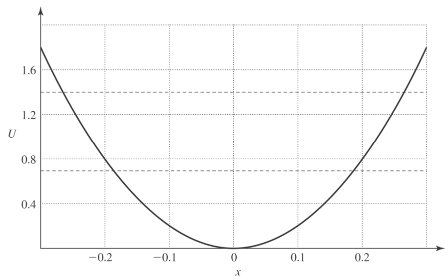
Splits at $x = A$ where all the energy is potential energy in the spring, so E doesn't change.

$E = \frac{1}{2}kA^2$ so A stays the same.

$T = 2\pi\sqrt{m/k}$ so $T \to T/\sqrt{2}$, as in part (a).

c) In example 13.5, the mass increased. This means that T increases rather than decreases. When the mass is added at $x = 0$, the energy and amplitude change. When the mass is added at $x = \pm A$, the energy and amplitude remain the same. This is the same as in this problem.

13.66: a)



For space considerations, this figure is not precisely to the scale suggested in the problem. The following answers are found algebraically, to be used as a check on the graphical method.

b) $$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.200\,\text{J})}{(10.0\,\text{N/m})}} = 0.200\,\text{m}.$$

c) $\frac{E}{4} = 0.050\,\text{J}$. d) If $U = \frac{1}{2}E$, $x = \frac{A}{\sqrt{2}} = 0.141\,\text{m}$. e) From Eq. (13.18), using $v_0 = -\sqrt{\frac{2K_0}{m}}$ and $x_0 = \sqrt{\frac{2U_0}{k}}$,

$$-\frac{v_0}{\omega\, x_0} = \frac{\sqrt{\frac{2K_0}{m}}}{\sqrt{\frac{k}{m}}\sqrt{\frac{2U_0}{k}}} = \sqrt{\frac{K_0}{U_0}} = \sqrt{0.429}$$

and $\phi = \arctan\left(\sqrt{0.429}\right) = 0.580\,\text{rad}$.

13.67: a) The quantity Δl is the amount that the origin of coordinates has been moved from the unstretched length of the spring, so the spring is stretched a distance $\Delta l - x$ (see Fig. (13.16 (c))) and the elastic potential energy is $U_{\text{el}} = (1/2)k(\Delta l - x)^2$.

b) $$U = U_{\text{el}} + mg(x - x_0) = \frac{1}{2}kx^2 + \frac{1}{2}(\Delta l)^2 - k\Delta lx + mgx - mgx_0.$$

Since $\Delta l = mg/k,$ the two terms proportional to x cancel, and

$$U = \frac{1}{2}kx^2 + \frac{1}{2}k(\Delta l)^2 - mgx_0.$$

c) An additive constant to the mechanical energy does not change the dependence of the force on $x,\ F_x = -\frac{dU}{dx},$ and so the relations expressing Newton's laws and the resulting equations of motion are unchanged.

13.68: The "spring constant" for this wire is $k = \frac{mg}{\Delta l},$ so

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta l}} = \frac{1}{2\pi}\sqrt{\frac{9.80\ \text{m/s}^2}{2.00\times 10^{-3}\ \text{m}}} = 11.1\,\text{Hz}.$$

13.69: a) $\frac{2\pi A}{T} = 0.150\ \text{m/s}.$ b) $a = -(2\pi/T)^2 x = -0.112\ \text{m/s}^2.$ The time to go from equilibrium to half the amplitude is $\sin\omega t = (1/2),$ or $\omega t = \pi/6\ \text{rad},$ or one-twelfth of a period. The needed time is twice this, or one-sixth of a period, 0.70 s.
d) $\Delta l = \frac{mg}{k} = \frac{g}{\omega^2} = \frac{g}{(2\pi/r)^2} = 4.38\,\text{m}.$

13.70: Expressing Eq. (13.13) in terms of the frequency, and with $\phi = 0$, and taking two derivatives,

$$x = (0.240\,\text{m})\cos\left(\frac{2\pi t}{1.50\,\text{s}}\right)$$

$$v_x = -\left(\frac{2\pi(0.240\,\text{m})}{(1.50\,\text{s})}\right)\sin\left(\frac{2\pi t}{1.50\,\text{s}}\right) = -(1.00530\,\text{m/s})\sin\left(\frac{2\pi t}{1.50\,\text{s}}\right)$$

$$a_x = -\left(\frac{2\pi}{1.50\,\text{s}}\right)^2(0.240\,\text{m})\cos\left(\frac{2\pi t}{1.50\,\text{s}}\right) = -(4.2110\,\text{m/s}^2)\cos\left(\frac{2\pi t}{1.50\,\text{s}}\right).$$

a) Substitution gives $x = -0.120\,\text{m}$, or using $t = \frac{T}{3}$ gives $x = A\cos 120° = \frac{-A}{2}$.
b) Substitution gives
$ma_x = +(0.0200\,\text{kg})(2.106\,\text{m/s}^2) = 4.21\times10^{-2}$ N, in the $+x$-direction.
c) $t = \frac{T}{2\pi}\arccos\left(\frac{-3A/4}{A}\right) = 0.577\,\text{s}.$
d) Using the time found in part (c), $v = 0.665\,\text{m/s}$ (Eq.(13.22) of course gives the same result).

13.71: a) For the totally inelastic collision, the final speed v in terms of the initial speed $V = \sqrt{2gh}$ is

$v = V\frac{M}{m+M} = \sqrt{2(9.80\,\text{m/s}^2)(0.40\,\text{m})}\left(\frac{2.2}{2.4}\right) = 2.57$ m/s, or 2.6 m/s to two figures. b) When the steak hits, the pan is $\frac{Mg}{k}$ above the new equilibrium position. The ratio $\frac{v_0^2}{\omega^2}$ is $v^2/(k/(m+M)) = 2ghM^2/k(m+M)$, and so the amplitude of oscillation is

$$\begin{aligned} A &= \sqrt{\left(\frac{Mg}{k}\right)^2 + \frac{2ghM^2}{k(m+M)}} \\ &= \sqrt{\left(\frac{(2.2\,\text{kg})(9.80\,\text{m/s}^2)}{(400\,\text{N/m})}\right)^2 + \frac{2(9.80\,\text{m/s}^2)(0.40\,\text{m})(2.2\,\text{kg})^2}{(400\,\text{N/m})(2.4\,\text{kg})}} \\ &= 0.206\,\text{m}. \end{aligned}$$

(This avoids the intermediate calculation of the speed.) c) Using the total mass, $T = 2\pi\sqrt{(m+M)/k} = 0.487\,\text{s}.$

13.72: $f = 0.600\ \text{Hz}, m = 400\ \text{kg};\ f = \frac{1}{2}\sqrt{\frac{k}{m}}$ gives $k = 5685\ \text{N/m}$.

This is the effective force constant of the two springs.

a) After the gravel sack falls off, the remaining mass attached to the springs is 225 kg. The force constant of the springs is unaffected, so $f = 0.800\ \text{Hz}$.

To find the new amplitude use energy considerations to find the distance downward that the beam travels after the gravel falls off.

Before the sack falls off, the amount x_0 that the spring is stretched at equilibrium is given by $mg - kx_0$, so $x_0 = mg/k = (400\ \text{kg})(9.80\ \text{m/s}^2)/(5685\ \text{N/m}) = 0.6895\ \text{m}$. The maximum upward displacement of the beam is $A = 0.400\ \text{m}$. above this point, so at this point the spring is stretched 0.2895 m.

With the new mass, the mass 225 kg of the beam alone, at equilibrium the spring is stretched $mg/k = (225\ \text{kg})\ (9.80\ \text{m/s}^2)/(5685\ \text{N/m}) = 0.6895\ \text{m}$. The new amplitude is therefore $0.3879\ \text{m} - 0.2895\ \text{m} = 0.098\ \text{m}$. The beam moves 0.098 m above and below the new equilibrium position. Energy calculations show that $v = 0$ when the beam is 0.098 m above and below the equilibrium point.

b) The remaining mass and the spring constant is the same in part (a), so the new frequency is again $0.800\ \text{Hz}$.

The sack falls off when the spring is stretched 0.6895 m. And the speed of the beam at this point is $v = A\sqrt{k/m} = (0.400\ \text{m}) = \sqrt{(5685\ \text{N/m})/(400\ \text{kg})} = 1.508\ \text{m/s}.$. Take $y = 0$ at this point. The total energy of the beam at this point, just after the sack falls off, is $E = K + U_{\text{el}} + U_{\text{g}} = \frac{1}{2}(225\ \text{kg})(1.508\ \text{m/s}^2) + \frac{1}{2}(5695\ \text{N/m})(0.6895\ \text{m})^2 + 0 = 1608\ \text{J}$. Let this be point 1. Let point 2 be where the beam has moved upward a distance d and where $v = 0$. $E_2 = \frac{1}{2}k(0.6985\ \text{m} - d)^2 + mgd$. $E_1 = E_2$ gives $d = 0.7275\ \text{m}$. At this end point of motion the spring is compressed 0.7275 m – 0.6895 m =0.0380 m. At the new equilibrium position the spring is stretched 0.3879 m, so the new amplitude is 0.3789 m + 0.0380 m = 0.426 m. Energy calculations show that v is also zero when the beam is 0.426 m below the equilibrium position.

13.73: The pendulum swings through $\frac{1}{2}$ cycle in 1.42 s, so $T = 2.84\ \text{s}$. $L = 1.85\ \text{m}$.

Use T to find g:

$T = 2\pi\sqrt{L/g}$ so $g = L(2\pi/T)^2 = 9.055\ \text{m/s}^2$

Use g to find the mass M_{p} of Newtonia:

$$g = GM_{\text{p}}/R_{\text{p}}^2$$

$$2\pi R_{\text{p}} = 5.14\times10^7\ \text{m, so}\ R_{\text{p}} = 8.18\times10^6\ \text{m}$$

$$m_{\text{p}} = \frac{gR_{\text{p}}^2}{G} = 9.08\times10^{24}\ \text{kg}$$

13.74: a) Solving Eq. (13.12) for m, and using $k = \frac{F}{\Delta l}$

$$m = \left(\frac{T}{2\pi}\right)^2 \frac{F}{\Delta l} = \left(\frac{1}{2\pi}\right)^2 \frac{40.0\ \text{N}}{0.250\ \text{m}} = 4.05\ \text{kg}.$$

b) $t = (0.35)T$, and so $x = -A\sin 2\pi(0.35) = -0.0405\ \text{m}$. Since $t > \frac{T}{4}$, the mass has already passed the lowest point of its motion, and is on the way up.

c) Taking upward forces to be positive, $F_{\text{spring}} - mg = -kx$, where x is the displacement from equilibrium , so

$$F_{\text{spring}} = -(160\ \text{N/m})(-0.030\ \text{m}) + (4.05\ \text{kg})(9.80\ \text{m/s}^2) = 44.5\ \text{N}.$$

13.75: Of the many ways to find the time interval, a convenient method is to take $\phi = 0$ in Eq. (13.13) and find that for $x = A/2, \cos\omega t = \cos(2\pi t/T) = \frac{1}{2}$ and so $t = T/6$. The time interval available is from $-t$ to t, and $T/3 = 1.17\ \text{s}$.

13.76: See Problem 12.84; using x as the variable instead of r,

$$F(x) = -\frac{dU}{dx} = -\frac{GM_{\text{E}}m}{R_{\text{E}}^3}x, \text{ so } \omega^2 = \frac{GM_{\text{E}}}{R_{\text{E}}^3} = \frac{g}{R_{\text{E}}}.$$

The period is then

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R_{\text{E}}}{g}} = 2\pi\sqrt{\frac{6.38\times 10^6\ \text{m}}{9.80\ \text{m/s}^2}} = 5070\ \text{s},$$

or 84.5 min.

13.77: Take only the positive root (to get the least time), so that

$$\frac{dx}{dt} = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}\text{, or}$$

$$\frac{dx}{\sqrt{A^2 - x^2}} = \sqrt{\frac{k}{m}}dt$$

$$\int_0^A \frac{dx}{\sqrt{A^2 - x^2}} = \sqrt{\frac{k}{m}}\int_0^{t_1} dt = \sqrt{\frac{k}{m}}(t_1)$$

$$\arcsin(1) = \sqrt{\frac{k}{m}}t_1$$

$$\frac{\pi}{2} = \sqrt{\frac{k}{m}}t_1,$$

where the integral was taken from Appendix C. The above may be rearranged to show that $t_1 = \frac{\pi}{2\sqrt{\frac{k}{m}}} = \frac{T}{4}$, which is expected.

13.78: a) $$U = -\int_0^x F\,dx = c\int_0^x x^3\,dx = \frac{c}{4}x^4.$$

a) From conservation of energy, $\frac{1}{2}mv^2 = \frac{c}{4}(A^4 - x^4)$, and using the technique of Problem 13.77, the separated equation is

$$\frac{dx}{\sqrt{A^4 - x^4}} = \sqrt{\frac{c}{2m}}dt.$$

Integrating from 0 to A with respect to x and from 0 to $T/4$ with respect to t,

$$\int_0^A \frac{dx}{\sqrt{A^4 - x^4}} = \sqrt{\frac{c}{2m}}\frac{T}{4}.$$

To use the hint, let $u = \frac{x}{A}$, so that $dx = a\,du$ and the upper limit of the $u-$integral is $u = 1.$ Factoring A^2 out of the square root,

$$\frac{1}{A}\int_0^1 \frac{du}{\sqrt{1-u^4}} = \frac{1.31}{A} = \sqrt{\frac{c}{32m}}T,$$

which may be expressed as $T = \frac{7.41}{A}\sqrt{\frac{m}{c}}.$ c) The period does depend on amplitude, and the motion is not simple harmonic.

13.79: As shown in Fig. (13.5(b)), $v = -v_{\tan}\sin\theta.$ With $v_{\tan} = A\omega$ and $\theta = \omega t + \phi,$ this is Eq. (13.15).

13.80: a) Taking positive displacements and forces to be upwad, $n - mg = ma, a = -(2\pi f)^2 x,$ so

$$n = m\left(g - (2\pi f)^2 A\cos((2\pi f)t + \phi)\right).$$

a) The fact that the ball bounces means that the ball is no longer in contact with the lens, and that the normal force goes to zero periodically. This occurs when the amplitude of the acceleration is equal to g, or when

$$g = (2\pi f_{\mathrm{b}})^2 A.$$

13.81: a) For the center of mass to be at rest, the total momentum must be zero, so the momentum vectors must be of equal magnitude but opposite directions, and the momenta can be represented as $\vec{\boldsymbol{p}}$ and $-\vec{\boldsymbol{p}}$.

b) $$K_{\text{tot}} = 2 \times \frac{p^2}{2m} = \frac{p^2}{2(m/2)}.$$

c) The argument of part (a) is valid for any masses. The kinetic energy is

$$K_{\text{tot}} = \frac{p^2}{2m_1} + \frac{p^2}{2m_2} = \frac{p^2}{2}\left(\frac{m_1 + m_2}{m_1 m_2}\right) = \frac{p^2}{2(m_1 m_2/(m_1 + m_2))}.$$

13.82: a) $$F_r = -\frac{dU}{dr} = A\left[\left(\frac{R_0^7}{r^9}\right) - \frac{1}{r^2}\right].$$

b) Setting the above expression for F_r equal to zero, the term in square brackets vanishes, so that $\frac{R_0^7}{r^9} = \frac{1}{r^2}$, or $R_0^7 = r^7$, and $r = R_0$.

c) $$U(R_0) = -\frac{7A}{8R_0} = -7.57 \times 10^{-19}\ \text{J}.$$

d) The above expression for F_r can be expressed as

$$\begin{aligned}
F_r &= \frac{A}{R_0^2}\left[\left(\frac{r}{R_0}\right)^{-9} - \left(\frac{r}{R_0}\right)^{-2}\right] \\
&= \frac{A}{R_0^2}\left[(1 + (x/R_0))^{-9} - (1 + (x/R_0))^{-2}\right] \\
&\approx \frac{A}{R_0^2}\left[(1 - 9(x/R_0)) - (1 - 2(x/R_0))\right] \\
&= \frac{A}{R_0^2}(-7x/R_0) \\
&= -\left(\frac{7A}{R_0^3}\right)x.
\end{aligned}$$

e) $$f = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{\frac{7A}{R_0^3 m}} = 8.39 \times 10^{12}\,\text{Hz}.$$

13.83: a) $$F_r = -\frac{dU}{dx} = A\left[\frac{1}{r^2} - \frac{1}{(r-2R_0)^2}\right].$$

b) Setting the term in square brackets equal to zero, and ignoring solutions with $r < 0$ or $r > 2R_0, r = 2R_0 - r$, or $r = R_0$.

c) The above expression for F_r may be written as

$$\begin{aligned} F_r &= \frac{A}{R_0^2}\left[\left(\frac{r}{R_0}\right)^{-2} - \left(\frac{r}{R_0} - 2\right)^{-2}\right] \\ &= \frac{A}{R_0^2}\left[(1+(x/R_0))^{-2} - (1-(x/R_0))^{-2}\right] \\ &\approx \frac{A}{R_0^2}\left[(1-2(x/R_0)) - (1-(-2)(x/R_0))\right] \\ &= -\left(\frac{4A}{R_0^3}\right)x, \end{aligned}$$

corresponding to a force constant of $k = 4A/R_0^{\;3}$. d) The frequency of small oscillations would be $f = (1/2\pi)\sqrt{k/m} = (1/\pi)\sqrt{A/mR_0^{\;3}}$.

13.84: a) As the mass approaches the origin, the motion is that of a mass attached to a spring of spring constant k, and the time to reach the origin is $\frac{\pi}{2}\sqrt{m/k}$. After passing through the origin, the motion is that of a mass attached to a spring of spring constant $2k$ and the time it takes to reach the other extreme of the motions is $\frac{\pi}{2}\sqrt{m/2k}$. The period is twice the sum of these times, or $T = \pi\sqrt{\frac{m}{k}}\left(1+\frac{1}{\sqrt{2}}\right)$. The period does not depend on the amplitude, but the motion is not simple harmonic. B) From conservation of energy, if the negative extreme is $A', \frac{1}{2}kA^2 = \frac{1}{2}(2k)A'^2$, so $A' = -\frac{A}{\sqrt{2}}$; the motion is not symmetric about the origin.

13.85: There are many equivalent ways to find the period of this oscillation. Energy considerations give an elegant result. Using the force and torque equations, taking torques about the contact point, saves a few intermediate steps. Following the hint, take torques about the cylinder axis, with positive torques counterclockwise; the direction of positive rotation is then such that $\alpha = Ra$, and the friction force f that causes this torque acts in the $-x$-direction. The equations to solve are then

$$Ma_x = -f - kx, \qquad fR = I_{\text{cm}}\alpha, \quad a = \mathrm{R}\alpha,$$

Which are solved for

$$a_x = \frac{kx}{M + I/R^2} = -\frac{k}{(3/2)\,M}x,$$

where $I = I_{\text{cm}} = (1/2)MR^2$ has been used for the combination of cylinders. Comparison with Eq. (13.8) gives $T = \frac{2\pi}{\omega} = 2\pi\sqrt{3M/2k}.$

13.86: Energy conservation during downward swing:

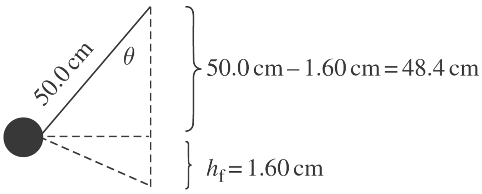
$$m_2 g h_0 = \tfrac{1}{2} m_2 v^2$$
$$v = \sqrt{2gh_0} = \sqrt{2(9.8\ \mathrm{m/s^2})(0.100\ \mathrm{m})} = 1.40\ \mathrm{m/s}$$

Momentum conservation during collision:

$$m_2 v = (m_2 + m_3)V$$
$$V = \frac{m_2 v}{m_2 + m_3} = \frac{(2.00\ \mathrm{kg})(1.40\ \mathrm{m/s})}{5.00\ \mathrm{kg}} = 0.560\ \mathrm{m/s}$$

Energy conservation during upward swing:

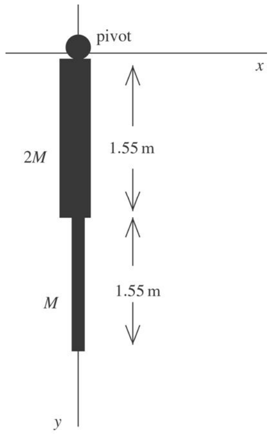
$$Mgh_\mathrm{f} = \frac{1}{2} MV^2$$
$$h_\mathrm{f} = V^2/2g = \frac{(0.560\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s^2})} = 0.0160\ \mathrm{m} = 1.60\ \mathrm{cm}$$



$$\cos\theta = \frac{48.4\ \ \mathrm{cm}}{50.0\ \mathrm{cm}}$$
$$\theta = 14.5°$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{9.80\ \mathrm{m/s^2}}{0.500\ \mathrm{m}}} = 0.705\ \mathrm{Hz}$$

13.87:



$$T = 2\pi\sqrt{I/mgd},\ m = 3M$$

$$d = y_{\text{cg}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$d = \frac{2M([1.55\,\text{m}]/2) + M(1.55\,\text{m} + [1.55\,\text{m}]/2)}{3M} = 1.292\,\text{m}$$

$$I + I_1 + I_2$$

$$I_1 = \tfrac{1}{3}(2M)(1.55\,\text{m})^2 = (1.602\,\text{m}^2)M$$

$$I_{2,\text{cm}} = \tfrac{1}{12}M(1.55\,\text{m})^2$$

The parallel-axis theorem (Eq. 9.19) gives

$$I_2 = I_{2,\text{cm}} + M(1.55\,\text{m} + [1.55\,\text{m}]/2)^2 = (5.06\,\text{m}^2)M$$

$$I = I_1 + I_2 = (7.208\,\text{m}^2)M$$

Then $T = 2\pi\sqrt{I/mgd} = 2\pi\sqrt{\dfrac{(7.208\,\text{m}^2)M}{(3M)(9.80\,\text{m/s}^2)(1.292\,\text{m})}} = 2.74\,\text{s}.$

This is smaller than $T = 2.9\,\text{s}$ found in Example 13.10.

13.88: The torque on the rod about the pivot (with angles positive in the direction indicated in the figure) is $\tau = -\left(k\frac{L}{2}\theta\right)\frac{L}{2}$. Setting this equal to the rate of change of angular momentum, $I\alpha = I\frac{d^2\theta}{dt^2}$,

$$\frac{d^2\theta}{dt^2} = -k\frac{L^2/4}{I}\theta = -\frac{3k}{M}\theta,$$

where the moment of inertia for a slender rod about its center, $I = \frac{1}{12}ML^2$ has been used. It follows that $\omega^2 = \frac{3K}{M}$, and $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{3k}}$.

13.89: The period of the simple pendulum (the clapper) must be the same as that of the bell; equating the expression in Eq. (13.34) to that in Eq. (13.39) and solving for *L* gives $L = I/md = (18.0\ \text{kg}\cdot\text{m}^2)/((34.0\ \text{kg})(0.60\ \text{m})) = 0.882\ \text{m}$. Note that the mass of the bell, not the clapper, is used. As with any simple pendulum, the period of small oscillations of the clapper is independent of its mass.

13.90: The moment of inertia about the pivot is $2(1/3)ML^2 = (2/3)\,ML^2$, and the center of gravity when balanced is a distance $d = L/(2\sqrt{2})$ below the pivot (see Problem 8.95). From Eq. (13.39), the frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{3g}{4\sqrt{2}L}} = \frac{1}{4\pi}\sqrt{\frac{3g}{\sqrt{2}L}}.$$

13.91: a) $L = g(T/2\pi)^2 = 3.97\ \text{m}$. b)There are many possibilities. One is to have a uniform thin rod pivoted about an axis perpendicular to the rod a distance *d* from its center. Using the desired period in Eq. (13.39) gives a quadratic in *d*, and using the maximum size for the length of the rod gives a pivot point a distance of 5.25 mm, which is on the edge of practicality. Using a “dumbbell,” two spheres separated by a light rod of length *L* gives a slight improvement to *d*=1.6 cm (neglecting the radii of the spheres in comparison to the length of the rod; see Problem 13.94).

13.92: Using the notation $\frac{b}{2m} = \gamma, \frac{k}{m} = \omega^2$ and taking derivatives of Eq. (13.42) (setting the phase angle $\phi = 0$ does not affect the result),

$$\begin{aligned} x &= Ae^{-\gamma t} \cos\omega' t \\ v_x &= -A\mathrm{e}^{-\gamma t}(\omega' \sin \omega' t + \gamma \cos \omega' t) \\ a_x &= -A\mathrm{e}^{-\gamma t}((\omega'^2 - \gamma^2) \cos \omega' t - 2\omega' \gamma \sin \omega' t). \end{aligned}$$

Using these expression in the left side of Eq. (13.41),

$$\begin{aligned} -kx - bv_x &= Ae^{-\gamma t}(-k \cos \omega' t + (2\gamma m)\omega' \sin \omega' \mathrm{t} + 2m\gamma^2 \cos \omega' t) \\ &= mA\, e^{-\gamma t}((2\gamma^2 - \omega^2) \cos \omega' t + 2\gamma\omega' \sin \omega' t). \end{aligned}$$

The factor $(2\gamma^2 - \omega^2)$ is $\gamma^2 - \omega'^2$ (this is Eq. (13.43)), and so

$$-kx - bv_x = mA\, e^{-\gamma t}((\gamma^2 - \omega'^2) \cos \omega' t + 2\gamma\omega' \sin \omega' t) = ma_x.$$

13.93: a) In Eq. (13.38), d=x and from the parallel axis theorem, $I = m(L^2/12 + x^2)$, so $\omega^2 = \frac{gx}{(L^2/12) + x^2}$. b) Differentiating the ratio $\omega^2/g = \frac{x}{(L^2/12) + x^2}$ with respect to x and setting the result equal to zero gives

$$\frac{1}{(L^2/12) + x^2} = \frac{2x^2}{((L^2/12) + x^2)^2}, \text{ or } 2x^2 = x^2 + L^2/12,$$

Which is solved for $x = L/\sqrt{12}.$

c) When x is the value that maximizes ω the ratio $\frac{\omega^2}{g} = \frac{L/\sqrt{12}}{2(L^2/12)} = \frac{6}{L\sqrt{12}} = \frac{\sqrt{3}}{L},$

so the length is $L = \frac{\sqrt{3g}}{\omega^2} = 0.430\,\mathrm{m}.$

13.94: a) From the parellel axis theorem, the moment of inertia about the pivot point is $M\left(L^2+(2/5)R^2\right)$.

Using this in Eq. (13.39), With $d=L$ gives.

$$T=2\pi\sqrt{\frac{L^2+(2/5)R^2}{gL}}=2\pi\sqrt{\frac{L}{g}}\sqrt{1+2R^2/5L^2}=T_{\text{sp}}\sqrt{1+2R^2/5L^2}.$$

b) Letting $\sqrt{1+2R^2/5L^2}=1.001$ and solving for the ratio L/R (or approximating the square root as $1+R^2/5L^2$) gives $\frac{L}{R}=14.1.$
c) $(14.1)(1.270\text{ cm})=18.0\text{ cm}.$

13.95: a) The net force on the block at equilibrium is zero, and so one spring (the one with $k_1=2.00\text{ N/m}$) must be stretched three times as much as the one with $k_2=6.00\text{ N/m}$. The sum of the elongations is 0.200 m, and so one spring stretches 0.150 m and the other stretches 0.050 m, and so the equilibrium lengths are 0.350 m and 0.250 m. b) There are many ways to approach this problem, all of which of course lead to the result of Problem 13.96(b). The most direct way is to let $\Delta x_1=0.150\text{ m}$ and $x_2=0.050\text{ m},$ the results of part (a). When the block in Fig.(13.35) is displaced a distance *x* to the right, the net force on the block is

$$-k_1(\Delta x_1+x)+k_2(\Delta x_2-x)=[k_1\Delta x_1-k_2\Delta x_2]-(k_1+k_2)x.$$

From the result of part (a), the term in square brackets is zero, and so the net force is $-(k_1+k_2)x,$ the effective spring constant is $k_{\text{eff}}=k_1+k_2$ and the period of vibration is $T=2\pi\sqrt{\frac{0.100\text{ kg}}{8.00\text{ N/m}}}=0.702\text{ s}.$

13.96: In each situation, imagine the mass moves a distance $\Delta x,$ the springs move distances Δx_1 and $\Delta x_2,$ with forces $F_1 = -k_1 \Delta x_1, F_2 = -k_2 \Delta x_2.$
a) $\Delta x = \Delta x_1 = \Delta x_2, F = F_1 + F_2 = -(k_1 + k_2)\Delta x,$ so $k_{\text{eff}} = k_1 + k_2.$
b) Despite the orientation of the springs, and the fact that one will be compressed when the other is extended, $\Delta x = \Delta x_1 + \Delta x_2,$ and the above result is still valid; $k_{\text{eff}} = k_1 + k_2.$
c) For massless springs, the force on the block must be equal to the tension in any point of the spring combination, and $F = F_1 = F_2,$ and so $\Delta x_1 = -\frac{F}{k_1}, \Delta x_2 = -\frac{F}{k_2},$ and

$$\Delta x = -\left(\frac{1}{k_1} + \frac{1}{k_2}\right) F = -\frac{k_1 + k_2}{k_1 k_2} F$$

and $\kappa_{\text{eff}} = \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$ d) The result of part (c) shows that when a spring is cut in half, the effective spring constant doubles, and so the frequency increases by a factor of $\sqrt{2}.$

13.97: a) Using the hint,

$$T + \Delta T \approx 2\pi\sqrt{L}\left(g^{-1/2} - \frac{1}{2} g^{-3/2} \Delta g\right) = T - T\frac{\Delta g}{2g},$$

so $\Delta T = -(1/2)(T/g)\Delta g.$ This result can also be obtained from $T^2 g = 4\pi^2 L,$ from which

$(2T\Delta T)g + T^2 \Delta g = 0.$ Therefore, $\frac{\Delta T}{T} = -\frac{1}{2}\frac{\Delta g}{g}.$ b) The clock runs slow; $\Delta T > 0,$ $\Delta g < 0$ and $g + \Delta g = g\left(1 - \frac{2\Delta T}{T}\right) = \left(9.80\ \text{m/s}^2\right)\left(1 - \frac{2(4.00\ \text{s})}{(86{,}400\ \text{s})}\right) = 9.7991\ \text{m/s}^2.$

13.98: Denote the position of a piece of the spring by l; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l) = v\frac{l}{L}$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time). a) $dm = \frac{M}{L}dl$, and so

$$dK = \frac{1}{2}dm\,u^2 = \frac{1}{2}\frac{Mv^2}{L^3}l^2\,dl,$$

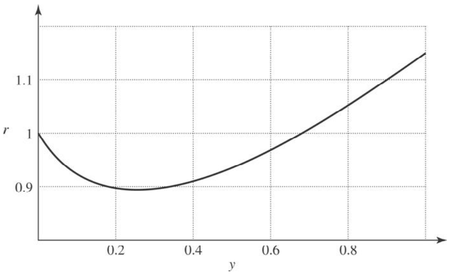
and

$$K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2\,dl = \frac{Mv^2}{6}.$$

b) $mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$, or $ma + kx = 0$, which is Eq.(13.4). c) m is replaced by $\frac{M}{3}$, so $\omega = \sqrt{\frac{3k}{M}}$ and $M' = \frac{M}{3}$.

13.99: a) With $I = (1/3)ML^2$ and $d = L/2$ in Eq.(13.39), $T_0 = 2\pi\sqrt{2L/3g}$. With the addedmass, $I = M\left(\left(L^2/3\right) + y^2\right)$, $m = 2M$ and $d = (L/4) + y/2$, $T = 2\pi$ $\times\sqrt{\left(L^2/3 + y^2\right)/\left(g(L/2 + y)\right)}$ and

$$r = \frac{T}{T_0} = \sqrt{\frac{L^2 + 3y^2}{L^2 + 2yL}}.$$



b) From the expression found in part a), $T = T_0$ when $y = \frac{2}{3}L$. At this point, a simple pendulum with length y would have the same period as the meter stick without the added mass; the two bodies oscillate with the same period and do not affect the other's motion.

13.100: Let the two distances from the center of mass be d_1 and d_2. There are then two relations of the form of Eq. (13.39); with $I_1 = I_{\text{cm}} + md_1^2$ and $I_2 = I_{\text{cm}} + md_2^2$, these relations may be rewritten as

$$mgd_1T^2 = 4\pi^2\left(I_{\text{cm}} + md_1^2\right)$$
$$mgd_2T^2 = 4\pi^2\left(I_{\text{cm}} + md_2^2\right).$$

Subtracting the expressions gives

$$mg(d_1 - d_2)T^2 = 4\pi^2 m\left(d_1^2 - d_2^2\right) = 4\pi^2 m(d_1 - d_2)(d_1 + d_2),$$

and dividing by the common factor of $m(d_1 - d_2)$ and letting $d_1 + d_2 = L$ gives the desired result.

13.101: a) The spring, when stretched, provides an inward force; using $\omega'^2 l$ for the magnitude of the inward radial acceleration,

$$m\omega' l = k(l - l_0), \text{ or } l = \frac{kl_0}{k - m\omega'^2}.$$

b) The spring will tend to become unboundedly long.

13.102: Let $r = R_0 + x$, so that $r - R_0 = x$ and

$$F = A[e^{-2bx} - e^{-bx}].$$

When x is small compared to b^{-1}, expanding the exponential function gives

$$F \approx A\left[(1 - 2bx) - (1 - bx)\right] = -Abx,$$

corresponding to a force constant of $Ab = 579.2\ \text{N/m}$ or $579\ \text{N/m}$ to three figures. This is close to the value given in Exercise 13.40.

14.1: $w = mg = \rho V g$

$$= \left(7.8\times10^{3}\ \text{kg/m}^{3}\right)\left(0.858\,\text{m}\right)\pi\left(1.43\times10^{-2}\ \text{m}\right)^{2}\left(9.80\,\text{m/s}^{2}\right) = 41.8\ \text{N}$$

or 42 N to two places. A cart is not necessary.

14.2: $$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^{3}} = \frac{\left(7.35\times10^{22}\ \text{kg}\right)}{\frac{4}{3}\pi\left(1.74\times10^{6}\ \text{m}\right)^{3}} = 3.33\times10^{3}\ \text{kg/m}^{3}.$$

14.3: $\rho = \frac{m}{V} = \frac{(0.0158\ \text{kg})}{(5.0\times15.0\times30.0)\text{mm}^{3}} = 7.02\times10^{3}\ \text{kg/m}^{3}$. You were cheated.

14.4: The length L of a side of the cube is

$$L = V^{\frac{1}{3}} = \left(\frac{m}{\rho}\right)^{\frac{1}{3}} = \left(\frac{40.0\,\text{kg}}{21.4\times10^{3}\ \text{kg/m}^{3}}\right)^{\frac{1}{3}} = 12.3\,\text{cm}.$$

14.5: $m = \rho V = \frac{4}{3}\pi r^{3}\rho$

Same mass means $r_{\text{a}}^{3}\rho_{\text{a}} = r_{1}^{3}\rho_{1}\left(\text{a} = \text{aluminum}, 1 = \text{lead}\right)$

$$\frac{r_{\text{a}}}{r_{1}} = \left(\frac{\rho_{1}}{\rho_{\text{a}}}\right)^{1/3} = \left(\frac{11.3\times10^{3}}{2.7\times10^{3}}\right)^{1/3} = 1.6$$

14.6: a) $$D = \frac{M_{\text{sun}}}{V_{\text{sun}}} = \frac{1.99\times10^{30}\ \text{kg}}{\frac{4}{3}\pi\left(6.96\times10^{8}\,\text{m}\right)^{3}} = \frac{1.99\times10^{30}\ \text{kg}}{1.412\times10^{27}\ \text{m}^{3}}$$

$$= 1.409\times10^{3}\ \text{kg/m}^{3}$$

b) $$D = \frac{1.99\times10^{30}\ \text{kg}}{\frac{4}{3}\pi\left(2.00\times10^{4}\ \text{m}\right)^{3}} = \frac{1.99\times10^{30}\ \text{kg}}{3.351\times10^{13}\ \text{m}^{3}} = 0.594\times10^{17}\ \text{kg/m}^{3}$$

$$= 5.94\times10^{16}\ \text{kg/m}^{3}$$

14.7: $p - p_{0} = \rho g h$

$$h = \frac{p - p_{0}}{\rho g} = \frac{1.00\times10^{5}\ \text{Pa}}{(1030\,\text{kg/m}^{3})\,(9.80\,\text{m/s}^{2})} = 9.91\text{m}$$

14.8: The pressure difference between the top and bottom of the tube must be at least 5980 Pa in order to force fluid into the vein:

$$\rho g h = 5980\,\text{Pa}$$

$$h = \frac{5980\text{Pa}}{gh} = \frac{5980\,\text{N/m}^2}{(1050\,\text{kg/m}^3)\,(9.80\,\text{m/s}^2)} = 0.581\text{m}$$

14.9: a) $\rho g h = \left(600\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(0.12\text{m}\right) = 706\text{Pa}.$

b) $706\text{Pa} + \left(1000\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(0.250\text{m}\right) = 3.16 \times 10^3\,\text{Pa}.$

14.10: a) The pressure used to find the area is the gauge pressure, and so the total area is

$$\frac{(16.5 \times 10^3\ \text{N})}{(205 \times 10^3\ \text{Pa})} = 805\,\text{cm}^2\,.$$

b) With the extra weight, repeating the above calculation gives $1250\,\text{cm}^2$.

14.11: a) $\rho g h = (1.03 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(250\,\text{m}) = 2.52 \times 10^6\ \text{Pa}.$ b) The pressure difference is the gauge pressure, and the net force due to the water and the air is $(2.52 \times 10^6\ \text{Pa})(\pi(0.15\,\text{m})^2) = 1.78 \times 10^5\ \text{N}.$

14.12: $p = \rho g h = (1.00 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(640\ \text{m}) = 6.27 \times 10^6\ \text{Pa} = 61.9\,\text{atm}.$

14.13: a) $p_a + \rho g y_2 = 980 \times 10^2\ \text{Pa} + (13.6 \times 10^3\ \text{kg/m}^3)(9.80\,\text{m/s}^2)(7.00 \times 10^{-2}\ \text{m}) =$ $1.07 \times 10^5\ \text{Pa}.$ b) Repeating the calcultion with $y = y_2 - y_1 = 4.00\,\text{cm}$ instead of y_2 gives $1.03 \times 10^5\ \text{Pa}.$ c) The absolute pressure is that found in part (b), $1.03 \times 10^5\ \text{Pa}.$ d) $(y_2 - y_1)\rho g = 5.33 \times 10^3\ \text{Pa}$ (this is not the same as the difference between the results of parts (a) and (b) due to roundoff error).

14.14: $\rho g h = (1.00 \times 10^3\text{kg/m}^3)(9.80\,\text{m/s}^2)(6.1\,\text{m}) = 6.0 \times 10^4\ \text{Pa}.$

14.15: With just the mercury, the gauge pressure at the bottom of the cylinder is $p = p_0 + p_{\mathrm{m}} g h_{\mathrm{m}}$. With the water to a depth h_{w}, the gauge pressure at the bottom of the cylinder is $p = p_0 + \rho_{\mathrm{m}} g h_{\mathrm{m}} + p_{\mathrm{w}} g h_{\mathrm{w}}$. If this is to be double the first value, then $\rho_{\mathrm{w}} g h_{\mathrm{w}} = \rho_{\mathrm{m}} g h_{\mathrm{m}}$.

$$h_{\mathrm{w}} = h_{\mathrm{m}}(\rho_{\mathrm{m}}/\rho_{\mathrm{w}}) = (0.0500\,\mathrm{m})(13.6\times10^3/1.00\times10^3) = 0.680\,\mathrm{m}$$

The volume of water is

$$V = h\mathrm{A} = (0.680\,\mathrm{m})(12.0\times10^{-4}\ \mathrm{m}^2) = 8.16\times10^{-4}\ \mathrm{m}^3 = 816\,\mathrm{cm}^3$$

14.16: a) Gauge pressure is the excess pressure above atmospheric pressure. The pressure difference between the surface of the water and the bottom is due to the weight of the water and is still 2500 Pa after the pressure increase above the surface. But the surface pressure increase is also transmitted to the fluid, making the total difference from atmospheric 2500 Pa+1500 Pa = 4000 Pa.

b) The pressure due to the water alone is 2500 Pa= $\rho g h$. Thus

$$h = \frac{2500\,\mathrm{N/m^2}}{(1000\ \mathrm{kg/m^3})\,(9.80\ \mathrm{m/s^2})} = 0.255\,\mathrm{m}$$

To keep the bottom gauge pressure at 2500 Pa after the 1500 Pa increase at the surface, the pressure due to the water's weight must be reduced to 1000 Pa:

$$h = \frac{1000\,\mathrm{N/m^2}}{(1000\,\mathrm{kg/m^3})(9.80\,\mathrm{m/s^2})} = 0.102\,\mathrm{m}$$

Thus the water must be lowered by $0.255\,\mathrm{m} - 0.102\,\mathrm{m} = 0.153\,\mathrm{m}$

14.17: The force is the difference between the upward force of the water and the downward forces of the air and the weight. The difference between the pressure inside and out is the gauge pressure, so

$$F = (\rho g h)\,A - w = (1.03\times10^3)\,(9.80\ \mathrm{m/s^2})\,(30\,\mathrm{m})\,(0.75\,\mathrm{m^2}) - 300\ \mathrm{N} = 2.27\times10^5\ \mathrm{N}.$$

14.18: $\left[130\times10^3\ \mathrm{Pa} + (1.00\times10^3\ \mathrm{kg/m^3})(3.71\,\mathrm{m/s^2})(14.2\,\mathrm{m}) - 93\times10^3\ \mathrm{Pa}\right](2.00\,\mathrm{m^2})$
$= 1.79\times10^5\ \mathrm{N}.$

14.19: The depth of the kerosene is the difference in pressure, divided by the product $\rho g = \frac{mg}{V}$,

$$h = \frac{(16.4\times10^3\ \text{N})/(0.0700\,\text{m}^2) - 2.01\times10^5\ \text{Pa}}{(205\,\text{kg})(9.80\,\text{m/s}^2)\ /(0.250\,\text{m}^3)} = 4.14\,\text{m}.$$

14.20: $$p = \frac{F}{A} = \frac{mg}{\pi(d/2)^2} = \frac{(1200\,\text{kg})(9.80\ \text{m/s}^2)}{\pi(0.15\,\text{m})^2} = 1.66\times10^5\ \text{Pa} = 1.64\,\text{atm}.$$

14.21: The buoyant force must be equal to the total weight; $\rho_{\text{water}}V\text{g} = \rho_{\text{ice}}V\text{g} + mg$, so

$$V = \frac{m}{\rho_{\text{water}} - \rho_{ice}} = \frac{45.0\ \text{kg}}{1000\,\text{kg/m}^3 - 920\,\text{kg/m}^3} = 0.563\,\text{m}^3,$$

or $0.56\,\text{m}^3$ to two figures.

14.22: The buoyant force is $B = 17.50\ \text{N} - 11.20\ \text{N} = 6.30\ \text{N},$ and

$$V = \frac{B}{\rho_{\text{water}}g} = \frac{(6.30\ \text{N})}{(1.00\times10^3\ \text{kg/m}^3)(9.80\,\text{m/s}^2)} = 6.43\times10^{-4}\ \text{m}^3.$$

The density is

$$\rho = \frac{m}{V} = \frac{w/g}{B/\rho_{\text{water}}g} = \rho_{\text{water}}\frac{w}{B} = (1.00\times10^3\ \text{kg/m}^3)\left(\frac{17.50}{6.30}\right) = 2.78\times10^3\ \text{kg/m}^3.$$

14.23: a) The displaced fluid must weigh more than the object, so $\rho < \rho_{\text{fluid}}$. b) If the ship does not leak, much of the water will be displaced by air or cargo, and the average density of the floating ship is less than that of water. c) Let the portion submerged have volume *V*, and the total volume be V_0. Then, $\rho V_o = \rho_{\text{fluid}} V$, so $\frac{V}{V_0} = \frac{\rho}{\rho_{\text{fluid}}}\cdot$ The fraction above the fluid is then $1 - \frac{p}{p_{\text{fluid}}}$. If $p \to 0$, the entire object floats, and if $\rho \to \rho_{\text{fluid}}$, none of the object is above the surface. d) Using the result of part (c),

$$1 - \frac{\rho}{\rho_{\text{fluid}}} = 1 - \frac{(0.042\,\text{kg})/(5.0 \times 4.0 \times 3.0 \times 10^{-6}\,\text{m}^3)}{1030\,\text{kg/m}^3} = 0.32 = 32\%.$$

14.24: a) $B = \rho_{\text{water}} gV = (1.00 \times 10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(0.650\,\text{m}^3) = 6370\,\text{N}.$

b) $m = \frac{w}{g} = \frac{B-T}{g} = \frac{6370\,\text{N-}900\,\text{N}}{9.80\,\text{m/s}^2} = 558\,\text{kg}.$

c) (See Exercise 14.23.) If the submerged volume is V',

$$V' = \frac{w}{\rho_{\text{water}} g} \text{ and } \frac{V'}{V} = \frac{w}{\rho_{\text{water}} gV} = \frac{5470\,\text{N}}{6370\,\text{N}} = 0.859 = 85.9\%.$$

14.25: a) $\rho_{\text{oil}}\, gh_{\text{oil}} = 116\,\text{Pa}.$

b) $((790\,\text{kg/m}^3)(0.100\,\text{m}) + (1000\,\text{kg/m}^3)(0.0150\,\text{m}))(9.80\,\text{m/s}^2) = 921\,\text{Pa}.$

c) $$m = \frac{w}{g} = \frac{(p_{\text{bottom}} - p_{\text{top}})A}{g} = \frac{(805\,\text{Pa})(0.100\,\text{m})^2}{(9.80\,\text{m/s}^2)} = 0.822\,\text{kg}.$$

The density of the block is $p = \frac{0.822\,\text{kg}}{(0.10\,\text{m})^3} = 822\frac{\text{kg}}{\text{m}^3}$. Note that is the same as the average density of the fluid displaced, $(0.85)(790\,\text{kg/m}^3) + (0.15)(1000\,\text{kg/m}^3)$.

14.26: a) Neglecting the density of the air,

$$V = \frac{m}{\rho} = \frac{w/g}{\rho} = \frac{w}{g\rho} = \frac{(89\,\mathrm{N})}{(9.80\,\mathrm{m/s^2})(2.7\times10^3\,\mathrm{kg/m^3})} = 3.36\times10^{-3}\,\mathrm{m^3},$$

or $3.4\times10^{-3}\mathrm{m^3}$ to two figures.

$$\text{b)}\, T = w - B = w - g\rho_{\text{water}}V = \omega\left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{aluminum}}}\right) = (89\,\mathrm{N})\left(1 - \frac{1.00}{2.7}\right) = 56.0\,\mathrm{N}.$$

14.27: a) The pressure at the top of the block is $p = p_0 + \rho gh,$ where h is the depth of the top of the block below the surface. h is greater for block B, so the pressure is greater at the top of block B.

b) $B = \rho_{\text{fl}}V_{\text{obj}}g.$ The blocks have the same volume V_{obj} so experience the same buoyant force.

c) $T - w + B = 0$ so $T = w - B.$

$w = \rho Vg.$ The object have the same V but ρ is larger for brass than for aluminum so w is larger for the brass block. B is the same for both, so T is larger for the brass block, block B.

14.28: The rock displaces a volume of water whose weight is $39.2\,\mathrm{N} - 28.4\,\mathrm{N} = 10.8\,\mathrm{N}.$ The mass of this much water is thus $10.8\,\mathrm{N}/9.80\,\mathrm{m/s^2} = 1.102\,\mathrm{kg}$ and its volume, equal to the rock's volume, is

$$\frac{1.102\,\mathrm{kg}}{1.00\times10^3\,\mathrm{kg/m^3}} = 1.102\times10^{-3}\mathrm{m^3}$$

The weight of unknown liquid displaced is $39.2\,\mathrm{N} - 18.6\,\mathrm{N} = 20.6\,\mathrm{N},$ and its mass is $20.6\,\mathrm{N}/9.80\,\mathrm{m/s^2} = 2.102\,\mathrm{kg}.$ The liquid's density is thus $2.102\mathrm{kg}/1.102\times10^{-3}\,\mathrm{m^3}$ $= 1.91\times10^3\,\mathrm{kg/m^3},$ or roughly twice the density of water.

14.29: $v_1A_1 = v_2A_2, v_2 = v_1(A_1/A_2)$

$$A_1 = \pi(0.80\,\mathrm{cm})^2,\ A_2 = 20\pi(0.10\,\mathrm{cm})^2$$

$$v_2 = (3.0\,\mathrm{m/s})\frac{\pi(0.80)^2}{20\pi(0.10)^2} = 9.6\,\mathrm{m/s}$$

14.30: $v_2 = v_1 \dfrac{A_1}{A_2} = \dfrac{(3.50\ \text{m/s})(0.0700\ \text{m}^2)}{A_2} = \dfrac{0.245\ \text{m}^3/\text{s}}{A_2}.$

a) (i) $A_2 = 0.1050\ \text{m}^2, v_2 = 2.33\ \text{m/s}.$ (ii) $A_2 = 0.047\ \text{m}^2, v_2 = 5.21\ \text{m/s}.$

b) $v_1 A_1 t = \upsilon_2 A_2 t = (0.245\ \text{m}^3/\text{s})\,(3600\text{s}) = 882\ \text{m}^3.$

14.31: a) $v = \dfrac{dV/dt}{A} = \dfrac{(1.20\ \text{m}^3/\text{s})}{\pi(0.150\ \text{m})^2} = 16.98.$

b) $r_2 = r_1\sqrt{v_1/v_2} = \sqrt{(dV/dt)/\pi v_2} = 0.317\ \text{m}.$

14.32: a) From the equation preceding Eq. (14.10), dividing by the time interval *dt* gives Eq. (14.12). b) The volume flow rate decreases by 1.50% (to two figures).

14.33: The hole is given as being “small,”and this may be taken to mean that the velocity of the seawater at the top of the tank is zero, and Eq. (14.18) gives

$$\begin{aligned} v &= \sqrt{2(gy + (p/\rho))} \\ &= \sqrt{2((9.80\ \text{m/s}^2)(11.0\ \text{m}) + (3.00)(1.013\times10^5\ \text{Pa})\big/(1.03\times10^3\ \text{kg/m}^3))} \\ &= 28.4\ \text{m/s}. \end{aligned}$$

Note that $y = 0$ and $p = p_\text{a}$ were used at the bottom of the tank, so that p was the given gauge pressure at the top of the tank.

14.34: a) From Eq. (14.18), $v = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(14.0\ \text{m})} = 16.6\ \text{m/s}.$

b) $vA = (16.57\ \text{m/s})(\pi(0.30\times10^{-2}\ \text{m})^2) = 4.69\times10^{-4}\ \text{m}^3/\text{s}.$ Note that an extra figure was kept in the intermediate calculation.

14.35: The assumption may be taken to mean that $v_1 = 0$ in Eq. (14.17). At the maximum height, $v_2 = 0,$ and using gauge pressure for p_1 and $p_2, p_2 = 0$ (the water is open to the atmosphere), $p_1 = \rho g y_2 = 1.47\times10^5\ \text{Pa}.$

14.36: Using $v_2 = \frac{1}{4}v_1$ in Eq. (14.17),

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2) = p_1 + \rho\left[\left(\frac{15}{32}\right)v_1^2 + g(y_1 - y_2)\right]$$
$$= 5.00\times10^4\ \text{Pa} + (1.00\times10^3\ \text{kg/m}^3)\left(\frac{15}{32}(3.00\ \text{m/s})^2 + (9.80\ \text{m/s}^2)(11.0\ \text{m})\right)$$
$$= 1.62\times10^5\ \text{Pa}.$$

14.37: Neglecting the thickness of the wing (so that $y_1 = y_2$ in Eq. (14.17)), the pressure difference is $\Delta p = (1/2)\rho(v_2^2 - v_1^2) = 780\ \text{Pa}$. The net upward force is then $(780\ \text{Pa})\times(16.2\ \text{m}^2) - (1340\ \text{kg})(9.80\ \text{m/s}^2) = -496\ \text{N}$.

14.38: a) $\frac{(220)(0.355\ \text{kg})}{60.0\ \text{s}} = 1.30\ \text{kg/s}$. b) The density of the liquid is $\frac{0.355\ \text{kg}}{0.355\times10^{-3}\ \text{m}^3} = 1000\ \text{kg/m}^3$, and so the volume flow rate is $\frac{1.30\ \text{kg/s}}{1000\ \text{kg/m}^3} = 1.30\times10^{-3}\ \text{m}^3/\text{s} = 1.30\ \text{L/s}$. This result may also be obtained from $\frac{(220)(0.355\ \text{L})}{60.0\ \text{s}} = 1.30\ \text{L/s}$. c) $v_1 = \frac{1.30\times10^{-3}\ \text{m}^3/\text{s}}{2.00\times10^{-4}\ \text{m}^2}$ $= 6.50\ \text{m/s}, v_2 = v_1/4 = 1.63\ \text{m/s}$.

d) $$p_1 = p_2 + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g\left(y_2 - y_1\right)$$
$$= 152\ \text{kPa} + (1/2)\left(1000\ \text{kg/m}^3\right)\left((1.63\ \text{m/s})^2 - (6.50\ \text{m/s})^2\right)$$
$$+ \left(1000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(-1.35\ \text{m}\right)$$
$$= 119\ \text{kPa}$$

14.39: The water is discharged at a rate of $v_1 = \frac{4.65\times10^{-4}\ \text{m}^3/\text{s}}{1.32\times10^{-3}\ \text{m}^2} = 0.352\ \text{m/s}$. The pipe is given as horizonatal, so the speed at the constriction is $v_2 = \sqrt{v_1^2 + 2\Delta p/\rho} = 8.95\ \text{m/s}$, keeping an extra figure, so the cross-section are at the constriction is $\frac{4.65\times10^{-4}\ \text{m}^3/\text{s}}{8.95\ \text{m/s}} = 5.19\times10^{-5}\ \text{m}^2$, and the radius is $r = \sqrt{A/\pi} = 0.41\ \text{cm}$.

14.40: From Eq. (14.17), with $y_1 = y_2$,

$$p_2 = p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = p_1 + \frac{1}{2}\rho\left(v_1^2 - \frac{v_1^2}{4}\right) = p_1 + \frac{3}{8}\rho v_1^2$$

$$= 1.80\times10^4 \text{ Pa} + \frac{3}{8}\left(1.00\times10^3 \text{ kg/m}^3\right)\left(2.50 \text{ m/s}\right)^2 = 2.03\times10^4 \text{ Pa},$$

where the continutity relation $v_2 = \dfrac{v_1}{2}$ has been used.

14.41: Let point 1 be where $r_1 = 4.00\,\text{cm}$ and point 2 be where $r_2 = 2.00\,\text{cm}$. The volume flow rate has the value $7200\,\text{cm}^3/\text{s}$ at all points in the pipe.

$$v_1 A_1 = v_1 \pi r_1^2 = 7200\,\text{cm}^3, \text{so } v_1 = 1.43 \text{ m/s}$$

$$v_2 A_2 = v_2 \pi r_2^2 = 7200\,\text{cm}^3, \text{so } v_2 = 5.73 \text{ m/s}$$

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$y_1 = y_2 \text{ and } p_2 = 2.40\times10^5 \text{ Pa, so } p_2 = p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 2.25\times10^5 \text{ Pa}$$

14.42: a) The cross-sectional area presented by a sphere is $\pi\frac{D^2}{4}$, therefore $F = \left(p_0 - p\right)\pi\frac{D^2}{4}$. b) The force on each hemisphere due to the atmosphere is $\pi\left(5.00\times10^{-2} \text{ m}\right)^2\left(1.013\times10^5 \text{ Pa}\right)\left(0.975\right) = 776\text{N}$.

14.43: a) $\rho g h = \left(1.03\times10^3 \text{ kg/m}^3\right)\left(9.80\times \text{m/s}^2\right)\left(10.92\times10^3 \text{ m}\right) = 1.10\times10^8 \text{ Pa}$.
b) The fractional change in volume is the negative of the fractional change in density. The density at that depth is then

$$\rho = \rho_0\left(1 + k\Delta p\right) = \left(1.03\times10^3 \text{ kg/m}^3\right)\left(1 + \left(1.16\times10^8 \text{ Pa}\right)\left(45.8\times10^{-11} \text{ Pa}^{-1}\right)\right)$$
$$= 1.08\times10^3 \text{ kg/m}^3,$$

A fractional increase of 5.0%. Note that to three figures, the gauge pressure and absolute pressure are the same.

14.44: a) The weight of the water is

$$\rho gV = (1.00\times10^{3}\ \text{kg/m}^{3})(9.80\ \text{m/s}^{2})((5.00\ \text{m})(4.0\ \text{m})(3.0\ \text{m})) = 5.88\times10^{5}\ \text{N},$$

or $5.9\times10^{5}\ \text{N}$ to two figures. b) Integration gives the expected result the force is what it would be if the pressure were uniform and equal to the pressure at the midpoint;

$$\begin{aligned} F &= \rho gA\frac{d}{2} \\ &= (1.00\times10^{3}\ \text{kg/m}^{3})(9.80\ \text{m/s}^{2})((4.0\ \text{m})(3.0\ \text{m}))(1.50\ \text{m}) = 1.76\times10^{5}\ \text{N}, \end{aligned}$$

or $1.8\times10^{5}\ \text{N}$ to two figures.

14.45: Let the width be w and the depth at the bottom of the gate be H. The force on a strip of vertical thickness dh at a depth h is then $dF = \rho gh(wdh)$ and the torque about the hinge is $d\tau = \rho gwh(h - H/2)dh$; integrating from $h = 0$ to $h = H$ gives $\tau = \rho g\omega H^{3}/12 = 2.61\times10^{4}\ \text{N}\cdot\text{m}.$

14.46: a) See problem 14.45; the net force is $\int dF$ from $h = 0$ to $h = H, F = \rho g\omega H^{2}/2 = \rho gAH/2,$ where $A = \omega H$. b) The torque on a strip of vertical thickness dh about the bottom is $d\tau = dF(H - h) = \rho gwh(H - h)dh,$ and integrating from $h = 0$ to $h = H$ gives $\tau = \rho gwH^{3}/6 = \rho gAH^{2}/6.$ c) The force depends on the width and the square of the depth, and the torque about the bottom depends on the width and the cube of the depth; the surface area of the lake does not affect either result (for a given width).

14.47: The acceleration due to gravity on the planet is

$$g = \frac{\Delta p}{\rho d} = \frac{\Delta p}{\frac{m}{V}d}$$

and so the planet's mass is

$$M = \frac{gR^{2}}{G} = \frac{\Delta pVR^{2}}{mGd}$$

14.48: The cylindrical rod has mass M, radius R, and length L with a density that is proportional to the square of the distance from one end, $\rho = Cx^2$.

a) $M = \int \rho dV = \int Cx^2 dV$. The volume element $dV = \pi R^2 dx$. Then the integral becomes $M = \int_0^L Cx^2 \pi R^2 dx$. Integrating gives $M = C\pi R^2 \int_0^L x^2 dx = C\pi R^2 \dfrac{L^3}{3}$. Solving for $C, C = 3M/\pi R^2 L^3$.

b) The density at the $x = L$ end is $\rho = Cx^2 = \left(\frac{3M}{\pi R^2 L^3}\right)\left(L^2\right) = \left(\frac{3M}{\pi R^2 L}\right)$. The denominator is just the total volume V, so $\rho = 3M/V$, or three times the average density, M/V. So the average density is one-third the density at the $x = L$ end of the rod.

14.49: a) At $r = 0$, the model predicts $\rho = A = 12{,}700\ \text{kg/m}^3$ and at $r = R$, the model predicts

$\rho = A - BR = 12{,}700\ \text{kg/m}^3 - (1.50\times10^{-3}\ \text{kg/m}^4)(6.37\times10^6\ \text{m}) = 3.15\times10^3\ \text{kg/m}^3.$

b), c)

$$M = \int dm = 4\pi\int_0^R [A - Br]r^2 dr = 4\pi\left[\frac{AR^3}{3} - \frac{BR^4}{4}\right] = \left(\frac{4\pi R^3}{3}\right)\left[A - \frac{3BR}{4}\right]$$

$$= \left(\frac{4\pi(6.37\times10^6\ \text{m})^3}{3}\right)\left[12{,}700\ \text{kg/m}^3 - \frac{3(1.50\times10^{-3}\ \text{kg/m}^4)(6.37\times10^6\ \text{m})}{4}\right]$$

$$= 5.99\times10^{24}\ \text{kg},$$

which is within 0.36% of the earth's mass. d) If $m\ (r)$ is used to denote the mass contained in a sphere of radius r, then $g = Gm(r)/r^2$. Using the same integration as that in part (b), with an upper limit of r instead of R gives the result.

e) $g = 0$ at $r = 0$, and g at $r = R$, $g = Gm(R)/R^2 = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)$ $(5.99\times10^{24}\ \text{kg})/(6.37\times10^6\ \text{m})^2 = 9.85\ \text{m/s}^2.$

f) $$\frac{dg}{dr} = \left(\frac{4\pi G}{3}\right)\frac{d}{dr}\left[Ar - \frac{3Br^2}{4}\right] = \left(\frac{4\pi G}{3}\right)\left[A - \frac{3Br}{2}\right];$$

setting ths equal to zero gives $r = 2A/3B = 5.64\times10^6\ \text{m}$, and at this radius

$$g = \left(\frac{4\pi G}{3}\right)\left(\frac{2A}{3B}\right)\left[A - \left(\frac{3}{4}\right)B\left(\frac{2A}{3B}\right)\right]$$

$$= \frac{4\pi GA^2}{9B}$$

$$= \frac{4\pi(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(12{,}700\ \text{kg/m}^3)^2}{9(1.50\times10^{-3}\ \text{kg/m}^4)} = 10.02\ \text{m/s}^2.$$

14.50: a) Equation (14.4), with the radius r instead of height y, becomes $dp = -\rho g\, dr = -\rho g_s (r/R) dr.$ This form shows that the pressure decreases with increasing radius. Integrating, with $p = 0$ at $r = R$,

$$p = -\frac{\rho g_s}{R}\int_R^r r\, dr = \frac{\rho g_s}{R}\int_r^R r\, dr = \frac{\rho g_s}{2R}(R^2 - r^2).$$

b) Using the above expression with $r = 0$ and $\rho = \frac{M}{V} = \frac{3M}{4\pi R^3}$,

$$p(0) = \frac{3(5.97\times10^{24}\ \text{kg})(9.80\ \text{m/s}^2)}{8\pi(6.38\times10^6\ \text{m})^2} = 1.71\times10^{11}\ \text{Pa}.$$

c) While the same order of magnitude, this is not in very good agreement with the estimated value. In more realistic density models (see Problem 14.49 or Problem 9.99), the concentration of mass at lower radii leads to a higher pressure.

14.51: a) $\rho_{\text{water}} g h_{\text{water}} = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(15.0\times10^{-2}\ \text{m}) = 1.47\times10^3\ \text{Pa}.$ b) The gauge pressure at a depth of 15.0 cm $- h$ below the top of the mercury column must be that found in part (a); $\rho_{\text{Hg}} g(15.0\,\text{cm} - h) = \rho_{\text{water}} g(15.0\,\text{cm}),$ which is solved for $h = 13.9\,\text{cm}.$

14.52: Following the hint,

$$F = \int_0^h (\rho g y)(2\pi R)dy = \rho g \pi R h^2$$

where R and h are the radius and height of the tank (the fact that $2R = h$ is more or less coincidental). Using the given numerical values gives $F = 5.07\times10^8$ N.

14.53: For the barge to be completely submerged, the mass of water displaced would need to be $\rho_{\text{water}} V = (1.00\times10^3\ \text{kg/m}^3)(22\times40\times12\ \text{m}^3) = 1.056\times10^7\ \text{kg}.$ The mass of the barge itself is

$$(7.8\times10^3\,\text{kg/m}^3)((2(22+40)\times12 + 22\times40)\times4.0\times10^{-2}\ \text{m}^3) = 7.39\times10^5\ \text{kg},$$

so the barge can hold 9.82×10^6 kg of coal. This mass of coal occupies a solid volume of $6.55\times10^3\ \text{m}^3$, which is less than the volume of the interior of the barge $(1.06\times10^4\ \text{m}^3)$, but the coal must not be too loosely packed.

14.54: The difference between the densities must provide the "lift" of 5800 N (see Problem 14.59). The average density of the gases in the balloon is then

$$\rho_{\text{ave}} = 1.23\ \text{kg/m}^3 - \frac{(5800\ \text{N})}{(9.80\ \text{m/s}^2)(2200\ \text{m}^3)} = 0.96\ \text{kg/m}^3.$$

14.55: a) The submerged volume V' is $\frac{w}{\rho_{\text{water}} g}$, so

$$\frac{V'}{V} = \frac{w/\rho_{\text{water}} g}{V} = \frac{m}{\rho_{\text{water}} V} = \frac{(900\ \text{kg})}{(1.00 \times 10^3\ \text{kg/m}^3)\,(3.0\ \text{m}^3)} = 0.30 = 30\% \cdot$$

b) As the car is about to sink, the weight of the water displaced is equal to the weight of the car plus the weight of the water inside the car. If the volume of water inside the car is V'',

$$V\rho_{water}\, g = w + V''\rho_{water}\, g, \text{ or } \frac{V''}{V} = 1 - \frac{w}{V\rho_{water}\, g} = 1 - 0.30 = 0.70 = 70\% \cdot$$

14.56: a) The volume displaced must be that which has the same weight and mass as the ice, $\frac{9.70\ \text{gm}}{1.00\ \text{gm/cm}^3} = 9.70\ \text{cm}^3$ (note that the choice of the form for the density of water avoids conversion of units). b) No; when melted, it is as if the volume displaced by the $9.70\ \text{gm}$ of melted ice displaces the same volume, and the water level does not change. c) $\frac{9.70\ \text{gm}}{1.05\ \text{gm/cm}^3} = 9.24\ \text{cm}^3 \cdot$ d) The melted water takes up more volume than the salt water displaced, and so $0.46\ \text{cm}^3$ flows over. A way of considering this situation (as a thought experiment only) is that the less dense water "floats" on the salt water, and as there is insufficient volume to contain the melted ice, some spills over.

14.57: The total mass of the lead and wood must be the mass of the water displaced, or

$$V_{\mathrm{Pb}}\rho_{\mathrm{Pb}}+V_{\mathrm{wood}}\rho_{\mathrm{wood}}=(V_{\mathrm{Pb}}+V_{\mathrm{wood}})\rho_{\mathrm{water}};$$

solving for the volume V_{Pb},

$$\begin{aligned}V_{\mathrm{Pb}} &= V_{\mathrm{wood}}\frac{\rho_{\mathrm{water}}-\rho_{\mathrm{wood}}}{\rho_{\mathrm{Pb}}-\rho_{\mathrm{water}}}\\ &= (1.2\times10^{-2}\ \mathrm{m}^3)\frac{1.00\times10^3\ \mathrm{kg/m^3}-600\ \mathrm{kg/m^3}}{11.3\times10^3\ \mathrm{kg/m^3}-1.00\times10^3\ \mathrm{kg/m^3}}\\ &= 4.66\times10^{-4}\ \mathrm{m}^3,\end{aligned}$$

which has a mass of 5.27 kg.

14.58: The fraction f of the volume that floats above the fluid is $f=1-\dfrac{\rho}{\rho_{\mathrm{fluid}}}$, where ρ is the average density of the hydrometer (see Problem 14.23 or Problem 14.55), which can be expressed as $\rho_{\mathrm{fluid}}=\rho\dfrac{1}{1-f}$. Thus, if two fluids are observed to have floating fraction f_1 and f_2, $\rho_2=\rho_1\dfrac{1-f_1}{1-f_2}$. In this form, it's clear that a larger f_2 corresponds to a larger density; more of the stem is above the fluid. Using $f_1=\frac{(8.00\ \mathrm{cm})(0.400\ \mathrm{cm}^2)}{(13.2\ \mathrm{cm}^3)}=0.242$, $f_2=\frac{(3.20\ \mathrm{cm})(0.400\ \mathrm{cm}^2)}{(13.2\ \mathrm{cm}^3)}=0.097$ gives $\rho_{\mathrm{alcohol}}=(0.839)\,\rho_{\mathrm{water}}=839\ \mathrm{kg/m^3}$.

14.59: a) The "lift" is $V(\rho_{\mathrm{air}}-\rho_{\mathrm{H_2}})g$, from which

$$V=\frac{120{,}000\ \mathrm{N}}{(1.20\ \mathrm{kg/m^3}-0.0899\ \mathrm{kg/m^3})(9.80\ \mathrm{m/s^2})}=11.0\times10^3\ \mathrm{m}^3.$$

b) For the same volume, the "lift" would be different by the ratio of the density differences,

$$(120{,}000\ \mathrm{N})\left(\frac{\rho_{\mathrm{air}}-\rho_{\mathrm{He}}}{\rho_{\mathrm{air}}-\rho_{\mathrm{H_2}}}\right)=11.2\times10^4\ \mathrm{N}.$$

This increase in lift is not worth the hazards associated with use of hydrogen.

14.60: a) Archimedes' principle states $\rho gLA = Mg$, so $L = \dfrac{M}{\rho A}$.

b) The buoyant force is $\rho gA(L + x) = Mg + F$, and using the result of part (a) and solving for x gives $x = \frac{F}{\rho gA}$.

c) The "spring constant," that is, the proportionality between the displacement x and the applied force F, is $k = \rho gA$, and the period of oscillation is

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M}{\rho gA}}.$$

14.61: a) $x = \dfrac{w}{\rho gA} = \dfrac{mg}{\rho gA} = \dfrac{m}{\rho A} = \dfrac{(70.0\,\text{kg})}{(1.03\times 10^3\,\text{kg/m}^3)\pi(0.450\,\text{m})^2} = 0.107\,\text{m}.$

b) Note that in part (c) of Problem 14.60, M is the mass of the buoy, not the mass of the man, and A is the cross-section area of the buoy, not the amplitude. The period is then

$$T = 2\pi\sqrt{\frac{(950\,\text{kg})}{(1.03\times 10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)\pi(0.450\,\text{m})^2}} = 2.42\,\text{s}.$$

14.62: To save some intermediate calculation, let the density, mass and volume of the life preserver be ρ_0, m and v, and the same quantities for the person be ρ_1, M and V. Then, equating the buoyant force and the weight, and dividing out the common factor of g,

$$\rho_{\text{water}}\left((0.80)V + v\right) = \rho_0 v + \rho_1 V,$$

Eliminating V in favor of ρ_1 and M, and eliminating m in favor of ρ_0 and v,

$$\rho_0 v + M = \rho_{\text{water}}\left((0.80)\frac{M}{\rho_1} + v\right).$$

Solving for ρ_0,

$$\begin{aligned}
\rho_0 &= \frac{1}{v}\left(\rho_{\text{water}}\left((0.80)\frac{M}{\rho_1} + v\right) - M\right) \\
&= \rho_{\text{water}} - \frac{M}{v}\left(1 - (0.80)\frac{\rho_{\text{water}}}{\rho_1}\right) \\
&= 1.03\times 10^3\ \text{kg/m}^3 - \frac{75.0\ \text{kg}}{0.400\ \text{m}^3}\left(1 - (0.80)\frac{1.03\times 10^3\ \text{kg/m}^3}{980\ \text{kg/m}^3}\right) \\
&= 732\ \text{kg/m}^3 \cdot
\end{aligned}$$

14.63: To the given precision, the density of air is negligible compared to that of brass, but not compared to that of the wood. The fact that the density of brass may not be known the three-figure precision does not matter; the mass of the brass is given to three figures. The weight of the brass is the difference between the weight of the wood and the buoyant force of the air on the wood, and canceling a common factor of g, $V_{\text{wood}}\,(\rho_{\text{wood}} - \rho_{\text{air}}) = M_{\text{brass,}}$ and

$$\begin{aligned}
M_{\text{wood}} = \rho_{\text{wood}} V_{\text{wood}} &= M_{\text{brass}}\frac{\rho_{\text{wood}}}{\rho_{\text{wood}} - \rho_{\text{air}}} = M_{\text{brass}}\left(1 - \frac{\rho_{\text{air}}}{\rho_{\text{wood}}}\right)^{-1} \\
&= (0.0950\ \text{kg})\left(1 - \frac{1.20\ \text{kg/m}^3}{150\ \text{kg/m}^3}\right)^{-1} = 0.0958\ \text{kg}.
\end{aligned}$$

14.64: The buoyant force on the mass *A*, divided by g, must be $7.50\,\text{kg} - 1.00\,\text{kg} - 1.80\,\text{kg} = 4.70\,\text{kg}$ (see Example 14.6), so the mass block is $4.70\,\text{kg} + 3.50\,\text{kg} = 8.20\,\text{kg}$. a) The mass of the liquid displaced by the block is $4.70\,\text{kg}$, so the density of the liquid is $\frac{4.70\ \text{kg}}{3.80\times10^{-3}\ \text{m}^3} = 1.24\times10^3\ \text{kg/m}^3$. b) Scale *D* will read the mass of the block, $8.20\,\text{kg}$, as found above. Scale *E* will read the sum of the masses of the beaker and liquid, $2.80\,\text{kg}$.

14.65: Neglecting the buoyancy of the air, the weight in air is

$$g(\rho_{\text{Au}}V_{\text{Au}} + \rho_{\text{A1}}V_{\text{A1}}) = 45.0\ \text{N}.$$

and the buoyant force when suspended in water is

$$\rho_{\text{water}}(V_{\text{Au}} + V_{\text{A1}})g = 45.0\ \text{N} - 39.0\ \text{N} = 6.0\ \text{N}.$$

These are two equations in the two unknowns V_{Au} and V_{A1}. Multiplying the second by ρ_{A1} and the first by ρ_{water} and subtracting to eliminate the V_{A1} term gives

$$\begin{aligned}
\rho_{\text{water}}V_{\text{Au}}g\,(\rho_{\text{Au}} - \rho_{\text{A1}}) &= \rho_{\text{water}}(45.0\ \text{N}) - \rho_{\text{A1}}(6.0\ \text{N}) \\
w_{\text{Au}} = \rho_{\text{Au}}gV_{\text{Au}} &= \frac{\rho_{\text{Au}}}{\rho_{\text{water}}(\rho_{\text{Au}} - \rho_{\text{A1}})}(\rho_{\text{water}}(45.0\ \text{N}) - \rho_{\text{Au}}(6.0)) \\
&= \frac{(19.3)}{(1.00)(19.3 - 2.7)}((1.00)(45.0\ \text{N}) - (2.7)(6.0\ \text{N})) \\
&= 33.5\ \text{N}.
\end{aligned}$$

Note that in the numerical determination of w_{Au}, specific gravities were used instead of densities.

14.66: The ball's volume is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (12.0\ \text{cm})^3 = 7238\ \text{cm}^3$$

As it floats, it displaces a weight of water equal to its weight. a) By pushing the ball under water, you displace an additional amount of water equal to 84% of the ball's volume or $(0.84)(7238\,\text{cm}^3) = 6080\,\text{cm}^3$. This much water has a mass of $6080\,g = 6.080\,\text{kg}$ and weighs $(6.080\,\text{kg})(9.80\ \text{m/s}^2) = 59.6\ \text{N}$, which is how hard you'll have to push to submerge the ball.

b) The upward force on the ball in excess of its own weight was found in part (a): $59.6\ \text{N}$. The ball's mass is equal to the mass of water displaced when the ball is floating:

$$(0.16)(7238\,\text{cm}^3)(1.00\ \text{g/cm}^3) = 1158\,\text{g} = 1.158\,\text{kg},$$

and its acceleration upon release is thus

$$a = \frac{F_{\text{net}}}{m} = \frac{59.6\ \text{N}}{1.158\,\text{kg}} = 51.5\ \text{m/s}^2$$

14.67: a) The weight of the crown of its volume V is $w = \rho_{\text{crown}} gV$, and when suspended the apparent weight is the difference between the weight and the buoyant force,

$$fw = f\rho_{\text{crown}} gV = (\rho_{\text{crown}} - \rho_{\text{water}})gV.$$

Dividing by the common factors leads to

$$-\rho_{\text{water}} + \rho_{\text{crown}} = f\rho_{\text{crown}} \text{ or } \frac{\rho_{\text{crown}}}{\rho_{\text{water}}} = \frac{1}{1-f}.$$

As $f \to 0$, the apparent weight approaches zero, which means the crown tends to float; from the above result, the specific gravity of the crown tends to 1. As $f \to 1$, the apparent weight is the same as the weight, which means that the buoyant force is negligble compared to the weight, and the specific gravity of the crown is very large, as reflected in the above expression. b) Solving the above equations for f in terms of the specific gravity, $f = 1 - \frac{\rho_{\text{water}}}{\rho_{\text{crown}}}$, and so the weight of the crown would be $(1-(1/19.3))(12.9\ \text{N}) = 12.2\ \text{N}$. c) Approximating the average density by that of lead for a "thin" gold plate, the apparent weight would be $(1-(1/11.3))(12.9\ \text{N}) = 11.8\ \text{N}$.

14.68: a) See problem 14.67. Replacing f with, respectively, w_{water}/w and w_{fluid}/w gives

$$\frac{\rho_{\text{steel}}}{\rho_{\text{fluid}}} = \frac{w}{w - w_{\text{fluid}}}, \frac{\rho_{\text{steel}}}{\rho_{\text{fluid}}} = \frac{w}{w - w_{\text{water}}},$$

and dividing the second of these by the first gives

$$\frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \frac{w - w_{\text{fluid}}}{w - w_{\text{water}}}.$$

b) When w_{fluid} is greater than $w_{\text{water,}}$ the term on the right in the above expression is less than one, indicating that the fluids is less dense than water, and this is consistent with the buoyant force when suspended in liquid being less than that when suspended in water. If the density of the fluid is the same as that of water $w_{\text{fluid}} = w_{\text{water}}$, as expected. Similarly, if w_{fluid} is less than w_{water}, the term on the right in the above expression is greater than one, indicating the the fluid is denser than water. c) Writing the result of part (a) as

$$\frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \frac{1 - f_{\text{fluid}}}{1 - f_{\text{water}}}.$$

and solving for f_{fluid},

$$f_{\text{fluid}} = 1 - \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}}(1 - f_{\text{water}}) = 1 - (1.220)(0.128) = 0.844 = 84.4\%.$$

14.69: a) Let the total volume be V; neglecting the density of the air, the buoyant force in terms of the weight is

$$B = \rho_{\text{water}} gV = \rho_{\text{water}} g\left(\frac{(w/g)}{\rho_{\text{m}}} + V_0\right),$$

or

$$V_0 = \frac{B}{\rho_{water}\, g} - \frac{w}{\rho_w g}.$$

b) $\frac{B}{\rho_{\text{water}} g} - \frac{w}{\rho_{\text{Cu}} g} = 2.52 \times 10^{-4}\ \text{m}^3$. Since the total volume of the casting is $\frac{B}{\rho_{\text{water}} g}$, the cavities are 12.4% of the total volume.

14.70: a) Let d be the depth of the oil layer, h the depth that the cube is submerged in the water, and L be the length of a side of the cube. Then, setting the buoyant force equal to the weight, canceling the common factors of g and the cross-section area and supressing units,
$(1000)h + (750)d = (550)L.d$, h and L are related by $d + h + (0.35)L = L$, so $h = (0.65)L - d$. Substitution into the first relation gives $d = L\frac{(0.65)(1000)-(550)}{(1000)-(750)} = \frac{2L}{5.00} = 0.040\,\text{m}$. b) The gauge pressure at the lower face must be sufficient to support the block (the oil exerts only sideways forces directly on the block), and
$p = \rho_{\text{wood}}gL = (550\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.100\,\text{m}) = 539\,\text{Pa}$. As a check, the gauge pressure, found from the depths and densities of the fluids, is
$((0.040\,\text{m})(750\ \text{kg/m}^3) + (0.025\,\text{m})(1000\ \text{kg/m}^3))(9.80\ \text{m/s}^2) = 539\,\text{Pa}$.

14.71: The ship will rise; the total mass of water displaced by the barge-anchor combination must be the same, and when the anchor is dropped overboard, it displaces some water and so the barge itself displaces less water, and so rises.

To find the amount the barge rises, let the original depth of the barge in the water be $h_0 = (m_{\text{b}} + m_{\text{a}})/(\rho_{\text{water}}A)$, where m_{b} and m_{a} are the masses of the barge and the anchor, and A is the area of the bottom of the barge. When the anchor is dropped, the buoyant force on the barge is less than what it was by an amount equal to the buoyant force on the anchor; symbolically,

$$h'\rho_{\text{water}}Ag = h_0\rho_{\text{water}}Ag - (m_{\text{a}}/\rho_{\text{steel}})\rho_{\text{water}}g,$$

which is solved for

$$\Delta h = h_0 - h' = \frac{m_{\text{a}}}{\rho_{\text{steel}}A} = \frac{(35.0\,\text{kg})}{(7860\ \text{kg/m}^3)(8.00\,\text{m}^2)} = 5.57\times10^{-4}\ \text{m},$$

or about 0.56 mm.

14.72: a) The average density of a filled barrel is $\rho_{\text{oil}} + \frac{m}{V} = 750\ \text{kg}/\text{m}^3 + \frac{15.0\ \text{kg}}{0.120\ \text{m}^3} = 875\ \text{kg}/\text{m}^3$, which is less than the density of seawater, so the barrel floats.

b) The fraction that floats (see Problem 14.23) is

$$1 - \frac{\rho_{\text{ave}}}{\rho_{\text{water}}} = 1 - \frac{875\ \text{kg}/\text{m}^3}{1030\ \text{kg}/\text{m}^3} = 0.150 = 15.0\%.$$

c) The average density is $910\frac{\text{kg}}{\text{m}^3} + \frac{32.0\ \text{kg}}{0.120\ \text{m}^3} = 1172\frac{\text{kg}}{\text{m}^3}$ which means the barrel sinks. In order to lift it, a tension $T = (1177\frac{\text{kg}}{\text{m}^3})(0.120\,\text{m}^3)(9.80\frac{\text{m}}{\text{s}^2}) - (1030\frac{\text{kg}}{\text{m}^3})(0.120\,\text{m}^3)(9.80\frac{\text{m}}{\text{s}^2}) = 173\,\text{N}$ is required.

14.73: a) See Exercise 14.23; the fraction of the volume that remains unsubmerged is $1 - \frac{\rho B}{\rho L}$. b) Let the depth of the liquid be x and the depth of the water be y. Then $\rho Lgx + \rho wgy = \rho_B gL$ and $x + y = L$. Therefore $x = L - y$ and $y = \frac{(\rho_L - \rho_B)L}{\rho_L - \rho_\omega}$. c) $y = \frac{13.6-7.8}{13.6-1.0}(0.10\,\text{m}) = 0.046\,\text{m}$.

14.74: a) The change is height Δy is related to the displaced volume ΔV by $\Delta y = \dfrac{\Delta V}{A}$, where A is the surface area of the water in the lock. ΔV is the volume of water that has the same weight as the metal, so

$$\Delta y = \frac{\Delta V}{A} = \frac{w/\rho_{\text{water}}g}{A} = \frac{w}{\rho_{\text{water}}gA}$$
$$= \frac{(2.50\times10^6\ \text{N})}{(1.00\text{x}10^3\ \text{kg}/\text{m}^3)(9.80\ \text{m}/\text{s}^2)((60.0\,\text{m})(20.0\,\text{m}))} = 0.213\,\text{m}.$$

b) In this case, ΔV is the volume of the metal; in the above expression, ρ_{water} is replaced by $\rho_{\text{metal}} = 9.00\rho_{\text{water}}$, which gives $\Delta y' = \frac{\Delta y}{9}$, and $\Delta y - \Delta y' = \frac{8}{9}\Delta y = 0.189\,\text{m}$; the water sinks by this amount.

14.75: a) Consider the fluid in the horizontal part of the tube. This fluid, with mass ρAl, is subject to a net force due to the pressure difference between the ends of the tube, which is the difference between the gauge pressures at the bottoms of the ends of the tubes. This difference is $\rho g(y_{\mathrm{L}} - y_{\mathrm{R}})$, and the net force on the horizontal part of the fluid is

$$\rho g(y_{\mathrm{L}} - y_{\mathrm{R}})A = \rho Ala,$$

or

$$(y_{\mathrm{L}} - y_{\mathrm{R}}) = \frac{a}{g}l.$$

b) Again consider the fluid in the horizontal part of the tube. As in part (a), the fluid is accelerating; the center of mass has a radial acceleration of magnitude $a_{\mathrm{rad}} = \omega^2 l/2$, and so the difference in heights between the columns is $(\omega^2 l/2)(l/g) = \omega^2 l^2/2g$.

Anticipating Problem, 14.77, an equivalent way to do part (b) is to break the fluid in the horizontal part of the tube into elements of thickness *dr*; the pressure difference between the sides of this piece is $dp = \rho(\omega^2 r)dr$ (see Problem 14.78), and integrating from $r = 0$ to $r = l$ gives $\Delta p = \rho\omega^2 l^2/2$, giving the same result.

c) At any point, Newton's second law gives $dpA = pAdla$ from which the area A cancels out. Therefore the cross-sectional area does not affect the result, even if it varies. Integrating the above result from 0 to l gives $\Delta p = pal$ between the ends. This is related to the height of the columns through $\Delta p = pg\Delta y$ from which p cancels out.

14.76: a) The change in pressure with respect to the vertical distance supplies the force necessary to keep a fluid element in vertical equilibrium (opposing the weight). For the rotating fluid, the change in pressure with respect to radius supplies the force necessary to keep a fluid element accelerating toward the axis; specifically, $dp = \frac{\partial p}{\partial p} dr = \rho a \, dr$, and using $a = \omega^2 r$ gives $\frac{\partial p}{\partial p} = \rho\omega^2 r$. b) Let the pressure at $y = 0, r = 0$ be p_a (atmospheric pressure); integrating the expression for $\frac{\partial p}{\partial p}$ from part (a) gives

$$p(r, y = 0) = p_{\mathrm{a}} + \frac{\rho\omega^2}{2} r^{2.}$$

c) In Eq. (14.5), $p_2 = p_{\mathrm{a}}, p_1 = p(r, y = 0)$ as found in part (b), $y_1 = 0$ and $\mathrm{y}_2 = h(r)$, the height of the liquid above the $y = 0$ plane. Using the result of part (b) gives $h(r) = \omega^2 r^2 / 2g$.

14.77: a) The net inward force is $(p + dp)A - pA = Adp$, and the mass of the fluid element is $\rho Adr'$. Using Newton's second law, with the inward radial acceleration of $\omega^2 r'$, gives $dp = \rho\omega^2 r' dr'$. b) Integrating the above expression,

$$\int_{p0}^{p} dp = \int_{r0}^{r} \rho\omega^2 r' dr'$$

$$p - p_0 = \left(\frac{\rho\omega^2}{2}\right)\left(r^2 - r^2{}_0\right),$$

which is the desired result. c) Using the same reasoning as in Section 14.3 (and Problem 14.78), the net force on the object must be the same as that on a fluid element of the same shape. Such a fluid element is accelerating inward with an acceleration of magnitude $\omega^2 R_{\mathrm{cm},}$ and so the force on the object is $\rho V\omega^2 R_{\mathrm{cm}}$. d) If $\rho \mathrm{R}_{\mathrm{cm}} > \rho_{\mathrm{ob}} R_{cmob,}$ the inward force is greater than that needed to keep the object moving in a circle with radius R_{cmob} at angular frequency ω, and the object moves inward. If $\rho R_{\mathrm{cm}} < \rho_{\mathrm{ob}} R_{cmob,}$, the net force is insufficient to keep the object in the circular motion at that radius, and the object moves outward. e) Objects with lower densities will tend to move toward the center, and objects with higher densities will tend to move away from the center.

14.78: (Note that increasing x corresponds to moving toward the back of the car.)

a) The mass of air in the volume element is $\rho dV = \rho A dx$, and the net force on the element in the forward direction is $(p+dp)A - pA = Adp.$ From Newton's second law, $Adp = (\rho A\,dx)a,$ from which $dp = \rho adx.$ b) With ρ given to be constant, and with $p = p_0 at\ x = 0,\ p = p_0 + \rho ax.$ c) Using $\rho = 1.2\ \text{kg/m}^3$ in the result of part (b) gives $(1.2\ \text{kg/m}^3)(5.0\ \text{m/s}^2)(2.5\ \text{m}) = 15.0\ \text{Pa} \sim 15\times10^{-5}\ p_{atm}$, so the fractional pressure difference is negligble. d) Following the argument in Section 14-4, the force on the balloon must be the same as the force on the same volume of air; this force is the product of the mass ρV and the acceleration, or $\rho Va.$ e) The acceleration of the balloon is the force found in part (d) divided by the mass $\rho_{\text{bal}}V, or(\rho/\rho_{\text{bal}})a.$ The acceleration relative to the car is the difference between this acceleration and the car's acceleration, $a_{\text{rel}} = [(\rho/\rho_{\text{bal}}) - 1]a.$ f) For a balloon filled with air, $(\rho/\rho_{\text{bal}}) < 1$ (air balloons tend to sink in still air), and so the quantity in square brackets in the result of part (e) is negative; the balloon moves to the back of the car. For a helium balloon, the quantity in square brackets is positive, and the balloon moves to the front of the car.

14.79: If the block were uniform, the buoyant force would be along a line directed through its geometric center, and the fact that the center of gravity is not at the geometric center does not affect the buoyant force. This means that the torque about the geometric center is due to the offset of the center of gravity, and is equal to the product of the block's weight and the horizontal displacement of the center of gravity from the geometric center, $(0.075\ \text{m})/\sqrt{2}.$ The block's mass is half of its volume times the density of water, so the net torque is

$$\frac{(0.30\ \text{m})^3(1000\ \text{kg/m}^3)}{2}(9.80\ \text{m/s}^2)\frac{0.075\ \text{m}}{\sqrt{2}} = 7.02\ \text{N}\cdot\text{m},$$

or $7.0\ \text{N}\cdot\text{m}$ to two figures. Note that the buoyant force and the block's weight form a couple, and the torque is the same about any axis.

14.80: a) As in Example 14.8, the speed of efflux is $\sqrt{2gh}.$ After leaving the tank, the water is in free fall, and the time it takes any portion of the water to reach the ground is $t = \sqrt{\frac{2(H-h)}{g}},$ in which time the water travels a horizontal distance $R = vt = 2\sqrt{h(H-h)}.$

b) Note that if $h' = H - h, h'(H - h') = (H - h)h,$ and so $h' = H - h$ gives the same range. A hole $H - h$ below the water surface is a distance h above the bottom of the tank.

14.81: The water will rise until the rate at which the water flows out of the hole is the rate at which water is added;

$$A\sqrt{2gh} = \frac{dV}{dt},$$

which is solved for

$$h = \left(\frac{dV/dt}{A}\right)^2 \frac{1}{2g} = \left(\frac{2.40\times10^{-4}\ \text{m}^3/\text{s}}{1.50\times10^{-4}\ \text{m}^2}\right)^2 \frac{1}{2(9.80\ \text{m/s}^2)} = 13.1\,\text{cm}.$$

Note that the result is independent of the diameter of the bucket.

14.82: a) $v_3 A_3 = \sqrt{2g(y_1 - y_3)}A_3 = \sqrt{2(9.80\ \text{m/s}^2)(8.00\ \text{m})}(0.0160\,\text{m}^2) = 0.200\ \text{m}^3/\text{s}.$

b) Since p_3 is atmospheric, the gauge pressure at point 2 is

$$p_2 = \frac{1}{2}\rho(v_3^2 - v_2^2) = \frac{1}{2}\rho v_3^2\left(1 - \left(\frac{A_3}{A_2}\right)^2\right) = \frac{8}{9}\rho g(y_1 - y_3),$$

using the expression for v_3 found above. Subsititution of numerical values gives $p_2 = 6.97\times10^4$ Pa.

14.83: The pressure difference, neglecting the thickness of the wing, is $\Delta p = (1/2)\ \rho(v_{\text{top}}^2 - v_{\text{bottom}}^2),$ and solving for the speed on the top of the wing gives

$$v_{\text{top}} = \sqrt{(120\ \text{m/s})^2 + 2(2000\ \text{Pa})/(1.20\ \text{kg/m}^3)} = 133\ \text{m/s}.$$

The pressure difference is comparable to that due to an altitude change of about 200 m, so ignoring the thickness of the wing is valid.

14.84: a) Using the constancy of angular momentum, the product of the radius and speed is constant, so the speed at the rim is about $(200\ \text{km/h})\left(\dfrac{30}{350}\right) = 17\ \text{km/h}$. b) The pressure is lower at the eye, by an amount

$$\Delta p = \frac{1}{2}(1.2\ \text{kg/m}^3)((200\ \text{km/h})^2 - (17\ \text{km/h})^2)\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)^2 = 1.8 \times 10^3\ \text{Pa}.$$

c) $\frac{v^2}{2g} = 160\,\text{m}$ to two figures. d) The pressure at higher altitudes is even lower.

14.85: The speed of efflux at point D is $\sqrt{2gh_1}$, and so is $\sqrt{8gh_1}$ at C. The gauge pressure at C is then $\rho g h_1 - 4\rho g h_1 = -3\rho g h_1$, and this is the gauge pressure at E. The height of the fluid in the column is $3h_1$.

14.86: a) $v = \frac{dV/dt}{A}$, so the speeds are

$$\frac{6.00 \times 10^{-3}\ \text{m}^3/\text{s}}{10.0 \times 10^{-4}\ \text{m}^2} = 6.00\ \text{m/s} \text{ and } \frac{6.00 \times 10^{-3}\ \text{m}^3/\text{s}}{40.0 \times 10^{-4}\ \text{m}^2} = 1.50\ \text{m/s}.$$

b) $\Delta p = \frac{1}{2}\rho(v_1^2 - v_2^2) = 1.688 \times 10^4\ \text{Pa}$, or $1.69 \times 10^4\ \text{Pa}$ to three figures.

c) $\Delta h = \frac{\Delta p}{\rho_{\text{Hg}} g} = \frac{(1.688 \times 10^4\ \text{Pa})}{(13.6 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 12.7\ \text{cm}.$

14.87: a) The speed of the liquid as a function of the distance *y* that it has fallen is $v = \sqrt{v_0^2 + 2gy}$, and the cross-section area of the flow is inversely proportional to this speed. The radius is then inversely proportional to the square root of the speed, and if the radius of the pipe is r_0, the radius *r* of the stream a distance *y* below the pipe is

$$r = \frac{r_0\sqrt{v_0}}{(v_0^2 + 2gy)^{1/4}} = r_0\left(1 + \frac{2gy}{v_0^2}\right)^{-1/4}.$$

b) From the result of part (a), the height is found from $(1 + 2gy/v_0^2)^{1/4} = 2$, or

$$y = \frac{15v_0^2}{2g} = \frac{15(1.2 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 1.10 \text{ m}.$$

14.88: a) The volume *V* of the rock is

$$V = \frac{B}{\rho_{\text{water}}g} = \frac{w - T}{\rho_{\text{water}}g} = \frac{((3.00 \text{ kg})(9.80 \text{ m/s}^2) - 21.0 \text{ N})}{(1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 8.57\times10^{-4} \text{ m}^3.$$

In the accelerated frames, all of the quantities that depend on *g* (weights, buoyant forces, gauge pressures and hence tensions) may be replaced by $g' = g + a$, with the positive direction taken upward. Thus, the tension is $T = mg' - B' = (m - \rho V)g' = T_0 \frac{g'}{g}$, where $T_0 = 21.0 \text{ N}$.

b) $g' = g + a$; for $a = 2.50 \text{ m/s}^2, T = (21.0 \text{ N})\frac{9.80+2.50}{9.80} = 26.4 \text{ N}$.

c) For $a = -2.50 \text{ m/s}^2, T = (21.0 \text{ N})\frac{9.80-2.50}{9.80} = 15.6 \text{ N}$.

d) If $a = -g, g' = 0$ and $T = 0$.

14.89: a) The tension in the cord plus the weight must be equal to the buoyant force, so

$$\begin{aligned} T &= Vg(\rho_{\text{water}} - \rho_{\text{foam}}) \\ &= (1/2)(0.20\,\text{m})^2(0.50\,\text{m})(9.80\ \text{m/s}^2)(1000\ \text{kg/m}^3 - 180\,\text{kg/m}^3) \\ &= 80.4\,\text{N}. \end{aligned}$$

b) The depth of the bottom of the styrofoam is not given; let this depth be h_0. Denote the length of the piece of foam by L and the length of the two sides by l. The pressure force on the bottom of the foam is then $(p_0 + \rho g h_0)L\left(\sqrt{2}l\right)$ and is directed up. The pressure on each side is not constant; the force can be found by integrating, or using the result of Problem 14.44 or Problem 14.46. Although these problems found forces on vertical surfaces, the result that the force is the product of the average pressure and the area is valid. The average pressure is $p_0 + \rho g(h_0 - (l/(2\sqrt{2})))$, and the force on one side has magnitude

$$(p_0 + \rho g(h_0 - l/(2\sqrt{2})))Ll$$

and is directed perpendicular to the side, at an angle of $45.0°$ from the vertical. The force on the other side has the same magnitude, but has a horizontal component that is opposite that of the other side. The horizontal component of the net buoyant force is zero, and the vertical component is

$$B = (p_0 + \rho g h_0)Ll\sqrt{2} - 2(\cos 45.0°)(p_0 + \rho g(h_0 - l/(2\sqrt{2})))Ll = \rho g \frac{Ll^2}{2},$$

the weight of the water displaced.

14.90: When the level of the water is a height y above the opening, the efflux speed is $\sqrt{2gy}$, and $\frac{dV}{dt} = \pi(d/2)^2\sqrt{2gy}$. As the tank drains, the height decreases, and

$$\frac{dy}{dt} = -\frac{dV/dt}{A} = -\frac{\pi(d/2)^2\sqrt{2gy}}{\pi(D/2)^2} = -\left(\frac{d}{D}\right)^2\sqrt{2gy}.$$

This is a separable differential equation, and the time T to drain the tank is found from

$$\frac{dy}{\sqrt{y}} = -\left(\frac{d}{D}\right)^2\sqrt{2g}\,dt,$$

which integrates to

$$\left[2\sqrt{y}\,\right]_H^0 = -\left(\frac{d}{D}\right)^2\sqrt{2g}\,T,$$

or

$$T = \left(\frac{D}{d}\right)^2\frac{2\sqrt{H}}{\sqrt{2g}} = \left(\frac{D}{d}\right)^2\sqrt{\frac{2H}{g}}.$$

14.91: a) The fact that the water first moves upwards before leaving the siphon does not change the efflux speed, $\sqrt{2gh}$. b) Water will not flow if the absolute (not gauge) pressure would be negative. The hose is open to the atmosphere at the bottom, so the pressure at the top of the siphon is $p_a - \rho g(H+h)$, where the assumption that the cross-section area is constant has been used to equate the speed of the liquid at the top and bottom. Setting $p = 0$ and solving for H gives $H = (p_a/\rho g) - h$.

14.92: Any bubbles will cause inaccuracies. At the bubble, the pressure at the surfaces of the water will be the same, but the levels need not be the same. The use of a hose as a level assumes that pressure is the same at all point that are at the same level, an assumption that is invalidated by the bubble.

15.1: a) The period is twice the time to go from one extreme to the other, and $v = f\lambda = \lambda/T = (6.00\,\text{m})/(5.0\,\text{s}) = 1.20\ \text{m/s}$, or $1.2\ \text{m/s}$ to two figures. b) The amplitude is half the total vertical distance, $0.310\,\text{m}$. c) The amplitude does not affect the wave speed; the new amplitude is $0.150\,\text{m}$. d) For the waves to exist, the water level cannot be level (horizontal), and the boat would tend to move along a wave toward the lower level, alternately in the direction of and opposed to the direction of the wave motion.

15.2:

$$f\lambda = v$$

$$f = \frac{v}{\lambda} = \frac{1500\ \text{m/s}}{0.001\,\text{m}} = 1.5\times10^{6}\ \text{Hz}$$

15.3: a) $\lambda = v/f = (344\ \text{m/s})/(784\,\text{Hz}) = 0.439\,\text{m}.$

b) $f = v/\lambda = (344\ \text{m/s})/(6.55\times10^{-5}\ \text{m}) = 5.25\times10^{6}\ \text{Hz}.$

15.4: Denoting the speed of light by c, $\lambda = \frac{c}{f}$, and

a) $\frac{3.00\times10^{8}\ \text{m/s}}{540\times10^{3}\ \text{Hz}} = 556\,\text{m}.$ b) $\frac{3.00\times10^{8}\ \text{m/s}}{104.5\times10^{6}\ \text{Hz}} = 2.87\ \text{m}.$

15.5: a) $\lambda_{\text{max}} = (344\ \text{m/s})/(20.0\,\text{Hz}) = 17.2\,\text{m}$, $\lambda_{\text{min}} = (344\ \text{m/s})/(20{,}000\,\text{Hz})$ $= 1.72\ \text{cm}.$

b) $\lambda_{\text{max}} = (1480\ \text{m/s})/(20.0\,\text{Hz}) = 74.0\,\text{m}, \lambda_{\text{min}} = (1480\ \text{m/s})/(20{,}000\,\text{Hz}) = 74.0\,\text{mm}.$

15.6: Comparison with Eq. (15.4) gives a) $6.50\,\text{mm}$, b) $28.0\,\text{cm}$, c) $f = \frac{1}{T} = \frac{1}{0.0360\ \text{s}} = 27.8\,\text{Hz}$ and from Eq. (15.1), d) $v = (0.280\,\text{m})(27.8\,\text{Hz}) = 7.78\ \text{m/s}$, e) $+x$ direction.

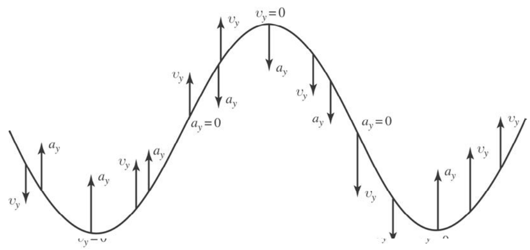
15.7: a) $f = v/\lambda = (8.00\text{ m/s})/(0.320\text{ m}) = 25.0\text{ Hz}$,
$T = 1/f = 1/(25.0\text{ Hz}) = 4.00\times10^{-2}\text{ s}$, $k = 2\pi/\lambda = (2\pi)/(0.320\text{ m}) = 19.6\text{ rad/m}$.

b) $y(x,t) = (0.0700\text{ m})\cos 2\pi\left(t(25.0\text{ Hz}) + \dfrac{x}{0.320\text{ m}}\right)$.

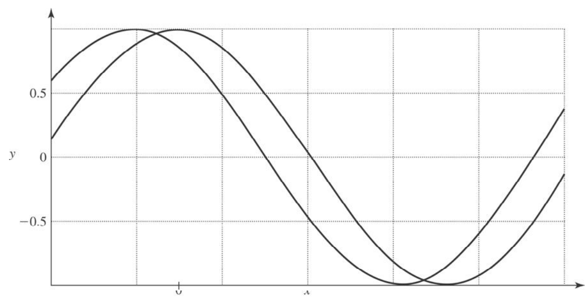
c) $(0.0700\text{ m})\cos\left[2\pi((0.150\text{ s})(25.0\text{ Hz}) + (0.360\text{ m})/(0.320\text{ m}))\right] = -4.95\text{ cm}$.

d) The argument in the square brackets in the expression used in part (c) is $2\pi(4.875)$, and the displacement will next be zero when the argument is 10π; the time is then $T(5 - x/\lambda) = (1/25.0\text{ Hz})(5 - (0.360\text{ m})/(0.320\text{ m})) = 0.1550\text{ s}$ and the elapsed time is 0.0050 s, e) $T/2 = 0.02\text{ s}$.

15.8: a)



b)



15.9: a) $$\frac{\partial y}{\partial x} = -Ak\sin(kx+\omega t) \qquad \frac{\partial^2 y}{\partial x^2} = -Ak^2\cos(kx+\omega t)$$

$$\frac{\partial y}{\partial t} = -A\omega\sin(kx+\omega t) \qquad \frac{\partial^2 y}{\partial t^2} = -A\omega^2\cos(kx+\omega t),$$

and so $\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2}$, and $y(x,t)$ is a solution of Eq. (15.12) with $v = \omega/k$.

b) $$\frac{\partial y}{\partial x} = +Ak\cos(kx+\omega t) \qquad \frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(kx+\omega t)$$

$$\frac{\partial y}{\partial t} = +A\omega\cos(kx+\omega t) \qquad \frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx+\omega t),$$

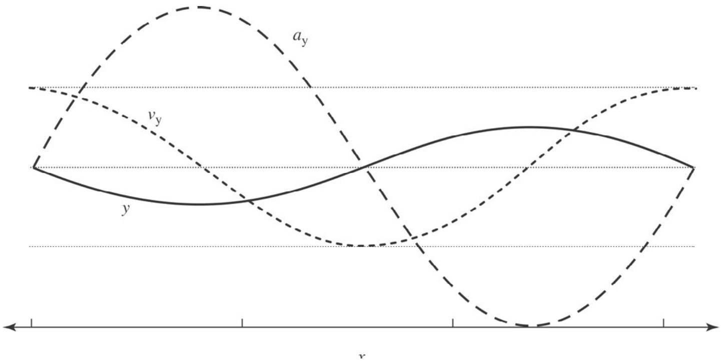
and so $\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2}$, and $y(x,t)$ is a solution of Eq. (15.12) with $v = \omega/k$. c) Both waves are moving in the $-x$-direction, as explained in the discussion preceding Eq. (15.8).
d) Taking derivatives yields $v_y(x,t) = -\omega A\cos(kx+\omega t)$ and $a_y(x,t) = -\omega^2 A\sin(kx+\omega t)$.

15.10: a) The relevant expressions are

$$y(x,t) = A\cos(kx - \omega t)$$

$$v_y = \frac{\partial y}{\partial t} = \omega A \sin(kx - \omega t)$$

$$a_y = \frac{\partial^2 y}{\partial t^2} = \frac{\partial v_y}{\partial t} = -\omega^2 A\cos(\omega t - kx).$$



b) (Take A, k and ω to be positive. At $^x t = 0,$ the wave is represented by (19.7(a)); point (i) in the problem corresponds to the origin, and points (ii)-(vii) correspond to the points in the figure labeled 1-7.) (i) $v_y = \omega A\cos(0) = \omega A,$ and the particle is moving upward (in the positive y-direction). $a_y = -\omega^2 A\sin(0) = 0,$ and the particle is instantaneously not accelerating. (ii) $v_y = \omega A\cos(-\pi/4) = \omega A/\sqrt{2}$, and the particle is moving up.
$a_y = -\omega^2 A\sin(-\pi/4) = \omega^2 A/\sqrt{2}$, and the particle is speeding up.
(iii) $v_y = \omega A\cos(-\pi/2) = 0,$ and the particle is instantaneously at rest.
$a_y = -\omega^2 A\sin(-\pi/2) = \omega^2 A,$ and the particle is speeding up.
(iv) $v_y = \omega A\cos(-3\pi/4) = -\omega A/\sqrt{2}$, and the particle is moving down.
$a_y = -\omega^2 A\sin(-3\pi/4) = \omega^2 A/\sqrt{2}$, and the particle is slowing down (v_y is becoming less negative). (v) $v_y = \omega A\cos(-\pi) = -\omega A$ and the particle is moving down.
$a_y = -\omega^2 A\sin(-\pi) = 0,$ and the particle is instantaneously not accelerating.
(vi) $v_y = \omega A\cos(-5\pi/4) = -\omega A/\sqrt{2}$ and the particle is moving down.
$a_y = -\omega^2 A\sin(-5\pi/4) = -\omega^2 A/\sqrt{2}$ and the particle is speeding up (v_y and a_y have the same sign). (vii) $v_y = \omega A\cos(-3\pi/2) = 0,$ and the particle is instantaneously at rest.
$a_y = -\omega^2 A\sin(-3\pi/2) = -\omega^2 A$ and the particle is speeding up.
(viii) $v_y = \omega A\cos(-7\pi/4) = \omega A/\sqrt{2}$, and the particle is moving upward.
$a_y = -\omega^2 A\sin(-7\pi/4) = -\omega^2 A/\sqrt{2}$ and the particle is slowing down (v_y and a_y have opposite signs).

15.11: Reading from the graph, a) $A = 4.0\,\text{mm}$, b) $T = 0.040\,\text{s}$. c) A displacement of 0.090 m corresponds to a time interval of 0.025 s; that is, the part of the wave represented by the point where the red curve crosses the origin corresponds to the point where the blue curve crosses the *t*-axis $(y = 0)$ at $t = 0.025\,\text{s}$, and in this time the wave has traveled 0.090 m, and so the wave speed is $3.6\ \text{m/s}$ and the wavelength is $vT = (3.6\ \text{m/s})(0.040\,\text{s}) = 0.14\,\text{m}$. d) $0.090\,\text{m}/0.015\,\text{s} = 6.0\,\text{m/s}$ and the wavelength is 0.24 m. d) No; there could be many wavelengths between the places where $y(t)$ is measured.

15.12: a)

$$A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) = +A\cos\frac{2\pi}{\lambda}\left(x - \frac{\lambda}{T}t\right)$$

$$= +A\cos\frac{2\pi}{\lambda}(x - vt),$$

where $\frac{\lambda}{T} = \lambda f = v$ has been used.

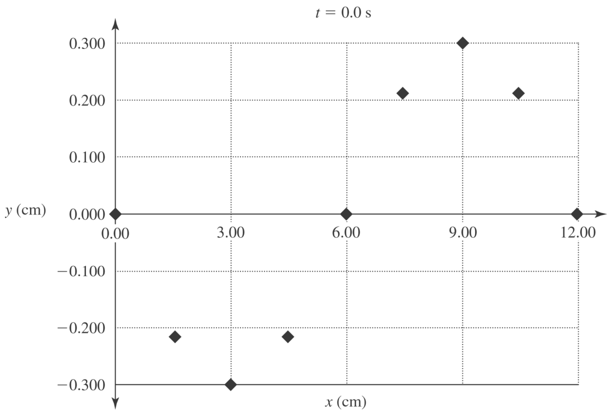
b)

$$v_y = \frac{\partial y}{\partial t} = \frac{2\pi v}{\lambda} A\sin\frac{2\pi}{\lambda}(x - vt).$$

c) The speed is the greatest when the cosine is 1, and that speed is $2\pi vA/\lambda$. This will be equal to v if $A = \lambda/2\pi$, less than v if $A < \lambda/2\pi$ and greater than v if $A > \lambda/2\pi$.

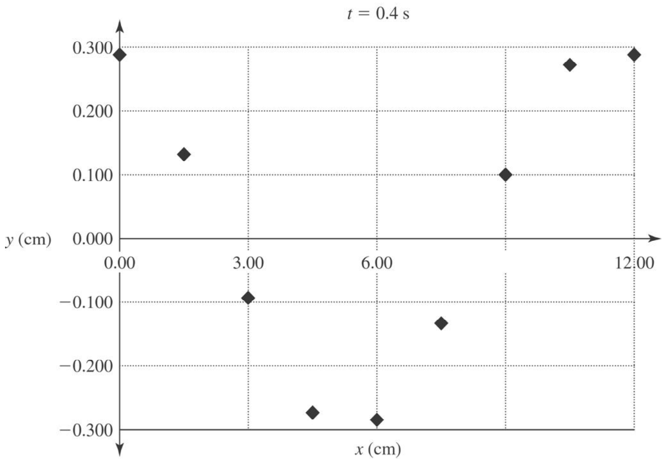
15.13: a) $t = 0$:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	0.000	–0.212	–0.300	–0.212	0.000	0.212	0.300	0.212	0.000



b) i) t = 0.400 s:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	0.285	0.136	–0.093	–0.267	–0.285	–0.136	0.093	0.267	0.285



ii) $t = 0.800\,\text{s}$:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	0.176	0.296	0.243	0.047	–0.176	–0.296	–0.243	–0.047	0.176

15.14: Solving Eq. (15.13) for the force F,

$$F = \mu v^2 = \mu(f\lambda)^2 = \left(\frac{0.120\,\text{kg}}{2.50\,\text{m}}\right)((40.0\,\text{Hz})(0.750\,\text{m}))^2 = 43.2\,\text{N}.$$

15.15: a) Neglecting the mass of the string, the tension in the string is the weight of the pulley, and the speed of a transverse wave on the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{(1.50\,\text{kg})(9.80\,\text{m/s}^2)}{(0.0550\,\text{kg/m})}} = 16.3\,\text{m/s}.$$

b) $\lambda = v/f = (16.3\,\text{m/s})/(120\,\text{Hz}) = 0.136\,\text{m}.$ c) The speed is proportional to the square root of the tension, and hence to the square root of the suspended mass; the answers change by a factor of $\sqrt{2}$, to $23.1\,\text{m/s}$ and 0.192 m.

15.16: a) $v = \sqrt{F/\mu} = \sqrt{(140.0\,\text{N})(10.0\,\text{m})/(0.800\,\text{kg})} = 41.8\,\text{m/s}.$
b) $\lambda = v/f = (41.8\,\text{m/s})/(1.20\,\text{Hz}) = 34.9\,\text{m}.$ c) The speed is larger by a factor of $\sqrt{2}$, and so for the same wavelength, the frequency must be multiplied by $\sqrt{2}$, or 1.70 Hz.

15.17: Denoting the suspended mass by M and the string mass by m, the time for the pulse to reach the other end is

$$t = \frac{L}{v} = \frac{L}{\sqrt{Mg/(m/L)}} = \sqrt{\frac{mL}{Mg}} = \sqrt{\frac{(0.800\text{ kg})(14.0\text{ m})}{(7.50\text{ kg})(9.80\text{ m/s}^2)}} = 0.390\,\text{s}.$$

15.18: a) The tension at the bottom of the rope is due to the weight of the load, and the speed is the same 88.5 m/s as found in Example 15.4 b) The tension at the middle of the rope is $(21.0\text{ kg})(9.80\text{ m/s}^2) = 205.8\text{ N}$ (keeping an extra figure) and the speed of the rope is 90.7 m/s. c) The tension at the top of the rope is $(22.0\,\text{kg})(9.80\text{ m/s}^2) = 215.6\text{ m/s}$ and the speed is 92.9 m/s. (See Challenge Problem (15.80) for the effects of varying tension on the time it takes to send signals.)

15.19: a) $v = \sqrt{F/\mu} = \sqrt{(5.00\ \text{N})/(0.0500\ \text{kg/m})} = 10.0\ \text{m/s}$

b) $\lambda = v/f = (10.0\ \text{m/s})/(40.0\ \text{Hz}) = 0.250\ \text{m}$

c) $y(x,t) = A\cos(kx - \omega t)$ (Note: $y(0.0) = A$, as specified.)

$k = 2\pi/\lambda = 8.00\pi\ \text{rad/m};\ \omega = 2\pi f = 80.0\pi\ \text{rad/s}$ -

$y(x,t) = (3.00\ \text{cm})\cos[\pi(8.00\ \text{rad/m})x - (80.0\pi\ \text{rad/s})t]$

d) $v_y = +A\omega\sin(kx - \omega t)$ and $a_y = -A\omega^2\cos(kx - \omega t)$

$a_{y,\max} = A\omega^2 = A(2\pi f)^2 = 1890\ \text{m/s}^2$

e) $a_{y,\max}$ is much larger than g, so ok to ignore gravity.

15.20: a) Using Eq.(15.25),

$$P_{\text{ave}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$$
$$= \frac{1}{2}\sqrt{\left(\frac{3.00\times 10^{-3}\ \text{kg}}{0.80\ \text{m}}\right)(25.0\ \text{N})}(2\pi(120.0\ \text{Hz}))^2(1.6\times 10^{-3}\ \text{m})^2$$
$$= 0.223\ \text{W},$$

or 0.22 W to two figures. b) Halving the amplitude quarters the average power, to 0.056 W.

15.21: Fig. 15.13 plots $P(x,t) = \sqrt{\mu F}\omega^2 A^2 \sin^2(kx - \omega t)$ at $x = 0$.

For $x = 0$, $P(x,t) = \sqrt{\mu F}\omega^2\ A^2\ \sin^2(\omega t) = P_{\max}\ \sin^2(\omega t)$

When $x = \lambda/4, kx = (2\pi/\lambda)(\lambda/4) = \pi/2$.

$\sin\ (\pi/2 - \omega t) = \cos\omega t$, so $P(\lambda/4, t) = P_{\max}\ \cos^2\ \omega t$

The graph is shifted by $T/4$ but is otherwise the same. The instantaneous power is still never negative and $P_{\text{av}} = \frac{1}{2}P_{\max}$, the same as at $x = 0$.

15.22: $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (7.5\ \text{m})\sqrt{\frac{0.11\ \text{W/m}^2}{1.0\ \text{W/m}^2}} = 2.5\ \text{m}$, so it is possible to move $r_1 - r_2 = 7.5\ \text{m} - 2.5\ \text{m} = 5.0\ \text{m}$ closer to the source.

15.23: a) $I_1 r_1^2 = I_2 r_2^2$

$$I_2 = I_1 (r_1/r_2)^2 = (0.026\ \text{W/m}^2)(4.3\text{m}/3.1\,\text{m})^2 = 0.050\ \text{W/m}^2$$

b) $P = 4\pi r^2 I = 4\pi(4.3\text{m})^2(0.026\ \text{W/m}^2) = 6.04\ \text{W}$

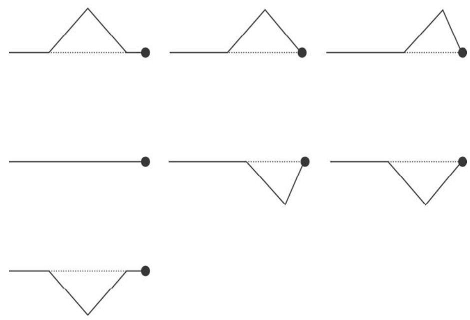
Energy = $Pt = (6.04\ \text{W})(3600\,\text{s}) = 2.2\times 10^4\ \text{J}$

15.24: (a) $A = 2.30\,\text{mm}$. (b) $f = \frac{\omega}{2\pi} = \frac{742\ \text{rad/s}}{2\pi} 118\,\text{Hz}$. (c) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.98\ \text{rad/m}} = 0.90\,\text{m}$. (d) $v = \frac{\omega}{k} = \frac{742\ \text{rad/s}}{6.98\ \text{rad/m}} = 106\ \text{m/s}$. (e) The wave is traveling in the –*x* direction because the phase of $y\,(x,t)$ has the form $kx + \omega t$. (f) The linear mass density is $\mu = (3.38\times 10^{-3}\ \text{kg})/(1.35\ \text{m}) = 2.504\times 10^{-3}\ \text{kg/m}$, so the tension is $F = \mu v^2 = (2.504\times 10^{-3}\ \text{kg/m})(106.3\ \text{m/s})^2 = 28.3\ \text{N}$ (keeping an extra figure in v for accuracy). (g) $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \frac{1}{2}\sqrt{(2.50\times 10^{-3}\ \text{kg/m})(28.3\,\text{N})}(742\ \text{rad/s})^2$ $(2.30\times 10^{-3}\ \text{m})^2 = 0.39\,\text{W}$.

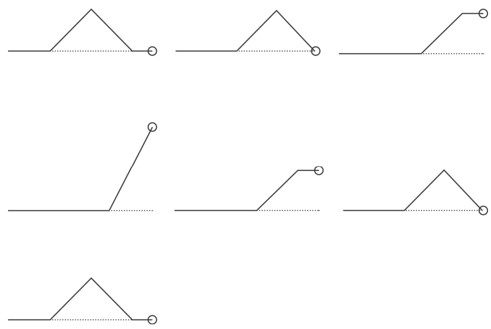
15.25: $I = 0.250\ \text{W/m}^2$ at $r = 15.0\,\text{m}$

$$P = 4\pi r^2 I = 4\pi(15.0\,\text{m})^2(0.250\ \text{W/m}^2) = 707\ \text{W}$$

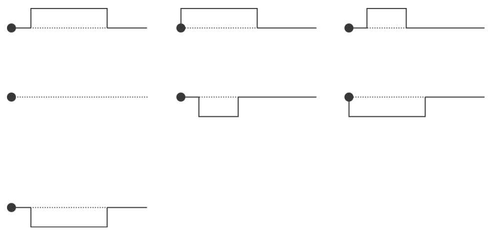
15.26: a) The wave form for the given times, respectively, is shown.



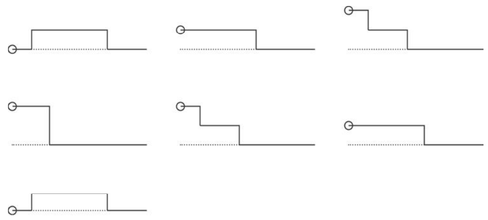
b)



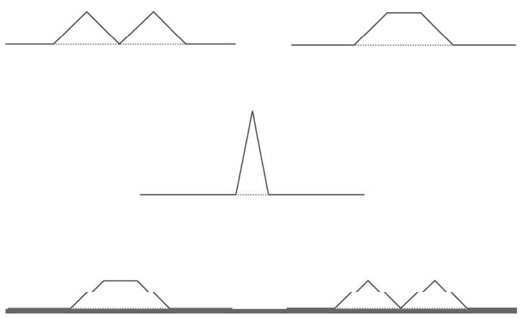
15.27: a) The wave form for the given times, respectively, is shown.



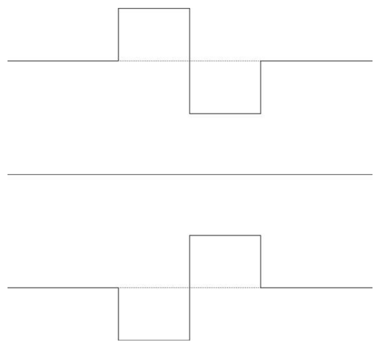
b)



15.28:



15.29:



15.30: Let the wave traveling in the $+x$ direction be $y_1(x,t) = A\cos(kx - \omega t)$. The wave traveling in the $-x$ direction is inverted due to reflection from the fixed end of the string at $x = 0$, so it has the form $y_2(x,t) = -A\cos(kx + \omega t)$. The wave function of the resulting standing wave is then $y(x,t) = y_1(x,t) + y_2(x,t)$, where $A = 2.46\ \text{mm}, \omega = 2\pi/T = 2\pi/(3.65\times10^{-3}\ \text{s}) = 1.72\times10^{3}\ \text{rad/s},\ k = \omega/v =$ $(1.72\times10^{3}\ \text{rad/s})(111\ \text{m/s}) = 15.5\,\text{rad/m}.$

15.31: a) The nodes correspond to the places where $y = 0$ for all t in Eq. (15.1); that is, $\sin kx_{\text{node}} = 0$ or $kx_{\text{node}} = n\pi$, n an integer. With $k = 0.75\pi$ rad/m, $x_{\text{node}} = (1.333\,\text{m})n$ and for $n = 0, 1, 2, \ldots,\ x_{\text{node}} = 0, 1.333\,\text{m}, 2.67\,\text{m}, 4.00\,\text{m}, 5.33\,\text{m}, 6.67\,\text{m}, \ldots$ b) The antinodes correspond to the points where $\cos kx = 0$, which are halfway between any two adjacent nodes, at 0.667 m, 2.00 m, 3.33 m, 4.67 m, 6.00 m, ...

15.32: a) $\dfrac{\partial^2 y}{\partial x^2} = -k^2[A_{\text{sw}} \sin \omega t]\sin kx,\ \dfrac{\partial^2 y}{\partial t^2} = -\omega^2[A_{\text{sw}} \sin \omega t]\sin kx,$

so for $y(x,t)$ to be a solution of Eq. (15.12), $-k^2 = \dfrac{-\omega^2}{v^2}$, and $v = \dfrac{\omega}{k}$.

b) A standing wave is built up by the superposition of traveling waves, to which the relationship $v = \omega/k$ applies.

15.33: a) The amplitude of the standing wave is $A_{sw} = 0.85\,\text{cm}$, the wavelength is twice the distance between adjacent antinodes, and so Eq. (15.28) is

$$y(x,t) = (0.85\,\text{cm})\sin((2\pi/0.075\,\text{s})t)\sin(2\pi x/30.0\,\text{cm}).$$

b) $v = \lambda f = \lambda/T = (30.0\,\text{cm})/(0.0750\,\text{s}) = 4.00\,\text{m/s}.$

c) $(0.850\,\text{cm})\sin(2\pi(10.5\,\text{cm})/(30.0\,\text{cm})) = 0.688\,\text{cm}.$

15.34:
$$\begin{aligned} y_1 + y_2 &= A\,[-\cos(kx+\omega t) + \cos(kx-\omega t)] \\ &= A\,[-\cos kx\cos\omega t + \sin kx\,\sin\omega t + \cos kx\cos\omega t + \sin kx\sin\omega t] \\ &= 2A\sin kx\sin\omega t. \end{aligned}$$

15.35: The wave equation is a linear equation, as it is linear in the derivatives, and differentiation is a linear operation. Specifically,

$$\frac{\partial y}{\partial x} = \frac{\partial(y_1+y_2)}{\partial x} = \frac{\partial y_1}{\partial x} + \frac{\partial y_2}{\partial x}.$$

Repeating the differentiation to second order in both x and t,

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y_1}{\partial x^2} + \frac{\partial^2 y_2}{\partial x^2}, \qquad \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y_1}{\partial t^2} + \frac{\partial^2 y_2}{\partial t^2}.$$

The functions y_1 and y_2 are given as being solutions to the wave equation; that is,

$$\begin{aligned} \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y_1}{\partial x^2} + \frac{\partial^2 y_2}{\partial x^2} &= \left(\frac{1}{v^2}\right)\frac{\partial^2 y_1}{\partial t^2} + \left(\frac{1}{v^2}\right)\frac{\partial^2 y_2}{\partial t^2} \\ &= \left(\frac{1}{v^2}\right)\left[\frac{\partial^2 y_1}{\partial t^2} + \frac{\partial^2 y_2}{\partial t^2}\right] \\ &= \left(\frac{1}{v^2}\right)\frac{\partial^2 y}{\partial t^2} \end{aligned}$$

and so $y = y_1 + y_2$ is a solution of Eq. (15.12).

15.36: a) From Eq. (15.35),

$$f_1 = \frac{1}{2L}\sqrt{\frac{FL}{m}} = \frac{1}{2(0.400\,\text{m})}\sqrt{\frac{(800\,\text{N})(0.400\,\text{m})}{(3.00\times10^{-3}\,\text{kg})}} = 408\text{ Hz.}$$

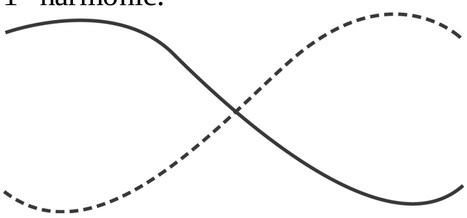
b) $\frac{10{,}000\text{ Hz}}{408\text{ Hz}} = 24.5$, so the 24^{th} harmonic may be heard, but not the 25^{th}.

15.37: a) In the fundamental mode,
$\lambda = 2L = 1.60\,\text{m}$ and so $v = f\lambda = (60.0\,\text{Hz})(1.60\,\text{m}) = 96.0\,\text{m/s}$.

b) $F = v^2\mu = v^2\, m/L = (96.0\,\text{m/s})^2\,(0.0400\,\text{kg})/(0.800\,\text{m}) = 461\,\text{N}$.

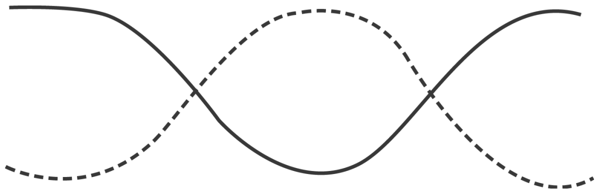
15.38: The ends of the stick are free, so they must be displacement antinodes.

1^{st} harmonic:



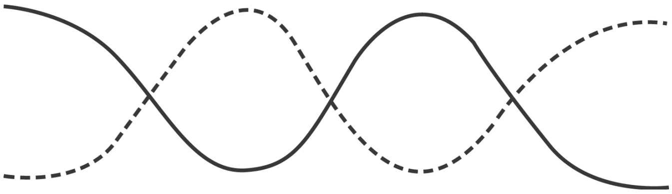
$$L = \frac{1}{2}\lambda_1 \rightarrow \lambda_1 = 2L = 4.0\,\text{m}$$

2^{nd} harmonic:



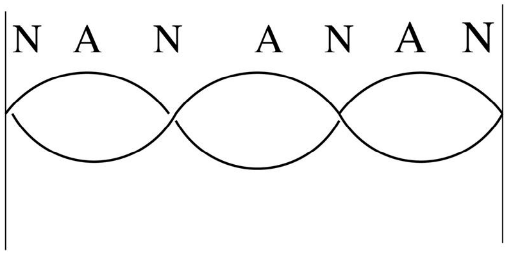
$$L = 1\lambda_2 \rightarrow \lambda_2 = L = 2.0\,\text{m}$$

3^{rd} harmonic:



$$L = \frac{3}{2}\lambda_3 \rightarrow \lambda_3 = \frac{2L}{3} = 1.33\,\text{m}$$

15.39: a)



b) Eq. (15.28) gives the general equation for a standing wave on a string:
$y(x,t) = (A_{sw} \sin\ kx) \sin\omega t$

$A_{sw} = 2A$, so $A = A_{SW}/2 = (5.60\text{ cm})/2 = 2.80\text{ cm}$

c) The sketch in part (a) shows that $L = 3(\lambda/2)$

$k = 2\pi/\lambda,\ \lambda = 2\pi/k$

Comparison of $y(x,t)$ given in the problem to Eq.(15.28) gives
$k = 0.0340\text{ rad/cm}$.

So, $\lambda = 2\pi/(0.0340\text{ rad/cm}) = 184.8\text{ cm}$

$L = 3(\lambda/3) = 277\text{ cm}$

d) $\lambda = 185\text{ cm}$, from part (c)

$\omega = 50.0\text{ rad/s}$ so $f = \omega/2\pi = 7.96\text{ Hz}$

period $T = 1/f = 0.126\text{ s}$

$v = f\lambda = 1470\text{ cm/s}$

e) $v_y = dy/dt = \omega A_{sw} \sin kx \cos\omega t$
$v_{y,\max} = \omega A_{SW} = (50.0\text{ rad/s})(5.60\text{ cm}) = 280\text{ cm/s}$

f) $f_3 = 7.96\text{ Hz} = 3f_1$, so $f_1 = 2.65\text{ Hz}$ is the fundamental

$f_8 = 8f_1 = 21.2\text{ Hz};\ \omega_8 = 2\pi f_8 = 133\text{ rad/s}$

$\lambda = v/f = (1470\text{cm/s})/(21.2\text{Hz}) = 69.3\text{cm}$ and $k = 2\pi/\lambda = 0.0906\text{rad/cm}$.

$y(x,t) = (5.60\text{ cm}) \sin([0.0906\text{ rad/cm}]x) \sin([133\text{ rad/s}]t)$

15.40: (a) $A = \frac{1}{2}A_{SW} = \frac{1}{2}(4.44\,\text{mm}) = 2.22\,\text{mm}$. (b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{32.5\ \text{rad/m}} = 0.193\,\text{m}$.
(c) $f = \frac{\omega}{2\pi} = \frac{754\ \text{rad/m}}{2\pi} = 120\,\text{Hz}$. (d) $v = \frac{\omega}{k} = \frac{754\ \text{rad/m}}{32.5\ \text{rad/m}} = 23.2\ \text{m/s}$. (e) If the wave traveling in the $+x$ direction is written as $y_1(x,t) = A\cos(kx - \omega t)$, then the wave traveling in the $-x$ direction is $y_2(x,t) = -A\cos(kx + \omega t)$, where $A = 2.22\,\text{mm}$ from (a), and $k = 32.5\,\text{rad/m}$ and $\omega = 754\ \text{rad/s}$. (f) The harmonic cannot be determined because the length of the string is not specified.

15.41: a) The traveling wave is $y(x,t) = (2.30\,\text{m})\cos([6.98\,\text{rad/m}]x) + [742\,\text{rad/s}]t)$

$A = 2.30\,\text{mm}$ so $A_{SW} = 4.60\,\text{mm}$; $k = 6.98\,\text{rad/m}$ and $\omega = 742\,\text{rad/s}$

The general equation for a standing wave is $y(x,t) = (A_{SW}\sin kx)\sin\omega t$, so

$y(x,t) = (4.60\,\text{mm})\sin([6.98\,\text{rad/m}]x)\sin([742\,\text{rad/s}]t)$

b) $L = 1.35\,\text{m}$ (from Exercise 15.24)

$\lambda = 2\pi/k = 0.900\,\text{m}$

$L = 3(\lambda/2)$, so this is the 3^{rd} harmonic

c) For this 3rd harmonic, $f = \omega/2\pi = 118\,\text{Hz}$

$f_3 = 3f_1$ so $f_1 = (118\,\text{Hz})/3 = 39.3\,\text{Hz}$

15.42: The condition that $x = L$ is a node becomes $k_n L = n\pi$. The wave number and wavelength are related by $k_n\lambda_n = 2\pi$, and so $\lambda_n = 2L/n$.

15.43: a) The product of the frequency and the string length is a constant for a given string, equal to half of the wave speed, so to play a note with frequency $587\,\text{Hz}$, $x = (60.0\,\text{cm})(440\,\text{Hz})/(587\,\text{Hz}) = 45.0\,\text{cm}$.

b) Lower frequency requires longer length of string free to vibrate. Full length of string gives $440\,\text{Hz}$, so this is the lowest note possible.

15.44: a) (i) $x = \frac{\lambda}{2}$ is a node, and there is no motion. (ii) $x = \frac{\lambda}{4}$ is an antinode, and $v_{max} = A(2\pi f) = 2\pi f A$, $a_{max} = (2\pi f)v_{max} = 4\pi^2 f^2 A$. (iii) $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, and this factor multiplies the results of (ii), so $v_{max} = \sqrt{2}\pi fA$, $a_{max} = 2\sqrt{2}\pi^2 f^2 A$. b) The amplitude is $A\sin kx$, or (i) 0, (ii) A, (iii) $A/\sqrt{2}$. c) The time between the extremes of the motion is the same for any point on the string (although the period of the zero motion at a node might be considered indeterminate) and is $\frac{1}{2f}$.

15.45: a) $\lambda_1 = 2L = 3.00\ \text{m}, f_1 = \frac{v}{2L} = \frac{(48.0\ \text{m/s})}{2(1.50\ \text{m})} = 16.0\ \text{Hz}.$
b) $\lambda_3 = \lambda_1/3 = 1.00\ \text{m},\ f_2 = 3f_1 = 48.0\ \text{Hz}.$ c) $\lambda_4 = \lambda_1/4 = 0.75\ \text{m},\ f_3 = 4f_1 = 64.0\ \text{Hz}.$

15.46: a) For the fundamental mode, the wavelength is twice the length of the string, and $v = f\lambda = 2fL = 2(245\ \text{Hz})(0.635\ \text{m}) = 311\ \text{m/s}.$ b) The frequency of the fundamental mode is proportional to the speed and hence to the square root of the tension; $(245\ \text{Hz})\sqrt{1.01} = 246\ \text{Hz}.$ c) The frequency will be the same, $245\ \text{Hz}.$ The wavelength will be $\lambda_{\text{air}} = v_{\text{air}}/f = (344\ \text{m/s})/(245\ \text{Hz}) = 1.40\ \text{m},$ which is larger than the wavelength of standing wave on the string by a factor of the ration of the speeds.

15.47: a) $f = v/\lambda = (36.0\ \text{m/s})/(1.80\ \text{m}) = 20.0\ \text{Hz}, \omega = 2\pi f = 126\ \text{rad/s},$
$k = \omega/v = 2\pi/\lambda = 3.49\ \text{rad/m}.$
b) $y(x,t) = A\cos(kx - \omega t) = (2.50\ \text{mm})\cos[(3.49\ \text{rad/m})x - (126\ \text{rad/s})t].$

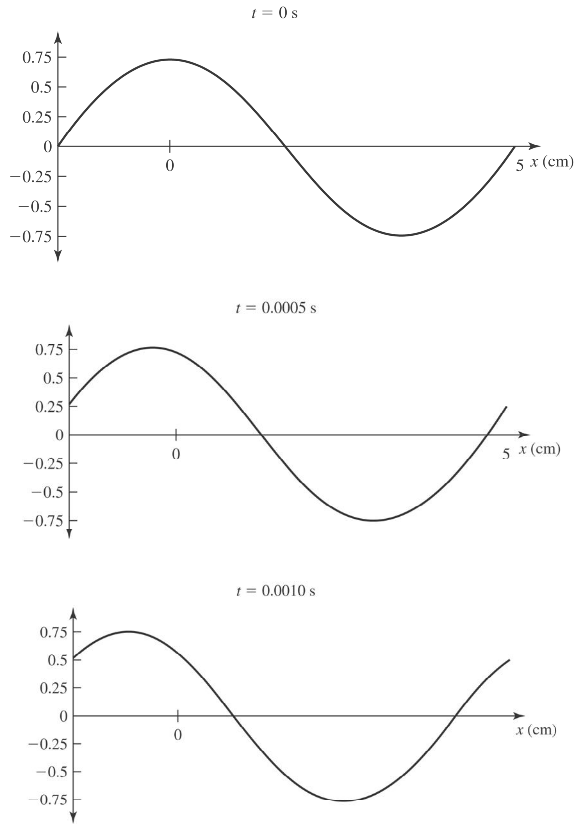
c)At $x = 0, y(0,t) = A\cos\omega t = (2.50\ \text{mm})\cos[(126\ \text{rad/s})t].$ From this form it may be seen that at $x = 0, t = 0, \frac{\partial y}{\partial t} > 0.$ d) At $x = 1.35\ \text{m} = 3\lambda/4, kx = 3\pi/2$ and

$$y(3\lambda/4, t) = A\cos[3\pi/2 - \omega t].$$

e) See Exercise 15.12; $\omega A = 0.315\ \text{m/s}.$ f) From the result of part (d), $y = 0\ \text{mm}.\ v_y = -0.315\ \text{m/s}.$

15.48: a) From comparison with Eq. (15.4), $A = 0.75\,\text{cm}, \lambda = \frac{2}{0.400/\text{cm}} = 5.00\,\text{cm}$, $f = 125\,\text{Hz}, T = \frac{1}{f} = 0.00800\,\text{s}$ and $v - \lambda f = 6.25\,\text{m/s}$.

b)



c) To stay with a wavefront as t increases, x and so the wave is moving in the $-x$-direction. d) From Eq. (15.13), the tension is $F = \mu v^2 = (0.50\,\text{kg/m})\,(6.25\,\text{m/s})^2 = 19.5\,\text{N}$.

e) $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = 54.2\,\text{W}$.

15.49: a) Speed in each segment is $v = \sqrt{F/\mu}$. The time to travel through a segment is $t = L/v$. The travel times then, are $t_1 = L\sqrt{\frac{\mu_1}{F}}, t_2 = L\sqrt{\frac{4\mu_1}{F}}$, and $t_3 = L\sqrt{\frac{\mu_1}{4F}}$. Adding gives $t_{\text{total}} = L\sqrt{\frac{\mu_1}{F}} + 2L\sqrt{\frac{\mu_1}{F}} + \frac{1}{2}L\sqrt{\frac{\mu_1}{F}} = \frac{7}{2}L\sqrt{\frac{\mu_1}{F}}$.

b) No, because the tension is uniform throughout each piece.

15.50: The amplitude given is not needed, it just ensures that the wave disturbance is small. Both strings have the same tension F, and the same length $L = 1.5\,\text{m}$. The wave takes different times t_1 and t_2 to travel along each string, so the design requirements is $t_1 + t_2 = 0.20\,\text{s}$. Using $t = L/v$ and $v = \sqrt{F/\mu} = \sqrt{FL/m}$ gives $\left(\sqrt{m_1} + \sqrt{m_2}\right)\sqrt{L/F} = 0.20\,\text{s}$, with $m_1 = 90 \times 10^{-3}\,\text{kg}$ and $m_1 = 10 \times 10^{-3}\,\text{kg}$. Solving for F gives $F = 6.0\,\text{N}$.

15.51: a) $y(x,t) = A\cos(kx - \omega t)$

$$v_y = dy/dt = +A\omega\sin(kx - \omega t)$$

$$v_{y,\max} = A\omega = 2\pi f A$$

$$f = \frac{v}{\lambda} \text{ and } v = \sqrt{\frac{F}{(m/L)}}, \text{ so } f = \left(\frac{1}{\lambda}\right)\sqrt{\frac{FL}{M}}$$

$$v_{y,\max} = \left(\frac{2\pi A}{\lambda}\right)\sqrt{\frac{FL}{M}}$$

b) To double $v_{y,\max}$ increase F by a factor of 4

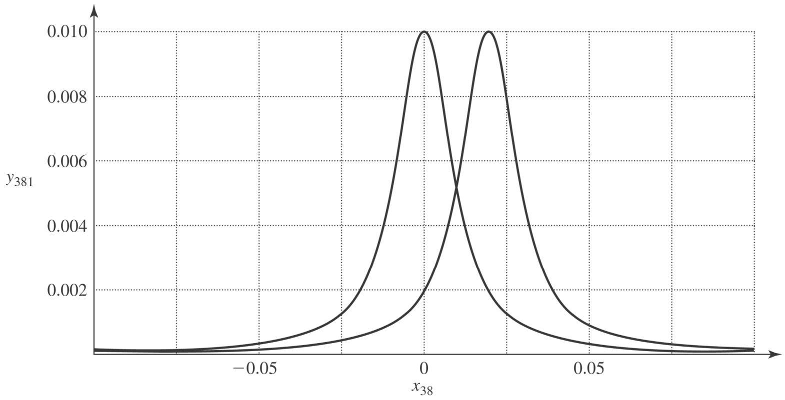
15.52: The maximum vertical acceleration must be at least g. Because $a = \omega^2 A, g = \omega^2 A_{\min}$ and thus $A_{\min} = g/\omega^2$. Using $\omega = 2\pi f = 2\pi v/\lambda$ and $v = \sqrt{F/\mu}$, this becomes $A_{\min} = \frac{g\lambda^2\mu}{4\pi^2 F}$.

15.53: a) See Exercise 15.10; $a_y = \frac{\partial^2 y}{\partial t^2} = -\omega^2 y$, and so $k' = \Delta m\omega^2 = \Delta x\mu\,\omega^2$.

b)
$$\omega^2 = (2\pi f)^2 = \left(\frac{2\pi v}{\lambda}\right)^2 = \frac{4\pi^2 F}{\mu\lambda^2}$$

and so $k' = (4\pi^2 F/\lambda^2)\Delta x$. The effective force constant k' is independent of amplitude, as for a simple harmonic oscillator, and is proportional to the tension that provides the restoring force. The factor of $1/\lambda^2$ indicates that the curvature of the string creates the restoring force on a segment of the string. More specifically, one factor of $1/\lambda$ is due to the curvature, and a factor of $1/(\lambda\mu)$ represents the mass in one wavelength, which determines the frequency of the overall oscillation of the string. The mass $\Delta m = \mu\Delta x$ also contains a factor of μ, and so the effective spring constant per unit length is independent of μ.

15.54: a), b)



c) The displacement is a maximum when the term in parentheses in the denominator is zero; the denominator is the sum of two squares and is minimized when $x = vt$, and the maximum displacement is A. At $x = 4.50\,\text{cm}$, the displacement is a maximum at $t = (4.50\times10^{-2}\ \text{m})/(20.0\ \text{m/s}) = 2.25\times10^{-3}\ \text{s}$. The displacement will be half of the maximum when $(x - vt)^2 = A^2$, or $t = (x \pm A)/v = 1.75\times10^{-3}$ s and 2.75×10^{-3} s.

d) Of the many ways to obtain the result, the method presented saves some algebra and minor calculus, relying on the chain rule for partial derivatives. Specifically, let $u = u(x,t) = x - vt$, so that if $f(x,t) = g(u)$, $\frac{\partial f}{\partial x} = \frac{dg}{du}\frac{\partial u}{\partial t} = \frac{dg}{du}$ and $\frac{\partial f}{\partial t} = \frac{dg}{du}\frac{\partial u}{\partial t} = -\frac{dg}{du}v.$ (In this form it may be seen that any function of this form satisfies the wave equation; see Problem 15.59.) In this case, $y(x,t) = A^3(A^2 + u^2)^{-1}$, and so

$$\frac{\partial y}{\partial x} = \frac{-2A^3u}{(A^2+u^2)^2}, \qquad \frac{\partial^2 y}{\partial x^2} = -\frac{2A^3(A^2-3u^2)}{(A^2+u^2)^3}$$

$$\frac{\partial y}{\partial t} = v\frac{2A^3u}{(A^2+u^2)^2}, \qquad \frac{\partial^2 y}{\partial t^2} = -v^2\,\frac{2A^3(A^2-3u^2)}{(A^2+u^2)^2},$$

and so the given form for $y(x,t)$ is a solution to the wave equation with speed v.

15.55: a) and b) (1): The curve appears to be horizontal, and $v_y = 0$. As the wave moves, the point will begin to move downward, and $a_y < 0$. (2): As the wave moves in the $+x$-direction (to the right in Fig. (15.34)), the particle will move upward so $v_y > 0$. The portion of the curve to the left of the point is steeper, so $a_y > 0$. (3) The point is moving down, and will increase its speed as the wave moves; $v_y < 0$, $a_y < 0$. (4) The curve appears to be horizontal, and $v_y = 0$. As the wave moves, the point will move away from the x-axis, and $a_y > 0$. (5) The point is moving downward, and will increase its speed as the wave moves; $v_y < 0, a_y < 0$. (6) The particle is moving upward, but the curve that represents the wave appears to have no curvature, so $v_y > 0$ and $a_y = 0$. c) The accelerations, which are related to the curvatures, will not change. The transverse velocities will all change sign.

15.56: (a) The wave travels a horizontal distance d in a time

$$t = \frac{d}{v} = \frac{d}{\lambda f} = \frac{8.00 \text{ m}}{(0.600\,\text{m})(40.0\,\text{Hz})} = 0.333\,\text{s}.$$

(b) A point on the string will travel a vertical distance of $4A$ each cycle. Although the transverse velocity $v_y(x,t)$ is not constant, a distance of $h = 8.00\,\text{m}$ corresponds to a whole number of cycles, $n = h/(4A) = (8.00\,\text{m})/((4(5.00\times10^{-3}\,\text{m})) = 400$, so the amount of time is $t = nT = n/f = (400)/(40.0\,\text{Hz}) = 10.0\,\text{s}$.

(c) The answer for (a) is independent of amplitude. For (b), the time is halved if the amplitude is doubled.

15.57: a) $y^2(x,y) + z^2(x,y) = A^2$

The trajectory is a circle of radius A.

At $t = 0, y(0,0) = A, z\,(0,0) = 0.$

At $t = \pi/2\omega, y(0,\pi/2\omega) = 0, z(0,\pi/2\omega) = -A.$

At $t = \pi/\omega, y(0,\pi/\omega) = -A, z(0,\pi/2\omega) = 0.$

At $t = 3\pi/2\omega, y(0,3\pi/2\omega) = 0, z(0,3\pi/2\omega) = +A$

b) $v_y = dy/dt = +A\omega\sin(kx - \omega t), v_z = dz/dt = -A\omega\cos(kx - \omega t)$

$v = \sqrt{v_y^2 + v_z^2} = A\omega,$ so the speed is constant.

$\vec{\boldsymbol{r}} = y\hat{\boldsymbol{j}} + z\hat{\boldsymbol{k}}$

$\vec{\boldsymbol{r}} \cdot \vec{\boldsymbol{v}} = yv_y + zv_z = A^2\omega\sin\,(kx - \omega t)\cos(kx - \omega t) - A^2\omega\cos(kx - \omega t)\sin(kx - \omega t)$

$\vec{\boldsymbol{r}} \cdot \vec{\boldsymbol{v}} = 0,$ so $\vec{\boldsymbol{v}}$ is tangent to the circular path.

c) $a_y = dv_y/dt = -A\omega^2\cos(kx - \omega t), a_z = dv_z/dt = -A\omega^2\sin(kx - \omega t)$

$\vec{\boldsymbol{r}} \cdot \vec{\boldsymbol{a}} = ya_y + za_z = -A^2\omega^2[\cos^2(kx - \omega t) + \sin^2(kx - \omega t)] = -A^2\omega^2$

$r = A, a = A\omega^2,$ so $\vec{\boldsymbol{r}} \cdot \vec{\boldsymbol{a}} = -ra$

$\vec{\boldsymbol{r}} \cdot \vec{\boldsymbol{a}} = ra\cos\phi$ so $\phi = 180°$ and $\vec{\boldsymbol{a}}$ is opposite in direction to $\vec{\boldsymbol{r}}; \vec{\boldsymbol{a}}$ is radially inward.

$y^2 + z^2 = A^2,$ so the path is again circular, but the particle rotates in the opposite sense compared to part (a).

15.58: The speed of light is so large compared to the speed of sound that the travel time of the light from the lightning or the radio signal may be neglected. Them, the distance from the storm to the dorm is $(344\,\text{m/s})(4.43\,\text{s}) = 1523.92\,\text{m}$ and the distance from the storm to the ballpark is $(344\text{m/s})(3.00\,\text{s}) = 1032\,\text{m}.$ The angle that the direction from the storm to the ballpark makes with the north direction is found from these distances using the law of cosines;

$$\theta = \arccos\left[\frac{(1523.92\,\text{m})^2 - (1032\,\text{m})^2 - (1120\,\text{m})^2}{-2(1032\,\text{m})\,(1120\,\text{m})}\right] = 90.07°,$$

so the storm can be considered to be due west of the park.

15.59: a) As time goes on, someone moving with the wave would need to move in such a way that the wave appears to have the same shape. If this motion can be described by $x = vt + c$, with c a constant (not the speed light),then $y(x,t) = f(c)$,
and the waveform is the same to such observer. b) See Problem 15.54. The derivation is completed by taking the second partials,

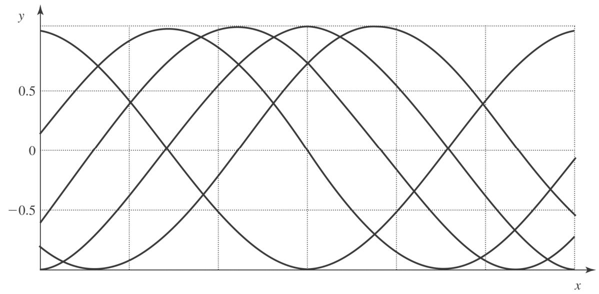
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{d^2 f}{du^2}, \qquad \frac{\partial^2 y}{\partial t^2} = \frac{d^2 f}{du^2},$$

so $y(x,t) = f(t - x/v)$ is a solution to the wave equation with wave speed v. c) This is of the form $y(x,t) = f(u)$, with $u = t - x/v$ and

$$f(u) = De^{-C^2(t-(B/c)x)^2},$$

and the result of part (b) may be used to determine the speed $v = C/B$ immediately.

15.60: a)



b) $\frac{\partial y}{\partial t} = \omega A \sin(kx - \omega t + \phi)$. c) No; $\phi = \pi/4$ or $\phi = 3\pi/4$ would both give $A/\sqrt{2}$. If the particle is known to be moving downward, the result of part b) shows that $\cos\phi < 0$, and so $\phi = 3\pi/4$. d) To identify ϕ uniquely, the quadrant in which ϕ is must be known. In physical terms, the signs of both the position and velocity, and the magnitude of either, are necessary to determine ϕ (within additive multiples of 2π).

15.61: a) $\sqrt{\mu F} = F\sqrt{\mu/F} = F/v = F\,k/\omega$ and substituting this into Eq. (15.33) gives the result.

b) Quadrupling the tension for F to $F' = 4F$ increases the speed $v = \sqrt{F/\mu}$ by a factor of 2, so the new frequency ω' and new wave number k' are related to ω and k by $(\omega'/k') = 2(\omega/k)$. For the average power to be the same, we must have $Fk\omega = F'k'\omega'$, so $k\omega = 4k'\omega'$ and $k'\omega' = k\omega/4$.

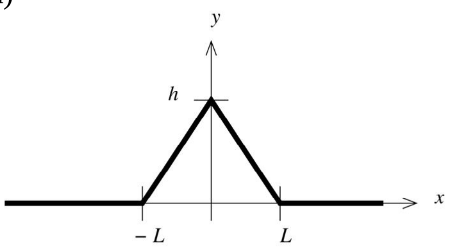
Multiplying the first and second equations together gives

$$\omega'^2 = \omega^2/2, \text{ so } \omega' = \omega/\sqrt{2}.$$

Thus, the frequency must decrease by a factor of $\sqrt{2}$. Dividing the second equation by the first equation gives

$$k'^2 = k^2/8, \text{ so } k' = k/\sqrt{8}.$$

15.62: (a)



(b) The wave moves in the $+x$ direction with speed $v,$ so in the expression for $y(x,0)$ replace x with $x-vt$:

$$y(x,t)=\begin{cases}0 & \text{for} \quad (x-vt)<-L\\ h(L+x-vt)/L & \text{for} -L<(x-vt)<0\\ h(L-x+vt)/L & \text{for} \quad 0<(x-vt)<L\\ 0 & \text{for} \quad (x-vt)>L\end{cases}$$

(c) From Eq. (15.21):

$$P(x,t)=-F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}=\begin{cases}-F(0)(0)=0 & \text{for}(x-vt)<-L\\ -F(h/L)(-hv/L)=Fv(h/L)^2 & \text{for} -L<(x-vt)<0\\ -F(-h/L)(hv/L)=Fv(h/L)^2 & \text{for} \quad 0<(x-vt)<L\\ -F(0)(0)=0 & \text{for} \quad (x-vt)>L\end{cases}$$

Thus the instantaneous power is zero except for $-L<(x-vt)<L,$ where it has the constant value $Fv(h/L)^2.$

15.63: a) $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$

$v = \sqrt{F/\mu}$ so $\sqrt{F} = v\sqrt{\mu}$

$\omega = 2\pi f = 2\pi(v/\lambda)$

Using these two expressions to replace $\sqrt{F}$ and ω gives

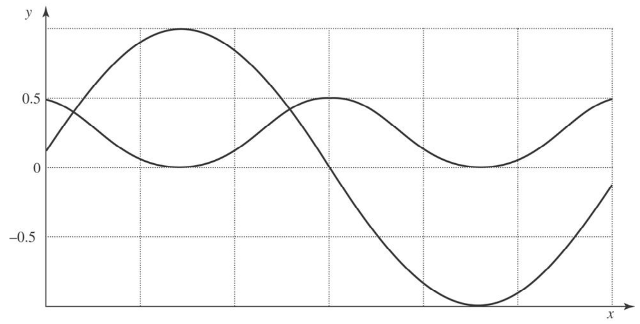
$P_{\text{av}} = 2\mu\pi^2 v^3 A^2/\lambda^2$; $\mu = (6.00\times10^{-3}\text{ kg})/(8.00\text{ m})$

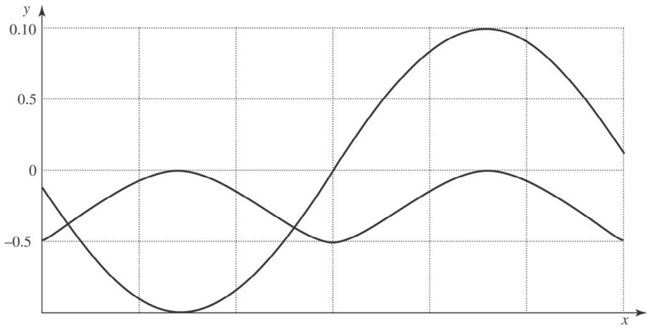
$$A = \left(\frac{2\lambda^2 P_{\text{av}}}{4\pi^2 v^3 \mu}\right)^{1/2} = 7.07\text{ cm}$$

b) $P_{\text{av}} \sim v^3$ so doubling v increases P_{av} by a factor of 8.

$P_{\text{av}} = 8(50.0\text{ W}) = 400.0\text{ W}$

15.64: a) , d)





b)The power is a maximum where the displacement is zero, and the power is a minimum of zero when the magnitude of the displacement is a maximum. c) The direction of the energy flow is always in the same direction. d) In this case, $\frac{\partial y}{\partial x} = -kA\sin(kx + \omega t)$, and so Eq. (15.22) becomes

$$P(x,t) = -Fk\omega A^2 \sin^2(kx + \omega t).$$

The power is now negative (energy flows in the $-x$-direction), but the qualitative relations of part (b) are unchanged.

15.65: $$v_1^2 = \frac{F_1}{\mu},\ v_2^2 = \frac{F_2}{\mu} = \frac{F_1 - YA\alpha\Delta T}{\mu}.$$

Solving for α,

$$\alpha = \frac{v_1^2 - v_2^2}{Y(A/\mu)\Delta T} = \frac{v_1^2 - v_2^2}{(Y/\rho)\Delta T}.$$

15.66: (a) The string vibrates through $1/2$ cycle in $4 \times \frac{1}{5000}\,\text{min}$, so

$$\frac{1}{2}T = \frac{4}{5000}\,\text{min} \rightarrow T = 1.6 \times 10^{-3}\,\text{min} = 9.6 \times 10^{-2}\,\text{s}$$

$$f = 1/T = 1/9.6 \times 10^{-2}\,\text{s} = 10.4\,\text{Hz}$$

$$\lambda = L = 50.0\,\text{cm} = 0.50\,\text{m}$$

(b) Second harmonic

(c) $v = f\lambda = (10.4\,\text{Hz})(0.50\,\text{m}) = 5.2\,\text{m/s}$

(d) (i)Maximum displacement, so $v = 0$ (ii) $v_y = \frac{\partial y}{\partial t} = \frac{\partial}{\partial t}(1.5\,\text{cm} \sin kx \sin \omega t)$

Speed $= |v_y| = \omega(1.5\,\text{cm}) \sin kx \sin \omega t$

at maximum speed, $\sin kx = \sin \omega t = 1$

$$|v_y| = \omega(1.5\,\text{cm}) = 2\pi f(1.5\,\text{cm}) = 2\pi(10.4\,\text{Hz})(1.5\,\text{cm})$$
$$= 98\,\text{cm/s} = 0.98\,\text{m/s}$$

(e) $v = \sqrt{F/\mu} \rightarrow \mu = F/v^2$

$$M = \mu L = \frac{F}{v^2}L = \frac{(1.00\,\text{N})(0.500\,\text{m})}{(5.2\,\text{m/s})^2} = 1.85 \times 10^{-2}\,\text{kg}$$
$$= 18.5\ \text{g}$$

15.67: There is a node at the post and there must be a node at the clothespin. There could be additional nodes in between. The distance between adjacent nodes is $\lambda/2$, so the distance between any two nodes is $n\,(\lambda/2)$ for $n = 1, 2, 3, \ldots$

$45.0\,\text{cm} = n(\lambda/2)$, $\lambda = v/f$, so

$f = n[v/(90.0\,\text{cm})] = (0.800\,\text{Hz})n, n = 1, 2, 3, \ldots$

15.68: (a) The displacement of the string at any point is $y(x,t) = (A_{\mathrm{SW}} \sin kx)\sin \omega t$. For the fundamental mode $\lambda = 2L$, so at the midpoint of the string $\sin kx = \sin(2\pi/\lambda)(L/2) = 1$, and $y = A_{\mathrm{SW}} \sin \omega t$. Taking derivatives gives $v_y = \frac{\partial y}{\partial t} = \omega A_{\mathrm{SW}} \cos \omega t$, with maximum value $v_{y\,\max} = \omega A_{\mathrm{SW}}$, and $a_y = \frac{\partial v_y}{\partial t} = -\omega^2 A_{\mathrm{SW}} \sin \omega t$, with maximum value $a_{y\,\max} = \omega^2 A_{\mathrm{SW}}$.

Dividing these gives

$\omega = a_{y\,\max} / v_{y\,\max} = (8.40\times 10^3 \text{ m/s}^2)/(3.80 \text{ m/s}) = 2.21\times 10^3 \text{ rad/s}$, and then

$A_{\mathrm{SW}} = v_{y\,\max}/\omega = (3.80\,\text{m/s})/(2.21\times 10^3 \text{ rad/s}) = 1.72\times 10^{-3}\text{ m}$.

(b) $v = \lambda f = (2L)(\omega/2\pi) = L\omega/\pi = (0.386\,\text{m})(2.21\times 10^3 \text{ rad/s})/\pi = 272\,\text{m/s}$.

15.69: a) To show this relationship is valid, take the second time derivative:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2}{\partial t^2}[(A_{\mathrm{SW}} \sin kx)\cos \omega t],$$

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\omega \frac{\partial}{\partial t}[(A_{\mathrm{SW}} \sin kx)\sin \omega t]$$

$$\frac{\partial^2 y(x,\mathrm{t})}{\partial \mathrm{t}^2} = -\omega^2[(A_{\mathrm{sw}}\sin kx)\cos \omega \mathrm{t}],$$

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 y(x,t), \text{Q.E.D.}$$

The displacement of the harmonic oscillator is periodic in both time and space.

b) Yes, the travelling wave is also a solution of the wave equation.

15.70: a) The wave moving to the left is inverted and reflected; the reflection means that the wave moving to the left is the same function of $-x$, and the inversion means that the function is $-f(-x)$. More rigorously, the wave moving to the left in Fig. (15.17) is obtained from the wave moving to the right by a rotation of $180°$, so both the coordinates (f and x) have their signs changed. b). The wave that is the sum is $f(x) - f(-x)$ (an inherently odd function), and for any f, $f(0) - f(-0) = 0$. c) The wave is reflected but not inverted (see the discussion in part (a) above), so the wave moving to the left in Fig. (15.18) is $+f(-x)$.

d) $$\frac{dy}{dx} = \frac{d}{dx}(f(x) + f(-x)) = \frac{df(x)}{dx} + \frac{df(-x)}{dx} = \frac{df(x)}{dx} + \frac{df(-x)}{d(-x)}\frac{d(-x)}{dx}$$

$$= \frac{df}{dx} - \left.\frac{df}{dx}\right|_{x=-x}.$$

At $x = 0$, the terms are the same and the derivatives is zero. (See Exercise 20-2 for a situation where the derivative of f is not finite, so the string is not always horizontal at the boundary.)

15.71: a)
$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ &= A[\cos(kx + \omega t) + \cos(kx - \omega t)] \\ &= A[\cos\omega t \cos kx - \sin\omega t \sin kx + \cos\omega t \cos kx + \sin\omega t \sin kx] \\ &= (2\mathrm{A}) \cos\omega t \cos kx. \end{aligned}$$

b) At $x = 0$, $y(0,t) = (2A)\cos\omega t$, and so $x = 0$ is an antinode. c) The maximum displacement is, front part (b), $A_{\mathrm{SW}} = 2A$, the maximum speed is $\omega A_{\mathrm{SW}} = 2\omega A$ and the magnitude of the maximum acceleration is $\omega^2 A_{\mathrm{sw}} = 2\omega^2 A.$

15.72: a) $\lambda = v/f = (192.0\ \mathrm{m/s})/(240.0\ \mathrm{Hz}) = 0.800\ \mathrm{m}$, and the wave amplitude is $A_{\mathrm{SW}} = 0.400\ \mathrm{cm}$. The amplitude of the motion at the given points is (i) $(0.400\ \mathrm{cm})\sin(\pi) = 0$ (a node), (ii) $(0.400\ \mathrm{cm})\sin(\pi/2) = 0.004\ \mathrm{cm}$ (an antinode) and (iii) $(0.400\ \mathrm{cm})\sin(\pi/4) = 0.283\ \mathrm{cm}$. b). The time is half of the period, or $1/(2f) = 2.08\times10^{-3}\ \mathrm{s}$. c) In each case, the maximum velocity is the amplitude multiplied by $\omega = 2\pi f$ and the maximum acceleration is the amplitude multiplied by $\omega^2 = 4\pi^2 f^2$, or (i) 0, 0; (ii) $6.03\ \mathrm{m/s}$, $9.10\times10^3\ \mathrm{m/s^2}$; (iii) $4.27\ \mathrm{m/s}$, $6.43\times10^3\ \mathrm{m/s^2}$.

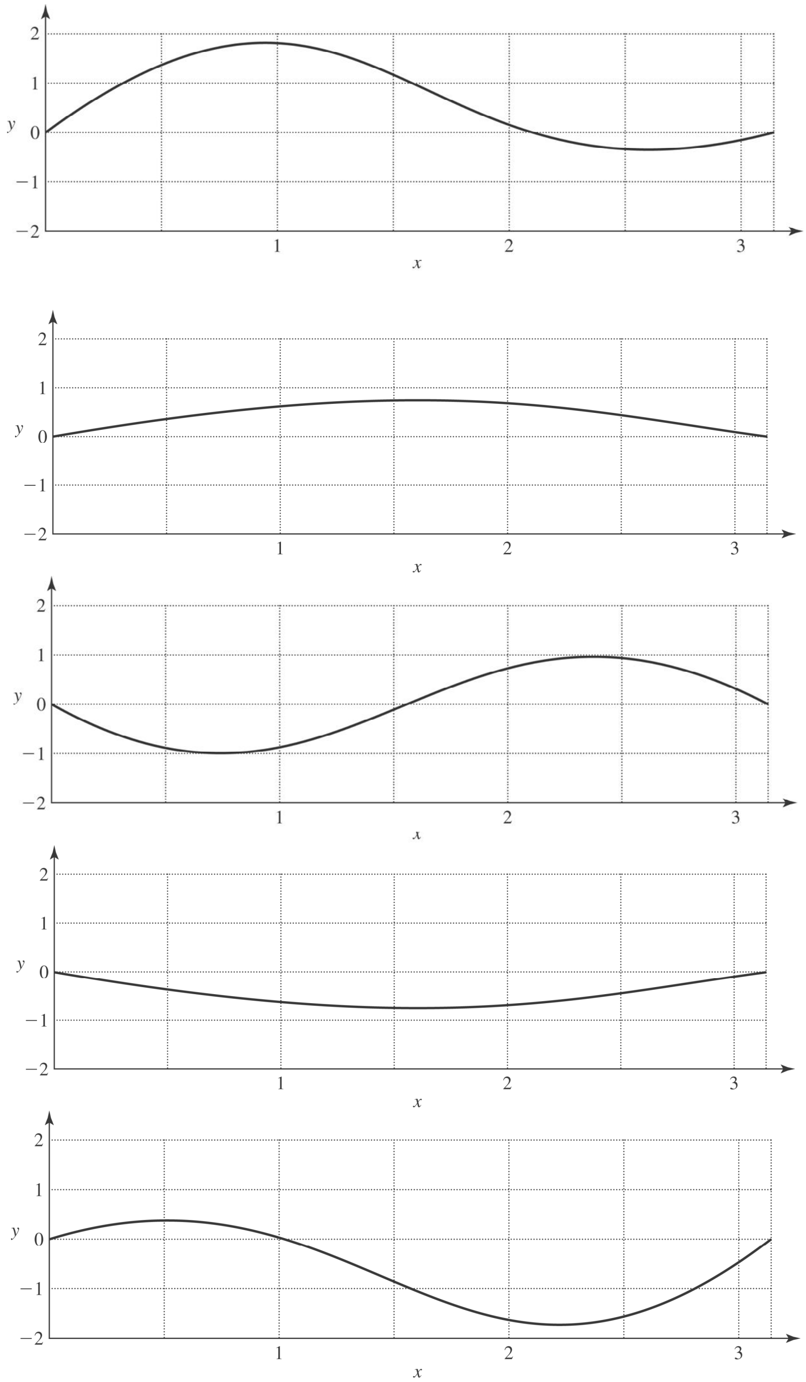
15.73: The plank is oscillating in its fundamental mode, so $\lambda = 2L = 10.0\,\text{m}$, with a frequency of $2.00\,\text{Hz}$. a) $v = f\lambda = 20.0\ \text{m/s}$. b) The plank would be its first overtone, with twice the frequency, or $4\ \text{jumps/s}$.

15.74: (a) The breaking stress is $\frac{F}{\pi r^2} = 7.0\times10^8\ \text{N/m}^2$, and the maximum tension is $F = 900\,\text{N}$, so solving for r gives the minimum radius $r = \sqrt{\frac{900\ \text{N}}{\pi(7.0\times10^8\ \text{N/m}^2)}} = 6.4\times10^{-4}\ \text{m}$. The mass and density are fixed, $\rho = \frac{M}{\pi r^2 L}$, so the minimum radius gives the maximum length $L = \frac{M}{\pi r^2 \rho} = \frac{4.0\times10^{-3}\ \text{kg}}{\pi(6.4\times10^{-4}\ \text{m})^2(7800\ \text{kg/m}^3)} = 0.40\,\text{m}$.

(b) The fundamental frequency is $f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{2L}\sqrt{\frac{F}{M/L}} = \frac{1}{2}\sqrt{\frac{F}{ML}}$. Assuming the maximum length of the string is free to vibrate, the highest fundamental frequency occurs when $F = 900\ \text{N}$, $f_1 = \frac{1}{2}\sqrt{\frac{900\ \text{N}}{(4.0\times10^{-3}\ \text{kg})(0.40\ \text{m})}} = 376\,\text{Hz}$.

15.75: a) The fundamental has nodes only at the ends, $x = 0$ and $x = L.$ b) For the second harmonic, the wavelength is the length of the string, and the nodes are at $x = 0, x = L/2$ and $x = L.$

b)



d) No; no part of the string except for $x = L/2,$ oscillates with a single frequency.

15.76:

a) The new tension F' in the wire is

$$F' = F - B = w - \frac{(1/3w)\rho_{\text{water}}}{\rho_{A1}} = w\left(1 - \frac{1}{3}\frac{\rho_{\text{water}}}{\rho_{A1}}\right)$$

$$= w\left(1 - \frac{(1.00\times 10^3\ \text{kg/m}^3)}{3(2.7\times 10^3\ \text{kg/m}^3)}\right) = (0.8765)w = (0.87645)F.$$

The frequency will be proportional to the square root of the tension, and so $f' = (200\,\text{Hz})\sqrt{0.8765} = 187\,\text{Hz}.$

b) The water does not offer much resistance to the transverse waves in the wire, and hencethe node will be located a the point where the wire attaches to the sculpture and not at the surface of the water.

15.77: a) Solving Eq. (15.35) for the tension *F*,

$$F = 4L^2 f_1^2 \mu = 4mLf_1^2 = 4(14.4\times 10^{-3}\ \text{kg})(0.600\,\text{m})(65.4\,\text{Hz})^2 = 148\,\text{N}.$$

b) The tension must increase by a factor of $(\frac{73.4}{65.4})^2$, and the percent increase is $(73.4/65.4)^2 - 1 = 26.0\%.$

15.78: a) Consider the derivation of the speed of a longitudinal wave in Section 16.2. Instead of the bulk modulus *B*, the quantity of interest is the change in force per fractional length change. The force constant k' is the change in force force per length change, so the force change per fractional length change is $k'L,$ the applied force at one end is $F = (k'L)(v_y/v)$ and the longitudinal impulse when this force is applied for a time *t* is $k'Ltv_y/v.$ The change in longitudinal momentum is $((vt)m/L)v_y$ and equating the expressions, canceling a factor of *t* and solving for v gives $v^2 = L^2 k'/m.$

An equivalent method is to use the result of Problem 11.82(a), which relates the force constant k' and the "Young's modulus" of the Slinky^{TM}, $k' = YA/L$, or $Y = k'L/A$. The mass density is $\rho = m/(AL)$, and Eq. (16.8) gives the result immediately.

b) $(2.00\,\text{m})\sqrt{(1.50\ \text{N/m})/(0.250\,\text{kg})} = 4.90\ \text{m/s}.$

15.79: a) $u_{\mathrm{k}} = \frac{\Delta K}{\Delta x} = \frac{(1/2)\Delta m v_y^2}{\Delta m/\mu} = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2.$

b) $\frac{\partial y}{\partial t} = \omega A \sin(kx - \omega t)$ and so

$$u_k = \frac{1}{2}\mu\omega^2 A^2 \sin^2(kx - \omega t).$$

c) The piece has width Δx and height $\Delta x \frac{\partial y}{\partial x}$, and so the length of the piece is

$$\left((\Delta x)^2 + \left(\Delta x \frac{\partial y}{\partial x}\right)^2\right)^{1/2} = \Delta x\left(1 + \left(\frac{\partial y}{\partial x}\right)^2\right)^{1/2}$$

$$\approx \Delta x\left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right],$$

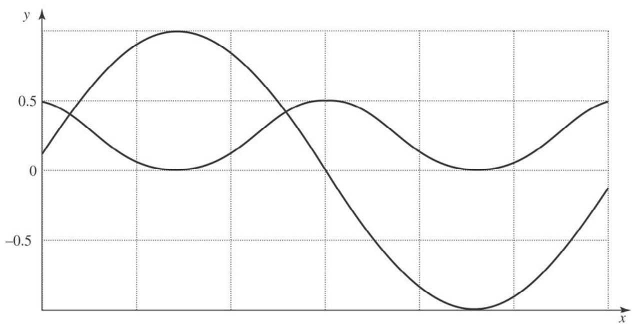
where the relation given in the hint has been used.

d) $$u_{\mathrm{p}} = F\frac{\Delta x\left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right] - \Delta x}{\Delta x} = \frac{1}{2}F\left(\frac{\partial y}{\partial x}\right)^2.$$

e) $\frac{\partial y}{\partial x} = -kA\sin(kx - \omega t)$, and so

$$u_{\mathrm{p}} = \frac{1}{2}Fk^2 A^2 \sin^2(kx - \omega t)$$

and f) comparison with the result of part (c)with $k^2 = \omega^2/v^2 = \omega^2\,\mu/F$, shows that for a sinusoidal wave $u_{\mathrm{k}} = uv_{\mathrm{p}}$. g) In this graph, u_{k} and u_{p} coincide, as shown in part (f).



15.80: a) The tension is the difference between the diver's weight and the buoyant force,

$$F = (m - \rho_{\text{water}}V)g = (120\,\text{kg} - (1000\ \text{kg/m}^3)(0.0800\,\text{m}^3)(9.80\ \text{m/s}^2)) = 392\ \text{N}.$$

b) The increase in tension will be the weight of the cable between the diver and the point at *x*, minus the buoyant force. This increase in tension is then

$$\begin{aligned}(\mu x - \rho(Ax))g &= (1.10\ \text{kg/m} - (1000\ \text{kg/m}^3)\pi(1.00\times10^{-2}\ \text{m})^2)(9.80\ \text{m/s}^2)x \\ &= (7.70\ \text{N/m})x\end{aligned}$$

The tension as a function of *x* is then $F(x) = (392\,\text{N}) + (7.70\ \text{N/m})x.$ c) Denote the tension as $F(x) = F_0 + ax,$ where $F_0 = 392\,\text{N}$ and $a = 7.70\ \text{N/m}.$ Then, the speed of transverse waves as a function of *x* is $v = \frac{dx}{dt} = \sqrt{(F_0 + ax)/\mu}$ and the time *t* needed for a wave to reach the surface is found from

$$t = \int dt = \int \frac{dx}{dx/dt} = \int \frac{\sqrt{\mu}}{\sqrt{F_0 + ax}}dx.$$

Let the length of the cable be *L*, so

$$\begin{aligned}t &= \sqrt{\mu}\int_0^L \frac{dx}{\sqrt{F_0 + ax}} = \sqrt{\mu}\frac{2}{a}\sqrt{F_0 + ax}\Big|_0^L \\ &= \frac{2\sqrt{\mu}}{a}\left(\sqrt{F_0 + aL} - \sqrt{F_0}\right)\end{aligned}$$

$$= \frac{2\sqrt{1.10\ \text{kg/m}}}{7.70\ \text{N/m}}(\sqrt{392\,\text{N} + (7.70\ \text{N/m})(100\,\text{m})} - \sqrt{392\,\text{N}}) = 3.98\,\text{s}.$$

15.81: The tension in the rope will vary with radius r. The tension at a distance r from the center must supply the force to keep the mass of the rope that is further out than r accelerating inward. The mass of this piece in $m\frac{L-r}{L}$, and its center of mass moves in a circle of radius $\frac{L+r}{2}$, and so

$$T(r) = \left[m\frac{L-r}{L}\right]\omega^2\left[\frac{L+r}{L}\right] = \frac{m\omega^2}{2L}(L^2 - r^2).$$

An equivalent method is to consider the net force on a piece of the rope with length dr and mass $dm = dr\ m/L$. The tension must vary in such a way that $T(r) - T(r + dr) = -\omega^2 r\, dm$, or $\frac{dT}{dr} = -(m\omega^2/L) r\, dr$. This is integrated to obtained $T(r) = -(m\omega^2/2L)r^2 + C$, where C is a constant of integration. The tension must vanish at $r = L$, from which $C = (m\omega^2 L/2)$ and the previous result is obtained.

The speed of propagation as a function of distance is

$$v(r) = \frac{dr}{dt} = \sqrt{\frac{T(r)}{\mu}} = \sqrt{\frac{TL}{m}} = \frac{\omega}{\sqrt{2}}\sqrt{L^2 - r^2},$$

where $\frac{dr}{dt} > 0$ has been chosen for a wave traveling from the center to the edge. Separating variables and integrating, the time t is

$$t = \int dt = \frac{\sqrt{2}}{\omega}\int_0^L \frac{dr}{\sqrt{L^2 - r^2}}.$$

The integral may be found in a table, or in Appendix B. The integral is done explicitly by letting $r = L\sin\theta, dr = L\cos\theta\, d\theta, \sqrt{L^2 - r^2} = L\cos\theta,$ so that

$$\int \frac{dr}{\sqrt{L^2 - r^2}} = \theta = \arcsin\frac{r}{L}, \text{ and}$$

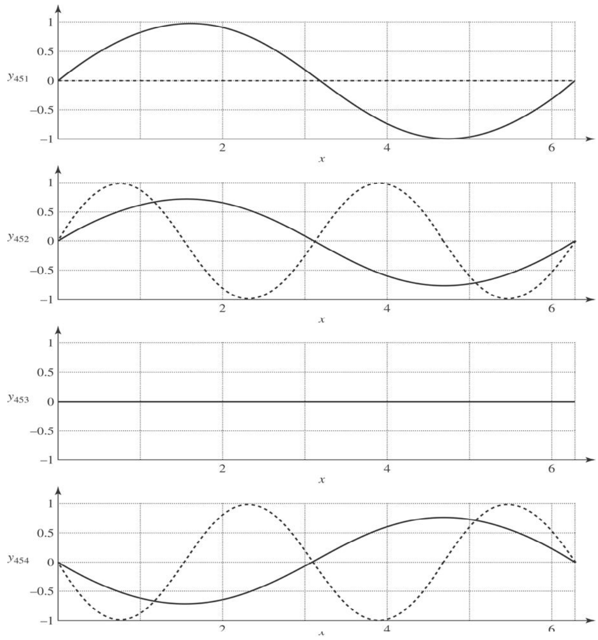
$$t = \frac{\sqrt{2}}{\omega}\arcsin(1) = \frac{\pi}{\omega\sqrt{2}}.$$

15.82: a) $\frac{\partial y}{\partial x} = kA_{SW}\cos kx \sin\omega t, \frac{\partial y}{\partial t} = -\omega A_{SW}\,\omega \sin kx \cos\omega t,$ and so the instantaneous power is

$$P = FA^2{}_{sw}\,\omega k(\sin\ kx \cos kx)(\sin\ \omega t \cos\omega t)$$

$$= \frac{1}{4}FA^2{}_{sw}\,\omega k \sin(2kx)\sin(2\omega t).$$

b) The average value of P is proportional to the average value of $\sin(2\omega t)$, and the average of the sine function is zero; $P_{av} = 0.$ c) The waveform is the solid line, and the power is the dashed line. At time $t = \pi/2\omega$, $y = 0$ and $P = 0$ and the graphs coincide. d) When the standing wave is at its maximum displacement at all points, all of the energy is potential, and is concentrated at the places where the slope is steepest (the nodes). When the standing wave has zero displacement, all of the energy is kinetic, concentrated where the particles are moving the fastest (the antinodes). Thus, the energy must be transferred from the nodes to the antinodes, and back again, twice in each cycle. Note that $|P|$ is greatest midway between adjacent nodes and antinodes, and that P vanishes at the nodes and antinodes.



15.83: a) For a string, $f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$ and in this case, $n = 1$. Rearranging this and solving for F gives $F = \mu 4L^2 f^2$. Note that $\mu = \pi r^2 \rho$,
so $\mu = \pi(.203\times10^{-3}\ \text{m})^2(7800\ \text{kg/m}^3) = 1.01\times10^{-3}\ \text{kg/m}$. Substituting values,
$F = (1.01\times10^{-3}\ \text{kg/m})4(.635\ \text{m})^2(247.0\ \text{Hz})^2 = 99.4\ \text{N}$.

b) To find the fractional change in the frequency we must take the ration of Δf to f:

$$f = \frac{1}{2L}\sqrt{\frac{F}{\mu}},$$

$$\Delta(f) = \Delta\left(\frac{1}{2L}\sqrt{\frac{F}{\mu}}\right) = \Delta\left(\frac{1}{2L\sqrt{\mu}}F^{\frac{1}{2}}\right),$$

$$\Delta f = \frac{1}{2L\sqrt{\mu}}\Delta\left(F^{\frac{1}{2}}\right)$$

$$\Delta f = \frac{1}{2L\sqrt{\mu}}\frac{1}{2}\frac{\Delta F}{\sqrt{F}}.$$

Now divide both sides by the original equation for f and cancel terms:

$$\frac{\Delta f}{f} = \frac{\frac{1}{2L\sqrt{\mu}}\frac{1}{2}\frac{\Delta F}{\sqrt{F}}}{\frac{1}{2L}\sqrt{\frac{F}{\mu}}}$$

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta F}{F}$$

c) From Section 17.4, $\Delta F = -Y\alpha A\Delta T$, so
$\Delta F = -\left(2.00\times10^{11}\ \text{Pa}\right)\left(1.20\times10^{-5}/\text{C}°\right)\times(\pi(.203\times10^{-3}\text{m})^2)(11°\text{C}) = 3.4\ \text{N}$. Then, $\Delta F/F =$ -0.034, $\Delta f/f = -0.017$, and finally, $\Delta f = -4.2\ \text{Hz}$, or the pitch falls. This also explains the constant the constant tuning in the string sections of symphonic orchestras.

16.1: a) $\lambda = v/f = (344 \text{ m/s})/(100 \text{ Hz}) = 0.344 m$. b) if $p \to 1000 p_0$, then $A \to 1000 A_0$ Therefore, the amplitude is 1.2×10^{-5} m. c) Since $p_{\max} = BkA$, increasing $p_{\max}$ while keeping A constant requires decreasing k, and increasing π, by the same factor. Therefore the new wavelength is $(0.688 \text{ m})(20) = 6.9 \text{ m}$, $f_{\text{new}} = \frac{344 \text{ m/s}}{6.9 \text{ m}} = 50 \text{ Hz}$.

16.2: $A = \frac{p_{\max} v}{2\pi Bf} = \frac{(3.0\times10^{-2} \text{ Pa})(1480 \text{ m/s})}{2\pi(2.2\times10^{9} \text{ Pa})(1000 \text{ Hz})}$, or $A = 3.21 \times 10^{-12}$ m. The much higher bulk modulus increases both the needed pressure amplitude and the speed, but the speed is proportional to the square root of the bulk modulus. The overall effect is that for such a large bulk modulus, large pressure amplitudes are needed to produce a given displacement.

16.3: From Eq. (16.5), $p_{\max} = BkA = 2\pi BA/\lambda = 2\pi BA f/v$.

a) $2\pi(1.42\times10^{5} \text{ Pa})(2.00\times10^{-5} \text{ m})(150 \text{ Hz})/(344 \text{ m/s}) = 7.78 \text{ Pa}$.

b) $10 \times 7.78 \text{ Pa} = 77.8 \text{ Pa}$. c) $100 \times 7.78 \text{ Pa} = 778 \text{ Pa}$.

The amplitude at 1500 Hz exceeds the pain threshold, and at 15,000 Hz the sound would be unbearable.

16.4: The values from Example 16.8 are $B = 3.16 \times 10^4$ Pa, $f = 1000$ Hz, $A = 1.2 \times 10^{-8}$ m. Using Example 16.5, $v = 344 \text{ m/s}\sqrt{\frac{216\text{K}}{293\text{K}}} = 295 \text{ m/s}$, so the pressure amplitude of this wave is $p_{\max} = BkA = B\frac{2\pi f}{v}A = (3.16\times10^4 \text{ Pa})$.

$\frac{2\pi(1000 \text{ Hz})}{295 \text{ m/s}}(1.2\times10^{-8} \text{ m}) = 8.1\times10^{-3}$ Pa. This is $(8.1\times10^{-3} \text{ Pa})/(3.0\times10^{-2} \text{ Pa}) = 0.27$ times smaller than the pressure amplitude at sea level (Example 16-1), so pressure amplitude decreases with altitude for constant frequency and displacement amplitude.

16.5: a) Using Equation (16.7), $B = v^2\rho = (\lambda f)^2$, so $B = [(8 \text{ m})(400/\text{s})]^2 \times (1300 \text{ kg/m}^3) = 1.33 \times 10^{10}$ Pa.

b) Using Equation (16.8), $Y = v^2\rho = (L/t)^2\rho = [(1.5 \text{ m})/(3.9\times10^{-4} \text{ s}]^2 \times (6400 \text{ kg/m}^3) = 9.47\times10^{10}$ Pa.

16.6: a) The time for the wave to travel to Caracas was $9\,\text{min}\ 39\ \text{s} = 579\ \text{s}$ and the speed was $1.085 \times 10^4\ \text{m/s}$ (keeping an extra figure). Similarly, the time for the wave to travel to Kevo was 680 s for a speed of $1.278 \times 10^4\ \text{m/s}$, and the time to travel to Vienna was 767 s for a speed of $1.258 \times 10^4\ \text{m/s}$. The average speed for these three measurements is $1.21 \times 10^4\ \text{m/s}$. Due to variations in density, or reflections (a subject addressed in later chapters), not all waves travel in straight lines with constant speeds.
b) From Eq. (16.7), $B = v^2 \rho$, and using the given value of $\rho = 3.3 \times 10^3\ \text{kg/m}^3$ and the speeds found in part (a), the values for the bulk modulus are, respectively, $3.9 \times 10^{11}\ \text{Pa}$, $5.4 \times 10^{11}\ \text{Pa}$ and $5.2 \times 10^{11}\ \text{Pa}$. These are larger, by a factor of 2 or 3, than the largest values in Table (11-1).

16.7: Use $v_{\text{water}} = 1482\,\text{m/s}$ at $20°\text{C}$, as given in Table (16.1) The sound wave travels in water for the same time as the wave travels a distance $22.0\ \text{m} - 1.20\ \text{m} = 20.8\,\text{m}$ in air, and so the depth of the diver is

$$(20.8\,\text{m})\frac{v_{\text{water}}}{v_{\text{air}}} = (20.8\,\text{m})\frac{1482\,\text{m/s}}{344\,\text{m/s}} = 89.6\,\text{m}.$$

This is the depth of the diver; the distance from the horn is $90.8\,\text{m}$.

16.8: a), b), c) Using Eq. (16.10),

$$v_{\text{H2}} = \sqrt{\frac{(1.41)(8.3145\,\text{J/mol}\cdot\text{K})(300.15\,\text{K})}{(2.02 \times 10^{-3}\ \text{kg/mol})}} = 1.32 \times 10^3\ \text{m/s}$$
$$v_{\text{He}} = \sqrt{\frac{(1.67)(8.3145\,\text{J/mol}\cdot\text{K})(300.15\,\text{K})}{(4.00 \times 10^{-3}\ \text{kg/mol})}} = 1.02 \times 10^3\ \text{m/s}$$
$$v_{\text{Ar}} = \sqrt{\frac{(1.67)(8.3145\,\text{J/mol}\cdot\text{K})(300.15\,\text{K})}{(39.9 \times 10^{-3}\ \text{kg/mol})}} = 323\,\text{m/s}.$$

d) Repeating the calculation of Example 16.5 at $T = 300.15\,\text{K}$ gives $v_{\text{air}} = 348\ \text{m/s}$, and so $v_{\text{H}_2} = 3.80\,v_{\text{air}}$, $v_{\text{He}} = 2.94\,v_{\text{air}}$ and $v_{\text{Ar}} = 0.928\,v_{\text{air}}$.

16.9: Solving Eq. (16.10) for the temperature,

$$T = \frac{Mv^2}{\gamma R} = \frac{(28.8\times10^{-3}\ \text{kg/mol})\left(\left(\dfrac{850\ \text{km/h}}{0.85}\right)\left(\dfrac{1\ \text{m/s}}{3.6\ \text{km/hr}}\right)\right)^2}{(1.40)(8.3145\,\text{J/mol}\cdot\text{K})} = 191\ \text{K},$$

or $-82°\text{C}$. b) See the results of Problem 18.88, the variation of atmospheric pressure with altitude, assuming a non-constant temperature. If we know the altitude we can use the result of Problem 18.88, $p = p_0\left(1-\dfrac{\alpha y}{T_0}\right)^{\left(\frac{Mg}{R\alpha}\right)}$. Since $T = T_o - \alpha y$,

for $T = 191\text{K}$, $\alpha = .6\times10^{-2}\ °\text{C/m}$, and $T_0 = 273\text{K}$, $y = 13{,}667\ \text{m}$ (44,840 ft.). Although a very high altitude for commercial aircraft, some military aircraft fly this high. This result assumes a uniform decrease in temperature that is solely due to the increasing altitude. Then, if we use this altitude, the pressure can be found:

$$p = \text{p}_\text{o}\left(1-\frac{(.6\times10^{-2}\ °\text{C/m})\ (13{,}667\text{m})}{273\text{K}}\right)^{\left(\frac{(28.8\times10^{-3}\ \text{kg/mol})(9.8\ \text{m/s}^2)}{(8.315\ \text{J/mol}\cdot\text{K})(.6\times10^{-2}°\text{C/m})}\right)},$$

and $p = \text{p}_\text{o}(.70)^{5.66} = .13\text{p}_\text{o}$, or about .13 atm. Using an altitude of 13,667 m in the equation derived in Example 18.4 gives $p = .18\text{p}_\text{o}$, which overestimates the pressure due to the assumption of an isothermal atmosphere.

16.10: As in Example 16-5, with $T = 21°\text{C} = 294.15\text{K}$,

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.04)(8.3145\ \text{J/mol}\cdot\text{K})(294.15\ \text{K})}{28.8\times10^{-3}\ \text{kg/mol}}} = 344.80\ \text{m/s}.$$

The same calculation with $T = 283.15\text{K}$ gives $344.22\ \text{m/s}$, so the increase is $0.58\ \text{m/s}$.

16.11: Table 16.1 suggests that the speed of longitudinal waves in brass is much higher than in air, and so the sound that travels through the metal arrives first. The time difference is

$$\Delta t = \frac{L}{v_\text{air}} - \frac{L}{v_\text{Brass}} = \frac{80.0\ \text{m}}{344\ \text{m/s}} - \frac{80.0\ \text{m}}{\sqrt{(0.90\times10^{11}\ \text{Pa})/(8600\ \text{kg/m}^3)}} = 0.208\,\text{s}.$$

16.12: $\sqrt{\dfrac{(1.40)(8.3145\ \text{J/mol}\cdot\text{K})(300.15\ \text{K})}{(28.8\times10^{-3}\ \text{kg/mol})}} - \sqrt{\dfrac{(1.40)(8.3145\ \text{J/mol}\cdot\text{K})(260.15\ \text{K})}{(28.8\times10^{-3}\ \text{kg/mol})}}$

$= 24\ \text{m/s}.$

(The result is known to only two figures, being the difference of quantities known to three figures.)

16.13: The mass per unit length μ is related to the density (assumed uniform) and the cross-section area A by $\mu = A\rho,$ so combining Eq. (15.13) and Eq. (16.8) with the given relations between the speeds,

$$\frac{Y}{\rho} = 900\frac{F}{A\rho}\ \text{ so } F/A = \frac{Y}{900}\cdot$$

16.14: a) $\lambda = \dfrac{v}{f} = \dfrac{\sqrt{Y/\rho}}{f} = \dfrac{\sqrt{(11.0\times10^{10}\ \text{Pa})/(8.9\times10^{3}\ \text{kg/m}^3)}}{220\ \text{Hz}} = 16.0\ \text{m}.$

b) Solving for the amplitude A (as opposed to the area $a = \pi r^2$) in terms of the average power $P_{\text{av}} = Ia,$

$$A = \sqrt{\frac{(2P_{\text{av}}/a)}{\sqrt{\rho Y\omega^2}}}$$

$$= \sqrt{\frac{2(6.50\times10^{-6})\ \text{W})/(\pi(0.800\times10^{-2}\ \text{m})^2)}{\sqrt{(8.9\times10^{3}\ \text{kg/m}^3)(11.0\times10^{10}\ \text{Pa})}\,(2\pi(220\ \text{Hz}))^2}} = 3.29\times10^{-8}\ \text{m}.$$

c) $\omega\text{A} = 2\pi\ f\ A = 2\pi(220\ \text{Hz})(3.289\times10^{-8}\ \text{m}) = 4.55\times10^{-5}\ \text{m/s}.$

16.15: a) See Exercise 16.14. The amplitude is

$$A = \sqrt{\frac{2I}{\sqrt{\rho B \omega^2}}}$$

$$= \sqrt{\frac{2(3.00\times10^{-6}\ \mathrm{W/m^2})}{\sqrt{(1000\ \mathrm{kg/m^3})(2.18\times10^{9}\ \mathrm{Pa})}(2\pi(3400\ \mathrm{Hz}))^2}} = 9.44\times10^{-11}\ \mathrm{m}.$$

The wavelength is

$$\lambda = \frac{v}{f} = \frac{\sqrt{B/\rho}}{f} = \frac{\sqrt{(2.18\times10^{9}\ \mathrm{Pa})/(1000\ \mathrm{kg/m^3})}}{3400\ \mathrm{Hz}} = 0.434\ \mathrm{m}.$$

b) Repeating the above with $B = \gamma p = 1.40\times10^{5}$ Pa and the density of air gives $A = 5.66\times10^{-9}$ m and $\lambda = 0.100$ m. c) The amplitude is larger in air, by a factor of about 60. For a given frequency, the much less dense air molecules must have a larger amplitude to transfer the same amount of energy.

16.16: From Eq. (16.13), $I = vp_{\mathrm{max}}^2/2B$, and from Eq. (19.21), $v^2 = B/\rho$. Using Eq. (16.7) to eliminate $v, I = \left(\sqrt{B/\rho}\right)p_{\mathrm{max}}^2/2B = p_{\mathrm{max}}^2/2\sqrt{\rho B}$. Using Eq. (16.7) to eliminate B, $I = vp_{\mathrm{max}}^2/2(v^2\rho) = p_{\mathrm{max}}^2/2\rho v$.

16.17: a) $p_{\mathrm{max}} = BkA = \frac{2\pi BfA}{v} = \frac{2\pi(1.42\times10^{5}\ \mathrm{Pa})(150\ \mathrm{Hz})(5.00\times10^{-6}\ \mathrm{m})}{(344\ \mathrm{m/s})} = 1.95\ \mathrm{Pa}.$

b) From Eq. (16.14),

$I = p_{\mathrm{max}}^2/2\rho v = (1.95\ \mathrm{Pa})^2/(2\times(1.2\ \mathrm{kg/m^3})(344\ \mathrm{m/s})) = 4.58\times10^{-3}\ \mathrm{W/m^2}.$

c) $10\times\log\left(\frac{4.58\times10^{-3}}{10^{-12}}\right) = 96.6\ \mathrm{dB}.$

16.18: (a) The sound level is
$\beta = (10\,\text{dB})\log\frac{I}{I_0}$, so $\beta = (10\,\text{dB})\log\frac{0.500\,\mu\text{W/m}^2}{10^{-12}\,\text{W/m}^2}$, or $\beta = 57\,\text{dB}$.

b) First find *v*, the speed of sound at $20.0\,°\text{C}$, from Table 16.1, $v = 344\ \text{m/s}$. The density of air at that temperature is $1.20\ \text{kg/m}^3$. Using Equation (16.14),
$I = \dfrac{p_{\max}^2}{2\rho v} = \dfrac{(0.150\ \text{N/m}^2)^2}{2(1.20\ \text{kg/m}^3)(344\ \text{m/s})}$, or $I = 2.73\times10^{-5}\ \text{W/m}^2$. Using this in Equation (16.15), $\beta = (10\,\text{dB})\log\dfrac{2.73\times10^{-5}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}$, or $\beta = 74.4\,\text{dB}$.

16.19: a) As in Example 16.6, $I = \frac{(6.0\times10^{-5}\ \text{Pa})^2}{2(1.20\ \text{kg/m}^3)(344\ \text{m/s})} = 4.4\times10^{-12}\ \text{W/m}^2$. $\beta = 6.40\,\text{dB}$.

16.20: a) $10\times\log\left(\frac{4I}{I}\right) = 6.0\,\text{dB}$. b) The number must be multiplied by four, for an increase of 12 kids.

16.21: Mom is five times further away than Dad, and so the intensity she hears is $\frac{1}{25} = 5^{-2}$ of the intensity that he hears, and the difference in sound intensity levels is $10\times\log(25) = 14\,\text{dB}$.

16.22:

$$\Delta(\text{Sound level}) = 75\,\text{dB} - 90\,\text{dB} = -25\,\text{dB}$$

$$\Delta(\text{Sound level}) = 10\log\frac{I_f}{I_0} - 10\log\frac{I_i}{I_0} = 10\log\frac{I_f}{I_i}$$

Therefore

$$-25\,\text{dB} = 10\log\frac{I_f}{I_i}$$

$$\frac{I_f}{I_i} = 10^{-2.5} = 3.2\times10^{-3}$$

16.23: $\beta = (10\ \text{dB})\log\frac{I}{I_0}$, or $13\,\text{dB} = (10\ \text{dB})\log\frac{I}{I_0}$. Thus, $I/I_0 = 20.0$, or the intensity has increased a factor of 20.0.

16.24: Open Pipe:

$$\lambda_1 = 2L = \frac{v}{f_1} = \frac{v}{594\,\text{Hz}}$$

Closed at one end:

$$\lambda_1 = 4L = \frac{v}{f}$$

Taking ratios:

$$\frac{2L}{4L} = \frac{v/594\,\text{Hz}}{v/f}$$

$$f = \frac{594\,\text{Hz}}{2} = 297\,\text{Hz}$$

16.25: a) Refer to Fig. (16.18). i) The fundamental has a displacement node at $\frac{L}{2} = 0.600\,\text{m}$, the first overtone mode has displacement nodes at $\frac{L}{4} = 0.300\,\text{m}$ and $\frac{3L}{4} = 0.900\,\text{m}$ and the second overtone mode has displacement nodes at $\frac{L}{6} = 0.200\,\text{m}, \frac{L}{2} = 0.600\,\text{m}$ and $\frac{5L}{6} = 1.000\,\text{m}$. ii) Fundamental: $0, L = 1.200\,\text{m}$. First: $0, \frac{L}{2} = 0.600\,\text{m}, L = 1.200\,\text{m}$. Second: $0, \frac{L}{3} = 0.400\,\text{m}, \frac{2L}{3} = 0.800\,\text{m}, L = 1.200\,\text{m}$.

b) Refer to Fig. (16.19); distances are measured from the right end of the pipe in the figure. Pressure nodes at: Fundamental: $L = 1.200\,\text{m}$. First overtone: $L/3 = 0.400\,\text{m}, L = 1.200\,\text{m}$. Second overtone: $L/5 = 0.240\,\text{m}$, $3L/5 = 0.720\,\text{m}, L = 1.200\,\text{m}$. Displacement nodes at Fundamental: 0. First overtone: $0, 2L/3 = 0.800\,\text{m}$. Second overtone: $0,\ 2L/5 = 0.480\,\text{m},\ 4L/5 = 0.960\,\text{m}$

16.26: a) $f_1 = \frac{v}{2L} = \frac{(344\,m/s)}{2(0.450\,\text{m})} = 382\,\text{Hz},\quad 2f_1 = 764\,\text{Hz},\ f_3 = 3f_1 = 1147\,\text{Hz},$ $f_4 = 4f_1 = 1529\,\text{Hz}.$

b) $f_1 = \frac{v}{4L} = 191\,\text{Hz},\ f_3 = 3f_1 = 573\,\text{Hz},\ f_5 = 5f_1 = 956\,\text{Hz},\ f_7 = 7f_1 = 1338\,\text{Hz}.$ Note that the symbol "f_1" denotes different frequencies in the two parts. The frequencies are not always exact multiples of the fundamental, due to rounding.

c) Open: $\frac{20{,}000}{f_1} = 52.3,$ so the 52nd harmonic is heard. Stopped; $\frac{20{,}000}{f_1} = 104.7,$ so 103 rd highest harmonic heard.

16.27: $f_1 = \frac{(344\,\text{m/s})}{4(0.17\,\text{m})} = 506\,\text{Hz},\ f_2 = 3f_1 = 1517\,\text{Hz},\ \ f_3 = 5f_1 = 2529\,\text{Hz}.$

16.28: a) The fundamental frequency is proportional to the square root of the ratio $\frac{\gamma}{M}$ (see Eq. (16.10)), so

$$f_{\text{He}} = f_{\text{air}} = \sqrt{\frac{\gamma_{\text{He}}}{\gamma_{\text{air}}} \cdot \frac{M_{\text{air}}}{M_{\text{He}}}} = (262\,\text{Hz})\sqrt{\frac{(5/3)}{(7/5)} \cdot \frac{28.8}{4.00}} = 767\,\text{Hz},$$

b) No; for a fixed wavelength , the frequency is proportional to the speed of sound in the gas.

16.29: a) For a stopped pipe, the wavelength of the fundamental standing wave is $4L = 0.56\,\text{m}$, and so the frequency is $f_1 = (344\,\text{m/s})/(0.56\,\text{m}) = 0.614\,\text{kHz}$. b) The length of the column is half of the original length, and so the frequency of the fundamental mode is twice the result of part (a), or 1.23 kHz.

16.30: For a string fixed at both ends, Equation (15.33), $f_n = \frac{nv}{2L}$, is useful. It is important to remember the second *overtone* is the third *harmonic*. Solving for $v, v = \frac{2 f_n L}{n}$, and inserting the data, $v = \frac{(2)(.635\,\text{m})(588\,/\text{s})}{3}$, and $v = 249\,\text{m/s}$.

16.31: a) For constructive interference, the path difference $d = 2.00\,\text{m}$ must be equal to an integer multiple of the wavelength, so $\lambda_n = d/n$,

$$f_n = \frac{v}{\lambda_n} = \frac{vn}{d} = n\left(\frac{v}{d}\right) = n\frac{344\,\text{m/s}}{2.00\,\text{m}} = n(172\,\text{Hz}).$$

Therefore, the lowest frequency is 172 Hz.

b) Repeating the above with the path difference an odd multiple of half a wavelength, $f_n = \left(n + \frac{1}{2}\right)(172\,\text{Hz})$. Therefore, the lowest frequency is $86\,\text{Hz}\ (\text{n} = 0)$.

16.32: The difference in path length is $\Delta x = (L - x) - x = L - 2x$, or $x = (L - \Delta x)/2$. For destructive interference, $\Delta x = (n + (1/2))\lambda$, and for constructive interference, $\Delta x = n\lambda$. The wavelength is $\lambda = v/f = (344\ \text{m/s})/(206\ \text{Hz}) = 1.670\ \text{m}$ (keeping an extra figure), and so to have $0 \le x \le L, -4 \le n \le 3$ for destructive interference and $-4 \le n \le 4$ for constructive interference. Note that neither speaker is at a point of constructive or destructive interference.

a) The points of destructive interference would be at $x = 0.58\ \text{m}, 1.42\ \text{m}$.

b) Constructive interference would be at the points $x = 0.17\ \text{m},\ 1.00\ \text{m},\ 1.83\ \text{m}$.

c) The positions are very sensitive to frequency, the amplitudes of the waves will not be the same (except possibly at the middle), and exact cancellation at any frequency is not likely. Also, treating the speakers as point sources is a poor approximation for these dimensions, and sound reaches these points after reflecting from the walls , ceiling, and floor.

16.33: $\lambda = v/f = (344\ \text{m/s})/(688\ \text{Hz}) = 0.500\ \text{m}$

To move from constructive interference to destructive interference, the path difference must change by $\lambda/2$. If you move a distance x toward speaker B, the distance to B gets shorter by x and the difference to A gets longer by x so the path difference changes by *2x.*

$2x = \lambda/2$ and $x = \lambda/4 = 0.125\ \text{m}$

16.34: We are to assume $v = 344\ \text{m/s}$, so $\lambda = v/f = (344\ \text{m/s})/(172\ \text{Hz}) = 2.00\ \text{m}$. If $r_A = 8.00\ m$ and r_B are the distances of the person from each speaker, the condition for destructive interference is $r_B - r_A = (n + \frac{1}{2})\lambda$, where *n* is any integer. Requiring $r_B = r_A + (n + \frac{1}{2})\lambda > 0$ gives $n + \frac{1}{2} > -r_A/\lambda = -(8.00\ \text{m})/(2.00\ \text{m}) = -4$, so the smallest value of r_B occurs when $n = -4$, and the closest distance to *B* is $r_B = 8.00\ \text{m} + (-4 + \frac{1}{2})(2.00\ \text{m}) = 1.00\ \text{m}$.

16.35: $\lambda = v/f = (344\text{ m/s})/(860\text{ Hz}) = 0.400\text{ m}$

The path difference is $13.4\text{ m} - 12.0\text{ m} = 1.4\text{ m}$.

$$\frac{\text{path difference}}{\lambda} = 3.5$$

The path difference is a half-integer number of wavelengths, so the interference is destructive.

16.36: a) Since $f_{\text{beat}} = f_a - f_b$, the possible frequencies are $440.0\text{ Hz} \pm 1.5\text{ Hz} = 438.5\text{ Hz or } 441.5\text{ Hz}$ b) The tension is proportional to the square of the frequency. Therefore $T \propto f^2$ and $\Delta T \propto 2f\Delta f$. So $\frac{\Delta T}{T} = \frac{2\Delta f}{f}$. i) $\frac{\Delta T}{T} = \frac{2(1.5\text{ Hz})}{440\text{ Hz}} = 6.82\times10^{-3}$. ii) $\frac{\Delta T}{T} = \frac{2(-1.5\text{ Hz})}{440\text{ Hz}} = -6.82\times10^{-3}$.

16.37: a) A frequency of $\frac{1}{2}(108\text{ Hz} + 112\text{ Hz}) = 110\text{ Hz}$ will be heard, with a beat frequency of 112 Hz–108 Hz = 4 beats per second. b) The maximum amplitude is the sum of the amplitudes of the individual waves, $2(1.5\times10^{-8}\text{ m}) = 3.0\times10^{-8}\text{ m}$. The minimum amplitude is the difference, zero.

16.38: Solving Eq. (16.17) for v, with $v_{\text{L}} = 0$, gives

$$v = \frac{f_{\text{L}}}{f_{\text{S}} - f_{\text{L}}} v_{\text{S}} = \left(\frac{1240\text{ Hz}}{1200\text{ Hz} - 1240\text{Hz}}\right)(-25.0\text{ m/s}) = 775\text{ m/s},$$

or 780 m/s to two figures (the difference in frequency is known to only two figures). Note that $v_{\text{S}} < 0$, since the source is moving toward the listener.

16.39: Redoing the calculation with $+20.0\text{ m/s}$ for v_{S} and -20.0 m/s for v_{L} gives 267 Hz.

16.40: a) From Eq. (16.17), with $v_{\text{S}} = 0, v_{\text{L}} = -15.0\text{ m/s}$, $f_A' = 375\text{ Hz}$.

b) With $v_{\text{S}} = 35.0\text{ m/s}$, $v_{\text{L}} = 15.0\text{ m/s}$, $f_B' = 371\text{ Hz}$.

c) $f_A' - f_B' = 4\text{ Hz}$ (keeping an extra figure in f_A'). The difference between the frequencies is known to only one figure.

16.41: In terms of wavelength, Eq. (16.29) is

$$\lambda_L = \frac{v+v_s}{v+v_L}\lambda_S .$$

a) $v_L = 0, v_S = -25.0\text{ m}$ and $\lambda_L = \left(\frac{319}{344}\right)(344\text{ m/s})/(400\text{ Hz}) = 0.798\text{ m}.$ This is, of course, the same result as obtained directly from Eq. (16.27).
$v_S = 25.0\text{ m/s}$ and $v_L = (369\text{ m/s})/(400\text{ Hz}) = 0.922\text{ m}.$ The frequencies corresponding to these wavelengths are c) 431 Hz and d) 373 Hz.

16.42: a) In terms of the period of the source, Eq. (16.27) becomes

$$v_S = v - \frac{\lambda}{T_S} = 0.32\text{ m/s} - \frac{0.12\text{ m}}{1.6\text{ s}} = 0.25\text{ m/s}.$$

b) Using the result of part (a) in Eq. (16.18), or solving Eq. (16.27) for v_S and substituting into Eq. (16.28) (making sure to distinguish the symbols for the different wavelengths) gives $\lambda = 0.91\text{ m}.$

16.43: $f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S$

a) The direction from the listener to source is positive, so $v_S = -v/2$ and $v_L = 0.$

$$f_L = \left(\frac{v}{v-v/2}\right) f_S = 2 f_S = 2.00\text{ kHz}$$

b) $v_S = 0,\ v_L = +v/2$

$$f_L = \left(\frac{v+v/2}{v}\right) f_S = \tfrac{3}{2} f_S = 1.50\text{ kHz}$$

This is less than the answer in part (a).

The waves travel in air and what matters is the velocity of the listener or source relative to the air, not relative to each other.

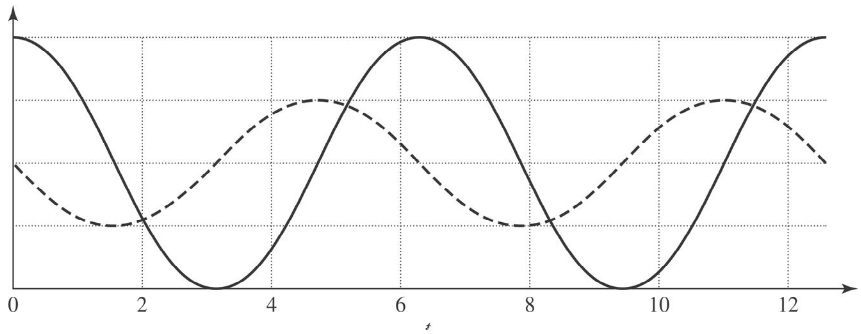
16.44: For a stationary source, $v_S = 0,$ so $f_L = \frac{v+v_L}{v+v_S} f_S = (1 + v_L/v) f_S,$ which gives $v_L = v\left(\frac{f_L}{f_S} - 1\right) = (344\text{ m/s})\left(\frac{490\text{ Hz}}{520\text{ Hz}} - 1\right) = -19.8\text{ m/s}.$
This is negative because the listener is moving away from the source.

16.45: a) $v_L = 18.0\ \text{m/s}, v_S = -30.0\ \text{m/s}$, and Eq. (16.29) gives $f_L = \left(\frac{362}{314}\right)(262\ \text{Hz})$ $= 302\ \text{Hz}$. b) $v_L = -18.0\ \text{m/s}, v_S = 30.0\ \text{m/s}$ and $f_L = 228\ \text{Hz}$.

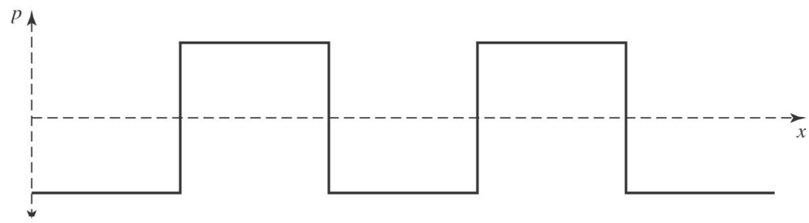
16.46: a) In Eq. (16.31), $v/v_S = 1/1.70 = 0.588$ and $\alpha = \arcsin(0.588) = 36.0°$.
b) As in Example 16.20,

$$t = \frac{(950\ \text{m})}{(1.70)\,(344\ \text{m/s})\,(\tan(36.0°))} = 2.23\ \text{s}.$$

16.47: a) Mathematically, the waves given by Eq. (16.1) and Eq. (16.4) are out of phase. Physically, at a displacement node, the air is most compressed or rarefied on either side of the node, and the pressure gradient is zero. Thus, displacement nodes are pressure antinodes. b) (This is the same as Fig. (16.3).) The solid curve is the pressure and the dashed curve is the displacement.



c)



The pressure amplitude is not the same. The pressure gradient is either zero or undefined. At the places where the pressure gradient is undefined mathematically (the "cusps" of the y - x plot), the particles go from moving at uniform speed in one direction to moving at the same speed in the other direction. In the limit that Fig. (16.43) is an accurate depiction, this would happen in a vanishing small time, hence requiring a very large force, which would result from a very large pressure gradient. d) The statement is true, but incomplete. The pressure is indeed greatest where the displacement is zero, but the pressure is equal to its largest value at points other than those where the displacement is zero.

16.48: The altitude of the plane when it passes over the end of the runway is $(1740\,\text{m} - 1200\,\text{m})\tan 15° = 145\,\text{m}$, and so the sound intensity is $1/(1.45)^2$ of what the intensity would be at 100 m. The intensity level is then

$$100.0\,\text{dB} - 10 \times \log\left[(1.45)^2\right] = 96.8\,\text{dB},$$

so the airliner is not in violation of the ordinance.

16.49: a) Combining Eq. (16.14) and Eq. (16.15),

$$p_{\max} = \sqrt{2\rho v I_0 10^{(\beta/10)}} = \sqrt{2(1.20\ \mathrm{kg/m^3})(344\ \mathrm{m/s})(10^{-12}\ \mathrm{W/m^2})10^{5.20}}$$
$$= 1.144\times10^{-2}\ \mathrm{Pa},$$

or $= 1.14\times10^{-2}\ \mathrm{Pa}$, to three figures. b) From Eq. (16.5), and as in Example 16.1,

$$A = \frac{p_{\max}}{Bk} = \frac{p_{\max} v}{B 2\pi f} = \frac{(1.144\times10^{-2}\ \mathrm{Pa})\,(344\ \mathrm{m/s})}{2\pi(1.42\times10^{5}\ \mathrm{Pa})(587\ \mathrm{Hz})} = 7.51\times10^{-9}\ \mathrm{m}.$$

c) The distance is proportional to the reciprocal of the square root of the intensity, and hence to 10 raised to half of the sound intensity levels divided by 10. Specifically,

$$(5.00\ \mathrm{m})10^{(5.20-3.00)/2} = 62.9\ \mathrm{m}.$$

16.50: a) $p = IA = I_0 10^{(\beta/10\,\mathrm{dB})} A.$ b) $\left(1.00\times10^{-12}\ \mathrm{W/m^2}\right)(10^{5.50})(1.20\ \mathrm{m^2})$
$= 3.79\times10^{-7}\ \mathrm{W}.$

16.51: For the flute, the fundamental frequency is

$$f_{1\mathrm{f}} = \frac{v}{4L} = \frac{344.0\ \mathrm{m/s}}{4(0.1075\ \mathrm{m})} = 800.0\ \mathrm{Hz}$$

For the flute and string to be in resonance,

$n_\mathrm{f} f_{1\mathrm{f}} = n_\mathrm{s} f_{1\mathrm{s}}$, where $f_{1\mathrm{s}} = 600.0\ \mathrm{Hz}$ is the fundamental frequency for the string.

$n_\mathrm{s} = n_\mathrm{f}\,(f_{1\mathrm{f}}/f_{1\mathrm{s}}) = \frac{4}{3} n_\mathrm{f}$

n_s is an integer when $n_\mathrm{f} = 3N$, $N = 1, 3, 5\ldots$ (the flute has only odd harmonics)

$n_\mathrm{f} = 3N$ gives $n_\mathrm{s} = 4N$

Flute harmonic $3N$ resonates with string harmonic $4N$, $N = 1,3,5,\ldots$

16.52: (a) The length of the string is $d = L/10,$ so its third harmonic has frequency $f_3^{\text{string}} = 3\frac{1}{2d}\sqrt{F/\mu}.$ The stopped pipe has length L, so its first harmonic has frequency $f_1^{\text{pipe}} = \frac{v_s}{4L}.$ Equating these and using $d = L/10$ gives $F = \frac{1}{3600}\mu v_s^2.$

(b) If the tension is doubled, all the frequencies of the string will increase by a factor of $\sqrt{2}$. In particular, the third harmonic of the string will no longer be in resonance with the first harmonic of the pipe because the frequencies will no longer match, so the sound produced by the instrument will be diminished.

(c) The string will be in resonance with a standing wave in the pipe when their frequencies are equal. Using $f_1^{\text{pipe}} = 3f_1^{\text{string}}$, the frequencies of the pipe are $nf_1^{\text{pipe}} = 3nf_1^{\text{string}}$, (where n=1, 3, 5…). Setting this equal to the frequencies of the string $n'f_1^{\text{string}},$ the harmonics of the string are $n' = 3n = 3, 9, 15, \ldots$

16.53: a) For an open pipe, the difference between successive frequencies is the fundamental, in this case 392 Hz, and all frequencies would be integer multiples of this frequency. This is not the case, so the pipe cannot be an open pipe. For a stopped pipe, the difference between successive frequencies is twice the fundamental, and each frequency is an odd integer multiple of the fundamental. In this case, $f_1 = 196\,\text{Hz},$ and $1372\,\text{Hz} = 7f_1,$ $1764\,\text{Hz} = 9f_1.$ b) $n = 7$ for $1372\,\text{Hz}$, $n = 9$ for $1764\,\text{Hz}$.
c) $f_1 = v/4L,$ so $L = v/4f_1 = (344\,\text{m/s})/(784\,\text{Hz}) = 0.439\,\text{m}.$

16.54: The steel rod has standing waves much like a pipe open at both ends, as shown in Figure (16.18). An integral number of half wavelengths must fit on the rod, that is, $f_n = \frac{nv}{2L}.$

a) The ends of the rod are antinodes because the ends of the rod are free to ocsillate.

b) The fundamental can be produced when the rod is held at the middle because a node is located there.

c) $f_1 = \frac{(1)(5941\,\text{m/s})}{2(1.50\,\text{m})} = 1980\,\text{Hz}.$

d) The next harmonic is $n = 2,$ or $f_2 = 3961\,\text{Hz}.$ We would need to hold the rod at an n = 2 node, which is located at $L/4$ from either end, or at 0.375 m from either end.

16.55: The shower stall can be modeled as a pipe closed at both ends, and hence there are nodes at the two end walls. Figure (15.23) shows standing waves on a *string* fixed at both ends but the sequence of harmonics is the same, namely that an integral number of half wavelengths must fit in the stall.

a) The condition for standing waves is $f_n = \frac{nv}{2L}$, so the first three harmonics are n = 1, 2, 3.

b) A particular physics professor's shower has a length of $L = 1.48\,\text{m}$. Using $f_n = \frac{nv}{2L}$, the resonant frequencies can be found when $v = 344\,\text{m/s}$.

n	***f(*Hz*)***
1	116
2	232
3	349

Note that the fundamental and second harmonic, which would have the greatest amplitude, are frequencies typically in the normal range of male singers. Hence, men do sing better in the shower! (For a further discussion of resonance and the human voice, see Thomas D. Rossing , *The Science of Sound*, Second Edition, Addison-Wesley, 1990, especially Chapters 4 and 17.)

16.56: a) The cross-section area of the string would be $a = (900\,\text{N})/(7.0\times10^{8}\,\text{Pa}) = 1.29\times10^{-6}\,\text{m}^2$, corresponding to a radius of $0.640\,\text{mm}$ (keeping extra figures). The length is the volume divided by the area,

$$L = \frac{V}{a} = \frac{m/\rho}{a} = \frac{(4.00\times10^{-3}\,\text{kg})}{(7.8\times10^{3}\,\text{kg/m}^3)(1.29\times10^{-6}\,\text{m}^2)} = 0.40\,\text{m}.$$

b) Using the above result in Eq. (16.35) gives $f_1 = 377\,\text{Hz}$, or $380\,\text{Hz}$ to two figures.

16.57: a) The second distance is midway between the first and third, and if there are no other distances for which resonance occurs, the difference between the first and third positions is the wavelength $\lambda = 0.750\,\text{m}$. (This would give the first distance as $\lambda/4 = 18.75\,\text{cm}$, but at the end of the pipe, where the air is not longer constrained to move along the tube axis, the pressure node and displacement antinode will not coincide exactly with the end). The speed of sound in the air is then $v = f\lambda = (500\,\text{Hz})(0.750\,\text{m}) = 375\,\text{m/s}$.

b) Solving Eq. (16.10) for γ,

$$\gamma = \frac{Mv^2}{RT} = \frac{(28.8\times10^{-3}\ \text{kg/mol})(375\,\text{m/s})^2}{(8.3145\,\text{J/mol}\cdot\text{K})(350.15\,\text{K})} = 1.39.$$

c) Since the first resonance should occur at $\tau/4 = 0.875\,\text{m}$ but actually occurs at $0.18\,\text{m}$, the difference is $0.0075\,\text{m}$.

16.58: a) Considering the ear as a stopped pipe with the given length, the frequency of the fundamental is $f_1 = v/4L = (344\ \text{m/s})/(0.10\,\text{m}) = 3440\,\text{Hz}$; $3500\,\text{Hz}$ is near the resonant frequency, and the ear will be sensitive to this frequency. b) The next resonant frequency would be $10{,}500\,\text{Hz}$ and the ear would be sensitive to sounds with frequencies close to this value. But $7000\,\text{Hz}$ is not a resonant frequency for an open pipe and the ear is not sensitive at this frequency.

16.59: a) From Eq. (15.35), with *m* the mass of the string and *M* the suspended mass,

$$f_1 = \sqrt{\frac{F}{4mL}} = \sqrt{\frac{Mg}{\pi d^2 L^2 \rho}} =$$
$$\sqrt{\frac{(420.0\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(225\times10^{-6}\ \text{m})^2(0.45\,\text{m})^2(21.4\times10^3\ \text{kg/m}^3)}} = 77.3\,\text{Hz}$$

and the tuning fork frequencies for which the fork would vibrate are integer multiples of $77.3\,\text{Hz}$. b) The ratio $m/M \approx 9\times10^{-4}$, so the tension does not vary appreciably along the string.

16.60: a) $L = \lambda/4 = v/4f = (344\ \mathrm{m/s})/(4(349\ \mathrm{Hz})) = 0.246\ \mathrm{m}$. b) The frequency will be proportional to the speed, and hence to the square root of the Kelvin temperature. The temperature necessary to have the frequency be higher is

$$(293.15\ \mathrm{K})(1.060)^2 = 329.5\ \mathrm{K},$$

which is $56.3°\ \mathrm{C}$.

16.61: The wavelength is twice the separation of the nodes, so

$$v = \lambda f = 2Lf = \sqrt{\frac{\gamma RT}{M}}.$$

Solving for γ,

$$\gamma = \frac{M}{RT}(2Lf)^2 = \frac{(16.0\times10^{-3}\ \mathrm{kg})}{(8.3145\ \mathrm{J/mol\cdot K})\,(293.15\ \mathrm{K})}\big(2(0.200\ \mathrm{m})(1100\ \mathrm{Hz})\big)^2 = 1.27.$$

16.62: If the separation of the speakers is denoted h, the condition for destructive interference is

$$\sqrt{x^2 + h^2} - x = \beta\lambda,$$

where β is an odd multiple of one-half. Adding x to both sides, squaring, cancelling the x^2 term from both sides and solving for x gives

$$x = \frac{h^2}{2\beta\lambda} - \frac{\beta}{2}\lambda.$$

Using $\lambda = \frac{v}{f}$ and h from the given data yields 9.01 m $\left(\beta = \frac{1}{2}\right)$, 2.71 m$\left(\beta = \frac{3}{2}\right)$, 1.27 m $\left(\beta = \frac{5}{2}\right)$, 0.53 m $\left(\beta = \frac{7}{2}\right)$ and 0.026 m $\left(\beta = \frac{9}{2}\right)$. These are the only allowable values of β that give positive solutions for x. (Negative values of x may be physical, depending on speaker design, but in that case the difference between path lengths is $\sqrt{x^2 + h^2} + x$.)
b) Repeating the above for integral values of β, constructive interference occurs at 4.34 m, 1.84 m, 0.86 m, 0.26 m. Note that these are between, but not midway between, the answers to part (a). c) If $h = \lambda/2$, there will be destructive interference at speaker B. If $\lambda/2 > h$, the path difference can never be as large as $\lambda/2$. (This is also obtained from the above expression for x, with $x = 0$ and $\beta = \frac{1}{2}$.) The minimum frequency is then $v/2h = (344\ \text{m/s})/(4.0\ \text{m}) = 86\ \text{Hz}$.

16.63: a) The wall serves as the listener, want $f_L = 600\ \text{Hz}$.

$$f_S = \left(\frac{v + v_S}{v + v_L}\right) f_L$$

$$v_L = 0,\ v_S = -30\ \text{m/s},\ v = 344\ \text{m/s}$$

$$f_S = 548\ \text{Hz}$$

b) Now the wall serves as a stationary source with $f_s = 600\ \text{Hz}$

$$f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$$

$$v_S = 0,\ v_L = +30\ \text{m/s},\ v = 344\ \text{m/s}$$

$$f_L = 652\ \text{Hz}$$

16.64: To produce a 10.0 Hz beat, the bat hears 2000 Hz from its own sound plus 2010 Hz coming from the wall. Call v the magnitude of the bat's speed, f_{w} the frequency the wall receives (and reflects), and V the speed of sound.

Bat is moving source and wall is stationary observer:

$$\frac{V}{f_{\mathrm{w}}} = \frac{V - v}{2000\,\mathrm{Hz}} \tag{1}$$

Bat is moving observer and wall is stationary source:

$$\frac{V + v}{2010\,\mathrm{Hz}} = \frac{V}{f_{\mathrm{w}}} \tag{2}$$

Solve (1) and (2) together:

$$v = 0.858\ \mathrm{m/s}$$

16.65: a) $A = \Delta R \cdot p_{\max} = BkA = \dfrac{2\pi BA}{\lambda} = \dfrac{2\pi BAf}{v}$. In air $v = \sqrt{\dfrac{B}{\rho}}$. Therefore

$$p_{\max} = 2\pi\sqrt{\rho B}\, f\Delta R, I = \frac{p^2{}_{\max}}{2\sqrt{\rho B}} = 2\pi^2\sqrt{\rho B}\, f^2(\Delta R)^2.$$

b) $P_{\mathrm{Tot}} = 4\pi R^2 I = 8\pi^3\sqrt{\rho B}\, f^2 R^2(\Delta R)^2$

c) $I = \frac{P_{\mathrm{Tot}}}{4\pi d^2} = \frac{2\pi^2\sqrt{\rho B}\, f^2 R^2 (\Delta R)^2}{d^2}$,

$$p_{\max} = (2\sqrt{\rho B I})^{1/2} = \frac{2\pi\sqrt{\rho B}\, fR(\Delta R)}{d}, A = \frac{p_{\max}}{2\pi\sqrt{\rho B}\, f} = \frac{R(\Delta R)}{d}.$$

16.66: (See also Problems 16.70 and 16.74). Let $f_0 = 2.00\,\text{MHz}$ be the frequency of the generated wave. The frequency with which the heart wall receives this wave is $f_H = \frac{v+v_H}{v} f_0$, and this is also the frequency with which the heart wall re-emits the wave. The detected frequency of this reflected wave is $f' \frac{v}{v-v_H}, f_H$, with the minus sign indicating that the heart wall, acting now as a source of waves, is moving toward the receiver. Combining, $f' \frac{v+v_H}{v-v_H} f_0$, and the beat frequency is

$$f_{\text{beat}} = f' - f_0 = \left(\frac{v+v_H}{v-v_H} - 1\right) f_0 = \frac{2v_H}{v-v_H} f_0.$$

Solving for v_H,

$$v_H = v\left(\frac{f_{\text{beat}}}{2\,f_0 + f_{\text{beat}}}\right) = (1500\ \text{m/s})\left(\frac{85\,\text{Hz}}{2(2.00\times10^6\ \text{Hz}) + (85\,\text{Hz})}\right)$$
$$= 3.19\times10^{-2}\ \text{m/s}.$$

Note that in the denominator in the final calculation, f_{beat} is negligible compared to f_0.

16.67: a) $\lambda = v/f = (1482\,\text{m/s})/(22.0\times10^3\ \text{Hz}) = 6.74\times10^{-2}\ \text{m}.$ b) See Problem 16.66 or Problem 16.70; the difference in frequencies is

$$\Delta f = f_S\left(\frac{2v_W}{v - v_W}\right) = \left(22.0\times10^3\,\text{Hz}\right)\frac{2(4.95\,\text{m/s})}{(1482\ \text{m/s}) - (4.95\,\text{m/s})} = 147\,\text{Hz}.$$

The reflected waves have higher frequency.

16.68: a) The maximum velocity of the siren is $\omega_P A_P = 2\pi f_P A_P$. You hear a sound with frequency $f_L = f_{\text{siren}}\, v/(v + v_S)$, where v_S varies between $+\,2\pi f_P A_P$ and $-\,2\pi f_P A_P$.
So $f_{L-\max} = f_{\text{siren}}\, v/(v - 2\pi f_P A_P)$ and $f_{L-\min} = f_{\text{siren}}\, v/(v + 2\pi f_P A_P)$.
b) The maximum (minimum) frequency is heard when the platform is passing through equilibrium and moving up (down).

16.69: a) Let v_b be the speed of the bat, v_i the speed of the insect and f_i the frequency with which the sound waves both strike and are reflected from the insect. The frequencies at which the bat sends and receives the signals are related by

$$f_\mathrm{L} = f_\mathrm{i}\left(\frac{v+v_\mathrm{b}}{v-v_\mathrm{i}}\right) = f_\mathrm{s}\left(\frac{v+v_\mathrm{i}}{v-v_\mathrm{b}}\right)\left(\frac{v+v_\mathrm{b}}{v-v_\mathrm{i}}\right).$$

Solving for v_i,

$$v_\mathrm{i} = v\left[\frac{1-\frac{f_\mathrm{S}}{f_\mathrm{L}}\left(\frac{v+v_\mathrm{b}}{v-v_\mathrm{b}}\right)}{1+\frac{f_\mathrm{S}}{f_\mathrm{L}}\left(\frac{v+v_\mathrm{b}}{v-v_\mathrm{b}}\right)}\right] = v\left[\frac{f_\mathrm{L}(v-v_\mathrm{b})-f_\mathrm{S}(v+v_\mathrm{b})}{f_\mathrm{L}(v-v_\mathrm{b})+f_\mathrm{S}(v+v_\mathrm{b})}\right].$$

Letting $f_\mathrm{L} = f_\mathrm{refl}$ and $f_\mathrm{S} = f_\mathrm{bat}$ gives the result.

b) If $f_\mathrm{bat} = 80.7\ \mathrm{kHz}$, $f_\mathrm{refl} = 83.5\ \mathrm{kHz}$, and $v_\mathrm{bat} = 3.9\ \mathrm{m/s}$, $v_\mathrm{insect} = 2.0\ \mathrm{m/s}$.

16.70: (See Problems 16.66, 16.74, 16.67). a) In a time t, the wall has moved a distance $v_1 t$ and the wavefront that hits the wall at time t has traveled a distance vt, where $v = f_0\lambda_0$, and the number of wavecrests in the total distance is $\frac{(v+v_1)t}{\lambda_0}$. b) The reflected wave has traveled vt and the wall has moved $v_1 t$, so the wall and the wavefront are separated by $(v-v_1)t$. c) The distance found in part (b) must contain the number of reflected waves found in part (a), and the ratio of the quantities is the wavelength of the reflected wave, $\lambda_0\frac{v-v_1}{v+v_1}$. d) The speed v divided by the result of part (c), expressed in terms of f_0 is $f_0\frac{v-v_1}{v+v_1}$. This is what is predicted by the problem-solving strategy.

e) $f_0\frac{v+v_1}{v-v_1} - f_0 = f_0\frac{2v_1}{v-v_1}$.

16.71: a)

$$f_R = f_L\sqrt{\frac{c-v}{c+v}} = f_S\frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}} = f_S\left(1-\frac{v}{c}\right)^{1/2}\left(1+\frac{v}{c}\right)^{-1/2}.$$

b) For small x, the binomial theorem (see Appendix B) gives $(1-x)^{1/2} \approx 1-x/2$, $(1+x)^{-1/2} \approx 1-x/2$, so

$$f_L \approx f_S\left(1-\frac{v}{2c}\right)^2 \approx f_S\left(1-\frac{v}{c}\right)$$

where the binomial theorem has been used to approximate $(1-x/2)^2 \approx 1-x$.

The above result may be obtained without resort to the binomial theorem by expressing f_R in terms of f_S as

$$f_R = f_S\frac{\sqrt{1-(v/c)}}{\sqrt{1+(v/c)}}\frac{\sqrt{1-(v/c)}}{\sqrt{1-(v/c)}} = f_S\frac{1-(v/c)}{\sqrt{1-(v/c)^2}}.$$

To first order in v/c, the square root in the denominator is 1, and the previous result is obtained. c) For an airplane, the approximation $v << c$ is certainly valid, and solving the expression found in part (b) for v,

$$v = c\frac{f_S - f_R}{f_S} = c\frac{f_{\text{beat}}}{f_S} = (3.00\times10^8\ \text{m/s})\frac{46.0\ \text{Hz}}{2.43\times10^8\ \text{Hz}} = 56.8\ \text{m/s},$$

and the approximation $v << c$ is seen to be valid. Note that in this case, the frequency *difference* is known to three figures, so the speed of the plane is known to three figures.

16.72: a) As in Problem 16.71,

$$v = c\frac{f_S - f_R}{f_S} = (3.00\times10^8\ \text{m/s})\frac{-0.018\times10^{14}\ \text{Hz}}{4.568\times10^8\,\text{Hz}} = -1.2\times10^6\ \text{m/s},$$

with the minus sign indicating that the gas is approaching the earth, as is expected since $f_R > f_S$. b) The radius is (952 yr) $(3.156\times10^7\ \text{s/yr})(1.2\times10^6\ \text{m/s}) = 3.6\times10^{16}$ $\text{m} = 3.8\ \text{ly}$. This may also be obtained from (952 yr) $\frac{f_R - f_S}{f_S}$. c) The ratio of the width of the nebula to 2π times the distance from the earth is the ratio of the angular width (taken as 5 arc minutes) to an entire circle, which is 60×360 arc minutes. The distance to the nebula is then (keeping an extra figure in the intermediate calculation)

$$2\times3.75\,\text{ly}\frac{(60)\times(360)}{5} = 5.2\times10^3\ \text{ly},$$

so the explosion actually took place about 4100 B.C

16.73: a) The frequency is greater than 2800 MHz; the thunderclouds, moving toward the installation, encounter more wavefronts per time than would a stationary cloud, and so an observer in the frame of the storm would detect a higher frequency. Using the result of Problem 16.71, with $v = -42.0\ \text{Km/h}$,

$$f_R - f_S = f_S\frac{-v}{c} = (2800\times10^6\text{Hz})\frac{(42.0\,\text{km/h})\ (3.6\,\text{km/h/1}\,\text{m/s})}{(3.00\times10^8\ \text{m/s})} = 109\,\text{Hz}.$$

b) The waves are being sent at a higher frequency than 2800 MHz from an approaching source, and so are received at a higher frequency. Repeating the above calculation gives the result that the waves are detected at the installation with a frequency 109 Hz greater than the frequency with which the cloud received the waves, or 218 Hz higher than the frequency at which the waves were originally transmitted at the receiver. Note that in doing the second calculation, $f_S = 2800\,\text{MHz} + 109\,\text{Hz}$ is the same as 2800 MHz to three figures.

16.74: a) (See also Example 16.19 and Problem 16.66.) The wall will receive and reflect pulses at a frequency $\frac{v}{v - v_{\mathrm{w}}} f_0$, and the woman will hear this reflected wave at a frequency

$$\frac{v + v_{\mathrm{w}}}{v} \cdot \frac{v}{v - v_{\mathrm{w}}} f_0 = \frac{v + v_{\mathrm{w}}}{v - v_{\mathrm{w}}} f_0;$$

The beat frequency is

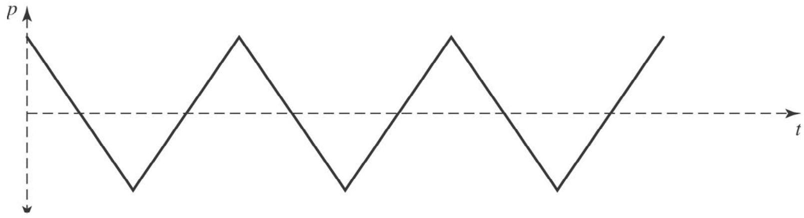
$$f_{\mathrm{beat}} = f_0\left(\frac{v + v_{\mathrm{w}}}{v - v_{\mathrm{w}}} - 1\right) = f_0\left(\frac{2v_{\mathrm{w}}}{v - v_{\mathrm{w}}}\right).$$

b) In this case, the sound reflected from the wall will have a lower frequency, and using $f_0(v - v_{\mathrm{w}})/(v + v_{\mathrm{w}})$ as the detected frequency (see Example 21-12; v_{w} is replaced by $-v_{\mathrm{w}}$ in the calculation of part (a)),

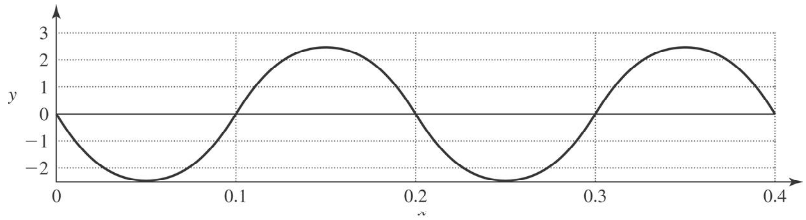
$$f_{\mathrm{beat}} = f_0\left(1 - \frac{v - v_{\mathrm{w}}}{v + v_{\mathrm{w}}}\right) = f_0\left(\frac{2v_{\mathrm{w}}}{v + v_{\mathrm{w}}}\right).$$

16.75: Refer to Equation (16.31) and Figure (16.38). The sound travels a distance vT and the plane travels a distance $v_{\mathrm{s}}T$ before the boom is found. So, $h^2 = (vT)^2 + (v_{\mathrm{s}}T)^2$, or $v_{\mathrm{s}}T = \sqrt{h^2 - v^2T^2}$. From Equation (16.31), $\sin\alpha = \frac{v}{v_{\mathrm{s}}}$. Then, $v_{\mathrm{s}} = \frac{hv}{\sqrt{h^2 - v^2T^2}}$.

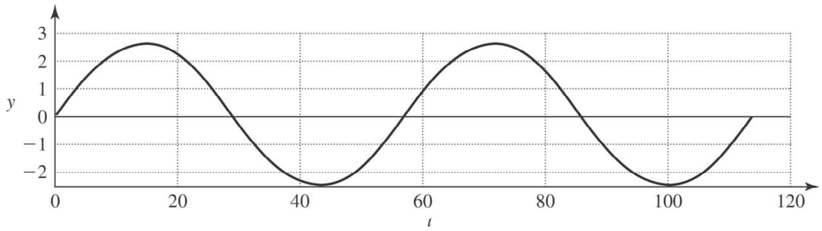
16.76: a)



b) From Eq. (16.4), the function that has the given $p(x, 0)$ at $t = 0$ is given graphically as shown. Each section is a parabola, not a portion of a sine curve. The period is $\lambda/v = (0.200\,\text{m})/(344\,\text{m/s}) = 5.81\times10^{-4}$ s and the amplitude is equal to the area under the $p - x$ curve between $x = 0$ and $x = 0.0500$ m divided by B, or 7.04×10^{-6} m.



c) Assuming a wave moving in the $+x$-direction, $y(0,t)$ is as shown.



d) The maximum velocity of a particle occurs when a particle is moving throughout the origin, and the particle speed is $v_y = -\frac{\partial y}{\partial x} v = \frac{pv}{B}$. The maximum velocity is found from the maximum pressure, and $v_{y\text{max}} = (40\,\text{Pa})(344\,\text{m/s})/(1.42\times10^5\,\text{Pa}) = 9.69\,\text{cm/s}$. The maximum acceleration is the maximum pressure gradient divided by the density,

$$a_{\text{max}} = \frac{(80.0\,\text{Pa})/(0.100\,\text{m})}{(1.20\,\text{kg/m}^3)} = 6.67\times10^2\ \text{m/s}^2.$$

e) The speaker cone moves with the displacement as found in part (c); the speaker cone alternates between moving forward and backward with constant magnitude of acceleration (but changing sign). The acceleration as a function of time is a square wave with amplitude $667\,\text{m/s}^2$ and frequency $f = v/\lambda = (344\,\text{m/s})/(0.200\,\text{m}) = 1.72\,\text{kHz}$.

16.77: Taking the speed of sound to be $344\ \text{m/s}$, the wavelength of the waves emitted by each speaker is $2.00\ \text{m}$. a) Point C is two wavelengths from speaker A and one and one-half from speaker B, and so the phase difference is $180° = \pi$ rad.

b) $$I = \frac{P}{4\pi r^2} = \frac{8.00\times10^{-4}\,\text{W}}{4\pi(4.00\ \text{m})^2} = 3.98\times10^{-6}\ \text{W/m}^2,$$

and the sound intensity level is $(10\ \text{dB})\log(3.98\times10^{6}) = 66.0\ \text{dB}.$ Repeating with $P = 6.00\times10^{-5}\ \text{W}$ and $r = 3.00\ \text{m}$ gives $I = 5.31\times10^{-7}\,\text{W}$ and $\beta = 57.2\text{dB}.$ c) With the result of part (a), the amplitudes, either displacement or pressure, must be subtracted. That is, the intensity is found by taking the square roots of the intensities found in part (b), subtracting, and squaring the difference. The result is that $I = 1.60\times10^{-6}\ \text{W}$ and $\beta = 62.1\text{dB}.$

17.1: From Eq. (17.1), a) $(9/5)(-62.8)+32=-81.0°\text{F}$. b) $(9/5)(56.7)+32=134.1°\text{F}$. c) $(9/5)(31.1)+32=88.0°\text{F}$.

17.2: From Eq. (17.2), a) $(5/9)(41.0-32)=5.0°\text{C}$. b) $(5/9)(107-32)=41.7°\text{C}$. c) $(5/9)(-18-32)=-27.8°\text{C}$.

17.3: $1\,\text{C}°=\frac{9}{5}\,\text{F}°$, so $40.0=72.0\,\text{F}°$

$T_2=T_1+70.0\,\text{F}°=140.2°\text{F}$

17.4: a) $(5/9)\,(45.0-(-4.0))=27.2°\,\text{C}$. b) $(5/9)\,(-56.0-44)=-55.6°\,\text{C}$.

17.5: a) From Eq. (17.1), $(9/5)(40.2)+32=104.4°\text{F}$, which is cause for worry.

b) $(9/5)(12)+32=53.6°\text{F}$, or $54°\text{F}$ to two figures.

17.6: $(9/5)(11.8)=21.2\,\text{F}°$

17.7: $1\,\text{K}=1\,\text{C}°=\frac{9}{5}\,\text{F}°$, so a temperature increase of 10 K corresponds to an increase of $18\,\text{F}°$. Beaker B has the higher temperature.

17.8: For (b), $\Delta T_\text{C}=\Delta T_\text{K}=-10.0\,\text{C}°$. Then for (a), $\Delta T_\text{F}=\frac{9}{5}\Delta T_\text{C}=\frac{9}{5}(-10.0\,\text{C}°)$ $=-18.0\,\text{F}°$.

17.9: Combining Eq. (17.2) and Eq. (17.3),

$$T_K=\frac{5}{9}(T_F-32°)+273.15,$$

and substitution of the given Fahrenheit temperatures gives a) 216.5 K, b) 325.9 K, c) 205.4 K.

17.10: (In these calculations, extra figures were kept in the intermediate calculations to arrive at the numerical results.) a) $T_\text{C}=400-273.15=127°\text{C}$, $T_\text{F}=(9/5)(126.85)+32=260°\text{F}$. b) $T_\text{C}=95-273.15=-178°\text{C}$, $T_\text{F}=(9/5)(-178.15)+32=-289°\text{F}$.
c) $T_\text{C}=1.55\times10^7-273.15=1.55\times10^7°\text{C}$, $T_\text{F}=(9/5)(1.55\times10^7)+32=2.79\times10^7°\text{F}$.

17.11: From Eq. (17.3), $T_{\text{K}} = (-245.92°\text{C}) + 273.15 = 27.23\,\text{K}$.

17.12: From Eq. (17.4), $(7.476)(273.16\,\text{K}) = 2042.14\,\text{K} - 273.15 = 1769°\text{C}$.

17.13: From Eq. (17.4), $(325.0\,\text{mm})\left(\frac{373.15\,\text{K}}{273.16\,\text{K}}\right) = 444\,\text{mm}$.

17.14: On the Kelvin scale, the triple point is 273.16 K, so $°\text{R} = (9/5)273.15\,\text{K} = 491.69°\text{R}$. One could also look at Figure 17.7 and note that the Fahrenheit scale extends from $-460°\text{F}$ to $+32°\text{F}$ and conclude that the triple point is about $492\,°\text{R}$.

17.15: From the point-slope formula for a straight line (or linear regression, which, while perhaps not appropriate, may be convenient for some calculators),

$$(0.01°\text{C}) - (100.0°\text{C})\frac{4.80\times10^4\ \text{Pa}}{6.50\times10^4\ \text{Pa} - 4.80\times10^4\ \text{Pa}} = -282.33°\text{C},$$

which is $-282°\text{C}$ to three figures.

b) Equation (17.4) was not obeyed precisely. If it were, the pressure at the triple point would be $P = (273.16)\left(\frac{6.50\times10^4\ \text{Pa}}{373.15}\right) = 4.76\times10^4\ \text{Pa}$.

17.16: $\Delta T = (\Delta L)/(\alpha L_0) = (25\times10^{-2}\ \text{m})/((2.4\times10^{-5}(\text{C°})^{-1}(62.1\,\text{m})) = 168°\,\text{C}$, so the temperature is $183°\,\text{C}$.

17.17: $\alpha L_0 \Delta T = (1.2\times10^{-5}(\text{C°})^{-1})(1410\text{m})(18.0°\,\text{C} - (-5.0)°\text{C}) = +0.39\,\text{m}$.

17.18:
$$\begin{aligned} d + \Delta d &= d(1+\alpha\Delta T) \\ &= (0.4500\text{cm})(1+(2.4\times10^{-5}(\text{C°})^{-1})(23.0°\,\text{C} - (-78.0°\text{C}))) \\ &= 0.4511\,\text{cm} = 4.511\text{mm}. \end{aligned}$$

17.19: a) $\alpha D_0 \Delta T = (2.6\times10^{-5}\,(\text{C°})^{-1})(1.90\text{cm})(28.0°\text{C}) = 1.4\times10^{-3}\,\text{cm}$, so the diameter is 1.9014 cm. b) $\alpha D_0 \Delta T = -3.6\times10^{-3}$ cm, so the diameter is 1.8964 cm.

17.20: $\alpha\Delta T = (2.0\times10^{-5}\,(\text{C}°)^{-1})(5.00°\text{C} - 19.5°\text{C}) = -2.9\times10^{-4}$.

17.21: $\alpha = (\Delta L)/(L_0\Delta T) = \left(2.3\times10^{-4}\text{ m}\right)/\left(\left(40.125\times10^{-2}\text{ m}\right)\left(25.0\text{ C}°\right)\right)$
$= 2.3\times10^{-5}\left(\text{C}°\right)^{-1}$.

17.22: From Eq. (17.8), $\Delta T = \frac{\Delta V/V_0}{\beta} = \frac{1.50\times10^{-3}}{5.1\times10^{-5}\text{ K}^{-1}} = 29.4°\text{C}$, so $T = 49.4°\text{C}$.

17.23 $\beta V_0\Delta T = \left(75\times10^{-5}\left(\text{C}°\right)^{-1}\right)\left(1700\text{ L}\right)\left(-9.0°\text{C}\right) = -11\text{ L}$, so there is 11 L of air.

17.24: The temperature change is $\Delta T = 18.0°\text{ C} - 32.0°\text{ C} = -14.0\text{ C}°$. The volume of ethanol contracts more than the volume of the steel tank does, so the additional amount of ethanol that can be put into the tank is $\Delta V_{\text{steel}} - \Delta V_{\text{ethanol}} = \left(\beta_{\text{steel}} - \beta_{\text{ethanol}}\right)V_0\Delta T$
$= \left(3.6\times10^{-5}\left(\text{C}°\right)^{-1} - 75\times10^{-5}\left(\text{C}°\right)^{-1}\right)\left(2.80\text{ m}^3\right)\left(-14.0\text{ C}°\right) = 0.0280\text{ m}^3$

17.25: The amount of mercury that overflows is the difference between the volume change of the mercury and that of the glass;

$$\beta_{\text{glass}} = 18.0\times10^{-5}\text{ K}^{-1} - \frac{\left(8.95\text{ cm}^3\right)}{\left(1000\text{ cm}^3\right)\left(55.0°C\right)} = 1.7\times10^{-5}\left(\text{C}°\right)^{-1}.$$

17.26: a) $A = L^2, \Delta A = 2L\Delta L = 2\frac{\Delta L}{L}L^2 = 2\frac{\Delta L}{L}A_0$. But $\frac{\Delta L}{L} = \alpha\Delta T$, and so $\Delta A = 2\alpha\Delta TA_0 = \left(2\alpha\right)A_0\Delta T$.

b) $\Delta A = \left(2\alpha\right)A_\circ\Delta T = \left(2\right)(2.4\times10^{-5}(\text{C}°)^{-1})\left(\pi\times\left(.275\text{ m}\right)^2\right)\left(12.5°\text{C}\right) = 1.4\times10^{-4}\text{ m}^2$.

17.27: a) $A_0 = \dfrac{\pi D^2}{4} = \dfrac{\pi}{4}\left(1.350\text{ cm}\right)^2 = 1.431\text{ cm}^2$.

b) $A = A_0\left(1 + 2\alpha\,\Delta T\right) = \left(1.431\text{ cm}^2\right)\left(1 + \left(2\right)\left(1.20\times10^{-5}°\text{C}\right)\left(150°\text{C}\right)\right) = 1.437\text{ cm}^2$.

17.28: (a) No, the brass expands more than the steel.

(b) call $D_\circ$ the inside diameter of the steel cylinder at $20°\text{C}$ At $150°\text{C}: D_{\text{ST}} = D_{\text{BR}}$

$$
\begin{aligned}
D_\circ + \Delta D_{\text{ST}} &= 25.000\,\text{cm} + \Delta \text{D}_{\text{BR}} \\
D_\circ + \alpha_{\text{ST}} D_\circ \Delta \text{T} &= 25\,\text{cm} + \alpha_{\text{BR}}(25\,\text{cm})\Delta T \\
D_\circ &= \frac{25\,\text{cm}(1+\alpha_{\text{BR}}\Delta T)}{1+\alpha_{\text{ST}}\Delta T} \\
&= \frac{(25\,\text{cm})\left[1+(2.0\times10^{-5}(\text{C}°)^{-1})(130\text{C}°)\right]}{1+(1.2\times10^{-5}(\text{C}°)^{-1})(130\,\text{C}°)} \\
&= 25.026\,\text{cm}
\end{aligned}
$$

17.29: The aluminum ruler expands to a new length of

$L = L_0(1+\alpha\Delta T) = (20.0\,\text{cm})[1+(2.4\times10^{-5}(\text{C}°)^{-1})(100\,\text{C}°)] = 20.048\,\text{cm}$

The brass ruler expands to a new length of

$L = L_0(1+\alpha\Delta T) = (20.0\,\text{cm})[1+(2.0\times10^{-5}(\text{C}°)^{-1})(100\,\text{C}°)] = 20.040\,\text{cm}$

The section of the aluminum ruler will be longer by 0.008 cm

17.30: From Eq. (17.12),

$$
\begin{aligned}
F &= -Y\alpha\Delta TA \\
&= -(0.9\times10^{11}\ \text{Pa})(2.0\times10^{-5}(\text{C}°)^{-1})(-110°\text{C})(2.01\times10^{-4}\ \text{m}^2) \\
&= 4.0\times10^4\ \text{N}.
\end{aligned}
$$

17.31: a) $\alpha = (\Delta L/L_0\Delta T) = (1.9\times10^{-2}\ \text{m})/\big((1.50\,\text{m})(400\,\text{C}°)\big) = 3.2\times10^{-5}(\text{C}°)^{-1}.$

b) $Y\alpha\Delta T = Y\Delta L/L_0 = (2.0\times10^{11}\ \text{Pa})(1.9\times10^{-2}\ \text{m})/(1.50\ \text{m}) = 2.5\times10^9\ \text{Pa}.$

17.32: a) $\Delta L = \alpha\Delta TL = (1.2\times10^{-5}\ \text{K}^{-1})(35.0\,\text{K})(12.0\,\text{m}) = 5.0\times10^{-3}\ \text{m}.$

b) Using absolute values in Eq. (17.12),

$$\frac{F}{A} = Y\alpha\Delta T = (2.0\times10^{11}\ \text{Pa})(1.2\times10^{-5}\ \text{K}^{-1})(35.0\ \text{K}) = 8.4\times10^7\ \text{Pa}.$$

17.33: a) $(37°\text{C} - (-20°\text{C}))(0.50\ \text{L})(1.3\times10^{-3}\ \text{kg/L})\ (1020\text{J/kg}\cdot\text{K}) = 38\ \text{J}$

b) There will be 1200 breaths per hour, so the heat lost is $(1200)(38\ \text{J}) = 4.6\times\times10^{4}\ \text{J}$.

17.34: $t = \dfrac{Q}{P} = \dfrac{mc\Delta T}{P} = \dfrac{(70\ \text{kg})(3480\ \text{J/kg}\cdot\text{K})(7\ \text{C°})}{(1200\ \text{W})} = 1.4\times10^{3}\ \text{s},\ $ about 24 min.

17.35: Using Q=mgh in Eq. (17.13) and solving for ΔT gives

$$\Delta T = \frac{gh}{c} = \frac{(9.80\ \text{m/s}^2)(225\ \text{m})}{(4190\ \text{J/kg.K})} = 0.53\ \text{C°}.$$

17.36: a) The work done by friction is the loss of mechanical energy,

$$mgh + \frac{1}{2}m(v_1^2 - v_2^2) = (35.0\ \text{kg})\left((9.80\ \text{m/s}^2)(8.00\ \text{m})\sin 36.9° - \frac{1}{2}(2.50\ \text{m/s})^2\right)$$
$$= 1.54\times10^{3}\ \text{J}.$$

b) Using the result of part (a) for Q in Eq. (17.13) gives

$$\Delta T = \left(1.54\times10^{3}\ \text{J}\right)/\left((35.0\ \text{kg})(3650\ \text{J/kg}\cdot\text{K})\right) = 1.21\times10^{-2}\ \text{C°}.$$

17.37: $(210°\ \text{C} - 20°\text{C})\left((1.60\ \text{kg})(910\ \text{J/kg}\cdot\text{K}) + (0.30\ \text{kg})(470\ \text{J/kg}\cdot\text{K})\right) = 3.03\times10^{5}\ \text{J}.$

17.38: Assuming $Q = (0.60)\times10\times K$,

$$\Delta T = (0.60)\times10\times\frac{K}{mc} = 6\frac{\frac{1}{2}MV^2}{mc} = \frac{(6)\frac{1}{2}(1.80\ \text{kg})(7.80\text{m/s})^2}{(8.00\times10^{-3}\ \text{kg})(910\ \text{J/kg}\cdot\text{K})} = 45.1\ \text{C°}.$$

17.39: $(85.0°\ \text{C} - 20.0°\ \text{C})\left((1.50\ \text{kg})(910\text{J/kg}\cdot\text{K}) + (1.80\ \text{kg})(4190\text{J/kg}\cdot\text{K})\right)$
$= 5.79\times10^{5}\ \text{J}.$

17.40: a) $Q = mc\Delta T = (0.320\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(60.0\,\text{K}) = 8.05\times10^{4}\ \text{J}.$

b) $t = \frac{Q}{P} = \frac{8.05\times10^{4}\ \text{J}}{200\ \text{W}} = 402\ \text{s}.$

17.41: a) $c = \dfrac{Q}{m\Delta T} = \dfrac{(120\,\text{s})(65.0\,\text{W})}{(0.780\,\text{kg})(22.54°\,\text{C} - 18.55°\,\text{C})} = 2.51\times10^{3}\ \text{J/kg}\cdot\text{K}.$

b) An overstimate; the heat Q is in reality less than the power times the time interval.

17.42: The temperature change is $\Delta T = 18.0\,\text{K}$, so

$$c = \frac{Q}{m\Delta T} = \frac{gQ}{w\Delta T} = \frac{(9.80\,\text{m/s}^2)(1.25\times10^{4}\ \text{J})}{(28.4\,\text{N})(18.0\,\text{K})} = 240\,\text{J/kg}\cdot\text{K}.$$

17.43: a) $Q = mc\Delta T,\ c = 470\,\text{J/kg}\cdot\text{K}$

We need to find the mass of 3.00 mol:

$m = nM = (3.00\,\text{mol})(55.845\times10^{-3}\ \text{kg/mol}) = 0.1675\,\text{kg}$

$\Delta T = Q/mc = (8950\,\text{J})/[(0.1675\,\text{kg})(470\,\text{J/kg}\cdot\text{K})] = 114\,\text{K} = 114\,\text{C}°$

b) For $m = 3.00\,\text{kg},\ \Delta T = Q/mc = 6.35\,\text{C}°$

c) The result of part (a) is much larger; 3.00 kg is more material than 3.00 mol.

17.44: (a) $L_F = \dfrac{Q_{\text{melt}}}{m} = \dfrac{(10{,}000\,\text{J/min})(1.5\,\text{min})}{0.50\,\text{kg}} = 30{,}000\,\text{J/kg}$

(b) $Liquid: Q = mc\Delta T \rightarrow c = \dfrac{Q}{m\Delta T}$

$$c = \frac{(10{,}000\,\text{J/min})(1.5\,\text{min})}{(0.50\,\text{kg})(30\text{C}°)} = 1{,}000\,\text{J/kg}\cdot\text{C}°$$

$Solid: c = \dfrac{Q}{m\Delta T} = \dfrac{(10{,}000\,\text{J/min})(1.0\,\text{min})}{(0.50\,\text{kg})(15\,\text{C}°)} = 1300\,\text{J/kg}\cdot\text{C}°$

17.45: a) $Q_{\text{water}} + Q_{\text{metal}} = 0$

$$m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}} + m_{\text{metal}} c_{\text{metal}} \Delta T_{\text{metal}} = 0$$
$$(1.00\text{kg})(4190\ \text{J/kg}\cdot\text{K})(2.0\ \text{C}°) + (0.500\ \text{kg})(c_{\text{metal}})(-78.0\ \text{C}°) = 0$$
$$c_{metal} = 215\ \text{J/kg}\cdot\text{K}$$

b) Water has a larger specific heat capacity so stores more heat per degree of temperature change.

c) If some heat went into the styrofoam then Q_{metal} should actually be larger than in part (a), so the true c_{metal} is larger than we calculated; the value we calculated would be smaller than the true value.

17.46: a) Let the man be designated by the subscript m and the "'water" by w, and T is the final equilibrium temperature.

$$-m_{\text{m}} C_{\text{m}} \Delta T_{\text{m}} = m_{\text{w}} C_{\text{w}} \Delta T_{\text{w}}$$
$$-m_{\text{m}} C_{\text{m}} (T - T_{\text{m}}) = m_{\text{w}} C_{\text{w}} (T - T_{\text{w}})$$
$$m_{\text{m}} C_{\text{m}} (T_{\text{m}} - T) = m_{\text{w}} C_{\text{w}} (T - T_{\text{w}})$$

Or solving for T, $T = \frac{m_{\text{m}} C_{\text{m}} T_{\text{m}} + m_{\text{w}} C_{\text{w}} T_{\text{w}}}{m_{\text{m}} C_{\text{m}} + m_{\text{w}} C_{\text{w}}}$. Inserting numbers, and realizing we can change K to $°\text{C}$, and the mass of water is .355 kg, we get

$$T = \frac{(70.0\ \text{kg})\ (3480\ \text{J/kg}\cdot\text{K})\ (37.0°C) + (0.355\ \text{kg})\ (4190\ \text{J/kg}\cdot°C)\ (12.0°C)}{(70.0\ \text{kg})(3480\ \text{J/kg}.\ °C) + (0.355\ \text{kg})\ (4190\ \text{J/kg}\cdot°C)}$$

Thus, $T = 36.85°C$.

b) It is possible a sensitive digital thermometer could measure this change since they can read to .1°C. It is best to refrain from drinking cold fluids prior to orally measuring a body temperature due to cooling of the mouth.

17.47: The rate of heat loss is $\Delta Q/\Delta t.\left(\frac{\Delta Q}{\Delta t}\right) = \frac{mC\Delta T}{\Delta t}$, or $\Delta t = \frac{mC\Delta t}{(\frac{\Delta Q}{\Delta t})}$. Interestin g numbers, $\Delta t = \frac{(70.355\ \text{kg})(3480\ \text{J/kg.°C})(0.15°\text{C})}{7\times10^{6}\ \text{J/day}} = 0.005\,\text{d}$, or $\Delta t = 7.6$ minutes. This may acount for mothers taking the temperature of a sick child several minutes *after* the child has something to drink.

17.48: $Q = m(c\Delta T + L_{\text{f}})$
$$= (0.350\ \text{kg})\big((4190\ \text{J/kg}\cdot\text{K})(18.0\ \text{K}) + 334\times10^{3}\ \text{J/kg}\big)$$
$$= 1.43\times10^{5}\ \text{J} = 34.2\ \text{kcal} = 136\ \text{Btu}.$$

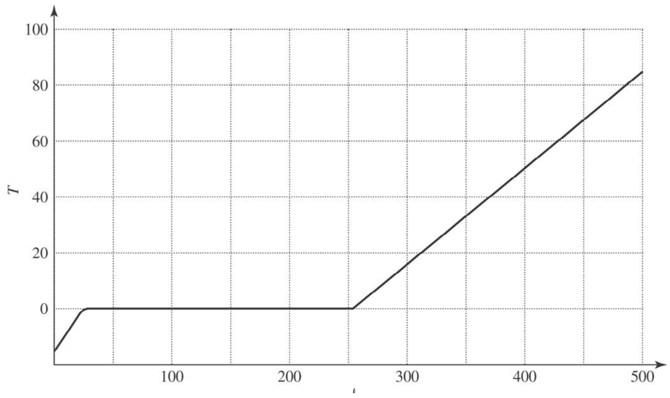
17.49: $Q = m(c_{\text{ice}}\Delta T_{\text{ice}} + L_{\text{f}} + c_{\text{water}}\Delta T_{\text{water}} + L_{\text{V}})$

$$= (12.0\times 10^{-3}\ \text{kg})\begin{pmatrix}(2100\ \text{J/kg}\cdot\text{K})(10.0\ \text{C}°) + 334\times 10^{3}\ \text{J/kg} \\ + (100\ \text{C}°)(4190\ \text{J/kg}\cdot\text{K}) + 2256\times 10^{3}\ \text{J/kg}\end{pmatrix}$$

$$= 3.64\times 10^{4}\ \text{J} = 8.69\ \text{kcal} = 34.5\ \text{Btu}.$$

17.50: a) $t = \dfrac{Q}{P} = \dfrac{mc\Delta T}{P} = \dfrac{(0.550\ \text{kg})(2100\ \text{J/kg}\cdot\text{K})(15.0\ \ \text{K})}{(800\ \ \text{J/min})} = 21.7\,\text{min}.$

b) $\frac{mL_{\text{f}}}{P} = \frac{(0.550\ \text{kg})(334\times 10^{3}\ \text{J/kg})}{(800\ \text{J/min})} = 230\,\text{min},$ so the time until the ice has melted is $21.7\ \text{min} + 230\ \text{min} = 252\ \text{min}.$



17.51: $\dfrac{\big((4000\ \text{lb})/2.205\ \text{lb/kg})\big)(334\times 10^{3}\ \text{J/kg})}{(86{,}400\ \text{s})} = 7.01\,\text{kW} = 2.40\times 10^{4}\ \text{Btu/hr}.$

17.52: a) $m(c\Delta T + L_{\text{v}}) = (25.0\times 10^{-3}\ \text{kg})\big((4190\ \text{J/kg}\cdot\text{K})(66.0\ \text{K}) + 2256\times 10^{3}\ \text{J/kg}\big) =$ $6.33\times 10^{4}\ \text{J}.$ b) $mc\Delta T = (25.0\times 10^{-3}\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(66.0\ \text{K}) = 6.91\times 10^{3}\ \text{J}.$

c) Steam burns are far more severe than hot-water burns.

17.53: With $Q = m(c\Delta T + L_{\text{f}})$ and $K = (1/2)mv^{2}$, setting $Q = K$ and solving for v gives $v = \sqrt{2\big((130\ \text{J/Kg}\cdot\text{K})(302.3\text{C}°) + 24.5\times 10^{3}\ \text{J/kg}\big)} = 357\ \text{m/s}.$

17.54: a) $m_{\text{sweat}} = \dfrac{Mc\Delta T}{L_{\text{v}}} = \dfrac{(70.0\,\text{kg})(3480\,\text{J/kg}\cdot\text{K})(1.00\,\text{K})}{(2.42\times10^{6}\ \text{J/kg})} = 101\,\text{g}.$

b) This much water has a volume of $101\ \text{cm}^3$, about a third of a can of soda.

17.55: The mass of water that the camel saves is

$$\frac{Mc\Delta T}{L_{\text{v}}} = \frac{(400\,\text{kg})(3480\,\text{J/kg}\cdot\text{K})(6.0\,\text{K})}{(2.42\times10^{6}\ \text{J/kg})} = 3.45\,\text{kg},$$

which is a volume of 3.45 L.

17.56: For this case, the algebra reduces to

$$T = \frac{\begin{pmatrix}((200)(3.00\times10^{-3}\ \text{kg}))(390\ \text{J/kg}\cdot\text{K})(100.0\,\text{C}°)\\ +(0.240\,\text{kg})(4190\text{J/kg})(20.0\,\text{C}°)\end{pmatrix}}{\begin{pmatrix}((200)(3.00\times10^{-3}\ \text{kg})(390\ \text{J/kg}\cdot\text{K})\\ +(0.240\,\text{kg})(4190\ \text{J/kg}\cdot.\text{K})\end{pmatrix}} = 35.1°\text{C}.$$

17.57: The algebra reduces to

$$T = \frac{\begin{pmatrix}((0.500\,\text{kg})(390\,\text{J/kg}\cdot\text{K}) + (0.170\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}))(20.0°\,\text{C}\\ +(0.250\,\text{kg})(470\text{J/kg}\cdot\text{K})(85.0°\text{C})\end{pmatrix}}{\begin{pmatrix}((0.500\,\text{kg})(390\,\text{J/kg}\cdot\text{K}) + (0.170\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}))\\ +(0.250\,\text{kg})(470\,\text{J/kg}\cdot\text{K})\end{pmatrix}} = 27.5°\text{C}$$

17.58: The heat lost by the sample is the heat gained by the calorimeter and water, and the heat capacity of the sample is

$$c = \frac{Q}{m\Delta T} = \frac{((0.200\,\text{kg})(4190\ \text{J/kg}\cdot\text{K}) + (0.150\,\text{kg})(390\ \text{J/kg}\cdot\text{K}))(7.1\text{C}°)}{(0.0850\,\text{kg})(73.9\,\text{C}°)}$$

$$=1010\ \text{J/kg}\cdot\text{K},$$

or $1000\ \text{J/kg}\cdot\text{K}$ to the two figures to which the temperature change is known.

17.59: The heat lost by the original water is

$$-Q = (0.250\,\text{kg})(4190\ \text{J/kg}\cdot\text{K})(45.0\,\text{C}°) = 4.714\times10^{4}\ \text{J},$$

and the mass of the ice needed is

$$\begin{aligned} m_{\text{ice}} &= \frac{-Q}{c_{\text{ice}}\Delta T_{ice} + L_{\text{f}} + c_{\text{water}}\Delta T_{\text{water}}} \\ &= \frac{(4.714\times10^{4})\ \text{J}}{(2100\ \text{J/kg}\cdot\text{K})(20.0\,\text{C}°) + (334\times10^{3}\ \text{J/kg}) + (4190\ \text{J/kg}\cdot\text{K})(30.0\,\text{C}°)} \\ &= 9.40\times10^{-2}\ \text{kg} = 94.0\,\text{g}. \end{aligned}$$

17.60: The heat lost by the sample (and vial) melts a mass *m*, where

$$m = \frac{Q}{L_{\text{f}}} = \frac{((16.0\,\text{g})(2250\ \text{J/kg}\cdot\text{K}) + (6.0\,\text{g})(2800\ \text{J/kg}\cdot\text{K}))(19.5\text{K})}{(334\times10^{3}\ \text{J/kg})} = 3.08\,\text{g}.$$

Since this is less than the mass of ice, not all of the ice melts, and the sample is indeed cooled to $0°\text{C}$. Note that conversion from grams to kilograms was not necessary.

17.61: $$\frac{(4.00\,\text{kg})(234\ \text{J/kg}\cdot\text{K})(750\,\text{C}°)}{(334\times10^{3}\ \text{J/kg})} = 2.10\,\text{kg}.$$

17.62: Equating the heat lost by the lead to the heat gained by the calorimeter (including the water-ice mixtue),

$$m_{\text{Pb}}c_{\text{Pb}}(200°C - T) = (m_{\text{w}} + m_{\text{ice}})c_{w}T + m_{\text{cu}}c_{\text{cu}}T + m_{\text{ice}}L_{\text{f}}.$$

Solving for the final temperature *T* and using numerical values,

$$T = \frac{\begin{pmatrix}(0.750\,\text{kg})(130\,\text{J/kg}\cdot\text{K})(255\,\text{C}°) \\ -(0.018\,\text{kg})(334\times10^{3}\,\text{J/kg})\end{pmatrix}}{\begin{pmatrix}(0.750\,\text{kg})(130\cdot\text{J/kgK}) \\ +(0.178\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}) \\ +(0.100\,\text{kg})(390\,\text{J/kg}\cdot\text{K})\end{pmatrix}} = 21.4°\text{C}.$$

(The fact that a positive Celsius temperature was obtained indicates that all of the ice does indeed melt.)

17.63: The steam both condenses and cools, and the ice melts and heats up along with the original water; the mass of steam needed is

$$m_{\text{steam}} = \frac{(0.450\,\text{kg})(334\times10^3\ \text{J/kg}) + (2.85\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(28.0\,\text{C}°)}{2256\times10^3\ \text{J/kg} + (4190\,\text{J/kg})(72.0\,\text{C}°)}$$

$$= 0.190\,\text{kg}.$$

17.64: The SI units of H *and* $\frac{dQ}{dt}$ are both watts, the units of area are m^2, temperature difference is in K, length in meters, so the SI units for thermal conductivity are

$$\frac{[\text{W}][\text{m}]}{[\text{m}^2][\text{K}]} = \frac{\text{W}}{\text{m}\cdot\text{K}}.$$

17.65: a) $\frac{100\ \text{K}}{0.450\ \text{m}} = 222\,\text{K/m}.$ b) $(385\,\text{W/m}\cdot\text{K})(1.25\times10^{-4}\ \text{m}^2)(400\,\text{K/m}) = 10.7\,\text{W}.$ c) $100.0°\text{C} - (222\,\text{K/m})(12.00\times10^{-2}\ \text{m}) = 73.3°\text{C}.$

17.66: Using the chain rule, $H = \frac{dQ}{dt} = L_{\text{f}}\,\frac{dm}{dt}$ and solving Eq. (17.21) for k,

$$k = L_{\text{f}}\,\frac{dm}{dt}\frac{L}{A\Delta T}$$

$$= (334\times10^3\ \text{J/kg})\frac{(8.50\times10^{-3}\ \text{kg})}{(600\,\text{s})}\frac{(60.0\times10^{-2}\ \text{s})}{(1.250\times10^{-4}\ \text{m}^2)(100\,\text{K})}$$

$$= 227\,\text{W/m}\cdot\text{K}.$$

17.67: (Although it may be easier for some to solve for the heat flow per unit area, part (b), first the method presented here follows the order in the text.) a) See Example 17.13; as in that example, the area may be divided out, and solving for temperature T at the boundary,

$$T = \frac{(k_{\text{foam}}/L_{\text{foam}})T_{\text{in}} + (k_{\text{wood}}/L_{\text{wood}})T_{\text{out}}}{N(k_{\text{foam}}/L_{\text{foam}}) + (k_{\text{wood}}/L_{\text{wood}})}$$

$$\frac{((0.010\ \text{W/m}\cdot\text{K})/(2.2\ \text{cm}))(19.0°\ \text{C}) + ((0.080\ \text{W/m}\cdot\text{K})/(3.0\ \text{cm}))(-10.0°\ \text{C})}{((0.010\ \text{W/m}\cdot\text{K})/(2.2\ \text{cm})) + ((0.080\ \text{W/m}\cdot\text{K})/(3.0\ \text{cm}))}$$

$$= -5.8°\text{C}.$$

Note that the conversion of the thickness to meters was not necessary. b) Keeping extra figures for the result of part, (a), and using that result in the temperature difference across either the wood or the foam gives

$$\frac{H_{\text{foam}}}{A} = \frac{H_{\text{wood}}}{A} = (0.010\ \text{W/m}\cdot K)\frac{(19.0°\ \text{C} - (-5.767°\ \text{C}))}{2.2\times10^{-2}\ \text{m}}$$

$$= (0.080\ \text{W/m}\cdot\text{K})\frac{(-5.767°\text{C} - (-10.0°\text{C}))}{3.0\times10^{-2}\ \text{m}}$$

$$= 11\ \text{W/m}^2.$$

17.68: a) From Eq. (17.21),

$$H = (0.040\ \text{W/m}\cdot\text{K})(1040\ \text{m}^2)\frac{(140\ \text{K})}{(4.0\times10^{-2}\ \text{m})} = 196\ \text{W},$$

or $200\ \text{W}$ to two figures. b) The result of part (a) is the needed power input.

17.69: From Eq. (17.23), the energy that flows in time Δt is

$$H\Delta t = \frac{A\Delta T}{R}\Delta t = \frac{(125\ \text{ft}^2)(34\text{F}°)}{(30\ \text{ft}^2\cdot\text{F}°\cdot\text{h/Btu})}(5.0\ \text{h}) = 708\ \text{Btu} = 7.5\times10^5\ \text{J}.$$

17.70: a) The heat current will be the same in both metals; since the length of the copper rod is known,

$$H = (385.0\ \text{W/m}\cdot\text{K})(400\times10^{-4}\ \text{m}^2)\frac{(35.0\ \text{K})}{(1.00\ \text{m})} = 5.39\ \text{W}.$$

b) The length of the steel rod may be found by using the above value of H in Eq. (17.21) and solving for L_2, or, since H and A are the same for the rods,

$$L_2 = L\frac{k_2}{k}\frac{\Delta T_2}{\Delta T} = (1.00\ \text{m})\frac{(50.2\ \text{W/m}\cdot\text{K})(65.0\ \text{K})}{(385.0\ \text{W/m}\cdot\text{K})(35.0\ \text{K})} = 0.242\ \text{m}.$$

17.71: Using $H = L_v \frac{dm}{dt}$ (see Problem 17.66) in Eq. (17.21),

$$\begin{aligned}\Delta T &= L_v \frac{dm}{dt}\frac{L}{kA}\\ &= (2256\times10^3\ \text{J/kg})\frac{(0.390\ \text{kg})}{(180\ \text{s})}\frac{(0.85\times10^{-2}\ \text{m})}{(50.2\ W/\text{m}\cdot\text{K})(0.150\ \text{m}^2)} = 5.5\ \text{C}^\circ,\end{aligned}$$

and the temperature of the bottom of the pot is $100^\circ\ \text{C} + 6\ \text{C}^\circ = 106^\circ\ \text{C}$.

17.72:

$$\begin{aligned}\frac{\Delta Q}{\Delta t} &= kA\frac{\Delta T}{L}\\ 150\ \text{J/s} &= \left(50.2\frac{\text{W}}{\text{m. K}}\right)A\left(\frac{300\ \text{K}}{0.500\ \text{m}}\right)\\ \text{A} &= 4.98\times10^{-3}\ \text{m}^2\\ A &= \pi R^2 = \pi\left(\frac{\text{D}}{2}\right)^2\\ \text{D} &= \sqrt{4A/\pi}\\ &= \sqrt{4(4.98\times10^{-3}\ \text{m}^2)/\pi}\\ &= 8.0\times10^{-2}\ \text{m} = 8.0\ \text{cm}\end{aligned}$$

17.73: $H_a = H_b$ (a = aluminum, b = brass)

$$H_a = k_a \frac{A(150.0°\text{C} - T)}{L_a}, \quad H_b = k_b \frac{A(T - 0°\text{ C})}{L_b}$$

(It has been assumed that the two sections have the same cross-sectional area.)

$$k_a \frac{A(150.0°\text{C} - T)}{L_a} = k_b \frac{A(T - 0°\text{C})}{L_b}$$

$$\frac{(2050\ \text{W/m} \cdot \text{K})(150.0°\text{C} - T)}{0.800\ \text{m}} = \frac{(109.0\ \text{W/m} \cdot \text{K})(T - 0°\text{C})}{0.500\ \text{m}}$$

Solving for T gives $T = 90.2°\text{C}$

17.74: From Eq. (17.25), with $e = 1$,

a) $(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(273\ \text{K})^4 = 315\ \text{W/m}^2$.

b) A factor of ten increase in temperature results in a factor of 10^4 increase in the output; $3.15 \times 10^6\ \text{W/m}^2$.

17.75: Repeating the calculation with $T_s = 273\ \text{K} + 5.0\ \text{C}° = 278\ \text{K}$ gives $H = 167\ \text{W}$.

17.76: The power input will be equal to H_{net} as given in Eq. (17.26);

$$\begin{aligned} \text{P} &= Ae\sigma(T^4 - T_s^4) \\ &= (4\pi(1.50 \times 10^{-2}\ \text{m})^2)(0.35)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)((3000\ \text{K})^4 - (290\ \text{K})^4) \\ &= 4.54 \times 10^3\ \text{W}. \end{aligned}$$

17.77: $$A = \frac{H}{e\sigma T^4} = \frac{150\text{W}}{(0.35)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(2450\text{K})^4} = 2.10\ \text{cm}^2$$

17.78: The radius is found from

$$R=\sqrt{\frac{A}{4\pi}}=\sqrt{\frac{H/\left(\sigma T^{2}\right)}{4\pi}}=\sqrt{\frac{H}{4\pi\sigma}}\frac{1}{T^{2}}.$$

Using the numerical values, the radius for parts (a) and (b) are

$$R_{\mathrm{a}}=\sqrt{\frac{\left(2.7\times10^{32}\ \mathrm{W}\right)}{4\pi\left(5.67\times10^{-8}\ \mathrm{W/m^{2}\cdot K^{4}}\right)}}\frac{1}{\left(11{,}000\ \mathrm{K}\right)^{2}}=1.61\times10^{11}\ \mathrm{m}$$

$$R_{\mathrm{b}}=\sqrt{\frac{\left(2.10\times10^{23}\ \mathrm{W}\right)}{4\pi\left(5.67\times10^{-8}\ \mathrm{W/m^{2}\cdot K^{4}}\right)}}\frac{1}{\left(10{,}000\ \mathrm{K}\right)^{2}}=5.43\times10^{6}\ \mathrm{m}$$

c) The radius of Procyon B is comparable to that of the earth, and the radius of Rigel is comparable to the earth-sun distance.

17.79: a) normal melting point of mercury: $-30^\circ\,\mathrm{C}=0.0^\circ\mathrm{M}$
normal boiling point of mercury: $357^\circ\,\mathrm{C}=100.0^\circ\mathrm{M}$
$100\,\mathrm{M}^\circ=396\,\mathrm{C}^\circ$ so $1\,\mathrm{M}^\circ=3.96\,\mathrm{C}^\circ$

Zero on the M scale is -39 on the C scale, so to obtain T_{C} multiple T_{M} by 3.96 and then subtract $39^\circ: T_{\mathrm{C}}=3.96T_{\mathrm{M}}-39^\circ$

Solving for T_{M} gives $T_{\mathrm{M}}=\dfrac{1}{3.96}\left(T_{\mathrm{C}}+39^\circ\right)$

The normal boiling point of water is $100^\circ\mathrm{C}; T_{\mathrm{M}}=\dfrac{1}{3.96}\left(100^\circ+39^\circ\right)=35.1^\circ\mathrm{M}$

b) $10.0\,\mathrm{M}^\circ=39.6^\circ\,\mathrm{C}^\circ$

17.80: All linear dimensions of the hoop are increased by the same factor of $\alpha\Delta T,$ so the increase in the radius of the hoop would be

$$R\alpha\Delta T=\left(6.38\times10^{6}\ \mathrm{m}\right)\left(1.2\times10^{-5}\ \mathrm{K}^{-1}\right)\left(0.5\ \mathrm{K}\right)=38\,\mathrm{m}.$$

17.81: The tube is initially at temperature T_0, has sides of length L_0 volume V_0, density ρ_0, and coefficient of volume expansion β.

a) When the temperature increase to $T_0 + \Delta T$, the volume changes by an amount ΔV, where $\Delta V = \beta V_0 \Delta T$. Then, $\rho = \dfrac{m}{V_0 + \Delta V}$, or eliminating $\Delta V, \rho = \dfrac{m}{V_0 + \beta V_0 \Delta T}$.

Divide the top and bottom by V_0 and substitute $\rho_0 = m/V_0$. Then

$\rho = \dfrac{m/V_0}{V_0/V_0 + \beta V_0 \Delta T/V_0}$ or $\rho = \dfrac{\rho_0}{1 + \beta\Delta T}$. This can be rewritten as $\rho = \rho_0 (1 + \beta \Delta T)^{-1}$. Then using the expression $(1 + x)^n \approx 1 + nx$, where $n = -1, \rho = \rho_0 (1 - \beta \Delta T)$.

b) The copper cube has sides of length
$1.25\,\text{cm} = .0125\,\text{m}$, and $\Delta T = 70.0°\,\text{C} - 20.0°\,\text{C} = 50.0°\,\text{C}$.
$\Delta V = \beta V_0 \Delta T = (5.1 \times 10^{-5}/°\,\text{C})(.0125\,\text{m})^3 (50.0°\,\text{C}) = 5 \times 10^{-9}\ \text{m}^3$.

Similarly, $\rho = 8.9 \times 10^3\ \text{kg/m}^3 (1 - (5.1 \times 10^{-5}/°\,\text{C})(50.0°\,\text{C}))$, or $\rho = 8.877 \times 10^3\ \text{kg/m}^3$; extra significant figures have been keep. So $\Delta\rho = -23\,\frac{\text{kg}}{\text{m}^3}$.

17.82: (a) We can use differentials to find the frequency change because all length changes are small percents . Let *m* be the mass of the wire

$$v = \sqrt{F/\mu} = \sqrt{F/(m/L)} = \sqrt{FL/m}$$

$$f = \frac{v}{\lambda} \text{and } \lambda = 2L(\text{fundamental})$$

$$f = v/\lambda = \sqrt{\frac{FL/m}{2L}} = \frac{1}{2}\sqrt{\frac{F}{mL}}$$

$$\Delta f \approx \frac{\partial F}{\partial L}\Delta L \text{ (only } L \text{ changes due to heating)}$$

$$\left|\frac{\Delta f}{f}\right| \approx \left|\frac{\frac{1}{2}(\frac{1}{2})(F/mL)^{-1/2}(-\frac{F}{mL^2})\Delta L}{\frac{1}{2}\sqrt{\frac{F}{mL}}}\right| = \frac{1}{2}\frac{\Delta L}{L}$$

$$\Delta f \approx \frac{1}{2}(\alpha\Delta T)\, f = \frac{1}{2}(1.7\times10^{-5}(\text{C}°)^{-1})(40\text{C}°)(440\text{ C}°)(440\text{ Hz}) = 0.15\text{ Hz}$$

The frequency decreases since the length increases

(b) $v = \sqrt{F/\mu} = \sqrt{FL/m}$

$$\frac{\Delta v}{v} = \frac{\frac{1}{2}(FL/m)^{-1/2}(F/m)\Delta L}{\sqrt{FL/m}} = \frac{\Delta L}{2L} = \frac{\alpha\Delta T}{2}$$

$$= \frac{1}{2}(1.7\times10^{-5}(\text{C}°)^{-1})(40\text{C}°) = 3.4\times10^{-4} = 0.034\%$$

(c) $\lambda = 2L \rightarrow \Delta\lambda = 2\Delta L \rightarrow \frac{\Delta\lambda}{\lambda} = \frac{2\Delta L}{2L} = \frac{\Delta L}{L} = \alpha\Delta T$

$$\frac{\Delta\lambda}{\lambda} = (1.7\times10^{-5°}\text{C}^{-1})(40\text{C}°) = 6.8\times10^{-4} = 0.068\%:$$

it increases

17.83: Both the volume of the cup and the volume of the olive oil increase when the temperature increases, but β is larger for the oil so it expands more. When the oil starts to overflow,

$\Delta V_{\text{oil}} = \Delta V_{\text{glass}} + (1.00\times10^{-3}\text{ m})A$, where A is the cross-sectional area of the cup.

$$\Delta V_{\text{oil}} = V_{0,\text{oil}}\beta_{\text{oil}}\Delta T = (9.9\text{ cm})A\beta_{\text{oil}}\Delta T$$

$$\Delta V_{\text{glass}} = V_{0,\text{ glass}}\beta_{glass}\Delta T = (10.0\text{ cm})A\beta_{\text{glass}}\Delta T$$

$$(9.9\text{ cm})A\beta_{\text{oil}}\Delta T = (10.0\text{ cm})A\beta_{\text{glass}}\Delta T + (1.00\times10^{-3}\text{ m})A$$

The area A divides out. Solving for ΔT gives $\Delta T = 15.5\text{C}°$

$$T_2 = T_1 + \Delta T = 37.5°\text{C}$$

17.84: Volume expansion: $dV = \beta V\ dT$

$$\beta = \frac{dV/dT}{V} = \frac{\text{Slope of graph}}{V}$$

Construct the tangent to the graph at $2°\text{C}$ and $8°\text{C}$ and measure the slope of this line.

$At\ 22°\text{C}: \text{Slope} \approx -\frac{0.10\ \text{cm}^3}{3C°}$ and $V \approx 1000\,\text{cm}^3$

$$\beta \approx -\frac{0.10\,\text{cm}^3/3\text{C}°}{1000\,\text{cm}^3} \approx -3\times10^{-5}(\text{C}°)^{-1}$$

The slope in negative, as the water contracts or it is heated. At

$8°\ \text{C}: \text{slope} \approx \frac{0.24\,\text{cm}^3}{4\text{C}°}$ and $V \approx 1000\,\text{cm}^3$

$$\beta \approx \frac{0.24\,\text{cm}^3/4\text{C}°}{1000\,\text{cm}^3} \approx 6\times10^{-5}(\text{C}°)^{-1}$$

The water now expands when heated.

17.85: $\Delta L_\text{a} + \Delta L_\text{s} = 0.40\,\text{cm}\ \ (\text{a} = \text{aluminum}, \text{s} = \text{steel})$

$$\Delta L = L_0 \alpha\, \Delta T,\ \text{so}$$
$$(24.8\,\text{cm})(2.4\times10^{-5}(C°)^{-1}))\Delta T + (34.8\,\text{cm})(1.2\times10^{-5}(C°)^{-1}))\Delta T = 0.40\,\text{cm}$$
$$\Delta T = 395\ \text{C}°$$
$$T_2 = T_1 + \Delta T = 415°\text{C}$$

17.86: a) The change in height will be the difference between the changes in volume of the liquid and the glass, divided by the area. The liquid is free to expand along the column, but not across the diameter of the tube, so the increase in volume is reflected in the change in the length of the columns of liquid in the stem.

b) $$\Delta h = \frac{\Delta V_\text{liquid} - \Delta V_\text{glass}}{A} = \frac{V}{A}(\beta_\text{liquid} - \beta_\text{glass})\Delta T$$
$$= \frac{(100\times10^{-6}\ \text{m}^3)}{(50.0\times10^{-6}\ \text{m}^2)}(8.00\times10^{-4}\ \text{K}^{-1} - 2.00\times10^{-5}\ \text{K}^{-1})(30.0\ \text{K})$$
$$= 4.68\times10^{-2}\ \text{m}.$$

17.87: To save some intermediate calculation, let the third rod be made of fractions f_1 and f_2 of the original rods; then $f_1 + f_2 = 1$ and $f_1(0.0650)$ $+ f_2(0.0350) = 0.0580.$ These two equations in f_1 and f_2 are solved for

$$f_1 = \frac{0.0580 - 0.0350}{0.0650 - 0.0350}, \quad f_2 = 1 - f_1,$$

and the lengths are $f_1(30.0\,\text{cm}) = 23.0\,\text{cm}$ and $f_2(30.0\,\text{cm}) = 7.00\,\text{cm}$

17.88: a) The lost volume, 2.6 L, is the difference between the expanded volume of the fuel and the tanks, and the maximum temperature difference is

$$\begin{aligned}\Delta T &= \frac{\Delta V}{(\beta_{\text{fuel}} - \beta_{\text{A1}})V_0} \\ &= \frac{(2.6\times10^{-3}\ \text{m}^3)}{(9.5\times10^{-4}(\text{C}°)^{-1} - 7.2\times10^{-5}(\text{C}°)^{-1})(106.0\times10^{-3}\ \text{m}^3)} \\ &= 2.78\,\text{C}°,\end{aligned}$$

or $28°\text{C}$ to two figures; the maximum temperature was $32°\text{C}.$ b) No fuel can spill if the tanks are filled just before takeoff.

17.89: a) The change in length is due to the tension and heating $\frac{\Delta L}{L_0} = \frac{F}{AY} + \alpha\Delta T.$ Solving for F/A, $\frac{F}{A} = Y\left(\frac{\Delta L}{L_0} - \alpha\Delta T\right).$

b) The brass bar is given as "heavy" and the wires are given as "fine," so it may be assumed that the stress in the bar due to the fine wires does not affect the amount by which the bar expands due to the temperature increase. This means that in the equation preceding Eq. (17.12), ΔL is not zero, but is the amount $\alpha_{\text{brass}} L_0 \Delta T$ that the brass expands, and so

$$\begin{aligned}\frac{F}{A} &= Y_{\text{steel}}(\alpha_{\text{brass}} - \alpha_{\text{steel}})\Delta T \\ &= 20\times10^{10}\ \text{Pa})(2.0\times10^{-5}\ (\text{C}°)^{-1} - 1.2\times10^{-5}\ (\text{C}°)^{-1})(120°\text{C}) \\ &= 1.92\times10^{8}\ \text{Pa}.\end{aligned}$$

17.90: In deriving Eq. (17.12), it was assumed that $\Delta L = 0;$ if this is not the case when there are both thermal and tensile stresses, Eq. (17.12) becomes

$$\Delta L = L_0\left(\alpha\Delta T + \frac{F}{AY}\right).$$

For the situation in this problem, there are two length changes which must sum to zero, and so Eq. (17.12) may be extended to two materials *a* and *b* in the form

$$L_{0a}\left(\alpha_a\Delta T + \frac{F}{AY_a}\right) + L_{0b}\left(\alpha_b\Delta T + \frac{F}{AY_b}\right) = 0.$$

Note that in the above, $\Delta T, F$ and A are the same for the two rods. Solving for the stress $F/A,$

$$\frac{F}{A} = -\frac{\alpha_a L_{0a} + \alpha_b L_{0b}}{((L_{oa}/Y_a) + (L_{0b}/Y_b))}\Delta T$$

$$= \frac{(1.2\times10^{-5}\ (\text{C}°)^{-1})(0.350\ \text{m}) + (2.4\times10^{-5}\ (\text{C}°)^{-1})(0.250\ \text{m})}{((0.350\ \text{m})/20\times10^{10}\ \text{Pa}) + (0.250\ \text{m}/7\times10^{10}\ \text{Pa}))}(60.0\ \text{C}°)$$

$$= -1.2\times10^{8}\ \text{Pa}$$

to two figures.

17.91: a) $\Delta T = \frac{\Delta R}{\alpha R_0} = \frac{(0.0020\ \text{in.})}{(1.2\times10^{-5}\ (\text{C}°)^{-1}(2.5000\ \text{in.})} = 67\ \text{C}°$ to two figures, so the ring should be warmed to $87°\text{C}$. b) the difference in the radii was initially 0.0020 in., and this must be the difference between the amounts the radii have shrunk. Taking R_0 to be the same for both rings, the temperature must be lowered by an amount

$$\Delta T = \frac{\Delta R}{(\alpha_{\text{brass}} - \alpha_{\text{steel}})R_0}$$

$$= \frac{(0.0020\ \text{in.})}{(2.0\times10^{-5}(\text{C}°)^{-1} - 1.2\times10^{-5}(\text{C}°)^{-1})(2.50\ \text{in.})} = 100\ \text{C}°$$

to two figures, so the final temperature would be $-80°\text{C}$.

17.92: a) The change in volume due to the temperature increase is $\beta V\Delta T,$ and the change in volume due to the pressure increase is $-\frac{V}{B}\Delta p$ (Eq. (11.13)). Setting the net change equal to zero, $\beta V\Delta T = V\frac{\Delta p}{B}$, or $\Delta p = B\beta\Delta V.$ b) From the above, $\Delta p = (1.6\times10^{11}\ \text{Pa})(3.0\times10^{-5}\ \text{K}^{-1})(15.0\ \text{K}) = 8.64\times10^{7}\ \text{Pa}.$

17.93: As the liquid is compressed, its volume changes by an amount $\Delta V = -\Delta pkV_0$. When cooled, the difference between the decrease in volume of the liquid and the decrease in volume of the metal must be this change in volume, or $(\alpha_1 - \alpha_m)V_0\Delta T = \Delta V$. Setting the expressions for ΔV equal and solving for ΔT gives

$$\Delta T = \frac{\Delta pk}{\alpha_m - \alpha_1} = \frac{(5.065 \times 10^6 \text{ Pa})(8.50 \times 10^{-10} \text{ Pa}^{-1})}{(3.90 \times 10^{-5} \text{ K}^{-1} - 4.8 \times 10^{-4} \text{ K}^{-1})} = -9.76\,\text{C°},$$

so the temperature is $20.2°\text{C}$.

17.94: Equating the heat lost be the soda and mug to the heat gained by the ice and solving for the final temperature $T =$

$$\frac{\begin{pmatrix}((2.00\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}) + (0.257\,\text{kg})(910\,\text{J/kg}\cdot\text{K}))(20.0\text{C°}) \\ -(0.120\,\text{kg})((2100\,\text{J/kg}\cdot\text{K})(15.0\,\text{C°}) + 334 \times 10^3\,\text{J/kg})\end{pmatrix}}{(2.00\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}) + (0.257\,\text{kg})(910\,\text{J/kg}\cdot\text{K}) + (0.120\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})}$$

$= 14.1°\text{C}$. Note that the mass of the ice $(0.120\,\text{kg})$ appears in the denominator of this expression multiplied by the heat capacity of water; after the ice melts, the mass of the melted ice must be raised further to T.

17.95: a) $\dfrac{K}{Q} = \dfrac{(1/2)mv^2}{cm\Delta T} = \dfrac{v^2}{2c\Delta T} = \dfrac{(7700\,\text{m/s})^2}{2(910\,\text{J/kg}\cdot\text{K})(600\text{C°})} = 54.3.$

b) Unless the kinetic energy can be converted into forms other than the increased heat of the satellite cannot return intact.

17.96: a) The capstan is doing work on the rope at a rate

$$P = \tau\omega = \Delta Fr\frac{2\pi}{T} = (520\,\text{N})(5.0 \times 10^{-2}\,\text{m})\frac{2\pi}{(0.90\,\text{s})} = 182\,\text{W},$$

or $180\,\text{W}$ to two figures. The net torque that the rope exerts on the capstan, and hence the net torque that the capstan exerts on the rope, is the difference between the forces of the ends times the radius. A larger number of turns might increase the force, but for given forces, the torque is independent of the number of turns.

b) $$\frac{dT}{dt} = \frac{dQ/dt}{mc} = \frac{P}{mc} = \frac{(182\,\text{W})}{(6.00\,\text{kg})(470\,\text{J/mol}\cdot\text{K})} = 0.064\,\text{C°/s}.$$

17.97: a) Replacing m with nM and nMc with nC,

$$Q = \int dQ = \frac{nk}{\Theta^3}\int_{T_1}^{T_2} T^3\, dt = \frac{nk}{4\Theta^3}(T_2^4 - T_1^4).$$

For the given temperatues,

$$Q = \frac{(1.50\,\text{mol})(1940\,\text{J/mol}\cdot\text{K})}{4(281\,\text{K})^3}((40.0\,\text{K})^4 - (10.0\,\text{K})^4) = 83.6\,\text{J}.$$

b) $\frac{Q}{n\Delta T} = \frac{(83.6\ \text{J})}{(1.50\ \text{mol})\ (30.0\ \text{K})} = 1.86\,\text{J/mol}\cdot\text{K}.$

c) $\text{C} = (1940\,\text{J/mol}\cdot\text{K})\,(40.0\,\text{K}/281\,\text{K})^3 = 5.60\,\text{J/mol}\cdot\text{K}.$

17.98: Setting the decrease in internal energy of the water equal to the final gravitational potential energy, $L_\text{f}\rho_\text{w}V_\text{w} + C_\text{w}\rho_\text{w}V_\text{w}\Delta T = mgh.$ Solving for h, and inserting numbers:

$$\begin{aligned} h &= \frac{\rho_\text{w}V_\text{w}(L_\text{f} + C_\text{w}\Delta T)}{mg} \\ &= \frac{(1000\,\text{kg/m}^3)(1.9\times.8\times.1\,\text{m}^3)\left[334\times10^3\ \text{J/kg} + (4190\,\text{J/kg}\cdot{}^\circ\text{C})(37^\circ\,\text{C})\right]}{(70\,\text{kg})(9.8\,\text{m/s}^2)} \\ &= 1.08\times10^5\ \text{m} = 108\,\text{km}. \end{aligned}$$

17.99: a) $(90)(100\,\text{W})(3000\,\text{s}) = 2.7\times10^7\ \text{J}.$

b) $\Delta T = \dfrac{Q}{cm} = \dfrac{Q}{c\rho V} = \dfrac{2.7\times10^7\ \text{J}}{(1020\ \text{J/kg}\cdot\text{K})(1.20\ \text{kg/m}^3)(3200\ \text{m}^3)} = 6.89\ \text{C}^\circ,$

or $6.9\,\text{C}^\circ$ to the more apropriate two figures. c) The answers to both parts (a) and (b) are multiplied by 2.8, and the temperature rises by $19.3\,\text{C}^\circ$.

17.100: See Problem 17.97. Denoting C by $C = a + bT$, a and b independent of temperature, integration gives.

$$Q = n\left(\text{a}(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2)\right).$$

In this form, the temperatures for the linear part may be expressed in terms of Celsius temperatures, but the quadratic *must* be converted to Kelvin temperatures, $T_1 = 300\,\text{K}$ and $T_2 = 500\,\text{K}$. Insertion of the given values yields

$$\begin{aligned} Q &= (3.00\,\text{mol})(29.5\,\text{J/mol}\cdot\text{K})(500\,\text{K} - 300\,\text{K}) \\ &\quad + (4.10\times10^{-3}\ \text{J/mol}\cdot\text{K}^2)((500\,\text{K})^2 - (300\,\text{K})^2)) \\ &= 1.97\times10^4\ \text{J}. \end{aligned}$$

17.101: a) To heat the ice cube to $0.0°\text{C}$, heat must be lost by the water, which means that some of the water will freeze. The mass of this water is

$$m_{\text{water}} = \frac{m_{\text{ice}} C_{\text{ice}} \Delta T_{\text{ice}}}{L_{\text{f}}} = \frac{(0.075\,\text{kg})(2100\ \text{J/kg}\cdot\text{K})(10.0\,\text{C}°)}{(334\times10^3\ \text{J/kg})} = 4.72\times10^{-3}\ \text{kg} = 4.72\,\text{g}.$$

b) In theory, yes, but it takes 16.7 kg of ice to freeze 1 kg of water, so this is impractical.

17.102: The ratio of the masses is

$$\frac{m_{\text{s}}}{m_{\text{w}}} = \frac{C_{\text{w}} \Delta T_{\text{w}}}{C_{\text{w}} \Delta T_{\text{s}} + L_{\text{v}}} = \frac{(4190\ \text{J/kg}\cdot\text{K})(42.0\,\text{K})}{(4190\ \text{J/kg}\cdot\text{K})(65.0\,\text{K}) + 2256\times10^3\ \text{J/kg}} = 0.0696,$$

so $0.0696\,\text{kg}$ of steam supplies the same heat as $1.00\,\text{kg}$ of water. Note the heat capacity of water is used to find the heat lost by the condensed steam.

17.103: a) The possible final states are steam, water and copper at $100°\text{C}$, water, ice and copper at $0.0°\text{C}$ or water and copper at an intermediate temperature. Assume the last possibility; the final temperature would be

$$T = \frac{\begin{pmatrix}(0.0350\,\text{kg})((4190\ \text{J/kg}\cdot\text{K})(100\,\text{C}°) + 2256\times10^3\ \text{J/kg}) \\ -(0.0950\,\text{kg})(334\times10^3\ \text{J/kg})\end{pmatrix}}{\begin{pmatrix}(0.0350\,\text{kg})(4190\ \text{J/kg}\cdot\text{K}) + (0.446\,\text{kg})(390\ \text{J/kg}\cdot\text{K}) \\ +(0.0950\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})\end{pmatrix}} = 86.1°\text{C}$$

This is indeed a temperature intermediate between the freezing and boiling points, so the reasonable assumption was a valid one. b) There are $0.13\,\text{kg}$ of water.

17.104: a) The three possible final states are ice at a temperature below $0.0°\text{C}$, an ice-water mixture at $0.0°\,\text{C}$ or water at a temperature above $0.0°\text{C}$. To make an educated guess at the final possibility, note that $(0.140\,\text{kg})(2100\ \text{J/kg}\cdot\text{K})(15.0\,\text{C}°) = 4.41\,\text{kJ}$ are needed to heat the ice to $0.0°\text{C}$, and $(0.190\,\text{kg})(4190\ \text{J/kg}\cdot\text{K})(35.0\,\text{C}°) = 27.9\,\text{kJ}$ must removed to cool the water to $0.0°\text{C}$, so the water will not freeze. Melting all of the ice would require an additional $(0.140\,\text{kg})(334\times10^3\ \text{J/kg}) = 46.8\,\text{kJ}$, so some of the ice melts but not all; the final temperature of the system is $0.0°\text{C}$.

Considering the other possibilities would lead to contradictions, as either water at a temperature below freezing or ice at a temperature above freezing.

b) The ice will absorb 27.9 kJ of heat energy to cool the water to $0°\text{C}$. Then, $m = \frac{(27.9\ \text{kJ} - 4.41\ \text{kJ})}{334\times10^3\ \text{J/kg}} = 0.070$ kg will be converted to water. There will be 0.070 kg of ice and 0.260 kg of water.

17.105: a) If all of the steam were to condense, the energy available to heat the water would be $(0.0400\,\text{kg})(2256\times10^3\ \text{J/kg}) = 9.02\times10^4\ \text{J}$. If all of the water were to be heated to $100.0°\text{C}$, the needed heat would be $(0.200\,\text{kg})(4190\ \text{J/kg}\cdot\text{K})(50.0\,\text{C°}) = 4.19\times10^4\ \text{J}$. Thus, the water heats to $100.0°\text{C}$ and some of the steam condenses; the temperature of the final state is $100°\text{C}$.

b) Because the steam has more energy to give up than it takes to raise the water temperature, we can assume that some of the steam is converted to water:

$$m = \frac{4.19\times10^4\ \text{J}}{2256\times10^3\ \text{J/kg}} = 0.019\,\text{kg}.$$

Thus in the final state, there are 0.219 kg of water and 0.021 kg of steam.

17.106: The mass of the steam condensed $0.525\,\text{kg} - 0.490\,\text{kg} = 0.035\,\text{kg}$. The heat lost by the steam as it condenses and cools is

$$(0.035\,\text{kg})L_\text{v} + (0.035\,\text{kg})(4190\ \text{J/kg}\cdot\text{K})(29.0\,\text{K}),$$

and the heat gained by the original water and calorimeter is

$$((0.150\,\text{kg})(420\ \text{J/kg}\cdot\text{K}) + (0.340\,\text{kg})(4190\ \text{J/kg}\cdot\text{K}))(56.0\,\text{K}) = 8.33\times10^4\ \text{J}.$$

Setting the heat lost equal to the heat gained and solving for L_v gives $2.26\times10^6\ \text{J/kg}$, or $2.3\times10^6\ \text{J/kg}$ to two figures (the mass of steam condensed is known to only two figures).

17.107: a) The possible final states are in ice-water mix at $0.0°\text{C}$, a water-steam mix at $100.0°\text{C}$ or water at an intermediate temperature. Due to the large latent heat of vaporization, it is reasonable to make an initial guess that the final state is at $100.0°\text{C}$. To check this, the energy lost by the steam if all of it were to condense would be $(0.0950\,\text{kg})(2256\times10^3\ \text{J/kg}) = 2.14\times10^5\ \text{J}.$ The energy required to melt the ice and heat it to $100°\text{C}$ is $(0.150\,\text{kg})(334\times10^3\ \text{J/kg} + (4190\ \text{J/kg}\cdot\text{K})(100\,\text{C°})) = 1.13\times10^5$ J, and the energy required to heat the origianl water to $100°\text{C}$ is $(0.200\,\text{kg})(4190\ \text{J/kg.K})$ $(50.0\,\text{C°}) = 4.19\times10^4\ \text{J}.$ Thus, some of the steam will condense, and the final state of the system wil be a water-steam mixture at $100.0°\text{C}$.

b) All of the ice is converted to water, so it adds 0.150 kg to the mass of water. Some of the steam condenses giving up1.55×10^3 J of energy to melt the ice and raise the temperature. Thus, $m = \frac{1.55\times10^5\ \text{J}}{2256\times10^3\ \text{J/kg}} = 0.69\,\text{kg}$ and the final mass of steam is 0.026 kg, and of the water, $.150\ \text{kg} + .069\,\text{kg} + .20\,\text{kg} = 0.419\,\text{kg}$.

c) Due to the much larger quantity of ice, a reasonable initial guess is an ice-water mix at $0.0°\,\text{C}.$ The energy required to melt all of the ice would be $(0.350\,\text{kg})\,(334\times10^3\ \text{J/kg}) = \ 1.17\times10^5\ \text{J}$. The maximum energy that could be transferred to the ice would be if all of the steam would condense and cool to $0.0°\text{C}$ and if all of the water would cool to $0.0\,\text{C}$,

$$(0.0120)\,\text{kg}\,(2256\times10^3\ \text{J/kg} + (4190\ \text{J/kg}\cdot\text{K})(100.0\,\text{C°}))$$
$$+ (0.200\,\text{kg})(4190\ \text{J/kg}\cdot\text{K})(40.0\,\text{C°}) = 6.56\times10^4\ \text{J}.$$

This is insufficient to melt all of the ice, so the final state of the system is an ice-water mixture at $0.0°\,\text{C}.$ 6.56×10^4 J of energy goes into melting the ice. So, $m = \frac{6.56\times10^4\ \text{J}}{334\times10^3\ \text{J/kg}}$ $= 0.196\,\text{kg}.$ So there is $0.154\,\text{kg}$ of ice, and $0.012\,\text{kg} + 0.196\,\text{kg} + 0.20\,\text{kg} = 0.408\,\text{kg}$ of water.

17.108: Solving Eq. (17.21) for *k*,

$$k = H\frac{\Delta T}{A\Delta T} = (180\,\text{W})\frac{(3.9\times10^{-2}\ \text{m})}{(2.18\,\text{m}^2)(65.0\,\text{K})} = 5.0\times10^{-2}\ \text{W/m}\cdot\text{K}.$$

17.109: a) $H = kA\dfrac{\Delta T}{L} = (0.120\ \text{J/mol.K})\,(2.00\times 0.95\,\text{m}^2)\left(\dfrac{28.0\,\text{C}°}{5.0\times 10^{-2}\ \text{m} + 1.8\times 10^{-2}\ \text{m}}\right)$

$= 93.9\,\text{W}.$

b) The flow through the wood part of the door is reduced by a factor of $1 - \frac{(0.50)^2}{(2.00\times 0.95)} = 0.868$ to $81.5\,\text{W}$. The heat flow through the glass is

$$H_{\text{glass}} = (0.80\ \text{J/mol}\cdot\text{K})(0.50\,\text{m})^2\left(\frac{28.0\,\text{C}°}{12.45\times 10^{-2}\ \text{m}}\right) = 45.0\,\text{W},$$

and so the ratio is $\frac{81.5+45.0}{93.9} = 1.35.$

17.110: $R_1 = \frac{L}{k_1}, R_2 = \frac{L}{k_2}, H_1 = H_2,$ and so $\Delta T_1 = \frac{H}{A}R_1,\ \Delta T_2 = \frac{H}{A}R_2.$ The temperature difference across the combination is

$$\Delta T = \Delta T_1 + \Delta T_2 = \frac{H}{A}(R_1 + R_2) = \frac{H}{A}R,$$

so, $R = R_1 + R_2.$

17.111: The ratio will be the inverse of the ratio of the total thermal resistance, as given by Eq. (17.24). With two panes of glass with the air trapped in between, compared to the single pane, the ratio of the heat flows is

$$\frac{(2(L_{\text{glass}}/k_{\text{glass}}) + R_0 + (L_{\text{air}}/k_{\text{air}})}{(L_{\text{glass}}/k_{\text{glass})} + R_0},$$

where R_0 is the thermal resistance of the air films. Numerically, the ratio is

$$\frac{\left(2((4.2\times 10^{-3}\ \text{m})/(0.80\ W/\text{m}\cdot\text{K})) + 0.15\,\text{m}^2\cdot\text{K/W} + ((7.0\times 10^{-3}\ \text{m})/(0.024\ \text{W/m}\cdot\text{K}))\right)}{(4.2\times 10^{-3}\ \text{m})/(0.80\ \text{W/m}\cdot\text{K}) + 0.15\,\text{m}^2\cdot\text{K/W}} = 2$$

17.112: Denote the quantites for copper, brass and steel by 1, 2 and 3, respectively, and denote the temperature at the junction by T_0.

a) $H_1 = H_2 + H_3$, and using Eq. (17.21) and dividing by the common area,

$$\frac{k_1}{L_1}(100°\text{C} - T_0) = \frac{k_2}{L_2}T_0 + \frac{k_3}{L_3}T_0.$$

Solving for T_0 gives

$$T_0 = \frac{(k_1/L_1)}{(k_1/L_1) + (k_2/L_2) + (k_3/L_3)}(100°\text{C}).$$

Substitution of numerical values gives $T_0 = 78.4°\text{C}$.

b) Using $H = \frac{kA}{L}\Delta T$ for each rod, with $\Delta T_1 = 21.6\,\text{C}°$, $\Delta T_2 = \Delta T_3 = 78.4°\text{C}$ gives $H_1 = 12.8\,\text{W}$, $H_2 = 9.50\,\text{W}$ and $H_3 = 3.30\,\text{W}$. If higher precision is kept, H_1 is seen to be the sum of H_2 and H_3.

17.113: a) See Figure 17.11. As the temperature approaches $0.0°\text{C}$, the coldest water rises to the top and begins to freeze while the slightly warmer water, which is more dense, will be beneath the surface. b) (As in part (c), a constant temperature difference is assumed.) Let the thickness of the sheet be x, and the amount the ice thickens in time dt be dx. The mass of ice added per unit area is then $\rho_{\text{ice}}dx$, meaning a heat transfer of $\rho_{\text{ice}}L_{\text{f}}\,dx$. This must be the product of the heat flow per unit area times the time, $(H/A)dt = (k\Delta T/x)dt$. Equating these expressions,

$$\rho_{\text{ice}}L_{\text{f}}\,dx = \frac{k\Delta T}{x}dt \quad\text{or}\quad xdx = \frac{k\Delta T}{\rho_{\text{ice}}L_{\text{f}}}dt.$$

This is a separable differential equation; integrating both sides, setting $x = 0$ at $t = 0$, gives

$$x^2 = \frac{2k\Delta T}{\rho_{\text{ice}}L_{\text{f}}}t.$$

The square of the thickness is propotional to the time, so the thickness is propotional to the square root of the time. c) Solving for the time in the above expression,

$$t = \frac{(920\,\text{kg/m}^3)(334\times10^3\ \text{J/kg})}{2(1.6\,\text{J/mol}\cdot\text{K})(10°\text{C})}(0.25\,\text{m})^2 = 6.0\times10^5\ \text{s}.$$

d) Using $x = 40\,\text{m}$ in the above calculation gives $t = 1.5\times10^{10}\ \text{s}$, about 500 y, a very long cold spell.

17.114: Equation(17.21) becomes $H = kA\frac{\partial T}{\partial x}$.

a) $H = (380\,\mathrm{J/kg\cdot K})(2.50\times10^{-4}\,\mathrm{m^2})(140\,\mathrm{C°/m}) = 13.3\,\mathrm{W}$.

b) Denoting the points as 1 and 2, $H_2 - H_1 = \frac{dQ}{dt} = mc\frac{\partial T}{\partial t}$. Solving for $\frac{\partial T}{\partial x}$ at 2,

$$\left.\frac{\partial T}{\partial x}\right|_2 = \left.\frac{\partial T}{\partial x}\right|_1 + \frac{mc}{kA}\frac{\partial T}{\partial t}.$$

The mass m is $\rho A\Delta x$, so the factor multiplying $\frac{\partial T}{\partial t}$ in the above expression is $\frac{c\rho}{k}\Delta x = 137\,\mathrm{s/m}$. Then,

$$\left.\frac{\partial T}{\partial x}\right|_2 = 140\,\mathrm{C°/m} + (137\,\mathrm{s/m})(0.250\,\mathrm{C°/s}) = 174\,\mathrm{C°/m}.$$

17.115: The mass of ice per unit area will be the product of the density and the thickness x, and the energy needed per unit area to melt the ice is product of the mass per unit area and the heat of fusion. The time is then

$$t = \frac{\rho x L_f}{P/A} = \frac{(920\,\mathrm{kg/m^3})(2.50\times10^{-2}\,\mathrm{m})(334\times10^3\,\mathrm{L/kg})}{(0.70)(600\,\mathrm{W/m^2})}$$
$$= 18.3\times10^3\,\mathrm{s} = 305\,\mathrm{min}.$$

17.116: a) Assuimg no substantial energy loss in the region between the earth and the sun, the power per unit area will be inversely proportional to the square of the distance from the center of the sun, and so the energy flux at the surface of the sun is $(1.50\times10^3\,\mathrm{W/m^2})\left(\frac{1.50\times10^{11}\,\mathrm{m}}{6.96\times10^8\,\mathrm{m}}\right)^2 = 6.97\times10^7\,\mathrm{W/m^2}$. b) Solving Eq. (17.25) with $e = 1$,

$$T = \left[\frac{H}{A}\frac{1}{\sigma}\right]^{\frac{1}{4}} = \left[\frac{6.97\times10^7\,\mathrm{W/m^2}}{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}}\right]^{\frac{1}{4}} = 5920\,\mathrm{K}.$$

17.117: The rate at which the helium evaporates is the heat gained from the surroundings by radiation divided by the heat of vaporization. The heat gained from the surroundings come from both the side and the ends of the cylinder, and so the rate at which the mass is lost is

$$\frac{\left(h\pi\, d + 2\pi(d/2)^2\right)\sigma e\left(T_s^4 - T^4\right)}{L_v}$$

$$= \frac{\left(\begin{array}{l}\left((0.250\,\text{m})\pi(0.090\,\text{m}) + 2\pi(0.045\,\text{m})^2(.200)\right. \\ \left.\times\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left((77.3\,\text{K})^4 - (4.22\,\text{K})^4\right)\right)\end{array}\right)}{\left(2.09\times10^4\ \text{J/kg}\right)}$$

$$= 1.62\times10^{-6}\ \text{kg/s},$$

which is $5.82\,\text{g/h}$.

17.118: a) With $\Delta p = 0$,

$$p\Delta V = nR\Delta T = \frac{pV}{T}\Delta T,$$

or

$$\frac{\Delta V}{V} = \frac{\Delta T}{T}, \quad \text{and} \quad \beta = \frac{1}{T}.$$

b) $$\frac{\beta_{\text{air}}}{\beta_{\text{copper}}} = \frac{1}{(293\,\text{K})(5.1\times10^{-5}\ \text{K}^{-1})} = 67.$$

17.119: a) At steady state, the input power all goes into heating the water, so $P = H = \frac{dm}{dt}c\Delta T$ and

$$\Delta T = \frac{P}{c(dm/dt)} = \frac{(1800\,\text{W})}{(4190\ \text{J/kg}\cdot\text{K})\,(0.500\ \text{kg/min})/(60\ \text{s/min})} = 51.6\,\text{K},$$

and the output temperature is $18.0°\text{C} + 51.6°\text{C}$. b) At steady state, the apparatus will neither remove heat from nor add heat to the water.

17.120: a) The heat generated by the hamster is the heat added to the box;

$$P = mc\frac{dT}{dt} = (1.20\ \text{kg/m}^3)(0.0500\ \text{m}^3)(1020\ \text{J/mol}\cdot\text{K})(1.60\ \text{C°/h}) = 97.9\ \text{J/h}.$$

b) Taking the efficiency into account,

$$\frac{M}{t} = \frac{P_0}{L_c} = \frac{P/(10\%)}{L_c} = \frac{979\ \text{J/h}}{24\ \text{J/g}} = 40.8\ \text{g/h}.$$

17.121: For a spherical or cylindrical surface, the area A in Eq. (17.21) is not constant, and the material must be considered to consist of shells with thickness dr and a temperature difference between the inside and outside of the shell dT. The heat current will be a constant, and must be found by integrating a differential equation. a)Equation (17.21) becomes

$$H = k(4\pi r^2)\frac{dT}{dr} \quad \text{or} \quad \frac{H\,dr}{4\pi r^2} = k\,dT.$$

Integrating both sides between the appropriate limits,

$$\frac{H}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right) = k(T_2 - T_1).$$

In this case the "appropriate limits" have been chosen so that if the inner temperature T_2 is at the higher temperature T_1, the heat flows outward; that is, $\frac{dT}{dr} < 0$. Solving for the heat current,

$$H = \frac{k4\pi ab(T_2 - T_1)}{b-a}.$$

b) Of the many ways to find the temperature, the one presented here avoids some intermediate calculations and avoids (or rather sidesteps) the sign ambiguity mentioned above. From the model of heat conduction used, the rate of changed of temperature with radius is of the form $\frac{dT}{dr} = \frac{B}{r^2}$, with B a constant. Integrating from $r = a$ to r and from $r = a$ to $r = b$ gives

$$T(r) - T_2 = B\left(\frac{1}{a}-\frac{1}{r}\right) \text{ and } T_1 - R_2 = B\left(\frac{1}{a}-\frac{1}{b}\right).$$

Using the second of these to eliminate B and solving $T(r)$ (and rearranging to eliminate compound fractions) gives

$$T(r) = T_2 - (T_2 - T_1)\left(\frac{r-a}{b-a}\right)\left(\frac{b}{r}\right).$$

There are, of course, many equivalent forms. As a check, note that at $r = a, T = T_2$ and at $r = b, T = T_1$. c) As in part (a), the expression for the heat current is

$$H = k(2\pi rL)\frac{dT}{dr} \text{ or } \frac{\mathrm{H}}{2\pi r} = kLdT,$$

which integrates, with the same condition on the limits, to

$$\frac{H}{2\pi}\ln(b/a) = kL(T_2 - T_1) \text{ or } \mathrm{H} = \frac{2\pi kL(T_2 - T_1)}{\ln(b/a)}.$$

d) A method similar (but slightly simpler) than that use in part (b) gives

$$T(r) = T_2 + (T_1 - T_2)\frac{\ln(r/a)}{\ln(b/a)}.$$

e) For the sphere: Let $b - a = l$, and approximate $b \sim a$, with a the common radius. Then the surface area of the sphere is $A = 4\pi a^2$, and the expression for H is that of Eq. (17.21) (with l instead of L, which has another use in this problem). For the cylinder: with the same notation, consider

17.122: From the result of Problem 17.121, the heat current through each of the jackets is related to the temperature difference by $H = \frac{2\pi lk}{\ln(b/a)}\Delta T$, where *l* is the length of the cylinder and *b* and *a* are the inner and outer radii of the cylinder. Let the temperature across the cork be ΔT_1 and the temperature across the styrofoam be ΔT_2, with similar notation for the thermal conductivities and heat currents. Then, $\Delta T_1 + \Delta T_2 = \Delta T =$ $125\ \text{C}°$. Setting $H_1 = H_2 = H$ and canceling the common factors,

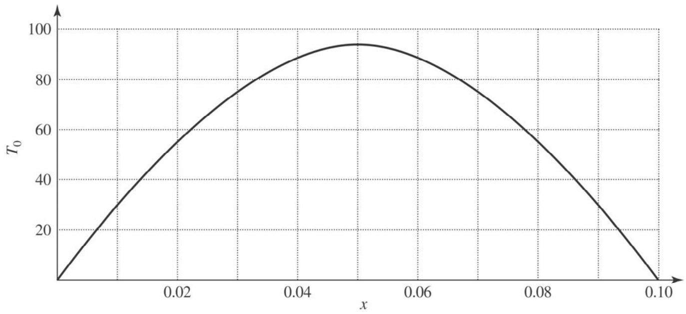
$$\frac{\Delta T_1 k_1}{1\text{n}\,2} = \frac{\Delta T_2 k_2}{1\text{n}\,1.5}$$

Eliminating ΔT_2 and solving for ΔT_1 gives $\Delta T_1 = \Delta T\left(1 + \frac{k_1}{k_2}\frac{\ln 1.5}{\ln 2}\right)^{-1}$.

Substitution of numerical values gives $\Delta T_1 = 37\ \text{C}°$, and the temperature at the radius where the layers meet is $140°\text{C} - 37°\text{C} = 103°\text{C}$. b) Substitution of this value for ΔT_1 into the above expression for $H_1 = H$ gives

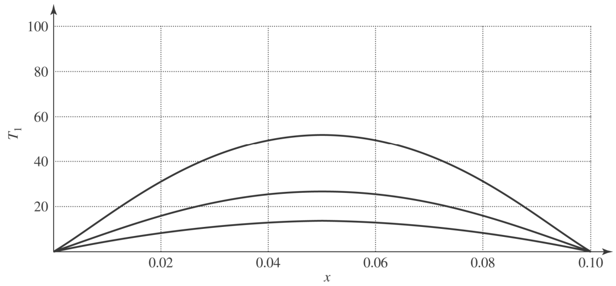
$$H = \frac{2\pi(2.00\ \text{m})(0.04\ \text{J/mol}\cdot\text{K})}{\ln 2}(37\ \text{C}°) = 27\ \text{W}.$$

17.123: a)



b) After a very long time, no heat will flow, and the entire rod will be at a uniform temperature which must be that of the ends, $0°\text{C}$.

c)



d) $\frac{\partial T}{\partial x} = (100°\text{C})(\pi/L)\cos \pi x/L.$ At the ends, $x = 0$ and $x = L,$ the cosine is ± 1 and the temperature gradient is $\pm(100°\text{C})(\pi/0.100\ m) = \pm 3.14\times 10^3\ \text{C}°/\text{m}.$ e) Taking the phrase "into the rod" to mean an absolute value, the heat current will be $kA\frac{\partial T}{\partial x} =$ $(385.0\ W/\text{m}\cdot\text{K})\,(1.00\times 10^{-4}\ \text{m}^2)(3.14\times 10^3\ \text{C}°/\text{m}) = 121\,\text{W}.$ f) Either by evaluating $\frac{\partial T}{\partial x}$ at the center of the rod, where $\pi x/L = \pi/2$ and $\cos(\pi/2) = 0,$ or by checking the figure in part (a), the temperature gradient is zero, and no heat flows through the center; this is consistent with the symmetry of the situation. There will not be any heat current at the center of the rod at any later time. g) See Problem 17.114;

$$\frac{k}{\rho c} = \frac{(385\ \text{W/m}\cdot\text{K})}{(8.9\times 10^3\ \text{kg/m}^3)(390\ \text{J/kg}\cdot\text{K})} = 1.1\times 10^{-4}\ \text{m}^2/\text{s}.$$

h) Although there is no net heat current, the temperature of the center of the rod is decreasing; by considering the heat current at points just to either side of the center, where there is a non-zero temperature gradient, there must be a net flow of heat out of the region around the center. Specifically,

$$\begin{aligned} H((L/2)+\Delta x) - H((L/2)-\Delta x) &= \rho A \Delta x c \frac{\partial T}{\partial t} \\ &= kA\left(\left.\frac{\partial T}{\partial x}\right|_{(L/2)+\Delta x} - \left.\frac{\partial T}{\partial x}\right|_{(L/2)-\Delta x}\right) \\ &= kA\frac{\partial^2 t}{\partial x^2}\Delta x, \end{aligned}$$

17.124: a) In hot weather, the moment of inertia I and the length d in Eq. (13.39) will both increase by the same factor, and so the period will be longer and the clock will run slow (lose time). Similarly, the clock will run fast (gain time) in cold weather. (An ideal pendulum is a special case of physical pendulum.) b) $\frac{\Delta L}{L_0} = \alpha \Delta T = (1.2\times10^{-5}(\text{C}°)^{-1} \times (10.0\,\text{C}°) = 1.2\times10^{-4}$. c) See Problem 13.97; to avoid possible confusion, denote the pendulum period by τ. For this problem, $\frac{\Delta\tau}{\tau} = \frac{1}{2}\frac{\Delta L}{L} = 6.0\times10^{-5}$, so in one day the clock will gain $(86{,}400\,\text{s})(6.0\times10^{-5}) = 5.2\,\text{s}$ so two figures. d) $\left|\frac{\Delta\tau}{\tau}\right| = (1/2)\alpha\Delta T < (86{,}400)^{-1}$, so $\Delta T < 2((1.2\times10^{-5}\,(\text{C}°)^{-1})\times(86{,}400))^{-1} = 1.93\,\text{C}°$.

17.125: The rate at which heat is aborbed at the blackened end is the heat current in the rod,

$$A\sigma(T_S^4 - T_2^4) = \frac{kA}{L}(T_2 - T_1),$$

where $T_1 = 20.00\,\text{K}$ and T_2 is the temperature of the blackened end of the rod. If this were to be solved exactly, the equation would be a quartic, very likely not worth the trouble. Following the hint, approximate T_2 on the left side of the above expression as T_1 to obtain

$$T_2 = T_1 + \frac{\sigma L}{k}(T_s^2 - T_1^4) = T_1 + (6.79\times10^{-12}\,\text{K}^{-3})(T_s^4 - T_1^4) = T_1 + 0.424\,\text{K}.$$

This approximation for T_2 is indeed only slightly than $T_{1,}$ and is a good estimate of the temperature. Using this for T_2 in the original expression to find a better value of ΔT gives the same ΔT to eight figures, and further, and further iterations are not worth – while.

A numerical program used to find roots of the quartic equation returns a value for ΔT that differed from that found above in the eighth place; this, of course, is more precision than is warranted in this problem.

17.126: a) The rates are:

(i) $280\ \text{W}$,

(ii) $(54\ \text{J/h}\cdot\text{C}°\cdot\text{m}^2)(1.5\ \text{m}^2)(11\text{C}°)/(3600\ \text{s/h}) = 0.248\ \text{W}$,

(iii) $(1400\ \text{W/m}^2)(1.5\ \text{m}^2) = 2.10\times10^3\ \text{W}$,

(iv) $(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.5\ \text{m}^2)(320\ \text{K})^4 - (309\ \text{K}^4) = 116\ \text{W}$.

The total is $2.50\ \text{kW}$, with the largest portion due to radiation from the sun.

b) $\dfrac{P}{\rho L_\text{v}} = \dfrac{2.50\times10^3\ \text{W}}{(1000\ \text{kg/m}^3)(2.42\times10^6\ \text{J/kg}\cdot\text{K})} = 1.03\times10^{-6}\ \text{m}^3/\text{s} = 3.72\ \text{L/h}$.

c) Redoing the above calculations with $e = 0$ and the decreased area gives a power of 945 W and a corresponding evaporation rate of $1.4\ \text{L/h}$. Wearing reflective clothing helps a good deal. Large areas of loose weave clothing also facilitate evaporation.

In doing the numerical calculations for the exercises and problems for this chapter, the values of the ideal-gas constant have been used with the precision given on page 501 of the text,

$$R = 8.3145\,\mathrm{J/mol\cdot K} = 0.08206\,\mathrm{L\cdot atm/mol\cdot K}.$$

Use of values of these constants with either greater or less precision may introduce differences in the third figures of some answers.

18.1: a) $n = m_{\mathrm{tot}}/M = (0.225\,\mathrm{kg})/(400\times10^{-3}\,\mathrm{kg/mol}) = 56.3\,\mathrm{mol}$. b) Of the many ways to find the pressure, Eq. (18.3) gives

$$p = \frac{nRT}{V} = \frac{(56.3\,\mathrm{mol})(0.08206\,\mathrm{L\cdot atm/mol\cdot K})(291.15\,\mathrm{K})}{(20.0\,\mathrm{L})}$$
$$= 67.2\,\mathrm{atm} = 6.81\times10^{6}\,\mathrm{Pa}.$$

18.2: a) The final temperature is four times the initial Kelvin temperature, or 4(314.15 K) $-273.15 = 983°\mathrm{C}$ to the nearest degree.

b) $$m_{\mathrm{tot}} = nM = \frac{MpV}{RT} = \frac{(4.00\times10^{-3}\,\mathrm{kg/mol})(1.30\,\mathrm{atm})(2.60\,\mathrm{L})}{(0.08206\,\mathrm{L\cdot atm/mol\cdot K})(314.15\,\mathrm{K})} = 5.24\times10^{-4}\,\mathrm{kg}.$$

18.3: For constant temperature, Eq. (18.6) becomes

$$p_2 = p_1(V_1/V_2) = (3.40\,\mathrm{atm})(0.110/0.390) = 0.96\,\mathrm{atm}.$$

18.4: a) Decreasing the pressure by a factor of one-third decreases the Kelvin temperature by a factor of one-third, so the new Celsius temperatures is 1/3(293.15 K) – $273.15 = -175°\mathrm{C}$ rounded to the nearest degree. b) The net effect of the two changes is to keep the pressure the same while decreasing the Kelvin temperature by a factor of one-third, resulting in a decrease in volume by a factor of one-third, to 1.00 L.

18.5: Assume a room size of 20 ft X 20 ft X 20 ft

$V = 4000\,\text{ft}^3 = 113\,\text{m}^3$. Assume a temperature of $20°\text{C}$.

$$pV = nRT \text{ so } n = \frac{pV}{RT} = \frac{(1.01\times10^5\ \text{Pa})(113\,\text{m}^3)}{(8.315\,\text{J/mol}\cdot\text{K})(293\,\text{K})} = 4685\,\text{mol}$$

$$N = nN_A = 2.8\times10^{27}\ \text{molecules}$$

b) $$\frac{N}{V} = \frac{2.8\times10^{27}\ \text{molecules}}{113\times10^6\ \text{cm}^3} = 2.5\times10^{19}\ \text{molecules/cm}^3$$

18.6: The temperature is $T = 22.0°\text{C} = 295.15\text{K}$. (a) The average molar mass of air is $M = 28.8\times10^{-3}\ \text{kg/mol}$, so

$$m_{\text{tot}} = nM = \frac{pV}{RT}M = \frac{(1.00\,\text{atm})(0.900\,\text{L})(28.8\times10^{-3}\ \text{kg/mol})}{(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(295.15\,\text{K})} = 1.07\times10^{-3}\ \text{kg}.$$

(b) For Helium $M = 4.00\times10^{-3}\,\text{kg/mol}$, so

$$m_{\text{tot}} = nM = \frac{pV}{RT}M = \frac{(1.00\,\text{atm})(0.900\,\text{L})(4.00\times10^{-3}\ \text{kg/mol})}{(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(295.15\,\text{K})} = 1.49\times10^{-4}\ \text{kg}.$$

18.7: From Eq. (18.6),

$$T_2 = T_1\left(\frac{p_2V_2}{p_1V_1}\right) = (300.15\,\text{K})\left(\frac{(2.821\times10^6\ \text{Pa})(46.2\,\text{cm}^3)}{(1.01\times10^5\ \text{Pa})(499\,\text{cm}^3)}\right) = 776\,\text{K} = 503°\text{C}.$$

18.8: a) $$m_{\text{tot}} = \frac{MpV}{RT} = \frac{(32.0\times10^{-3}\,\text{kg/mol})(4.013\times10^5\ \text{Pa})(0.0750\,\text{m}^3)}{(8.3145\,\text{J/mol}\cdot\text{K})(310.15\,\text{K})} = 0.373\,\text{kg}.$$

b) Using the final pressure of 2.813×10^5 Pa and temperature of $295.15\,\text{K}$, $m' = 0.275\,\text{kg}$,
so the mass lost is $0.098\,\text{kg}$ where extra figures were kept in the intermediate calculation of m_{tot}.

18.9: From Eq. (18.6),

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) = (1.50\times10^5\ \text{Pa})\left(\frac{430.15\,\text{K}}{300.15\,\text{K}}\right)\left(\frac{0.750\,\text{m}^3}{0.48\,\text{m}^3}\right) = 3.36\times10^5\ \text{Pa}.$$

18.10: a) $$n=\frac{pV}{RT}=\frac{(1.00\,\text{atm})(140\times10^{3}\text{ L})}{(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K})\,(295.15\,\text{K})}=5.78\times10^{3}\text{ mol.}$$

b) $(32.0\times10^{-3}\text{ kg/mol})(5.78\times10^{3}\text{ mol})=185\,\text{kg.}$

18.11: $V_2=V_1(T_2/T_1)=(0.600\,\text{L})(77.3/292.15)=0.159\,\text{L.}$

18.12: a) $nRT/V=7.28\times10^{6}$ Pa while Eq. (18.7) gives 5.87×10^{6} Pa. b) The van der Waals equation, which accounts for the attraction between molecules, gives a pressure that is 20% lower.
c) 7.28×10^{5} Pa, 7.13×10^{5} Pa, 2.1%. d) As n/V decreases, the formulas and the numerical values are the same.

18.13: At constant temperature, $p_2=p_1(V_1/V_2)=(1.0\,\text{atm})(6.0/5.7)=1.1\,\text{atm.}$

18.14: a) $\frac{V_2}{V_1}=\frac{p_1}{p_2}\frac{T_2}{T_1}=(3.50)(\frac{296\text{K}}{277\text{K}})=3.74.$ b) Lungs cannot withstand such a volume change; breathing is a good idea.

18.15: a) $$T_2=\frac{p_2V}{nR}=\frac{(100\,\text{atm})(3.10\,\text{L})}{(11.0\,\text{mol})(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K})}=343\,\text{K}=70.3°\text{C}.$$

b) This is a very small temperature increase and the thermal expansion of the tank may be neglected; in this case, neglecting the expansion means not including expansion in finding the highest safe temperature, and including the expansion would tend to relax safe standards.

18.16: (a) The force of any side of the cube is $F = pA = (nRT/V)A = (nRT)/L,$ since the ratio of area to volume is $A/V = 1/L.$ For $T = 20.0°\,\text{C} = 293.15\text{K}.$

$$F = \frac{nRT}{L} = \frac{(3\,\text{mol})\,(8.3145\,\text{J/mol}\cdot\text{K})\,(293.15\,\text{K})}{0.200\,\text{m}} = 3.66\times10^{4}\ \text{N}.$$

b) For $T = 100.00°\text{C} = 373.15\,\text{K},$

$$F = \frac{nRT}{L} = \frac{(3\,\text{mol})(8.3145\,\text{J/mol}\cdot\text{K})(373.15\,\text{K})}{0.200\,\text{m}} = 4.65\times10^{4}\ \text{N}.$$

18.17: Example 18.4 assumes a temperature of $0°\,\text{C}$ at all altitudes and neglects the variation of g with elevation.

With these approximations, $p = p_0 e^{-Mgy/\text{RT}}$

We want y for $p = 0.90p_0$ so $0.90 = e^{-Mgy/\text{RT}}$ and

$$y = -\frac{RT}{Mg}\ln(0.90) = 850\ \text{m}$$

(We have used $M = 28.8\times10^{-3}$ kg/mol for air.)

18.18: From example 18.4, the pressure at elevation y above sea level is $p = p_0 e^{-Mgy/\text{RT}}$. The average molar mass of air is $M = 28.8\times10^{-3}$ kg/mol, so at an altitude of 100 m,

$$\frac{Mgy_1}{RT} = \frac{(28.8\times10^{-3}\ \text{kg/mol})(9.80\ \text{m/s}^2)(100\,\text{m})}{(8.3145\,\text{J/mol}\cdot\text{K})(273.15\,\text{K})} = 0.01243,$$

and the percent decrease in pressure is $1 - p/p_0 = 1 - e^{-0.01243} = 0.0124 = 1.24\%.$ At an altitude of $1000\,\text{m},$ $Mgy_2/RT = 0.1243,$ and the percent decrease in pressure is $1 - e^{-0.1243} = 0.117 = 11.7\%.$ These answers differ by a factor of $11.7\%/1.24\% = 9.44,$ which is less than 10 because the variation of pressure with altitude is exponential rather than linear.

18.19: $p = p_0 e^{-Myg/RT}$ from Example 18.4.

Eq. (18.5) says $p = (\rho/M)RT$. Example 18.4 assumes a constant $T = 273\,\text{K}$, so p and ρ are directly proportional and we can write

$$\rho = \rho_0 e^{-Mgy/RT}$$

For $y = 100\,\text{m}$, $\dfrac{Mgy}{RT} = 0.0124$, so $\rho = \rho_0 e^{-0.0124} = 0.988\rho_0$

The density at sea level is 1.2% larger than the density at $100\,\text{m}$.

18.20: Repeating the calculation of Example 18.4 (and using the same numerical values for R and the temperature gives) $p = (0.537)\,p_{\text{atm}} = 5.44\times10^4\ \text{Pa}$.

18.21:

$$p = \rho RT/M = \left(0.364\,\text{kg/m}^3\right)\left(8.3145\,\text{J/mol}\cdot\text{K}\right)\left(273.15\,\text{K} - 56.5\,\text{K}\right)/(28.8\times10^{-3}\ \text{kg/mol})$$
$$= 2.28\times10^4\ \text{Pa}.$$

18.22:

$$M = N_{\text{A}} m = (6.02\times10^{23}\ \text{molecules/mol})(1.41\times10^{-21}\ \text{kg/molecule}) = 849\,\text{kg/mol}.$$

18.23: Find the mass: $m = nM = (3.00\,\text{mol})(63.546\times10^{-3}\ \text{kg/mol}) = 0.1906\,\text{kg}$

$$V = \frac{m}{\rho} = \frac{0.1906\,\text{kg}}{8.9\times10^3\,\text{kg/m}^3} = 2.14\times10^{-5}\,\text{m}^3 = 21.4\,\text{cm}^3$$

18.24: $$N = nN_A = \frac{pV}{RT} N_A$$

$$= \frac{(9.119\times10^{-9}\ \text{Pa})(1.00\times10^{-6}\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(300\,\text{K})}(6.023\times10^{23}\ \text{molecules/mol})$$
$$= 2.20\times10^6\ \text{molecules}.$$

18.25: a)

$$p = \frac{nRT}{V} = \frac{N}{V}\frac{RT}{N_\text{a}} = \left(80\times10^{3}\ \frac{\text{molecules}}{\text{L}}\right)\frac{(0.08206\,\text{L}\cdot\text{atm}/\text{mol}\cdot\text{K})(7500\,\text{K})}{(6.023\times10^{23}\ \text{molecules}/\text{mol})}$$

$$= 8.2\times10^{-17}\ \text{atm},$$

about 8.2×10^{-12} Pa. This is much lower, by a factor of a thousand, than the pressures considered in Exercise 18.24. b) Variations in pressure of this size are not likely to affect the motion of a starship.

18.26: Since this gas is at standard conditions, the volume will be $V = (22.4\times10^{-3}\ \text{m}^3)\ \frac{N}{N_A} = 2.23\times10^{-16}\ \text{m}^3$, and the length of a side of a cube of this volume is $(2.23\times10^{-16}\ \text{m}^3)^{\frac{1}{3}} = 6.1\times10^{-6}$ m.

18.27: $\frac{1000\,\text{g}}{18.0\ \text{g}/\text{mol}} = 55.6\,\text{mol}$, which is $(55.6\,\text{mol})(6.023\times10^{23}\,\text{molecules}/\text{mol}) = 3.35\times10^{25}$ molecules.

18.28: a) The volume per molecule is

$$\frac{V}{N} = \frac{nRT/p}{nN_A} = \frac{RT}{N_{Ap}}$$
$$= \frac{(8.3145\ \text{J}/\text{mol}\cdot\text{K})(300.15\,\text{K})}{(6.023\times10^{23}\,\text{molecules}/\text{mol})(1.013\times10^{5}\ \text{Pa})}$$
$$= 4.091\times10^{-26}\text{m}^3.$$

If this volume were a cube of side *L*,

$$L = \left(4.091\times10^{-26}\ \text{m}^3\right)^{\frac{1}{3}} = 3.45\times10^{-9}\ \text{m},$$

which is (b) a bit more than ten times the size of a molecule.

18.29:
$a) V = m/\rho = nM/\rho = (5.00\,\text{mol})(18.0\text{g/mol})/(1.00\,\text{g/cm}^3) = 90.0\,\text{cm}^3 = 9.00\times10^{-5}\,\text{m}^3.$
b) See Excercise 18.28;

$$\left(\frac{V}{N}\right)^{1/3} = \left(\frac{V}{nN_\text{A}}\right)^{1/3} = \left(\frac{9.00\times10^{-5}\text{m}^3}{(5.00\,\text{mol})(6.023\times10^{23}\text{molecules/mol})}\right)^{1/3}$$
$$= 3.10\times10^{-10}\text{ m}.$$

c) This is comparable to the size of a water molecule.

18.30: a) From Eq. (18.16), the average kinetic energy depends only on the temperature, not on the mass of individual molecules, so the average kinetic energy is the same for the molecules of each element. b) Equation (18.19) also shows that the rms speed is proportional to the inverse square root of the mass, and so

$$\frac{v_{\text{rms Kr}}}{v_{\text{rms Ne}}} = \sqrt{\frac{20.18}{83.80}} = 0.491, \frac{v_{\text{rms Rn}}}{v_{\text{rms Ne}}} = \sqrt{\frac{20.18}{222.00}} = 0.301 \quad \text{and}$$
$$\frac{v_{\text{rms Rn}}}{v_{\text{rms Kr}}} = \sqrt{\frac{83.80}{222.00}} = 0.614.$$

18.31: a) At the same temperature, the average speeds will be different for the different isotopes; a stream of such isotopes would tend to separate into two groups.
b) $\sqrt{\frac{0.352}{0.349}} = 1.004.$

18.32: (Many calculators have statistics functions that are preprogrammed for such calculations as part of a statistics application. The results presented here were done on such a calculator.) a) With the multiplicity of each score denoted by n_1, the average is $\left(\frac{1}{150}\right)\sum n_i x_i = 54.6$ b) $\left[\left(\frac{1}{150}\right)\sum n_i x_i^2\right]^{1/2} = 61.1.$ (Extra significant figures are warranted because the sums are known to higher precision.)

18.33: We known that $V_A = V_B$ and that $T_A > T_B$.

a) $p = nRT/V;$ we don't know n for each box, so either pressure could be higher.

b) $pV = \left(\frac{N}{N_A}\right)RT$ so $N = \frac{pVN_A}{RT}$, where N_A is Avogadro's number. We don't know how the pressures compare, so either N could be larger.

c) $pV = (m/M)RT.$ We don't know the mass of the gas in each box, so they could contain the same gas or different gases.

d) $\frac{1}{2}m\left(v^2\right)_{\text{av}} = \frac{3}{2}kT$

$T_A > T_B$ and the average kinetic energy per molecule depends only on T, so the statement **must** be true.

e) $v_{rms} = \sqrt{3kT/m}$

We don't know anything about the masses of the atoms of the gas in each box, so either set of molecules could have a larger v_{rms}.

18.34: Box *A* has higher pressure than *B*. This could be due to higher temperature and/or higher particle density in *A*. Since we know nothing more about these gases, none of the choices is *necessarily* true, although each of them *could* be true.

18.35: *a*) $m = m_{\text{P}} + m_{\text{n}} = 3.348 \times 10^{-27}$ kg; $T = 300 \times 10^6$ K

$v_{\text{rms}} = \sqrt{3kT/m} = 1.9 \times 10^6$ m/s; $v_{\text{rms}}/c = 0.64\%$

b) $T = mv_{\text{rms}}^2/3k$

For $v_{\text{mms}} = 3.0 \times 10^7\ \text{m/s}, T = 7.3 \times 10^{10}$ K

18.36: From $pV = nRT,$ the temperature increases by a factor of 4 if the pressure and volume are each doubled. Then the rms speed $v_{\text{rms}} = \sqrt{3RT/M}$ increases by a factor of $\sqrt{4} = 2,$ so the final rms speed is $2(250\ \text{m/s}) = 500\,\text{m/s}.$

18.37: a) $\frac{3}{2}kT = (3/2)(1.381\times10^{-23}\ \text{J/K})(300\,\text{K}) = 6.21\times10^{-21}\ \text{J}.$

b) $$\frac{2K_{\text{ave}}}{m} = \frac{2(6.21\times10^{-21}\ \text{J})}{(32.0\times10^{-3}\ \text{kg/mol})/(6.023\times10^{23}\ \text{molecules/mol})} = 2.34\times10^{5}\ \text{m}^2/s^2.$$

c) $$v\text{s} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.3145\ \text{J/mol}\cdot\text{K})(300\,\text{K})}{(32.0\times10^{-3}\ \text{kg/mol})}} = 4.84\times10^{2}\ \text{m/s},$$

which is of course the square root of the result of part (b).

d) $$mv\text{s} = \left(\frac{M}{N_A}\right)v\text{s} = \frac{(32.0\times10^{-3}\ \text{kg/mol})}{(6.023\times10^{23}\,\text{molecules/mol})}(4.84\times10^{2}\ \text{m/s})$$
$$= 2.57\times10^{-23}\ \text{kg}\cdot\text{m}$$

This may also be obtained from

$$\sqrt{2mK_{\text{ave}}} = \sqrt{\frac{2(6.21\times10^{-21}\ \text{J})(32.0\times10^{-3}\ \text{kg/mol})}{(6.023\times10^{23}\ \text{molecules/mol})}}$$

e) The average force is the change in momentum of the atom, divided by the time between collisions. The magnitude of the momentum change is twice the result of part (d) (assuming an elastic collision), and the time between collisions is twice the length of a side of the cube, divided by the speed. Numerically,

$$F_{\text{ave}} = \frac{2mv\text{s}}{2L/v\text{s}} = \frac{mv\text{s}^2}{L} = \frac{2K\text{s}}{L} = \frac{2(6.21\times10^{-21}\ \text{J})}{(0.100\,\text{m})} = 1.24\times10^{-19}\ \text{N}.$$

f) $$p_{\text{ave}} = F_{\text{ave}}/L^2 = 1.24\times10^{-17}\ \text{Pa}.$$

g) $$P/P_{\text{ave}} = \left(1.013\times10^{5}\ \text{Pa}\right)/\left(1.24\times10^{-17}\ \text{Pa}\right) = 8.15\times10^{21}\ \text{molecules}.$$

h) $$N = nN_{\text{A}} = \frac{pV}{RT}N_{\text{A}}$$
$$= \left(\frac{(1.00\,\text{atm})(1.00\,\text{L})}{(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(300\,\text{K})}\right)\left(6.023\times10^{23}\ \text{molecules/mol}\right)$$
$$= 2.45\times10^{22}.$$

i) The result of part (g) was obtained by assuming that all of the molecules move in the same direction, and that there was a force on only two of the sides of the cube.

18.38: This is the same calculation done in Example 16-9, but with $p = 3.50 \times 10^{-13}$ atm, giving $\lambda = 1.6 \times 10^5$ m.

18.39: The rms speeds will be the same if the Kelvin temperature is proportional to the molecular mass; $T_{N_2} = T_{H_2}(M_{N_2}/M_{H_2}) = (293.15\ \text{K})(28.0/2.02)$
$= 4.06 \times 10^3\ \text{K} = 3.79 \times 10^3\ °\text{C}$.

18.40: a) $\sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3(1.381 \times 10^{-23}\ \text{J/K})(300\ \text{K})}{(3.00 \times 10^{-16}\ \text{kg})}} = 6.44 \times 10^{-3}\ \text{m/s}$. b) If the particle is in thermal equilibrium with its surroundings, its motion will depend only on the surrounding temperature, not the mass of the individual particles.

18.41: a) The six degress of freedom would mean a heat capacity at constant volume of $6\left(\frac{1}{2}\right)R = 3R = 24.9\ \text{J/mol} \cdot \text{K}$. $\frac{3R}{M} = \frac{3(8.3145\ \text{J/mol}\cdot\text{K})}{(18.0 \times 10^{-3}\ \text{kg/mol})} = 1.39 \times 10^3\ \text{J/kg} \cdot \text{K}$, b) vibrations do contribute to the heat capacity.

18.42: a) $C_v = (C)$ (molar mass), so $(833\ \text{J/kg} \cdot °\text{C})(0.018\ \text{kg/mol}) = 15.0\ \text{J/mol} \cdot °\text{C}$ at $-180°\text{C}$, $(1640\ \text{J/kg} \cdot °\text{C})(0.018\ \text{kg/mol}) = 29.5\ \text{J/kg} \cdot °\text{C}$ at $-60°\text{C}$, $(2060\ \text{J/kg} \cdot °\text{C}) \times (0.018\ \text{kg/mol}) = 37.1\ \text{J/mol} \cdot °\text{C}$ at $-5.0°\text{C}$. b) Vibrational degrees of freedom become more important. c) C_V exceeds $3R$ because H_2O also has rotational degrees of freedom.

18.43: a) Using Eq. (18.26), $Q = (2.50\ \text{mol})(20.79\ \text{J/mol} \cdot \text{K})(30.0\ \text{K}) = 1.56\ \text{kJ}$.
b) From Eq. (18.25), $\frac{3}{5}$ of the result of part (a), 936 J.

18.44: a) $$c = \frac{C_V}{M} = \frac{20.76\ \text{J/mol}\cdot\text{K}}{28.0\times10^{-3}\ \text{kg/mol}} = 741\ \text{J/kg}\cdot\text{K},$$

which is $\frac{741}{4190} = 0.177$ times the specific heat capacity of water.

b) $m_{\text{N}}C_{\text{N}}\Delta T_{\text{N}} = m_{\text{w}}C_{\text{w}}\Delta T_{\text{w}}$, or $m_{\text{N}} = \frac{m_{\text{w}}C_{\text{w}}}{C_{\text{N}}}$. Inserting the given data and the result from part (a) gives $m_{\text{N}} = 5.65$ kg. To find and volume, use $pV = nRT$, or $V = \frac{nRT}{p} =$

$$\frac{\left[(5.65\ \text{kg})/(0.028\ \text{kg/mol})\right](0.08206\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(293\ \text{K})}{1\ \text{atm}} = 4855\ \text{L}.$$

18.45: From Table (18.2), the speed is $(1.60)v$ s, and so

$$v\,\text{s}^2 = \frac{3kT}{m} = \frac{3RT}{M} = \frac{v^2}{(1.60)^2}$$

(see Exercise 18.48), and so the temperature is

$$T = \frac{Mv^2}{3(1.60)^2R} = \frac{(28.0\times10^{-3}\ \text{kg/mol})}{3(1.60)^2(8.3145\ \text{J/mol}\cdot\text{K})}v^2 = (4.385\times10^{-4}\ \text{K}\cdot\text{s}^2/\text{m}^2)v^2.$$

a) $(4.385\times10^{-4}\ \text{K}\cdot\text{s}^2/\text{m}^2)(1500\ \text{m/s})^2 = 987\,\text{K}$

b) $(4.385\times10^{-4}\ \text{K}\cdot\text{s}^2/\text{m}^2)(1000\ \text{m/s})^2 = 438\,\text{K}$

c) $(4.385\times10^{-4}\ \text{K}\cdot\text{s}^2/\text{m}^2)(500\ \text{m/s})^2 = 110\,\text{K}.$

18.46: Making the given substitution $\varepsilon = \frac{1}{2}mv^2$,

$$f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{2\varepsilon}{m}e^{-\varepsilon/kT} = \frac{8\pi}{m}\left(\frac{m}{2\pi kT}\right)^{3/2}\varepsilon e^{-\varepsilon/kT}.$$

18.47: Express Eq. (18.33) as $f = A\varepsilon e^{-\varepsilon/kT}$, with *A* a constant. Then,

$$\frac{df}{de} = A\left[e^{-\varepsilon/kT} - \frac{\varepsilon}{kT}e^{-\varepsilon/kT}\right] = Ae^{-\varepsilon/kT}\left[1 - \frac{\varepsilon}{kT}\right].$$

Thus, *f* will be a maximum when the term in square brackets is zero, or $\varepsilon = \frac{1}{2}mv^2 = kT$, which is Eq. (18.34).

18.48: Note that $\frac{k}{m} = \frac{R/N_A}{M/N_A} = \frac{R}{M}$.

a) $\sqrt{2(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})/(44.0\times10^{-3}\ \text{kg/mol})} = 3.37\times10^2\ \text{m/s}.$

b) $\sqrt{8(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})/(\pi(44.0\times10^{-3}\ \text{kg/mol}))} = 3.80\times10^2\ \text{m/s}.$

c) $\sqrt{3(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})/(44.0\times10^{-3}\ \text{kg/mol})} = 4.12\times10^2\ \text{m/s}.$

18.49: Ice crystals will form if $T = 0.0°\text{C}$; using this in the given relation for temperature as a function of altitude gives $y = 2.5\times10^3\ \text{m} = 2.5\ \text{km}$.

18.50: a) The pressure must be above the triple point, $p_1 = 610\ \text{Pa}$. If $p < p_1$, the water cannot exist in the liquid phase, and the phase transition is from solid to vapor (sublimation). b) p_2 is the critical pressure, $p_2 = p_c = 221\times10^5$ Pa. For pressures below p_2 but above p_1, the phase transition is the most commonly observed sequence, solid to liquid to vapor, or ice to water to steam.

18.51: The temperature of $0.00°\ \text{C}$ is just below the triple point of water, and so there will be no liquid. Solid ice and water vapor at $0.00°\text{C}$ will be in equilibrium.

18.52: The atmospheric pressure is below the triple point pressure of water, and there can be no liquid water on Mars. The same holds true for CO_2

18.53: a) $\Delta V = \beta V_0 \Delta T = (3.6\times10^{-5}/°\text{C})\,(11\,\text{L})(21°\text{C}) = 0.0083\,\text{L}$

$$\Delta V = -kV_0\Delta p = (6.25\times10^{-12}/\text{Pa})(11\,\text{L})\,(2.1\times10^{7}\ \text{Pa}) = -0.0014\,\text{L}$$

So the total change in volume is $\Delta V = 0.0083\,\text{L} - 0.0014\,\text{L} = 0.0069\,\text{L}$. b) Yes; ΔV is much less than the original volume of 11.0 L.

18.54: $m = nM = \dfrac{MpV}{RT}$

$$= \frac{(28.0\times10^{-3}\ \text{kg/mol})(2.026\times10^{-8}\ \text{Pa})(3000\times10^{-6}\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(295.15\,\text{K})}$$

$$= 6.94\times10^{-16}\ \text{kg}.$$

18.55: $\Delta m = \Delta nM = \dfrac{\Delta pVM}{RT}$

$$= \frac{(1.05\times10^{6}\ \text{Pa})((1.00\,\text{m})\pi(0.060\,\text{m})^2)(44.10\times10^{-3}\ \text{kg/mol})}{(8.3145\ \text{J/mol}\cdot\text{K})(295.15\,\text{K})} = 0.213\,\text{kg}.$$

18.56: a) The height h' at this depth will be proportional to the volume, and hence inversely proportional to the pressure and proportional to the Kelvin temperature;

$$h' = h\frac{p}{p'}\frac{T'}{T} = h\frac{p_{\text{atm}}}{p_{\text{atm}} + \rho gy}\frac{T'}{T}$$

$$= (2.30\,\text{m})\frac{(1.013\times10^{5}\ \text{Pa})}{(1.013\times10^{5}\ \text{Pa}) + (1030\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(73.0\,\text{m})}\left(\frac{280.15\,\text{K}}{300.15\,\text{K}}\right)$$

$$= 0.26\,\text{m},$$

so $\Delta h = h - h' = 2.04\,\text{m}$. b) The necessary gauge pressure is the term ρgy from the above calculation, $p_g = 7.37\times10^{5}\ \text{Pa}$.

18.57: The change in the height of the column of mercury is due to the pressure of the air. The mass of the air is

$$m_{\mathrm{air}} = nM = \frac{PV}{RT}M = \frac{\rho_{\mathrm{Hg}} g \Delta h V}{RT} M$$

$$= \left(\frac{(13.6\times10^{3}\ \mathrm{kg/m^{3}})(9.80\ \mathrm{m/s^{2}})(0.060\,\mathrm{m}) \times ((0.900\,\mathrm{m} - 0.690\,\mathrm{m}))(0.620\times10^{-4}\ \mathrm{m^{2}})}{(8.3145\ \mathrm{J/mol\cdot K})(293.15\,\mathrm{K})} \right)(28.8\ \mathrm{g/mol})$$

$$= 1.23\times10^{-3}\,\mathrm{g}.$$

18.58: The density ρ' of the hot air must be $\rho' = \rho - \frac{m}{V}$, where ρ is the density of the ambient air and *m* is the load. The density is inversely proportional to the temperature, so

$$T' = T\frac{\rho}{\rho'} = \frac{\rho}{\rho - (m/V)} = T\left(1 - \frac{m}{\rho V}\right)^{-1}$$

$$= (288.15\,\mathrm{K})\left(1 - \frac{(290\,\mathrm{kg})}{(1.23\ \mathrm{kg/m^{3}})(500\,\mathrm{m^{3}})}\right)^{-1} = 545\,\mathrm{K},$$

which is $272°\mathrm{C}$.

18.59: $p_2 = p_1\left(\frac{V_1 T_2}{V_2 T_1}\right) = (2.72\,\mathrm{atm})\left(\frac{(0.0150\,\mathrm{m^{3}})(318.15\,\mathrm{K})}{(0.0159\,\mathrm{m^{3}})(278.15\,\mathrm{K})}\right) = 2.94\,\mathrm{atm},$

so the gauge pressure is $1.92\,\mathrm{atm}$.

18.60: (Neglect the thermal expansion of the flask.) a) $p_2 = p_1(T_2/T_1) =$ $(1.013\times10^5\ \text{Pa})(300/380) = 8.00\times10^4\ \text{Pa}.$

b) $$m_{\text{tot}} = nM = \left(\frac{p_2V}{RT_2}\right)M$$

$$= \left(\frac{(8.00\times10^4\ \text{Pa})(1.50\ \text{L})}{(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})}\right)(30.1\ \text{g/mol}) = 1.45\ \text{g}.$$

18.61: a) The absolute pressure of the gas in a cylinder is $(1.20\times10^6 + 1.013\times10^5)$ Pa $= 1.30\times10^6$ Pa. At atmospheric pressure, the volume of hydrogen will increase by a factor of $\frac{1.30\times10^6}{1.01\times10^5}$, so the number of cylinders is

$$\frac{750\ \text{m}^3}{(1.90\ \text{m}^3)((1.30\times10^6)/(1.01\times10^5))} = 31.$$

b) The difference between the weight of the air displaced and the weight of the hydrogen is

$$(\rho_{\text{air}} - \rho_{\text{H}_2})Vg = \left(\rho_{\text{air}} - \frac{pM_{\text{H}_2}}{RT}\right)Vg$$

$$= \left(1.23\ \text{kg/m}^3 - \frac{(1.01\times10^5\ \text{Pa})(2.02\times10^{-3}\ \text{kg/mol})}{(8.3145\ \text{J/mol}\cdot\text{K})(288.15\ \text{K})}\right)$$

$$\times(9.80\ \text{m/s}^2)(750\text{m}^3)$$

$$= 8.42\times10^3\ \text{N}.$$

c) Repeating the above calculation with $M = 4.00\times10^{-3}$ kg/mol gives a weight of 7.80×10^3 N.

18.62: If the original height is h and the piston descends a distance y, the final pressure of the air will be $p_{\mathrm{atm}}\left(\frac{h}{h-y}\right)$. This must be the same as the pressure at the bottom of the mercury column, $p_{\mathrm{atm}}+(\rho g)y$. Equating these two, performing some minor algebra and solving for y gives

$$y = h - \frac{p_{\mathrm{atm}}}{\rho \mathrm{g}} = (0.900\ \mathrm{m}) - \frac{(1.013\times10^{5}\ \mathrm{Pa})}{(13.6\times10^{3}\ \mathrm{kg/m^{3}})(9.80\ \mathrm{m/s^{2}})} = 0.140\,\mathrm{m}.$$

18.63: a) The tank is given as being "large," so the speed of the water at the top of the surface in the tank may be neglected. The efflux speed is then obtained from

$$\frac{1}{2}\rho v^2 = \rho g \Delta h + \Delta p, \quad \text{or}$$

$$v = \sqrt{2\left(g\Delta h + \frac{\Delta p}{\rho}\right)} = \sqrt{2\left((9.80\ \text{m/s}^2)(2.50\ \text{m}) + \frac{(3.20\times 10^5\ \text{Pa})}{(1000\ \text{kg/m}^3)}\right)}$$
$$= 26.2\ \text{m/s}.$$

b) Let $h_0 = 3.50\ \text{m}$ and $p_0 = 4.20\times 10^5\ \text{Pa}$. In the above expression for $v, \Delta h = h - 1.00\ \text{m}$ and $\Delta p = p_0\left(\frac{4.00\ \text{m} - h_0}{4.00\ \text{m} - h}\right) - p_\text{a}$. Repeating the calculation for $h = 3.00\ \text{m}$ gives $v = 16.1\ \text{m/s}$ and with $h = 2.00\ \text{m}$, $v = 5.44\ \text{m/s}$. c) Setting $v^2 = 0$ in the above expression gives a quadratic equation in h which may be re-expressed as

$$(h - 1.00\ \text{m}) = \frac{p_\text{a}}{\rho \text{g}} - \frac{p_0}{\rho g}\frac{0.50\ \text{m}}{4.00\ \text{m} - h}.$$

Denoting $\frac{p_\text{a}}{\rho \text{g}} = y = 10.204\ \text{m}$ and $\frac{p_0(0.50\ \text{m})}{\rho g} = z^2 = 21.43\ \text{m}^2$, this quadratic becomes

$$h^2 - (5.00\ \text{m} + y)h + ((4.00\ \text{m})y + (4.00\ \text{m}^2) - z^2) = 0,$$

which has as its solutions $h = 1.737\ \text{m}$ and $h = 13.47\ \text{m}$. The larger solution is unphysical (the height is greater than the height of the tank), and so the flow stops when $h = 1.74\ \text{m}$.

Although use of the quadratic formula is correct, for this problem it is more efficient for those with programmable calculators to find the solution to the quadratic by iteration. Using $h = 2.00\ \text{m}$ (the lower height in part (b)) gives convergence to three figures after four iterations. (The larger root is not obtained by a convergent iteration.)

18.64: a) $$\frac{N}{\Delta t}=\frac{nN_A}{\Delta t}=\frac{pVN_A}{RT\Delta t}=\frac{(1.00\,\text{atm})(14.5\,\text{L})(6.023\times10^{23}\text{ molecules/mol})}{(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(293.15\,\text{K})(3600\,\text{s})}$$
$$=1.01\times10^{20}\text{ molecule}\left[-36pt\right]$$

b) $$\frac{(14.5\,\text{L})/60\,\text{min})}{(0.5\,\text{L})(0.210-0.163)}=10/\text{min}.$$

c) The density of the air has decreased by a factor of $(0.72\,\text{atm}/1.00\,\text{atm})\times(293\,\text{K}/273\,\text{K})=0.773,$ and so the respiration rate must increase by a factor of $\frac{1}{0.733}$, to 13 breaths/min. If the breathing rate is not increased, one would experience "shortness of breath."

18.65: $3N=3nN_\text{A}=3(m/M)N_\text{A}=3\dfrac{(6.023\times10^{23}\text{ molecules/mol})(50\,\text{kg})}{(18.0\times10^{-3}\text{ kg/mol})}$

$$=5.0\times10^{27}\text{ atoms.}$$

18.66: The volume of gas per molecule (see Problem 18.28) is $\frac{RT}{N_{\text{A}p}}$, and the volume of a molecule is about $V_0=\dfrac{4}{3}\pi(2.0\times10^{-10}\text{ m})^3=3.4\times10^{-29}\text{ m}^3$. Denoting the ratio of these volumes as *f*,

$$p=f\frac{RT}{N_\text{A}V_0}=f\frac{(8.3145\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{(6.023\times10^{23}\text{ molecules/mol})(3.4\times10^{-29}\text{ m}^3)}=(1.2\times10^8\text{ Pa})\,f.$$

"Noticeable deviations" is a subjective term, but *f* on the order of unity gives a pressure of 10^8 Pa. Deviations from ideality are likely to be seen at values of *f* substantially lower than this.

18.67: a) Dividing both sides of Eq. (18.7) by the product *RTV* gives the result. b) The algorithm described is best implemented on a programmable calculator or computer; for a calculator, the numerical procedure is an interation of

$$x = \left[\frac{(9.8\times10^5)}{(8.3145)(400.15)} + \frac{(0.448)}{(8.3145)(400.15)}x^2\right]\left[1-(4.29\times10^{-5})x\right]$$

Starting at $x = 0$ gives a fixed point at $x = 3.03\times10^2$ after four iterations. The number density is $3.03\times10^2\ \mathrm{mol/m^2}$. c) The ideal-gas equation is the result after the first iteration, $295\ \mathrm{mol/m^3}$. The vander Waals density is larger. The term corresponding to *a* represents the attraction of the molecules, and hence more molecules will be in a given volume for a given pressure.

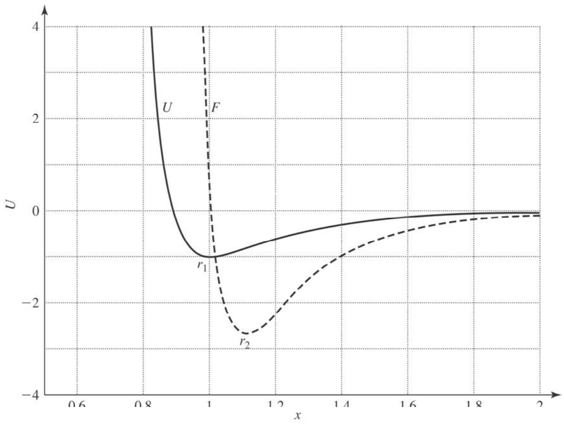
18.68:

a) $U = mgh = \dfrac{M}{N_A}gh = \left(\dfrac{28.0\times10^{-3}\ \mathrm{kg/mol}}{6.023\times10^{23}\ \mathrm{molecules/mol}}\right)(9.80\ \mathrm{m/s^2})(400\,\mathrm{m}) = 1.82\times10^{-22}\ \mathrm{J}.$

b) Setting $U = \dfrac{3}{2}kT,\ T = \dfrac{2}{3}\dfrac{1.82\times10^{-22}\ \mathrm{J}}{1.38\times10^{-23}\ \mathrm{J/K}} = 8.80\,\mathrm{K}.$ c) It is possible, but not at all likely for a molecule to rise to that altitude. This altitude is much larger than the mean free path.

18.69: a), b) (See figure.) The solid curve is $U(r)$, in units of U_0, and with $x = r/R_0$. The dashed curve is $F(r)$ in units of U_0/R_0. Note that $r_1 < r_2$.

c) When $U = 0, \left(\frac{R_0}{r_1}\right)^{12} = 2\left(\frac{R_0}{r_1}\right)^{6}$, or $r_1 = R_0/2^{1/6}$. Setting $F = 0$ in Eq. (18.26) gives $r_2 = R_0$ and $\frac{r_1}{r_2} = 2^{-1/6}$. d) $U(r_2) = U(R_0) = -U_0$, so the work required is U_0.



18.70: a) $\frac{3}{2}nRT = \frac{3}{2}pV = \frac{3}{2}(1.01\times10^5\ \text{Pa})(5.00\times10^{-3}\ \text{m}^3) = 758\ \text{J}$. b) The mass of the gas is $\frac{MpV}{RT}$, and so the ratio of the energies is

$$\frac{1}{2}\frac{\frac{MpV}{RT}v^2}{\frac{3}{2}pV} = \frac{1}{3}\frac{Mv^2}{RT} = \frac{1}{3}\frac{(2.016\times10^{-3}\ \text{kg/mol})(30.0\ \text{m/s})^2}{(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})} = 2.42\times10^{-4} = 0.0242\%.$$

18.71: a) From Eq. (18.19),

$$v\text{s} = \sqrt{3(8.3145\ \text{J/mol}\cdot\text{K})\ (300.15\ \text{K})\ /(28.0\times10^{-3}\ \text{kg/mol})} = 517\ \text{m/s}.$$

b) $v\text{s}/\sqrt{3} = 299\ \text{m/s}$.

18.72: a) $\sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3(1.38\times10^{-23}\ \mathrm{J/K})\,(5800\,\mathrm{K})}{(1.67\times10^{-27}\ \mathrm{kg})}} = 1.20\times10^{4}\ \mathrm{m/s}.$

b) $\sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})\,(1.99\times10^{30}\ \mathrm{kg})}{(6.96\times10^{8}\ \mathrm{m})}} = 6.18\times10^{5}\ \mathrm{m/s}.$

c) The escape speed is about 50 times the rms speed, and any of Fig. (18.20), Eq. (18.32) or Table (18.2) will indicate that there is a negligibly small fraction of molecules with the escape speed.

18.73: a) To escape, the total energy must be positive, $K + U > 0$. At the surface of the earth, $U = -GmM/R = -mgR$, so to escape $K > mgR$. b) Setting the average kinetic energy equal to the expression found in part (a), $(3/2)kT = mgR$, or $T = (2/3)(mgR/k)$. For nitrogen, this is

$$T = \frac{2}{3}\frac{(28.0\times10^{-3}\ \mathrm{kg/mol})(9.80\ \mathrm{m/s^2})(6.38\times10^{6}\ \mathrm{m})}{(6.023\times10^{23}\ \mathrm{molecules/mol})(1.381\times10^{-23}\ \mathrm{J/K})}$$

$$= 1.40\times10^{5}\ \mathrm{K}$$

and for hydrogen the escape temperature is $(\frac{2.02}{28.0})$ times this, or $1.01\times10^{4}\ \mathrm{K}$. c) For nitrogen, $T = 6.36\times10^{3}\ \mathrm{K}$ and for hydrogen, $T = 459\ \mathrm{K}$. d) The escape temperature for hydrogen on the moon is comparable to the temperature of the moon, and so hydrogen would tend to escape until there would be none left. Although the escape temperature for nitrogen is higher than the moon's temperature, nitrogen would escape, and continue to escape, until there would be none left.

18.74: (See Example 12.5 for calculation of the escape speeds)

a) Jupiter:

$v\ \text{s} = \sqrt{3(8.3145\ \text{J/mol}\cdot\text{K})(140\ \text{K})/(2.02\times10^{-3}\ \text{kg/mol})} = 1.31\times10^{3}\ \text{m/s} = (0.0221)v_{\text{e}}.$

Earth:

$v\ \text{s} = \sqrt{3(8.3145\ \text{J/mol}\cdot\text{K})(220\ \text{K})/((2.02\times10^{-3}\ \text{kg/mol})} = 1.65\times10^{3}\ \text{m/s} = (0.146)v_{\text{e}}.$

b) Escape from Jupiter is not likely for any molecule, while escape from earth is possible for some and hence possible for all.

c) $v\ \text{s} = \sqrt{3(8.3145\ \text{J/mol}\cdot\text{K})(200\ \text{K})/(32.0\times10^{-3}\ \text{kg/mol})} = 395\ \text{m/s}.$ The radius of the asteroid is $R = (3M/4\pi\rho)^{1/3} = 4.68\times10^{5}\ \text{m},$ and the escape speed is $\sqrt{2GM/R} = 542\ \text{m/s},$ so there can be no such atmosphere.

18.75: a) From Eq. (18.19),

$$m = \frac{3kT}{v\ \text{s}^2} = \frac{3(1.381\times10^{-23}\ \text{J/K})(300\ \text{K})}{(0.001\ \text{m/s})^2} = 1.24\times10^{-14}\ \text{kg}.$$

b) $mN_A/M = (1.24\times10^{-14}\ \text{kg})(6.023\times10^{23}\ \text{molecules/mol})/(18.0\times10^{-3}\ \text{kg/mol})$
$= 4.16\times10^{11}$ molecules.

c) $$D = 2r = 2\left(\frac{3V}{4\pi}\right)^{1/3} = 2\left(\frac{3m/\rho}{4\pi}\right)^{1/3}$$
$$= 2\left(\frac{3(1.24\times10^{-14}\ \text{kg})}{4\pi(920\ \text{kg/m}^3)}\right)^{1/3} = 2.95\times10^{-6}\ \text{m},$$

which is too small to see.

18.76: From $x = A\cos\omega t, v = -\omega A\sin\omega t,$

$$U_{\text{ave}} = \frac{1}{2}kA^2(\cos^2\omega t)_{\text{ave}}, \qquad K_{\text{ave}} = \frac{1}{2}m\omega^2A^2(\sin^2\omega t)_{ave}.$$

Using $(\sin^2\theta)_{\text{ave}} = (\cos^2\theta)_{\text{ave}} = \frac{1}{2}$ and $m\omega^2 = k$ shows that $K_{\text{ace}} = U_{\text{ave}}$.

18.77: a) In the same manner that Eq. (18.27) was obtained, the heat capacity of the two-dimensional solid would be $2R = 16.6\ \text{J/mol}\cdot\text{K}$. b) The heat capcity would behave qualitatively like those in Fig. (18.18), and heat capacity would decrease with decreasing temperature.

18.78: a) The two degrees of freedom associated with the rotation for a diatomic molecule account for two-fifths of the total kinetic energy, so $K_{\text{rot}} = nRT = (1.00)$ $(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K}) = 2.49\times10^{3}\ \text{J}$.

b) $$I = 2m(\text{L}/2)^2 = 2\left(\frac{16.0\times10^{-3}\ \text{kg/mol}}{6.023\times10^{23}\ \text{molecules/mol}}\right)(6.05\times10^{-11}\ \text{m})^2$$

$$= 1.94\times10^{-46}\ \text{kg}\cdot\text{m}^2.$$

c) Using the results of parts (a) and (b),

$$\omega\text{s} = \sqrt{\frac{2K_{\text{rot}}/N_A}{I}} = \sqrt{\frac{2(2.49\times10^{3}\ \text{J})}{(1.94\times10^{-46}\ \text{kg}\cdot\text{m}^2)(6.023\times10^{23}\ \text{molecules/mol})}}$$

$$= 6.52\times10^{12}\ \text{rad/s},$$

much larger than that of machinery.

18.79: For CO_2, the contribution to C_V other than vibration is

$$\frac{5}{2}R = 20.79\ \text{J/mol}\cdot\text{K}, \quad \text{and} \quad C_V - \frac{5}{2}R = 0.270\,C_V$$

For both SO_2 and H_2S, the contribution to C_V other than vibration is

$$\frac{6}{2}R = 24.94\ \text{J/mol}\cdot\text{K},$$

and the respective fractions of C_V are 0.25 and 0.039.

18.80: a)

$$\int_0^\infty f(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^2\,e^{-mv^2/2kT}\,dv$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{1}{4(m/2KT)}\right)\sqrt{\frac{\pi}{m/2KT}} = 1$$

where the tabulated integral (given in Problem18.81) has been used. b) $f(v)dv$ is the probability that a particle has speed between v and $v + dv$; the probability that the particle has some speed is unity, so the sum (integral) of $f(v)\,dv$ must be 1.

18.81: With $n = 2$ and $a = m/2kT$, the integral is

$$4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{3}{2^3(m/2kT)^2}\right)\sqrt{\frac{\pi}{(m/2kT)}} = \frac{3kT}{m},$$

which is Eq. (18.16).

18.82:

$$\int_0^\infty f(v)dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^3 e^{-mv^2/2kT}\,dv.$$

Making the suggested change of variable, $v^2 = x,\ 2v\ \ dv = dx,\ v^3dv = (1/2)x\,dx,$ the integral becomes

$$\int_0^\infty vf(v)dv = 2\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty xe^{-mx/2kT}\,dx$$

$$= 2\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2kT}{m}\right)^2$$

$$= \frac{2}{\sqrt{\pi}}\sqrt{\frac{2KT}{m}} = \sqrt{\frac{8KT}{\pi m}},$$

which is Eq. (18.35).

18.83: a) See Problem 18.80. Because $f(v)dv$ is the probability that a particle has a speed between v and $v+dv$, $f(v)dv$ is the fraction of the particles that have speed in that range. The number of particles with speeds between v and $v+dv$ is therefore $dN = Nf(v)dv$ and

$$\Delta N = N\int_v^{v+\Delta v} f(v)dv.$$

b) $v_{\text{mp}} = \sqrt{\frac{2kT}{m}}$, and

$$f(v_{\text{mp}}) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2kT}{m}\right)e^{-1} = \frac{4}{e\sqrt{\pi}v_{\text{mp}}}.$$

For oxygen gas at $300\,\text{K}$, $v_{\text{mp}} = 3.95\times10^2\ \text{m/s}$, and $f(v)\Delta v = 0.0421$, keeping an extra figure. c) Increasing v by a factor of 7 changes f by a factor of $7^2 e^{-48}$, and $f(v)\Delta v = 2.94\times10^{-21}$. d) Multiplying the temperature by a factor of 2 increases the most probable speed by a factor of $\sqrt{2}$, and the answers are decreased by $\sqrt{2}$; 0.0297 and 2.08×10^{-21}. e) Similarly, when the temperature is one-half what it was parts (b) and (c), the fractions increase by $\sqrt{2}$ to 0.0595 and 4.15×10^{-21}. f) At lower temperatures, the distribution is more sharply peaked about the maximum (the most probable speed), as is shown in Fig. (18.20).

18.84: a) $(0.60)(2.34\times10^3\ \text{Pa}) = 1.40\times10^3\ \text{Pa}.$

$$\text{b)}\ m = \frac{MpV}{RT} = \frac{(18.0\times10^{-3}\ \text{kg/mol})(1.40\times10^3\ \text{Pa})(1.00\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(293.15\ \text{K})} = 10\ g.$$

18.85: The partial pressure of water in the room is the vapor pressure at which condensation occurs. The relative humidity is $\frac{1.81}{4.25} = 42.6\%.$

18.86: a) The partial pressure is $(0.35)(3.78\times10^{3}\ \text{Pa}) = 1.323\times10^{3}\ \text{Pa}$. This is close to the vapor pressure at $12°\text{C}$, which would be at an altitude $(30°\text{C} - 12°\text{C})/(0.6°\text{C}/100\ \text{m}) = 3\ \text{km}$ above the ground (more precise interpolation is not warranted for this estimate).

b) The vapor pressure will be the same as the water pressure at around $24°\text{C}$, corresponding to an altitude of about $1\ \text{km}$.

18.87: a) From Eq. (18.21),

$$\lambda = (4\pi\sqrt{2}r^{2}(N/V))^{-1} = (4\pi\sqrt{2}(5.0\times10^{-11}\ \text{m})^{2}(50\times10^{6}\ \text{m}^{-3}))^{-1}$$

$$= 4.5\times10^{11}\ \text{m}.$$

b) $\sqrt{3(8.3145\ \text{J/mol}\cdot\text{K})(20\ \text{K})/(1.008\times10^{-3}\ \text{kg/mol})} = 703\ \text{m/s}$, and the time between collisions is then $(4.5\times10^{11}\ \text{m})/(703\ \text{m/s}) = 6.4\times10^{8}\ \text{s}$, about 20 yr. Collisions are not very important. c) $p = (N/V)kT = (50\times10^{6}\ \text{m}^{-3})(1.381\times10^{-23}\ \text{J/K})(20\ \text{K}) = 1.4\times10^{-14}\ \text{Pa}.$

d) $$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G(Nm/V)(4\pi R^{3}/3)}{R}} = \sqrt{(8\pi/3)G(N/V)mR}$$

$$= \sqrt{(8\pi/3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^{2}/\text{kg}^{2})(50\times10^{6}\ \text{m}^{-3})(1.67\times10^{-27}\ \text{kg})\times(10\times9.46\times10^{15}\ \text{m})}$$

$$= 650\ \text{m/s}.$$

This is lower than v s, and the cloud would tend to evaporate. e) In equilibrium (clearly not *thermal* equilibrium), the pressures will be the same; from $pV = NkT$,

$$kT_{\text{ISM}}(N/V)_{\text{ISM}} = kT_{\text{nebula}}(N/V)_{\text{nebula}}$$

and the result follows. f) With the result of part (e),

$$T_{\text{ISM}} = T_{\text{nebula}}\left(\frac{(V/N)_{\text{nebula}}}{(V/N)_{\text{ISM}}}\right) = (20\ \text{K})\left(\frac{50\times10^{6}\ \text{m}^{3}}{(200\times10^{-6}\ \text{m}^{3})^{-1}}\right) = 2\times10^{5}\ \text{K},$$

more than three times the temperature of the sun. This indicates a high average kinetic energy, but the thinness of the ISM means that a ship would not burn up.

18.88: a) Following Example 18.4, $\frac{dP}{dy} = -\frac{pM}{RT}$, which in this case becomes

$$\frac{dp}{p} = -\frac{Mg}{R}\frac{dy}{T_0 - \alpha y},$$

which integrates to

$$\ln\left(\frac{p}{p_0}\right) = \frac{Mg}{R\alpha}\ln\left(1 - \frac{\alpha y}{T_0}\right), \text{ or } p = p_0\left(1 - \frac{\alpha y}{T_0}\right)^{\frac{Mg}{R\alpha}}.$$

b) Using the first equation above, for sufficiently small $\alpha, \ln(1 - \frac{\alpha y}{T_0}) \approx -\frac{\alpha y}{T_0}$, and this gives the expression derived in Example 18.4.

c) $$\left(1 - \frac{(0.6\times10^{-2}\ \text{C}°/\text{m})(8863\,\text{m})}{(288\,\text{K})}\right) = 0.8154,$$

$$\frac{Mg}{R\alpha} = \frac{(28.8\times10^{-3})(9.80\ \text{m/s}^2)}{(8.3145\ \text{J/mol}\cdot\text{K})(0.6\times10^{-2}\ \text{C}°/\text{m})} = 5.6576$$

(the extra significant figures are needed in exponents to reduce roundoff error), and $p_0(0.8154)^{5.6576} = 0.315\,\text{atm}$, which is 0.95 of the result found in Example 18.4. Note: for calculators without the x^y function, the pressure in part (c) must be found from $p = p_0 \exp((5.6576)\ln(0.8154))$.

18.89: a) A positive slope $\frac{\partial p}{\partial V}$ would mean that an increase in pressure causes an increase in volume, or that decreasing volume results in a decrease in pressure, which cannot be the case for any real gas. b) See Fig. (18.5). From part (a), p cannot have a positive slope along an isotherm, and so can have no extremes (maxima or minima) along an isotherm. When $\frac{\partial p}{\partial V}$ vanishes along an isotherm, the point on the curve in a p-V diagram must be an inflection point, and $\frac{\partial^2 p}{\partial V^2} = 0$

c) $$p = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

$$\frac{\partial p}{\partial V} = -\frac{nRT}{(V - nb)^2} + \frac{2an^2}{V^3}$$

$$\frac{\partial^2 p}{\partial V^2} = \frac{2nRT}{(V - nb)^3} - \frac{6an^2}{V^4}.$$

Setting the last two of these equal to zero gives

$$V^3 nRT = 2an^2 (V - nb)^2, \qquad V^4 nRT = 3an^2 (V - nb)^3.$$

c) Following the hint, $V = (3/2)(V - nb),$ which is solved for $(V/n)_c = 3b.$ Substituting this into either of the last two expressions in part (c) gives $T_c = 8a/27Rb.$

d) $$p_c = \frac{RT}{(V/n)_c - b} - \frac{a}{(V/n)_c} = \frac{R(\frac{8a}{27Rb})}{2b} - \frac{a}{9b^2} = \frac{a}{27b^2}.$$

e) $$\frac{RT_c}{p_c (V/n)_c} = \frac{\frac{8}{27}\frac{a}{b}}{\frac{a}{27b^2} 3b} = \frac{8}{3}.$$

g) $H_2 : 3.28.$ $N_2 : 3.44.$ $H_2O : 4.35.$ h) While all are close to $8/3,$ the agreement is not good enough to be useful in predicting critical point data. The van der Waals equation models certain gases, and is not accurate for substances near critical points.

18.90: a) $v_{av} = \frac{1}{2}(v_1 + v_2)$ and $v_{rms} = \frac{1}{\sqrt{2}}\sqrt{v_1^2 + v_2^2}$, and

$$v_{rms}^2 - v_{av}^2 = \frac{1}{2}(v_1^2 + v_2^2) - \frac{1}{4}(v_1^2 + v_2^2 + 2v_1v_2)$$

$$= \frac{1}{4}(v_1^2 + v_2^2 - 2v_1v^2)$$
$$= \frac{1}{4}(v_1 - v^2)^2.$$

This shows that $v_{rms} \geq v_{av}$, with equality holding if and only if the particles have the same speeds.

b) $v_{rms}'^2 = \frac{1}{N+1}(Nv_{rms}^2 + u^2), v_{av}' = \frac{1}{N+1}(Nv_{av} + u)$, and the given forms follow immediately.

c) The algebra is similar to that in part (a); it helps somewhat to express

$$v_{av}'^2 = \frac{1}{(N+1)^2}(N((N+1)-1)v_{av}^2 + 2Nv_{av}u + ((N+1)-N)u^2)$$
$$= \frac{N}{N+1}v_{av}^2 + \frac{N}{(N+1)^2}(-v_{av}^2 + 2v_{av}u - u^2) + \frac{1}{N+1}u^2.$$

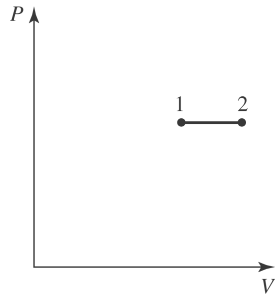
Then,

$$v_{rms}'^2 - v_{av}'^2 = \frac{N}{(N+1)}(v_{rms}^2 - v_{av}^2) + \frac{N}{(N+1)^2}(v_{av}^2 - 2v_{av}u + u^2)$$
$$= \frac{N}{N+1}(v_{rms}^2 - v_{av}^2) + \frac{N}{(N+1)^2}(v_{av} - u)^2.$$

If $v_{rms} > v_{av}$, then this difference is necessarily positive, and $v_{rms}' > v_{av}'$.

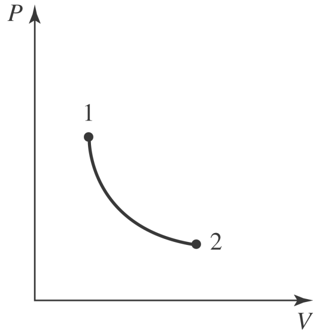
d) The result has been shown for $N = 1$, and it has been shown that validity for N implies validity for $N + 1$; by induction, the result is true for all N.

19.1: a)



b) $p\Delta V = nR\Delta T = (2.00\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(80\ \text{C}°) = 1.33\times10^{3}\ \text{J}.$

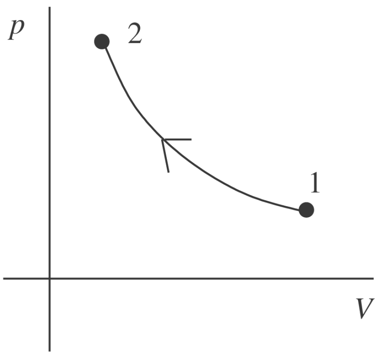
19.2: a)



b) If the pressure is reduced to 40.0% of its original value, the final volume is $(5/2)$ of its original value. From Eq. (19.4),

$$W = nRT\ln\frac{V_2}{V_1} = (3)(8.3145\ \text{J/mol}\cdot\text{K})(400.15\ \text{K})\ln\left(\frac{5}{2}\right) = 9.15\times10^{3}\ \text{J}.$$

19.3:



$pV = nRT$

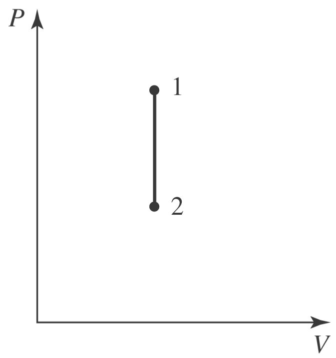
T constant, so when p increases, V decrease

19.4: At constant pressure, $W = p\Delta V = nR\Delta T$, so

$$\Delta T = \frac{W}{nR} = \frac{1.75\times10^{3}\ \text{J}}{(6\ \text{mol})\ (8.3145\ \text{J/mol}\cdot\text{K})} = 35.1\,\text{K}$$

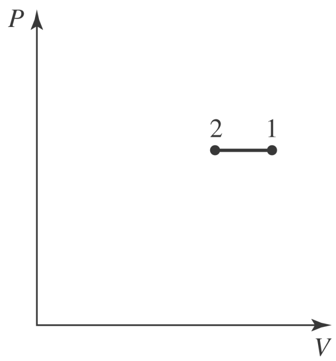
and $\Delta T_{\text{K}} = \Delta \text{T}_{\text{C}}$, so $T_2 = 27.0°\text{C} + 35.1°\text{C} = 62.1°\text{C}$.

19.5: a)



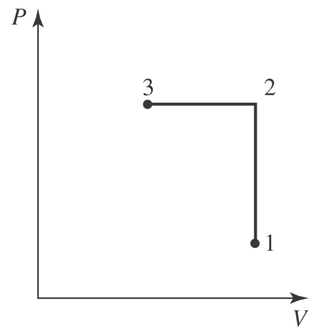
b) At constant volume, $dV = 0$ and so $W = 0$.

19.6:



b) $p\Delta V = (1.50\times10^{5}\ \text{Pa})\,(0.0600\ \text{m}^3 - 0.0900\ \text{m}^3) = -4.50\times10^{3}\ \text{J}$.

19.7: a)



b) In the first process, $W_1 = p\Delta V = 0$. In the second process, $W_2 = p\Delta V = (5.00 \times 10^5 \text{ Pa})(-0.080 \text{ m}^3) = -4.00 \times 10^4 \text{ J}.$

19.8: a) $W_{13} = p_1(V_2 - V_1), W_{32} = 0, W_{24} = p_2(V_1 - V_2)$ and $W_{41} = 0.$ The total work done by the system is $W_{13} + W_{32} + W_{24} + W_{41} = (p_1 - p_2)(V_2 - V_1),$ which is the area in the *p*- *V* plane enclosed by the loop. b) For the process in reverse, the pressures are the same, but the volume changes are all the negatives of those found in part (a), so the total work is negative of the work found in part (a).

19.9: $Q = 254 \text{ J}, W = -73 \text{ J}$ (work is done *on* the system), and so $\Delta U = Q - W = 327 \text{ J}.$

19.10: a) $p\Delta V = (1.80 \times 10^5 \text{ Pa})(0.210 \text{ m}^2) = 3.78 \times 10^4 \text{ J}.$

b) $\Delta U = Q - W = 1.15 \times 10^5 \text{ J} - 3.78 \times 10^4 \text{ J} = 7.72 \times 10^4 \text{ J}.$

c) The relations $W = p\Delta V$ and $\Delta U = Q - W$ hold for any system.

19.11: The type of process is not specified. We can use $\Delta U = Q - W$ because this applies to all processes.

Q is positive since heat goes into the gas; $Q = +1200\,\mathrm{J}$

W positive since gas expands; $W = +2100\,\mathrm{J}$

$\Delta U = 1200\,\mathrm{J} - 2100\,\mathrm{J} = -900\,\mathrm{J}$

We can also use $\Delta U = n\left(\frac{3}{2}R\right)\Delta T$ since this is true for any process for an ideal gas.

$$\Delta T = \frac{2\Delta U}{3nR} = \frac{2(-900\,\mathrm{J})}{3(5.00\,\mathrm{mol})(8.3145\,\mathrm{J/mol\cdot K})} = -14.4\,\mathrm{C}^\circ$$

$$T_2 = T_1 + \Delta T = 127^\circ\mathrm{C} - 14.4\,\mathrm{C}^\circ = 113^\circ\mathrm{C}$$

19.12: At constant volume, the work done by the system is zero, so $\Delta U = Q - W = Q.$ Because heat flows into the system, Q is positive, so the internal energy of the system increases.

19.13: a) $p\Delta V = (2.30\times10^5\,\mathrm{Pa})(-0.50\,\mathrm{m}^3) = -1.15\times10^5\,\mathrm{J}$. (b $Q = \Delta U + W = -1.40\times10^5\,\mathrm{J} + (-1.15\times10^5\,\mathrm{J}) = -2.55\times10^5\,\mathrm{J}$ (heat flows out of the gas). c) No; the first law of thermodynamics is valid for any system.

19.14: a) The greatest work is done along the path that bounds the largest area above the V-axis in the p- V plane (see Fig. (19.8)), which is path 1. The least work is done along path 3. b) $\mathrm{W} > 0$ in all three cases; $Q = \Delta U + W,$ so $Q > 0$ for all three, with the greatest Q for the greatest work, that along path 1. When $Q > 0,$ heat is absorbed.

19.15: a) The energy is

$$(2.0\ \mathrm{g})(4.0\ \mathrm{kcal/g}) + (17.0\ \mathrm{g})(4.0\ \mathrm{kcal/g}) + (7.0\ \mathrm{g})(9.0\ \mathrm{kcal/g}) = 139\ \mathrm{kcal},$$

and the time required is $(139\,\mathrm{kcal})/(510\ \mathrm{kcal/h}) = 0.273\,\mathrm{h} = 16.4\,\mathrm{min}.$ b) $v = \sqrt{2K/m} = \sqrt{2(139\times10^3\ \mathrm{cal})(4.186\ \mathrm{J/cal})/(60\,\mathrm{kg})} = 139\ \mathrm{m/s} = 501\ \mathrm{km/h}.$

19.16: a) The container is said to be well-insulated, so there is no heat transfer. b) Stirring requires work. The stirring needs to be irregular so that the stirring mechanism moves against the water, not with the water. c) The work mentioned in part (b) is work done *on* the system, so $W < 0$, and since no heat has been transferred, $\Delta U = -W > 0$.

19.17: The work done is positive from *a* to *b* and negative from *b* to *a*; the net work is the area enclosed and is positive around the clockwise path. For the closed path $\Delta U = 0$, so $Q = W > 0$. A positive value for Q means heat is absorbed.

b) $|Q| = 7200\,\text{J}$, and from part (a), $Q > 0$ and so $Q = W = 7200\,\text{J}$.

c) For the counterclockwise path, $Q = W < 0$. $W = -7200\,\text{J}$, so $Q = -7200\,\text{J}$ and heat is liberated, with $|Q| = 7200$ J.

19.18: a), b) The clockwise loop (I) encloses a larger area in the *p*-*V* plane than the counterclockwise loop (II). Clockwise loops represent positive work and counterclockwise loops negative work, so $W_{\text{I}} > 0$ and $W_{\text{II}} < 0$. Over one complete cycle, the net work $W_{\text{I}} + W_{\text{II}} > 0$, and the net work done by the system is positive. c) For the complete cycle, $\Delta U = 0$ and so $W = Q$. From part (a), $W > 0$ so $Q > 0$, and heat flows into the system. d) Consider each loop as beginning and ending at the intersection point of the loops. Around each loop, $\Delta U = 0$, so $Q = W$; then, $Q_{\text{I}} = W_{\text{I}} > 0$ and $Q_{\text{II}} = W_{\text{II}} < 0$. Heat flows into the system for loop I and out of the system for loop II.

19.19: a) Yes; heat has been transferred form the gasses to the water (and very likely the can), as indicated by the temperature rise of the water. For the system of the gasses, $Q < 0$.

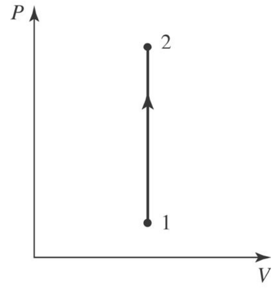
b) The can is given as being constant-volume, so the gasses do no work. Neglecting the thermal expansion of the water, no work is done. c) $\Delta U = Q - W = Q < 0$.

19.20: a) $p\Delta V = (2.026\times10^{5}\ \text{Pa})(0.824\,\text{m}^3 - 1.00\times10^{-3}\ \text{m}^3) = 1.67\times10^{5}\ \text{J}.$

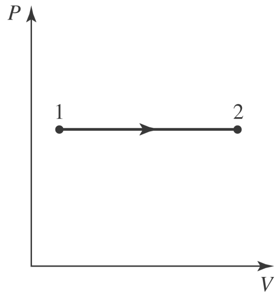
b) $\Delta U = Q - W = mL_{\text{v}} - W$

$$= (1.00\,\text{kg})(2.20\times10^{6}\ \text{J/kg}) - 1.67\times10^{5}\ \text{J} = 2.03\times10^{6}\ \text{J}.$$

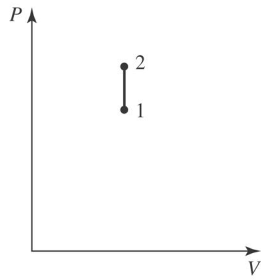
19.21: a) Using Equation (19.12), $\mathrm{d}T = \frac{\mathrm{d}Q}{nC_V} = \frac{645\,\mathrm{J}}{(0.185\,\mathrm{mol})(20.76\,\mathrm{J/mol\cdot K})} = 167.9\,\mathrm{K}$, or $T = 948$ K.



b) Using Equation (19.14), $\mathrm{d}T = \frac{\mathrm{d}Q}{nC_p} = \frac{645\ \mathrm{J}}{(0.185\ \mathrm{mol})(29.07\ \mathrm{J/mol.K})} = 119.9\,\mathrm{K}$, or $T = 900\,\mathrm{K}$.



19.22: a) $nC_V \Delta T = (0.0100\,\mathrm{mol})(12.47\ \mathrm{J/mol \cdot K})(40.0\,\mathrm{C°}) = 4.99\,\mathrm{J}$.



19.23: $n = 5.00\,\text{mol}$. $\Delta T = +30.0\,\text{C}°$

a) For constant *p*,

$Q = nC_p \Delta T = (5.00\ \text{mol})(20.78\ \text{J/mol}\cdot\text{K})(30.0\ \text{C}°) = +3120\ \text{J}$

$Q > 0$ so heat goes into gas.

b) For constant *V*,

$Q = nC_v \Delta T = (5.00\ \text{mol})(12.47\ \text{J/mol}\cdot\text{K})(30.0\ \text{C}°) = +1870\ \text{J}$

$Q > 0$ so heat goes into gas.

c) For constant *p*,

$Q = nC_p \Delta T = (5.00\ \text{mol})(36.94\ \text{J/mol}\cdot\text{K})(30.0\ \text{C}°) = +5540\ \text{J}$

$Q > 0$ so heat goes into gas.

19.24: For an ideal gas, $\Delta U = C_V \Delta T$, and at constant pressure, $p\Delta V = nR\Delta T$. Using $C_V = \frac{3}{2}R$ for a monatomic gas,

$$\Delta U = n\left(\frac{3}{2}R\right)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}(4.00\times 10^4\ \text{Pa})(8.00\times 10^{-3}\ \text{m}^3 - 2.00\times 10^{-3}\ \text{m}^3) = 360\ \text{J}.$$

19.25: For constant $p, Q = nC_p \Delta T$

Since the gas is ideal, $pV = nRT$ and for constant $p, p\Delta V = nR\Delta T$.

$$Q = nC_p\left(\frac{p\Delta V}{nR}\right) = \left(\frac{C_p}{R}\right)p\Delta V$$

Since the gas expands, $\Delta V > 0$ and therefore $Q > 0$. $Q > 0$ means heat goes into gas.

19.26: For an ideal gas, $\Delta U = C_V \Delta T$, and at constant pressure, $W = p\Delta V = nR\Delta T$. Using $C_V = \frac{3}{2}R$ for a monatomic gas, $\Delta U = n(\frac{3}{2}R)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}W$. Then $Q = \Delta U + W = \frac{5}{2}W$, so $W/Q = \frac{2}{5}$.

19.27: a) For an isothermal process,

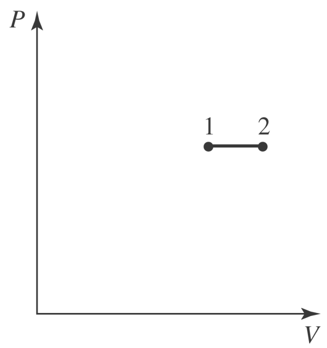
$$W = nRT \ln (V_2/V_1) = (0.150\,\text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(350.15\text{K})\ln(1/4)$$
$$= -605\,\text{J}.$$

b) For an isothermal process for an ideal gas, $\Delta T = 0$ and $\Delta U = 0$. c) For a process with $\Delta U = 0$, $Q = W = -605\,\text{J}$; 605 J are liberated.

19.28: For an isothermal process, $\Delta U = 0$, so $W = Q = -335$ J.

19.29: For an ideal gas $\gamma = C_\text{p}/C_\text{V} = 1 + R/C_\text{V}$, and so $C_\text{V} = R/(\gamma - 1) =$ $(8.3145\ \text{J/mol}\cdot\text{K})/(0.127) = 65.5\ \text{J/mol}\cdot\text{K}$ and $C_p = C_V + R = 73.8\ \text{J/mol}\cdot\text{K}$.

19.30: a)



b) $pV_2 - pV_1 = nR(T_2 - T_1)$
$$= (0.250\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(100.0\ \text{K}) = 208\ \text{J}.$$

c) The work is done on the piston.
d) Since Eq. (19.13) holds for any process,

$$\Delta U = nC_V\Delta T = (0.250\,\text{mol})(28.46\,\text{J/mol}\cdot\text{K})(100.0\,\text{K}) = 712\,\text{J}.$$

e) Either $Q = nC_P\Delta T$ or $Q = \Delta U + W$ gives $Q = 924\times10^3$ J to three significant figures.
f) The lower pressure would mean a correspondingly larger volume, and the net result would be that the work done would be the same as that found in part (b).

19.31: a) $C_p = R/(1-(1/\gamma))$, and so

$$Q = nC_p\Delta T = \frac{(2.40\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(5.0\text{ C}°)}{1-1/1.220} = 553\text{ J}.$$

b) $nC_V\Delta T = nC_P\Delta T/\gamma = (553\text{ J})/(1.220) = 454\text{ J}.$ (An extra figure was kept for these calculations.)

19.32: a) See also Exercise 19.36;

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = (1.50\times10^5\text{ Pa})\left(\frac{0.0800\text{ m}^3}{0.0400\text{ m}^3}\right)^{\frac{5}{3}} = 4.76\times10^5\text{ Pa}.$$

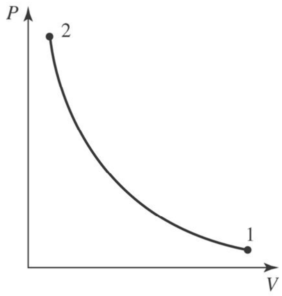
b) This result may be substituted into Eq. (19.26), or, substituting the above form for p_2,

$$W = \frac{1}{\gamma-1}p_1V_1\left(1-\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right)$$

$$= \frac{3}{2}(1.50\times10^5\text{ Pa})(0.0800\text{ m}^3)\left(1-\left(\frac{0.0800}{0.0400}\right)^{\frac{2}{3}}\right) = -1.60\times10^4\text{ J}.$$

c) From Eq. (19.22), $(T_2/T_1) = (V_2/V_1)^{\gamma-1} = (0.0800/0.0400)^{2/3} = 1.59$, and since the final temperature is higher than the initial temperature, the gas is heated (see the note in Section 19.8 regarding “heating” and “cooling.”)

19.33: a)



b) (Use $\gamma = 1.400$, as in Example 19.6) From Eq. (19.22),

$$T_2 = T_1 (V_1/V_2)^{\gamma-1} = (293.15\ \text{K})(11.1)^{0.400} = 768\ \text{K} = 495°\text{C}$$

and from Eq. (19.24), $p_2 = p_1 (V_1/V_2)^{\gamma} = (1.00\ \text{atm})(11.1)^{1.400} = 29.1\ \text{atm}.$

19.34: $\gamma = 1.4$ for ideal diatomic gas $Q = \Delta U + W = 0$ for adiabatic process

$$\Delta U = -W = -\int P dV$$
$$PV^{\gamma} = \text{const} = P_i V_i^{\gamma}$$

$$\begin{aligned}\Delta \text{U} &= -\int_{30\text{L}}^{10\text{L}} \frac{\text{P}_\text{i} \text{V}_\text{i}^{\gamma}}{\text{V}^{\gamma}} \text{dV} = -\text{P}_\text{i} \text{V}_\text{i}^{\gamma} \left(\frac{\text{V}^{-\gamma+1}}{-\gamma+1}\right)\Bigg|_{30\text{L}}^{10\text{L}1} \\ &= -(1.2\ \text{atm})\ (30\ \text{L})^{1.4} \left[\frac{(10\text{L})^{1\text{-}1.4} - (30\text{L})^{1\text{-}1.4}}{1-1.4}\right] \\ &= 50\ \text{L} \cdot \text{atm} = 5.1 \times 10^{3}\ \ \text{J}.\end{aligned}$$

The internal energy increases because work is done *on* the gas $(\Delta U > 0)$.
The temperature increases because the internal energy has increased.

19.35: For an ideal gas $\Delta U = nC_V \Delta T$. The sign of ΔU is the same as the sign of ΔT.

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ and $V = nRT/p$ so,
$T_1^{\gamma} p_1^{1-\gamma} = T_2^{\gamma} p_2^{1-\gamma}$ and $T_2^{\gamma} = T_1^{\gamma} (p_2/p_1)^{\gamma-1}$
$p_2 < p_1$ and $\gamma - 1$ is positive so $T_2 < T_1$. ΔT is negative so ΔU is negative; the energy of the gas decreases.

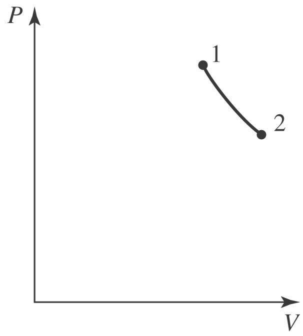
19.36: Equations (19.22) and (19.24) may be re-expressed as

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}, \quad \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}.$$

$a)\ \gamma = \frac{5}{3},\ p_2 = (4.00 \text{ atm})(2/3)^{\frac{5}{3}} = 2.04 \text{ atm},\ T_2 = (350 \text{ K})(2/3)^{\frac{2}{3}} = 267 \text{ K}.$

b) $\gamma = \frac{7}{5},\ p_2 = (4.00 \text{ atm})\,(2/3)^{\frac{7}{5}} = 2.27 \text{ atm},\ T_2 = (350 \text{ K})\,(2/3)^{\frac{2}{5}} = 298 \text{ K}.$

19.37: a)



b) From Eq. (19.25), $W = nC_V \Delta T = (0.450 \text{ mol})\,(12.47 \text{ J/mol} \cdot \text{K})\,(40.0\,\text{C}°)$

$= 224$ J. For an adiabatic process, $Q = 0$ and there is no heat flow.
$\Delta U = Q - W = -W = -224$ J.

19.38: a) $T = \frac{pV}{nR} = \frac{(1.00\times10^5\ \text{Pa})\,(2.50\times10^{-3}\ \text{m}^3)}{(0.1\,\text{mol})\,(8.3145\ \text{J/mol}\cdot\text{K})} = 301\,\text{K}.$

b) i) Isothermal: If the expansion is *isothermal*, the process occurs at constant temperature and the final temperature is the same as the initial temperature, namely $301\,\text{K}$.

ii) Isobaric:

$$T = \frac{pV}{nR} = \frac{(1.00\times10^5\ \text{Pa})\,(5.00\times10^{-3}\ \text{m}^3)}{(0.100\ \text{mol})\,(8.3145\ \text{J/mol}\cdot\text{K})}$$
$$T = 601\,\text{K}.$$

iii) Adiabatic: Using Equation (19.22), $T_2 = \frac{T_1 V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{(301\,K)(V_1^{.67})}{(2V_1^{.67})} = (301\,\text{K})(\tfrac{1}{2})^{.67} = 189\,K.$

19.39: See Exercise 19.32. a) $p_2 = p_1(V_1/V_2)^\gamma = (1.10\times10^5\ \text{Pa})$
$((5.00\times10^{-3}\ \text{m}^3/1.100\times10^{-2}\ \text{m}^3))^{1.29} = 4.50\times10^4\ \text{Pa}.$

b) Using Equation (19.26),

$$W = \frac{(p_1V_1 - p_2V_2)}{\gamma - 1}$$
$$= \frac{[(1.1\times10^5\ \text{N/m}^3)(5.0\times10^{-3}\ \text{m}^3) - (4.5\times10^4\ \text{N/m}^3)(1.0\times10^{-2}\ \text{m}^3)]}{(1.29-1)},$$

and thus $W = 345\,\text{J}$

c) $(T_2/T_1) = (V_2/V_1)^{\gamma-1} = ((5.00\times10^{-3}\ \text{m}^3)/(1.00\times10^{-2}\ \text{m}^3))^{0.29} = 0.818.$ The final temperature is lower than the initial temperature, and the gas is cooled.

19.40: a) The product pV increases, and even for a non-ideal gas, this indicates a temperature increase. b) The work is the area in the $p-V$ plane bounded by the blue line representing the process and the vericals at V_a and $V_{\text{b.}}$ The area of this trapeziod is

$$\frac{1}{2}(p_b + p_a)(V_b - V_a) = \frac{1}{2}(2.40\times10^5\ \text{Pa})\,(0.0400\ \text{m}^3) = 4800\ \text{J}.$$

19.41: $|w|$ is the area under the path from A to B in the pV-graph. The volume decreases, so $W < 0$. $W = -\frac{1}{2}(500\times10^3\ \text{Pa} + 150\times10^3\ \text{Pa})(0.60\ \text{m}^3) = -1.95\times10^5\ \text{J}$

$\Delta U = nC_V\,\Delta T$

$T_1 = \dfrac{p_1V_1}{nR},\ T_2 = \dfrac{P_2V_2}{nR}$ so $\Delta T = T_2 - T_1 = \dfrac{p_2V_2 - p_1V_1}{nR}$

$\Delta U = (C_V/R)(p_2V_2 - p_1V_1)$

$\Delta U = (20.85/8.315)[(500\times10^3\ \text{Pa})(0.20\ \text{m}^3) - (150\times10^3\ \text{Pa})(0.80\ \text{m}^3)] = -5.015\times1$

Then $\Delta U = Q - W$ gives

$Q = \Delta U + W = -5.015\times10^4\ \text{J} - 1.95\times10^5\ \text{J} = -2.45\times10^5\ \text{J}$

Q is negative, so heat flows out of the gas.

19.42: (a) $Q_{abc} = \Delta U_{ac} + W_{abc} = nC_v \Delta T_{ac} + W_{abc}$
get $\Delta T_{ac} : PV = nRT \rightarrow T = PV/nR$

$$\Delta T_{ac} = T_c - T_a = \frac{P_c V_c}{nR} - \frac{P_a V_a}{nR} = \frac{P_c V_c - P_a V_a}{nR}$$

$$\Delta T_{ac} = \frac{(1.0\times10^5 \text{ Pa})(0.010 \text{ m}^3) - (1.0\times10^5 \text{ Pa})(0.0020 \text{ m}^3)}{(\frac{1}{3} \text{ mole})(8.31 \text{ J/mole K})} = 289 \text{ K}$$

$$W_{abc} = \text{Area under } PV \text{ graph} = \frac{1}{2}(0.010 - 0.002)\,\text{m}^3 (2.5\times10^5 \text{ Pa})$$
$$+ (0.010 - 0.002)\,\text{m}^3 (1.0\times10^5 \text{ Pa})$$

$$W_{abc} = 1.80\times10^3 \text{ J}$$

$$\Delta U_{\text{ac}} = nC_v \Delta T_{ac} = n\left(\frac{3}{2}R\right)\Delta T_{ac}$$

$$= \left(\frac{1}{3}\text{mole}\right)\left(\frac{3}{2}\right)\left(8.31\frac{\text{J}}{\text{mole K}}\right)(289\,\text{K}) = 1.20\times10^3 \text{ J}$$

$$Q_{abc} = 1.20\times10^3 \text{ J} + 1.8\times10^3 \text{ J} = 3000 \text{ J into the gas}$$

(b) ΔU_{ac} in the same = 1200 J

$$W_{ac} = \text{area} = (0.010 - 0.002)\text{m}^3 (1.0\times10^5 \text{ Pa}) = 800\,\text{J}$$
$$Q_{ac} = \Delta U_{ac} + W_{ac} = 1200\,\text{J} + 800\,\text{J} = 2000\,\text{J into the gas}$$

(c) More heat is transfered in *abc* than in *ac* because more work is done in *abc*.

19.43: a) $\Delta U = Q - W = (90.0\,\text{J}) - (60.0\,\text{J}) = 30.0\,\text{J}$ for any path between *a* and *b*. If *W*=15.0 J along path *abd*, then $Q = \Delta U + W = 30.0\,\text{J} + 15.0\,\text{J} = 45.0\,\text{J}.$ b) Along the return path, $\Delta U = -30.0\,\text{J}$, and $Q = \Delta U + W = (-30.0\,\text{J}) + (-35.0\,\text{J}) = -65.0\,\text{J}$; the negative sign indicates that the system liberates heat.
c) In the process $db, dV = 0$ and so the work done in the process *ad* is 15.0 J; $Q_{ad} = (U_d - U_a) + W_{ad} = (8.00\,\text{J}) + (15.0\,\text{J}) = 23.0\,\text{J}$. In the process *db*, $W = 0$ and so $Q_{db} = U_b - U_d = 30.0\,\text{J} - 8.0\,\text{J} = 22.0\,\text{J}$.

19.44: For each process, $Q = \Delta U + W$. No work is done in the processes *ab* and *dc*, and so $W_{bc} = W_{abc}$ and $W_{ad} = W_{adc}$, and the heat flow for each process is: for $ab, Q = 90\ \text{J}$: for bc, $Q = 440\ \text{J} + 450\ \text{J} = 890\ \text{J}$: for $ad, Q = 180\ \text{J} + 120\ \text{J} = 300\ \text{J}$: for $dc, Q = 350\ \text{J}.$ for Q = 350 each process, heat is absorbed in each process. Note that the arrows representing the processes all point the direction of increasing temperature (increasing *U*).

19.45: We will need to use Equations (19.3), $W = p(V_2 - V_1)$ and (17 - 4), $\Delta U = Q - W$.

a) The work done by the system during the process: Along *ab* or *cd, W*=0. Along *bc,* $W_{bc} = p_c(V_c - V_a)$. Along $ad, W_{ad} = p_a(V_c - V_a)$.

b) The heat flow into the system during the process: $Q = \Delta U + W$.

$\Delta U_{ab} = U_b - U_a$, so $Q_{ab} = U_b - U_a + 0.$
$\Delta U_{bc} = U_c - U_b$, so $Q_{bc} = (U_c - U_b) + p_c(V_c - V_a)$.
$\Delta U_{ad} = U_d - U_a$, so $Q_{ad} = (U_d - U_a) + p_a(V_c - V_a)$.
$\Delta U_{dc} = U_c - U_d$, so $Q_{dc} = (U_c - U_d) + 0.$

c) From state *a* to state *c* along path *abc*:

$W_{abc} = p_c(V_c - V_a)$. $Q_{abc} = U_b - U_a + (U_c - U_b) + p_c(V_c - V_a) = (U_c - U_a) + p_c(V_c - V_a)$

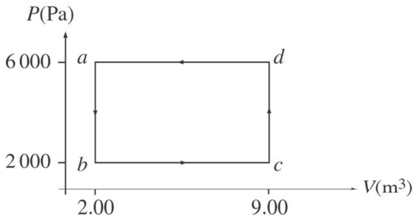
From state *a* to state *c* along path *adc*:

$W_{adc} = p_a(V_c - V_a)$. $Q_{adc} = (U_c - U_a) + p_a(V_c - V_a)$

Assuming $p_c > p_a$, $Q_{abc} > Q_{adc}$, and $W_{abc} > W_{adc}$.

d) To understand this difference, start from the relationship $Q = W + \Delta U$. The internal energy change ΔU is path independent and so it is the same for path *abc* and path *adc*. The work done by the system is the area *under* the path in the *pV*-plane and is *not* the same for the two paths. Indeed, it is larger for path *abc*. Since ΔU is the same and *W* is different, *Q* must be different for the two paths. The heat flow *Q* is path dependent.

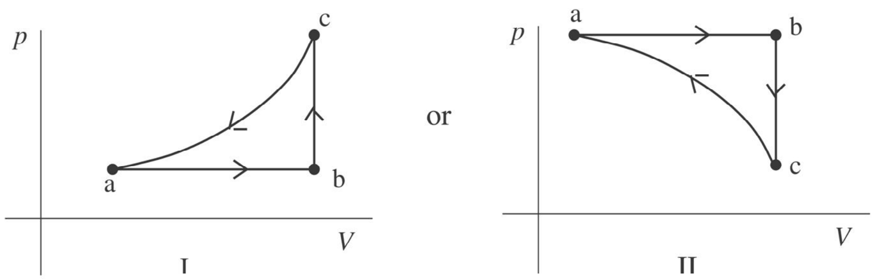
19.46: a)



b)
$$\begin{aligned} W &= W_{ab} + W_{bc} + W_{cd} + W_{da} \\ &= 0 + \text{area}(bc) + 0 + \text{area}(da) \\ &= (7.00\,\text{m}^3)(2000\,\text{Pa}) + (-7.00\,\text{m}^3)(6000\,\text{Pa}) \\ &= -28{,}000\,\text{J};\ \textit{on}\ \text{the gas since}\ W < 0 \end{aligned}$$

c) $Q = \Delta U + W = 0 + (-28{,}000\ \text{J}) = -28{,}000\ \text{J}$
Heat comes *out* of the gas since $Q < 0$.

19.47: a) We aren't told whether the pressure increases or decreases in process *bc*. The cycle could be



In cycle I, the total work is negative and in cycle II the total work is positive. For a cycle, $\Delta U = 0$, so $Q_{\text{tot}} = W_{\text{tot}}$

The net heat flow for the cycle is out of the gas, so heat $Q_{\text{tot}} < 0$ and $W_{\text{tot}} < 0$. Sketch I is correct.

b) $W_{\text{tot}} = Q_{\text{tot}} = -800\,\text{J}$

$W_{\text{tot}} = W_{ab} + W_{bc} + W_{ca}$

$W_{bc} = 0$ since $\Delta V = 0$.

$W_{ab} = p\Delta V$ since p is constant. But since it is an ideal gas, $p\Delta V = nR\Delta T$

$W_{ab} = nR(T_b - T_a) = 1660\,\text{J}$

$W_{ca} = W_{\text{tot}} - W_{ab} = -800\,\text{J} - 1660\,\text{J} = -2460\,\text{J}$

19.48: Path ac has constant pressure, so $W_{ac} = p\Delta V = nR\Delta T$, and

$$\begin{aligned} W_{ac} &= nR(T_c - T_a) \\ &= (3\,\text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(492\,\text{K} - 300\,\text{K}) = 4.789\times10^3\ \text{J}. \end{aligned}$$

Path cb is adiabatic $(Q = 0)$, so $W_{cb} = Q - \Delta U = -\Delta U = -nC_V\Delta T$, and using $C_V = C_p - R$,

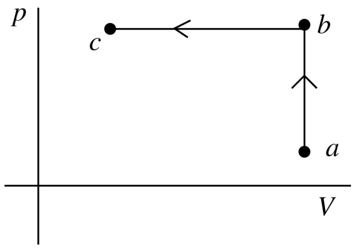
$$W_{cb} = -n(C_p - R)(T_b - T_c)$$

$$= -(3\,\text{mol})(29.1\ \text{J/mol}\cdot\text{K} - 8.3145\ \text{J/mol}\cdot\text{K})(600\,\text{K} - 492\,\text{K}) = -6.735\times10^3\ \text{J}.$$

Path ba has constant volume, so $W_{ba} = 0$. So the total work done is

$$\begin{aligned} W &= W_{ac} + W_{cb} + W_{ba} \\ &= 4.789\times10^3\ \text{J} - 6.735\times10^3\ \text{J} + 0 \\ &= -1.95\times10^3\ \text{J}. \end{aligned}$$

19.49: a)



$T_a = T_c$

19.50: a) $n = \frac{Q}{C_p\Delta T} = \frac{(+2.5\times10^4\ \text{J})}{(29.07\ \text{J/mol}\cdot\text{K})(40.0\ \text{K})} = 21.5\,\text{mol}.$

b) $\Delta U = nC_V\Delta T = Q\frac{C_V}{C_p} = (-2.5\times10^4\ \text{J})\frac{20.76}{29.07} = -1.79\times10^4\ \text{J}.$

c) $W = Q - \Delta U = -7.15\times10^3\ \text{J}.$

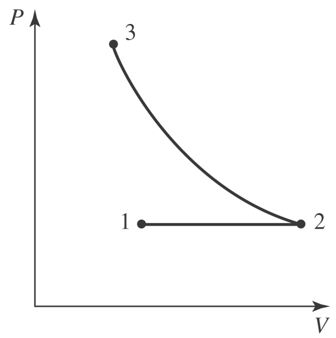
d) ΔU is the same for both processes, and if $dV = 0$, $W = 0$ and $Q = \Delta U = -1.79\times10^4\ \text{J}.$

19.51: $\Delta U = 0$, and so $Q = W = p\Delta V$ and

$$\Delta V = \frac{W}{p} = \frac{(-2.15\times10^5\ \text{J})}{(9.50\times10^5\ \text{Pa})} = -0.226\ \text{m}^3,$$

with the negative sign indicating a decrease in volume.

19.52: a)



b) At constant temperature, the product pV is constant, so $V_2 = V_1(p_1/p_2) = (1.5\ \text{L})\left(\frac{1.00\times10^5\ \text{Pa}}{2.50\times10^4\ \text{Pa}}\right) = 6.00\ \text{L}$. The final pressure is given as being the same as $p_3 = p_2 = 2.5\times10^4$ Pa. The final volume is the same as the initial volume, so $T_3 = T_1(p_3/p_1) = 75.0\ \text{K}$. c) Treating the gas as ideal, the work done in the first process is

$$\begin{aligned} nRT\ln(V_2/V_1) &= p_1V_1\ln(p_1/p_2) \\ &= (1.00\times10^5\ \text{Pa})(1.5\times10^{-3}\ \text{m}^3)\ln\left(\frac{1.00\times10^5\ \text{Pa}}{2.50\times10^4\ \text{Pa}}\right) \\ &= 208\ \text{J}, \end{aligned}$$

keeping an extra figure. For the second process,

$$\begin{aligned} p_2(V_3 - V_2) &= P_2(V_1 - V_2) = p_2V_1(1 - (p_1/p_2)) \\ &= (2.50\times10^4\ \text{Pa})(1.5\times10^{-3}\ \text{m}^3)\left(1 - \frac{1.00\times10^5\ \text{Pa}}{2.50\times10^4\ \text{Pa}}\right) = -113\ \text{J}. \end{aligned}$$

The total work done is $208\ \text{J} - 113\ \text{J} = 95\ \text{J}$. d) Heat at constant volume.

19.53: a) The fractional change in volume is

$$\Delta V = V_0\beta\Delta T = (1.20\times10^{-2}\ \text{m}^3)(1.20\times10^{-3}\ \text{K}^{-1})(30.0\,\text{K}) = 4.32\times10^{-4}\ \text{m}^3.$$

b) $p\Delta V = (F/A)\Delta V = ((3.00\times10^4\ \text{N})/0.0200\,\text{m}^2))(4.32\times10^{-4}\ \text{m}^3) = 648\,\text{J}.$

c) $Q = mC_p\Delta T = V_0\rho C_p\Delta T = (1.20\times10^{-2}\ \text{m}^3)(791\ \text{kg/m}^3)(2.51\times10^3\ \text{J/kg}\cdot\text{K})(30.0\,\text{K}$

$= 7.15\times10^5\ \text{J}.$

d) $\Delta U = Q - W = 7.15\times10^5$ J to three figures. e) Under these conditions, there is no substantial difference between c_V and c_p.

19.54: a) $\beta\Delta TV_0 = (5.1\times10^{-5}(\text{C°})^{-1})(70.0\,\text{C°})(2.00\times10^{-2})^3 = 2.86\times10^{-8}\ \text{m}^3.$

b) $p\Delta V = 2.88\times10^{-3}$ J.

c) $Q = mC\Delta T = \rho V_0 C\Delta T$

$= (8.9\times10^3\ \text{kg/m}^3)(8.00\times10^{-6}\ \text{m}^3)(390\ \text{J/kg}\cdot\text{K})(70.0\,\text{C°})$

$= 1944$ J.

a) To three figures, $\Delta U = Q = 1940$ J. e) Under these conditions, the difference is not substantial.

19.55: For a mass *m* of ejected spray, the heat of reaction *L* is related to the temperature rise and the kinetic energy of the spray by $mL = mC\Delta T - (1/2)mv^2$, or

$$L = C\Delta T - \frac{1}{2}v^2 = (4190\ \text{J/kg}\cdot\text{K})\,(80\,\text{C°}) - \frac{1}{2}(19\ \text{m/s})^2 = 3.4\times10^5\ \text{J/kg}.$$

19.56: Solving Equations (19.22) and (19.24) to eliminate the volumes,

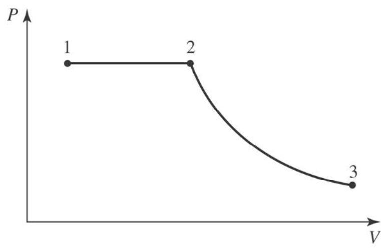
$$p_1^{\gamma-1}T_1^{\gamma} = p_2^{\gamma-1}T_2^{\gamma},\ \text{or}\ T_1 = T_2\left(\frac{p_1}{p_2}\right)^{1-\frac{1}{\gamma}}.$$

Using $\gamma = \frac{7}{5}$ for air, $T_1 = (273.15\ \text{K})(\frac{1.60\times10^6}{2.80\times10^5})^{\frac{2}{7}} = 449\,\text{K}$, which is $176°\text{C}$.

19.57: a) As the air moves to lower altitude its density increases; under an adiabatic compression, the temperature rises. If the wind is fast-moving, Q is not as likely to be signigicant, and modeling the process as adiabatic (no heat loss to the surroundings) is more accurate. b) See Problems 19.59 and 19.56: The temperature at the higher pressure is

$T_2 = (258.15\ \text{K})((8.12\times10^4\ \text{Pa})/(5.60\times10^4\ \text{Pa}))^{2/7} = 287.1\ \text{K}$, which is $13.9°\text{C}$ and so the temperature would rise by 11.9 C°.

19.58: a)



b) The work done is

$$W = p_0(2V_0 - V_0) + \frac{C_V}{R}(p_0(2V_0) - p_3(4V_0)).$$

$p_3 = p_0(2V_0/4V_0)^{\gamma}$ and so

$$W = p_0V_0\left[1 + \frac{C_V}{R}(2 - 2^{2-\gamma})\right]$$

Note that p_0 is the absolute pressure. c) The most direct way to find the temperature is to find the ratio of the final pressure and volume to the original and treat the air as an ideal gas;

$$T_3 = T_0\frac{p_3V_3}{p_1V_1} = T_0\left(\frac{V_2}{V_3}\right)^{\gamma}\left(\frac{V_3}{V_1}\right) = T_0\left(\frac{1}{2}\right)^{\gamma}4 = T_0(2)^{2-\gamma}$$

d) Since $n = \dfrac{p_0V_0}{RT_0}, Q = \dfrac{p_0V_0}{RT_0}(C_V + R)(2T_0 - T_0) = p_0V_0\left(\dfrac{C_V}{R} + 1\right)$. This amount of heat flows into the gas.

19.59: a) From constant cross-section area, the volume is proportional to the length, and Eq. (19.24) becomes $L_2 = L_1(p_1/p_2)^{1/\gamma}$ and the distance the piston has moved is

$$L_1 - L_2 = L_1\left(1-\left(\frac{p_1}{p_2}\right)^{1/\gamma}\right) = (0.250\ \text{m})\left(1-\left(\frac{1.01\times10^5\ \text{Pa}}{5.21\times10^5\ \text{Pa}}\right)^{1/1.400}\right)$$
$$= 0.173\,\text{m}.$$

b) Raising both sides of Eq. (19.22) to the power γ and both sides of Eq. (19.24) to the power $\gamma-1$, dividing to eliminate the terms $V_1^{\gamma(\gamma-1)}$ and $V_2^{\gamma(\gamma-1)}$ and solving for the ratio of the temperatures,

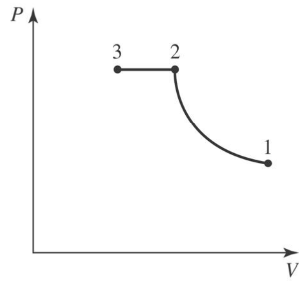
$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-(1/\gamma)} = (300.15\,\text{K})\left(\frac{5.21\times10^5\ \text{Pa}}{1.01\times10^5\ \text{Pa}}\right)^{1-(1/1.400)} = 480\,\text{K} = 206°\text{C}.$$

Using the result of part (a) to find L_2 and then using Eq. (19.22) gives the same result.
c) Of the many possible ways to find the work done, the most straightforward is to use the result of part (b) in Eq. (19.25),

$$W = nC_V\Delta T = (20.0\,\text{mol})(20.8\,\text{J/mol}\cdot\text{K})(179.0\,\text{C}°) = 7.45\times10^4\ \text{J},$$

where an extra figure was kept for the temperature difference.

19.60: a)



b) The final temperature is the same as the initial temperature, and the density is proportional to the absolute pressure. The mass needed to fill the cylinder is then

$$m = \rho_0 V \frac{p}{p_a} = (1.23\ \text{kg/m}^3)(575\times10^{-6}\ \text{m}^3)\frac{1.45\times10^5\,\text{Pa}}{1.01\times10^5\,\text{Pa}} = 1.02\times10^{-3}\ \text{kg}.$$

The increase in power is proportional to the increase in pressure; the percentage increases is $\frac{1.45}{1.01} - 1 = 0.44 = 44\%.$ c) The temperature of the compressed air is not the same as the original temperature; the density is proportional to the pressure, and for the process, and modeled as abiabatic, the volumes are related to the pressure by Eq. (19.24), and the mass of air needed to fill the cylinder is

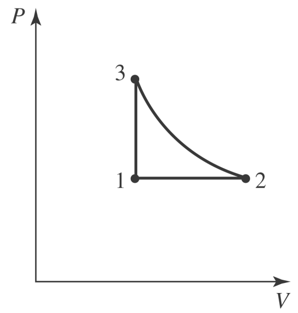
$$m = \rho_0 V\left(\frac{p}{p_a}\right)^{1/\gamma} = (1.23\ \text{kg/m}^3)(575\times10^{-6}\ \text{m}^3)\left(\frac{1.45\times10^5\ \text{Pa}}{1.01\times10^5\ \text{Pa}}\right)^{1/1.40}$$
$$= 9.16\times10^{-4}\ \text{kg},$$

an increase of $(1.45/1.01)^{1/1.04} - 1 = 0.29 = 29\%$

19.61: a) For as isothermal process for an ideal gas, $\Delta T = 0$ and $\Delta U = 0,$ so $Q = W = 300$ J. b) For an adiabatic process, $Q = 0$, and $\Delta U = -W = -300$ J. c) For isobaric, $W = pdV = nR\text{d}T$, or $\text{d}T = \frac{W}{nR}$. Then, $Q = nC_\text{p}\text{d}T$ and substituting for dT gives $Q = nC_p \frac{W}{nR} = C_p \frac{W}{R}$, or $Q = \frac{5}{2} R \frac{W}{R} = \frac{5}{2}(300\,\text{J})$. Thus, $Q = 750\,\text{J}$. To find ΔU, use $\Delta U = nC_V$

Substituting for dT and C_V, $\Delta U = n(\frac{3}{2}R)\frac{W}{nR} = \frac{3}{2}W = 450\,\text{J}$.

19.62: a)



b) The isobaric process doubles the temperature to 710 K, and this must be the temperature of the isothermal process. c) After the isothermal process, the oxygen is at its original volume but twice the original temperature, so the pressure is twice the original pressure, 4.80×10^{5} Pa. d) Break the process into three steps.

$W_1 = -nRT_o = -(0.25 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})(335 \text{ K}) = -738 \text{ J};$

$W_2 = nRT \ln(p_1/p_2) = nR(2T_o)\ \ln(1/2) = (0.250 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})(710 \text{ K})(.693$

$W_3 = 0 \text{ (because } dV = 0).$

Thus, $W = 285 \text{ J}.$

.63: a) During the expansion, the Kelvin temperature doubles and

$\Gamma = 300 \text{ K}. W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})(355 \text{ K}) = 738 \text{ J}, Q = nC_p\Delta T$

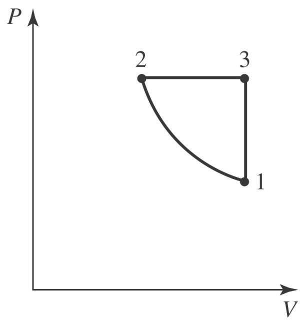
$.250 \text{ mol})(29.17 \text{ J/mol}\cdot\text{K})(355 \text{ K}) = 2590 \text{ J}$ and $\Delta U = nC_V\Delta T = Q - W = 1850 \text{ J}.$ b) The

al cooling is isochoric; $dV = 0$ and so $W = 0.$ The temperature change is

$\Gamma = -355 \text{ K},$ and $Q = \Delta U = nC_V\Delta T = -1850 \text{ J}.$ c) for the isothermal compression,

$\Gamma = 0$ and so $\Delta U = 0.$

.64: a)



b) At constant pressure, halving the volume halves the Kelvin temperature, and the nperature at the beginning of the adiabatic expansion is $150\,\text{K}$. The volume doubles ring the adiabatic expansion, and from Eq. (19.22), the temperature at the end of the pansion is $(150\,\text{K})(1/2)^{0.40} = 114\,\text{K}$. c) The minimum pressure occurs at the end of the iabatic expansion. During the heating the volume is held constant, so the minimum essure is proportional to the Kelvin temperature,
$_{\text{nin}} = (1.80\times10^{5}\ \text{Pa})(113.7\ \text{K}/300\,\text{K}) = 6.82\times10^{4}\ \text{Pa}.$

.65: a) $W = p\Delta V = nR\Delta T = (0.150\,\text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(-150\ \ \text{K}) = -187\,\text{J},$
$= nC_p\Delta T = (0.150\,\text{mol})(29.07\,\text{J/mol}\cdot\text{K})(-150\,\text{K}) = -654\,\text{J}, \Delta U = Q - W = -467\,\text{J}.$

b) From Eq. (19.24), using the expression for the temperature found in Problem .64,

$$W = \frac{1}{0.40}(0.150\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(150\,\text{K})(1-(1/2^{0.40}) = 113\,\text{J},$$

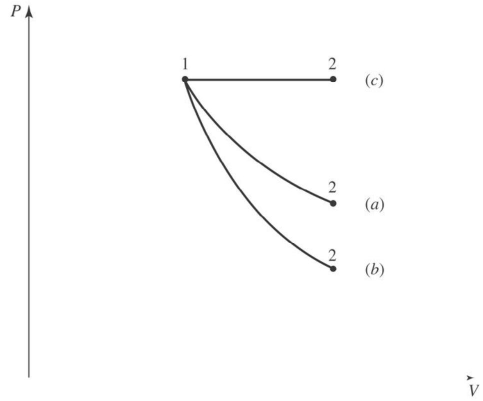
$= 0$ for an adiabatic process, and
$J = Q - W = -W = -113\,\text{J}.$ c) $dV = 0$, so $W = 0$. Using the temperature change as found Problem 19.64 and part (b),
$= nC_V\Delta T = (0.150\,\text{mol})(20.76\,\text{J/mol}\cdot\text{K})(300\,\text{K - }113.7\,\text{K}) = 580\,\text{J}$, and $\Delta U = Q - W = Q =$

.66: a) $W = nRT\ln\left(\frac{V_2}{V_1}\right) = nRT\ln(3) = 3.29\times10^3$ J.

b) See Problem 19.32(b); $nC_V T_1(1-(1/3)^{\frac{2}{3}}) = 2.33\times10^3$ J.

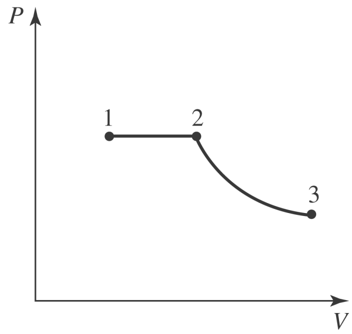
c) $V_2 = 3V_1$, so $p\Delta V = 2pV_1 = 2nRT_1 = 6.00\times10^3$ J.

d)



The most work done is in the isobaric process, as the pressure is maintained at its original value. The least work is done in the abiabatic process. e) The isobaric process involves the most work and the largest temperature increase, and so requires the most heat. Adiabatic processes involve no heat transfer, and so the magnitude is zero. f) The isobaric process doubles the Kelvin temperature, and so has the largest change in internal energy. The isothermal process necessarily involves no change in internal energy.

19.67: a)



b) No heat is supplied during the adiabatic expansion; during the isobaric expansion, the heat added is $nC_p\Delta T$. The Kelvin temperature doubles, so $\Delta T = 300.15\,\text{K}$ and $Q = (0.350\,\text{mol})(34.60\,\text{J/mol}\cdot\text{K})(300.15\,\text{K}) = 3.63\times10^{3}\ \text{J}$. c) For the entire process, $\Delta T = 0$ and so $\Delta U = 0$. d) If $\Delta U = 0$, $W = Q = 3.63\times10^{3}\ \text{J}$. e) During the isobaric expansion, the volume doubles. During the adiabatic expansion, the temperature decreases by a factor of two, and from Eq. (19.22) the volume changes by a factor of $2^{1/(\gamma-1)} = 2^{1/0.33}$, and the final volume is $(14\times10^{-3}\ \text{m}^3)2^{1/0.33} = 0.114\,\text{m}^3$.

19.68: a) The difference between the pressure, multiplied by the area of the piston, must be the weight of the piston. The pressure in the trapped gas is $p_0 + \frac{mg}{A} = p_0 + \frac{mg}{\pi r^2}$.

b) When the piston is a distance $h + y$ above the cylinder, the pressure in the trapped gas is

$$\left(p_0 + \frac{mg}{\pi r^2}\right)\left(\frac{h}{h+y}\right)$$

and for values of y small compared to $h, \dfrac{h}{h+y} = \left(1+\frac{y}{h}\right)^{-1} \sim 1 - \frac{y}{h}$. The net force, taking the positive direction to be upward, is then

$$\begin{aligned} F &= \left[\left(p_0 + \frac{mg}{\pi r^2}\right)\left(1 - \frac{y}{h}\right) - p_0\right]\left(\pi r^2\right) - mg \\ &= -\left(\frac{y}{h}\right)\left(p_0 \pi r^2 + mg\right). \end{aligned}$$

This form shows that for positive h, the net force is down; the trapped gas is at a lower pressure than the equilibrium pressure, and so the net force tends to restore the piston to equilibrium. c) The angular frequency of small oscillations would be

$$\omega^2 = \frac{\left(p_0 \pi r^2 + mg\right)/h}{m} = \frac{g}{h}\left(1 + \frac{p_0 \pi r^2}{mg}\right).$$

If the displacements are not small, the motion is not simple harmonic. This can be seen be considering what happens if $y \sim -h$; the gas is compressed to a very small volume, and the force due to the pressure of the gas would become unboundedly large for a finite displacement, which is not characteristic of simple harmonic motion. If $y >> h$ (but not so large that the piston leaves the cylinder), the force due to the pressure of the gas becomes small, and the restoring force due to the atmosphere and the weight would tend toward a constant, and this is not characteristic of simple harmonic motion.

19.69: a) Solving for p as a function of V and T and integrating with respect to V,

$$p = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

$$W = \int_{V_1}^{V_2} p dV = nRT \ln\left[\frac{V_2 - nb}{V_1 - nb}\right] + an^2\left[\frac{1}{V_2} - \frac{1}{V_1}\right].$$

When $a = b = 0$, $W = nRT \ln(V_2/V_1)$, as expected. b) Using the expression found in part (a),

$$\begin{aligned} \text{i)}\, W &= (1.80\,\text{mol})(8.3145\,\text{J/mol}\cdot\text{K})(300\,\text{K}) \\ &\quad \times \ln\left[\frac{(4.00\times10^{-3}\,\text{m}^3) - (1.80\,\text{mol})(6.38\times10^{-5}\,\text{m}^2/\text{mol})}{(2.00\times10^{-3}\,\text{m}^3) - (1.80\,\text{mol})(6.38\times10^{-5}\,\text{m}^2/\text{mol})}\right] \\ &\quad + (0.554\,\text{J}\cdot\text{m}^3/\text{mol}^2)(1.80\,\text{mol})^2\left[\frac{1}{4.00\times10^{-3}\,\text{m}^3} - \frac{1}{2.00\times10^{-3}\,\text{m}^3}\right] \\ &= 2.80\times10^3\,\text{J}. \end{aligned}$$

$$\text{ii)}\, nRT \ln(2) = 3.11\times10^3\,\text{J}.$$

c) 300 J to two figures, larger for the ideal gas. For this case, the difference due to nonzero a is more than that due to nonzero b. The presence of a nonzero a indicates that the molecules are attracted to each other and so do not do as much work in the expansion.

20.1: a) $2200\,\text{J} + 4300\,\text{J} = 6500\,\text{J}$. b) $\frac{2200}{6500} = 0.338 = 33.8\%$.

20.2: a) $9000\,\text{J} - 6400\,\text{J} = 2600\,\text{J}$. b) $\frac{2600\ \text{J}}{9000\ \text{J}} = 0.289 = 28.9\%$.

20.3: a) $\frac{3700}{16{,}100} = 0.230 = 23.0\%$.

b) $16{,}100\,\text{J} - 3700\,\text{J} = 12{,}400\,\text{J}$.

c) $\frac{16{,}100\ \text{J}}{4.60\times10^4\ \text{J/kg}} = 0.350\,\text{g}$.

d) $(3700\,\text{J})(60.0/\text{s}) = 222\,\text{kW} = 298\,\text{hp}$.

20.4: a) $Q = \frac{1}{e}Pt = \frac{(180\times10^3\,\text{W})(1.00\ \text{s})}{(0.280)} = 6.43\times10^5\ \text{J}$.

b) $Q - Pt = 6.43\times10^5\ \text{J} - (180\times10^3\ \text{W})(1.00\,\text{s}) = 4.63\times10^5\ \text{J}$.

20.5: a) $e = \frac{330\ \text{MW}}{1300\ \text{MW}} = 0.25 = 25\%$. b) $1300\,\text{MW} - 330\,\text{MW} = 970\,\text{MW}$.

20.6: Solving Eq. (20.6) for r,

$$(1-\gamma)\ln r = \ln(1-e) \ \text{ or}$$

$$r = (1-e)^{\frac{1}{1-\gamma}} = (0.350)^{-2.5} = 13.8.$$

If the first equation is used (for instance, using a calculator without the x^y function), note that the symbol "e" is the ideal efficiency , not the base of natural logarithms.

20.7: a) $T_b = T_a r^{\gamma-1} = (295.15\,\text{K})(9.5)^{0.40} = 726\,\text{K} = 453°\text{C}$.

b) $p_b = p_a r^{\gamma} = (8.50\times10^4\ \text{Pa})(9.50)^{\gamma} = 1.99\times10^6\ \text{Pa}$.

20.8: a) From Eq. (20.6), $e = 1 - r^{1-\gamma} = 1-(8.8)^{-0.40} = 0.58 = 58\%$.

b) $1-(9.6)^{-0.40} = 60\%$, an increase of 2%. If more figures are kept for the efficiencies, the difference is 1.4%.

20.9: a) $|W| = \frac{|Q_C|}{K} = \frac{3.40\times10^4\ \text{J}}{2.10} = 1.62\times10^4\ \text{J}$.

b) $|Q_\text{H}| = |Q_\text{C}| + |W| = |Q_\text{C}|(1+\frac{1}{K}) = 5.02\times10^4\ \text{J}$.

20.10: $P = \dfrac{W}{\Delta t} = \dfrac{|Q_{\mathrm{C}}|}{K\Delta t} = \dfrac{1}{K}\left(\dfrac{\Delta m}{\Delta t}\right)(L_{\mathrm{f}} + c_p|\Delta T|)$

$$= \frac{1}{2.8}\left(\frac{8.0\,\mathrm{kg}}{3600\,\mathrm{s}}\right)\big((1.60\times10^{5}\ \mathrm{J/kg}) + (485\ \mathrm{J/kg\cdot K})(2.5\,\mathrm{K})\big) = 128\,\mathrm{W}.$$

20.11: a) $\dfrac{1.44\times10^{5}\ \mathrm{J} - 9.80\times10^{4}\ \mathrm{J}}{60.0\,\mathrm{s}} = 767\,\mathrm{W}$. b) $EER = H/P$, or

$$EER = \frac{(9.8\times10^{4}\ \mathrm{J})/(60\,\mathrm{s})}{[(1.44\times10^{5}\ \mathrm{J})/(60\,\mathrm{s}) - (9.8\times10^{4}\ \mathrm{J})/(60\,\mathrm{s})]}(3.413) = \frac{1633\,\mathrm{W}}{767\,\mathrm{W}}(3.413) = 7.27.$$

20.12: a) $|Q_{\mathrm{C}}| = m(L_f + c_{\mathrm{ice}}|\Delta T_{\mathrm{ice}}| + c_{\mathrm{water}}|\Delta T_{\mathrm{water}}|)$

$$= (1.80\,\mathrm{kg})\big(334\times10^{3}\ \mathrm{J/kg} + (2100\,\mathrm{J/kg\cdot K})(5.0\,\mathrm{K}) + (4190\,\mathrm{J/kg\cdot K})(25.0\,\mathrm{K})\big)$$

$$= 8.90\times10^{5}\ \mathrm{J}.$$

b) $W = \frac{|Q_{\mathrm{C}}|}{K} = \frac{8.08\times10^{5}\ \mathrm{J}}{2.40} = 3.37\times10^{5}\ \mathrm{J}.$

c) $|Q_{\mathrm{H}}| = W + |Q_{\mathrm{C}}| = 3.37\times10^{5}\ \mathrm{J} + 8.08\times10^{5}\ \mathrm{J} = 1.14\times10^{6}\ \mathrm{J}$ (note that $|Q_{\mathrm{H}}| = |Q_{\mathrm{C}}|(1+\frac{1}{K})$.)

20.13: a) $|Q_{\mathrm{H}}| - |Q_{\mathrm{C}}| = 550\,\mathrm{J} - 335\,\mathrm{J} = 215\,\mathrm{J}.$

b) $T_{\mathrm{C}} = T_{\mathrm{H}}(|Q_{\mathrm{C}}|/|Q_{\mathrm{H}}|) = (620\,\mathrm{K})(335\,\mathrm{J}/550\,\mathrm{J}) = 378\,\mathrm{K}.$

c) $1 - (|Q_{\mathrm{C}}|/|Q_{\mathrm{H}}|) = 1 - (335\,\mathrm{J}/550\,\mathrm{J}) = 39\%.$

20.14: a) From Eq. (20.13), the rejected heat is $(\frac{300\,\mathrm{K}}{520\,\mathrm{K}})(6450\,\mathrm{J}) = 3.72\times10^{3}\ \mathrm{J}.$

b) $6450\,\mathrm{J} - 3.72\times10^{3}\ \mathrm{J} = 2.73\times10^{3}\ \mathrm{J}.$

c) From either Eq. (20.4) or Eq. (20.14), e=0.423=42.3%.

20.15: a) $|Q_{\mathrm{H}}| = |Q_{\mathrm{C}}|\dfrac{T_{\mathrm{H}}}{T_{\mathrm{C}}} = mL_{\mathrm{f}}\dfrac{T_{\mathrm{H}}}{T_{\mathrm{C}}}$

$$= (85.0\ \mathrm{kg})(334\times10^{3}\ \mathrm{J/kg})\frac{(287.15\,\mathrm{K})}{(273.15\,\mathrm{K})} = 3.088\times10^{7}\ \mathrm{J},$$

or 3.09×10^{7} J to two figures. b) $|W| = |Q_{\mathrm{H}}| - |Q_{\mathrm{C}}| = |Q_{\mathrm{H}}|(1 - (T_C/T_{\mathrm{H}})) =$ $(3.09\times10^{7}\ \mathrm{J})\times(1 - (273.15/297.15)) = 2.49\times10^{6}\ \mathrm{J}.$

20.16: a) From Eq. (20.13), $(\frac{320\ \text{K}}{270\ \text{K}})(415\ \text{J}) = 492\ \text{J}$. b) The work per cycle is $492\ \text{J} - 415\ \text{J} = 77\ \text{J}$, and $P = (2.75) \times \frac{77\ \text{J}}{1.00\ \text{s}} = 212\ \text{W}$, keeping an extra figure. c) $T_C/(T_H - T_C) = (270\ \text{K})/(50\ \text{K}) = 5.4$.

20.17: For all cases, $|W| = |Q_H| - |Q_C|$. a) The heat is discarded at a higher temperature, and a refrigerator is required; $|W| = |Q_C|\,((T_H/T_C) - 1) = (5.00 \times 10^3\ \text{J}) \times ((298.15/263.15) - 1) = 665\ \text{J}$. b) Again, the device is a refrigerator, and $|W| = |Q_C|\,((273.15/263.15) - 1) = 190\ \text{J}$. c) The device is an engine; the heat is taken form the hot reservoir, and the work done by the engine is $|W| = (5.00 \times 10^3\ \text{J}) \times ((248.15/263.15) - 1) = 285\ \text{J}$.

20.18: For the smallest amount of electrical energy, use a Carnot cycle.

$$\begin{aligned} Q_{\text{in}} &= Q_{\text{Cool water to } 0^\circ\text{C}} + Q_{\text{freeze water}} = mc\Delta T + mL_F \\ &= (5.00\ \text{kg})\left(4190\ \tfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(20\ \text{K}) + (5.00\ \text{kg})(334 \times 10^3\ \text{J/K}) \\ &= 2.09 \times 10^6\ \text{J} \end{aligned}$$

Carnot cycle: $\dfrac{Q_{\text{in}}}{T_{\text{cold}}} = \dfrac{Q_{\text{out}}}{T_{\text{hot}}} \rightarrow \dfrac{2.09 \times 10^6\ \text{J}}{268\text{K}} = \dfrac{Q_{\text{out}}}{293\text{K}}$

$$\begin{aligned} Q_{\text{out}} &= 2.28 \times 10^6\ \text{J(into the room)} \\ W &= Q_{\text{out}} - Q_{\text{in}} = 2.28 \times 10^6\ \text{J} - 2.09 \times 10^6\ \text{J} \\ W &= 1.95 \times 10^5\ \text{J(electrical energy)} \end{aligned}$$

20.19: The total work that must be done is

$W_{\text{tot}} = mgy = (500\ \text{kg})(9.80\ \text{m/s}^2)(100\ \text{m}) = 4.90 \times 10^5\ \text{J}$

$Q_H = 250\ \text{J}$ Find Q_C so can calculate work W done each cycle:

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$$

$$Q_C = -(T_C/T_H)Q_H = -(250\ \text{J})\left[(373.15\ \text{K})/(773.15\ \text{K})\right] = -120.7\ \text{J}$$

$$W = Q_C + Q_H = 129.3\ \text{J}$$

The number of cycles required is $\dfrac{W_{\text{tot}}}{W} = \dfrac{4.09 \times 10^5\ \text{J}}{129.3\ \text{J}} = 3790$ cycles.

20.20: For a heat engine, $Q_{\mathrm{H}} = -Q_{\mathrm{C}}/(1-e) = -(-3000\ \mathrm{J})/(1-0.600) = 7500\ \mathrm{J}$, and then $W = eQ_{\mathrm{H}} = (0.600)(7500\ \mathrm{J}) = 4500\ \mathrm{J}$. This does not make use of the given value of T_{H}. If T_{H} is used, then for a Carnot engine, $T_{\mathrm{C}} = T_{\mathrm{H}}(1-e) = (800\ \mathrm{K})(1-0.600) = 320\ \mathrm{K}$ and $Q_{\mathrm{H}} = -Q_{\mathrm{C}}T_{\mathrm{H}}/T_{\mathrm{C}}$, which gives the same result.

20.21: $Q_{\mathrm{C}} = -mL_{\mathrm{f}} = -(0.0400\mathrm{kg})(334\times10^{3}\ \mathrm{J/kg}) = -1.336\times10^{4}\ \mathrm{J}$

$$\frac{Q_{\mathrm{C}}}{Q_{\mathrm{H}}} = -\frac{T_{\mathrm{C}}}{T_{\mathrm{H}}}$$

$$Q_{\mathrm{H}} = -(T_{\mathrm{H}}/T_{\mathrm{C}})Q_{\mathrm{C}} = -(-1.336\times10^{4}\ \mathrm{J})[(373.15\mathrm{K})/(273.15\mathrm{K})] = +1.825\times10^{4}\ \mathrm{J}$$

$$W = Q_{\mathrm{C}} + Q_{\mathrm{H}} = 4.89\times10^{3}\ \mathrm{J}$$

20.22: The claimed efficiency of the engine is $\frac{1.51\times10^{8}\ \mathrm{J}}{2.60\times10^{8}\ \mathrm{J}} = 58\%$. While the most efficient engine that can operate between those temperatures has efficiency $e_{\mathrm{Carnot}} = 1 - \frac{250\ \mathrm{K}}{400\ \mathrm{K}} = 38\%$. The proposed engine would violate the second law of thermodynamics, and is not likely tc find a market among the prudent.

20.23: a) Combining Eq. (20.14) and Eq. (20.15),

$$K = \frac{T_{\mathrm{C}}/T_{\mathrm{H}}}{1-(T_{\mathrm{C}}/T_{\mathrm{H}})} = \frac{1-e}{(1-(1-e))} = \frac{1-e}{e}.$$

b) As $e \to 1$, $K \to 0$; a perfect $(e = 1)$ engine exhausts no heat $(Q_{\mathrm{C}} = 0)$, and this is useless as a refrigerator. As $e \to 0$, $K \to \infty$; a useless $(e = 0)$ engine does no work $(W = 0)$, and a refrigerator that requires no energy input is very good indeed.

20.24: a) $\dfrac{Q}{T_{\mathrm{C}}} = \dfrac{mL_{\mathrm{f}}}{T_{\mathrm{C}}} = \dfrac{(0.350\mathrm{kg})(334\times10^{3}\ \mathrm{J/kg})}{(273.15\mathrm{k})} = 428\ \mathrm{J/K}$.

b) $\dfrac{-1.17\times10^{5}\ \mathrm{J}}{298.15\mathrm{K}} = -392\ \mathrm{J/K}$.

c) $\Delta S = 428\ \mathrm{J/K} + (-392\ \mathrm{J/K}) = 36\ \mathrm{J/K}$. (If more figures are kept in the intermediate calculations, or if $\Delta S = Q((1/273.15\ \mathrm{K}) - (1/298.15\mathrm{K}))$ is used, $\Delta S = 35.6\ \mathrm{J/K}$.

20.25: a) Heat flows out of the $80.0°\text{C}$ water into the ocean water and the $80.0°\text{C}$ water cools to $20.0°\text{C}$ (the ocean warms, very, very slightly). Heat flow for an isolated system is always in this direction, from warmer objects into cooler objects, so this process is irreversible.

b) $0.100\,\text{kg}$ of water goes form $80.0°\text{C}$ to $20.0°\text{C}$ and the heat flow is $Q = mc\Delta T =$ $(0.100\,\text{kg})(4190\ \text{J/kg}\cdot\text{K})(-60.0\text{C°}) = -2.154\times10^4\ \text{J}$

This Q comes out of the 0.100 kg of water and goes into the ocean.

For the 0.100 kg of water,

$\Delta S = mc\ln(T_2/T_1) = (0.100\,\text{kg})(4190\ \text{J/kg}\cdot\text{K})\ln(293.15/353.15) = -78.02\ \text{J/K}$

For the ocean the heat flow is $Q = +2.154\times10^4\ \text{J}$ and occurs at constant T:

$$\Delta S = \frac{Q}{T} = \frac{2.154\times10^4\ \text{J}}{293.15\,\text{K}} = +85.76\ \text{J/K}$$

$$\Delta S_{\text{net}} = \Delta S_{\text{water}} + \Delta S_{\text{ocean}} = -78.02\ \text{J/K} + 85.76\ \text{J/K} = +7.7\ \text{J/K}$$

20.26: (a) Irreversible because heat will not spontaneously flow out of 15 kg of water into a warm room to freeze the water.

(b) $\Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{room}}$

$$= \frac{mL_{\text{F}}}{T_{\text{ice}}} + \frac{mL_{\text{F}}}{T_{\text{room}}}$$

$$= \frac{(15.0\,\text{kg})(334\times10^3\ \text{J/kg})}{273\,\text{K}} + \frac{-(15.0\,\text{kg})(334\times10^3\ \text{J/kg})}{293\,\text{K}}$$

$$= +1{,}250\ \text{J/K}$$

This result is consistent with the answer in (a) because $\Delta S > 0$ for irreversible processes.

20.27: The final temperature will be

$$\frac{(1.00\,\text{kg})(20.0°\text{C}) + (2.00\,\text{kg})(80.0°\text{C})}{(3.00\,\text{kg})} = 60°\text{C},$$

and so the entropy change is

$$(4190\ \text{J/kg}\cdot\text{K})\left[(1.00\,\text{kg})\ln\left(\frac{333.15\,\text{K}}{293.15\,\text{K}}\right) + (2.00\,\text{kg})\ln\left(\frac{333.15\,\text{K}}{353.15\,\text{K}}\right)\right] = 47.4\ \text{J/K}.$$

20.28: For an isothermal expansion,

$\Delta T = 0, \Delta U = 0$ and $Q = W$. The change of entropy is $\dfrac{Q}{T} = \frac{1850\ \text{J}}{293.15\ \text{K}} = 6.31\ \text{J/K}.$

20.29: The entropy change is $\Delta S = \dfrac{\Delta Q}{T}$, and $\Delta Q = mL_v$. Thus,

$$\Delta S = \frac{-mL_v}{T} = \frac{-(0.13\,\text{kg})(2.09\times10^4\ \text{J/kg})}{(4.216\,\text{K})} = -644\ \text{J/K}.$$

20.30: a) $\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = \frac{(1.00\ \text{kg})(2256\times10^3\ \text{J/kg})}{(373.15\ \text{K})} = 6.05\times10^3\ \text{J/K}.$ Note that this is the change of entropy of the water as it changes to steam. b) The magnitude of the entropy change is roughly five times the value found in Example 20.5. Water is less ordered (more random) than ice, but water is far less random than steam; a consideration of the density changes indicates why this should be so.

20.31: a) $$\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = \frac{(18.0\times10^{-3}\ \text{kg})(2256\times10^3\ \text{J/kg})}{(373.15\,\text{K})} = 109\ \text{J/K}.$$

b) $$\text{N}_2 : \frac{(28.0\times10^{-3}\ \text{kg})(201\times10^3\ \text{J/kg})}{(77.34\,\text{K})} = 72.8\ \text{J/K}$$

$$\text{Ag} : \frac{(107.9\times10^{-3}\ \text{kg})(2336\times10^3\ \text{J/kg})}{(2466\,\text{K})} = 102.2\ \text{J/K}$$

$$\text{Hg} : \frac{(200.6\times10^{-3}\ \text{kg})(272\times10^3\ \text{J/kg})}{(630\,\text{K})} = 86.6\ \text{J/K}$$

c) The results are the same order or magnitude, all around 100 J/K .The entropy change is a measure of the increase in randomness when a certain number (one mole) goes from the liquid to the vapor state. The entropy per particle for any substance in a vapor state is expected to be roughly the same, and since the randomness is much higher in the vapor state (see Exercise 20.30), the entropy change per molecule is roughly the same for these substances.

20.32: a) The final temperature, found using the methods of Chapter 17, is

$$T = \frac{(3.50\,\text{kg})(390\ \text{J/kg}\cdot\text{K})(100\,\text{C}°)}{(3.50\,\text{kg})(390\ \text{J/kg}\cdot\text{K}) + (0.800\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})} = 28.94°\text{C},$$

or $28.9°\text{C}$ to three figures. b) Using the result of Example 20.10, the total change in entropy is (making the conversion to Kelvin temperature)

$$\begin{aligned}\Delta S &= (3.50\,\text{kg})(390\ \text{J/kg}\cdot\text{K})\ln\left(\frac{302.09\,\text{K}}{373.15\,\text{K}}\right)\\ &+ (0.800\,\text{kg})(4190\ \text{J/kg}\cdot\text{K})\ln\left(\frac{302.09\,\text{K}}{273.15\,\text{K}}\right)\\ &= 49.2\ \text{J/K}.\end{aligned}$$

(This result was obtained by keeping even more figures in the intermediate calculation. Rounding the Kelvin temperature to the nearest $0.01\,\text{K}$ gives the same result.

20.33: As in Example 20.8,

$$\Delta S = nR\ln\left(\frac{V_2}{V_1}\right) = (2.00\,\text{mol})(8.3145\,\text{J/mol}\cdot\text{K})\ \ln\left(\frac{0.0420\,\text{m}^3}{0.0280\,\text{m}^3}\right) = 6.74\ \text{J/K}.$$

20.34: a) On the average, each half of the box will contain half of each type of molecule, 250 of nitrogen and 50 of oxygen. b) See Example 20.11. The total change in entropy is

$$\Delta S = kN_1\ln(2) + kN_2\ln(2) = (N_1 + N_2)k\ln(2)$$

$$= (600)(1.381\times10^{-23}\ \text{J/K})\ln(2) = 5.74\times10^{-21}\ \text{J/K}.$$

c) See also Exercise 20.36. The probability is $(1/2)^{500}\times(1/2)^{100} = (1/2)^{600} = 2.4\times10^{-181}$, and is not likely to happen.

The numerical result for part (c) above may not be obtained directly on some standard calculators. For such calculators, the result may be found by taking the log base ten of 0.5 and multiplying by 600, then adding 181 and then finding 10 to the power of the sum. The result is then $10^{-181}\times10^{0.87} = 2.4\times10^{-181}$.

20.35: a) No; the velocity distribution is a function of the mass of the particles, the number of particles and the temperature, none of which change during the isothermal expansion. b) As in Example 20.11, $w_1 = \frac{1}{3}^N w_2$ (the volume has increased, and $w_2 < w_1$); $\ln(w_2/w_1) = \ln(3^N) = N\ln(3)$, and $\Delta S = kN\ln(3) = knN_A\ln(3) = nR\ln(3) = 18.3\ \text{J/K}$.
c) As in Example 20.8, $\Delta S = nR\ln(V_2/V_1) = nR\ln(3)$, the same as the expression used in part (b), and $\Delta S = 18.3\ \text{J/K}$.

20.36: For those with a knowledge of elementary probability, all of the results for this exercise are obtained from

$$P(k) = \binom{n}{k}p^k(1-p)^{n-k} = \frac{4!}{k!(4-k)!}\left(\frac{1}{2}\right)^4,$$

where *P(k)* is the probability of obtaining *k* heads, $n = 4$ and $p = 1 - p = \frac{1}{2}$ for a fair coin. This is of course consistent with Fig. (20.18).

a) $\frac{4!}{4!0!}(1/2)^4 = \frac{4!}{0!4!}(1/2)^4 = \frac{1}{16}$ for all heads or all tails. b) $\frac{4!}{3!1!}(1/2)^4 = \frac{4!}{1!3!} = \frac{1}{4}$.
c) $\frac{4!}{2!2!}(1/2)^4 = \frac{3}{8}$. d) $2\times\frac{1}{16} + 2\times\frac{1}{4} + \frac{3}{8} = 1$. The number of heads must be one of 0, 1, 2, 3 or 4, and there must be unit probability of one and only one of these possibilities.

20.37: a) $Q_H = +400\ \text{J},\ W = +300\ \text{J}$

$W = Q_C + Q_H$, so $Q_C = W - Q_H = -100\ \text{J}$

Since it is a Carnot cycle, $\dfrac{Q_C}{Q_H} = -\dfrac{T_C}{T_H}$

$$T_C = -T_H(Q_C/Q_H) = -(800.15\ \text{K})[(-100\ \text{J})/(400\ \text{J})] = +200\ \text{K} = -73°\text{C}$$

b) Total Q_C required is $-mL_f = -(10.0\ \text{kg})(334\times10^3\ \text{J/kg}) = -3.34\times10^6\ \text{J}$

Q_C for one cycle is $-100\ \text{J}$, so the number of cycles required is

$$\frac{-3.34\times10^6\ \text{J}}{-100\ \text{J/cycle}} = 3.34\times10^4\ \text{cycles}$$

20.38: a) Solving Eq. (20.14) for $T_{\mathrm{H}}, T_{\mathrm{H}} = T_{\mathrm{C}} \dfrac{1}{1-e,}$ so the temperature change

$$T'_{\mathrm{H}} - T_{\mathrm{H}} = T_{\mathrm{C}}\left(\frac{1}{1-e'} - \frac{1}{1-e}\right) = (183.15\,\mathrm{K})\left(\frac{1}{0.55} - \frac{1}{0.600}\right) = 27.8\,\mathrm{K}.$$

b) Similarly, $T_{\mathrm{C}} = T_{\mathrm{H}}(1-e)$, and if $T'_{\mathrm{H}} = T_{\mathrm{H},}$

$$T'_{\mathrm{C}} - T_{\mathrm{C}} = T_{\mathrm{C}} \frac{e'-e}{1-e} = (183.15\,\mathrm{K})\left(\frac{0.050}{0.600}\right) = 15.3\,\mathrm{K}.$$

20.39: The initial volume is $V_1 = \frac{nRT_1}{p_1} = 8.62\times10^{-3}\,\mathrm{m}^3$. a) At point 1, the pressure is given as atmospheric, and $p_1 = 1.01\times10^5$ Pa, with the volume found above, $V_1 = 8.62\times10^{-3}\,\mathrm{m}^3 . V_2 = V_1 = 8.62\times10^{-3}\,\mathrm{m}^3$, and $p_2 = \frac{T_2}{T_1} p_1 = 2p_1 = 2.03\times10^5$ Pa (using $p_a = 1.013\times10^5$ Pa). $p_3 = p_1 = 1.01\times10^5$ Pa and $V_3 = V_1 \frac{T_3}{T_1} = 1.41\times10^{-2}\,\mathrm{m}^3$. b) Process 1 - 2 is isochoric, $\Delta V = 0$ so $W = 0$. $\Delta U = Q = nC_V\Delta T = (0.350\,\mathrm{mol})(5/2)\times(8.3145\,\mathrm{J/mol\cdot K})(300\mathrm{K}) = 2.18\times10^3\,\mathrm{J}$. The process 2 - 3 is adiabatic, $Q = 0$, and $\Delta U = -W = nC_V\Delta T = (0.350\,\mathrm{mol})(5/2)(8.3145\,\mathrm{J/mol\cdot K})(-108\,\mathrm{K}) = -786\,\mathrm{J}\ (W > 0)$. The process 3 - 1 is isobaric; $W = p\Delta V = nR\Delta T = (0.350\,\mathrm{mol})(8.3145\,\mathrm{J/mol\cdot K})(-192\,\mathrm{K}) = -559\,\mathrm{J}$, $\Delta U = nC_V\Delta T = n(5/2)(8.3145\,\mathrm{J/mol\cdot K})(-192\,\mathrm{K}) = -1397\,\mathrm{J}$ and $Q = nC_p\Delta T = (0.350\,\mathrm{mol})(7/2)(8.3145\,\mathrm{J/mol\cdot K})(-192\,\mathrm{K}) = -1956\,\mathrm{J} = \Delta U + W$. c) The net work done is 786 J – 559 J = 227 J. d) Keeping extra figures in the calculations for the process 1 - 2, the heat flow into the engine for one cycle is 2183 J – 1956 J = 227 J. e) $e = \frac{227\,\mathrm{J}}{2183\,\mathrm{J}} = 0.104 = 10.4\%$. For a Carnot - cycle engine operating between 300 K and 600 K, the thermal efficiency is $1 - \frac{300}{600} = 0.500 = 50\%$.

20.40: (a) The temperature at point *c* is $T_c = 1000\,\text{K}$ since from $pV = nRT$, the maximum temperature occurs when the pressure and volume are both maximum. So

$$n = \frac{p_c V_c}{RT_c} = \frac{(6.00\times10^5\,\text{Pa})(0.0300\,\text{m}^3)}{(8.3145\,\text{J/mol}\cdot\text{K})(1000\,\text{K})} = 2.16\,\text{mol}.$$

(b) Heat enters the gas along paths *ab* and *bc,* so the heat input per cycle is $Q_\text{H} = Q_{ac} = W_{ac} + \Delta U_{ac}$. Path *ab* has constant volume and path *bc* has constant pressure, so

$$W_{ac} = W_{ab} + W_{bc} = 0 + p_c(V_c - V_b) = (6.00\times10^5\,\text{Pa})(0.0300\,\text{m}^3 - 0.0100\,\text{m}^3) = 1.20\times10^4\ \text{J}.$$

For an ideal gas,
$\Delta U_{ac} = nC_V(\text{T}_c - T_a) = C_V(p_cV_c - p_aV_a)/R$, using $nT = pV/R$. For $\text{CO}_2, C_V = 28.46\ \text{J/mol.}$
so

$$\Delta U_{ac} = \frac{28.46\,\text{J/mol}\cdot\text{K}}{8.3145\,\text{J/mol}\cdot\text{K}}((6.00\times10^5\ \text{Pa})(0.0300\,\text{m}^3) - (2.00\times10^5\ \text{Pa})(0.0100\,\text{m}^3)) = 5.48\times$$

Then $Q_\text{H} = 1.20\times10^4\ \text{J} + 5.48\times10^4\ \text{J} = 6.68\times10^4\ \text{J}.$

(c) Heat is removed from the gas along paths *cd* and *da*, so the waste heat per cycle is $Q_C = Q_{ca} = W_{ca} + \Delta U_{ca}$. Path *cd* has constant volume and path *da* has constant pressure, so

$$W_{ca} = W_{cd} + W_{da} = 0 + p_d(V_a - V_d) = (2.00\times10^5\ \text{Pa})(0.0100\,\text{m}^3 - 0.0300\,\text{m}^3) = -0.400\times10^4$$

From (b),
$\Delta U_{ca} = -\Delta U_{ac} = -5.48\times10^4\ \text{J}$, so $Q_\text{C} = -0.400\times10^4\ \text{J} - 5.48\times10^4\ \text{J} = -5.88\times10^4\ \text{J}.$

(d) The work is the area enclosed by the rectangular path *abcd,*
$W = (p_c - p_a)(V_c - V_a)$, or $W = Q_\text{H} + Q_\text{C} = 6.68\times10^4\ \text{J} - 5.86\times10^4\ \text{J} = 8000\,\text{J}.$

(e) $e = W/Q_\text{H} = (8000\,\text{J})/(6.68\times10^4\ \text{J}) = 0.120.$

20.41: a) $W = 1.00\,\text{J},\ T_C = 268.15\,\text{K}, T_H = 290.15\,\text{K}$

For the heat pump $Q_C > 0$ and $Q_H < 0$

$W = Q_C + Q_H$; combining this with $\dfrac{Q_C}{Q_H} = -\dfrac{T_C}{T_H}$ gives

$$Q_H = \frac{W}{1 - T_C/T_H} = \frac{1.00\,\text{J}}{1-(268.15/290.15)} = 13.2\,\text{J}$$

b) Electrical energy is converted directly into heat, so an electrical energy input of 13.2 J would be required.

c) From part (a), $Q_H = \dfrac{W}{1 - T_C/T_H} \cdot Q_H$ decrease as T_C decreases.

The heat pump is less efficient as the temperature difference through which the heat has to be "pumped" increases. In an engine, heat flows from T_H to T_C and work is extracted. The engine is more efficient the larger the temperature difference through which the heat flows.

20.42: (a) $Q_{\text{in}} = Q_{ab} + Q_{bc}$

$Q_{\text{out}=}Q_{ca}$

$T_{\max} = T_b = T_c = 327°\text{C} = 600\,\text{K}$

$$\frac{P_aV_a}{T_a} = \frac{P_bV_b}{T_b} \rightarrow T_a = \frac{P_a}{P_b}T_b = \frac{1}{3}(600\,\text{K}) = 200\,\text{K}$$

$$P_bV_b = nRT_b \rightarrow V_b = \frac{nRT_b}{P_b} = \frac{(2\text{ moles})(8.31\,\frac{\text{J}}{\text{mole K}})(600\,\text{K})}{3.0\times10^5\text{ Pa}} = 0.0332\,\text{m}^3$$

$$\frac{P_bV_b}{T_b} = \frac{P_cV_c}{T_c} \rightarrow V_c = V_b\frac{P_b}{P_c} = (0.0332\,\text{m}^3)\left(\frac{3}{1}\right) = 0.0997\,\text{m}^3 = V_a$$

Monatomic gas: $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$

$$Q_{ab} = nC_V\Delta T_{ab} = (2\text{ moles})\left(\frac{3}{2}\right)\left(8.31\frac{\text{J}}{\text{mole K}}\right)(400\,\text{K}) = 9.97\times10^3\text{ J}$$

$$Q_{bc} = W_{bc} = \int_b^c PdV = \int_b^c \frac{nRT_b}{V}dV = nRT_b\ln\frac{V_c}{V_b} = nRT_b\ln 3$$

$$= (2.00\,\text{moles})\left(8.31\frac{\text{J}}{\text{mole K}}\right)(600\,\text{K})\ln 3 = 1.10\times10^4\text{ J}$$

$$Q_{\text{in}} = Q_{ab} + Q_{bc} = 2.10\times10^4\text{ J}$$

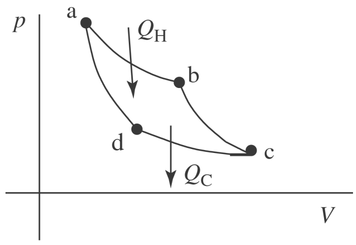
$$Q_{\text{out}} = Q_{ca} = nC_p\Delta T_{ca} = (2.00\,\text{moles})\left(\frac{5}{2}\right)\left(8.31\frac{\text{J}}{\text{mole K}}\right)(400\,\text{K}) = 1.66\times10^4\text{ J}$$

(b) $Q = \Delta U + w = 0 + W \rightarrow W = Q_{\text{in}} - Q_{\text{out}} = 2.10\times10^4\text{ J} - 1.66\times10^4\text{ J} = 4.4\times10^3\text{ J}$

$$e = W/Q_{\text{in}} = \frac{4.4\times10^3\text{ J}}{2.10\times10^4\text{ J}} = 0.21 = 21\%$$

(c) $e_{\max} = e_{\text{cannot}} = 1 - \frac{T_C}{T_h} = 1 - \frac{200\text{ K}}{600\text{ K}} = 0.67 = 67\%$

20.43: a)



b) $Q_H = +500\ \text{J}$

$W = mgy = (15.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m}) = 294\ \text{J}$

$W = Q_C + Q_H,\ Q_C = W - Q_H = 294\ \text{J} - 500\ \text{J} = -206\ \text{J}$

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$$

$T_C = -T_H(Q_C/Q_H) = -(773\ \text{K})[(-206\ \text{J})/(500\ \text{J})] = +318\ \text{K} = 45°\text{C}$

c) $e = W/Q_H = (294\ \text{J})/(500\ \text{J}) = 58.8\%$

d) $Q_C = -206\ \text{J}$; wastes 206 J of heat each cycle

e) From part (a), state a has the maximum pressure and minimum volume.

$$pV = nRT,\ p = \frac{nRT}{V} = \frac{(2.00\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(773\ \text{K})}{5.00\times10^{-3}\ \text{m}^3} = 2.57\times10^6\ \text{Pa}$$

20.44: a) $e = 1 - \frac{279.15\ \text{K}}{300.15\ \text{K}} = 7.0\%$. b) $\frac{P_{\text{out}}}{e} = \frac{210\ \text{kW}}{0.070} = 3.0\ \text{MW},\ 3.0\ \text{MW} - 210\ \text{kW} = (\frac{1}{e} - 1)(210\ \text{kW}) = 2.8\ \text{MW}$.

c) $$\frac{dm}{dt} = \frac{d|Q_C|/dt}{c\Delta T} = \frac{(2.8\times10^6\ \text{W})\,(3600\ \text{s/hr})}{(4190\ \text{J/kg}\cdot\text{K})\,(4\ \text{K})} = 6\times10^5\ \text{kg/hr} = 6\times10^5\ \text{L/hr}.$$

20.45: There are many equivalent ways of finding the efficiency; the method presented here saves some steps. The temperature at point 3 is $T_3 = 4T_0$, and so

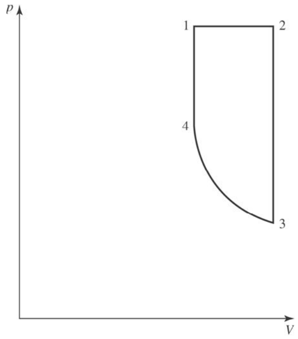
$$Q_H = \Delta U_{13} + W_{13} = nC_V(T_3 - T_0) + (2p_0)(2V_0 - V_0) = \frac{5}{2}nRT_0(3) + 2p_0V_0 = \frac{19}{2}p_0V_0,$$

where $nRT_0 = p_0V_0$ has been used for an ideal gas. The work done by the gas during one cycle is the area enclosed by the blue square in Fig. (20.22), $W = p_0V_0$, and so the efficiency is $e = \frac{W}{Q_H} = \frac{2}{19} = 10.5\%$.

20.46: a) $p_2 = p_1 = 2.00$ atm,

$V_2 = V_1 \frac{T_2}{T_1} = (4.00\,\text{L})(3/2) = 6.00\,\text{L}$. $V_3 = V_2 = 6.00\,\text{L}$, $p_3 = p_2 \frac{T_3}{T_2} = p_2(5/9) = 1.111\,\text{atm}$, $p_4 = p_3 \frac{V_3}{V_4} = p_3(3/2) = 1.67\,\text{atm}$. As a check, $p_1 = p_4 \frac{T_1}{T_4} = p_4(6/5) = 2.00$ atm. To summarize,

$$(p_1, V_1) = (2.00\text{ atm, }4.00\text{ L}) \qquad (p_2, V_2) = (2.00\text{ atm, }6.00\text{ L})$$
$$(p_3, V_3) = (1.111\text{ atm, }6.00\text{ L}) \qquad (p_4, V_4) = (1.67\text{ atm, }4.00\text{ L}).$$



b) The number of moles of oxygen is $n = \frac{p_1 V_1}{RT_1}$, and the heat capacities are those in Table (19.1). The product $p_1 V_1$ has the value $x = 810.4\,\text{J}$; using this and the ideal gas law,

i: $$Q = nC_P\Delta T = \frac{C_P}{R}x\left(\frac{T_2}{T_1} - 1\right) = (3.508)(810.4\,\text{J})(1/2) = 1422\,\text{J},$$

$$W = p_1\Delta V = x\left(\frac{T_2}{T_1} - 1\right) = (810.4\,\text{J})(1/2) = 405\,\text{J}.$$

ii: $$Q = nC_V\Delta T = \frac{C_V}{R}x\left(\frac{T_3 - T_2}{T_1}\right) = (2.508)(810.4\,\text{J})(-2/3) = -1355\text{ J, W} = 0.$$

iii: $$W = nRT_3\ln\left(\frac{V_4}{V_3}\right) = x\frac{T_3}{T_1}\ln\left(\frac{V_4}{V_3}\right) = (810.4\text{ J})(5/6)\ln(2/3) = -274\,\text{J},\ Q = W$$

iv: $$Q = nC_V\Delta T = \frac{C_V}{R}x\left(1 - \frac{T_4}{T_1}\right) = (2.508)(810.4\,\text{J})(1/6) = 339\,\text{J, W} = 0.$$

In the above, the terms are given to nearest integer number of joules to reduce roundoff error.

c) The net work done in the cycle is $405\,\text{J} - 274\,\text{J} = 131\,\text{J}$.

d) Heat is added in steps i and iv, and the added heat is $1422\,\text{J} + 339\,\text{J} = 1761\,\text{J}$ and the efficiency is $\frac{131\text{ J}}{1761\text{ J}} = 0.075$, or 7.5%. The efficiency of a Carnot-cycle engine operating between 250 K and 450 K is $1 - \frac{250}{450} = 0.44 = 44\%$.

20.47: a) $\Delta U = 1657\,\text{kJ} - 1005\,\text{kJ} = 6.52 \times 10^5\ \text{J}, W = p\Delta V = (363 \times 10^3\ \text{Pa}) \times (0.4513\,\text{m}^3 - 0.2202\,\text{m}^3) = 8.39 \times 10^4\ \text{J}$, and so $Q = \Delta U + W = 7.36 \times 10^5\ \text{J}$.

b) Similarly,

$$\begin{aligned} Q_\text{H} &= \Delta U - p\Delta V \\ &= (1171\,\text{kJ} - 1969\,\text{kJ}) + (2305 \times 10^3\ \text{Pa})(0.00946\,\text{m}^3 - 0.0682\,\text{m}^3) \\ &= -9.33 \times 10^5\ \text{J}. \end{aligned}$$

c) The work done during the adiabatic processes must be found indirectly (the coolant is not ideal, and is not always a gas). For the entire cycle, $\Delta U = 0$, and so the net work done by the coolant is the sum of the results of parts (a) and (b), $-1.97 \times 10^5\ \text{J}$. The work done by the motor is the negative of this, $1.97 \times 10^5\ \text{J}$. d) $K = \frac{|Q_c|}{|W|} = \frac{7.36 \times 10^5\ \text{J}}{1.97 \times 10^5\ \text{J}} = 3.74.$

20.48: For a monatomic ideal gas, $C_P = \frac{5}{2}R$ and $C_V = \frac{3}{2}R$.

a) *ab*: The temperature changes by the same factor as the volume, and so

$$Q = nC_P\Delta T = \frac{C_P}{R}p_a(V_a - V_b) = (2.5)(3.00 \times 10^5\ \text{Pa})(0.300\,\text{m}^3) = 2.25 \times 10^5\ \text{J}.$$

The work $p\Delta V$ is the same except for the factor of $\frac{5}{2}$, so $W = 0.90 \times 10^5\ \text{J}$.

$\Delta U = Q - W = 1.35 \times 10^5\ \text{J}$.

bc: The temperature now changes in proportion to the pressure change, and $Q = \frac{3}{2}(p_c - p_b)V_b = (1.5)(-2.00 \times 10^5\ \text{Pa})(0.800\,\text{m}^3) = -2.40 \times 10^5\ \text{J}$, and the work is zero $(\Delta V = 0)$. $\Delta U = Q - W = -2.40 \times 10^5\ \text{J}$.

ca: The easiest way to do this is to find the work done first; *W* will be the negative of area in the *p-V* plane bounded by the line representing the process *ca* and the verticals from points *a* and *c*. The area of this trapezoid is $\frac{1}{2}(3.00 \times 10^5\ \text{Pa} + 1.00 \times 10^5\ \text{Pa}) \times (0.800\,\text{m}^3 - 0.500\,\text{m}^3) = 6.00 \times 10^4\ \text{J}$, and so the work is $-0.60 \times 10^5\ \text{J}$. ΔU must be $1.05 \times 10^5\ \text{J}$ (since $\Delta U = 0$ for the cycle, anticipating part (b)), and so *Q* must be $\Delta U + W = 0.45 \times 10^5\ \text{J}$.

b) See above; $Q = W = 0.30 \times 10^5\ \text{J},\ \Delta U = 0.$

c) The heat added, during process *ab* and *ca*, is $2.25 \times 10^5\ \text{J} + 0.45 \times 10^5\ \text{J} = 2.70 \times 10^5\ \text{J}$ and the efficiency is $\frac{W}{Q_\text{H}} = \frac{0.30 \times 10^5}{2.70 \times 10^5} = 0.111 = 11.1\%.$

20.49: a) *ab*: For the isothermal process, $\Delta T = 0$ and $\Delta U = 0.W = nRT_1 \ln(V_b/V_a) = nRT_1 \ln(1/r) = -nRT_1 \ln(r)$, and $Q = W = -nRT_1 \ln(r)$. *bc:* For the isochoric process, $\Delta V = 0$ and $W = 0; Q = \Delta U = nC_V \Delta T = nC_V (T_2 - T_1)$. *cd:* As in the process *ab*, $\Delta U = 0$ and $W = Q = nRT_2 \ln(r)$. *da:* As in process *bc*, $\Delta V = 0$ and $W = 0$; $\Delta U = Q = nC_V (T_1 - T_2)$. b) The values of Q for the processes are the negatives of each other. c) The net work for one cycle is $W_{\text{net}} = nR(T_2 - T_1)\ln(r)$, and the heat added (neglecting the heat exchanged during the isochoric expansion and compression, as mentioned in part (b)) is $Q_{\text{cd}} = nRT_2 \ln(r)$, and the efficiency is $\frac{W_{net}}{Q_{cd}} = 1 - (T_1/T_2)$. This is the same as the efficiency of a Carnot-cycle engine operating between the two temperatures.

20.50: The efficiency of the first engine is $e_1 = \frac{T_{\text{H}} - T'}{T_{\text{H}}}$ and that of the second is $e_2 = \frac{T' - T_{\text{C}}}{T'}$, and the overall efficiency is

$$e = e_1 e_2 = \left[\frac{T_{\text{H}} - T'}{T_{\text{H}}}\right]\left[\frac{T' - T_{\text{C}}}{T'}\right].$$

The first term in the product is necessarily less than the original efficiency since $T' > T_{\text{C}}$, and the second term is less than 1, and so the overall efficiency has been reduced.

20.51: a) The cylinder described contains a mass of air $m = \rho(\pi d^2/4)L$, and so the total kinetic energy is $K = \rho(\pi/8)d^2 L v^2$. This mass of air will pass by the turbine in a time $t = L/v$, and so the maximum power is

$$P = \frac{K}{t} = \rho(\pi/8)d^2 v^3.$$

Numerically, the product $\rho_{\text{air}}(\pi/8) \approx 0.5\,\text{kg/m}^3 = 0.5\,\text{W}\cdot\text{s}^4/\text{m}^5$.

b) $v = \left(\frac{P/e}{kd^2}\right)^{1/3} = \left(\frac{(3.2\times10^6\ \text{W})/(0.25)}{(0.5\ \text{W}\cdot\text{s}^4/\text{m}^5)(97\ \text{m})^2}\right)^{1/3} = 14\ \text{m/s} = 50\ \text{km/h}.$

c) Wind speeds tend to be higher in mountain passes.

20.52: a) $(105\,\text{km/h})\left(\dfrac{1\,\text{gal}}{25\,\text{mi}}\right)\left(\dfrac{1\,\text{mi}}{1.609\,\text{km}}\right)\left(\dfrac{3.788\,\text{L}}{1\,\text{gal}}\right) = 9.89\,\text{L/h}.$

b) From Eq. (20.6), $e = 1 - r^{1-\gamma} = 1 - (8.5)^{-0.40} = 0.575 = 57.5\%.$

c) $\left(\dfrac{9.89\,\text{L/h}}{3600\,\text{s/hr}}\right)(0.740\,\text{kg/L})(4.60\times10^{7}\ \text{J/kg})(0.575) = 5.38\times10^{4}\,\text{W} = 72.1\,\text{hp}.$

d) Repeating the calculation gives $1.4\times10^{4}\,\text{W} = 19\,\text{hp},$ about 8% of the maximum power.

20.53: (Extra figures are given in the numerical answers for clarity.) a) The efficiency is $e = 1 - r^{-0.40} = 0.611$, so the work done is $Q_H e = 122\ \text{J}$ and $|Q_C| = 78\ \text{J}$. b) Denote the length of the cylinder when the piston is at point *a* by L_0 and the stroke as *s*. Then, $\frac{L_0}{L_0 - s} = r,\ L_0 = \frac{r}{r-1} s$ and volume is

$$L_0 A = \frac{r}{r-1} sA = \frac{10.6}{9.6}(86.4\times10^{-3}\ \text{m})\pi(41.25\times10^{-3}\ \text{m})^2 = 51.0\times10^{-4}\ \text{m}^3.$$

c) The calculations are presented symbolically, with numerical values substituted at the end. At point *a*, the pressure is $p_a = 8.50\times10^4$ Pa, the volume is $V_a = 5.10\times10^{-4}\ \text{m}^3$ as found in part (b) and the temperature is $T_a = 300\ \text{K}$. At point *b*, the volume is $V_b = V_a/r$, the pressure after the adiabatic compression is $p_b = p_a r^{\gamma}$ and the temperature is $T_b = T_a r^{\gamma-1}$. During the burning of the fuel, from *b* to *c*, the volume remains constant and so $V_c = V_b = V_a/r$. The temperature has changed by an amount

$$\Delta T = \frac{Q_\text{H}}{nC_V} = \frac{Q_\text{H}}{(p_a V_a/RT_a)C_V} = \frac{RQ_\text{H}}{p_a V_a C_V} T_a$$
$$= \frac{(8.3145\ \text{J/mol}\cdot\text{K})(200\ \text{J})}{(8.50\times10^4\ \text{Pa})(5.10\times10^{-4}\ \text{m}^3)(20.5\ \text{J/mol}\cdot\text{K})} T_a = f\,T_a,$$

where *f* is a dimensionless constant equal to 1.871 to four figures. The temperature at *c* is then $T_c = T_b + f\,T_a = T_a(r^{\gamma-1} + f)$. The pressure is found from the volume and temperature, $p_c = p_a r(r^{\gamma-1} + f)$. Similarly, the temperature at point *d* is found by considering the temperature change in going from *d* to *a*,
$\left|\frac{Q_\text{C}}{nC_V}\right| = (1-e)\frac{Q_\text{H}}{nC_V} = (1-e) f\,T_a$, so $T_d = T_a(1 + (1-e) f)$. The process from *d* to *a* is isochoric, so $V_d = V_a$, and $p_d = p_a(1 + (1-e) f)$. As a check, note that $p_d = p_c r^{-\gamma}$. To summarize,

	p	V	T
a	p_a	V_a	T_a
b	$p_a r^{\gamma}$	V_a/r	$T_a r^{\gamma-1}$
c	$p_a r(r^{\gamma-1} + f)$	V_a/r	$T_a(r^{\gamma-1} + f)$
d	$p_a(1 + (1-e)f)$	V_a	$T_a(1 + (1-e) f)$

Using numerical values (and keeping all figures in the intermediate calculations),

20.54: (a) $\dfrac{\Delta Q}{\Delta t} = k\ A\dfrac{\Delta T}{L}$ for furnace and water

$$\begin{aligned}
\frac{\Delta S}{\Delta t} &= \frac{\Delta S_{\text{furnace}}}{\Delta t} + \frac{\Delta S_{\text{water}}}{\Delta t} \\
&= -\frac{kA\Delta T/L}{T_{\text{f}}} + \frac{kA\Delta T/L}{T_{\text{w}}} \\
&= \frac{kA\Delta T}{L}\left(-\frac{1}{T_{\text{f}}} + \frac{1}{T_{\text{w}}}\right) \\
&= \frac{(79.5\ \text{W/m}\cdot\text{K})}{0.65\text{m}}\left[15\ \text{cm}^2\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2\right](210\ \text{K})\left(-\frac{1}{523\ \text{K}} + \frac{1}{313\ \text{K}}\right) \\
&= +0.0494\ \text{J/K}\cdot\text{s}
\end{aligned}$$

(b) $\Delta S > 0$ means that this process is irreversible. Heat will not flow spontaneously from the cool water into the hot furnace.

20.55: a) Consider an infinitesimal heat flow dQ_{H} that occurs when the temperature of the hot reservoir is T':

$$dQ_{\mathrm{C}} = -(T_{\mathrm{C}} / T')dQ_{\mathrm{H}}$$
$$\int dQ_{\mathrm{C}} = -T_{\mathrm{C}} \int \frac{dQ_{\mathrm{H}}}{T'}$$
$$|Q_{\mathrm{C}}| = T_{\mathrm{C}} \left| \int \frac{dQ_{\mathrm{H}}}{T'} \right| = T_C \, |\Delta S_{\mathrm{H}}|$$

b) The 1.00 kg of water (the high-temperature reservoir) goes from 373 K to 273 K.

$$Q_{\mathrm{H}} = mc\Delta T = (1.00\,\mathrm{kg})(4190\,\mathrm{J/kg \cdot K})(100\,\mathrm{K}) = 4.19 \times 10^{5}\ \mathrm{J}$$
$$\Delta \mathrm{S}_h = mc\ln(T_2/T_1) = (1.00\,\mathrm{kg})(4190\,\mathrm{J/kg \cdot K})\ln(273/373) = -1308\,\mathrm{J/K}$$
The result of part (a) gives $|Q_{\mathrm{C}}| = (273\,\mathrm{K})(1308\,\mathrm{J/K}) = 3.57 \times 10^{5}$ J

Q_{C} comes out of the engine, so $Q_{\mathrm{C}} = -3.57 \times 10^{5}$ J

Then $W = Q_{\mathrm{C}} + Q_{\mathrm{H}} = -3.57 \times 10^{5}\ \mathrm{J} + 4.19 \times 10^{5}\ \mathrm{J} = 6.2 \times 10^{4}$ J.

c) 2.00 kg of water goes from 323 K to 273 K

$$Q_{\mathrm{H}} = mc\Delta T = (2.00\,\mathrm{kg})(4190\,\mathrm{J/kg \cdot K})(50\,\mathrm{K}) = 4.19 \times 10^{5}\ \mathrm{J}$$
$$\Delta S_h = mc\ln(T_2/T_1) = (2.00\,\mathrm{kg})(4190\,\mathrm{J/kg \cdot K})\ln(273/323) = -1.41 \times 10^{3}\ \mathrm{J/K}$$
$$Q_{\mathrm{C}} = -T_{\mathrm{C}}\,|\Delta S_{\mathrm{h}}| = -3.85 \times 10^{5}\ \mathrm{J}$$
$$W = Q_{\mathrm{C}} + Q_{\mathrm{H}} = 3.4 \times 10^{4}\ \mathrm{J}$$

d) More work can be extracted from 1.00 kg of water at 373 K than from 2.00 kg of water at 323 K even though the energy that comes out of the water as it cools to 273 K is the same in both cases. The energy in the 323 K water is less available for conversion into mechanical work.

20.56: See Figure (20.15(c)), and Example 20.8.

a) For the isobaric expansion followed by the isochoric process, follow a path from T to $2T$ to T. Use $\mathrm{d}Q = nC_V\mathrm{d}T$ or $\mathrm{d}Q = nC_p\mathrm{d}T$ to get $\Delta S = nC_p\ln 2 + nC_V \ln\frac{1}{2} = n(C_p - C_V)\ln 2 = nR\ln 2.$

b) For the isochoric cooling followed by the isobaric expansion, follow a path from T to $T/2$ to T. Then $\Delta S = nC_V\ln\frac{1}{2} + nC_p\ln 2 = n(C_p - C_V)\ln = nR\ln 2.$

20.57: The much larger mass of water suggests that the final state of the system will be water at a temperature between $0°\text{C}$ and $60.0°\text{C}$. This temperature would be

$$T = \frac{\begin{pmatrix}(0.600\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(45.0\text{C}°) \\ -(0.0500\,\text{kg})((2100\,\text{J/kg}\cdot\text{K})(15.0\text{C}°) \\ +334\times 10^3\ \text{J/kg})\end{pmatrix}}{(0.650\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})} = 34.83°\text{C},$$

keeping an extra figure. The entropy change of the system is then

$$\Delta S = (0.600\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})\ln\left(\frac{307.98}{318.15}\right)$$

$$+(0.0500\,\text{kg})\begin{bmatrix}(2100\,\text{J/kg}\cdot\text{K})\ln\left(\frac{273.15}{258.15}\right) \\ +\frac{334\times 10^3\ \text{J/kg}}{273.15\,\text{K}} \\ +(4190\,\text{J/kg}\cdot\text{K})\ln\left(\frac{307.98}{273.15}\right)\end{bmatrix} = 10.5\,\text{J/K}.$$

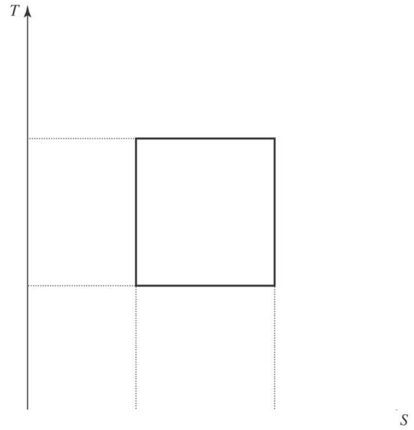
(Some precision is lost in taking the logarithms of numbers close to unity.)

20.58: a) For constant-volume processes for an ideal gas, the result of Example 20.10 may be used; the entropy changes are $nC_V\ \ln(T_c/T_b)$ and $nC_V\ \ln(T_a/T_d)$. b) The total entropy change for one cycle is the sum of the entropy changes found in part (a); the other processes in the cycle are adiabatic, with $Q = 0$ and $\Delta S = 0$. The total is then

$$\Delta S = nC_V\ \ln\frac{T_c}{T_b} + nC_V\ \ln\frac{T_a}{T_d} = nC_V\ \ln\left(\frac{T_cT_a}{T_aT_d}\right).$$

From the derivation of Eq. (20.6), $T_b = r^{\gamma-1}T_a$ and $T_c = r^{\gamma-1}T_d$, and so the argument of the logarithm in the expression for the net entropy change is 1 identically, and the net entropy change is zero. c) The system is not isolated, and a zero change of entropy for an irreversible system is certainly possible.

20.59: a)



b) From Eq. (20.17), $dS = \frac{dQ}{T}$, and so $dQ = T\ dS$, and

$$Q = \int dQ = \int T\ dS$$

which is the area under the curve in the *TS* plane. c) Q_H is the area under the rectangle bounded by the horizontal part of the rectangle at T_H and the verticals. $|Q_C|$ is the area bounded by the horizontal part of the rectangle at T_C and the verticals. The net work is then $Q_H - |Q_C|$, the area bounded by the rectangle that represents the process. The ratio of the areas is the ratio of the lengths of the vertical sides of the respective rectangles, and the efficiency is $e = \frac{W}{Q_H} = \frac{T_H - T_C}{T_H}$. d) As explained in problem 20.49, the substance that mediates the heat exchange during the isochoric expansion and compression does not leave the system, and the diagram is the same as in part (a). As found in that problem, the ideal efficiency is the same as for a Carnot-cycle engine.

20.60: a) $\Delta S = \frac{Q}{T} = -\frac{mL_f}{T} = -\frac{(0.160\ \text{kg})\ (334{\times}10^3\ \text{J/kg})}{(373.15\ \text{K})} = -143\ \text{J/K}.$

b) $\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{(0.160\ \text{kg})(334\times10^3\ \text{J/kg})}{(273.15\ \text{K})} = 196\ \text{J/K}.$

c) From the time equilbrium has been reached, there is no heat exchange between the rod and its surroundings (as much heat leaves the end of the rod in the ice as enters at the end of the rod in the boiling water), so the entropy change of the copper rod is zero. d) $196\ \text{J/K} - 143\ \text{J/K} = 53\ \text{J/K}.$

20.61: a)

$$\begin{aligned}\Delta S &= mc\ln(T_2/T_1)\\ &= (250\times10^{-3}\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})\ln(338.15\ \text{K}/293.15\ \text{K})\\ &= 150\ \text{J/K}.\end{aligned}$$

b) $\Delta S = \frac{-mc\Delta T}{T_{\text{element}}} = \frac{-(250\times10^{-3}\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(338.15\ \text{K}-293.15\ \text{K})}{393.15\ \text{K}} = -120\ \text{J/K}.$ c) The sum of the result of parts (a) and (b) is $\Delta S_{\text{system}} = 30\ \text{J/K}.$ d) Heating a liquid is not reversible. Whatever the energy source for the heating element, heat is being delivered at a higher temperature than that of the water, and the entropy loss of the source will be less in magnitude than the entropy gain of the water. The net entropy change is positive.

20.62: a) As in Example 20.10, the entropy change of the first object is $m_1c_1\ln(T/T_1)$ and that of the second is $m_2c_2\ln(T'/T_2)$, and so the net entropy change is as given. Neglecting heat transfer to the surroundings, $Q_1 + Q_2 = 0, m_1c_1(T - T_1) + m_2c_2(T' - T_2) = 0$, which is the given expression. b) Solving the energy-conservation relation for T' and substituting into the expression for ΔS gives

$$\Delta S = m_1c_1\ln\left(\frac{T}{T_1}\right) + m_2c_2\ln\left(1 - \frac{m_1c_1}{m_2c_2}\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right).$$

Differentiating with respect to T and setting the derivative equal to 0 gives

$$0 = \frac{m_1c_1}{T} + \frac{(m_2c_2)(m_1c_1/m_2c_2)(-1/T_2)}{\left(1-(m_1c_1/m_2c_2)\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right)}.$$

This may be solved for

$$T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2},$$

which is the same as T' when substituted into the expression representing conservation of energy.

Those familiar with Lagrange multipliers can use that technique to obtain the relations

$$\frac{\partial}{\partial T}\Delta S = \lambda\frac{\partial Q}{\partial T}, \qquad \frac{\partial}{\partial T'}\Delta S = \lambda\frac{\partial Q}{\partial T'}$$

and so conclude that $T = T'$ immediately; this is equivalent to treating the differentiation as a related rate problem, as

$$\frac{d}{dT'}\Delta S = \frac{m_1c_1}{T} + \frac{m_2c_2}{T'}\frac{dT'}{dT} = 0$$

and using $\frac{dT'}{dT} = -\frac{m_1c_1}{m_2c_2}$ gives $T = T'$ with a great savings of algebra.

c) The final state of the system will be that for which no further entropy change is possible. If $T < T'$, it is possible for the temperatures to approach each other while increasing the total entropy, but when $T = T'$, no further spontaneous heat exchange is possible.

20.63: a) For an ideal gas, $C_P = C_V + R$, and taking air to be diatomic, $C_P = \frac{7}{2}R, C_V = \frac{5}{2}R$ and $\gamma = \frac{7}{5}$. Referring to Fig. (20.6), $Q_{\mathrm{H}} = n\frac{7}{2}R(T_c - T_b) = \frac{7}{2}(p_cV_c - p_bV_b)$. Similarly, $Q_{\mathrm{C}} = n\frac{5}{2}R(p_aV_a - p_dV_d)$. What needs to be done is to find the relations between the product of the pressure and the volume at the four points.

For an ideal gas, $\frac{p_cV_c}{T_c} = \frac{p_bV_b}{T_b}$, so $p_cV_c = p_aV_a\left(\frac{T_c}{T_a}\right)$. For a compression ratio *r*, and given that for the Diesel cycle the process *ab* is adiabatic,

$$p_bV_b = p_aV_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = p_aV_ar^{\gamma-1}.$$

Similarly, $p_dV_d = p_cV_c\left(\frac{V_c}{V_a}\right)^{\gamma-1}$. Note that the last result uses the fact that process *da* is isochoric, and $V_d = V_a$; also, $p_c = p_b$ (process *bc* is isobaric), and so $V_c = V_b\left(\frac{T_c}{T_a}\right)$. Then,

$$\begin{aligned}\frac{V_c}{V_a} &= \frac{T_c}{T_b}\cdot\frac{V_b}{V_a} = \frac{T_b}{T_a}\cdot\frac{T_a}{T_b}\cdot\frac{V_a}{V_b}\\ &= \frac{T_c}{T_a}\cdot\left(\frac{T_aV_a^{\gamma-1}}{T_bV_b^{\gamma-1}}\right)\left(\frac{V_a}{V_b}\right)^{-\gamma}\\ &= \frac{T_c}{T_a}r^{\gamma}\end{aligned}$$

Combining the above results,

$$p_dV_d = p_aV_a\left(\frac{T_c}{T_a}\right)^{\gamma} r^{\gamma-\gamma^2}$$

Subisitution of the above results into Eq. (20.4) gives

$$\begin{aligned}e &= 1 - \frac{5}{7}\left[\frac{\left(\frac{T_c}{T_a}\right)^{\gamma} r^{\gamma-\gamma^2} - 1}{\left(\frac{T_c}{T_a}\right) - r^{\gamma-1}}\right]\\ &= 1 - \frac{1}{1.4}\left[\frac{(5.002)r^{-0.56} - 1}{(3.167) - r^{0.40}}\right],\end{aligned}$$

where $\frac{T_c}{T_a} = 3.167, \gamma = 1.4$ have been used. Substitution of $r = 21.0$ yields $e = 0.708 = 70.8\%$.

21.1: $m_{\text{lead}} = 8.00\text{ g and charge} = -3.20\times10^{-9}\text{ C}$

a) $n_{\text{e}} = \dfrac{-3.20\times10^{-9}\text{ C}}{-1.6\times10^{-19}\text{ C}} = 2.0\times10^{10}.$

b) $n_{\text{lead}} = N_A \times \dfrac{8.00\text{ g}}{207} = 2.33\times10^{22}$ and $\dfrac{n_{\text{e}}}{n_{\text{lead}}} = 8.58\times10^{-13}.$

21.2: $\text{current} = 20{,}000\text{ C/s and } t = 100\ \mu\text{s} = 10^{-4}\text{ s}$

$Q = It = 2.00\text{ C}$

$n_{\text{e}} = \dfrac{Q}{1.60\times10^{-19}\text{ C}} = 1.25\times10^{19}.$

21.3: The mass is primarily protons and neutrons of $m = 1.67\times10^{-27}$ kg, so:

$n_{\text{p and n}} = \dfrac{70.0\text{ kg}}{1.67\times10^{-27}\text{ kg}} = 4.19\times10^{28}$

About one-half are protons, so $n_{\text{p}} = 2.10\times10^{28} = n_{\text{e}}$ and the charge on the electrons is given by: $Q = (1.60\times10^{-19}\text{ C})\times(2.10\times10^{28}) = 3.35\times10^{9}\text{ C}.$

21.4: Mass of gold = 17.7 g and the atomic weight of gold is 197 g/mol. So the number of atoms $N_A\times\text{mol} = (6.02\times10^{23})\times\left(\frac{17.7\text{ g}}{197\text{ g/mol}}\right) = 5.41\times10^{22}.$

a) $n_{\text{p}} = 79\times5.41\times10^{22} = 4.27\times10^{24}$

$q = n_{\text{p}}\times1.60\times10^{-19}\text{ C} = 6.83\times10^{5}\text{ C}$

b) $n_{\text{e}} = n_{\text{p}} = 4.27\times10^{24}.$

21.5: $1.80\text{ mol} = 1.80\times6.02\times10^{23}\text{ H atoms} = 1.08\times10^{24}\text{ electrons}.$

$\text{charge} = -1.08\times10^{24}\times1.60\times10^{-19}\text{ C} = -1.73\times10^{5}\text{ C}.$

21.6: First find the total charge on the spheres:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \Rightarrow q = \sqrt{4\pi\varepsilon_0 F r^2} = \sqrt{4\pi\varepsilon_0(4.57\times10^{-21})(0.2)^2} = 1.43\times10^{-16}\text{ C}$$

And therefore, the total number of electrons required is

$n = q/e = 1.43\times10^{-16}\text{ C}/1.60\times10^{-19}\text{ C} = 890.$

21.7: a) Using Coulomb's Law for equal charges, we find:

$$F = 0.220\ \text{N} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(0.150\ \text{m})^2} \Rightarrow q = \sqrt{5.5\times10^{-13}\ \text{C}^2} = 7.42\times10^{-7}\ \text{C}.$$

b) When one charge is four times the other, we have:

$$F = 0.220\ \text{N} = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{(0.150\ \text{m})^2} \Rightarrow q = \sqrt{1.375\times10^{-13}\ \text{C}^2} = 3.71\times10^{-7}\ \text{C}$$

So one charge is 3.71×10^{-7} C, and the other is 1.484×10^{-6} C.

21.8: a) The total number of electrons on each sphere equals the number of protons.

$$n_\text{e} = n_\text{p} = 13\times N_A \times \frac{0.0250\ \text{kg}}{0.026982\ \text{kg/mol}} = 7.25\times10^{24}.$$

b) For a force of 1.00×10^4 N to act between the spheres,

$$F = 10^4\ \text{N} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \Rightarrow q = \sqrt{4\pi\varepsilon_0\ (10^4\ \text{N})\ (0.08\ \text{m})^2} = 8.43\times10^{-4}\ \text{C}.$$

$$\Rightarrow n'_\text{e} = q/e = 5.27\times10^{15}$$

c) n'_e is 7.27×10^{-10} of the total number.

21.9: The force of gravity must equal the electric force.

$$mg = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \Rightarrow r^2 = \frac{1}{4\pi\varepsilon_0}\frac{(1.60\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(9.8\ \text{m/s})} = 25.8\ \text{m}^2 \Rightarrow r = 5.08\ \text{m}.$$

21.10: a) Rubbing the glass rod removes electrons from it, since it becomes positive.

$$7.50\ \text{nC} = (7.50\times10^{-9}\ \text{C})\ (6.25\times10^{18}\ \text{electrons/C}) = 4.69\times10^{10}\ \text{electrons}$$

$$(4.69\times10^{10}\ \text{electrons})\ (9.11\times10^{-31}\ \text{kg/electron}) = 4.27\times10^{-20}\ \text{kg}.$$

The rods mass decreases by 4.27×10^{-20} kg.

b) The number of electrons transferred is the same, but they are *added* to the mass of the
plastic rod, which increases by 4.27×10^{-20} kg.

21.11: $\vec{\boldsymbol{F}}_2$ is in the $+x$-direction, so $\vec{\boldsymbol{F}}_1$ must be in the $-x$-direction and q_1 is positive.

$$F_1 = F_2, \qquad k\frac{q_1 q_3}{r_{13}^2} = k\frac{|q_2| q_3}{r_{23}^2}$$

$$q_1 = (0.0200/0.0400)^2 |q_2| = 0.750 \text{ nC}$$

21.12: a) $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \Rightarrow 0.200 \text{ N} = \frac{1}{4\pi\varepsilon_0}\frac{(0.550\times10^{-6}\text{ C})q_2}{(0.30\text{ m})^2}$$

$$\Rightarrow q_2 = +3.64\times10^{-6}\text{ C.}$$

b) $F = 0.200$ N, and is attractive.

21.13: Since the charges are equal in sign the force is repulsive and of magnitude:

$$F = \frac{kq^2}{r^2} = \frac{(3.50\times10^{-6}\text{ C})^2}{4\pi\varepsilon_0(0.800\text{ m})^2} = 0.172\text{ N}$$

21.14: We only need the y-components, and each charge contributes equally.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(2.0\times10^{-6}\text{ C})\,(4\times10^{-6}\text{ C})}{(0.500\text{ m})^2}\sin\alpha = 0.173\text{ N (since } \sin\alpha = 0.6\text{).}$$

Therefore, the total force is $2F = 0.35$ N, downward.

21.15: $\vec{\boldsymbol{F}}_2$ and $\vec{\boldsymbol{F}}_3$ are both in the $+x$-direction.

$$F_2 = k\frac{|q_1 q_2|}{r_{12}^2} = 6.749\times10^{-5}\text{ N}, \quad F_3 = k\frac{|q_1 q_3|}{r_{13}^2} = 1.124\times10^{-4}\text{ N}$$

$$F = F_2 + F_3 = 1.8\times10^{-4}\text{ N, in the } +x\text{-direction.}$$

21.16: $F_{21} = \dfrac{(9\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(20.\times 10^{-6}\ \text{C})\,(2.0\times 10^{-6}\ \text{C})}{(0.60\text{m})^2} = 0.100\ \text{N}$

F_{Q1} is equal and opposite to F_{1Q} (Ex. 21.4), so

$$(F_{Q1})_x = -0.23\,\text{N}$$
$$(F_{Q1})_y = 0.17\ \text{N}$$

Overall:

$$F_x = -0.23\ \text{N}$$
$$F_y = 0.100\ \text{N} + 0.17\ \text{N} = 0.27\ \text{N}$$

The magnitude of the total force is $\sqrt{(0.23\ \text{N})^2 + (0.27\ \text{N})^2} = 0.35\ \text{N}$. The direction of the force, as measured from the $+y$ axis is

$$\theta = \tan^{-1}\frac{0.23}{0.27} = 40^\circ$$

21.17: $\vec{F}_2$ is in the $+x-$direction.

$$F_2 = k\frac{|q_1 q_2|}{r_{12}^2} = 3.37\ \text{N},\quad \text{so } F_{2x} = +3.37\ \text{N}$$
$$F_x = F_{2x} + F_{3x} \text{ and } F_x = -7.00\ \text{N}$$
$$F_{3x} = F_x - F_{2x} = -7.00\ \text{N} - 3.37\ \text{N} = -10.37\ \text{N}$$

For F_{3x} to be negative, q_3 must be on the $-x$-axis.

$$F_3 = k\frac{|q_1 q_3|}{x^2},\ \text{so } |x| = \sqrt{\frac{k|q_1 q_3|}{F_3}} = 0.144\ \text{m},\ \text{so } x = -0.144\ \text{m}$$

21.18: The charge q_3 must be to the right of the origin; otherwise both q_2 and q_3 would exert forces in the $+x$ direction. Calculating the magnitude of the two forces:

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2} = \frac{(9\times10^9\ \mathrm{N\cdot m^2/C^2})(3.00\times10^{-6}\ \mathrm{C})(5.00\times10^{-6}\ \mathrm{C})}{(0.200\ \mathrm{m})^2}$$

$= 3.375$ N in the $+x$ direction.

$$F_{31} = \frac{(9\times10^9\ \mathrm{N\cdot m^3/C^2})\,(3.00\times10^{-6}\ \mathrm{C})\,(8.00\times10^{-6}\ \mathrm{C})}{r_{13}^2}$$

$$= \frac{0.216\ \mathrm{N\cdot m^2}}{r_{13}^2}\ \text{in the } -x \text{ direction}$$

We need $F_{21} - F_{31} = -7.00\ \mathrm{N}$:

$$3.375\ \mathrm{N} - \frac{0.216\ \mathrm{N\cdot m^2}}{r_{13}^2} = -7.00\ \mathrm{N}$$

$$r_{13}^2 = \frac{0.216\ \mathrm{N\cdot m^2}}{3.375\ \mathrm{N} + 7.00\ \mathrm{N}} = 0.0208\ \mathrm{m}^2$$

$$r_{13} = 0.144\ \text{m to the right of the origin}$$

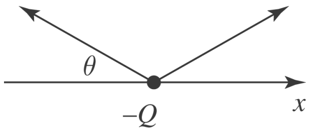
21.19: $\vec{F} = \vec{F}_1 + \vec{F}_2$ and $F = F_2 + F_1$ since they are acting in the same direction at $y = -0.400$ m so,

$$F = \frac{1}{4\pi\varepsilon_0}(5.00\times10^{-9}\ \mathrm{C})\left(\frac{1.50\times10^{-9}\ \mathrm{C}}{(0.200\ \mathrm{m})^2} + \frac{3.20\times10^{-9}\ \mathrm{C}}{(0.400\ \mathrm{m})^2}\right) = 2.59\times10^{-6}\ \text{N downward.}$$

21.20: $\vec{F} = \vec{F}_1 + \vec{F}_2$ and $F = F_1 - F_2$ since they are acting in opposite directions at $x = 0$ so,

$$F = \frac{1}{4\pi\varepsilon_0}(6.00\times10^{-9}\ \mathrm{C})\left(\frac{4.00\times10^{-9}\ \mathrm{C}}{(0.200\ \mathrm{m})^2} + \frac{5.00\times10^{-9}\ \mathrm{C}}{(0.300\ \mathrm{m})^2}\right) = 2.4\times10^{-6}\ \text{N to the right.}$$

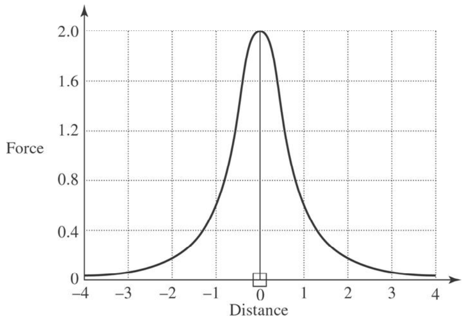
21.21: a)



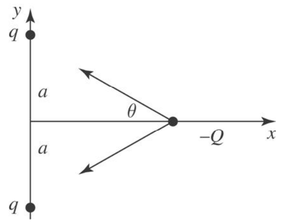
b) $F_x = 0$, $F_y = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{(a^2 + x^2)}\sin\theta \, \dfrac{1}{4\pi\varepsilon_0}\dfrac{2qQa}{(a^2 + x^2)^{3/2}}$

c) At $x = 0$, $F_y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2qQ}{a^2}$ in the $+\,y$ direction.

d)



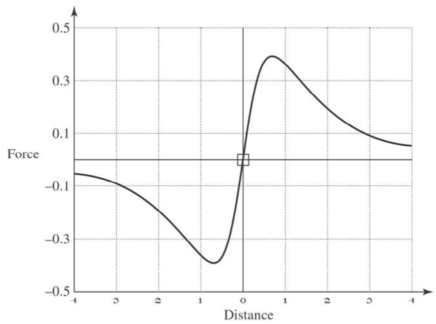
21.22: a)



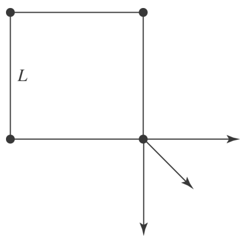
b) $F_x = -2\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{(a^2 + x^2)}\cos\theta = \dfrac{1}{4\pi\varepsilon_0}\dfrac{-2qQx}{(a^2 + x^2)^{3/2}}$, $F_y = 0$

c) At $x = 0$, $F = 0$.

d)



21.23:



b) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{2L^2} + \sqrt{2}\,\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{L^2} = \left(1 + 2\sqrt{2}\right)\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{2L^2}$ at an angle of $45°$ below the positive x-axis

21.24: a) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\dfrac{1}{4\pi\varepsilon_0}\dfrac{(3.00\times10^{-9}\text{ C})}{(0.250\text{ m})^2} = 432\text{ N/C}$, down toward the particle.

b) $E = 12.00\text{ N/C} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} \Rightarrow r = \sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{(3.00\times10^{-9}\text{ C})}{(12.0\text{ N/C})}} = 1.50\text{ m}.$

21.25: Let +*x*-direction be to the right. Find a_x :

$v_{0x} = +1.50 \times 10^3 \text{ m/s}, v_x = -1.50 \times 10^3 \text{ m/s}, t = 2.65 \times 10^{-6} \text{ s}, a_x = ?$

$v_x = v_{0x} + a_x t$ gives $a_x = -1.132 \times 10^9 \text{ m/s}^2$

$F_x = ma_x = -7.516 \times 10^{-18} \text{ N}$

$\vec{F}$ is to the left $(-x\text{-direction})$, charge is positive, so $\vec{E}$ is to the left.

$E = F/q = (7.516 \times 10^{-18} \text{ N})/[(2)(1.602 \times 10^{-19} \text{ C})] = 23.5 \text{ N/C}$

21.26: (a) $x = \frac{1}{2}at^2$

$$a = \frac{2x}{t^2} = \frac{2(4.50 \text{ m})}{(3.00 \times 10^{-6} \text{ s})^2} = 1.00 \times 10^{12} \text{ m/s}^2$$

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^{12} \text{ m/s}^2)}{1.6 \times 10^{-19} \text{ C}}$$

$= 5.69 \text{ N/C}$

The force is up, so the electric field must be *downward* since the electron is negative.
(b) The electron's acceleration is $\sim 10^{11}$ g, so gravity must be negligibly small compared to the electrical force.

21.27: a) $|q|E = mg \Rightarrow |q| = \dfrac{(0.00145 \text{ kg})(9.8 \text{ m/s}^2)}{650 \text{ N/C}} = 2.19 \times 10^{-5}$ C, sign is negative.

b) $qE = mg \Rightarrow E = \dfrac{(1.67 \times 10^{-27} \text{ kg})(9.8 \text{ m/s}^2)}{1.60 \times 10^{-19} \text{ C}} = 1.02 \times 10^{-7}$ N/C, upward.

21.28: a) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(26 \times 1.60 \times 10^{-19} \text{ C})}{(6.00 \times 10^{-10} \text{ m})^2} = 1.04 \times 10^{11} \text{ N/C}.$

b) $E_{\text{proton}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(1.60 \times 10^{-19} \text{ C})}{(5.29 \times 10^{-11} \text{ m})^2} = 5.15 \times 10^{11} \text{ N/C}.$

21.29: a) $q = -55.0 \times 10^{-6}$ C, and F is downward with magnitude 6.20×10^{-9} N. Therefore, $E = F/q = 1.13 \times 10^{-4}$ N/C, upward.

b) If a copper nucleus is placed at that point, it feels an upward force of magnitude $F = qE = (29) \cdot 1.6 \times 10^{-19} \text{C} \cdot 1.13 \times 10^{-4} \text{ N/C} = 5.24 \times 10^{-22} \text{ N}.$

21.30: a) The electric field of the Earth points toward the ground, so a NEGATIVE charge will hover above the surface.

$$mg = qE \Rightarrow q = -\frac{(60.0\ \text{kg})\ (9.8\ \text{m/s}^2)}{150\ \text{N/C}} = -3.92\ \text{C}.$$

b) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(3.92\ \text{C})^2}{(100.00\ \text{m})^2} = 1.38\times 10^7\ \text{N}.$ The magnitude of the charge is too great for practical use.

21.31: a) Passing between the charged plates the electron feels a force upward, and just misses the top plate. The distance it travels in the y-direction is 0.005 m. Time of flight $= t = \frac{0.0200\ \text{m}}{1.60\times 10^6\ \text{m/s}} = 1.25\times 10^{-8}\ \text{s}$ and initial y-velocity is zero. Now,

$y = v_{0y}t + \frac{1}{2}at^2$ so $0.005\,\text{m} = \frac{1}{2}a(1.25\times 10^{-8}\ \text{s})^2 \Rightarrow a = 6.40\times 10^{13}\ \text{m/s}^2$. But also

$a = \frac{F}{m} = \frac{eE}{m_e} \Rightarrow E = \frac{(9.11\times 10^{-31}\ \text{kg})(6.40\times 10^{13}\ \text{m/s}^2)}{1.60\times 10^{-19}\ \text{C}} = 364\ \text{N/C}.$

b) Since the proton is more massive, it will accelerate less, and NOT hit the plates. To find the vertical displacement when it exits the plates, we use the kinematic equations again:

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m_\text{p}}(1.25\times 10^{-8}\ \text{s})^2 = 2.73\times 10^{-6}\ \text{m}.$$

c) As mention in b), the proton will not hit one of the plates because although the electric force felt by the proton is the same as the electron felt, a smaller acceleration results for the more massive proton.

d) The acceleration produced by the electric force is much greater than g; it is reasonable to ignore gravity.

21.32: a)

$$\vec{E}_1 = \frac{q_1}{4\pi\varepsilon_0 r_1^{\,2}}\hat{j} = \frac{(9\times10^9\ \mathrm{N\cdot m^2/C^2})\,(-5.00\times10^{-9}\ \mathrm{C})}{(0.0400\ \mathrm{m})^2} = (-2.813\times10^4\ \mathrm{N/C})\hat{j}$$

$$\left|\vec{E}_2\right| = \frac{q_2}{r_2^2} = \frac{(9\times10^9\ \mathrm{N\cdot m^2/C^2})\,(3.00\times10^{-9}\ \mathrm{C})}{(0.0300\mathrm{m})^2 + (0.0400\ \mathrm{m})^2} = 1.08\times10^4\ \mathrm{N/C}$$

The angle of $\vec{E}_2$, measured from the x-axis, is $180 - \tan^{-1}\left(\frac{4.00\ \mathrm{cm}}{3.00\ \mathrm{cm}}\right) = 126.9°$ Thus

$$\vec{E}_2 = (1.080\times10^4\ \mathrm{N/C})\,(\hat{i}\cos 126.9° + \hat{j}\sin 126.9°)$$
$$= (-6.485\times10^3\ \mathrm{N/C})\,\hat{i} + (8.64\times10^3\ \mathrm{N/C})\,\hat{j}$$

b) The resultant field is

$$\vec{E}_1 + \vec{E}_2 = (-6.485\times10^3\ \mathrm{N/C})\,\hat{i} + (-2.813\times10^4\ \mathrm{N/C} + 8.64\times10^3\ \mathrm{N/C})\,\hat{j}$$
$$= (-6.485\times10^3\ \mathrm{N/C})\,\hat{i} - (1.95\times10^4\ \mathrm{N/C})\,\hat{j}$$

21.33: Let $+x$ be to the right and $+y$ be downward.
Use the horizontal motion to find the time when the electron emerges from the field:

$x - x_0 = 0.0200\ \mathrm{m},\ a_x = 0,\ v_{0x} = 1.60\times10^6\ \mathrm{m/s},\ t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = 1.25\times10^{-8}\ \mathrm{s}$

$v_x = 1.60\times10^6\ \mathrm{m/s}$

$y - y_0 = 0.0050\ \mathrm{m},\ v_{0y} = 0,\ t = 1.25\times10^{-8}\ \mathrm{s},\ v_y = ?$

$y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ gives $v_y = 8.00\times10^5\ \mathrm{m/s}$

$v = \sqrt{v_x^2 + v_y^2} = 1.79\times10^6\ \mathrm{m/s}$

21.34: a) $\vec{E} = -11\ \mathrm{N/C}\hat{i} + 14\ \mathrm{N/C}\hat{j}$, so $E = \sqrt{(-11)^2 + (14)^2} = 17.8\ \mathrm{N/C}$.

$\theta = \tan^{-1}(-14/11) = -51.8°$, so $\theta = 128°$ counterclockwise from the x-axis

b) $\vec{F} = \vec{E}q$ so $F = (17.8\ \mathrm{N/C})\,(2.5\times10^{-9}\ \mathrm{C}) = 4.45\times10^{-8}\ \mathrm{N}$, i) at $-52°$ (repulsive) ii) at $+128°$ (repulsive).

21.35: a) $F_{\mathrm{g}} = m_{\mathrm{e}} g = (9.11 \times 10^{-31}\ \mathrm{kg})\ (9.8\ \mathrm{m/s^2}) = 8.93 \times 10^{-30}\ \mathrm{N}$. $F_{\mathrm{e}} = eE =$ $(1.60 \times 10^{-19}\ \mathrm{C})\ (1.00 \times 10^{4}\ \mathrm{N/C}) = 1.60 \times 10^{-15}\ \mathrm{N}$. Yes, ok to neglect F_{g} because $F_{\mathrm{e}} >> F_{\mathrm{g}}$.

b) $E = 10^{4}\ \mathrm{N/C} \Rightarrow F_{\mathrm{e}} = 1.6 \times 10^{-15}\ \mathrm{N} = mg \Rightarrow m = 1.63 \times 10^{-16}\ \mathrm{kg}$

$$\Rightarrow m = 1.79 \times 10^{14} m_{\mathrm{e}}.$$

c) No. The field is uniform.

21.36: a) $x = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m_p}t^2 \Rightarrow E = \frac{2(0.0160\ \mathrm{m})\ (1.67 \times 10^{-27}\ \mathrm{kg})}{(1.60 \times 10^{-19}\ \mathrm{C})\ (1.50 \times 10^{-6}\ s)^2} = 148\ \mathrm{N/C}.$

b) $v = v_0 + at = \frac{eE}{m_p}t = 2.13 \times 10^{4}\ \mathrm{m/s}.$

21.37: a) $\tan^{-1}\left(\frac{-1.35}{0}\right) = -\frac{\pi}{2}, = \vec{r} - \hat{\boldsymbol{j}}$ b) $\tan^{-1}\left(\frac{12}{.2}\right) = \frac{\pi}{4}, \hat{\boldsymbol{r}} = \frac{\sqrt{2}}{2}\hat{\boldsymbol{i}} + \frac{\sqrt{2}}{2}\hat{\boldsymbol{j}}$

c) $\tan^{-1}\left(\frac{2.6}{+1.10}\right) = 1.97\ \text{radians} = 112.9°, \hat{\boldsymbol{r}} = -0.39\hat{\boldsymbol{i}} + 0.92\hat{\boldsymbol{j}}$ (Second quadrant).

21.38: a) $E = 614\ \mathrm{N/C},\ F = qE = 9.82 \times 10^{-17}\ \mathrm{N}.$

b) $F = e^2 / 4\pi\varepsilon_0\ (1.0 \times 10^{-10})^2 = 2.3 \times 10^{-8}\ \mathrm{N}.$

c) Part (b) >> Part (a), so the electron hardly notices the electric field. A person in the electric field should notice nothing if physiological effects are based solely on magnitude.

21.39: a) Let $+x$ be east.

$\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

$F_x = |q|E = (1.602\times10^{-19}\ \text{C})\,(1.50\ \text{V/m}) = 2.403\times10^{-19}\text{N}$

$a_x = F_x/m = (2.403\times10^{-19}\ \text{N})/(9.109\times10^{-31}\text{kg}) = +2.638\times10^{11}\ \text{m/s}^2$

$v_{0x} = +4.50\times10^5\ \text{m/s},\ a_x = +2.638\times10^{11}\ \text{m/s}^2,\ x - x_0 = 0.375\ \text{m},\ v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 6.33\times10^5\ \text{m/s}$

b) $q > 0$ so $\vec{F}$ is west and the proton slows down.

$F_x = -|q|E = -(1.602\times10^{-19}\ \text{C})\,(1.50\ \text{V/m}) = -2.403\times10^{-19}\ \text{N}$

$a_x = F_x/m = (-2.403\times10^{-19}\ \text{N})/(1.673\times10^{-27}\text{kg}) = -1.436\times10^8\ \text{m/s}^2$

$v_{0x} = +1.90\times10^4\ \text{m/s},\ a_x = -1.436\times10^8\ \text{m/s}^2,\ x - x_0 = 0.375\ \text{m},\ v_x = ?$

$v^2{}_x = v^2{}_{0x} + 2a_x(x - x_0)$ gives $v_x = 1.59\times10^4\ \text{m/s}$

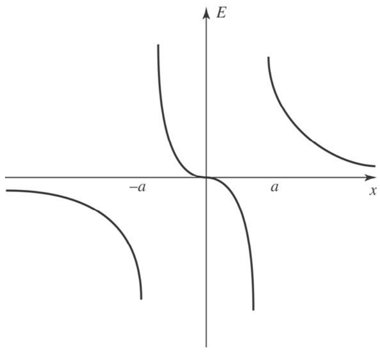
21.40: Point charges q_1 (0.500 nC) and q_2 (8.00 nC) are separated by $x = 1.20$ m. The electric field is zero when $E_1 = E_2 \Rightarrow \frac{kq_1}{r_1^2} = \frac{kq_2}{(1.20 - r_1)^2} \Rightarrow q_2 r_1^2 = q_1(1.2 - r_1)^2 =$
$q_1 r_1^2 - 2q_1(1.2)r_1 + 1.2^2 q_1 \Rightarrow (q_2 - q_1)r_1^2 + 2(1.2)q_1 r_1 - (1.2)^2 q_1 = 0$ or $7.5r_1^2 + 1.2r_1 - 0.72 =$
$r_1 = +0.24, -0.4$ $r_1 = 0.24$ is the point between.

21.41: Two positive charges, q, are on the x-axis a distance a from the origin.

a) Halfway between them, $E = 0$.

b) At any position x, $E = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right), & |x| < a \\ \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right), & x > a \\ \dfrac{-1}{4\pi\varepsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right), & x < -a \end{cases}$

For graph, see below.



21.42: The point where the two fields cancel each other will have to be closer to the negative charge, because it is smaller. Also, it cant't be between the two, since the two fields would then act in the same direction. We could use Coulomb's law to calculate the actual values, but a simpler way is to note that the 8.00 nC charge is twice as large as the –4.00 nC Charge. The zero point will therefore have to be a factor of $\sqrt{2}$ farther from the 8.00 nC charge for the two fields to have equal magnitude. Calling x the distance from the –4.00 nC charge:

$$1.20 + x = \sqrt{2}x$$
$$x = 2.90 \text{ m}$$

21.43: a) Point charge q_1 (2.00 nC) is at the origin and q_2 (-5.00 nC) is at $x = 0.800$ m.

i) At $x = 0.200$ m, $E = \dfrac{k\,|\,q_1\,|}{(0.200\text{ m})^2} + \dfrac{k\,|\,q_2\,|}{(0.600\text{ m})^2} = 575$ N/C right.

ii) At $x = 1.20$ m, $E = \dfrac{k\,|\,q_2\,|}{(0.400\text{ m})^2} + \dfrac{k\,|\,q_1\,|}{(1.20\text{ m})^2} = 269$ N/C left.

iii) At $x = -0.200$ m, $E = \dfrac{k\,|\,q_1\,|}{(0.200\text{ m})^2} + \dfrac{k\,|\,q_2\,|}{(1.00\text{ m})^2} = 405$ N/C left.

b) $F = -eE$ i) $F = 1.6\times10^{-19}\text{ C}\cdot 575\text{ N/C} = 9.2\times10^{-17}$ N left, ii) $F =$ $1.6\times10^{-19}\text{ C}\cdot 269\text{ N/C} = 4.3\times10^{-17}$ N right, iii) $F = 1.6\times10^{-19}\cdot 405 = 6.48\times10^{-17}$N right.

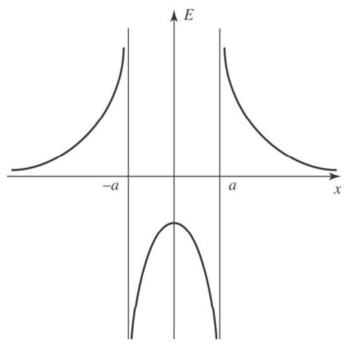
21.44: A positive and negative charge, of equal magnitude q, are on the x-axis, a distance a from the origin.

a) Halfway between them, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{a^2}$, to the left.

b) At any position x, $E = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right), & |\,x\,| < a \\ \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{-q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right), & x > a \\ \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right), & x < -a \end{cases}$

with "+" to the right.

This is graphed below.



21.45: a) At the origin, $E = 0$.

b) At $x = 0.3$ m, $y = 0$:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}(6.00\times10^{-9}\ \text{C})\left(\frac{1}{(0.15\ \text{m})^2} + \frac{1}{(0.45\text{m})^2}\right)\hat{i} = 2667\hat{i}\ \text{N/C}.$$

c) At $x = 0.15$ m, $y = -0.4$ m:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}(6.00\times10^{-9}\ \text{C})\left(\frac{-1}{(0.4\ \text{m})^2}\hat{j} + \frac{1}{(0.5\text{m})^2}\frac{0.3}{0.5}\hat{i} - \frac{1}{(0.5\ \text{m})^2}\frac{0.4}{0.5}\hat{j}\right)$$

$\Rightarrow \vec{E} = (129.6\hat{i} - 510.3\hat{j})\ \text{N/C} \Rightarrow E = 526.5\ \text{N/C}$ and $\theta = 75.7^\circ$ down from the x-axis.

d) $x = 0,\ y = 0.2\ \text{m}: \vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2(6.00\times10^{-9}\ \text{C})\cdot\left(\dfrac{0.2}{0.25}\right)}{(0.25\ \text{m})^2} = 1382\hat{j}\ \text{N/C}$

21.46: Calculate in vector form the electric field for each charge, and add them.

$$\vec{E}_- = \frac{-1}{4\pi\varepsilon_0}\frac{(6.00\times10^{-9}\ \text{C})}{(0.6\ \text{m})^2}\hat{i} = -150\hat{i}\ \text{N/C}$$

$$\vec{E}_+ = \frac{-1}{4\pi\varepsilon_0}(4.00\times10^{-9}\ \text{C})\left(\frac{1}{(1.00\ \text{m})^2}(0.6)\hat{i} + \frac{1}{(1.00\ \text{m})^2}(0.8)\hat{j}\right) = 21.6\hat{i} + 28.8\hat{j}\ \text{N/C}$$

$\Rightarrow E = \sqrt{(128.4)^2 + (28.8)^2} = 131.6\ \text{N/C}$, at $\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6^\circ$ up from $-x$ axis.

21.47: a) At the origin, $\vec{E} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{2(6.0\times10^{-9}\ \text{C})}{(0.15\ \text{m})^2}\hat{i} = -4800\hat{i}\ \text{N/C}.$

b) At $x = 0.3$ m, $y = 0$:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}(6.0\times10^{-9}\text{C})\left(\frac{1}{(0.15\ \text{m})^2} - \frac{1}{(0.45\ \text{m})^2}\right)\hat{i} = 2133\hat{i}\ \text{N/C}.$$

c) At $x = 0.15$ m, $y = -0.4$ m:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}(6.0\times10^{-9}\ \text{C})\left(\frac{-1}{(0.4\ \text{m})^2}\hat{j} - \frac{1}{(0.5\ \text{m})^2}\frac{0.3}{0.5}\hat{i} + \frac{1}{(0.5\ \text{m})^2}\frac{0.4}{0.5}\hat{j}\right)$$

$\Rightarrow \vec{E} = (-129.6\hat{i} - 164.5\hat{j})\ \text{N/C} \Rightarrow E = 209\ \text{N/C}$ and $\theta = 232°$ clockwise from $+x$-axis.

d) $x = 0, y = 0.2\ \text{m}: E_y = 0,\ \vec{E} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{2(6.00\times10^{-9}\ \text{C})\left(\frac{0.15}{0.25}\right)}{(0.25\ \text{m})^2} = -1037\hat{i}\ \text{N/C}$

21.48: For a long straight wire, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow r = \dfrac{1.5\times10^{-10}\ \mathrm{C/m}}{2\pi\varepsilon_0 (2.5\ \mathrm{N/C})} = 1.08\ \mathrm{m}.$

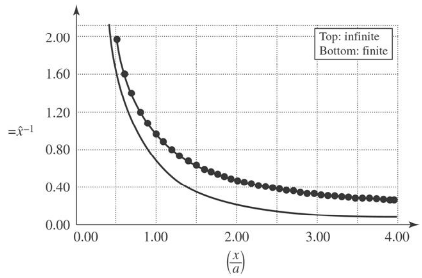
21.49: a) For a wire of length $2a$ centered at the origin and lying along the *y*-axis, the electric field is given by Eq. (21.10).

$$\vec{\boldsymbol{E}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x\sqrt{x^2/a^2+1}}\hat{\boldsymbol{i}}$$

b) For an infinite line of charge:

$$\vec{\boldsymbol{E}} = \frac{\lambda}{2\pi\varepsilon_0 x}\hat{\boldsymbol{i}}$$

Graphs of electric field versus position for both are shown below.



21.50: For a ring of charge, the electric field is given by Eq. (21.8).

a) $\vec{\boldsymbol{E}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}\hat{\boldsymbol{i}}$ so with

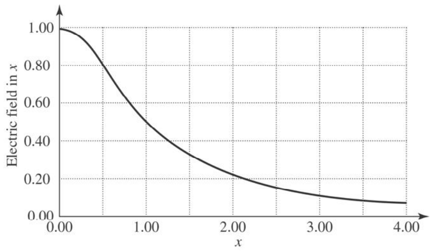
$Q = 0.125\times10^{-9}$ C, $a = 0.025$ m and $x = 0.4$ m $\Rightarrow \vec{\boldsymbol{E}} = 7.0\hat{\boldsymbol{i}}$ N/C.

b) $\vec{\boldsymbol{F}}_{\text{on ring}} = -\vec{\boldsymbol{F}}_{\text{on q}} = -q\,\vec{\boldsymbol{E}} = -(-2.50\times10^{-6}\ \mathrm{C})(7.0\hat{\boldsymbol{i}}\ \mathrm{N/C}) = 1.75\times10^{-5}\hat{\boldsymbol{i}}\ \mathrm{N}.$

21.51: For a uniformly charged disk, the electric field is given by Eq. (21.11):

$$\vec{\boldsymbol{E}} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{R^2/x^2 + 1}}\right)\hat{\boldsymbol{i}}$$

The x-component of the electric field is shown below.



21.52: The earth's electric field is 150 N/C, directly downward. So,

$E = 150 = \dfrac{\sigma}{2\varepsilon_0} \Rightarrow \sigma = 300\varepsilon_0 = 2.66 \times 10^{-9}\ \text{C}/\text{m}^2$, and is negative.

21.53: For an infinite plane sheet, E is constant and is given by $E = \dfrac{\sigma}{2\varepsilon_0}$ directed perpendicular to the surface.

$$\sigma = 2.5 \times 10^6\ \frac{e^-}{\text{cm}^2}\left(-1.6 \times 10^{-19}\frac{\text{C}}{\text{e}^-}\right)\cdot\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^2 = -4\ \times 10^{-9}\ \frac{\text{C}}{\text{m}^2}$$

so $E = \dfrac{4 \times 10^{-9}\ \frac{\text{C}}{\text{m}^2}}{2\varepsilon_0} = 226\ \text{N}/\text{C}$ directed toward the surface.

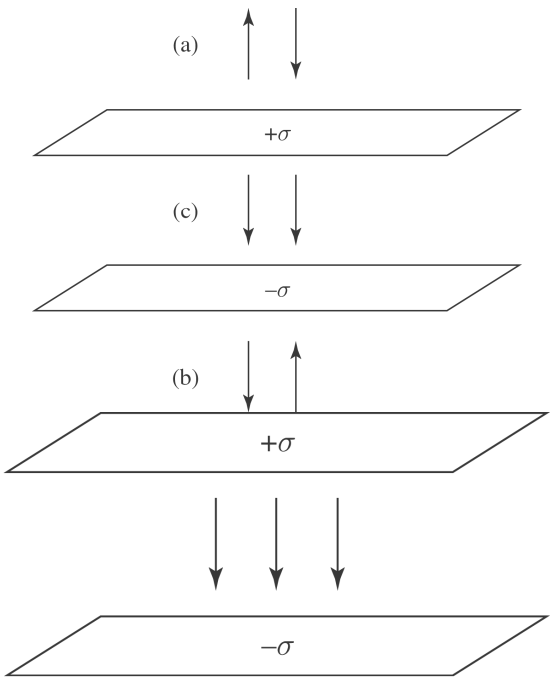
21.54: By superposition we can add the electric fields from two parallel sheets of charge.

a) $E = 0$.

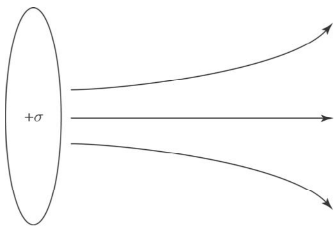
b) $E = 0$.

c) $E = 2\dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$, directed downward.

21.55:

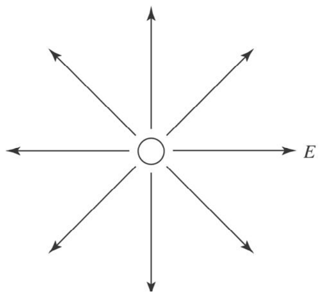


21.56: The field appears like that of a point charge a long way from the disk and an infinite plane close to the disk's center. The field is symmetrical on the right and left (not shown).



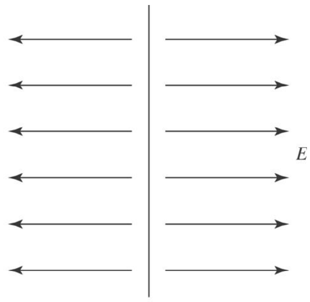
21.57: An infinite line of charge has a radial field in the plane through the wire, and constant in the plane of the wire, mirror-imaged about the wire:

Cross section through the wire:



Length of vector does not depend on angle.

Plane of the wire:



Length of vector gets shorter at points further away from wire.

21.58: a) Since field lines pass from positive charges and toward negative charges, we can deduce that the top charge is positive, middle is negative, and bottom is positive.

b) The electric field is the smallest on the horizontal line through the middle charge, at two positions on either side where the field lines are least dense. Here the *y*-components of the field are cancelled between the positive charges and the negative charge cancels the *x*-component of the field from the two positive charges.

21.59: a) $p = qd \Rightarrow (4.5 \times 10^{-9}\ \text{C})(0.0031\,\text{m}) = 1.4 \times 10^{-11}\ \text{C} \cdot \text{m}$, in the direction from and towards q_2.

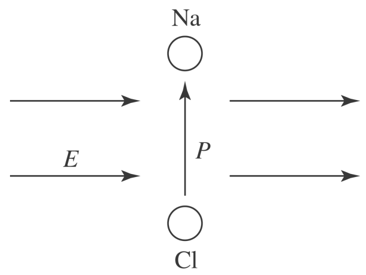
b) If $\vec{E}$ is at $36.9°$, and the torque $\tau = pE\sin\phi$, then:

$$E = \frac{\tau}{p\sin\phi} = \frac{7.2 \times 10^{-9}\ \text{N} \cdot \text{m}}{(1.4 \times 10^{-11}\ \text{C} \cdot \text{m})\sin 36.9°} = 856.5\ \text{N/C}.$$

21.60: a) $d = p/q = (8.9 \times 10^{-30}\ \text{C} \cdot \text{m})/(1.6 \times 10^{-19}\ \text{C}) = 5.56 \times 10^{-11}\ \text{m}.$

b) $\tau_{\text{max}} = pE = (8.9 \times 10^{-30}\ \text{C} \cdot \text{m})(6.0 \times 10^{5}\ \text{N/C}) = 5.34 \times 10^{-24}\ \text{N} \cdot \text{m}.$

Maximum torque:



21.61: a) Changing the orientation of a dipole from parallel to perpendicular yields:
$\Delta U = U_f - U_i = -(pE\cos 90° - pE\cos 0°) = +(5.0 \times 10^{-30}\ \text{C} \cdot \text{m})(1.6 \times 10^{6}\ \text{N/C}) =$
$+8 \times 10^{-24}\ \text{J}.$

b) $\frac{3}{2}kT = 8 \times 10^{-24}\ \text{J} \Rightarrow T = \frac{2(8 \times 10^{-24}\ \text{J})}{3(1.38 \times 10^{-23}\ \text{J/K})} = 0.384\ \text{K}.$

21.62: $E_{\text{dipole}}(x) = \frac{p}{2\pi\varepsilon_0 x^3} \Rightarrow E_{\text{dipole}}(3.00 \times 10^{-9}\ \text{m}) = \frac{6.17 \times 10^{-30}\ \text{C} \cdot \text{m}}{2\pi\varepsilon_0 (3.0 \times 10^{-9}\ \text{m})^3} = 4.11$
$\times 10^{6}\ \text{N/C}.$ The electric force
$F = qE = (1.60 \times 10^{-19}\ \text{C})(4.11 \times 10^{6}\ \text{N/C}) = 6.58 \times 10^{-13}\ \text{N}$ and is toward the water molecule (negative x-direction).

21.63: a) $\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2} = \frac{(y+d/2)^2 - (y-d/2)^2}{(y^2 - d^2/4)^2} = \frac{2yd}{(y^2 - d^2/4)^2}$

$\Rightarrow E_y = \frac{q}{4\pi\varepsilon_0} \frac{2yd}{(y^2 - d^2/4)^2} = \frac{qd}{2\pi\varepsilon_0} \frac{y}{(y^2 - d^2/4)^2} \approx \frac{p}{2\pi\varepsilon_0 y^3}.$

b) This also gives the correct expression for E_y since y appears in the full expression's denominator squared, so the signs carry through correctly.

21.64: a) The magnitude of the field the due to each charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(d/2)^2 + x^2)}\right),$$

where d is the distance between the two charges. The x-components of the forces due to the two charges are equal and oppositely directed and so cancel each other. The two fields have equal y-components, so:

$$E = 2E_y = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{(d/2)^2 + x^2}\right)\sin\theta$$

where θ is the angle below the x-axis for both fields.

$$\sin\theta = \frac{d/2}{\sqrt{(d/2)^2 + x^2}};$$

thus

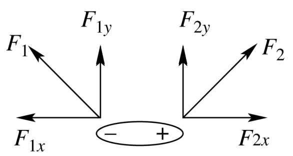
$$E_{\text{dipole}} = \left(\frac{2q}{4\pi\varepsilon_0}\right)\left(\frac{1}{(d/2)^2 + x^2}\right)\left(\frac{d/2}{\sqrt{(d/2)^2 + x^2}}\right) = \frac{qd}{4\pi\varepsilon_0((d/2)^2 + x^2)^{3/2}}$$

The field is the $-y$ directions.

b) At large x, $x^2 >> (d/2)^2$, so the relationship reduces to the approximations

$$E_{\text{dipole}} \approx \frac{qd}{4\pi\varepsilon_0 x^3}$$

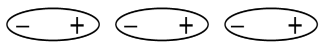
21.65:



The dipoles attract.

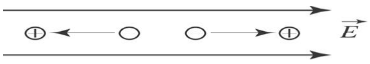
$$F_x = F_{1x} + F_{2x} = 0, \quad F_y = F_{1y} + F_{2y} = 2F_{1y}$$

b)



Opposite charges are closest so the dipoles attract.

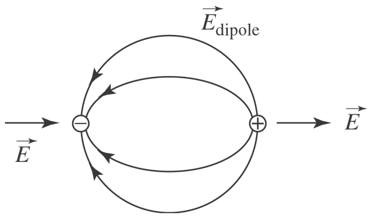
21.66: a)



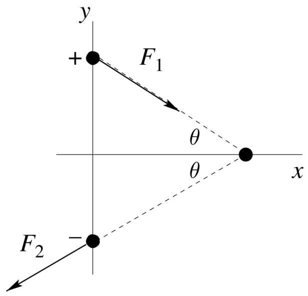
The torque is zero when $\vec{\boldsymbol{p}}$ is aligned either in the *same* direction as $\vec{\boldsymbol{E}}$ or in the *opposite* directions

b) The stable orientation is when $\vec{\boldsymbol{p}}$ is aligned in the *same* direction as $\vec{\boldsymbol{E}}$

c)



21.67:



$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$

Opposite charges attract and like charges repel.

$F_x = F_{1x} + F_{2x} = 0$

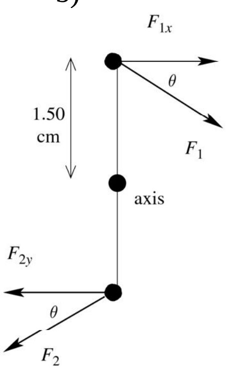
$$F_1 = k\frac{|qq'|}{r^2} = k\frac{(5.00\times10^{-6}\ \text{C})(10.0\times10^{-6}\ \text{C})}{(0.0200\ \text{m})^2} = 1.124\times10^3\ \text{N}$$

$F_{1y} = -F_1\sin\theta = -842.6$ N

$F_{2y} = -842.6$ N so $F_y = F_{1y} + F_{2y} = -1680$ N

(in the direction from the $+5.00$-μC charge toward the -5.00-μC charge).

b)



The $y-$components have zero moment arm and therefore zero torque.

F_{1x} and F_{2x} both produce clockwise torques.

$F_{1x} = F_1\cos\theta = 743.1$ N

$\tau = 2(F_{1x})(0.0150\ \text{m}) = 22.3\ \text{N}\cdot\text{m}$, clockwise

21.68: a) $\vec{\boldsymbol{F}}_{13} = +\left|\frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}^2}\right|\cos\theta\hat{\boldsymbol{i}} + \left|\frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}^2}\right|\sin\theta\hat{\boldsymbol{j}}$

$$\Rightarrow \vec{\boldsymbol{F}}_{13} = +\frac{1}{4\pi\varepsilon_0}\frac{(5.00\text{ nC})(6.00\text{ nC})}{((9.00+16.0)\times10^{-4}\text{ m})}\frac{4}{5}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{(5.00\text{ nC})(6.00\text{ nC})}{((9.00+16.0)\times10^{-4}\text{m})}\frac{3}{5}\hat{j}$$

$$\Rightarrow \vec{\boldsymbol{F}}_{13} = +(8.64\times10^{-5}\text{ N})\hat{\boldsymbol{i}} + (6.48\times10^{-5}\text{ N})\hat{\boldsymbol{j}}.$$

Similarly for the force from the other charge:

$$\vec{\boldsymbol{F}}_{23} = \frac{-1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_{23}^2}\hat{\boldsymbol{j}} = \frac{-1}{4\pi\varepsilon_0}\frac{(2.00\text{ nC})(6.00\text{ nC})}{(0.0300\text{ m})^2}\hat{\boldsymbol{j}} = -(1.20\times10^{-4}\ N)\hat{\boldsymbol{j}}$$

Therefore the two force components are:

$$F_x = 8.64\times10^{-5}\text{ N} \qquad F_y = 6.48\times10^{-5} - 12.0\times10^{-5} = -5.52\times10^{-5}\text{N}$$

b) Thus, $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(8.64\times10^{-5}\text{ N})^2 + (-5.52\times10^{-5}\text{ N})^2} = 1.03\times10^{-4}\text{ N}$, and the angle is $\theta = \arctan(F_y/F_x) = 32.6$, below the x axis

21.69: a) $F_q = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(a+x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(a-x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\left(\frac{1}{(1+x/a)^2} - \frac{1}{(1-x/a)^2}\right)$

$\Rightarrow F_q \approx \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}(1-2\frac{x}{a}\ldots-(1+2\frac{x}{a}\ldots)) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\left(-4\frac{x}{a}\right) = -\left(\frac{qQ}{\pi\varepsilon_0 a^3}\right)x.$ But this is the equation of a simple harmonic oscillator, so:

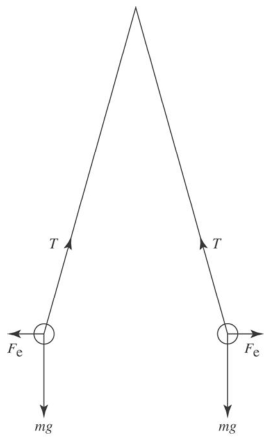
$$\omega = 2\pi f = \sqrt{\frac{qQ}{m\pi\,\varepsilon_0 a^3}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{qQ}{m\pi\,\varepsilon_0 a^3}} = \sqrt{\frac{kqQ}{m\pi^2 a^3}}.$$

b) If the charge was placed on the y-axis there would be no restoring force if q and Q had the same sign. It would move straight out from the origin along the y-axis, since the x-components of force would cancel.

21.70: Examining the forces: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$.

So $\frac{mg\sin\theta}{\cos\theta} = F_e = \frac{kq^2}{d^2}$ But $\tan\theta \approx \frac{d}{2L} \Rightarrow d^3 = \frac{2kq^2L}{mg} \Rightarrow d = \left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}$.

21.71: a)



b) Using the same force analysis as in problem **21.70,** we find:
$q^2 = 4\pi\varepsilon_0 d^2 mg \tan\theta$ and

$d = 2\cdot(1.2)\sin 25 \Rightarrow q = \sqrt{4\pi\varepsilon_0(2\cdot(1.2)\cdot\sin 25°)^2 \tan 25°(0.015\ \text{kg})(9.80\ \text{m}/\text{s}^2)} \Rightarrow q =$
2.79×10^{-6} C.

c) From Problem 21.70, $mg\ \tan\theta = \frac{kq^2}{d^2}$.

$$\sin\theta = \frac{d}{2L} \Rightarrow \tan\theta = \frac{kq^2}{mg(2L)^2\sin^2\theta} = \frac{(8.99\times 10^9\ \text{Nm}^2/\text{C})(2.79\times 10^{-6}\ \text{C})^2}{4(0.6\text{m})^2(0.015\ \text{kg})(9.8\ \text{m}/\text{s}^2)\sin^{-2}\theta}$$

Therefore $\tan\theta = \frac{0.331}{\sin^2\theta}$. Numerical solution of this transcendental equation leads to $\theta = 39.5°$.

21.72: a) Free body diagram as in **21.71.** Each charge still feels equal and opposite electric forces.

b) $T = mg/\cos 20° = 0.0834$ N so $F_e = T\sin 20° = 0.0285\ \text{N} = \dfrac{kq_1q_2}{r^2_1}$. (Note: $r_1 = 2(0.500\ \text{m})\sin 20° = 0.342$ m.)

c) From part (b), $q_1q_2 = 3.71\times 10^{-13}\ \text{C}^2$.

d) The charges on the spheres are made equal by connecting them with a wire, but we still have $F_e = mg\tan\theta = 0.0453\ \text{N} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r_2^2}$ where $Q = \frac{q_1+q_2}{2}$. But the separation r_2 is known: $r_2 = 2(0.500\ \text{m})\sin 30° = 0.500$ m. Hence: $Q = \frac{q_1+q_2}{2} = \sqrt{4\pi\varepsilon_0 F_e r^2{}_2}$ $= 1.12\times 10^{-6}$ C. This equation, along with that from part (b), gives us two equations in q_1 and q_2. $q_1 + q_2 = 2.24\times 10^{-6}$ C and $q_1q_2 = 3.70\times 10^{-13}\ \text{C}^2$. By elimination, substitution and after solving the resulting quadratic equation, we find: $q_1 = 2.06\times 10^{-6}$ C and $q_2 = 1.80\times 10^{-7}$ C.

21.73: a) $0.100\ \text{mol NaCl} \Rightarrow \text{m}_{\text{Na}} = (0.100\ \text{mol})(22.99\ \text{g/mol}) = 2.30\ \text{g}$

$$\Rightarrow \text{m}_{\text{Cl}} = (0.100\ \text{mol})(35.45\ \text{g/mol}) = 3.55\ \text{g}$$

Also the number of ions is $(0.100\ \text{mol})N_{\text{A}} = 6.02 \times 10^{22}$ so the charge is: $q = (6.02 \times 10^{22})(1.60 \times 10^{-19}\ \text{C}) = 9630\ \text{C}$. The force between two such charges is:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(9630)^2}{(0.0200\ \text{m})^2} = 2.09 \times 10^{21}\ \text{N}.$$

b) $a = F/m = (2.09 \times 10^{21}\ \text{N})/(3.55 \times 10^{-3}\ \text{kg}) = 5.89 \times 10^{23}\ \text{m/s}^2$.

c) With such a large force between them, it does not seem reasonable to think the sodium and chlorine ions could be separated in this way.

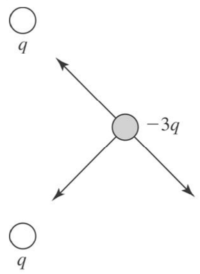
21.74: a) $F_3 = 4.0 \times 10^{-6}\ \text{N} = \frac{kq_1q_3}{r_{13}^2} + \left|\frac{kq_2q_3}{r_{23}^2}\right| = kq_3\left(\frac{(2.5 \times 10^{-9}\ \text{C})}{(-0.3\ \text{m})^2} + \frac{4.5 \times 10^{-9}\ \text{C}}{(+0.2\ \text{m})^2}\right) \Rightarrow$

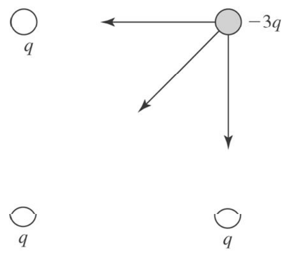
$$q_3 = \frac{4.0 \times 10^{-6}\ \text{N}}{(1262\ \text{N/C})} = 3.2\ \text{nC}.$$

b) The force acts on the middle charge to the right.

c) The force equals zero if the two forces from the other charges cancel. Because of the magnitude and size of the charges, this can only occur to the left of the negative charge q_2. Then: $F_{13} = F_{23} \Rightarrow \frac{kq_1}{(0.300 - x)^2} = \frac{kq_2}{(-0.200 - x)^2}$ where x is the distance from the origin. Solving for x we find: $x = -1.76\ \text{m}$. The other value of x was between the two charges and is not allowed.

21.75: a) $F = +\frac{1}{4\pi\varepsilon_0}\frac{q(3q)}{(L/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{6q^2}{L^2}$, toward the lower the left charge. The other two forces are equal and opposite.





b) The upper left charge and lower right charge have equal magnitude forces at right angles to each other, resulting in a total force of twice the force of one, directed toward the lower left charge. So, all the forces sum to:

$$F = \frac{1}{4\pi\varepsilon_0}\left(\frac{q(3q)\sqrt{2}}{L^2} + \frac{q(3q)}{(\sqrt{2}L)^2}\right) = \frac{q^2}{4\pi\varepsilon_0 L^2}\left(3\sqrt{2} + \frac{3}{2}\right)\text{N.}$$

21.76: a) $E(p) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{(y-a)^2} + \frac{q}{(y+a)^2} - \frac{2q}{y^2}\right).$

b) $E(p) = \frac{1}{4\pi\varepsilon_0}\frac{q}{y^2}((1-a/y)^{-2} + (1+a/y)^{-2} - 2).$ Using the binomial expansion:

$$\Rightarrow E(p) \approx \frac{1}{4\pi\varepsilon_0}\frac{q}{y^2}\left(1 + \frac{2a}{y} + \frac{3a^2}{y^2} + \cdots + 1 - \frac{2a}{y} + \frac{3a^2}{y^2} + \cdots - 2\right) = \frac{1}{4\pi\varepsilon_0}\frac{6qa^2}{y^4}.$$

Note that a point charge drops off like $\frac{1}{y^2}$ and a dipole like $\frac{1}{y^3}$.

21.77: a) The field is all in the x-direction (the rest cancels). From the $+q$ charges:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2+x^2} \Rightarrow E_x = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2+x^2}\frac{x}{\sqrt{a^2+x^2}} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2+x^2)^{3/2}}.$$

(Each $+q$ contributes this). From the $-2q$:

$$E_x = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{x^2} \Rightarrow E_{\text{total}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2qx}{(a^2+x^2)^{3/2}} - \frac{2q}{x^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{x^2}((a^2/x^2+1)^{-3/2}-1).$$

b) $$E_{\text{total}} \approx \frac{1}{4\pi\varepsilon_0}\frac{2q}{x^2}\left(1-\frac{3a^2}{2x^2}+\cdots-1\right) = \frac{1}{4\pi\varepsilon_0}\frac{3qa^2}{x^4}, \text{ for } x >> a..$$

Note that a point charge drops off like $\frac{1}{x^2}$ and a dipole like $\frac{1}{x^3}$.

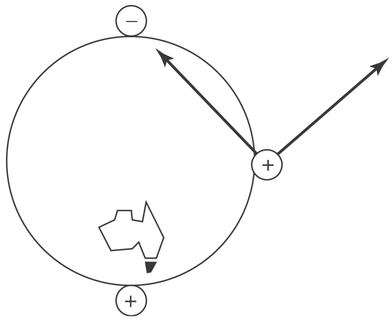
21.78: a) 20.0 g carbon $\Rightarrow \dfrac{20.0\ g}{12.0\ \text{g/mol}} = 1.67$ mol carbon $\Rightarrow 6(1.67) = 10.0$ mol electrons $\Rightarrow q = (10.0)N_A(1.60\times10^{-19}\text{C}) = 0.963\times10^6$ C. This much charge is placed at the earth's poles (negative at north, positive at south), leading to a force:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R_{\text{earth}})^2} = \frac{1}{4\pi\varepsilon_0}\frac{(0.963\times10^6\text{C})^2}{(1.276\times10^7\text{m})^2} = 5.13\times10^7\text{N}.$$

b) A positive charge at the equator of the same magnitude as above will feel a force in the south-to-north direction, perpendicular to the earth's surface:

$$F = 2\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R_{\text{earth}})^2}\sin 45^\circ$$

$$\Rightarrow F = 2\frac{1}{4\pi\varepsilon_0}\frac{4}{\sqrt{2}}\frac{(0.963\times10^6\text{C})^2}{(1.276\times10^7\text{m})^2} = 1.44\times10^8\ \text{N}.$$

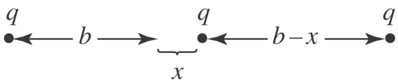


21.79: a) With the mass of the book about 1.0 kg, most of which is protons and neutrons, we find: #protons $= \frac{1}{2}(1.0\text{ kg})/(1.67\times10^{-27}\text{ kg}) = 3.0\times10^{26}$. Thus the charge difference present if the electron's charge was 99.999% of the proton's is $\Delta q = (3.0\times10^{26})(0.00001)(1.6\times10^{-19}\text{ C}) = 480\text{ C}$.

b) $F = k(\Delta q)^2/r^2 = k(480\text{ C})^2/(5.0\text{ m})^2 = 8.3\times10^{13}\text{ N}$ – repulsive. The acceleration $a = F/m = (8.3\times10^{13}\text{ N})/(1\text{ kg}) = 8.3\times10^{13}\text{ m/s}^2$.

c) Thus even the slightest charge imbalance in matter would lead to explosive repulsion!

21.80: (a)



$$F_{\text{net}}\text{(on central charge)} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(b-x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(b+x)^2}$$

$$= \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{(b-x)^2} - \frac{1}{(b+x)^2}\right]$$

$$= \frac{q^2}{4\pi\varepsilon_0}\frac{(b+x)^2-(b-x)^2}{(b-x)^2(b+x)^2} = \frac{q^2}{4\pi\varepsilon_0}\frac{4bx}{(b-x)^2(b+x)^2}$$

For $x << b$, this expression becomes

$$F_{\text{net}} \approx \frac{q^2}{\pi\varepsilon_0}\frac{bx}{b^2b^2} = \frac{q^2}{\pi\varepsilon_0 b^3}x \text{ Direction is opposite to } x.$$

(b) $\sum F = ma : -\frac{-q^2}{\pi\varepsilon_0 b^3}x = m\frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} = -\left(\frac{q^2}{m\pi\varepsilon_0 b^3}\right)x$$

$$\omega = \sqrt{\frac{q^2}{m\pi\varepsilon_0 b^3}} = 2\pi\ f \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{q^2}{m\pi\varepsilon_0 b^3}}$$

(c) $q = e, b = 4.0\times10^{-10}\text{m},\ m = 12\text{ amu} = 12(1.66\times10^{-27}\text{ kg})$

$$f = \frac{1}{2\pi}\sqrt{\frac{(1.6\times10^{-19}\text{ C})^2}{12(1.66\times10^{-27}\text{ kg})\pi(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)(4.0\times10^{-10}\text{ m})^3}} = 4.28\times10^{12}\text{Hz}$$

21.81: a) $m = \rho V = \rho(\frac{4}{3}\pi r^3) = (8.9\times 10^3\ \text{kg}/\text{m}^3)(\frac{4}{3}\pi)(1.00\times 10^{-3}\ \text{m})^3 =$

$3.728\times 10^{-5}\ \text{kg}$

$n = m/M = (3.728\times 10^{-5}\ \text{kg})(63.546\times 10^{-3}\ \text{kg}/\text{mol}) = 5.867\times 10^{-4}\ \text{mol}$

$N = nN_{\text{A}} = 3.5\times 10^{20}\ \text{atoms}$

(b) $N_{\text{e}} = (29)(3.5\times 10^{20}) = 1.015\times 10^{22}$ electrons and protons

$q_{\text{net}} = eN_{\text{e}} - (0.99900)eN_{\text{e}} = (0.100\times 10^{-2})(1.602\times 10^{-19}\ \text{C})(1.015\times 10^{22}) = 1.6\ \text{C}$

$$F = k\frac{q^2}{r^2} = k\frac{(1.6\ \text{C})^2}{(1.00\ \text{m})^2} = 2.3\times 10^{10}\ \text{N}$$

21.82: First, the mass of the drop:

$$m = \rho V = (1000\ \text{kg}/\text{m}^3)\left(\frac{4\pi(15.0\times 10^{-6}\ \text{m})^3}{3}\right) = 1.41\times 10^{-11}\ \text{kg}.$$

Next, the time of flight: $t = D/v = 0.02/20 = 0.00100\ \text{s}$ and the acceleration :

$$d = \frac{1}{2}at^2 \Rightarrow a = \frac{2d}{t^2} = \frac{2(3.00\times 10^{-4}\ \text{m})}{(0.001\ \text{s})^2} = 600\ \text{m}/\text{s}^2.$$

So:

$$a = F/m = qE/m \Rightarrow q = ma/E = \frac{(1.41\times 10^{-11}\ \text{kg})(600\ \text{m}/\text{s}^2)}{8.00\times 10^4\ \text{N}/\text{C}} = 1.06\times 10^{-13}\ \text{C}.$$

21.83: $F_y = eE \qquad F_x = 0$

$$a_y = \frac{F_y}{m_p} = \frac{eE}{m_p} \quad a_x = 0$$

a) $$v_y^2 = v_{0y}^2 + 2a_y\Delta y = v_0^2 \sin^2\alpha + \frac{2eE}{m_p}\Delta y \qquad |\Delta y| = h_{\max} \text{ when } v_y = 0$$

$$\Rightarrow h_{\max} = \frac{v_0^2 m_p \sin^2\alpha}{2eE}$$

b) $$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$t = t_{\text{orig}} \text{ when } \Delta y = 0$$

$$\Rightarrow 0 = \left(-v_0 \sin\alpha + \frac{1}{2}a_y t_{\text{orig}}\right)t_{\text{orig}}$$

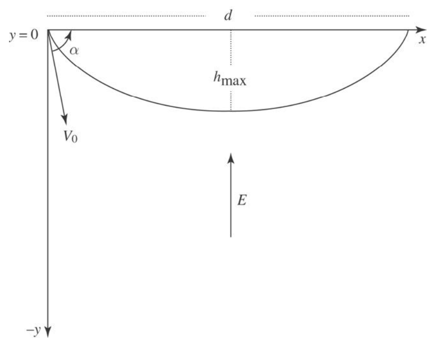
so $t_{\text{orig}} = 0, \dfrac{2v_0 \sin\alpha}{a_y}$

or

$$t_{\text{orig}} = \frac{2v_0 m_p \sin\alpha}{eE}$$

$$d = v_{0x}t_{\text{orig}} = \frac{2v_0^2 m_p}{eE}\cos\alpha \sin\alpha$$

c)



d) $$h_{\max} = \frac{(4\times10^5 \text{ m/s})^2(1.67\times10^{-27}\text{kg}) \sin^2 30°}{2(1.6\times10^{-19} \text{ C})(500 \text{ N/C})} = 0.42 \text{ m}$$

$$d = \frac{2(4\times10^5\text{m/s})^2(1.67\times10^{27} \text{ kg}) \cos 30° \sin 30°}{(1.6\times10^{-19} \text{ C})(500 \text{ N/C})} = 2.89 \text{ m}$$

21.84: a) $E = 50\ \text{N/C} = \left|\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\right| + \left|\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\right| = \frac{1}{4\pi\varepsilon_0}\left(\left|\frac{q_1}{r_1^2}\right| + \left|\frac{q_2}{r_2^2}\right|\right) \Rightarrow q_2 =$

$r_2^2\left(4\pi\varepsilon_0 E - \left|\frac{q_1}{r_1^2}\right|\right) \Rightarrow q_2 = (1.2\ \text{m})^2\left(4\pi\varepsilon_0 50.0\ \text{N/C} - \frac{(4.00\times10^{-9}\ \text{C})}{(0.6\ \text{m})^2}\right) = -8\times10^{-9}\ \text{C}.$

b) $E = -50\ \text{N/C} = \left|\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\right| + \left|\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\right| = \frac{1}{4\pi\varepsilon_0}\left(\left|\frac{q_1}{r_1^2}\right| + \left|\frac{q_2}{r_2^2}\right|\right) \Rightarrow q_2 =$

$r_2^2\left(\frac{-50}{k} - \left|\frac{q_1}{r_1^2}\right|\right) \Rightarrow q_2 = (1.2\ \text{m})^2\left(4\pi\varepsilon_0(-50.0) - \frac{(4.00\times10^{-9}\ \text{C})}{(0.6\ \text{m})^2}\right) = -24.0\times10^{-9}\ \text{C}.$

21.85: $E = 12.0\ \text{N/C} = \frac{-k(16.0\ \text{nC})}{(3.00\ \text{m})^2} + \frac{k(12.0\ \text{nC})}{(8.00\ \text{m})^2} + \frac{kq}{(5.00\ \text{m})^2}$

$\Rightarrow q = 25.0\ \text{m}^2\left(\frac{12}{k} + \frac{1.60\times10^{-8}\ \text{C}}{9.0\ \text{m}^2} - \frac{1.20\times10^{-8}\ \text{C}}{64.00\ \text{m}^2}\right) = +7.31\times10^{-8}\ \text{C} = +73.1\ \text{nC}.$

21.86: a) On the x-axis: $dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(a+r)^2} \Rightarrow E_x = \frac{1}{4\pi\varepsilon_0}\int_0^a \frac{Qdx}{a(a+r-x)^2} =$

$\frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\left(\frac{1}{r} - \frac{1}{a+r}\right)$. And $E_y = 0$.

b) If $a + r = x$, then $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right) \Rightarrow \vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right)\hat{\boldsymbol{i}}.$

c) For $x >> a$, $F = \frac{kqQ}{ax}((1 - a/x)^{-1} - 1) = \frac{kqQ}{ax}(1 + a/x + \cdots - 1) \approx \frac{kqQ}{x^2} \approx$

$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$. (Note that for $x >> a$, $r = x - a \approx x$.) Charge distribution looks like a point from far away.

21.87: a) $dE = \dfrac{k\,dq}{(x^2+y^2)} = \dfrac{kQ\,dy}{a(x^2+y^2)}$ with $dE_x = \dfrac{kQx\,dy}{a(x^2+y^2)^{3/2}}$ and $dE_y =$ $\dfrac{-KQy\,dy}{a(x^2+y^2)^{3/2}}$. Thus:

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{a}\frac{1}{(x^2+a^2)^{1/2}}\frac{a}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x(x^2+a^2)^{1/2}}$$

$$E_y = \frac{-1}{4\pi\varepsilon_0}\frac{Q}{a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = \frac{-1}{4\pi\varepsilon_0}\frac{Q}{a}\left(\frac{1}{x} - \frac{1}{(x^2+a^2)^{1/2}}\right)$$

b) $F_x = -qE_x$ and $F_y = -qE_y$ where E_x and E_y are given in (a).

c) For $x >> a$, $F_y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{ax}(1-(1+a^2/x^2)^{-1/2}) \approx \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{ax}\dfrac{a^2}{2x^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQa}{2x^3}$.

Looks dipole-like in y-direction $F_x = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{x^2}\left(1+\dfrac{a^2}{x^2}\right)^{-1/2} \approx \dfrac{qQ}{4\pi\varepsilon_0 x^2}$.

Looks point-like along x-direction

21.88: a) From $Eq.$ (22.9), $\vec{\boldsymbol{E}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}\hat{\boldsymbol{i}}$

$$\Rightarrow \vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_0}\frac{(-9.00\times10^{-9}\ C)}{(2.5\times10^{-3}\ \text{m})\sqrt{(2.5\times10^{-3}\ \text{m})^2+(0.025\,\text{m})^2}}\hat{\boldsymbol{i}} = (-1.29\times10^{6}\ \text{N/C})\hat{\boldsymbol{i}}.$$

(b) The electric field is less than that at the same distance from an infinite line of charge $(E_{a\to\infty} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\lambda}{x} = \dfrac{-1}{4\pi\varepsilon_0}\dfrac{2Q}{x2a} = -1.30\times10^6\ \text{N/C})$. This is because in the approximation, the terms left off were negative. $\dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{x\left(1+\frac{x^2}{a^2}\right)^{1/2}} \approx \dfrac{\lambda}{2\pi\varepsilon_0 x}\left(1-\dfrac{x^2}{2a^2}+\cdots\right) =$

$E_{\text{Line}} \infty -$ (Higher order terms).

c) For a 1% difference, we need the next highest term in the expansion that was left off to be less than 0.01:

$$\frac{x^2}{2a^2} < 0.01 \Rightarrow x < a\sqrt{2(0.01)} = 0.025\text{m}\sqrt{2(0.01)} \Rightarrow x < 0.35\,\text{cm}.$$

21.89: (a) From Eq. (22.9), $\vec{\boldsymbol{E}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}\hat{\boldsymbol{i}}$

$$\Rightarrow \vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_0}\frac{(-9.0\times10^{-9}\text{ C})}{(0.100\text{ m})\sqrt{(0.100\text{ m})^2+(0.025\text{ m})^2}} = (-7858\text{ N/C})\hat{\boldsymbol{i}}.$$

b) The electric field is less than that at the same distance from a point charge (8100 N/C). Since $\text{E}_{x\to\infty} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x^2}\left(1-\dfrac{a^2}{2x^2}+\cdots\right) = E_{\text{point}}$ –(Higher order terms).

c) For a 1% difference, we need the next highest term in the expansion that was left off to be less than 0.01:

$$\frac{a^2}{2x^2} \approx 0.01 \Rightarrow x \approx a\sqrt{1/(2(0.01))} = 0.025\sqrt{1/0.02} \Rightarrow x \approx 0.177\text{ m}.$$

21.90: (a) On the axis,

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\left(\frac{R^2}{x^2}+1\right)^{-1/2}\right] = \frac{4.00\text{ pC}/\pi(0.025\text{ m})^2}{2\varepsilon_0}\left[1-\left(\frac{(0.025\text{ m})^2}{(0.0020\text{ m})^2}+1\right)^{-1/2}\right]$$

$\Rightarrow E = 106\text{ N/C}$, in the $+x$-direction.

b) The electric field is less than that of an infinite sheet $E_\infty = \dfrac{\sigma}{2\varepsilon_0} = 115\text{ N/C}$.

c) Finite disk electric field can be expanded using the binomial theorem since the expansion terms are small: $\Rightarrow E \approx \dfrac{\sigma}{2\varepsilon_0}\left[1-\dfrac{x}{R}+\dfrac{x^3}{2R^3}-\cdots\right]$ So the difference between the infinite sheet and finite disk goes like $\dfrac{x}{R}$. Thus:

$$\Delta E(x=0.20\text{ cm}) \approx 0.2/2.5 = 0.08 = 8\% \text{ and } \Delta E(x=0.40\text{ cm})$$
$$\approx 0.4/2.5 = 0.16 = 16\%.$$

21.91: (a) As in **22.72:** $E=\frac{\sigma}{2\varepsilon_0}\left[1-\left(\frac{R^2}{x^2}+1\right)^{-1/2}\right]$

$$=\frac{4.00\ \text{pC}/\pi(0.025\ \text{m})^2}{2\varepsilon_0}\left[1-\left(\frac{(0.025\ \text{m})^2}{(0.200\ \text{m})^2}+1\right)^{-1/2}\right]\Rightarrow E$$

$$=0.89\ \text{N/C in the}+x\text{-direction.}$$

b) $x>>R,\ E=\frac{\sigma}{2\varepsilon_0}[1-(1-R^2/2x^2+3R^4/8x^4-\cdots)]$

$$\approx\frac{\sigma}{2\varepsilon_0}\frac{R^2}{2x^2}=\frac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2}=\frac{Q}{4\pi\varepsilon_0 x^2}.$$

c) The electric field of (a) is less than that of the point charge ($0.90\ \text{N/C}$) since the correction term that was omitted was negative.

d) From above $x=0.2\ \text{m}\frac{(0.9-0.89)}{0.89}=0.01=1\%.$ For $x=0.1\ \text{m}$

$$E_{\text{disk}}=3.43\ \text{N/C}$$
$$E_{\text{point}}=3.6\ \text{N/C}$$

so $\frac{(3.6-3.43)}{3.6}=0.047\approx5\%.$

21.92: a) $f(x)=f(-x):\int_{-a}^{a}f(x)dx=\int_{-a}^{0}f(x)dx+\int_0^a f(x)dx=\int_0^{-a}f(-x)d(-x)+$ $\int_0^a f(x)dx.$ Now replace $-x$ with $y:\Rightarrow\int_{-a}^{a}f(x)dx=\int_0^a f(y)d(y)+\int_0^a f(x)dx=$ $2\int_0^a f(x)dx.$

b) $g(x)=-g(-x):\int_{-a}^{a}g(x)dx=\int_{-a}^{0}g(x)dx+\int_0^a g(x)dx=-\int_0^{-a}-g(-x)(-d(-x))+$ $\int_0^a g(x)dx.$ Now replace $-x$ with $y:\Rightarrow\int_{-a}^{a}g(x)dx=-\int_0^a g(y)d(y)+\int_0^a g(x)dx=0.$

c) The integrand in E_y for Example 21.11 is odd, so E_y=0.

21.93: a) The y-components of the electric field cancel, and the x-components from both charges, as given in problem **21.87** is:

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{-2Q}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right) \Rightarrow \vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{-2Qq}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right)\hat{\boldsymbol{i}}.$$

$$\text{If } y >> a,\ \vec{F} \approx \frac{1}{4\pi\varepsilon_0}\frac{-2Qq}{ay}(1-(1-a^2/2y^2+\cdots))\hat{\boldsymbol{i}} = -\frac{1}{4\pi\varepsilon_0}\frac{Qqa}{y^3}.$$

b) If the point charge is now on the x-axis the two charged parts of the rods provide different forces, though still along the x-axis (see problem **21.86**).

$$\vec{F}_+ = q\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right)\hat{\boldsymbol{i}} \text{ and } \vec{F}_- = q\,\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}\left(\frac{1}{x} - \frac{1}{x+a}\right)\hat{\boldsymbol{i}}$$

So,

$$\vec{F} = \vec{F}_+ + \vec{F}_- = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{2}{x} + \frac{1}{x+a}\right)\hat{\boldsymbol{i}}$$

$$\text{For } x >> a,\ \vec{F} \approx \frac{1}{4\pi\varepsilon_0}\frac{Qq}{ax}\left(\left(1+\frac{a}{x}+\frac{a^2}{x^2}+\ldots\right) - 2 + \left(1-\frac{a}{x}+\frac{a^2}{x^2}-\ldots\right)\right)\hat{\boldsymbol{i}} = \frac{1}{4\pi\varepsilon_0}\frac{2Qqa}{x^3}\hat{\boldsymbol{i}}.$$

21.94: The electric field in the x-direction cancels the left and right halves of the semicircle. The remaining y-component points in the negative y-direction. The charge per unit length of the semicircle is:

$$\lambda = \frac{Q}{\pi a} \text{ and } dE = \frac{k\lambda\, dl}{a^2} = \frac{k\lambda\, d\theta}{a} \text{ but } dE_y = dE \sin\theta = \frac{k\lambda \sin\theta\, d\theta}{a}.$$

$$\text{So, } E_y = \frac{2k\lambda}{a}\int_0^{\pi/2} \sin\theta\, d\theta = \frac{2k\lambda}{a}[-\cos\theta]_0^{\pi/2} = \frac{2k\lambda}{a} = \frac{2kQ}{\pi a^2}, \text{ downward.}$$

21.95: By symmetry, $E_x = E_y$. For E_y, compared to problem **21.94**, the integral over the angle is halved but the charge density doubles—giving the same result. Thus,

$$E_x = E_y = \frac{2k\lambda}{a} = \frac{2kQ}{\pi a^2}.$$

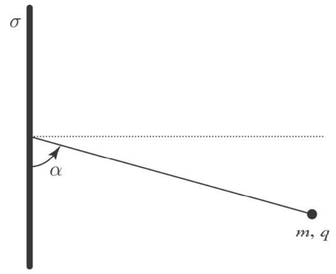
21.96:

$$\sum F_x = 0 \Rightarrow T\ \cos\alpha = mg \Rightarrow T = \frac{mg}{\cos\alpha}$$

$$\sum F_y = 0 \Rightarrow T\ \sin\alpha = \frac{q\sigma}{2\varepsilon_0} \Rightarrow T = \frac{q\sigma}{2\varepsilon_0 \sin\alpha}$$

$$\Rightarrow \frac{mg}{\cos\alpha} = \frac{q\sigma}{2\varepsilon_0 \sin\alpha} \Rightarrow \tan\alpha = \frac{q\sigma}{2\varepsilon_0 mg}$$

$$\Rightarrow \alpha = \arctan\left(\frac{q\sigma}{2\varepsilon_0 mg}\right)$$



21.97: a) $qE = 10\ mg \Rightarrow \frac{m}{q} = \frac{E}{10\text{g}} = \frac{1.4\times10^5\ \text{N/C}}{10(9.8\ \text{m/s}^2)} = 1429\ \text{kg/C}.$

b) $1429\ \frac{\text{kg}}{\text{C}}\cdot\frac{1\ \text{mol}}{12\times10^{-3}\ \text{kg}}\cdot\frac{6.02\times10^{23}\ \text{carbons}}{\text{mol}}\cdot\frac{1.6\times10^{-19}\ \text{C}}{\text{excess}\ e^-} = 1.15\times10^{10}\ \frac{\text{carbons}}{\text{excess}\ e^-}.$

21.98: a) $E_x = E_y$, and $E_x = 2E_{\text{length of wire } a, \text{ charge } Q} = 2\frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+\left(\frac{a}{2}\right)^2}}$, where

$x = \frac{a}{2} \Rightarrow E_x = -\frac{\sqrt{2}Q}{\pi\varepsilon_0 a^2},\ E_y = -\frac{\sqrt{2}Q}{\pi\varepsilon_0 a^2}.$

b) If all edges of the square had equal charge, the electric fields would cancel by symmetry at the center of the square.

21.99: a)

$$E(P) = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{|\sigma_2|}{2\varepsilon_0} + \frac{|\sigma_3|}{2\varepsilon_0} = -\frac{0.0200\ \mathrm{C/m^2}}{2\varepsilon_0} - \frac{0.0100\ \mathrm{C/m^2}}{2\varepsilon_0} + \frac{0.0200\ \mathrm{C/m^2}}{2\varepsilon_0}$$

$$\Rightarrow E(P) = \frac{0.0100\ \mathrm{C/m^2}}{2\varepsilon_0} = 5.65\times10^8\ \mathrm{N/C},\ \text{in the} - x\text{-direction.}$$

b) $$E(R) = +\frac{|\sigma_1|}{2\varepsilon_0} - \frac{|\sigma_2|}{2\varepsilon_0} + \frac{|\sigma_3|}{2\varepsilon_0} = +\frac{0.0200\ \mathrm{C/m^2}}{2\varepsilon_0} - \frac{0.0100\ \mathrm{C/m^2}}{2\varepsilon_0} + \frac{0.0200\ \mathrm{C/m^2}}{2\varepsilon_0}$$

$$\Rightarrow E(R) = \frac{0.0300\ \mathrm{C/m^2}}{2\varepsilon_0} = 1.69\times10^9\ \mathrm{N/C},\ \text{in the} + x\text{-direction.}$$

c) $$E(S) = +\frac{|\sigma_1|}{2\varepsilon_0} + \frac{|\sigma_2|}{2\varepsilon_0} + \frac{|\sigma_3|}{2\varepsilon_0} = +\frac{0.0200\ \mathrm{C/m^2}}{2\varepsilon_0} + \frac{0.0100\ \mathrm{C/m^2}}{2\varepsilon_0} + \frac{0.0200\ \mathrm{C/m^2}}{2\varepsilon_0}$$

$$\Rightarrow E(S) = \frac{0.0500\ \mathrm{C/m^2}}{2\varepsilon_0} = 2.82\times10^9\ \mathrm{N/C},\ \text{in the} + x\text{-direction.}$$

d) $$E(T) = +\frac{|\sigma_1|}{2\varepsilon_0} + \frac{|\sigma_2|}{2\varepsilon_0} - \frac{|\sigma_3|}{2\varepsilon_0} = +\frac{0.0200\ \mathrm{C/m^2}}{2\varepsilon_0} + \frac{0.0100\ \mathrm{C/m^2}}{2\varepsilon_0} - \frac{0.0200\ \mathrm{C/m^2}}{2\varepsilon_0}$$

$$\Rightarrow E(S) = \frac{0.0100\ \mathrm{C/m^2}}{2\varepsilon_0} = 5.65\times10^8\ \mathrm{N/C},\ \text{in the} + x\text{-direction.}$$

21.100:

$$\frac{F_{\text{on I}}}{A} = \frac{qE_{\text{at I}}}{A} = \sigma_1\left(\frac{-|\sigma_2| + |\sigma_3|}{2\varepsilon_0}\right) = \frac{2.00\times10^{-4}\ \mathrm{C/m^2}}{2\varepsilon_0} = +1.13\times10^7\ \mathrm{N/m}.$$

$$\frac{F_{\text{on II}}}{A} = \frac{qE_{\text{at II}}}{A} = \sigma_2\left(\frac{+|\sigma_1| + |\sigma_3|}{2\varepsilon_0}\right) = \frac{4.00\times10^{-4}\ \mathrm{C/m^2}}{2\varepsilon_0} = +2.26\times10^7\ \mathrm{N/m}$$

$$\frac{F_{\text{on III}}}{A} = \frac{qE_{\text{at III}}}{A} = \sigma_3\left(\frac{+|\sigma_1| + |\sigma_2|}{2\varepsilon_0}\right) = \frac{-6.00\times10^{-4}\ \mathrm{C/m^2}}{2\varepsilon_0} = -3.39\times10^7\ \mathrm{N/m}$$

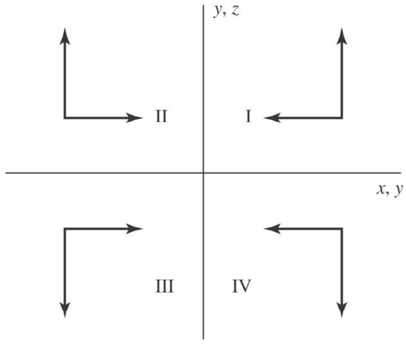
(Note that "+" means toward the right, and "–" is toward the left.)

21.101: By inspection the fields in the different regions are as shown below:

$$E_I = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\boldsymbol{i}} + \hat{\boldsymbol{k}}), \quad E_{II} = \left(\frac{\sigma}{2\varepsilon_0}\right)(+\hat{\boldsymbol{i}} + \hat{\boldsymbol{k}})$$

$$E_{III} = \left(\frac{\sigma}{2\varepsilon_0}\right)(+\hat{\boldsymbol{i}} - \hat{\boldsymbol{k}}), \; E_{IV} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\boldsymbol{i}} - \hat{\boldsymbol{k}})$$

$$\therefore \vec{\boldsymbol{E}} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\frac{|\,x\,|}{x}\hat{\boldsymbol{i}} + \frac{|\,z\,|}{z}\hat{\boldsymbol{k}}).$$



21.102: a) $Q = A\sigma = \pi(R_2^2 - R_2^2)\sigma$

b) Recall the electric field of a disk, Eq. (21.11): $E = \frac{\sigma}{2\varepsilon_0}\left[1 - 1\big/\sqrt{(R/x)^2 + 1}\right]$. So,

$$\vec{\boldsymbol{E}}\,(x) = \frac{\sigma}{2\varepsilon_0}\left(\left[1 - 1\big/\sqrt{(R_2/x)^2 + 1}\right] - \left[1 - 1\big/\sqrt{(R_1/x)^2 + 1}\right]\right)\frac{|x|}{x}\hat{\boldsymbol{i}} \Rightarrow E(x) = \frac{-\sigma}{2\varepsilon_0} \times$$

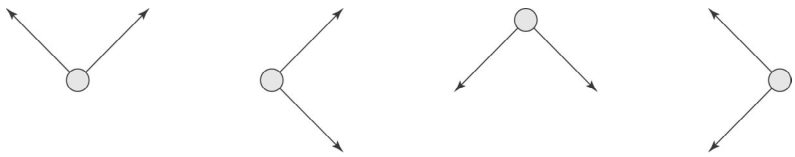
$$\left(1\big/\sqrt{(R_2/x)^2 + 1} - 1\big/\sqrt{(R_1/x)^2 + 1}\right)\frac{|x|}{x}\hat{\boldsymbol{i}}.$$

c) Note that $1\big/\sqrt{(R_1/x)^2 + 1} = \frac{|x|}{R_1}(1 + (x/R_1)^2)^{-1/2} \approx \frac{|x|}{R_1}\left(1 - \frac{(x/R_1)^2}{2} + \ldots\right)$

$\Rightarrow \vec{E}(x) = \frac{\sigma}{2\varepsilon_0}\left(\frac{x}{R_1} - \frac{x}{R_2}\right)\frac{x}{|x|}\hat{\boldsymbol{i}} = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{x}{|x^2|}\hat{\boldsymbol{i}}$, and sufficiently close means that $(x/R_1)^2 << 1$.

d) $F = qE(x) = -\frac{q\sigma}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)x = m\ddot{x} \Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\varepsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}.$

21.103: a) The four possible force diagrams are:



Only the last picture can result in an electric field in the –x-direction.

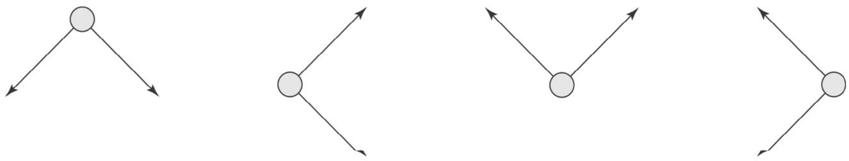
b) $q_1 = -2.00\ \mu\text{C}, q_3 = +4.00, \mu\text{C}, \text{and}\, q_2 > 0.$

c) $E_y = 0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{(0.0400\,\text{m})^2}\sin\theta_1 - \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(0.0300\,\text{m})^2}\sin\theta_2$

$\Rightarrow q_2 = \frac{9}{16}q_1\frac{\sin\theta_1}{\sin\theta_2} = \frac{9}{16}\; q_1\frac{3/5}{4/5} = \frac{27}{64}q_1 = 0.843\ \mu\text{C}.$

d) $F_3 = q_3 E_x = q_3\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{0.0016}\frac{4}{5} + \frac{q_2}{0.0009}\frac{3}{5}\right) = 56.2\ \text{N}$

21.104: (a) The four possible diagrams are:



The first diagram is the only one in which the electric field must point in the negative y-direction.

b) $q_1 = -3.00\ \mu\text{C},\ \text{and}\, q_2 < 0.$

c) $E_x = 0 = \frac{kq_1}{(0.050\ \text{m})^2}\frac{5}{13} - \frac{kq_2}{(0.120\ \text{m})^2}\frac{12}{13} \Rightarrow \frac{kq_2}{(0.120\ \text{m})^2} = \frac{kq_1}{(0.050\ \text{m})^2}\frac{5}{12}$

$E = E_y = \frac{kq_1}{(0.050\ \text{m})^2}\frac{12}{13} + \frac{kq_2}{(0.120\ \text{m})^2}\frac{5}{13} = \frac{kq_1}{(0.05\ \text{m})^2}\left(\frac{12}{13} + \left(\frac{5}{12}\right)\left(\frac{5}{13}\right)\right)$

$\Rightarrow E = E_y = 1.17\times 10^7\ \text{N/C}.$

21.105: a) For a rod in general of length L, $E=\frac{kQ}{L}\left(\frac{1}{r}-\frac{1}{L+r}\right)$ and here $r=x+\frac{a}{2}$.

So, $E_{\text{left rod}}=\frac{kQ}{L}\left(\frac{1}{x+a/2}-\frac{1}{L+x+a/2}\right)=\frac{2kQ}{L}\left(\frac{1}{2x+a}-\frac{1}{2L+2x+a}\right)$.

b) $dF=dq\,E\Rightarrow F=\int E\,dq=\int_{a/2}^{L+a/2}\frac{EQ}{L}\,dx=\frac{2kQ^2}{L^2}\int_{a/2}^{L+a/2}\times$

$$\left(\frac{1}{2x+a}-\frac{1}{2L+2x+a}\right)dx$$

$$\Rightarrow F=\frac{2kQ^2}{L^2}\frac{1}{2}\left([\ln(a+2x)]_{a/2}^{L+a/2}-[\ln(2L+2x+a)]_{a/2}^{L+a/2}\right)$$

$$\Rightarrow F=\frac{kQ^2}{L^2}\ln\left(\left(\frac{a+2L+a}{2a}\right)\left(\frac{2L+2a}{4L+2a}\right)\right)=\frac{kQ^2}{L^2}\ln\left(\frac{(a+L)^2}{a(a+2L)}\right).$$

c) For $a>>L:F=\frac{kQ^2}{L^2}\ln\left(\frac{a^2(1+L/a)^2}{a^2(1+2L/a)}\right)=\frac{kQ^2}{L^2}(2\ln(1+2L/a)-\ln(1+2L/a))$

$$\Rightarrow F\approx\frac{kQ^2}{L^2}\left(2\left(\frac{L}{a}-\frac{L^2}{2a^2}+\cdots\right)-\left(\frac{2L}{a}-\frac{2L^2}{a^2}+\cdots\right)\right)\Rightarrow F\approx\frac{kQ^2}{a^2}.$$

22.1: a) $\Phi = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}} = (14 \text{ N/C})\,(0.250 \text{ m}^2) \cos 60° = 1.75 \text{ Nm}^2/\text{C}.$

b) As long as the sheet is flat, its shape does not matter.

ci) The maximum flux occurs at an angle $\phi = 0°$ between the normal and field.

cii) The minimum flux occurs at an angle $\phi = 90°$ between the normal and field.

In part i), the paper is oriented to "capture" the most field lines whereas in ii) the area is oriented so that it "captures" no field lines.

22.2: a) $\Phi = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}} = EA\cos\theta$ where $\vec{\boldsymbol{A}} = A\hat{\boldsymbol{n}}$

$$\hat{\boldsymbol{n}}_{S_1} = -\hat{\boldsymbol{j}}\,(\text{left})\Phi_{S_1} = -(4\times10^3 \text{ N/C})\,(0.1\,\text{m})^2 \cos(90-36.9°) = -24 \text{ N}\cdot\text{m}^2/\text{C}$$

$$\hat{\boldsymbol{n}}_{S_2} = +\hat{\boldsymbol{k}}\,(\text{top})\Phi_{S_2} = -(4\times10^3 \text{ N/C})\,(0.1\,\text{m})^2 \cos 90° = 0$$

$$\hat{\boldsymbol{n}}_{S_3} = +\hat{\boldsymbol{j}}\,(\text{right})\Phi_{S_3} = +(4\times10^3 \text{ N/C})\,(0.1\,\text{m})^2 \cos(90°-36.9°) = +24 \text{ N}\cdot\text{m}^2/\text{C}$$

$$\hat{\boldsymbol{n}}_{S_4} = -\hat{\boldsymbol{k}}\,(\text{bottom})\Phi_{S_4} = (4\times10^3 \text{ N/C})\,(0.1\,\text{m})^2 \cos 90° = 0$$

$$\hat{\boldsymbol{n}}_{S_5} = +\hat{\boldsymbol{i}}\,(\text{front})\Phi_{S_5} = +(4\times10^3 \text{ N/C})\,(0.1\,\text{m})^2 \cos 36.9° = 32 \text{ N}\cdot\text{m}^2/\text{C}$$

$$\hat{\boldsymbol{n}}_{S_6} = -\hat{\boldsymbol{i}}\,(\text{back})\Phi_{S_6} = -(4\times10^3 \text{ N/C})\,(0.1\,\text{m})^2 \cos 36.9° = -32 \text{ N}\cdot\text{m}^2/\text{C}$$

b) The total flux through the cube must be zero; any flux entering the cube must also leave it.

22.3: a) Given that $\vec{\boldsymbol{E}} = -\text{B}\hat{\boldsymbol{i}} + \text{C}\hat{\boldsymbol{j}} - \text{D}\hat{\boldsymbol{k}},\ \Phi = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}},$ edge length L, and

$$\hat{\boldsymbol{n}}_{S_1} = -\hat{\boldsymbol{j}} \Rightarrow \Phi_1 = \vec{\boldsymbol{E}} \cdot A\hat{\boldsymbol{n}}_{S_1} = -\text{CL}^2.$$

$$\hat{\boldsymbol{n}}_{S_2} = +\hat{\boldsymbol{k}} \Rightarrow \Phi_2 = \vec{\boldsymbol{E}} \cdot A\hat{\boldsymbol{n}}_{S_2} = -\text{DL}^2.$$

$$\hat{\boldsymbol{n}}_{S_3} = +\hat{\boldsymbol{j}} \Rightarrow \Phi_3 = \vec{\boldsymbol{E}} \cdot A\hat{\boldsymbol{n}}_{S_3} = +\text{CL}^2.$$

$$\hat{\boldsymbol{n}}_{S_4} = -\hat{\boldsymbol{k}} \Rightarrow \Phi_4 = \vec{\boldsymbol{E}} \cdot A\hat{\boldsymbol{n}}_{S_4} = +\text{DL}^2.$$

$$\hat{\boldsymbol{n}}_{S_5} = +\hat{\boldsymbol{i}} \Rightarrow \Phi_5 = \vec{\boldsymbol{E}} \cdot A\hat{\boldsymbol{n}}_{S_5} = -\text{BL}^2.$$

$$\hat{\boldsymbol{n}}_{S_6} = -\hat{\boldsymbol{i}} \Rightarrow \Phi_6 = \vec{\boldsymbol{E}} \cdot A\hat{\boldsymbol{n}}_{S_6} = +\text{BL}^2.$$

b) Total flux $= \sum_{i=1}^{6} \Phi_\text{i} = 0$

22.4: $\Phi = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}} = (75.0 \text{ N/C})\,(0.240\,\text{m}^2) \cos 70° = 6.16 \text{ Nm}^2/\text{C}.$

22.5: a) $\Phi = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}} = \frac{\lambda}{2\pi\varepsilon_0 r}(2\pi r l) = \frac{\lambda l}{\varepsilon_0} = \frac{(6.00\times10^{-6}\text{ C/m})\,(0.400\text{ m})}{\varepsilon_0} = 2.71\times10^5\ \text{Nm}^2/\text{C}.$

b) We would get the same flux as in (a) if the cylinder's radius was made larger—the field lines must still pass through the surface.

c) If the length was increased to $l = 0.800$ m, the flux would increase by a factor of two: $\Phi = 5.42\times10^5\ \text{Nm}^2/\text{C}.$

22.6: a) $\Phi_{S_1} = q_1/\varepsilon_0 = (4.00\times10^{-9}\ \text{C})/\varepsilon_0 = 452\ \text{Nm}^2/\text{C}.$

b) $\Phi_{S_2} = q_2/\varepsilon_0 = (-7.80\times10^{-9}\ \text{C})/\varepsilon_0 = -881\ \text{Nm}^2/\text{C}.$

c) $\Phi_{S_3} = (q_1+q_2)/\varepsilon_0 = ((4.00-7.80)\times10^{-9}\ \text{C})/\varepsilon_0 = -429\ \text{Nm}^2/\text{C}.$

d) $\Phi_{S_4} = (q_1+q_2)/\varepsilon_0 = ((4.00+2.40)\times10^{-9}\ \text{C})/\varepsilon_0 = 723\ \text{Nm}^2/\text{C}.$

e) $\Phi_{S_5} = (q_1+q_2+q_3)/\varepsilon_0 = ((4.00-7.80+2.40)\times10^{-9}\ \text{C})/\varepsilon_0 = -158\ \text{Nm}^2/\text{C}.$

f) All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.7: a) $\Phi = q/\varepsilon_0 = (-3.60\times10^{-6}\ \text{C})/\varepsilon_0 = -4.07\times10^5\ \text{Nm}^2/\text{C}.$

b) $\Phi = q/\varepsilon_0 \Rightarrow q = \varepsilon_0\Phi = \varepsilon_0(780\ \text{Nm}^2/\text{C}) = 6.90\times10^{-9}\ \text{C}.$

c) No. All that matters is the total charge enclosed by the cube, not the details of where the charge is located.

22.8: a) No charge enclosed so $\Phi = 0$

b)
$$\Phi = \frac{q_2}{\varepsilon_0} = \frac{-6.00\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2} = -678\ \text{Nm}^2/\text{C}.$$

c)
$$\Phi = \frac{q_1+q_2}{\varepsilon_0} = \frac{(4.00-6.00)\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2} = -226\ \text{Nm}^2/\text{C}.$$

22.9: a) Since $\vec{\boldsymbol{E}}$ is uniform, the flux through a closed surface must be zero. That is: $\Phi = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int \rho\, dV = 0 \Rightarrow \int \rho\, dV = 0.$ But because we can choose any volume we want, ρ must be zero if the integral equals zero.

b) If there is no charge in a region of space, that does NOT mean that the electric field is uniform. Consider a closed volume close to, but not including, a point charge. The field diverges there, but there is no charge in that region.

22.10: a) If $\rho > 0$ and uniform, then q inside any closed surface is greater than zero. $\Rightarrow \Phi > 0 \Rightarrow \oint \vec{E} \cdot d\vec{A} > 0$ and so the electric field cannot be uniform, i.e., since an arbitrary surface of our choice encloses a non-zero amount of charge, E must depend on position.

b) However, inside a small bubble of zero density within the material with density ρ, the field CAN be uniform. All that is important is that there be zero flux through the surface of the bubble (since it encloses no charge). (See Exercise 22.61.)

22.11: $\Phi_{\text{6sides}} = q/\varepsilon_0 = (9.60 \times 10^{-6}\ \text{C})/\varepsilon_0 = 1.08 \times 10^6\ \text{Nm}^2/\text{C}$. But the box is symmetrical, so for one side, the flux is: $\Phi_{\text{1side}} = 1.80 \times 10^5\ \text{Nm}^2/\text{C}$.

b) No change. Charge enclosed is the same.

22.12: Since the cube is empty, there is no net charge enclosed in it. The net flux, according to Gauss's law, must be zero.

22.13: $\Phi_E = Q_{\text{encl}}/\varepsilon_0$

The flux through the sphere depends only on the charge within the sphere.

$Q_{\text{encl}} = \varepsilon_0 \Phi_E = \varepsilon_0 (360\ \text{N} \cdot \text{m}^2/\text{C}) = 3.19\ \text{nC}$

22.14: a) $E(r = 0.450\ \text{m} + 0.1\ \text{m}) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(2.50 \times 10^{-10}\ \text{C})}{(0.550\ \text{m})^2} = 7.44\ \text{N/C}$.

b) $\vec{E} = 0$ inside of a conductor or else free charges would move under the influence of forces, violating our electrostatic assumptions (i.e., that charges aren't moving).

22.15: a) $|E| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2} \Rightarrow r = \sqrt{\dfrac{1}{4\pi\varepsilon_0}\left|\dfrac{q}{E}\right|} = \sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{(0.180 \times 10^{-6}\ \text{C})}{614\ \text{N/C}}} = 1.62\ \text{m}$.

b) As long as we are outside the sphere, the charge enclosed is constant and the sphere acts like a point charge.

22.16: a) $\Phi = EA = q/\varepsilon_0 \Rightarrow q = \varepsilon_0 EA = \varepsilon_0\ (1.40 \times 10^5\ \text{N/C})\ (0.0610\ \text{m}^2) = 7.56 \times 10^{-8}\ \text{C}$.

b) Double the surface area: $q = \varepsilon_0 (1.40 \times 10^5\ \text{N/C})\ (0.122\ \text{m}^2) = 1.51 \times 10^{-7}\ \text{C}$.

22.17: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \Rightarrow q = 4\pi\varepsilon_0 E r^2 = 4\pi\varepsilon_0 (1150\ \text{N/C})\,(0.160\ \text{m})^2 = 3.27\times10^{-9}\ \text{C}$. So the number of electrons is: $n_\text{e} = \frac{3.27\times10^{-9}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 2.04\times10^{10}$.

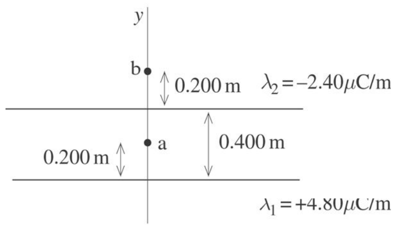
22.18: Draw a cylindrical Gaussian surface with the line of charge as its axis. The cylinder has radius 0.400 m and is 0.0200 m long. The electric field is then 840 N/C at every point on the cylindrical surface and directed perpendicular to the surface. Thus

$$\oint \vec{E}\cdot d\vec{s} = (E)(A_\text{cylinder}) = (E)(2\pi rL)$$
$$= (840\ \text{N/C})\,(2\pi)\,(0.400\ \text{m})\,(0.0200\ \text{m}) = 42.2\ \text{N}\cdot\text{m}^2/\text{C}$$

The field is parallel to the end caps of the cylinder, so for them $\oint \vec{E}\cdot d\vec{s} = 0$. From Gauss's law:

$$q = \varepsilon_0 \Phi_E = (8.854\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2})\,(42.2\frac{\text{N}\cdot\text{m}^2}{\text{C}})$$
$$= 3.74\times10^{-10}\ \text{C}$$

22.19:



$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

22.20: a) For points outside a uniform spherical charge distribution, all the charge can be considered to be concentrated at the center of the sphere. The field outside the sphere is thus inversely proportional to the square of the distance from the center. In this case:

$$E = (480\ \text{N/C})\left(\frac{0.200\ \text{cm}}{0.600\ \text{cm}}\right)^2 = 53\ \text{N/C}$$

b) For points outside a long cylindrically symmetrical charge distribution, the field is identical to that of a long line of charge:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r},$$

that is, inversely proportional to the distance from the axis of the cylinder. In this case

$$E = (480\ \text{N/C})\left(\frac{0.200\ \text{cm}}{0.600\ \text{cm}}\right) = 160\ \text{N/C}$$

c) The field of an infinite sheet of charge is $E = \sigma / 2\varepsilon_0$; i.e., it is independent of the distance from the sheet. Thus in this case $E = 480\ \text{N/C}$.

22.21: Outside each sphere the electric field is the same as if all the charge of the sphere were at its center, and the point where we are to calculate $\vec{E}$ is outside both spheres. $\vec{E}_1$ and $\vec{E}_2$ are both toward the sphere with negative charge.

$$E_1 = k\frac{|q_1|}{r_1^2} = k\frac{1.80\times 10^{-6}\ \text{C}}{(0.250\ \text{m})^2} = 2.591\times 10^5\ \text{N/C}$$

$$E_2 = k\frac{|q_2|}{r_2^2} = k\frac{3.80\times 10^{-6}\ \text{C}}{(0.250\ \text{m})^2} = 5.471\times 10^5\ \text{N/C}$$

$$E = E_1 + E_2 = 8.06\times 10^5\ \text{N/C, toward the negatively charged sphere.}$$

22.22: For points outside the sphere, the field is identical to that of a point charge of the same total magnitude located at the center of the sphere. The total charge is given by charge density $\times$ volume:

$$q = (7.50\ \text{nC/m}^3)(\frac{4}{3}\pi)(0.150\ \text{m})^3 = 1.60 \times 10^{-10}\ \text{C}$$

a) The field just outside the sphere is

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{(9\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(1.06\times 10^{-10}\ \text{C})}{(0.150\ \text{m})^2} = 42.4\ \text{N/C}$$

b) At a distance of 0.300 m from the center (double the sphere's radius) the field will be 1/4 as strong: 10.6 N/C

c) Inside the sphere, only the charge inside the radius in question affects the field. In this case, since the radius is half the sphere's radius, 1/8 of the total charge contributes to the field:

$$E = \frac{(9\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(1/8)\,(1.06\times 10^{-10}\ \text{C})}{(0.075\ \text{m})^2} = 21.2\ \text{N/C}$$

22.23: The point is inside the sphere, so $E = kQr/R^3$ (Example 22.9)

$$Q = \frac{ER^3}{kr} = \frac{(950\ \text{N/C})\,(0.220\ \text{m})^3}{k(0.100\ \text{m})} = 10.2\ \text{nC}$$

22.24: a) Positive charge is attracted to the inner surface of the conductor by the charge in the cavity. Its magnitude is the same as the cavity charge: $q_{\text{inner}} = +6.00\ \text{nC}$, since $\vec{E} = 0$ inside a conductor.

b) On the outer surface the charge is a combination of the net charge on the conductor and the charge "left behind" when the $+6.00$ nC moved to the inner surface:

$q_{\text{tot}} = q_{\text{inner}} + q_{\text{outer}} \Rightarrow q_{\text{outer}} = q_{\text{tot}} - q_{\text{inner}} = 5.00\ \text{nC} - 6.00\ \text{nC} = -1.00\ \text{nC}.$

22.25: S_2 and S_3 enclose no charge, so the flux is zero, and electric field outside the plates is zero. For between the plates, S_1 shows that: $EA = q/\varepsilon_0 = \sigma A/\varepsilon_0 \Rightarrow E = \sigma/\varepsilon_0$.

22.26: a) At a distance of 0.1 mm from the center, the sheet appears "infinite," so:

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = E\,2A = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{2\varepsilon_0 A} = \frac{7.50 \times 10^{-9}\,\mathrm{C}}{2\varepsilon_0 (0.800\ \mathrm{m})^2} = 662\ \mathrm{N/C}.$$

b) At a distance of 100 m from the center, the sheet looks like a point, so:

$$E \approx \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(7.50 \times 10^{-9}\ \mathrm{C})}{(100\ \mathrm{m})^2} = 6.75 \times 10^{-3}\ \mathrm{N/C}.$$

c) There would be no difference if the sheet was a conductor. The charge would automatically spread out evenly over both faces, giving it half the charge density on any as the insulator $(\sigma :)$. $E_c = \frac{\sigma}{\varepsilon_0} = \frac{\sigma :}{2\varepsilon_0}$ near one face. Unlike a conductor, the insulator *is* the charge density in some sense. Thus one shouldn't think of the charge as "spreading over each face" for an insulator. Far away, they both look like points with the same charge.

22.27: a)

$$\sigma = \frac{Q}{A} = \frac{Q}{2\pi RL} \Rightarrow \frac{Q}{L} = \sigma 2\pi R = \lambda.$$

b)

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = E(2\pi rL) = \frac{Q}{\varepsilon_0} = \frac{\sigma 2\pi RL}{\varepsilon_0} \Rightarrow E = \frac{\sigma R}{r\varepsilon_0}.$$

c) But from (a), $\lambda = \sigma 2\pi R$, so $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, same as an infinite line of charge.

22.28: All the σ's are absolute values.

(a) at $A: E_A = \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} + \frac{\sigma_4}{2\varepsilon_0} - \frac{\sigma_1}{2\varepsilon_0}$

$$E_A = \frac{1}{2\varepsilon_0}(\sigma_2 + \sigma_3 + \sigma_4 - \sigma_1)$$

$$= \frac{1}{2\varepsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2)$$

$$= 2.82 \times 10^5\ \text{N/C to the left.}$$

(b)

$$E_B = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} + \frac{\sigma_4}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_3 + \sigma_4 - \sigma_2)$$

$$= \frac{1}{2\varepsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2)$$

$$= 3.95 \times 10^5\ \text{N/C to the left.}$$

(c)

$$E_C = \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0} - \frac{\sigma_1}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(\sigma_2 + \sigma_3 - \sigma_4 - \sigma_1)$$

$$= \frac{1}{2\varepsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 - 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2)$$

$$= 1.69 \times 10^5\ \text{N/C to the left}$$

22.29: a) Gauss's law says +Q on inner surface, so $E = 0$ inside metal.

b) The outside surface of the sphere is grounded, so no excess charge.

c) Consider a Gaussian sphere with the –Q charge at its center and radius less than the inner radius of the metal. This sphere encloses net charge –Q so there is an electric field flux through it; there is electric field in the cavity.

d) In an electrostatic situation $E = 0$ inside a conductor. A Gaussian sphere with the $-Q$ charge at its center and radius greater than the outer radius of the metal encloses zero net charge (the $-Q$ charge and the $+Q$ on the inner surface of the metal) so there is no flux through it and $E = 0$ outside the metal.

e) No, $E = 0$ there. Yes, the charge has been shielded by the grounded conductor. There is nothing like positive and negative mass (the gravity force is always attractive), so this cannot be done for gravity.

22.30: Given $\vec{E} = (-5.00\ (\text{N/C})\cdot\text{m})x\hat{i} + (3.00\ (\text{N/C})\cdot\text{m})z\hat{k}$, edge length $L = 0.300$ m, $L = 0.300$ m, and $\hat{n}_{S_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E}\cdot\hat{n}_{S_1}A = 0.$

$\hat{n}_{S_1} = +\hat{k} \Rightarrow \Phi_2 = \vec{E}\cdot\hat{n}_{S_2}A = (3.00\ (\text{N/C})\cdot\text{m})(0.300\ \text{m})^2 z = (0.27\ (\text{N/C})\text{m})z =$ $(0.27\ (\text{N/C})\text{m})(0.300\ \text{m}) = 0.081\,(\text{N/C})\ \text{m}^2.$

$$\hat{n}_{S_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E}\cdot\hat{n}_{S_3}A = 0.$$

$$\hat{n}_{S_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E}\cdot\hat{n}_{S_4}A = -(0.27\ (\text{N/C})\cdot\text{m})z = 0\ (z=0).$$

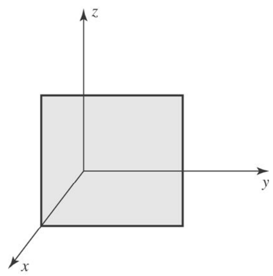
$$\hat{n}_{S_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E}\cdot\hat{n}_{S_5}A = (-5.00\ (\text{N/C})\cdot\text{m})(0.300\ \text{m})^2 x = -(0.45(\text{N/C})\cdot\text{m})x$$
$$= -(0.45\ (\text{N/C})\cdot\text{m})(0.300\ \text{m}) = -(0.135\,(\text{N/C})\cdot\text{m}^2).$$

$$\hat{n}_{S_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E}\cdot\hat{n}_{S_6}A = +(0.45\ (\text{N/C})\cdot\text{m})x = 0\ (x=0).$$

b) Total flux:

$$\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)\ (\text{N/C})\cdot\text{m}^2 = -0.054\ \text{Nm}^2/\text{C}$$
$$q = \varepsilon_0\Phi = -4.78\times10^{-13}\,\text{C}$$

22.31: a)



b) Imagine a charge q at the center of a cube of edge length $2L$. Then: $\Phi = q/\varepsilon_0$. Here the square is one 24th of the surface area of the imaginary cube, so it intercepts 1/24 of the flux. That is, $\Phi = q/24\varepsilon_0$.

22.32: a) $\Phi = EA = (125\ \text{N/C})(6.0\ \text{m}^2) = 750\ \text{N}\cdot\text{m}^2/\text{C}.$

b) Since the field is parallel to the surface, $\Phi = 0$.

c) Choose the Gaussian surface to equal the volume's surface. Then: 750 – $EA = q/\varepsilon_0 \Rightarrow E = \frac{1}{6.0\ \text{m}^2}(2.40\times10^{-8}\ \text{C}/\varepsilon_0 + 750) = 577\ \text{N/C}$, in the positive x-direction. Since $q < 0$ we must have some net flux flowing *in* so $EA \to -|EA|$ on second face.

d) $q < 0$ but we have E pointing *away* from face I. This is due to an external field that does not affect the flux but affects the value of E.

22.33: To find the charge enclosed, we need the flux through the parallelepiped:

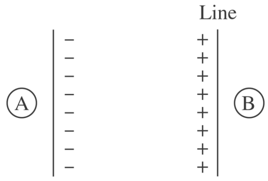
$$\Phi_1 = AE_1 \cos 60° = (0.0500 \text{ m})(0.0600 \text{ m})(2.50 \times 10^4 \text{ N/C}) \cos 60° = 37.5 \text{ N} \cdot \text{m}^2/\text{C}$$

$$\Phi_2 = AE_2 \cos 120° = (0.0500 \text{ m})(0.0600 \text{ m})(7.00 \times 10^4 \text{ N/C}) \cos 60° = -105 \text{ N} \cdot \text{m}^2/\text{C}$$

So the total flux is $\Phi = \Phi_1 + \Phi_2 = (37.5 - 105) \text{ N} \cdot \text{m}^2/\text{C} = -67.5 \text{ N} \cdot \text{m}^2/\text{C}$, and $q = \Phi\varepsilon_0 = (-67.5 \text{ N} \cdot \text{m}^2/\text{C})\varepsilon_0 = -5.97 \times 10^{-10} \text{ C}.$

b) There must be a net charge (negative) in the parallelepiped since there is a net flux flowing into the surface. Also, there must be an external field or all lines would point toward the slab.

22.34:



The α particle feels no force where the net electric field is zero. The fields can cancel only in regions A and B.

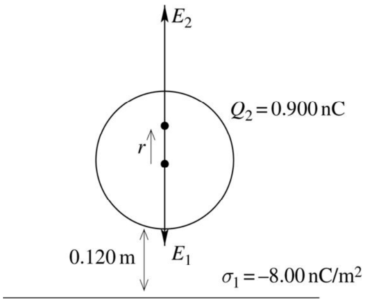
$$E_{line} = E_{sheet}$$

$$\frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma}{2\varepsilon_0}$$

$$r = \lambda/\pi\sigma = \frac{50 \ \mu\text{C/m}}{\pi(100 \ \mu\text{C/m}^2)} = 0.16\text{m} = 16\text{cm}$$

The fields cancel 16 cm from the line in regions A *and* B.

22.35:



The electric field $\vec{\boldsymbol{E}}_1$ of the sheet of charge is toward the sheet, so the electric field $\vec{\boldsymbol{E}}_2$ of the sphere must be away from the sheet. This is true above the center of the sphere. Let *r* be the distance above the center of the sphere for the point where the electric field is zero.

$$E_1 = E_2 \text{ so } \frac{\sigma_1}{2\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2 r}{R^3}$$

$$r = \frac{2\pi\sigma_1 R^3}{Q_2} = \frac{2\pi(8.00\times10^{-9}\ \text{C/m}^2)(0.120\ \text{m})^3}{0.900\times10^{-9}\ \text{C}} = 0.097\ \text{m}$$

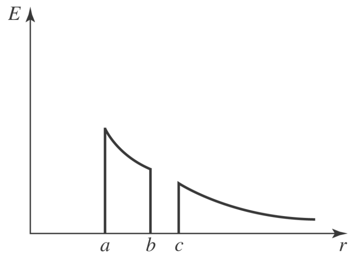
22.36: a) For $r < a, E = 0,$ since no charge is enclosed.

For $a < r < b, E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2},$ since there is $+q$ inside a radius r.

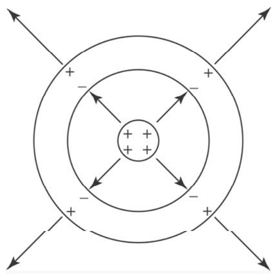
For $b < r < c, E = 0$, since now the $-q$ cancels the inner $+q$.

For $r > c, E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2},$ since again the total charge enclosed is $+q$.

b)

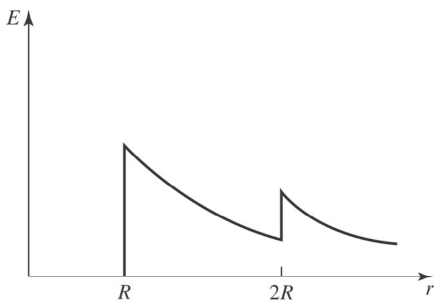


c) Charge on inner shell surface is $-q$.
d) Charge on outer shell surface is $+q$.
e)



22.37: a) $r < R, E = 0,$ since no charge is enclosed.

b) $R < r < 2R, E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2},$ since charge enclosed is Q. $r > 2R, E = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{r^2},$ since charge enclosed is $2Q$.



22.38: a) $r < a, E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2},$ since the charge enclosed is Q.

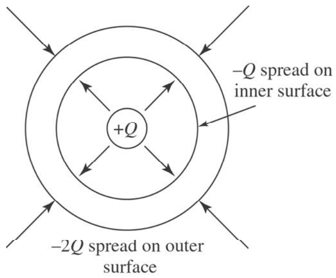
$a < r < b, E = 0,$ since the $-Q$ on the inner surface of the shell cancels the $+Q$ at the center of the sphere.

$r > b, E = -\frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$, since the total enclosed charge is $-2Q$.

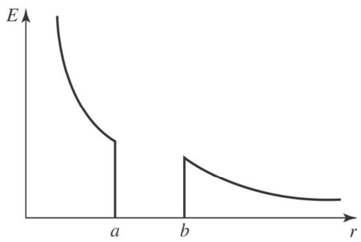
b) The surface charge density on inner surface: $\sigma = -\frac{Q}{4\pi a^2}$.

c) The surface charge density on the outer surface: $\sigma = -\frac{2Q}{4\pi b^2}$.

d)



e)



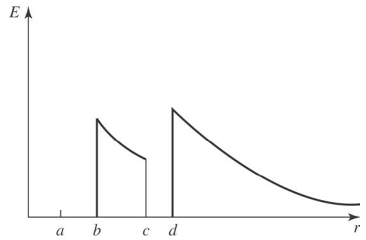
22.39: a)(i) $r < a, E = 0,$ since $Q = 0$

(ii) $a < r < b, E = 0,$ since $Q = 0.$

(iii) $b < r < c, E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2},$ since $Q = +2q.$

(iv) $c < r < d, E = 0,$ since $Q = 0.$

(v) $r > d, E = \frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2},$ since $Q = +6q.$



b)(i) small shell inner: $Q = 0$

(ii) small shell outer: $Q = +2q$

(iii) large shell inner: $Q = -2q$

(iv) large shell outer: $Q = +6q$

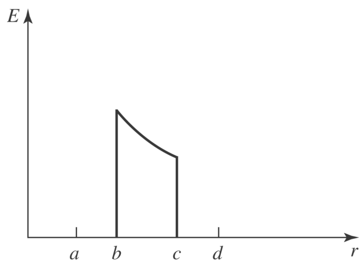
22.40: a)(i) $r < a, E = 0,$ since the charge enclosed is zero.

(ii) $a < r < b, E = 0,$ since the charge enclosed is zero.

(iii) $b < r < c, E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2},$ since charge enclosed is $+2q.$

(iv) $c < r < d, E = 0,$ since the net charge enclosed is zero.

(v) $r > d, E = 0,$ since the net charge enclosed is zero.



b)(i) small shell inner: $Q = 0$

(ii) small shell outer: $Q = +2q$

(iii) large shell inner: $Q = -2q$

(iv) large shell outer: $Q = 0$

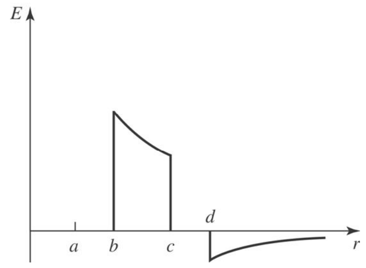
22.41: a)(i) $r < a, E = 0,$ since charge enclosed is zero.

(ii) $a < r < b, E = 0,$ since charge enclosed is zero.

(iii) $b < r < c, E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2},$ since charge enclosed is $+2q.$

(iv) $c < r < d, E = 0,$ since charge enclosed is zero.

(v) $r > d, E = -\frac{1}{4\pi\epsilon_0}\frac{2q}{r^2},$ since charge enclosed is $-2q.$



b)(i) small shell inner: $Q = 0$

(ii) small shell outer: $Q = +2q$

(iii) large shell inner: $Q = -2q$

(iv) large shell outer: $Q = -2q$

22.42: a) We need:

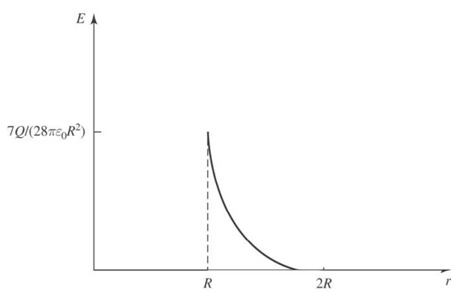
$$-Q = \frac{4\pi\rho}{3}((2R)^3 - R^3) \Rightarrow Q = \frac{-28\pi\rho R^3}{3} \Rightarrow \rho = -\frac{3Q}{28\pi R^3}.$$

b) $r < R, E = 0$ and $r > 2R, E = 0,$ since the net charges are zero.

$$R < r < 2R, \Phi = E(4\pi r^2) = \frac{Q}{\varepsilon_0} + \frac{4\pi\rho}{3\varepsilon_0}(r^3 - R^3) \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{\rho}{3\varepsilon_0 r^2}(r^3 - R^3).$$

Substituting ρ from (a) $E = \frac{2}{7\pi\epsilon_0}\frac{Q}{r^2} - \frac{Qr}{28\pi\epsilon_0 R^3}.$

c) We see a discontinuity in going from the conducting sphere to the insulator due to the thin surface charge of the conducting sphere—but we see a smooth transition from the uniform insulator to the outside.

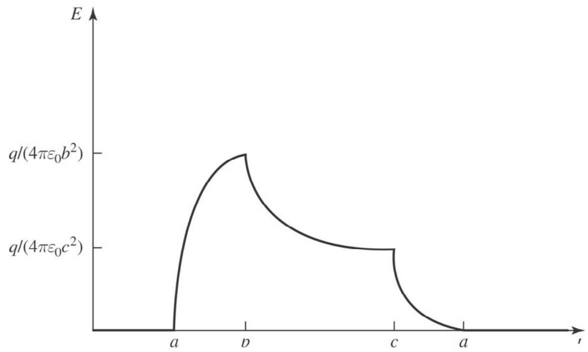


22.43: a) The sphere acts as a point charge on an external charge, so:
$F = qE = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$, radially inward.

(b) If the point charge was inside the sphere (where there is no electric field) it would feel zero force.

22.44: a) $\rho_{\text{inner}} = \dfrac{+q}{V_b - V_a} = \dfrac{q}{\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3} = \dfrac{3q}{4\pi}\left(\dfrac{1}{b^3 - a^3}\right)$

$$\rho_{\text{outer}} = \frac{-q}{V_d - V_c} = \frac{-q}{\frac{4}{3}\pi d^3 - \frac{4}{3}\pi c^3} = \frac{-3q}{4\pi}\left(\frac{1}{d^3 - c^3}\right)$$



b) (i) $r < a \oint \vec{E} \cdot d\vec{A} = 0 \Rightarrow E = 0.$

(ii) $a < r < b \oint \vec{E} \cdot d\vec{A} = \dfrac{1}{\varepsilon_0}\int \rho_{\text{inner}} dV \Rightarrow E 4\pi r^2 = \dfrac{4}{3\varepsilon_0}\pi(r^3 - a^3)\rho_{\text{inner}}$

$$E = \frac{1}{3\varepsilon_0}\rho_{\text{inner}}\frac{(r^3 - q^3)}{r^2} = \frac{q}{4\pi\varepsilon_0}\frac{(r^3 - a^3)}{(b^3 - a^3)}$$

(iii) $b < r < c \oint \vec{E} \cdot d\vec{A} = \dfrac{q}{\varepsilon_0} \Rightarrow E 4\pi r^2 = \dfrac{q}{\varepsilon_0} \Rightarrow E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

(iv) $c < r < d \oint \vec{E} \cdot d\vec{A} = \dfrac{q}{\varepsilon_0} + \dfrac{1}{\varepsilon_0}\int \rho_{\text{outer}} dV \Rightarrow$

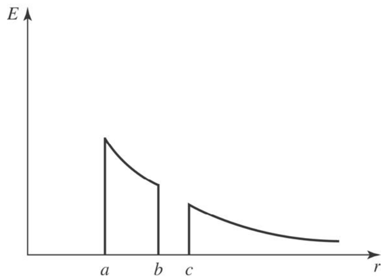
$$E 4\pi r^2 = \frac{q}{\varepsilon_0} + \frac{4\pi}{3\varepsilon_0}(r^3 - c^3)\rho_{\text{outer,}} \text{ so } E = \frac{q}{4\pi\varepsilon_0 r^2} - \frac{q(r^3 - c^3)}{4\pi\varepsilon_0 r^2 (d^3 - c^3)}$$

(v) $r > d \oint \vec{E} \cdot d\vec{A} = \dfrac{q}{\varepsilon_0} - \dfrac{q}{\varepsilon_0} = 0 \Rightarrow E = 0$

22.45: a) $a < r < b,\ E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\lambda}{r},$ radially outward, as in **22.48 (b)**.

b) $r > c,\ E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r},$ radially outward, since again the charge enclosed is the same as in part (a).

c)

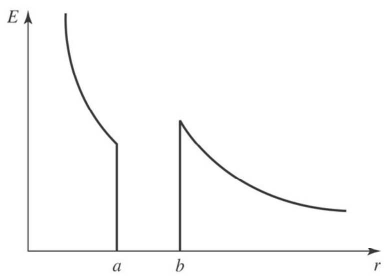


d) The inner and outer surfaces of the outer cylinder must have the same amount of charge on them: $\lambda l = -\lambda_{\text{inner}} l \Rightarrow \lambda_{\text{inner}} = -\lambda$, and $\lambda_{\text{outer}} = \lambda.$

22.46: a) (i) $r < a,\ E(2\pi rl) = \dfrac{q}{\varepsilon_0} = \dfrac{\alpha l}{\varepsilon_0} \Rightarrow E = \dfrac{\alpha}{2\pi\varepsilon_0 r}.$

(ii) $a < r < b,$ there is no net charge enclosed, so the electric field is zero.

(iii) $r > b,\ E(2\pi rl) = \dfrac{q}{\varepsilon_0} = \dfrac{2\alpha l}{\varepsilon_0} \Rightarrow E = \dfrac{\alpha}{\pi\varepsilon_0 r}.$

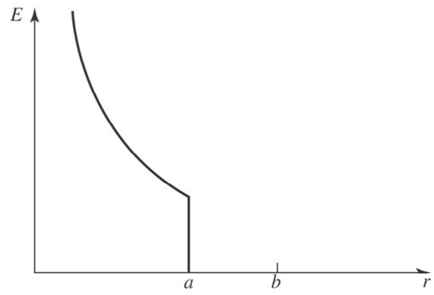


b) (i) Inner charge per unit length is $-\alpha.$ (ii) Outer charge per length is $+2\alpha.$

22.47: a) (i) $r < a,\ E(2\pi rl) = \frac{q}{\varepsilon_0} = \frac{\alpha l}{\varepsilon_0} \Rightarrow E = \frac{\alpha}{2\pi\varepsilon_0 r}$, radially outward.

(ii) $a < r < b,$ there is not net charge enclosed, so the electric field is zero.

(iii) $r > b,$ there is no net charge enclosed, so the electric field is zero.



b) (i) Inner charge per unit length is $-\alpha$.

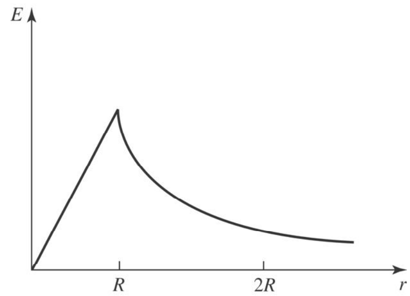
(ii) Outer charge per length is ZERO.

22.48: a) $r < R,\ E(2\pi rl) = \frac{q}{\varepsilon_0} = \frac{\rho\pi r^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{2\varepsilon_0}$, radially outward.

b) $r > R$, and $\lambda = \rho\pi R^2$, $E(2\pi rl) = \frac{q}{\varepsilon_0} = \frac{\rho\pi R^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$.

c) $r = R.$ the electric field for BOTH regions is $E = \frac{\rho R}{2\varepsilon_0}$, so they are consistent.

d)



22.49: a) The conductor has the surface charge density on BOTH sides, so it has twice the enclosed charge and twice the electric field.

b) We have a conductor with surface charge density σ on both sides. Thus the electric field outside the plate is $\Phi = E(2A) = (2\sigma A)/\varepsilon_0 \Rightarrow E = \sigma/\varepsilon_0$. To find the field inside the conductor use a Gaussian surface that has one face inside the conductor, and one outside.
Then:

$$\Phi = E_{\text{out}}A + E_{\text{in}}A = (\sigma A)/\varepsilon_0 \text{ but } E_{\text{out}} = \sigma/\varepsilon_0 \Rightarrow E_{\text{in}}A = 0 \Rightarrow E_{\text{in}} = 0.$$

22.50: a) If the nucleus is a uniform positively charged sphere, it is only at its very center where forces on a charge would balance or cancel

b) $\Phi = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \dfrac{q}{\varepsilon_0} \Rightarrow E4\pi r^2 = \dfrac{e}{\varepsilon_0}\left(\dfrac{r^3}{R^3}\right) \Rightarrow E = \dfrac{er}{4\pi\varepsilon_0 R^3}$

$$\Rightarrow F = qE = -\frac{1}{4\pi\varepsilon_0}\frac{e^2 r}{R^3}.$$

So from the simple harmonic motion equation:

$$F = -m\omega^2 r = -\frac{1}{4\pi\varepsilon_0}\frac{e^2 r}{R^3} \Rightarrow \omega = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mR^3}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mR^3}}.$$

c) If $f = 4.57\times 10^{14}\ \text{Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{mR^3}}$

$$\Rightarrow R = \sqrt[3]{\frac{1}{4\pi\varepsilon_0}\frac{(1.60\times 10^{-19}\ \text{C})^2}{4\pi^2(9.11\times 10^{-31}\ \text{kg})(4.57\times 10^{14}\ \text{Hz})^2}} = 3.13\times 10^{-10}\ \text{m}.$$

$r_{\text{actual}} / r_{\text{Thompson}} \approx 1$

d) If $r > R$ then the electron would still oscillate but not undergo simple harmonic motion, because for $r > R,\ F \propto 1/r^2$, and is not linear.

22.51: The electrons are separated by a distance $2d$, and the amount of the positive nucleus's charge that is within radius d is all that exerts a force on the electron. So:

$$F_e = \frac{ke^2}{(2d)^2} = F_{\text{nucleus}} = 2ke^2 \tfrac{d}{R^3} \Rightarrow d^3 = R^3/8 \Rightarrow d = R/2.$$

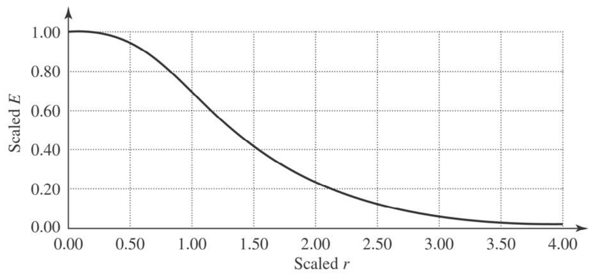
22.52: a) $Q(r) = Q - \int \rho dV = Q - \frac{Q}{\pi a_0^3} \int\int\int e^{-2r/a_0} r^2 dr \sin\theta d\theta \, d\phi = Q - \frac{4Q}{a_0^3} \int_0^r x^2 e^{-2x/a_0} \, dx$

$$\Rightarrow Q(r) = Q - \frac{4Qe^{-\alpha r}}{a_0{}^3 \alpha^3}(2e^{\alpha r} - \alpha^2 r^2 - 2\alpha r - 2) = Qe^{-2r/a_0}[2(r/a_0)^2 + 2(r/a_0) + 1].$$

Note if $r \to \infty$, $Q(r) \to 0$.

b) The electric field is radially outward, and has magnitude:

$$\Rightarrow E = \frac{kQe^{-2r/a_0}}{r^2}(2(r/a_0)^2 + 2(r/a_0) + 1).$$



22.53: a) At $r = 2R$, $F = q_e E = \frac{1}{4\pi\epsilon_0}\frac{q_e q_{Fe}}{4R^2} = \frac{1}{4\pi\epsilon_0}\frac{(82)(1.6\times10^{-19}\text{ C})^2}{4(7.1\times10^{-15}\text{ m})^2} = 94\text{ N}.$

So: $a = F/m = 94\text{ N}/9.11\times10^{-31}\text{ kg} = 1.0\times10^{32}\text{ m/s}^2.$

b) At $r = R$, $a = 4a_{(a)} = 4.1\times10^{32}\text{ m/s}^2.$

c) At $r = R/2$, $Q = \frac{1}{8}(82e)$ ($\frac{1}{8}$ because the charge enclosed goes like r^3) so with the radius decreasing by 2, the acceleration from the change in radius goes up by $(2)^2 = 4$, but the charge decreased by 8, so $a = \frac{4}{8}a_{(b)} = 2.1\times10^{32}\text{ m/s}^2.$

d) At $r = 0$, $Q = 0$, so $F = 0$.

22.54: a) The electric field of the slab must be zero by symmetry. There is no preferred direction in the y-z plane, so the electric field can only point in the x-direction. But at the origin in the x-direction, neither the positive nor negative directions should be singled out as special, and so the field must be zero.

b) Use a Gaussian surface that has one face of area A on in the y-z plane at $x = 0,$ and the other face at a general value $x.$ Then:

$$x \le d : \Phi = EA = \frac{Q_{encl}}{\varepsilon_0} = \frac{\rho A x}{\varepsilon_0} \Rightarrow E = \frac{\rho x}{\varepsilon_0},$$

with direction given by $\frac{x}{|x|}\hat{\boldsymbol{i}}.$

Note that E is zero at $x = 0.$

Now outside the slab, the enclosed charge is constant with x:

$$x \ge d : \Phi = EA = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho A d}{\varepsilon_0} \Rightarrow E = \frac{\rho d}{\varepsilon_0},$$

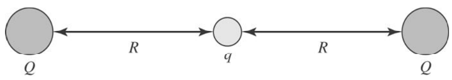
again with direction given by $\frac{x}{|x|}\hat{\boldsymbol{i}}.$

22.55: a) Again, E is zero at $x = 0$, by symmetry arguments.

b) $x \le d : \Phi = EA = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{\rho_0 A}{\varepsilon_0 d^2} \displaystyle\int_0^x x'^2 \, dx' = \dfrac{\rho_0 A x^3}{3\varepsilon_0 d^2} \Rightarrow E = \dfrac{\rho_0 x^3}{3\varepsilon_0 d^2}$, in $\dfrac{x}{|x|}\hat{\boldsymbol{i}}$ direction.

$x \ge d : \Phi = EA = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{\rho_0 A}{\varepsilon_0 d^2} \displaystyle\int_0^d x'^2 \, dx' = \dfrac{\rho_0 A d}{3\varepsilon_0} \Rightarrow E = \dfrac{\rho_o d}{3\varepsilon_0}$, *in* $\dfrac{x}{|x|}\hat{\boldsymbol{i}}$ direction.

22.56: a) We could place two charges $+Q$ on either side of the charge $+q$:



b) In order for the charge to be stable, the electric field in a neighborhood around it must always point back to the equilibrium position.

c) If q is moved to infinity and we require there to be an electric field always pointing in to the region where q had been, we could draw a small Gaussian surface there. We would find that we need a negative flux into the surface. That is, there has to be a negative charge in that region. However, there is none, and so we cannot get such a stable equilibrium.

d) For a negative charge to be in stable equilibrium, we need the electric field to always point away from the charge position. The argument in (c) carries through again, this time inferring that a positive charge must be in the space where the negative charge was if stable equilibrium is to be attained.

22.57: a) The total charge: $q = 4\pi \int_0^R \rho_0 (1 - r/R) r^2 dr = 4\pi [\int_0^R r^2 dr - \int_0^R r^3/R dr]$

$$\Rightarrow q = 4\pi\rho_0 \, [R^3/3 - R^3/4] = \frac{4\pi R^3 \rho_0}{12} = \frac{4\pi R^3}{12} \frac{3Q}{\pi R^3} = Q.$$

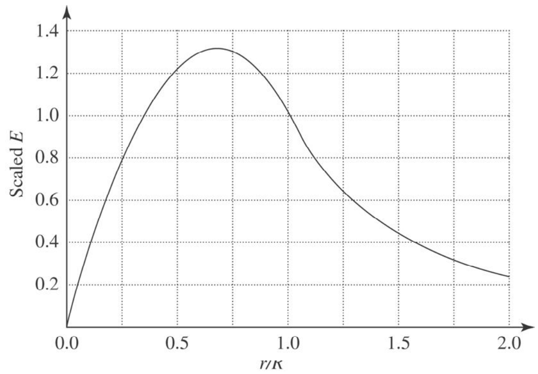
b) $r \geq R$, all the charge Q is enclosed, and: $\Phi = E(4\pi r^2) = Q/\varepsilon_0 \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, the same as a point charge.

c) $r \leq R$, then $Q(r) = q(r^3/R^3)$.

Also, $Q(r) = 4\pi \int \rho_0 (1 - r/R) r^2 dr = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R} \right)$.

$$\Rightarrow E(r) = \frac{12kQ}{r^2} \left(\frac{1}{3}\frac{r^3}{R^3} - \frac{1}{4}\frac{r^4}{R^4} \right) = \frac{kQ}{r^2} \left(\frac{4r^3}{R^3} - \frac{3r^4}{R^4} \right) = kQ \frac{r}{R^3} \left(4 - 3\frac{r}{R} \right)$$

d)



e) $\frac{\partial E}{\partial r} = 0 \; (r \leq R) \Rightarrow \frac{4kQ}{R^3} - \frac{6kQ}{R^4} r = 0 \Rightarrow r_{\text{max}} = \frac{2}{3} R.$ So $E_{\text{max}} = \frac{2}{3} \frac{kQ}{R^2} (4 - 2) = \frac{4kQ}{3R^2}$

22.58: a)

$$Q = 4\pi\int_0^\infty \rho(r)r^2dr = 4\pi\rho_0\int_0^R\left(1-\frac{4r}{3R}\right)r^2dr = 4\pi\rho_0\left[\int_0^R r^2dr - \frac{4}{3R}\int_0^R r^3dr\right]$$

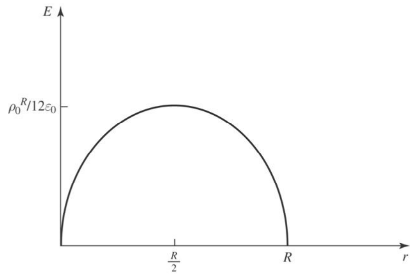
$$= 4\pi\rho_0\left[\frac{R^3}{3} - \frac{4}{3R}\cdot\frac{R^4}{4}\right] \Rightarrow Q = 0$$

b) $$r \geq R, \oint \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{A}} = \frac{q_{\text{encl}}}{\varepsilon_0} = 0 \Rightarrow E = 0$$

c) $$r \leq R, \oint \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{A}} = \frac{4\pi}{\varepsilon_0}\int_0^r \rho(r')r'^2dr' \Rightarrow E4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\left[\int_0^r r'^2dr' - \frac{4}{3R}\int_0^r r'^3dr'\right]$$

$$\Rightarrow E = \frac{\rho_0}{\varepsilon_0}\frac{1}{r^2}\left[\frac{r^3}{3} - \frac{r^4}{3R}\right] = \frac{\rho_0}{3\varepsilon_0}r\left[1-\frac{r}{R}\right]$$

d)



e) $$\frac{\partial E}{\partial r} = 0 \Rightarrow \frac{\rho_0}{3\varepsilon_0} - \frac{2\rho_0 r_{\text{max}}}{3\varepsilon_0 R} = 0 \; r_{\text{max}} = \frac{R}{2}$$

$$E\left(r-\frac{R}{2}\right) = \frac{\rho_0}{3\varepsilon_0}\frac{R}{2}\left[1-\frac{1}{2}\right] = \frac{\rho_0 R}{12\varepsilon_0}$$

22.59: a) $\Phi_g = \oint \vec{\boldsymbol{g}}\cdot d\vec{\boldsymbol{A}} = -Gm\oint\frac{r^2\sin\theta\; dr\; d\theta\; d\phi}{r^2} = -4\pi Gm.$

b) For any closed surface, mass OUTSIDE the surface contributes zero to the flux passing through the surface. Thus the formula above holds for any situation where m is the mass enclosed by the Gaussian surface.
That is: $\Phi_g = \oint \vec{\boldsymbol{g}}\cdot d\vec{\boldsymbol{A}} = -4\pi GM_{\text{encl.}}$

22.60: a) $\Phi_g = g4\pi r^2 = -4\pi GM \Rightarrow g = -\dfrac{GM}{r^2}$, which is the same as for a point mass.

b) Inside a hollow shell, the $M_{\text{encl}} = 0$, so $g = 0$.

c) Inside a uniform spherical mass:

$$\Phi_g = g4\pi r^2 = -4\pi GM_{\text{encl}} = -4\pi G\left(M\frac{r^3}{R^3}\right) \Rightarrow g = -\frac{GMr}{R^3},$$

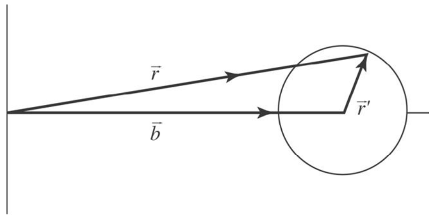
which is linear in r.

22.61: a) For a sphere NOT at the coordinate origin:

$$\vec{r}' = \vec{r} - \vec{b} \Rightarrow \Phi = 4\pi r'^2\ E = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\frac{4\pi r'^3}{3} \Rightarrow E = \frac{\rho r'}{3\varepsilon_0},$$

in the $\hat{r}'$- direction.

$$\Rightarrow \vec{E} = \frac{\rho(\vec{r} - \vec{b})}{3\varepsilon_0}.$$



b) The electric field inside a hole in a charged insulating sphere is:

$$\vec{E}_{\text{hole}} = \vec{E}_{\text{sphere}} - \vec{E}_{\text{(a)}} = \frac{\rho\vec{r}}{3\varepsilon_0} - \frac{\rho(\vec{r} - \vec{b})}{3\varepsilon_0} = \frac{\rho\vec{b}}{3\varepsilon_0}.$$

Note that $\vec{E}$ is uniform.

22.62: Using the technique of **22.61,** we first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields—that of a solid cylinder on-axis, and one off-axis.

$$\vec{r}' = \vec{r} - \vec{b} \Rightarrow \Phi = 2\pi r' l\ E = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0} l\pi r'^2 \Rightarrow E = \frac{\rho r'}{2\varepsilon_0} \Rightarrow \vec{E} = \frac{\rho(\vec{r} - \vec{b})}{2\varepsilon_0}.$$

$$\vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{E}_{\text{above}} = \frac{\rho\,\vec{r}}{2\varepsilon_0} - \frac{\rho(\vec{r} - \vec{b})}{2\varepsilon_0} = \frac{\rho\,\vec{b}}{2\varepsilon_0}.$$ Note that $\vec{E}$ is uniform.

22.63: a) $x = 0$: no field contribution from the sphere centered at the origin, and the other sphere produces a point-like field:

$$\vec{E}(x=0) = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}\hat{i} = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{4R^2}\hat{i}\,.$$

b) $x = R/2$: the sphere at the origin provides the field of a point charge of charge $q = Q/8$ since only one-eighth of the charge's volume is included. So:

$$\vec{E}\,(x=R/2) = \frac{1}{4\pi\varepsilon_0}\left(\frac{(Q/8)}{(R/2)^2} - \frac{Q}{(3R/2)^2}\right)\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}(1/2-4/9)\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{18R^2}\hat{i}.$$

c) $x = R$: the two electric fields cancel, so $\vec{E} = 0.$

d) $x = 3R$: now both spheres contribute fields pointing to the right:

$$\vec{E}\,(x=3R) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{(3R)^2} + \frac{Q}{R^2}\right)\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{10Q}{9R^2}\hat{i}.$$

22.64: (See Problem 22.63 with $Q \to -Q$ for terms associated with right sphere)

a) $\vec{E}\,(x=0) = +\frac{1}{4\pi\varepsilon_0}\frac{Q}{4R^2}\hat{i}$

b) $\vec{E}\left(x=\frac{R}{2}\right) = \frac{1}{4\pi\varepsilon_0}\left[\frac{(Q/8)}{(R/2)^2} + \frac{Q}{(3R/2)^2}\right]\hat{i} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{2R^2} + \frac{4Q}{9R^2}\right]\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{17Q}{18R^2}\hat{i}$

c) $$\vec{E}\,(x=R) = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R^2} + \frac{Q}{R^2}\right]\hat{i} = \frac{Q}{2\pi\varepsilon_0 R^2}\hat{i}$$

d) $\vec{E}\,(x=3R) = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{(3R)^2} - \frac{Q}{R^2}\right]\hat{i} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{9R^2} - \frac{Q}{R^2}\right]\hat{i} = \frac{-1}{4\pi\varepsilon_0}\frac{8Q}{9R^2}\hat{i}$

22.65: a) The charge enclosed:

$$Q = Q_i + Q_0, \text{ where } Q_i = \alpha \frac{4\pi (R/2)^3}{3} = \frac{\alpha\pi R^3}{6}, \text{ and } Q_0 = 4\pi(2\alpha)\int_{R/2}^{R} (r^2 - r^3/R)dr$$

$$= 8\alpha\pi\left(\frac{(R^3 - R^3/8)}{3} - \frac{(R^4 - R^4/16)}{4R}\right) = \frac{11\alpha\pi R^3}{24}$$

$$\Rightarrow Q = \frac{15\alpha\pi R^3}{24} \Rightarrow \alpha = \frac{8Q}{5\pi R^3}.$$

b) $r \le R/2 : \Phi = E4\pi r^2 = \dfrac{\alpha 4\pi r^3}{3\varepsilon_0} \Rightarrow E = \dfrac{\alpha r}{3\varepsilon_0} = \dfrac{8Qr}{15\pi\varepsilon_0 R^3}.$

$$R/2 \le r \le R : \Phi = E4\pi r^2 = \frac{Q_i}{\varepsilon_0} + \frac{1}{\varepsilon_0}\left(8\alpha\pi\left(\frac{(r^3 - R^3/8)}{3} - \frac{(r^4 - R^4/16)}{4R}\right)\right)$$

$$\Rightarrow E = \frac{\alpha\pi R^3}{24\varepsilon_0(4\pi r^2)}(64(r/R)^3 - 48(r/R)^4 - 1) = \frac{kQ}{15r^2}(64(r/R)^3 - 48(r/R)^4 - 1).$$

$r \ge R : E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, since all charge is enclosed.

c) $$\frac{Q_i}{Q} = \frac{(4Q/15)}{Q} = \frac{4}{15} = 0.267.$$

d) $r \le R/2 : F = -eE = -\frac{8eQ}{15\pi\varepsilon_0 R^3} r,$ so the restoring force depends upon displacement to the first power, and we have simple harmonic motion.

e) $F = -kr,\ k = \dfrac{8eQ}{15\pi\varepsilon_0 R^3},\ \omega = \sqrt{\dfrac{k}{m_e}} = \sqrt{\dfrac{8eQ}{15\pi\varepsilon_0 R^3 m_e}},\ T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{15\pi\varepsilon_0 R^3 m_e}{8eQ}}.$

f) If the amplitude of oscillation is greater than $R/2,$ the force is no longer linear in $r,$ and is thus no longer simple harmonic.

22.66: a) Charge enclosed:

$$Q = Q_i + Q_0 \text{ where } Q_i = 4\pi\int_0^{R/2} \frac{3\alpha r^3}{2R}dr = \frac{6\pi\alpha}{R}\frac{1}{4}\frac{R^4}{16} = \frac{3}{32}\pi\alpha R^3.$$

$$\text{and } Q_0 = 4\pi\alpha\int_{R/2}^{R} (1-(r/R)^2)r^2\,dr = 4\pi\alpha R^3\left(\frac{7}{24} - \frac{31}{160}\right) = \frac{47}{120}\pi\alpha R^3.$$

$$\text{Therefore, } Q = \left(\frac{3}{32} + \frac{47}{120}\right)\pi\alpha R^3 = \frac{233}{480}\pi\alpha R^3 \Rightarrow \alpha = \frac{480Q}{233\pi R^3}.$$

b) $$r \le R/2 : \Phi = E4\pi r^2 = \frac{4\pi}{\varepsilon_0}\int_0^r \frac{3\alpha r'^3}{2R}dr' = \frac{3\pi\alpha r^4}{2\varepsilon_0 R} \Rightarrow E = \frac{6\alpha r^2}{16\varepsilon_0 R} = \frac{180Qr^2}{233\pi\varepsilon_0 R^4}.$$

$$R/2 \le r \le R : \Phi = E4\pi r^2 = \frac{Q_i}{\varepsilon_0} + \frac{4\pi\alpha}{\varepsilon_0}\int_{R/2}^{r} (1-(r'/R)^2)r'^2\,dr'$$

$$= \frac{Q_i}{\varepsilon_0} + \frac{4\pi\alpha}{\varepsilon_0}\left(\frac{r^3}{3} - \frac{R^3}{24} - \frac{r^5}{5R^2} + \frac{R^3}{160}\right) = \frac{3}{128}\frac{4\pi\alpha R^3}{\varepsilon_0}$$

$$+ \frac{4\pi\alpha R^3}{\varepsilon_0}\left(\frac{1}{3}\left(\frac{r}{R}\right)^3 - \frac{1}{5}\left(\frac{r}{R}\right)^5 - \frac{17}{480}\right)$$

$$\Rightarrow E = \frac{480Q}{233\pi\varepsilon_0 r^2}\left(\frac{1}{3}\left(\frac{r}{R}\right)^3 - \frac{1}{5}\left(\frac{r}{R}\right)^5 - \frac{23}{1920}\right).$$

$r \ge R : E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, since all charge is enclosed.

c) The fraction of Q between $R/2 \le r \le R$:

$$\frac{Q_o}{Q} = \frac{47}{120}\frac{480}{233} = 0.807.$$

d) $E(r = R/2) = \frac{180}{233}\frac{Q}{4\pi\varepsilon_0 R^2}$, using either of the electric field expressions above, evaluated at $r = R/2$.

e) The force an electron would feel never is proportional to $-r$ which is necessary for simple harmonic oscillations. It is oscillatory since the force is always attractive, but it has the wrong power of r to be *simple* harmonic.

23.1: $\Delta U = kq_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = k(2.40\,\mu\text{C})(-4.30\,\mu\text{C})\left(\frac{1}{0.354\text{m}} - \frac{1}{0.150\text{m}}\right) = 0.357\text{J}$

$\Rightarrow W = -\Delta U = -0.357$ J.

23.2: $W = -1.9\times10^{-8}\text{J} = -\Delta U = U_i - U_f \Rightarrow U_f = 1.9\times10^{-8}\text{J} + 5.4\times10^{-8}\text{J} =$
$7.3\times10^{-8}\text{J}$

23.3: a)

$$E_i = K_i + U_i = \frac{1}{2}(0.0015\text{ kg})(22.0\text{ m/s})^2 + \frac{k(2.80\times10^{-6}\text{C})(7.50\times10^{-6}\text{C})}{0.800\text{ m}} = 0.608\text{ J}$$

$$E_i = E_f = \frac{1}{2}mv_f^2 + \frac{kq_1q_2}{r_f} \Rightarrow v_f = \sqrt{\frac{2(0.608\text{ J} - 0.491\text{ J})}{0.0015\text{ kg}}} = 12.5\text{ m/s}.$$

b) At the closest point, the velocity is zero:

$$\Rightarrow 0.608\text{ J} = \frac{kq_1q_2}{r} \Rightarrow r = \frac{k(2.80\times10^{-6}\text{C})(7.80\times10^{-6}\text{C})}{0.608\text{ J}} = 0.323\text{ m}.$$

23.4: $U = -0.400\text{ J} = \frac{kq_1q_2}{r} \Rightarrow r = \frac{-k(2.30\times10^{-6}\text{C})(7.20\times10^{-6}\text{C})}{-0.400\text{ J}} = 0.373\text{ m}.$

23.5: a) $U = \frac{kQq}{r} = \frac{k(4.60\times10^{-6}\text{C})\,(1.20\times10^{-6}\text{C})}{0.250\text{ m}} = 0.199\text{ J}.$

b) (i) $K_f = K_i + U_i - U_f$

$$= 0\text{ J} + k(4.60\times10^{-6}\text{C})\,(1.20\times10^{-6}\text{C})\left(\frac{1}{0.25\text{ m}} - \frac{1}{0.5\text{ m}}\right) = 0.0994\text{ J}$$

$$\Rightarrow K_f = 0.0994\text{ J} = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2(0.0994\text{ J})}{2.80\times10^{-4}\text{kg}}} = 26.6\text{ m/s}.$$

(ii) $K_f = 0.189\text{ J},\ v_f = 36.7\text{ m/s}.$

(iii) $K_f = 0.198\text{ J},\ v_f = 37.6\text{ m/s}.$

23.6: $U = \frac{kq^2}{0.500\text{m}} + \frac{2kq^2}{0.500\text{ m}} = 6kq^2 = 6k(1.2\times10^{-6}\text{C})^2 = 0.078\text{ J}.$

23.7: a)

$$U=k\left(\frac{q_1q_2}{r_{12}}+\frac{q_1q_2}{r_{13}}+\frac{q_1q_2}{r_{23}}\right)=k\left(\begin{array}{l}\frac{(4.00\text{ nC})(-3.00\text{ nC})}{(0.200\text{ m})}+\frac{(4.00\text{ nC})(2.00\text{ nC})}{(0.100\text{m})}\\ \qquad\qquad\qquad+\frac{(-3.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})}\end{array}\right)$$

$$=-3.60\times10^{-7}\text{ J.}$$

b) *If* $U=0, 0=k\left(\frac{q_1q_2}{r_{12}}+\frac{q_1q_3}{x}+\frac{q_2q_3}{r_{12}-x}\right)$. So solving for x we find:

$0=-60+\frac{8}{x}-\frac{6}{0.2-x}\Rightarrow 60x^2-26x+1.6=0\Rightarrow x=0.074\text{ m}, 0.360\text{ m}.$ Therefore $x=0.074\text{ m}$ since it is the only value between the two charges.

23.8: From Example 23.1, the initial energy E_i can be calculated:

$$E_i=K_i+U_i=\frac{1}{2}(9.11\times10^{-31}\text{kg})(3.00\times10^6\text{ m/s})^2$$
$$+\frac{k(-1.60\times10^{-19}\text{C})(3.20\times10^{-19}\text{C})}{10^{-10}\text{m}}$$
$$\Rightarrow E_i=-5.09\times10^{-19}\text{ J.}$$

When velocity equals zero, all energy is electric potential energy, so:

$$-5.09\times10^{-19}\text{ J}=-\frac{k2e^2}{r}\Rightarrow r=9.06\times10^{-10}\text{ m.}$$

23.9: Since the work done is zero, the sum of the work to bring in the two equal charges q must equal the work done in bringing in charge Q.

$$W_{qq}=W_{qQ}\Rightarrow -\frac{kq^2}{d}=\frac{2kqQ}{d}\Rightarrow Q=-\frac{q}{2}.$$

23.10: The work is the potential energy of the combination.

$$U = U_{p\propto} + U_{pe} + U_{e\propto}$$
$$= \frac{ke(2e)}{5\sqrt{2}\times10^{-10}\text{m}} + \frac{ke(-e)}{5\times10^{-10}\text{m}} + \frac{k(-e)(2e)}{5\times10^{-10}\text{m}}$$
$$= \frac{ke^2}{5\times10^{-10}\text{m}}\left(\frac{2}{\sqrt{2}} - 1 - 2\right)$$
$$= \frac{(9.0\times10^9\ \text{Nm}^2/\text{C}^2)\ (1.6\times10^{-19}\ \text{C})^2}{5\times10^{-10}\text{m}}\left(\frac{2}{\sqrt{2}} - 3\right)$$
$$= -7.31\times10^{-19}\ \text{J}$$

Since U is negative, we want do $+7.31\times10^{-19}$ J to separate the particles

23.11: $K_1 + U_1 = K_2 + U_2;\ K_1 = U_2 = 0$ so $K_2 = U_1$

$$U_1 = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{2}{r} + \frac{2}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{5e^2}{r}, \text{ with } r = 8.00\times10^{-10}\ \text{m}$$

$$U_1 = 1.44\times10^{-18}\ \text{J} = 9.00\ \text{eV}$$

23.12: Get closest distance γ. Energy conservation: $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{ke^2}{\gamma}$

$$\gamma = \frac{ke^2}{mv^2} = \frac{(9\times10^9\ \text{Nm}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(1.67\times10^{-27}\ \text{kg})(10^6\ \text{m/s})} = 1.38\times10^{-13}\ \text{m}$$

Maximum force:

$$F = \frac{ke^2}{\gamma^2}$$
$$= \frac{(9\times10^9\ \text{Nm}^2/\text{C}^2)\ (1.6\times10^{-19}\ \text{C})^2}{(1.38\times10^{-13}\text{m})^2}$$
$$= 0.012\ \text{N}$$

23.13: $K_A + U_A = K_B + U_B$

$U = qV$, so $K_A + qV_A = K_B + qV_B$

$$K_B = K_A + q(V_A - V_B) = 0.00250\ \text{J} + (-5.00\times10^{-6}\ \text{C})\ (200\text{V} - 800\ \text{V}) = 0.00550\ \text{J}$$

$$v_B = \sqrt{2K_B/m} = 7.42\ \text{m/s}$$

It is faster at *B;* a negative charge gains speed when it moves to higher potential.

23.14: Taking the origin at the center of the square, the symmetry means that the potential is the same at the two corners not occupied by the $+5.00\ \mu\text{C}$ charges (The work done in moving to either corner from infinity is the same). But this also means that no net work is done is moving from one corner to the other.

23.15: $\vec{E}$ points from high potential to low potential, so $V_B > V_A$ and $V_C < V_A$.

The force on a positive test charge is east, so no work is done on it by the electric force when it moves due south (the force and displacement are perpendicular); $V_D = V_A$.

23.16: a) $W = -\Delta U = qEd = \Delta K = 1.50 \times 10^{-6}\ \text{J}$.

b) The initial point was at a higher potential than the latter since any positive charge, when free to move, will move from greater to lesser potential.

$$\Delta V = \Delta U/q = (1.50 \times 10^{-6}\ \text{J})/(4.20\ \text{nC}) = 357\ \text{V}.$$

c) $qEd = 1.50 \times 10^{-6}\ \text{J} \Rightarrow E = \dfrac{1.50 \times 10^{-6}\ \text{J}}{(4.20\ \text{nC})(0.06\ \text{m})} = 5.95 \times 10^{3}\ \text{N/C}.$

23.17: a) Work done is zero since the motion is along an equipotential, perpendicular to the electric field.

b) $W = qEd = (28.0\ \text{nC})\left(4.00 \times 10^{4}\ \dfrac{\text{V}}{\text{m}}\right)(0.670\ \text{m}) = 7.5 \times 10^{-4}\ \text{J}$

c) $W = qEd = (28.0\ \text{nC})\left(4.00 \times 10^{4}\ \dfrac{\text{V}}{\text{m}}\right)(-2.60\cos 45°) = -2.06 \times 10^{-3}\ \text{J}$

23.18: Initial energy equals final energy:

$$E_i = E_f \Rightarrow -\frac{keq_1}{r_{1i}} - \frac{keq_2}{r_{2i}} = -\frac{keq_1}{r_{1f}} - \frac{keq_2}{r_{2f}} + \frac{1}{2}m_e v_f{}^2$$

$$E_i = k(-1.60\times10^{-19}\ \text{C})\left(\frac{(3.00\times10^{-9}\ \text{C})}{0.25\ \text{m}} + \frac{(2.00\times10^{-9}\ \text{C})}{0.25\ \text{m}}\right) = -2.88\times10^{-17}\ \text{J}$$

$$E_f = k(-1.60\times10^{-19}\ \text{C})\left(\frac{(3.00\times10^{-9}\ \text{C})}{0.10\ \text{m}} + \frac{(2.00\times10^{-9}\ \text{C}}{0.40\ \text{m}}\right) + \frac{1}{2}m_e v_f^2$$

$$= -5.04\times10^{-17}\ \text{J} + \frac{1}{2}m_e v_f{}^2$$

$$\Rightarrow v_f = \sqrt{\frac{2}{9.11\times10^{-31}\ \text{kg}}(5.04\times10^{-17}\ \text{J} - 2.88\times10^{-17}\ \text{J})}$$

$$= 6.89\times10^6\ \text{m/s}.$$

23.19: a) $V = \dfrac{kq}{r} \Rightarrow r = \dfrac{kq}{V} = \dfrac{k(2.50\times10^{-11}\ \text{C})}{90.0\ \text{V}} = 2.5\times10^{-3}\ \text{m}.$

b) $V = \dfrac{kq}{r} \Rightarrow r = \dfrac{kq}{V} = \dfrac{k(2.50\times10^{-11}\ \text{C})}{30.0\ \text{V}} = 7.5\times10^{-3}\ \text{m}.$

23.20: a) $V = \dfrac{kq}{r} \Rightarrow q = \dfrac{rV}{k} = \dfrac{(0.250\ \text{m})(48.0\ \text{V})}{k} = 1.33\times10^{-9}\ \text{C}.$

b) $V = \dfrac{k(1.33\times10^{-9}\ \text{C})}{(0.750\ \text{m})} = 16\ \text{V}$

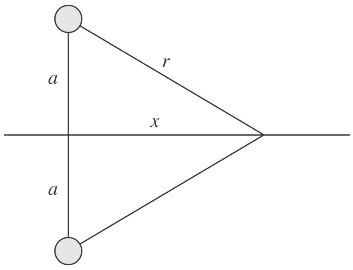
23.21: a) At A: $V_A = k\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2}\right) = k\left(\dfrac{2.40\times10^{-9}\ \text{C}}{0.05\ \text{m}} + \dfrac{-6.50\times10^{-9}\ \text{C}}{0.05\ \text{m}}\right) = -738\ \text{V}.$

b) At B: $V_B = k\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2}\right) = k\left(\dfrac{2.40\times10^{-9}\ \text{C}}{0.08\ \text{m}} + \dfrac{-6.50\times10^{-9}\ \text{C}}{0.06\ \text{m}}\right) = -705\ \text{V}.$

c) $W = q\Delta V = (2.50\times10^{-9}\ \text{C})(-33\ \text{V}) = -8.25\times10^{-8}\ \text{J}.$

The negative sign indicates that the work is done *on* the charge. So the work done by the field is 8.25×10^{-8} J.

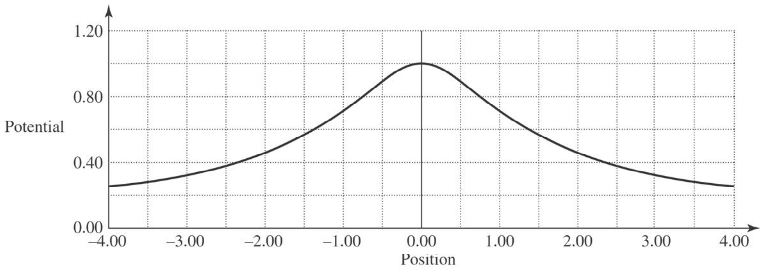
23.22: a)



b) $V = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a}.$

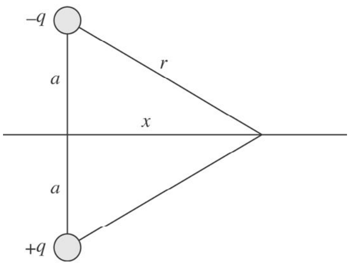
c) Looking at the diagram in (a): $V(x) = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r} = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\sqrt{a^2 + x^2}}$

d)



e) When $x >> a, V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{x}$, just like a point charge of charge $+2q$.

23.23: a)



b) $V_x = \dfrac{kq}{r} + \dfrac{k(-q)}{r} = 0.$

c) The potential along the x-axis is always zero, so a graph would be flat.
d) If the two charges are interchanged, then the results of (b) and (c) still hold.
The potential is zero

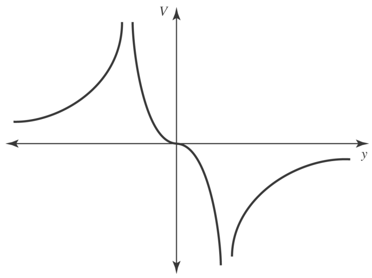
23.24: a) $|y| < a : V = \dfrac{kq}{(a+y)} - \dfrac{kq}{(a-y)} = \dfrac{2kqy}{y^2 - a^2}.$

$$y > a : V = \frac{kq}{(a+y)} - \frac{kq}{y-a} = \frac{-2kqa}{y^2 - a^2}.$$

$$y < -a : V = \frac{-kq}{(a+y)} - \frac{kq}{(-y+a)} = \frac{2kqa}{y^2 - a^2}.$$

Note: This can also be written as $V = k\left(\dfrac{-q}{|y-a|} + \dfrac{q}{|y+a|}\right)$

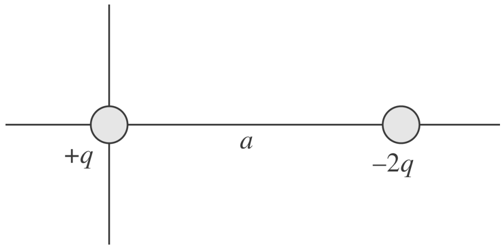
b)



c) $y >> a : V = \dfrac{kq}{(a+y)} - \dfrac{kq}{(y-a)} = \dfrac{-2kqa}{y^2}.$

d) If the charges are interchanged, then the potential is of the opposite sign.

23.25: a)



b) $$x > a : V = \frac{kq}{x} - \frac{2kq}{x-a} = \frac{-kq(x+a)}{x(x-a)}.$$

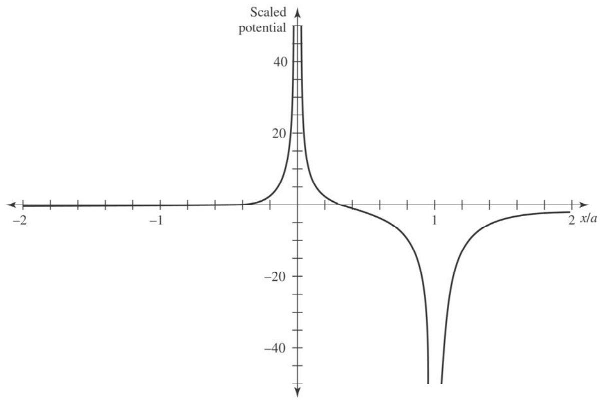
$$0 < x < a : V = \frac{kq}{x} - \frac{2kq}{a-x} = \frac{kq(3x-a)}{x(x-a)}.$$

$$x < 0 : V = \frac{-kq}{x} + \frac{2kq}{x-a} = \frac{kq(x+a)}{x(x-a)}.$$

Note: This can be also be written as $V = k(\frac{q}{|x|} - \frac{2q}{|x-a|})$

c) The potential is zero at $x = -a$ and $a/3$.

d)

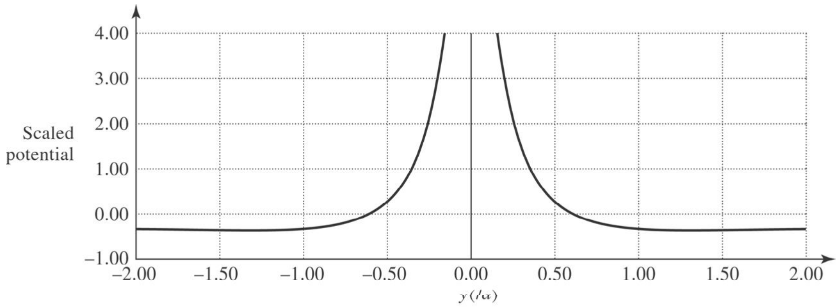


e) For $x >> a : V \approx \frac{-kqx}{x^2} = \frac{-kq}{x}$, which is the same as the potential of a point charge $-q$. (Note: The two charges must be added with the correct sign.)

23.26: a) $V = \dfrac{kq}{|y|} - \dfrac{2kq}{r} = kq\left(\dfrac{1}{|y|} - \dfrac{2}{\sqrt{a^2 + y^2}}\right).$

b) $V = 0$, when $y^2 = \dfrac{a^2 + y^2}{4} \Rightarrow 3y^2 = a^2 \Rightarrow y = \pm\dfrac{a}{\sqrt{3}}.$

c)



d) $y >> a : V \approx kq\left(\dfrac{1}{y} - \dfrac{2}{y}\right) = -\dfrac{kq}{y}$, which is the potential of a point charge $-q$.

23.27: $W = -\Delta U = -Vq = (295\text{ V})\ (1.60\times10^{-19}\text{ C}) = 4.72\times10^{-17}\text{ J}$. But also:

$$W = \Delta K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2(4.72\times10^{-17}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = 1.01\times10^{7}\text{ m/s}.$$

23.28: a) $E = \dfrac{V}{d} \Rightarrow d = \dfrac{V}{E} = \dfrac{4.98\text{ V}}{12.0\text{ N/C}} = 0.415\text{ m}.$

b) $V = \dfrac{kq}{d} \Rightarrow q = \dfrac{Vd}{k} = \dfrac{(4.98\text{ V})\ (0.415\text{ m})}{k} = 2.30\times10^{-10}\text{ C}.$

c) The electric field is directed away from q since it is a positive charge.

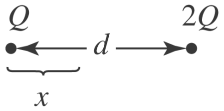
23.29: a) Point b has a higher potential since it is “upstream” from where the positive charge moves.

$$V_a - V_b = E(b-a) = -|E|(b-a) \Rightarrow V_b - V_a = |E|(b-a) > 0$$

b) $E = \dfrac{V}{d} = \dfrac{240\,V}{0.3\text{ m}} = 800\text{ N/C}.$

c) $W = -\Delta U = -q\Delta V = -(-0.20\times10^{-6}\text{ C})(-240\text{ V}) = -4.8\times10^{-5}\text{ J}.$

23.30:(a) $V = V_Q + V_{2Q} > 0,$ so V is zero nowhere except for infinitely far from the charges.

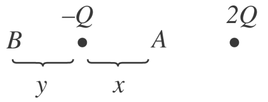


The fields can cancel only between the charges

$$E_Q = E_{2Q} \rightarrow \frac{kQ}{x^2} = \frac{k(2Q)}{(d-x)^2} \rightarrow (d-x)^2 = 2x^2$$

$x = \frac{d}{1+\sqrt{2}}$. The other root, $x = \frac{d}{1-\sqrt{2}}$, does not lie between the charges.

(b)



V can be zero in 2 places, A and B.

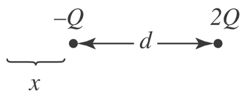
$$at\ A: \frac{k(-Q)}{x} + \frac{k(2Q)}{d-x} = 0 \rightarrow x = d/3$$

$$at\ B: \frac{k(-Q)}{y} + \frac{k(2Q)}{d+y} = 0 \rightarrow y = d$$

$E_Q = E_{2Q}$ to the left of $-Q$.

$$\frac{kQ}{x^2} = \frac{k(2Q)}{(d+x)^2} \rightarrow x = \frac{d}{\sqrt{2}-1}$$

(c)



Note that E and V are not zero at the same places.

23.31: a) $K_1 + qV_1 = K_2 + qV_2$

$q(V_1 - V_2) = K_2 - K_1;\qquad q = -1.602 \times 10^{-19}$ C

$K_1 = \frac{1}{2} m_e v_1^2 = 4.099 \times 10^{-18}$ J; $\qquad K_2 = \frac{1}{2} m_e v_2^2 = 2.915 \times 10^{-17}$ J

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = -156 \text{ V}$$

The electron gains kinetic energy when it moves to higher potential.

b) Now $K_1 = 2.915 \times 10^{-17}$ J, $K_2 = 0$

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = +182 \text{ V}$$

The electron loses kinetic energy when it moves to lower potential

23.32: a) $V = \frac{kq}{r} = \frac{k(3.50 \times 10^{-9}\ \text{C})}{0.48\ \text{m}} = 65.6\ \text{V}.$

b) $V = \frac{k(3.50 \times 10^{-9}\ \text{C})}{0.240\ \text{m}} = 131.3\ \text{V}$

c) Since the sphere is metal, its interior is an equipotential, and so the potential inside is 131.3 V.

23.33: a) The electron will exhibit simple harmonic motion for $x << a$, but will otherwise oscillate between ± 30.0 cm.

b) From Example 23.11,

$$V = \frac{kQ}{\sqrt{x^2 + a^2}} \Rightarrow \Delta V = kQ\left(\frac{1}{a} - \frac{1}{\sqrt{x^2 + a^2}}\right)$$

$$\Rightarrow \Delta V = k(24.0 \times 10^{-9}\ \text{C})\left(\frac{1}{0.150\ \text{m}} - \frac{1}{\sqrt{(0.300\ \text{m})^2 + (0.150\ \text{m})^2}}\right)$$

$$= 796\ \text{V}$$

But $W = -q\Delta V = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2(1.60 \times 10^{-19}\ \text{C})\ (796\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}} = 1.67 \times 10^7\ \text{m/s}.$

23.34: Energy is conserved:

$$\frac{1}{2}mv^2 = q\Delta V \Rightarrow \Delta V = \frac{(1.67 \times 10^{-27}\ \text{kg})\ (1500\ \text{m/s})^2}{2(1.60 \times 10^{-19}\ \text{C})} = 0.0117\ \text{V}.$$

But:

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln(r_0/r) \Rightarrow r_0 = r\ \exp\left(\frac{2\pi\varepsilon_0\Delta V}{\lambda}\right) \Rightarrow r = r_0\ \exp\left(-\frac{2\pi\varepsilon_0\Delta V}{\lambda}\right)$$

$$\Rightarrow r = (0.180\ \text{m})\exp\left(-\frac{2\pi\varepsilon_0(0.0117\ \text{V})}{5.00 \times 10^{-12}\ \text{C/m}}\right) = 0.158\ \text{m}.$$

23.35: a) $E = \frac{V}{d} = \frac{360\,V}{0.0450\ \text{m}} = 8000\ \text{N/C}.$

b) $F = Eq = (8000\ \text{N/C})\ (2.40 \times 10^{-9}\ \text{C}) = 1.92 \times 10^{-5}\ \text{N}.$

c) $W = Fd = (1.92 \times 10^{-5}\ \text{N})\ (0.0450\ \text{m}) = 8.64 \times 10^{-7}\ \text{J}.$

d) $\Delta U = \Delta Vq = (-360\ \text{V})\ (2.40 \times 10^{-9}\ \text{C}) = -8.64 \times 10^{-7}\ \text{J}.$

23.36: a) $V = Ed = (480\ \text{N/C})\ (3.8 \times 10^{-2}\ \text{m}) = 18.2\ \text{V}.$

b) The higher potential is at the positive sheet.

c) $E = \dfrac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0 (480\ \text{N/C}) = 4.25 \times 10^{-9}\ \text{C/m}^2.$

23.37: a) $E = \dfrac{V}{d} \Rightarrow d = \dfrac{V}{E} = \dfrac{4750\ \text{V}}{3.00 \times 10^6\ \text{V/m}} = 1.58 \times 10^{-3}\ \text{m}.$

b) $E = \dfrac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0 (3.00 \times 10^6\ \text{V/m}) = 2.66 \times 10^{-5}\ \text{C/m}^2.$

23.38: a) $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{47.0 \times 10^{-9}\ \text{C/m}^2}{\varepsilon_0} = 5311\ \text{N/C}.$

b) $V = Ed = (5311\ \text{N/C})\ (0.0220\ \text{m}) = 117\ \text{V}.$

c) The electric field stays the same if the separation of the plates doubles, while the potential between the plates doubles.

23.39: a) The electric field outside the shell is the same as for a point charge at the center of the shell, so the potential outside the shell is the same as for a point charge:

$$V = \frac{q}{4\pi\varepsilon_0 r} \text{ for } r > \text{R}.$$

The electric field is zero inside the shell, so no work is done on a test charge as it moves inside the shell and all points inside the shell are at the same potential as the surface of the shell: $V = \dfrac{q}{4\pi\varepsilon_0 R}$ for $r \le R.$

b) $V = \dfrac{kq}{R}$ so $q = \dfrac{RV}{k} = \dfrac{(0.15\ \text{m})\ (-1200\ \text{V})}{k} = -20\ \text{nC}$

c) No, the amount of charge on the sphere is very small.

23.40: For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

Therefore

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

and

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(3800\ \text{N/C})\,(0.200\ \text{m})^2}{9\times10^9\ \text{N.m}^2/C^2} = 1.69\times10^{-8}\,\text{C}$$

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking ∞ as zero, is

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{(9\times10^9\ \text{N.m}^2/\text{C}^2)\,(-1.69\times\ 10^{-8}\text{C})}{0.200\ \text{m}} = -760\ \text{V}$$

at the surface of the sphere. Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be -760 V.

23.41: a) $E = -\nabla V.$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(Axy - Bx^2 + Cy) = -Ay + 2Bx.$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(Axy - Bx^2 + Cy) = -Ax - C.$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}(Axy - Bx^2 + Cy) = 0.$$

b) $-Ay + 2Bx = 0 \Rightarrow y = \frac{2B}{A}x,\ -Ax - C = 0 \Rightarrow x = -\frac{C}{A}$ so $y = \frac{2B}{A}.\left(\frac{-C}{A}\right) =$ $\frac{-2BC}{A^2}$, $E = 0$ at $\left(\frac{-C}{A}, \frac{-2BC}{A^2}, z\right)$

23.42: a) $E = -\nabla V$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{kQ}{\sqrt{x^2+y^2+z^2}}\right) = \frac{kQx}{(x^2+y^2+z^2)^{3/2}} = \frac{kQx}{r^3}.$$

Similarly, $E_y = \dfrac{kQy}{r^3}$ and $E_z = \dfrac{kQz}{r^3}$.

b) So from (a), $E = \dfrac{kQ}{r^2}\left(\dfrac{x\hat{\boldsymbol{i}}}{r} + \dfrac{y\hat{\boldsymbol{j}}}{r} + \dfrac{z\hat{\boldsymbol{k}}}{r}\right) = \dfrac{kQ}{r^2}\hat{\boldsymbol{r}},$ which agrees with Equation (21.7).

23.43: a) There is no dependence of the potential on x or y, and so it has no components in those directions. However, there is z dependence:

$$E = -\nabla V \Rightarrow E_z = -\frac{\partial V}{\partial z} = -C \Rightarrow E = -C\hat{\boldsymbol{k}}, \ \ for \ \ 0 < z < d.$$

and $\vec{\boldsymbol{E}} = 0$, for $z > d$, since the potential is constant there.

(b) Infinite parallel plates of opposite charge could create this electric field, where the surface charge is $\sigma = \pm C\varepsilon_0$.

23.44: a)

(i) $r < r_a : V = \dfrac{kq}{r_a} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right).$

(ii) $r_a < r < r_b : V = \dfrac{kq}{r} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right).$

(iii) $r > r_b : V = 0,$ since outside a sphere the potential is the same as for point charge. Therefore we have the identical potential to two oppositely charged point charges at the same location. These potentials cancel.

b) $V_a = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{r_a} - \dfrac{q}{r_b}\right)$ and $V_b = 0 \Rightarrow V_{ab} = \dfrac{1}{4\pi\varepsilon_0} q\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$

c) $r_a < r < r_b : E = -\dfrac{\partial V}{\partial r} = -\dfrac{q}{4\pi\varepsilon_0}\dfrac{\partial}{\partial r}\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right) = +\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{V_{ab}}{\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)}\dfrac{1}{r^2}.$

d) From Equation (24.23): $E = 0,$ since V is zero outside the spheres.

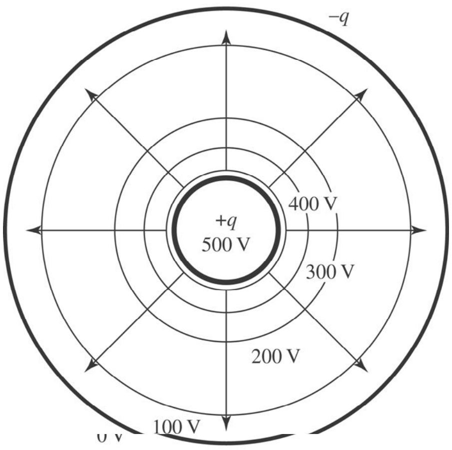
e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(q-Q)}{r}.$ All potentials inside the outer shell are just shifted by an amount $V = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r_b}.$ Therefore relative potentials within the shells are not affected. Thus (b) and (c) do not change. However, now that the potential does vary outside the spheres, there is an electric field there:

$$E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{kq}{r} + \frac{-kQ}{r}\right) = \frac{kq}{r^2}\left(1 - \frac{Q}{q}\right).$$

23.45: a) $$V_{\text{ab}} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = 500 \text{ V}$$

$$\Rightarrow q = \frac{500 \text{ V}}{k\left(\dfrac{1}{0.012 \text{ m}} - \dfrac{1}{0.096 \text{ m}}\right)} = 7.62 \times 10^{-10} \text{ C.}$$

b)



c) The equipotentials are closest when the electric field is largest.

23.46: a) $$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{kQ}{2a}\ln\left(\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right)\right)$$

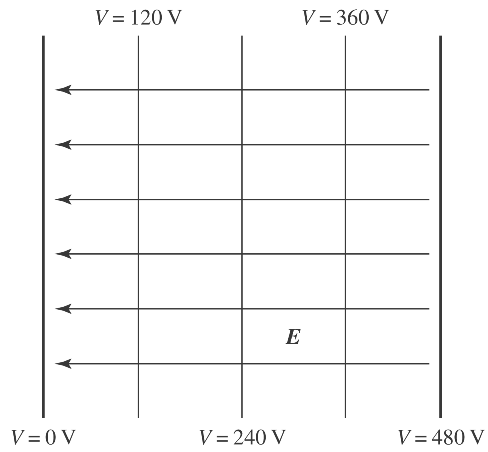
$$\Rightarrow E_x = -\frac{kQ}{2a}\left[\frac{\partial}{\partial x}\ln(\sqrt{a^2 + x^2} + a) - \frac{\partial}{\partial x}\ln(\sqrt{a^2 + x^2} - a)\right]$$

$$= -\frac{kQ}{2a}\left[\frac{x(a^2 + x^2)^{-1/2}}{\sqrt{a^2 + x^2} + a} - \frac{x(a^2 + x^2)^{-1/2}}{\sqrt{a^2 + x^2} - a}\right] = \frac{kQ}{x\sqrt{a^2 + x^2}}$$

$$\Rightarrow E_x = \frac{(2a\lambda)}{4\pi\varepsilon_0 xa\sqrt{1 + x^2/a^2}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x\sqrt{1 + x^2/a^2}}.$$

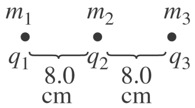
b) The potential was evaluated at y and z equal to zero, and thus shows no dependence on them. However, the electric field depends upon the derivative of the potential and the potential could still have a functional dependence on the variables y and z, and hence E_y and E_z may be non-zero.

23.47:



a) Equipotentials and electric field lines of two large parallel plates are shown above.
b) The electric field lines and the equipotential lines are mutually perpendicular.

23.48:



(a) $\Sigma F = m_1 a = F_{12} + F_{13}$

$$\frac{kq_1q_2}{r_{12}^2} + \frac{kq_1q_3}{r_{13}^2} = m_1 a$$

$$q_1 = q_2 = q_3 = q$$

$$ma = kq^2\left(\frac{1}{r_{12}^2} + \frac{1}{r_{13}^2}\right)$$

$$(0.02\ \text{kg})a = (9\times10^9\ \text{Nm}^2/\text{C}^2)\,(2.0\times10^{-6}\text{C})^2\left[\frac{1}{(0.08\ \text{m})^2} + \frac{1}{(0.16\ \text{m})^2}\right]$$

$$a = 352\ \text{m/s}^2$$

(b) Maximum speed occurs at "infinity". The center charge does not move since the forces on it balance. Energy conservation gives $U_\text{i} = K_\text{f}$.

$$\frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_3v_3^2.$$

$v_1 = v_3$, $m_1 = m_3$, and $q_1 = q_2 = q_3 = q$

$$v_1 = \sqrt{\frac{kq^2}{m_1}\left(\frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{23}}\right)}$$

$$= \sqrt{\frac{(9\times10^9\ \text{Nm}^2/\text{C}^2)\,(2\times10^{-6}\text{C})^2}{0.020\ \text{kg}}\left(\frac{1}{0.08\ \text{m}} + \frac{1}{0.16\ \text{m}} + \frac{1}{0.08\ \text{m}}\right)} = 7.5\ \text{m/s}$$

23.49: a) $W_E = \Delta K - W_F = 4.35\times10^{-5}\ \text{J} - 6.50\times10^{-5}\ \text{J} = -2.15\times10^{-5}\ \text{J}.$

b) $W_E = -q\Delta V \Rightarrow \Delta V = \dfrac{-W_E}{q} = \dfrac{2.15\times10^{-5}\ \text{J}}{7.60\times10^{-9}\ \text{C}} = +2829\ \text{V}.$ So the initial point is –2829 V with respect to the final point.

c) $E = \dfrac{V}{d} = \dfrac{2829\ \text{V}}{0.08\ \text{m}} = 3.54\times10^4\ \dfrac{\text{V}}{\text{m}}.$

23.50: a) $\dfrac{mv^2}{r} = \dfrac{ke^2}{r^2} \Rightarrow v = \sqrt{\dfrac{ke^2}{mr}}.$

b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\dfrac{ke^2}{r} = -\dfrac{1}{2}U.$

c) $E = K + U = \dfrac{1}{2}U = -\dfrac{1}{2}\dfrac{ke^2}{r} = -\dfrac{1}{2}\dfrac{k(1.60\times10^{-19}\ \text{C})^2}{5.29\times10^{-11}\ \text{m}} = -2.17\times10^{-18}\ \text{J} = -13.6\,\text{eV}.$

23.51: a) $V = Cx^{4/3} \Rightarrow C = (240\ \text{V})\ (0.0130\ \text{m})^{-4/3} = 7.85\times10^{4}\ \text{V/m}^{4/3}.$

b) $E = -\dfrac{\partial V}{\partial x} = -\dfrac{4}{3}Cx^{1/3} = -\dfrac{4}{3}(7.85\times10^{4}\ \text{V/m}^{4/3})x^{1/3} = \left(-1.05\times10^{5}\ \dfrac{\text{V}}{\text{m}^{4/3}}x^{1/3}\right)\text{V/m},$

toward cathode.

c) $F = -eE = ((1.05\times10^{5})\ (0.00650)^{1/3}\ \text{V/m})\ (1.60\times10^{-19}\ \text{C}) = 3.14\times10^{-15}\ \text{N},$

toward anode.

23.52: From Problem 22.51, the electric field of a sphere with radius R and q distributed uniformly over its volume is $E = \dfrac{qr}{4\pi\varepsilon_0 R^3}$ for $r \le R$ and $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ for $r \ge R$

$V_a - V_b = \int_a^b Edr$. Take b at infinity and $V_\infty = 0$. Let point a be a distance $r < R$ from the center of the sphere.

$$V_r = \int_r^R \frac{qr}{4\pi\varepsilon_0 R^3}dr + \int_R^\infty \frac{q}{4\pi\varepsilon_0 r^2}dr = \frac{q}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)$$

Set $q = +2e$ to get V_r for the sphere. The work done by the attractive force of the sphere when one electron is removed from $r = d$ to ∞ is

$$W_{\text{sphere}} = -eV_r = -\frac{2e^2}{8\pi\varepsilon_0 R}\left(3 - \frac{d^2}{R^2}\right)$$

The total work done by the attractive force of the sphere when both electrons are removed is twice this, $2W_{\text{sphere}}$. The work done by the repulsive force of the two electrons is $W_{ee} = \dfrac{e^2}{4\pi\varepsilon_0 (2d)}$ The total work done by the electrical forces is $2W_{\text{sphere}} + W_{ee}$. The energy required to remove the two electrons is the negative of this,

$$\frac{e^2}{2\pi\varepsilon_0 R}\left(3 - \frac{R}{4d} - \frac{d^2}{R^2}\right)$$

We can check this result in the special case of $d = R$, when the electrons initially sit on the surface of the sphere. The potential due to the sphere is the same as for a point charge $+2e$ at the center of the sphere.

$$W_{q\to b} = U_a - U_b$$

$$U_b = 0.\, U_a = 2\left(\frac{-2e^2}{4\pi\varepsilon_0 R}\right) + \frac{e^2}{4\pi\varepsilon_0 (2R)} = \frac{e^2}{4\pi\varepsilon_0 R}\left(-2 + \frac{1}{4}\right) = \frac{-7e^2}{8\pi\varepsilon_0 R}$$

The work done by the electric forces when the electrons are removed is $-7e^2/8\pi\varepsilon_0 R$ and the energy required to remove them is $7e^2/8\pi\varepsilon_0 R$. Setting $d = R$ in our general expression yields this same result.

23.53: a)

$$U = kq^2\left(-\frac{3}{d}+\frac{3}{\sqrt{2}d}-\frac{1}{\sqrt{3}d}\right)+kq^2\left(-\frac{2}{d}+\frac{3}{\sqrt{2}d}-\frac{1}{\sqrt{3}d}\right)+kq^2\left(-\frac{2}{d}+\frac{2}{\sqrt{2}d}-\frac{1}{\sqrt{3}}\right.$$

$$+kq^2\left(-\frac{1}{d}+\frac{2}{\sqrt{2}d}-\frac{1}{\sqrt{3}d}\right)+kq^2\left(-\frac{2}{d}+\frac{1}{\sqrt{2}d}\right)+kq^2\left(-\frac{1}{d}+\frac{1}{\sqrt{2}d}\right)+kq^2\left(-\right.$$

$$\Rightarrow U = kq^2\left(-\frac{12}{d}+\frac{12}{\sqrt{2}d}-\frac{4}{\sqrt{3}d}\right)=-\frac{12kq^2}{d}\left(1-\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right)=-1.46\,q^2/\pi\varepsilon_0 d$$

b) The fact that the electric potential energy is less than zero means that it is energetically favourable for the crystal ions to be together.

23.54: a) $U = 2kq^2\left(-\frac{1}{d}+\frac{1}{2d}-\frac{1}{3d}+\cdots\right)=-\frac{2kq^2}{d}\sum_{i=1}^{\infty}\frac{(-1)^{i-1}}{i}$.

b) $U=-\frac{2kq^2}{d}\ln(2)$

c) The potential energy is the same for the negative ions—the equations are identical if we examine (a).

d) If $d = 2.82\times10^{-10}$ m, then $U=-\frac{2k(1.60\times10^{-19}\text{ C})^2\ \ln(2)}{2.82\times10^{-10}\text{ m}}=-1.13\times10^{-18}$ J.

e) The real energy $(-0.80\times10^{-18}\text{ J})$ is about 70% of that calculated above.

23.55: a) $U_e=\frac{-2ke^2}{r}=\frac{-2k(1.60\times10^{-19}\text{ C})^2}{0.535\times10^{-10}\text{ m}}=-8.61\times10^{-18}$ J.

b) If all the kinetic energy goes into potential energy:

$$U_t = U_e + K = -8.61\times10^{-18}\text{ J}+1.02\times10^{-18}\text{ J}=\frac{2ke^2}{\sqrt{d^2+x^2}}=-7.59\times10^{-18}\text{ J}$$

$$\Rightarrow x^2=\frac{4k^2e^4}{U_t^2}-d^2=8.24\times10^{-22}\text{m}^2\ (d=5.35\times10^{-11}\text{ m})$$

(Note that we must be careful to keep all digits along the way.) $\Rightarrow x = 2.87\times10^{-11}$ m.

23.56: $F_e = mg\,\tan\theta = (1.50\times10^{-3}\text{ kg})\,(9.80\text{ m/s}^2)\,\tan(30°) = 0.0085\text{ N}$. (Balance forces in x and y directions.) But also:

$$F_e = Eq = \frac{Vq}{d} \Rightarrow V = \frac{Fd}{q} = \frac{(0.0085\text{ N})\,(0.0500\text{ m})}{8.90\times10^{-6}\text{ C}} = 47.8\text{ V}.$$

23.57: a) (i) $V = \dfrac{\lambda}{2\pi\varepsilon_0}(\ln(b/a) - \ln(b/b)) = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(b/a).$

(ii) $V = \dfrac{\lambda}{2\pi\varepsilon_0}(\ln(b/r) - \ln(b/b)) = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(b/r)$.

(iii) $V = 0.$

b) $V_{ab} = V\,(a) - V(b) = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(b/a).$

c) Between the cylinders:

$$V = \frac{\lambda}{2\pi\varepsilon_0}\ln(b/r) = \frac{V_{ab}}{\ln(b/a)}\ln(b/r)$$

$$\therefore E = -\frac{\partial V}{\partial r} = -\frac{V_{ab}}{\ln(b/a)}\frac{\partial}{\partial r}(\ln(b/r)) = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}.$$

d) The potential difference between the two cylinders is identical to that in part (b) even if the outer cylinder has no charge.

23.58: Using the results of Problem 23.57, we can calculate the potential difference:

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} \Rightarrow V_{ab} = E\ln(b/a)r$$

$$\Rightarrow V_{ab} = (2.00\times10^4\text{ N/C})\,(\ln(0.018\text{ m}/145\times10^{-6}\text{ m}))\,0.012\text{ m} = 1157\text{ V}.$$

23.59: a) $F = Eq = (1.10 \times 10^{3}\ \text{V/m})\ (1.60 \times 10^{-19}\ \text{C}) = 1.76 \times 10^{-16}\ \text{N}$, downward.

b) $a = F/m_e = (1.76 \times 10^{-16}\ \text{N})/(9.11 \times 10^{-31}\ \text{kg}) = 1.93 \times 10^{14}\ \text{m/s}^2$, downward.

c) $t = \dfrac{0.060\ \text{m}}{6.50 \times 10^{6}\ \text{m/s}} = 9.23 \times 10^{-9}\ \text{s},\ y - y_0 = \dfrac{1}{2}at^2 = \dfrac{1}{2}(1.93 \times 10^{14}\ \text{m/s}^2)\ (9.23 \times 10^{-9}\text{s}$

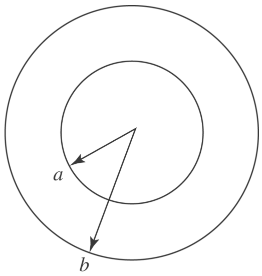
$= 8.22 \times 10^{-3}\ \text{m}.$

d) Angle $\theta = \arctan(v_y/v_x) = \arctan(at/v_x) = \arctan(1.78/6.50) = 15.3°.$

e) The distance below center of the screen is:

$$D = d_y + v_y t = 8.22 \times 10^{-3}\ \text{m} + (1.78 \times 10^{6}\ \text{m/s})\ \frac{0.120\ \text{m}}{6.50 \times 10^{6}\ \text{m/s}} = 0.0411\,\text{m}.$$

23.60:



(a) Use ΔV_{ab} to get $\lambda : \Delta V = \int_a^b E \cdot dl = \int_a^b \dfrac{\lambda}{2\pi\varepsilon_0} dr = \dfrac{\lambda}{2\pi\varepsilon_0} \ln \frac{b}{a}$

$$\lambda = \frac{2\pi\varepsilon_0 \Delta V}{\ln b/a}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2\pi\varepsilon_0 \Delta V / \ln b/a}{2\pi\varepsilon_0 r} = \frac{\Delta V}{r \ln b/a}$$

at outer surface of the wire, $r = a = \frac{0.127\ \text{mm}}{2}$

$$E = \frac{850\ \text{V}}{(\frac{0.000127\ \text{m}}{2})\ \ln\left[\frac{1.00\ \text{cm}}{(\frac{0.0127\ \text{cm}}{2})}\right]} = 2.65 \times 10^{6}\ \text{V/m}$$

(b) at the inner surface of the cylinder, $r = 1.00\ \text{cm}$, which gives

$$E = 1.68 \times 10^{4}\ \text{V/m}$$

23.61: a) From Problem 23.57,

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} = \frac{50{,}000\ \text{V}}{\ln(0.140/9.00\times10^{-5})}\frac{1}{0.070\ \text{m}}$$

$$\Rightarrow E = 9.72\times10^{4}\ \text{V/m}.$$

b) $F = Eq = 10\,mg \Rightarrow q = \dfrac{10\,(3.00\times10^{-8}\ \text{kg})(9.80\ \text{m/s}^2)}{9.72\times10^{4}\ \text{V/m}} = 3.02\times10^{-11}\ \text{C}.$

23.62: Recall from Example 23.12 for a line of charge of length a:

$$V = \frac{kQ}{a}\ln\left[\frac{\sqrt{a^2/4+x^2}+a/2}{\sqrt{a^2/4+x^2}-a/2}\right]$$

a) For a square with two sets of oppositely charged sides, the potentials cancel and $V = 0$.

b) If all sides have the same charge we have:

$$V = \frac{4kQ}{a}\ln\left[\frac{\sqrt{a^2/4+x^2}+a/2}{\sqrt{a^2/4+x^2}-a/2}\right],\ \text{but here } x = a/2,\ \text{so:}$$

$$\Rightarrow V = \frac{4kQ}{a}\ln\left[\frac{\sqrt{a^2+4x^2}+a}{\sqrt{a^2+4x^2}-a}\right] = \frac{4kQ}{a}\ln\left[\frac{(\sqrt{2}+1)}{(\sqrt{2}-1)}\right].$$

23.63: a)

$$dV = \frac{kQ}{\sqrt{x^2+r^2}}\left[\frac{2\pi r\,dr}{\pi R^2}\right] = \frac{2kQ}{R^2}\frac{r\,dr}{\sqrt{x^2+r^2}}$$

$$V = \int_0^R dV = \frac{2kQ}{R^2}\int_0^R \frac{r\,dr}{\sqrt{x^2+r^2}} = \frac{2kQ}{R^2}(z^{1/2})\Big|_{z=x^2}^{z=x^2+R^2} = \frac{2kQ}{R^2}\left[\sqrt{x^2+R^2}-x\right] =$$

$$\frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2+R^2}-x\right]$$

b) $$E_x = -\frac{\partial V}{\partial x} = -\frac{2kQ}{R^2}\left[\frac{x}{\sqrt{x^2+R^2}}-1\right] = \frac{\sigma}{2\epsilon_0}\left[1-\frac{1}{\sqrt{1+R^2/x^2}}\right].$$

23.64: a) From Example 23.12:

$$V(x) = \frac{kQ}{2a}\ln\left[\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right] = \frac{kQ}{2a}\ln\left[\frac{\sqrt{1 + a^2/x^2} + a/x}{\sqrt{1 + a^2/x^2} - a/x}\right]$$

If $a << x, \sqrt{1 + a^2/x^2} \pm a/x \approx 1 + \frac{1}{2}\left(\frac{a}{x}\right)^2 \pm \frac{a}{x} \approx 1 + \frac{a}{x},$ and $\ln(1+\alpha) \approx \alpha + \frac{1}{2}\alpha^2 + \cdots$

$$\Rightarrow V(x) \approx \frac{kQ}{2a}\left[\left(\frac{a}{x} + \frac{1}{2}\left(\frac{a}{x}\right)^2 + \cdots\right) - \left(-\frac{a}{x} + \frac{1}{2}\left(\frac{a}{x}\right)^2 + \cdots\right)\right] = \frac{kQ}{2a}\left[\frac{2a}{x}\right] = \frac{kQ}{x}.$$

That is, the finite rod acts like a point charge when you are a long way from it.

b) From Example 23.12:

$$V(x) = \frac{kQ}{2a}\ln\left[\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right] = \frac{kQ}{2a}\ln\left[\frac{\sqrt{1 + x^2/a^2} + 1}{\sqrt{1 + x^2/a^2} - 1}\right].$$

If $x << a, \sqrt{1 + x^2/a^2} \pm 1 \approx 1 \pm 1 + \frac{1}{2}\left(\frac{x}{a}\right)^2$, and $\ln(1+\alpha) \approx \alpha + \frac{1}{2}\alpha^2 + \cdots$

$$\Rightarrow V(x) \approx \frac{kQ}{2a}\left[\ln\left(\frac{(2 + x^2/2a^2)}{(x^2/2a^2)}\right)\right] = \frac{kQ}{2a}\left[\ln\left(\frac{4a^2}{x^2} + 1\right)\right] \approx \frac{kQ}{a}\,\mathrm{In}(2a/x) =$$

$$\frac{Q}{4\pi\varepsilon_0 a}\ln(2a/x) = \frac{\lambda}{2\pi\varepsilon_0}\ln\,(2a/x).$$

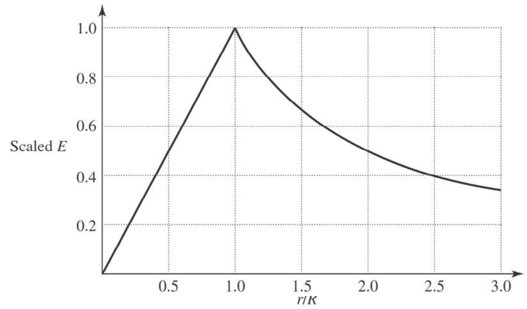
Thus $\lambda \equiv \frac{Q}{2a}$, and $R = 2a$, which is the only natural length in the problem.

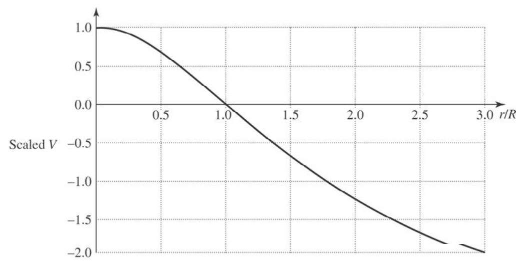
23.65: a) Recall: $r \le R : E = \dfrac{\rho r}{2\varepsilon_0} \Rightarrow V = -\int_R^r \vec{\boldsymbol{E}} \cdot d\,\vec{\boldsymbol{r}} = -\dfrac{\rho}{2\varepsilon_0}\int_R^r r\,dr = -\dfrac{\rho}{4\varepsilon_0}(r^2 - R^2)$

So with $\lambda = \pi R^2 \rho, V = -k\lambda(r^2 / R^2 - 1)$.

For $r \ge R : E = \dfrac{\rho R^2}{2\varepsilon_0 r} \Rightarrow V = -\int_R^r \vec{\boldsymbol{E}} \cdot d\,\vec{\boldsymbol{r}} = -\dfrac{\rho R^2}{2\varepsilon_0}\int_R^r \dfrac{dr}{r} = -\dfrac{\lambda}{2\pi\varepsilon_0}\ln\left[\frac{r}{R}\right] = -2k\lambda\ln\left[\frac{r}{R}\right]$.

b)





23.66: a) $V(0.03, 0) = \dfrac{k(5.00 \times 10^{-9}\ \text{C})}{0.0300\ \text{m}} + \dfrac{k(-2.00 \times 10^{-9}\ \text{C})}{0.01\ \text{m}} = -300\ \text{V}.$

$V(0.03, 0.05) = \dfrac{k(5.00 \times 10^{-9}\ \text{C})}{\sqrt{(0.0300^2 + 0.0500^2)}\ \text{m}} + \dfrac{k(-2.00 \times 10^{-9}\ \text{C})}{\sqrt{0.0100^2 + 0.0500^2}} = 419\ \text{V}.$

b) $W = -q\Delta V = -(+6.00 \times 10^{-9}\ \text{C})(718\ \text{V}) = -4.31 \times 10^{-6}\ \text{J}.$

Note that the work done by the field is negative, since the charge is moved AGAINST the electric field.

23.67: From Example 21.10, we have: $E_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{(x^2 + a^2)^{3/2}}$

$\Rightarrow V = -\dfrac{Q}{4\pi\,\varepsilon_0}\int_\infty^x \dfrac{x'}{(x'^2 + a^2)^{3/2}}dx' = \dfrac{Q}{4\pi\varepsilon_0}u^{-1/2}\Big|_{u=\infty}^{u=x^2+a^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\sqrt{x^2 + a^2}} = \text{Equation}$

(23.16).

23.68:

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi a}\frac{dl}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Q\, d\theta}{\pi a} \Rightarrow V = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\frac{Q\, d\theta}{\pi a} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}.$$

23.69: a) S_1 and $S_3 : V_{13} = -\int_0^{0.3}(-5x\hat{\boldsymbol{i}} + 3z\hat{\boldsymbol{k}})\cdot\hat{\boldsymbol{j}}\, dy = 0;$ S_1 and S_3 are at equal potentials.

b) S_2 and $S_4 : V_{24} = -\int_0^{0.3}(-5x\hat{\boldsymbol{i}} + 3z\hat{\boldsymbol{k}})\cdot\hat{\boldsymbol{k}}dz = -3\int_0^{0.3} z\, dz = \frac{-3}{2}z^2\,|_0^{0.3} = \frac{-3}{2}(0.3)^2 =$

-0.135 V . S_4 is higher.

c) S_5 and $S_6 : V_{56} = -\int_0^{0.3}(-5x\hat{\boldsymbol{i}} + 3z\hat{\boldsymbol{k}})\cdot\hat{\boldsymbol{i}}dx = 5\int_0^{0.3} x\, dx = \frac{5}{2}x^2\,|_0^{0.3} = \frac{5}{2}(0.3)^2 = 0.225$ V.

S_5 is higher.

23.70: From Example 22.9, we have:

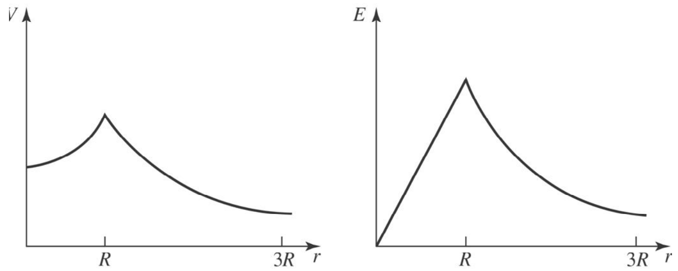
$$r > R : E = \frac{kQ}{r^2} \Rightarrow V = -kQ\int_\infty^r \frac{dr'}{r'^2} = \frac{kQ}{r}$$

$$r < R : E = \frac{kQr}{R^3} \Rightarrow V = -\int_\infty^R \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{r}}' - \int_R^r \vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{r}}' = \frac{kQ}{R} - \frac{kQ}{R^3}\int_R^r r'\, dr'$$

$$\Rightarrow V = \frac{kQ}{R} - \frac{kQ}{R^3}\frac{1}{2}r'^2\Big|_R^r = \frac{kQ}{R} + \frac{kQ}{2R} - \frac{kQr^2}{2R^3}$$

$$\therefore V = \frac{kQ}{2R}\left[3 - \frac{r^2}{R^2}\right]$$

b)



23.71: a) Problem 23.70 shows that

$$V_r = \frac{Q}{8\pi\varepsilon_0 R}(3 - r^2/R^2) \text{ for } r \le R \text{ and } V_r = \frac{Q}{4\pi\varepsilon_0 r} \text{ for } r \ge R$$

$$V_0 = \frac{3Q}{8\pi\varepsilon_0 R}, V_R = \frac{Q}{4\pi\varepsilon_0 R}, \text{ and } V_0 - V_R = \frac{Q}{8\pi\varepsilon_0 R}$$

b) If $Q > 0, V$ is higher at the center. If $Q < 0, V$ is higher at the surface.

23.72: (a) Points $a, b,$ and c are all at the same potential because $E = 0$ inside the spherical shell of charge on the outer surface. So $\Delta V_{ab} = \Delta V_{bc} = \Delta V_{ac} = 0.$

$$\Delta V_{c\infty} = \frac{kq}{R} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(150 \times 10^{-6} \text{ C})}{0.60 \text{ m}}$$

$$= 2.25 \times 10^6 \text{ V}.$$

(b) They are all at the same potential

(c) Only $\Delta V_{c\infty}$ would change; it would be -2.25×10^6 V.

23.73: a) The electrical potential energy for a spherical shell with uniform surface charge density and a point charge q outside the shell is the same as if the shell is replaced by a point charge at its center. Since $F_r = -dU/dr$, this means the force the shell exerts on the point charge is the same as if the shell were replaced by a point charge at its center. But by Newton's 3rd law, the force q exerts on the shell is the same as if the shell were a point charge. But q can be replaced by a spherical shell with uniform surface charge and the force is the same, so the force between the shells is the same as if they were both replaced by point charges at their centers. And since the force is the same as for point charges, the electrical potential energy for the pair of spheres is the same as for a pair of point charges.

b) The potential for solid insulating spheres with uniform charge density is the same outside of the sphere as for a spherical shell, so the same result holds.

c) The result doesn't hold for conducting spheres or shells because when two charged conductors are brought close together, the forces between them causes the charges to redistribute and the charges are no longer distributed uniformly over the surfaces.

23.74: Maximum speed occurs at "infinity" Energy conservation gives

$$\frac{kq_1q_2}{r} = \frac{1}{2}m_{50}v_{50}^2 + \frac{1}{2}m_{150}v_{150}^2$$

Momentum conservation: $m_{50}v_{50} = m_{150}v_{150}$ and $v_{50} = 3v_{150}$

Solve for v_{50} and v_{150}, where $r = 0.50$ m

$$v_{50} = 12.7 \text{ m/s}, v_{150} = 4.24 \text{ m/s}$$

Maximum acceleration occurs just after spheres are released. $\sum F = ma$ gives

$$\frac{kq_1q_2}{r^2} = m_{150}a_{150}$$

$$\frac{(9\times10^9 \text{ Nm}^2/\text{C}^2)\,(10^{-5}\text{ C})\,(3\times10^{-5}\text{ C})}{(0.50\text{ m})^2} = (0.15\text{ kg})a_{150}$$

$$a_{150} = 72.0 \text{ m/s}^2$$

$$a_{50} = 3a_{150} = 216 \text{ m/s}^2$$

23.75: Using the electric field from Problem 22.37, the potential difference between the conducting sphere and insulating shell is:

$$V = -\int_{2R}^{R} \vec{E}\cdot d\vec{r} = -\int_{2R}^{R}\frac{kQ}{r^2}dr = kQ\left[\frac{1}{R}-\frac{1}{2R}\right] \Rightarrow V = \frac{kQ}{2R}.$$

23.76: a) At $r = c : V = -\int_{\infty}^{c}\frac{kq}{r^2}dr = \frac{kq}{c}.$

b) At $r = b : V = -\int_{\infty}^{c}\vec{E}\cdot d\vec{r} - \int_{c}^{b}\vec{E}\cdot d\vec{r} = \frac{kq}{c} - 0 = \frac{kq}{c}.$

c) At $r = a : V = -\int_{\infty}^{c}\vec{E}\cdot d\vec{r} - \int_{c}^{b}\vec{E}\cdot d\vec{r} - \int_{b}^{a}\vec{E}\cdot d\vec{r} = \frac{kq}{c} - kq\int_{b}^{a}\frac{dr}{r^2} = kq\left[\frac{1}{c}-\frac{1}{b}+\frac{1}{a}\right].$

d) At $r = 0 : V = kq\left[\frac{1}{c}-\frac{1}{b}+\frac{1}{a}\right]$ since it is inside a metal sphere, and thus at the same potential as its surface.

23.77: Using the electric field from Problem 22.54, the potential difference between the two faces of the uniformly charged slab is:

$$V = -\int_{-d}^{d}\vec{E}\cdot d\vec{r} = -\int_{-d}^{d}\frac{\rho x}{2\varepsilon_0}dx = \frac{\rho}{2\varepsilon_0}\left(\frac{x^2}{2}\right)\Big|_{-d}^{d} \Rightarrow V = 0.$$

23.78: a) $V = \frac{kQ}{r} = \frac{k(-1.20 \times 10^{-12}\ \text{C})}{6.50 \times 10^{-4}\ \text{m}} = -16.6\ \text{V}.$

b) The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}} = \sqrt[3]{2}R = 8.19 \times 10^{-4}\ \text{m}$ and the new charge is $Q_{\text{new}} = 2Q = -2.40 \times 10^{-12}\ \text{C}$. So the new potential is:

$$V_{\text{new}} = \frac{kQ_{\text{new}}}{R_{\text{new}}} = \frac{k(-2.40 \times 10^{-12}\ \text{C})}{8.19 \times 10^{-4}\ \text{m}} = -26.4\ \text{V}.$$

23.79: a)

$$dV_p = \frac{kdq}{z+x} = \frac{kQ}{a}\frac{dz}{z+x} \Rightarrow V = \frac{kQ}{a}\int_0^a \frac{dz}{z+x} = \frac{kQ}{a}\ln\left(\frac{x+a}{x}\right) = \frac{kQ}{a}\ln\left(1+\frac{a}{x}\right).$$

b)

$$dV_R = \frac{kQ}{a}\frac{dz}{r} = \frac{kQ}{a}\frac{dz}{\sqrt{z^2+y^2}} \Rightarrow V_R = \frac{kQ}{a}\int_0^a \frac{dz}{\sqrt{z^2+y^2}} = \frac{kQ}{a}\ln\left(\frac{\sqrt{a^2+y^2}+a}{y}\right).$$

c)

$$x >> a : V_p \approx \frac{kQ}{a}\frac{a}{x} = \frac{kQ}{x}, \text{ Since } \ln(1+\alpha) \approx \alpha.$$

$$y >> a : V_R \approx \frac{kQ}{a}\frac{a}{y} = \frac{kQ}{y}, \text{ Since } \ln\left(\frac{\sqrt{a^2+y^2}+a}{y}\right) \approx \ln\left(\frac{y+a}{y}\right) = \ln\left(1+\frac{a}{y}\right) \approx \frac{a}{y}.$$

23.80: Set the alpha particle's kinetic energy equal to its potential energy:

$$K = U \Rightarrow 11.0\ \text{MeV} = \frac{k(2e)(82e)}{r} \Rightarrow r = \frac{k(164)(1.60 \times 10^{-19}\ \text{C})^2}{(11.0 \times 10^6\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})}$$
$$= 2.15 \times 10^{-14}\ \text{m}.$$

23.81: a) $V = \frac{kQ_B}{R_B} = \frac{kQ_A}{R_A} = \frac{kQ_B}{R_A/3} \Rightarrow Q_A = 3Q_B \Rightarrow \frac{Q_B}{Q_A} = \frac{1}{3}.$

b) $E_B = -\left.\frac{\partial V}{\partial r}\right|_{r=R_B} = \frac{kQ_B}{R_B^{\ 2}} = \frac{k(Q_A/3)}{(R_A/3)^2} = \frac{3kQ_A}{R_A^{\ 2}} = 3E_A \Rightarrow \frac{E_B}{E_A} = 3.$

23.82: a) From Problem 22.57 we have the electric field:

$$r \geq R : E = \frac{kQ}{r^2} \Rightarrow V = -\int_{\infty}^{r} \frac{kQ}{r'^2}\,dr' = \frac{kQ}{r},$$

which is the potential of a point charge.

b) $$r \leq R : E = \frac{kQ}{r^2}\left[4\frac{r^3}{R^3} - 3\frac{r^4}{R^4}\right] \Rightarrow V = -\int_{\infty}^{R} E\,dr' - \int_{R}^{r} E\,dr'$$

$$\Rightarrow V = \frac{kQ}{R}\left[1 - 2\frac{r^2}{R^2} + 2\frac{R^2}{R^2} + \frac{r^3}{R^3} - \frac{R^3}{R^3}\right] = \frac{kQ}{R}\left[\frac{r^3}{R^3} - 2\frac{r^2}{R^2} + 2\right].$$

23.83: a) $E = \dfrac{kQ_1}{R_1{}^2}, \quad V = \dfrac{kQ_1}{R_1}, \text{so}\, V = RE$

b) After electrostatic equilibrium is reached, with charge Q_1' now on the original sphere we have:

$$Q_1 = Q_1' + Q_2 \text{and}\, V_1 = V_2 \Rightarrow \frac{Q_1'}{R_1} = \frac{Q_2}{R_2} \Rightarrow Q_1' = Q_2\frac{R_1}{R_2}$$

$$Q_1 = Q_2\frac{R_1}{R_2} + Q_2 \Rightarrow Q_2 = \frac{Q_1}{(1 + \frac{R_1}{R_2})} = \frac{R_2 Q_1}{(R_2 + R_1)} \text{ and } Q_1' = \frac{(R_1/R_2)Q_1}{\left(1 + \frac{R_1}{R_2}\right)} = \frac{R_1 Q_1}{(R_2 + R_1)}$$

c) The new potential is the same at each sphere's surface:

$$V_1 = \frac{kQ_1'}{R_1} = \frac{kQ_1}{R_2\left(1 + \frac{R_1}{R_2}\right)} = \frac{kQ_1}{(R_2 + R_1)} = V_2$$

d) The new electric field is not the same at each sphere's surface:

$$E_1 = \frac{kQ_1'}{R_1{}^2} = \frac{kQ_1}{R_1 R_2 (1 + \frac{R_1}{R_2})} = \frac{kQ_1}{R_1(R_2 + R_1)}$$

$$E_2 = \frac{kQ_2}{R_2{}^2} = \frac{kQ_1}{R_2{}^2(1 + \frac{R_1}{R_2})} = \frac{kQ_1}{R_2(R_2 + R_1)}$$

23.84: a) We have $V(x, y, z) = A(x^2 - 3y^2 + z^2)$. So :

$$\boldsymbol{E} = -\frac{\partial V}{\partial x}\hat{\boldsymbol{i}} - \frac{\partial V}{\partial y}\hat{\boldsymbol{j}} - \frac{\partial V}{\partial z}\hat{\boldsymbol{k}} = -2Ax\hat{\boldsymbol{i}} + 6Ay\hat{\boldsymbol{j}} - 2Az\hat{\boldsymbol{k}}$$

b) A charge is moved in along the z-axis. So the work done is given by:

$$W = q\int_{z_0}^{0} \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{k}}\,dz = q\int_{z_0}^{0}(-2Az)\,dz = +(Aq){z_0}^2 \Rightarrow A = \frac{W}{q{z_0}^2}$$

$$A = \frac{6.00\times10^{-5}\ \text{J}}{(1.5\times10^{-6}\ \text{C})(0.250\ \text{m})^2} = 640\ \text{V/m}^2.$$

c) $\boldsymbol{E}(0, 0, 0.250) = -2(640\ \text{V/m}^2)\,(0.250\ \text{m})\hat{\boldsymbol{k}} = -320\ \text{V/m}\hat{\boldsymbol{k}}$.

d) In every plane parallel to the x-z plane, y is constant, so:

$$V(x, y, z) = Ax^2 + Az^2 - C \Rightarrow x^2 + z^2 = \frac{V + C}{A} \equiv R^2,$$

which is the equation for a circle since R is constant as long as we have constant potential on those planes.

e) $V = 1280$ V, and $y = 2$m: $x^2 + z^2 = \dfrac{1280\ \text{V} + 3(640\ \text{V/m}^2)(2.00\ \text{m})^2}{640\ \text{V/m}^2} = 14\ \text{m}^2$.

Thus the radius of the circle is 3.74 m.

23.85: a) $E_i = E_f \Rightarrow 2\left[\dfrac{1}{2}m_\text{p}v^2\right] = \dfrac{ke^2}{2r_\text{p}} \Rightarrow v = \sqrt{\dfrac{k(1.60\times10^{-19}\ \text{C})^2}{2(1.2\times10^{-15}\ \text{m})(1.67\times10^{-27}\ \text{kg})}}$

$\Rightarrow v = 7.58\times10^6\ \text{m/s}$.

b) For a helium-helium collision, the charges and masses change from (a):

$$v = \sqrt{\frac{k(2(1.60\times10^{-19}\ \text{C}))^2}{(3.5\times10^{-15}\ \text{m})(2.99)(1.67\times10^{-27}\ \text{kg})}} = 7.26\times10^6\ \text{m/s}.$$

c)

$$K = \frac{3kT}{2} = \frac{mv^2}{2} \Rightarrow T_\text{p} = \frac{m_\text{p}v^2}{3k} = \frac{(1.67\times10^{-27}\ \text{kg})(7.58\times10^6\ \text{m/s})^2}{3(1.38\times10^{-23}\ \text{J/K})} = 2.3\times10^9\ \text{K}$$

$$\Rightarrow T_\text{He} = \frac{m_\text{He}v^2}{3k} = \frac{(2.99)(1.67\times10^{-27}\ \text{kg})(7.26\times10^6\ \text{m/s})^2}{3(1.38\times10^{-23}\ \text{J/K})} = 6.4\times10^9\,\text{K}.$$

d) These calculations were based on the particles' average speed. The distribution of speeds ensures that there are always a certain percentage with a speed greater than the average speed, and these particles can undergo the necessary reactions in the sun's core.

23.86: a) The two daughter nuclei have half the volume of the original uranium nucleus, so their radii are smaller by a factor of the cube root of 2:

$$r = \frac{7.4 \times 10^{-15}\ \text{m}}{\sqrt[3]{2}} = 5.9 \times 10^{-15}\ \text{m}.$$

b) $U = \dfrac{k(46e)^2}{2r} = \dfrac{k(46)^2(1.60 \times 10^{-19}\ \text{C})^2}{1.17 \times 10^{-14}\ \text{m}} = 4.14 \times 10^{-11}\ \text{J}$

Each daughter has half of the potential energy turn into its kinetic energy when far from each other, so:

$$K = U/2 = (4.15 \times 10^{-11}\ \text{J})/2 = 2.07 \times 10^{-11}\ \text{J}.$$

c) If we have 10.0 kg of uranium, then the number of nuclei is:

$$n = \frac{10.0\ \text{kg}}{236\ \text{u}\ (1.66 \times 10^{-27}\ \text{kg/u})} = 2.55 \times 10^{25}\ \text{nuclei}.$$

And each releases energy $\text{U}: E = nU = (2.55 \times 10^{25})(4.15 \times 10^{-11}\ \text{J}) = 1.06 \times 10^{15}\ \text{J} =$ $253\ \text{kilotons of TNT}$.

d) We could call an atomic bomb an "electric" bomb since the electric potential energy provides the kinetic energy of the particles.

23.87: Angular momentum and energy must be conserved, so:

$$mv_1 b = mv_2 r_2 \ \textit{and}\ E_1 = E_2 \Rightarrow E_1 = \frac{1}{2}mv_2^{\ 2} + \frac{kq_1q_2}{r_2} \text{ and } E_1 = 11\ \text{MeV} = 1.76 \times 10^{-12}\ \text{J}.$$

Substituting in for v_2 we find:

$$E_1 = E_1\frac{b^2}{r_2^{\ 2}} + \frac{kq_1q_2}{r_2} \Rightarrow (E_1)r_2^{\ 2} - (kq_1q_2)r_2 - E_1 b^2 = 0, \text{and note } q_1 = 2e \text{ and } q_2 = 82e.$$

$(i)\ b = 10^{-12}\ \text{m} \Rightarrow r_2 = 1.01 \times 10^{-12}\ \text{m}$

$(\text{ii})\ b = 10^{-13}\ \text{m} \Rightarrow r_2 = 1.11 \times 10^{-13}\ \text{m}.$

$(\text{iii})\ b = 10^{-14}\ \text{m} \Rightarrow r_2 = 2.54 \times 10^{-14}\ \text{m}.$

23.88: a) $r \le a : V = \dfrac{\rho_0 a^2}{18\varepsilon_0}\left[1 - 3\dfrac{r^2}{a^2} + 2\dfrac{r^3}{a^3}\right]$ and $E = -\dfrac{\partial V}{\partial r}$

$$\Rightarrow E = -\frac{\rho_0 a^2}{18\varepsilon_0}\left[-6\frac{r}{a^2} + 6\frac{r^2}{a^3}\right] = \frac{\rho_0 a}{3\varepsilon_0}\left[\frac{r}{a} - \frac{r^2}{a^2}\right].$$

$r \ge a : V = 0$ and $E = -\dfrac{\partial V}{\partial r} = 0.$

b) $r \le a : E_r 4\pi r^2 = \dfrac{Q_r}{\varepsilon_0} = \dfrac{\rho_0 a}{3\varepsilon_0}\left[\dfrac{r}{a} - \dfrac{r^2}{a^2}\right]4\pi r^2$

$$E_{r+dr}4\pi(r^2 + 2rdr) = \frac{Q_{r+dr}}{\varepsilon_0} = \frac{\rho_0 a}{3\varepsilon_0}\left[\frac{r+dr}{a} - \frac{(r^2 + 2r\,dr)}{a^2}\right]4\pi(r^2 + 2r\,dr)$$

$$\Rightarrow \frac{Q_{r+dr} - Q_r}{\varepsilon_0} = \frac{\rho(r)4\pi r^2\,dr}{\varepsilon_0} \approx \frac{\rho_0 a 4\pi r^2 dr}{3\varepsilon_0}\left[-\frac{2r}{a^2} + \frac{2}{a} - \frac{2r}{a^2} + \frac{1}{a}\right]$$

$$\Rightarrow \rho(r) = \frac{\rho_0}{3}\left[3 - \frac{4r}{a}\right] = \rho_0\left[1 - \frac{4r}{3a}\right].$$

c) $r \ge a : \rho(r) = 0,$ so the total charge enclosed will be given by:

$$Q = 4\pi\int_0^a \rho(r)r^2 dr = 4\pi\rho_0\int_0^a\left[r^2 - \frac{4r^3}{3a}\right]dr = 4\pi\rho_0\left[\frac{1}{3}r^3 - \frac{r^4}{3a}\right]_0^a = 0.$$

Therefore, by Gauss's Law, the electric field must equal zero for any position $r \ge a$.

23.89: a) $F_g = mg = \dfrac{4\pi r^3}{3}\rho g = qV_{ab}/d = qE = F_e \Rightarrow q = \dfrac{4\pi}{3}\dfrac{\rho r^3 gd}{V_{ab}}.$

b) $F_g = mg = \dfrac{4\pi r^3}{3}\rho g = 6\pi\eta r v_t = F_v \Rightarrow r = \sqrt{\dfrac{9\eta v_t}{2\rho g}}$

$$\Rightarrow q = \frac{4\pi}{3}\frac{\rho gd}{V_{ab}}\left[\sqrt{\frac{9\eta v_t}{2\rho g}}\right]^3 = 18\pi\frac{d}{V_{ab}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

c) $q = 18\pi\dfrac{10^{-3}\text{ m}}{9.16\text{ V}}\sqrt{\dfrac{(1.81\times10^{-5}\text{ Ns/m}^2)^3(10^{-3}\text{ m}/39.3\text{ s})^3}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 4.80\times10^{-19}\text{ C} = 3e.$

$$r = \sqrt{\frac{9(1.81\times10^{-5}\text{ Ns/m}^2)(10^{-3}\text{ m}/39.3\text{ s})}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 5.07\times10^{-7}\text{ m}$$

23.90: For an infinitesimal slice of a finite cylinder, we have the potential:

$$dV = \frac{k\,dQ}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L}\frac{dz}{\sqrt{(x-z)^2 + R^2}}$$

$$\Rightarrow V = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dz}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L}\int_{-L/2-x}^{L/2-x}\frac{du}{\sqrt{u^2 + R^2}} \text{ where } u = x - z.$$

$$\Rightarrow V = \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2-x)^2 + R^2} + (L/2 - x)}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right] \text{on the cylinder's axis.}$$

b) For $L << R$:

$$V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{(\mathrm{L}/2-x)^2 + R^2} + L/2 - x}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right] \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{x^2 - xL + R^2} + L/2 - x}{\sqrt{x^2 + xL + R^2} - L/2 - x}\right]$$

$$\Rightarrow V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{1 - xL/(R^2+x^2)} + (L/2-x)/\sqrt{R^2+x^2}}{\sqrt{1 + xL/(R^2+x^2)} + (-L/2-x)/\sqrt{R^2+x^2}}\right]$$

$$\Rightarrow V \approx \frac{kQ}{L}\ln\left[\frac{1 - xL/2(R^2+x^2) + (L/2-x)/\sqrt{R^2+x^2}}{1 + xL/2(R^2+x^2) + (-L/2-x)/\sqrt{R^2+x^2}}\right]$$

$$\Rightarrow V \approx \frac{kQ}{L}\ln\left[\frac{1 + L/2\sqrt{R^2+x^2}}{1 - L/2\sqrt{R^2+x^2}}\right] = \frac{kQ}{L}\left(\ln\left[1 + \frac{L}{2\sqrt{R^2+x^2}}\right] - \ln\left[1 - \frac{L}{2\sqrt{R^2+x^2}}\right]\right)$$

$$\Rightarrow V \approx \frac{kQ}{L}\frac{2L}{2\sqrt{x^2+R^2}} = \frac{kQ}{\sqrt{x^2+R^2}},$$ which is the same as for a ring.

c) $$E = -\frac{\partial V}{\partial x} = \frac{2kQ\left(\sqrt{(L-2x)^2 + 4R^2} - \sqrt{(L+2x)^2 + 4R^2}\right)}{\sqrt{(L-2x)^2 + 4R^2}\cdot\sqrt{(L+2x)^2 + 4R^2}}.$$

23.91: a)

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(6\times10^{-5}\ \text{kg})(400\,\text{m/s}) + (3\times10^{-5}\ \text{kg})(1300\ \text{m/s})}{6.0\times10^{-5}\ \text{kg} + 3.0\times10^{-5}\ \text{kg}} = 700\ \text{m/s}$$

b) $E_{rel} = \frac{1}{2}m_1 v_1{}^2 + \frac{1}{2}m_2 v_2{}^2 + \frac{kq_1q_2}{r} - \frac{1}{2}(m_1 + m_2)v_{cm}{}^2.$

After expanding the center of mass velocity and collecting like terms:

$$\Rightarrow E_{rel} = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}[v_1{}^2 + v_2{}^2 - 2v_1 v_2] + \frac{kq_1q_2}{r} = \frac{1}{2}\mu(v_1 - v_2)^2 + \frac{kq_1q_2}{r}.$$

c) $E_{rel} = \frac{1}{2}(2.0\times10^{-5}\ \text{kg})(900\ \text{m/s})^2 + \frac{k(2.0\times10^{-6}\ \text{C})(-5.0\times10^{-6}\ \text{C})}{0.0090\,\text{m}} = -1.9\ \text{J}.$

d) Since the energy is less than zero, the system is "bound."

e) The maximum separation is when the velocity is zero:

$$-1.9\ \text{J} = \frac{kq_1q_2}{r} \Rightarrow r = \frac{k(2.0\times10^{-6}\ \text{C})(-5.0\times10^{-6}\ \text{C})}{-1.9\ \text{J}} = 0.047\ \text{m}.$$

f) Now using $v_1 = 400$ m/s and $v_2 = 1800$ m/s, we find : $E_{rel} = +9.6$ J. so the particles do escape, and the final relative velocity is :

$$|v_1 - v_2| = \sqrt{\frac{2E_{rel}}{\mu}} = \sqrt{\frac{2(9.6\ \text{J})}{2.0\times10^{-5}\ \text{kg}}} = 980\ \text{m/s}.$$

24.1: $Q = CV = (25.0\ \text{V})(7.28\ \mu\text{F}) = 1.82 \times 10^{-4}\ \text{C}.$

24.2: a) $C = \varepsilon_0 \dfrac{A}{d} = \varepsilon_0 \dfrac{0.00122\ \text{m}^2}{0.00328\ \text{m}} = 3.29\ \text{pF}.$

b) $V = \dfrac{Q}{C} = \dfrac{4.35 \times 10^{-8}\ \text{C}}{3.29 \times 10^{-12}\ \text{F}} = 13.2\ \text{kV}.$

c) $E = \dfrac{V}{d} = \dfrac{13.2 \times 10^{3}\ \text{V}}{0.00328\ \text{m}} = 4.02 \times 10^{6}\ \text{V/m}.$

24.3: a) $V = \dfrac{Q}{C} = \dfrac{0.148 \times 10^{-6}\ \text{C}}{2.45 \times 10^{-10}\ \text{F}} = 604\ \text{V}.$

b) $A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(2.45 \times 10^{-10}\ \text{F})(0.328 \times 10^{-3}\ \text{m})}{\varepsilon_0} = 0.0091\,\text{m}^2.$

c) $E = \dfrac{V}{d} = \dfrac{604\ \text{V}}{0.328 \times 10^{-3}\ \text{m}} = 1.84 \times 10^{6}\ \text{V/m}.$

d) $E = \dfrac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0 E = \varepsilon_0 (1.84 \times 10^{6}\ \text{V/m}) = 1.63 \times 10^{-5}\ \text{C/m}^2.$

24.4: $\Delta V = Ed = \dfrac{\sigma}{\varepsilon_0} d$

$$= \frac{(5.60 \times 10^{-12}\ \text{C/m}^2)(0.00180\ \text{m})}{8.85 \times 10^{-12}\ \text{C}^2/\text{Nm}^2}$$

$=1.14$ mV

24.5: a) $Q = CV = 120\ \mu C$

b) $C = \varepsilon_0\, A/d$

$d \to d/2$ means $C \to C/2$ and $Q \to Q/2 = 60\ \mu\text{C}$

c) $r \to 2r$ means $A \to 4A, C \to 4C$, and $Q \to 4Q = 480\ \mu\text{C}$

24.6: (a) 12.0 V since the plates remain charged.

(b) (i) $V = \frac{Q}{C}$

Q does not change since the plates are disconnected from the battery.

$$C = \frac{\varepsilon \cdot \text{A}}{d}$$

If d is doubled, $C \rightarrow \frac{1}{2}C$, so $V \rightarrow 2V = 24.0\,\text{V}$

(ii) $A = \pi r^2$, so if $r \rightarrow 2r$, then $A \rightarrow 4A$, and $C \rightarrow 4C$ which means that

$$V \rightarrow \frac{1}{4}V = 3.00 \text{ V}$$

24.7: Estimate $r = 1.0 \text{ cm}$

$$C = \frac{\varepsilon_0 A}{d} \text{ so } d = \frac{\varepsilon_0 \pi r^2}{C} = \frac{\varepsilon_0 \pi (0.010 \text{ m})^2}{1.00 \times 10^{-12} \text{ F}} = 2.8\,\text{mm}$$

The separation between the pennies is nearly a factor of 10 smaller than the diameter of a penny, so it is a reasonable approximation to treat them as infinite sheets.

24.8: (a) $\Delta V = Ed$

$100 \text{ V} = (10^4 \text{ N/C})d$

$d = 10^{-2} \text{ m} = 1.00\,\text{cm}$

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \pi R^2}{d}$$

$$R = \sqrt{\frac{Cd}{\pi \varepsilon_0}} = \sqrt{\frac{4Cd}{4\pi \varepsilon_0}}$$

$$R = \sqrt{4(5.00 \times 10^{-12}\text{F})(10^{-2}\text{m})(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})}$$

$$R = 4.24 \times 10^{-2}\,\text{m} = 4.24 \text{ cm}$$

(b) $Q = CV = (5\text{pF})(100 \text{ V}) = 500\,\text{pC}$

24.9: a) $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$

$$C = \frac{(0.180 \text{ m})2\pi\varepsilon_0}{\ln(5.00/0.50)} = 4.35 \times 10^{-12} \text{ F}$$

b) $V = Q/C = (10.0 \times 10^{-12}\text{C})/(4.35 \times 10^{-12} \text{ F}) = 2.30 \text{ V}$

24.10: a) $\dfrac{C}{L}=\dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}\Rightarrow \ln(r_b/r_a)=\dfrac{2\pi\varepsilon_0}{C/L}=\dfrac{2\pi\varepsilon_0}{31.5\times10^{-12}\ \text{F/m}}=1.77\Rightarrow\dfrac{r_b}{r_a}=5.84.$

b) $\dfrac{Q}{L}=V\dfrac{C}{L}=(2.60\,\text{V})(31.5\times10^{-12}\ \text{F/m})=8.19\times10^{-11}\ \text{C/m}.$

24.11: a) $C/L=\dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}=\dfrac{2\pi\varepsilon_0}{\ln(3.5\ \text{mm}/1.5\ \text{mm})}=6.56\times10^{-11}\ \text{F/m}.$

b) The charge on each conductor is equal but opposite. Since the inner conductor is at a higher potential it is positively charged, and the magnitude is:

$$Q=CV=\frac{2\pi\varepsilon_0 LV}{\ln(r_b/r_a)}=\frac{2\pi\varepsilon_0(2.8\ \text{m})(0.35\ \text{V})}{\ln(3.5\ \text{mm}/1.5\ \text{mm})}=6.43\times10^{-11}\text{C}.$$

24.12: a) For two concentric spherical shells, the capacitance is:

$$C=\frac{1}{k}\left(\frac{r_a r_b}{r_b-r_a}\right)\Rightarrow kCr_b-kCr_a=r_a r_b\Rightarrow r_b=\frac{kCr_a}{kC-r_a}$$

$$\Rightarrow r_b=\frac{k(116\times10^{-12}\ \text{F})(0.150\ \text{m})}{k(116\times10^{-12}\ \text{F})-0.150\ \text{m}}=0.175\ \text{m}.$$

b) $V=220$ V, and $Q=CV=(116\times10^{-12}\ \text{F})(220\ \text{V})=2.55\times10^{-8}$ C.

24.13: a) $C=\dfrac{1}{k}\left(\dfrac{r_b r_a}{r_b-r_a}\right)=\dfrac{1}{k}\left(\dfrac{(0.148\ \text{m})(0.125\ \text{m})}{0.148\ \text{m}-0.125\ \text{m}}\right)=8.94\times10^{-11}$ F.

b) The electric field at a distance of 12.6 cm:

$$E=\frac{kQ}{r^2}=\frac{kCV}{r^2}=\frac{k(8.94\times10^{-11}\ \text{F})(120\ \text{V})}{(0.126\ \text{m})^2}=6082\ \text{N/C}.$$

c) The electric field at a distance of 14.7 cm:

$$E=\frac{kQ}{r^2}=\frac{kCV}{r^2}=\frac{k(8.94\times10^{-11}\ \text{F})(120\ \text{V})}{(0.147\ \text{m})^2}=4468\ \text{N/C}.$$

d) For a spherical capacitor, the electric field is not constant between the surfaces.

24.14: a) $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1 + C_2} + \dfrac{1}{C_3} = \dfrac{1}{((3.0 + 5.0) \times 10^{-6}\text{ F})} + \dfrac{1}{(6.0 \times 10^{-6}\text{ F})}$

$$\Rightarrow C_{\text{eq}} = 3.42 \times 10^{-6}\text{ F.}$$

The magnitude of the charge for capacitors in series is equal, while the charge is distributed for capacitors in parallel. Therefore,

$$Q_3 = Q_1 + Q_2 = VC_{\text{eq}} = (24.0\text{ V})(3.42 \times 10^{-6}\text{ F}) = 8.21 \times 10^{-5}\text{ C.}$$

Since C_1 and C_2 are at the same potential, $\dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \Rightarrow Q_2 = \dfrac{C_2}{C_1} Q_1 = \dfrac{5}{3} Q,$

$$Q_3 = \tfrac{8}{3} Q_1 = 8.21 \times 10^{-5}\text{ C} \Rightarrow Q_1 = 3.08 \times 10^{-5}\text{ C, and } Q_2 = 5.13 \times 10^{-5}\text{ C.}$$

b) $V_2 = V_1 = Q_1/C_1 = (3.08 \times 10^{-5}\text{ C})/(3.00 \times 10^{-6}\text{ F}) = 10.3\text{ V}$. And $V_3 =$ $24.0\text{ V} - 10.3\text{ V} = 13.7\text{ V}$.

c) The potential difference between a and d: $V_{ad} = V_1 = V_2 = 10.3\text{ V}$.

24.15: a) $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{(\frac{1}{C_1} + \frac{1}{C_2}) + C_3} + \dfrac{1}{C_4} = \dfrac{1}{(2.00\ \mu\text{F} + 4.0\ \mu\text{F})} + \dfrac{1}{(4.0\ \mu\text{F})}$

$$\Rightarrow C_{\text{eq}} = 2.40\ \mu\text{F.}$$

Then, $Q_{12} + Q_3 = Q_4 = Q_{\text{total}} = C_{\text{eq}} V = (2.40 \times 10^{-6}\text{ F})(28.0\text{ V}) = 6.72 \times 10^{-5}\text{ C}$ and $2Q_{12} = Q_3 \Rightarrow Q_{12} = \dfrac{Q_{\text{total}}}{3} = \dfrac{6.72 \times 10^{-5}\text{ C}}{3} = 2.24 \times 10^{-5}\text{ C}$, and $Q_3 = 4.48 \times 10^{-5}\text{ C}$. But also, $Q_1 = Q_2 = Q_{12} = 2.24 \times 10^{-5}\text{ C}$.

b) $V_1 = Q_1/C_1 = (2.24 \times 10^{-5}\text{ C})/(4.00 \times 10^{-6}\text{ F}) = 5.60\text{ V} = V_2$

$V_3 = Q_3/C_3 = (4.48 \times 10^{-5}\text{C})/(4.00 \times 10^{-6}\text{ F}) = 11.2\text{ V.}$

$V_4 = Q_4/C_4 = (6.72 \times 10^{-5}\text{ C})/(4.00 \times 10^{-6}\text{ F}) = 16.8\text{ V.}$

c) $V_{ad} = V_{ab} - V_4 = 28.0\text{ V} - 16.8\text{ V} = 11.2\text{ V.}$

24.16: a)

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{(3.0 \times 10^{-6}\text{ F})} + \frac{1}{(5.0 \times 10^{-6}\text{ F})}$$

$$= 5.33 \times 10^5\text{ F}^{-1} \Rightarrow C_{\text{eq}} = 1.88 \times 10^{-6}\text{ F}$$

$$\Rightarrow Q = VC_{\text{eq}} = (52.0\text{ V})(1.88 \times 10^{-6}\text{ F}) = 9.75 \times 10^{-5}\text{ C}$$

b) $V_1 = Q/C_1 = 9.75 \times 10^{-5}\text{ C}/3.0 \times 10^{-6}\text{ F} = 32.5\text{ V.}$

$V_2 = Q/C_2 = 9.75 \times 10^{-5}\text{ C}/5.0 \times 10^{-6}\text{ F} = 19.5\text{ V.}$

24.17: a) $Q_1 = VC_1 = (52.0\text{ V})(3.0\times10^{-6}\text{ F}) = 1.56\times10^{-4}\text{ C}.$

$Q_2 = VC_2 = (52.0\text{ V})(5.0\times10^{-6}\text{ F}) = 2.6\times10^{-4}\text{ C}.$

b) For parallel capacitors, the voltage over each is the same, and equals the voltage source: 52.0 V.

24.18: $C_{\text{eq}} = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1} = \left(\frac{d_1}{\varepsilon_0 A}+\frac{d_2}{\varepsilon_0 A}\right)^{-1} = \frac{\varepsilon_0 A}{d_1+d_2}.$ So the combined capacitance for two capacitors in series is the same as that for a capacitor of area A and separation (d_1+d_2).

24.19: $C_{\text{eq}} = C_1 + C_2 = \frac{\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d} = \frac{\varepsilon_0 (A_1+A_2)}{d}.$ So the combined capacitance for two capacitors in parallel is that of a single capacitor of their combined area $(A_1 + A_2)$ and common plate separation d.

24.20: a) and b) The equivalent resistance of the combination is $6.0\ \mu\text{F}$, therefore the total charge on the network is: $Q = C_{\text{eq}}V_{\text{eq}}(6.0\ \mu\text{F})(36\text{ V}) = 2.16\times10^{-4}\text{C}.$ This is also the charge on the $9.0\ \mu\text{F}$ capacitor because it is connected in series with the point b. So:

$$V_9 = \frac{Q_9}{C_9} = \frac{2.16\times10^{-4}\text{ C}}{9.0\times10^{-6}\text{ F}} = 24\text{ V}.$$

Then $V_3 = V_{11} = V_{12} + V_6 = V - V_9 = 36\text{ V} - 24\text{ V} = 12\text{ V}.$

$\Rightarrow Q_3 = C_3V_3 = (3.0\ \mu\text{F})(12\text{ V}) = 3.6\times10^{-5}\text{ C}.$

$\Rightarrow Q_{11} = C_{11}V_{11} = (11\ \mu\text{F})(12\text{ V}) = 1.32\times10^{-4}\text{ C}.$

$\Rightarrow Q_6 = Q_{12} = Q - Q_3 - Q_{11}$

$= 2.16\times10^{-4}\text{ C} - 3.6\times10^{-5}\text{ C} - 1.32\times10^{-4}\text{ C}.$

$= 4.8\times10^{-5}\text{ C}.$

So now the final voltages can be calculated:

$$V_6 = \frac{Q_6}{C_6} = \frac{4.8\times10^{-5}\text{ C}}{6.0\times10^{-6}\text{ F}} = 8\text{ V}.$$

$$V_{12} = \frac{Q_{12}}{C_{12}} = \frac{4.8\times10^{-5}\text{ C}}{12\times10^{-6}\text{ F}} = 4\text{ V}.$$

c) Since the $3\ \mu\text{F}, 11\ \mu\text{F}$ and $6\ \mu\text{F}$ capacitors are connected in parallel and are in series with the $9\ \mu\text{F}$ capacitor, their charges must add up to that of the $9\ \mu\text{F}$ capacitor. Similarly, the charge on the $3\ \mu\text{F}, 11\ \mu\text{F}$ and $12\ \mu\text{F}$ capacitors must add up to the same as that of the $9\ \mu\text{F}$ capacitor, which is the same as the whole network. In short, charge is conserved for the whole system. It gets redistributed for capacitors in parallel and it is equal for capacitors in series.

24.21: Capacitances in parallel simply add, so:

$$\frac{1}{C_{\text{eq}}}=\frac{1}{8.0\ \mu\text{F}}=\left(\frac{1}{(11+4.0+x)\ \mu\text{F}}+\frac{1}{9.0\ \mu\text{F}}\right)\Rightarrow(15+x)\ \mu\text{F}=72\ \mu\text{F}\Rightarrow x=57\ \mu\text{F}.$$

24.22: a) C_1 and C_2 are in parallel and so have the same potential across them:

$$V=\frac{Q_2}{C_2}=\frac{40.0\times10^{-6}\text{C}}{3.00\times10^{-6}\ \text{F}}=13.33\ \text{V}$$

Thus $Q_1=VC_1=(13.33\ \text{V})(3.00\times10^{-6}\ \text{F})=80.0\times10^{-6}\ \text{C}$. Since Q_3 is in series with the parallel combination of C_1 and C_2, its charge must be equal to their combined charge: $40.0\times10^{-6}\text{C}+80.0\times10^{-6}\ \text{C}=120.0\times10^{-6}\text{C}$ b) The total capacitance is found from:

$$\frac{1}{C_{\text{tot}}}=\frac{1}{C_{\|}}+\frac{1}{C_3}=\frac{1}{9.00\times10^{-6}\ \text{F}}+\frac{1}{5.00\times10^{-6}\ \text{F}}$$

$$C_{\text{tot}}=3.21\ \mu\text{F}$$

and

$$V_{ab}=\frac{Q_{\text{tot}}}{C_{\text{tot}}}=\frac{120.0\times10^{-6}\ \text{C}}{3.21\times10^{-6}\ \text{F}}=37.4\ \text{V}$$

24.23: $V_1=Q_1/C_1=(150\ \mu\text{C})/(3.00\ \mu\text{F})=50\ \text{V}$

C_1 and C_2 are in parallel, so $V_2=50\ \text{V}$

$V_3=120\ \text{V}-V_1=70\ \text{V}$

24.24: a) $V=Q/C=(2.55\ \mu\text{C})/(920\times10^{-12}\ \text{F})=2772\ \text{V}.$

b) Since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to 5544 V.

c) $U=\frac{1}{2}CV^2=\frac{1}{2}(920\times10^{-12}\ \text{F})(2772\ \text{V})^2=3.53\times10^{-3}\ \text{J}.$ Now if the separation is doubled, the capacitance halves, and the energy stored doubles. So the amount of work done to move the plates equals the difference in energy stored in the capacitor, which is 3.53×10^{-3} J.

24.25: $E=V/d=(400\ \text{V})/(0.005\ \text{m})=8.00\times10^4\ \text{V/m}.$

And $u=\frac{1}{2}\varepsilon_0E^2=\frac{1}{2}\varepsilon_0(8.00\times10^4\ \text{V/m})^2=0.0283\ \text{J/m}^3.$

24.26: a) $C = Q/V = (0.0180\ \mu\text{C})/(200\ \text{V}) = 9.00 \times 10^{-11}\ \text{F}.$

b) $C = \dfrac{\varepsilon_0 A}{d} \Rightarrow A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(9.00 \times 10^{-11}\ \text{F})(0.0015\,\text{m})}{\varepsilon_0} = 0.0152\ \text{m}^2.$

c) $E_{\max} = V_{\max}/d \Rightarrow V_{\max} = E_{\max} d = (3.00 \times 10^6\ \text{V/m})(0.0015\ \text{m}) = 4500\ \text{V}.$

d) $U = \dfrac{Q^2}{2C} = \dfrac{(1.80 \times 10^{-8}\ \text{C})^2}{2(9.00 \times 10^{-11}\ \text{F})} = 1.80 \times 10^{-6}\ \text{J}.$

24.27: $U = \frac{1}{2}CV^2 = \frac{1}{2}(4.50 \times 10^{-4}\ \text{F})(295\ \text{V})^2 = 19.6\ \text{J}.$

24.28: a) $Q = CV_0$.

b) They must have equal potential difference, and their combined charge must add up to the original charge. Therefore:

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \text{ and also } Q_1 + Q_2 = Q = CV_0$$

$$C_1 = C \text{ and } C_2 = \frac{C}{2} \text{ so } \frac{Q_1}{C} = \frac{Q_2}{(C/2)} \Rightarrow Q_2 = \frac{Q_1}{2}$$

$$\Rightarrow Q = \frac{3}{2}Q_1 \Rightarrow Q_1 = \frac{2}{3}Q \text{ so } V = \frac{Q_1}{C} = \frac{2}{3}\frac{Q}{C} = \frac{2}{3}V_0$$

c) $U = \dfrac{1}{2}\left(\dfrac{Q_1^{\,2}}{C_1} + \dfrac{Q_2^{\,2}}{C_2}\right) = \dfrac{1}{2}\left[\dfrac{(\frac{2}{3}Q)^2}{C} + \dfrac{2(\frac{1}{3}Q)^2}{C}\right] = \dfrac{1}{3}\dfrac{Q^2}{C} = \dfrac{1}{3}CV_0^{\,2}$

d) The original U was $U = \frac{1}{2}CV_0^{\,2} \Rightarrow \Delta U = \frac{-1}{6}CV_0^{\,2}$.

e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

24.29: a) $U_0 = \dfrac{Q^2}{2C} = \dfrac{xQ^2}{2\varepsilon_0 A}.$

b) Increase the separation by $dx \Rightarrow U = \frac{(x+dx)Q^2}{2\varepsilon_0 A} = U_0(1 + dx/x).$ The change is then $\frac{Q^2}{2\varepsilon_0 A}dx$.

c) The work done in increasing the separation is given by:

$$dW = U - U_0 = \frac{dxQ^2}{2\varepsilon_0 A} = Fdx \Rightarrow F = \frac{Q^2}{2\varepsilon_0 A}.$$

d) The reason for the difference is that E is the field due to both plates. The force is QE if E is the field due to one plate is Q is the charge on the other plate.

24.30: a) If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2 / 2C.$ Therefore the new energy is 4.19 J.

b) If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.76 J, using $U = CV^2 / 2,$ when V is held constant throughout.

24.31: a) $U = Q^2 / 2C$

$$Q = \sqrt{2UC} = \sqrt{2(25.0\ \text{J})(5.00 \times 10^{-9}\ \text{F})} = 5.00 \times 10^{-4}\ \text{C}$$

The number of electrons N that must be removed from one plate and added to the other is $N = Q / e = (5.00 \times 10^{-4}\ \text{C})/(1.602 \times 10^{-19}\ \text{C}) = 3.12 \times 10^{15}$ electrons.

b) To double U while keeping Q constant, decrease C by a factor of 2.

$C = \varepsilon_0 A/d;$ halve the plate area or double the plate separation.

24.32: $C = \dfrac{Q}{V} = \dfrac{8.20 \times 10^{-12}\,\text{C}}{2.40\ \text{V}} = 3.417 \times 10^{-12}$ farad

Since $C = K\varepsilon_0\, A/d$ for a parallel plate capacitor

$$d = \frac{K\varepsilon_0 A}{C} = \frac{(1.00)(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.60 \times 10^{-3}\,\text{m}^2)}{3.417 \times 10^{-12}\ \text{farad}}$$

$$= 6.734 \times 10^{-3}\,\text{m}$$

The energy density is thus

$$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}(3.42 \times 10^{-12}\ \text{farad})(2.40\ \text{V})^2}{(2.60 \times 10^{-3}\text{m}^2)(6.734 \times 10^{-3}\text{m})} = 5.63 \times 10^{-7}\ \frac{\text{J}}{\text{m}^3}$$

24.33: a) $U = \dfrac{1}{2}QV \Rightarrow Q = \dfrac{2U}{V} = \dfrac{2(3.20 \times 10^{-9}\ \text{J})}{4.00\ \text{V}} = 1.60 \times 10^{-9}\ \text{C}.$

b) $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_a/r_b)} \Rightarrow \dfrac{r_a}{r_b} = \exp(2\pi\varepsilon_0\, L/C) = \exp(2\pi\varepsilon_0\, LV/Q)$

$$\Rightarrow \frac{r_a}{r_b} = \exp(2\pi\varepsilon_0 (15.0\ \text{m})(4.00\ \text{V}) / (1.60 \times 10^{-9}\ \text{C})) = 8.05.$$

24.34: a) For a spherical capacitor:

$$C = \frac{1}{k}\frac{r_a r_b}{r_b - r_a} = \frac{1\,(0.100\text{ m})(0.115\text{ m})}{k(0.115\text{ m} - 0.100\text{ m})} = 8.53\times10^{-11}\text{ F}$$

$$\Rightarrow V = Q/C = (3.30\times10^{-9}\text{ C})/(8.53\times10^{-11}\text{ F}) = 38.7\text{ V}.$$

b) $U = \frac{1}{2}CV^2 = \frac{(8.53\times10^{-11}\text{ F})(38.7\text{ V})^2}{2} = 6.38\times10^{-8}\text{ J}.$

24.35: a) $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\left(\frac{kq}{r^2}\right)^2 = \frac{\varepsilon_0}{2}\left(\frac{kVC}{r^2}\right)^2 = \frac{\varepsilon_0 k^2}{2}\frac{(120\text{ V})^2(8.94\times10^{-11}\text{ F})^2}{(0.126\text{ m})^4}$

$\Rightarrow u = 1.64\times10^{-4}\text{ J/m}^3.$

b) The same calculation for $r = 14.7\text{ cm} \Rightarrow u = 8.83\times10^{-5}\text{ J/m}^3.$

c) No, the electric energy density is NOT constant within the spheres.

24.36: a) $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2 = \frac{1}{32\pi^2\varepsilon_0}\frac{(8.00\times10^{-9}\text{ C})^2}{(0.120\text{ m})^4} = 1.11\times10^{-4}\text{ J/m}^3.$

b) If the charge was –8.00 nC, the electric field energy would remain the same since *U* only depends on the square of *E*.

24.37: Let the applied voltage be *V*. Let each capacitor have capacitance C. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage *V*.

a) **series**

Voltage across each capacitor is $V/2$. The total energy stored is

$$U_s = 2\left(\frac{1}{2}C[V/2]^2\right) = \tfrac{1}{4}CV^2$$

parallel

Voltage across each capacitor is *V*. The total energy stored is

$$U_p = 2\left(\tfrac{1}{2}CV^2\right) = CV^2$$

$U_p = 4U_s$ b) $Q = CV$ for a single capacitor with voltage V.

$Q_s = 2(C[V/2]) = CV; \quad Q_p = 2(CV) = 2CV; \quad Q_p = 2Q_s$

c) $E = V/d$ for a capacitor with voltage V

$E_s = V/2d; \quad E_p = V/d; \quad E_p = 2E_s$

24.38: a) $C = K\epsilon_0 A/d$ gives us the area of the plates:

$$A = \frac{Cd}{K\epsilon_0} \frac{(5.00\times10^{-12}\text{ farad})(1.50\times10^{-3}\text{ m})}{(1.00)(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)} = 8.475\times10^{-4}\text{ m}^2$$

We also have $C = K\epsilon_0 A/d = Q/V$, so $Q = K\epsilon_0 A(V/d)$. V/d is the electric field between the plates, which is not to exceed 3.00×10^4 N/C. Thus

$$Q = (1.00)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(8.475\times10^{-4}\text{ m}^2)(3.00\times10^4\text{ N/C})$$
$$= 2.25\times10^{-10}\text{ C}$$

b) Again, $Q = K\epsilon_0 A(V/d) = 2.70\epsilon_0 A(V/d)$. If we continue to think of V/d as the electric field, only K has changed from part (a); thus Q in this case is $(2.70)(2.25\times10^{-10}\text{ C}) = 6.08\times10^{-10}\text{C}$.

24.39: a) $\sigma_i = \epsilon_0((3.20-2.50)\times10^5\text{ V/m}) = 6.20\times10^{-7}\text{ C/m}^2$. The field induced in the dielectric creates the bound charges on its surface.

b) $K = \dfrac{E_0}{E} = \dfrac{3.20\times10^5\text{ V/m}}{2.50\times10^5\text{ V/m}} = 1.28.$

24.40: a) $E_0 = KE = (3.60)(1.20\times10^6\text{ V/m}) = 4.32\times10^6\text{ V/m} \Rightarrow \sigma = \epsilon_0 E_0 = 3.82\times10^{-5}\text{ C/m}^2$.

b) $\sigma_i = \sigma\left(1-\dfrac{1}{K}\right) = (3.82\times10^{-5}\text{ C/m}^2)(1-1/3.60) = 2.76\times10^{-5}\text{ C/m}^2.$

c) $U = \frac{1}{2}CV^2 = uAd = \frac{1}{2}K\epsilon_0 E^2 Ad$

$\Rightarrow U = \frac{1}{2}(3.60)\epsilon_0(1.20\times10^6\text{ V/m})^2(0.0018\text{ m})(2.5\times10^{-4}\text{ m}^2) = 1.03\times10^{-5}\text{ J}.$

24.41: $C = \dfrac{K\epsilon_0 A}{d} = \dfrac{K\epsilon_0 AE}{V} \Rightarrow A = \dfrac{CV}{K\epsilon_0 E} = \dfrac{(1.25\times10^{-9}\text{ F})(5500\text{ V})}{(3.60)\epsilon_0(1.60\times10^7\text{ V/m})} = 0.0135\text{ m}^2.$

24.42: Placing a dielectric between the plates just results in the replacement of ϵ for ϵ_0 in the derivation of Equation (24.20). One can follow exactly the procedure as shown for Equation (24.11).

24.43: a) $\varepsilon = K\varepsilon_0 = (2.6)\varepsilon_0 = 2.3\times10^{-11}\ \text{C}^2/\text{Nm}^2.$

b) $V_{\text{max}} = E_{\text{max}}d = (2.0\times10^7\ \text{V/m})(2.0\times10^{-3}\ \text{m}) = 4.0\times10^4\ \text{V}.$

c) $E = \dfrac{\sigma}{K\varepsilon_0} \Rightarrow \sigma = \varepsilon E = (2.3\times10^{-11}\ \text{C}^2/\text{Nm}^2)(2.0\times10^7\ \text{V/m}) = 0.46\times10^{-3}\ \text{C/m}^2.$

And $\sigma_i = \sigma\left(1-\dfrac{1}{K}\right) = (0.46\times10^{-3}\ \text{C/m}^2)(1-1/2.6) = 2.8\times10^{-4}\ \text{C/m}^2.$

24.44: a) $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times10^{-7}\ \text{F})(12\ \text{V}) =$ $6.3\times10^{-6}\ \text{C}.$

b) $Q_i = Q(1-\frac{1}{K}) = (9.3\times10^{-6}\ \text{C})(1-1/3.1) = 6.3\times10^{-6}\ \text{C}.$

c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

24.45: a) $U_0 = \dfrac{1}{2}C_0V^2 \Rightarrow V = \sqrt{\dfrac{2U_0}{C_0}} = \sqrt{\dfrac{2(1.85\times10^{-5}\ \text{J})}{(3.60\times10^{-7}\ \text{F})}} = 10.1\ \text{V}.$

b) $U = \dfrac{1}{2}KC_0V^2 \Rightarrow K = \dfrac{U}{C_0V^2} = \dfrac{2(2.32\times10^{-5} + 1.85\times10^{-5}\ \text{J})}{(3.60\times10^{-7}\ \text{F})(10.1\ \text{V})^2} = 2.27.$

24.46: a) The capacitance changes by a factor of K when the dielectric is inserted. Since V is unchanged (The battery is still connected),

$$\frac{C_{\text{after}}}{C_{\text{before}}} = \frac{Q_{\text{after}}}{Q_{\text{before}}} = \frac{45.0\,\text{pC}}{25.0\,\text{pC}} = K = 1.80$$

b) The area of the plates is $\pi r^2 = \pi(0.0300\text{ m})^2 = 2.827\times10^{-3}\text{m}^2$, and the separation between them is thus

$$d = \frac{K\varepsilon_0 A}{C} = \frac{(1.00)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\text{ m}^2)}{12.5\times10^{-12}\text{ farad}}$$

$$= 2.002\times10^{-3}\text{ m}$$

Before the dielectric is inserted,

$$C = \frac{K\varepsilon_0 A}{d} = \frac{Q}{V}$$

$$V = \frac{Qd}{K\varepsilon_0 A} = \frac{(25.0\times10^{-12}\text{ C})(2.00\times10^{-3}\text{ m})}{(1.00)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\text{ m}^2)}$$

$$= 2.000\text{ V}$$

The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

c) Before the dielectric is inserted,

$$E = \frac{Q}{\varepsilon_0 KA} = \frac{25.0\times10^{-12}\text{ C}}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1.00)(2.827\times10^{-3}\text{ m}^2)}$$

$$= 999\text{ N/C}$$

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

24.47: a) before: $V_0 = Q_0/C_0 = (9.00\times10^{-6}\text{ C})/(3.00\times10^{-6}\text{ F}) = 3.00\text{ V}$

after: $C = KC_0 = 15.0\text{ F}; Q = Q_0$

$V = Q/C = 0.600\text{ V}; V$ decreases by a factor of K

b) $E = V/d$, the same at all points between the plates (as long as far from the edges of the plates)

before: $E = (3.00\text{ V})/(2.00\times10^{-3}\text{ m}) = 1500\text{ V/m}$

after: $E = (0.600\text{ V})/(2.00\times10^{-3}\text{ m}) = 300\text{ V/m}$

24.48: a) $\oint K\vec{\mathbf{E}}\cdot\vec{\mathbf{A}}=\dfrac{Q_{free}}{\varepsilon_0}\Rightarrow KE4\pi d^2=\dfrac{q}{\varepsilon_0}\Rightarrow E=\dfrac{q}{4\pi\varepsilon d^2}.$

b) $\oint\vec{\boldsymbol{E}}\cdot d\vec{\mathbf{A}}=\dfrac{q_{total}}{\varepsilon_0}=\dfrac{q_f+q_b}{\varepsilon_0}\Rightarrow E4\pi d^2=\dfrac{q+q_b}{\varepsilon_0}\Rightarrow E=\dfrac{q+q_b}{4\pi\varepsilon_0 d^2}$

$\Rightarrow q_{total}=q+q_b=q/K.$

c) The total bound change is $q_{\mathrm{b}}=q\left(\frac{1}{K}-1\right).$

24.49: a) Equation (25.22): $\oint K\vec{E}\cdot d\vec{A}=\frac{Q_{free}}{\varepsilon_0}\Rightarrow KEA=\frac{Q}{\varepsilon_0}\Rightarrow E=\frac{Q}{K\varepsilon_0 A}=\frac{Q}{\varepsilon A}.$

b) $V=Ed=\dfrac{Qd}{K\varepsilon_0 A}=\dfrac{Qd}{\varepsilon A}.$

c) $C=\dfrac{Q}{V}=\dfrac{\varepsilon A}{d}=K\dfrac{\varepsilon_0 A}{d}=KC_0.$

24.50: a) $C=\dfrac{\varepsilon_0 A}{d}=\dfrac{\varepsilon_0(0.16\text{ m})^2}{4.7\times10^{-3}\text{ m}}=4.8\times10^{-11}\text{ F}.$

b) $Q=CV=(4.8\times10^{-11}\text{F})\,(12\text{ V})=0.58\times10^{-9}\text{ C}.$

c) $E{=}V/d={(12\text{ V})}/{(4.7\times10^{-3}\text{m})}=2553\text{ V/m}.$

d) $U=\frac{1}{2}CV^2=\frac{1}{2}(4.8\times10^{-11}\text{F})(12\text{ V})^2=3.46\times10^{-9}\text{ J}.$

e) If the battery is disconnected, so the charge remains constant, and the plates are pulled further apart to 0.0094 m, then the calculations above can be carried out just as before, and we find:

a) $C=2.41\times10^{-11}\text{ F}$ b) $Q=0.58\times10^{-9}\text{ C}.$

c) $E=2553\,\text{V/m}$ d) $U=\dfrac{Q^2}{2C}=\dfrac{(0.58\times10^{-9}\text{ C})^2}{2(2.41\times10^{-11}\text{ F})}=6.91\times10^{-9}\text{ J}.$

24.51: If the plates are pulled out as in Problem 24.50 the battery is connected, ensuring that the voltage remains constant. This time we find:

a) $C=2.4\times10^{-11}\text{ F}$ b) $Q=2.9\times10^{-10}\text{ C}$ c) $E=\dfrac{V}{d}=\dfrac{12\text{ V}}{0.0094}=1.3\times10^{3}\ \dfrac{\text{V}}{\text{m}}$

d) $U=\dfrac{CV^2}{2}=\dfrac{(2.4\times10^{-11}\text{ F})\,(12\text{ V})^2}{2}=1.73\times10^{-9}\text{ J}.$

24.52: a) System acts like two capacitors in series so $C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$

$$C_1 = C_2 = \frac{\varepsilon_0 L^2}{d} \text{ so } C_{\text{eq}} = \frac{\varepsilon_0 L^2}{2d}\cdot \quad U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{\left(\frac{\varepsilon_0 L^2}{2d}\right)} = \frac{Q^2 d}{\varepsilon_0 L^2}.$$

b) After rearranging, the E fields should be calculated. Use superposition recalling $E = \frac{Q}{2\varepsilon_0 A}$ for a single plate (not $\frac{Q}{\varepsilon_0 A}$ since charge Q is only on one face).

between 1 and 3: $E = \left(\frac{Q}{2\varepsilon_0 L^2}\right)_1 - \left(\frac{Q}{2\varepsilon_0 L^2}\right)_3 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_2 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_4 = \frac{Q}{\varepsilon_0 L^2}$

between 3 and 2: $E = \left(\frac{Q}{2\varepsilon_0 L^2}\right)_1 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_3 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_2 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_4 = \frac{2Q}{\varepsilon_0 L^2}$

between 2 and 4: $E = \left(\frac{Q}{2\varepsilon_0 L^2}\right)_1 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_3 - \left(\frac{Q}{2\varepsilon_0 L^2}\right)_2 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_4 = \frac{Q}{\varepsilon_0 L^2}$

$$U_{\text{new}} = \left(\frac{1}{2}\varepsilon_0 E^2\right) L^2 d = \frac{1}{2}\varepsilon_0\left(\frac{Q^2}{\varepsilon_0{}^2 L^4} + \frac{4Q^2}{\varepsilon_0{}^2 L^4} + \frac{Q^2}{\varepsilon_0{}^2 L^4}\right) L^2 d = \frac{3Q^2 d}{\varepsilon_0 L^2}$$

$$\Delta U = U_{\text{new}} - U = \frac{3Q^2 d}{\varepsilon_0 L^2} - \frac{Q^2 d}{\varepsilon_0 L^2} = \frac{2Q^2 d}{\varepsilon_0 L^2}$$

This is the work required to rearrange the plates.

24.53: a) The power output is 600 W, and 95% of the original energy is converted.
$\Rightarrow E = Pt = (2.70 \times 10^5\,\text{W})\,(1.48 \times 10^{-3}\,\text{s}) = 400\,\text{J} \therefore E_0 = \frac{400\,\text{J}}{0.95} = 421\,\text{J}.$

b) $U = \frac{1}{2}CV^2 \Rightarrow C = \frac{2U}{V^2} = \frac{2(421\,\text{J})}{(125\,\text{V})^2} = 0.054\,\text{F}$

24.54: $C_0 = \frac{A\varepsilon_0}{d} = \frac{(4.20 \times 10^{-5}\ \text{m}^2)\varepsilon_0}{7.00 \times 10^{-4}\ \text{m}} = 5.31 \times 10^{-13}\ \text{F}$

$$\Rightarrow C = C_0 + 0.25\ \text{pF} = 7.81 \times 10^{-13}\ \text{F}.$$

But $C = \frac{A\varepsilon_0}{d'} \Rightarrow d' = \frac{A\varepsilon_0}{C} = \frac{(4.20 \times 10^{-5}\ \text{m}^2)\varepsilon_0}{7.81 \times 10^{-13}\ \text{F}} = 4.76 \times 10^{-4}\text{m}.$

Therefore the key must be depressed by a distance of:

$$7.00 \times 10^{-4}\ \text{m} - 4.76 \times 10^{-4}\ \text{m} = 0.224\,\text{mm}.$$

24.55: a) $d << r_a : C = \dfrac{2\pi\varepsilon_0 L}{\ln(r_b/r_a)} = \dfrac{2\pi\varepsilon_0 L}{\ln((d+r_a)/r_a)} = \dfrac{2\pi\varepsilon_0 L}{\ln(1+d/r_a)} \approx \dfrac{2\pi r_a L\varepsilon_0}{d} = \dfrac{\varepsilon_0 A}{d}$.

b) At the scale of part (a) the cylinders appear to be flat, and so the capacitance should appear like that of flat plates.

24.56: Originally: $Q_1 = C_1V_1 = (9.0\ \mu\text{F})\ (28\ \text{V}) = 2.52\times10^{-4}\ \text{C}$; $Q_2 = C_2V_2 = (4.0\ \mu\text{F})\times(28\ \text{V}) = 1.12\times10^{-4}\ \text{C}$, and $C_{\text{eq}} = C_1 + C_2 = 13.0\ \mu\text{F}$. So the original energy stored is $U = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(13.0\times10^{-6}\ \text{F})\ (28\ \text{V})^2 = 5.10\times10^{-3}\ \text{J}$. Disconnect and flip the capacitors, so now the total charge is $Q = Q_2 - Q_1 = 1.4\times10^{-4}\ \text{C}$, and the equivalent capacitance is still the same, $C_{\text{eq}} = 13.0\ \mu\text{F}$. So the new energy stored is:

$$U = \frac{Q^2}{2C_{\text{eq}}} = \frac{(1.4\times10^{-4}\ \text{C})^2}{2(13.0\times10^{-6}\ \text{F})} = 7.54\times10^{-4}\ \text{J}$$

$$\Rightarrow \Delta U = 7.45\times10^{-4}\ \text{J} - 5.10\times10^{-3}\ \text{J} = -\,4.35\times10^{-3}\ \text{J}.$$

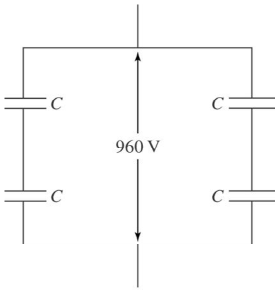
24.57: a) $C_{\text{eq}} = 4.00\ \mu F + 6.00\mu\text{F} = 10.00\ \mu\text{F}$, and $Q_{total} = C_{\text{eq}}\ V = (10.00\ \mu\text{F})\ (660\ \text{V}) = 6.6\times10^{-3}\text{C}$. The voltage over each is 660 V since they are in parallel. So:

$$Q_1 = C_1V_1 = (4.00\ \mu\text{F})\ (660\ \text{V}) = 2.64\times10^{-3}\ \text{C}.$$
$$Q_2 = C_2V_2 = (6.00\ \mu\text{F})\ (660\ \text{V}) = 3.96\times10^{-3}\ \text{C}.$$

b) $Q_{total} = 3.96\times10^{-3}\ \text{C} - 2.64\times10^{-3}\ \text{C} = 1.32\times10^{-3}\ \text{C}$, and still $C_{\text{eq}} = 10.00\ \mu\text{F}$, so the voltage is $V = Q/C = (1.32\times10^{-3}\ \text{C})/(10.00\ \mu\text{F}) = 132\,\text{V}$, and the new charges:

$$Q_1 = C_1V_1 = (4.00\ \mu\text{F})(132\ \text{V}) = 5.28\times10^{-4}\ \text{C}.$$
$$Q_2 = C_2V_2 = (6.00\ \mu\text{F})(132\ \text{V}) = 7.92\times10^{-4}\ \text{C}.$$

24.58: a)



$C_{eq} = \frac{C}{2} + \frac{C}{2} = C.$ So the total capacitance is the same as each individual capacitor, and the voltage is spilt over each so that $V = 480$ V. Another solution is two capacitors in parallel that are in series with two others in parallel.

b) If one capacitor is a moderately good conductor, then it can be treated as a "short" and thus removed from the circuit, and one capacitor will have greater than 600 V over it.

24.59: a) $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2 + \left(\frac{1}{C_3} + \frac{1}{C_4}\right)^{-1}} + \dfrac{1}{C_5} \Rightarrow C_1 = C_5 = 2C_2$ and

$C_2 = C_3 = C_4$ so $\dfrac{1}{C_{eq}} = \dfrac{2}{C_1} + \dfrac{2}{3C_2} = \dfrac{5}{3}C_2 \Rightarrow C_{eq} = \dfrac{3}{5}C_2 = 2.52\ \mu\text{F}.$

b) $Q = CV = (2.52\ \mu\text{F})(220\text{ V}) = 5.54 \times 10^{-4}\text{ C} = Q_1 = Q_5$

$\Rightarrow V_1 = V_5 = (5.54 \times 10^{-4}\text{ C}) / (8.4 \times 10^{-6}\text{ F}) = 66\text{ V}.$

So $V_2 = 220 - 2(66) = 88\text{ V} \Rightarrow Q_2 = (88\text{ V})(4.2\ \mu\text{F}) = 3.70 \times 10^{-4}$ C. Also $V_3 = V_4 =$ $\frac{1}{2}(88\text{ V}) = 44\text{ V} \Rightarrow Q_3 = Q_4 = (44\text{ V})(4.2\ \mu\text{F}) = 1.85 \times 10^{-4}\text{C}.$

24.60: a) With the switch open: $C_{\text{eq}} = \left(\left(\frac{1}{3\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}}\right)^{-1} + \left(\frac{1}{3\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}}\right)^{-1}\right) = 4.00\ \mu\text{F}$

$\Rightarrow Q_{total} = C_{\text{eq}}V = (4.00\ \mu\text{F})\ (210\ \text{V}) = 8.4 \times 10^{-4}\ \text{C}$. By symmetry, each capacitor carries 4.20×10^{-4} C. The voltages are then just calculated via $V=Q/C$. So: $V_{ad} = Q/C_3 = 140$ V, and $V_{ac} = Q/C_6 = 70\ \text{V} \Rightarrow V_{cd} = V_{ad} - V_{ac} = 70$ V.

b) When the switch is closed, the points c and d must be at the same potential, so the equivalent capacitance is:

$$C_{\text{eq}} = \left(\frac{1}{(3+6)\ \mu\text{F}} + \frac{1}{(3+6)\ \mu\text{F}}\right)^{-1} = 4.5\ \mu\text{F}.$$

$\Rightarrow Q_{total} = C_{\text{eq}}V = (4.50\ \mu\text{F})\ (210\ \text{V}) = 9.5 \times 10^{-4}$ C, and each capacitor has the same potential difference of 105 V (again, by symmetry)

c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the charge in $Q_2 - Q_1 = 0$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315\ \mu\text{C}$; $315\ \mu\text{C}$ of charge flowed through the switch.

24.61: a) $C_{\text{eq}} = \left(\frac{1}{8.4\ \mu\text{F}} + \frac{1}{8.4\ \mu\text{F}} + \frac{1}{4.2\ \mu\text{F}}\right)^{-1} = 2.1\ \mu\text{F}$

$\Rightarrow Q = C_{\text{eq}}V = (2.1\ \mu\ \text{F})\ (36\ \text{V}) = 7.50 \times 10^{-5}\ \text{C}.$

b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(2.1\ \mu\text{F})\ (36\ \text{V})^2 = 1.36 \times 10^{-3}\ \text{J}.$

c) If the capacitors are all in parallel, then:

$C_{\text{eq}} = (8.4\ \mu\text{F} + 8.4\ \mu\text{F} + 4.2\ \mu\text{F}) = 21\ \mu\text{F}$ and $Q = 3(7.56 \times 10^{-5}\text{C}) = 2.27 \times 10^{-4}$ C,

and $V = Q/C = (2.27 \times 10^{-4}\ \text{C})/(21\ \mu\text{F}) = 10.8$ V.

d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(21\ \mu\text{F})\ (10.8\ \text{V})^2 = 1.22 \times 10^{-3}\ \text{J}.$

24.62: a) $C_{\text{eq}} = \left(\dfrac{1}{4.0\ \mu\text{F}} + \dfrac{1}{6.0\ \mu\text{F}}\right)^{-1} = 2.4 \times 10^{-6}\ \text{F}$

$$\Rightarrow Q = C_{\text{eq}}V = (2.4 \times 10^{-6}\ \text{F})\ (600\ \text{V}) = 1.58 \times 10^{-3}\text{C}$$

and $V_2 = Q/C_2 = (1.58 \times 10^{-3}\ \text{C})/(4.0\ \mu\text{F}) = 395\ \text{V} \Rightarrow V_3 = 660\ \text{V} - 395\ \text{V} = 265\ \text{V}.$

b) Disconnecting them from the voltage source and reconnecting them to themselves we must have equal potential difference, and the sum of their charges must be the sum of the original charges:

$$Q_1 = C_1V \text{ and } Q_2 = C_2V \Rightarrow 2Q = Q_1 + Q_2 = (C_1 + C_2)\,V$$

$$\Rightarrow V = \frac{2Q}{C_1 + C_2} = \frac{2(1.58 \times 10^{-3}\ \text{C})}{10.0 \times 10^{-6}\ \text{F}} = 316\ \text{V}.$$

$$\Rightarrow Q_1 = (4.00 \times 10^{-6}\ \text{F})(316\ \text{V}) = 1.26 \times 10^{-3}\ \text{C}.$$

$$\Rightarrow Q_2 = (6.00 \times 10^{-6}\ \text{F})(316\ \text{V}) = 1.90 \times 10^{-3}\ \text{C}.$$

24.63: a) Reducing the furthest right leg yields $C = \left(\frac{1}{6.9\ \mu\text{F}} + \frac{1}{6.9\ \mu\text{F}} + \frac{1}{6.9\ \mu\text{F}}\right)^{-1} =$ $2.3\ \mu\text{F} = C_1/3.$ It combines in parallel with a $C_2 \Rightarrow C = 4.6\ \mu\text{F} + 2.3\ \mu\text{F} = 6.9\ \mu\text{F} = C_1.$ So the next reduction is the same as the first: $C = 2.3\ \mu\text{F} = C_1/3.$ And the next is the same as the second, leaving $3C_1$'s in series so $C_{\text{eq}} = 2.3\ \mu\text{F} = C_1/3.$

b) For the three capacitors nearest points a and b:

$$Q_{C_1} = C_{\text{eq}}V = (2.3 \times 10^{-6}\ \text{F})(420\ \text{V}) = 9.7 \times 10^{-4}\ \text{C}$$

and $Q_{C_2} = C_2V_2 = (4.6 \times 10^{-6}\ \text{F})(420\ \text{V})/3 = 6.44 \times 10^{-4}\ \text{C}.$

c) $V_{cd} = \frac{1}{3}\left(\frac{420}{3}\ \text{V}\right) = 46.7\ \text{V},$ since by symmetry the total voltage drop over the equivalent capacitance of the part of the circuit from the junctions between a, c and d, b is $\frac{420}{3}\ \text{V},$ and the equivalent capacitance is that of three equal capacitors C_1 in series. V_{cd} is the voltage over just one of those capacitors, i.e., $1/3$ of $\frac{420}{3}$ V.

24.64: (a) $C_{\text{equiv}} = C_1 + C_2 + C_3 = 60\ \mu\text{F}$

$$Q = CV = (60\ \mu\text{F})\ (120\ \text{V}) = 7200\ \mu\text{C}$$

(b) $\dfrac{1}{C_{\text{equiv}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

$$C_{\text{equiv}} = 5.45\ \mu\text{F}$$

$$Q = CV = (5.45\ \mu\text{F})(120\ \text{V}) = 654\ \mu\text{C}$$

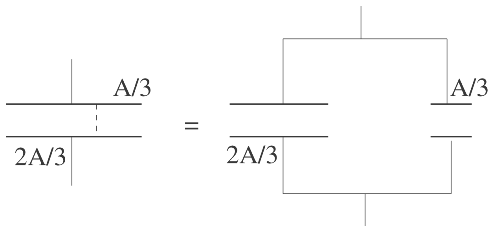
24.65: a) Q is constant.

with the dielectric: $V = Q/C = Q/(KC_0)$

without the dielectric: $V_0 = Q/C_0$

$V_0/V = K$, so $K = (45.0\text{ V})/(11.5\text{ V}) = 3.91$

b)



Let $C_0 = \varepsilon_0 A/d$ be the capacitance with only air between the plates. With the dielectric filling one-third of the space between the plates, the capacitor is equivalent to C_1 and C_2 in parallel, where C_1 has $A_1 = A/3$ and C_2 has $A_2 = 2A/3$

$$C_1 = KC_0/3,\ C_2 = 2C_0/3;\ C_{\text{eq}} = C_1 + C_2 = (C_0/3)\,(K+2)$$

$$V = \frac{Q}{C_{\text{eq}}} = \frac{Q}{C_0}\left(\frac{3}{K+2}\right) = V_0\left(\frac{3}{K+2}\right) = (45.0\text{ V})\left(\frac{3}{5.91}\right) = 22.8\text{ V}$$

24.66: a) This situation is analagous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d-a)$. Therefore $C = \left(\frac{1}{C_1} + \frac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\frac{\varepsilon_0 A}{(d-a)/2} = \frac{\varepsilon_0 A}{d-a}$.

b) $C = \dfrac{\varepsilon_0 A}{d-a} = \dfrac{\varepsilon_0 A}{d}\dfrac{d}{d-a} = C_0\dfrac{d}{d-a}$.

c) As $a \to 0, C \to C_0$. And as $a \to d, C \to \infty$.

24.67: a) One can think of "infinity" as a giant conductor with $V = 0$.

b) $C = \frac{Q}{V} = \frac{Q}{(Q/4\pi\varepsilon_0 R)} = 4\pi\varepsilon_0 R,$ where we've chosen $V = 0$ at infinity.

c) $C_{earth} = 4\pi\varepsilon_0 R_{earth} = 4\pi\varepsilon_0(6.4\times10^6\text{ m}) = 7.1\times10^{-4}\text{ F}.$ Larger than, but comparable to the capacitance of a typical capacitor in a circuit.

24.68: a) $r < R: u = \dfrac{1}{2}\varepsilon_0 E^2 = 0.$

b) $r > R: u = \frac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{Q}{4\pi\varepsilon_0 r^2}\right)^2 = \dfrac{Q^2}{32\pi^2\varepsilon_0 r^4}.$

c) $U = \int u dV = 4\pi\int_R^\infty r^2 u dr = \dfrac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \dfrac{dr}{r^2} = \dfrac{Q^2}{8\pi\varepsilon_0 R}.$

d) This energy is equal to $\frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R}$ which is just the energy required to assemble all the charge into a spherical distribution. (Note, being aware of double counting gives the factor of $1/2$ in front of the familiar potential energy formula for a charge Q a distance R from another charge Q.)

e) From Equation (24.9): $U = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\varepsilon_0 R}$ from part (c) $\Rightarrow C = 4\pi\varepsilon_0 R,$ as in Problem (24.67).

24.69: a) $r < R: u = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{kQr}{R^3}\right)^2 = \dfrac{kQ^2 r^2}{8\pi R^6}.$

b) $r > R: u = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{kQ}{r^2}\right)^2 = \dfrac{kQ^2}{8\pi r^4}.$

c) $r < R: U = \int u dV = 4\pi\int_0^R r^2 u dr = \dfrac{kQ^2}{2R^6}\int_0^R r^4 dr = \dfrac{kQ^2}{10R}.$

$r > R: U = \int u dV = 4\pi\int_R^\infty r^2 u dr = \dfrac{kQ^2}{2}\int_R^\infty \dfrac{dr}{r^2} = \dfrac{kQ^2}{2R} \Rightarrow U = \dfrac{3kQ^2}{5R}.$

24.70: a) $u = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{\lambda}{2\pi\varepsilon_0 r}\right)^2 = \dfrac{\lambda^2}{8\pi^2\varepsilon_0 r^2}.$

b) $U = \int u dV = 2\pi L\int u r dr = \dfrac{L\lambda^2}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\dfrac{dr}{r} \Rightarrow \dfrac{U}{L} = \dfrac{\lambda^2}{4\pi\varepsilon_0}\ln(r_b/r_a).$

c) Using Equation (24.9):

$$U = \frac{Q^2}{2C} = \frac{Q^2}{4\pi\varepsilon_0 L}\ln(r_b/r_a) = \frac{\lambda^2 L}{4\pi\varepsilon_0}\ln(r_b/r_a) = U \text{ of part (b).}$$

24.71: $C_{\text{eq}} = \left(\left(\dfrac{\varepsilon_1 A}{d/2} \right)^{-1} + \left(\dfrac{\varepsilon_2 A}{d/2} \right)^{-1} \right)^{-1} = \left(\left(\dfrac{d}{2\varepsilon_1 A} \right) + \left(\dfrac{d}{2\varepsilon_2 A} \right) \right)^{-1} = \left(\dfrac{d}{2\varepsilon_0 A} \left(\dfrac{1}{K_1} + \dfrac{1}{K_2} \right) \right)^{-1}$

$$\Rightarrow C_{\text{eq}} = \frac{2\varepsilon_0 A}{d} \left(\frac{K_1 K_2}{K_1 + K_2} \right).$$

24.72: This situation is analagous to having two capacitors in parallel, each with an area $\frac{A}{2}$. So:

$$C_{\text{eq}} = C_1 + C_2 = \frac{\varepsilon_1 A/2}{d} + \frac{\varepsilon_2 A/2}{d} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2).$$

24.73: a) $E = \dfrac{\sigma}{K\varepsilon_0} = \dfrac{0.50 \times 10^{-3}\ \text{C/m}^2}{(5.4)\varepsilon_0} = 1.0 \times 10^7\ \text{V/m}.$

b) $V = Ed = (1.0 \times 10^7\ \text{V/m})\,(5.0 \times 10^{-9}\ \text{m}) = 0.052\ \text{V}.$ The outside is at the higher potential.

c) volume $= 10^{-16}\ \text{m}^3 \Rightarrow R \approx 2.88 \times 10^{-6}\ \text{m}$

$\Rightarrow$ shell volume $= 4\pi R^2 d = 4\pi(2.88 \times 10^{-6}\ \text{m})^2 (5.0 \times 10^{-9}\text{m}) = 5.2 \times 10^{-19}\ \text{m}^3$

$\Rightarrow U = uV = (\frac{1}{2} K\varepsilon_0 E^2)\text{V} = \frac{1}{2}(5.4)\varepsilon_0 (1.0 \times 10^7\ \text{V/m})^2 (5.2 \times 10^{-19}\ \text{m}^3) = 1.36 \times 10^{-15}\ \text{J}.$

24.74: a) $Q = CV = \dfrac{K\varepsilon_0 A}{d} V = \dfrac{(2.50)\varepsilon_0 (0.200\ \text{m}^2)\,(3000\ \text{V})}{1.00 \times 10^{-2}\ \text{m}} = 1.33 \times 10^{-6}\ \text{C}.$

b) $Q_i = Q(1 - 1/K) = (1.33 \times 10^{-6}\ \text{C})\,(1 - 1/2.50) = 7.98 \times 10^{-7}\ \text{C}.$

c) $E = \dfrac{\sigma}{\varepsilon} = \dfrac{Q}{K\varepsilon_0 A} = \dfrac{1.33 \times 10^{-6}\ \text{C}}{(2.50)\,\varepsilon_0\,(0.200\ \text{m}^2)} = 3.01 \times 10^5\ \text{V/m}.$

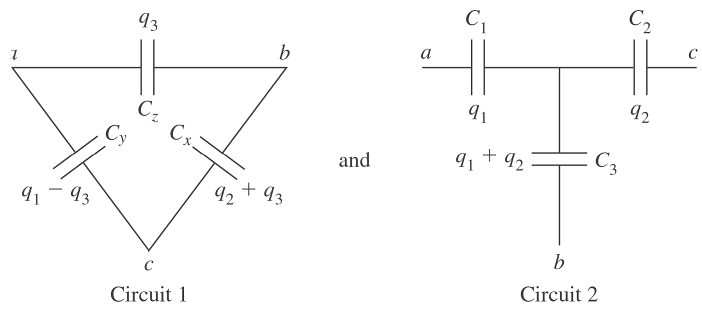
d) $U = \dfrac{1}{2} QV = \dfrac{1}{2}(1.33 \times 10^{-6}\ \text{C})\,(3000\ \text{V}) = 2.00 \times 10^{-3}\ \text{J}.$

e) $u = \dfrac{U}{Ad} = \dfrac{2.00 \times 10^{-3}\ \text{J}}{(0.200\ \text{m}^2)\,(0.0100\ \text{m})} = 1.00\ \text{J/m}^3 \Rightarrow$ or

$u = \frac{1}{2} K\varepsilon_0 E^2 = \frac{1}{2}(2.50)\varepsilon_0 (3.01 \times 10^5\ \text{V/m})^2 = 1.00\ \text{J/m}^3.$

f) In this case, one does work by pushing the slab into the capacitor since the constant potential requires more charges to be brought onto the plates. When the charge is kept constant, the field pulls the dielectric into the gap, with the field (or charges) doing the work.

24.75: a) We are to show the transformation from one circuit to the other:



From Circuit 1: $V_{ac} = \dfrac{q_1 - q_3}{C_y}$ and $V_{bc} = \dfrac{q_2 + q_3}{C_x}$, where q_3 is derived from V_{ab} :

$$V_{ab} = \frac{q_3}{C_z} = \frac{q_1 - q_3}{C_y} - \frac{q_2 - q_3}{C_x} \Rightarrow q_3 = \frac{C_x C_y C_z}{C_x + C_y + C_z}\left(\frac{q_1}{C_y} - \frac{q_2}{C_x}\right) \equiv K\left(\frac{q_1}{C_y} - \frac{q_2}{C_x}\right)$$

From Circuit 2: $V_{ac} = \dfrac{q_1}{C_1} + \dfrac{q_1 + q_2}{C_3} = q_1\left(\dfrac{1}{C_1} + \dfrac{1}{C_3}\right) + q_2\dfrac{1}{C_3}$ and

$$V_{bc} = \frac{q_2}{C_2} + \frac{q_1 + q_2}{C_3} = q_1\frac{1}{C_3} + q_2\left(\frac{1}{C_2} + \frac{1}{C_3}\right).$$

Setting the coefficients of the charges equal to each other in matching potential equations from the two circuits results in three independent equations relating the two sets of capacitances. The set of equations are:

$$\frac{1}{C_1} = \frac{1}{C_y}\left(1 - \frac{1}{KC_y} - \frac{1}{KC_x}\right), \frac{1}{C_2} = \frac{1}{C_x}\left(1 - \frac{1}{KC_y} - \frac{1}{KC_x}\right) \text{ and } \frac{1}{C_3} = \frac{1}{KC_y C_x}.$$

From these, subbing in the expression for K, we get:

$$C_1 = (C_x C_y + C_y C_z + C_z C_x)/C_x.$$

$$C_2 = (C_x C_y + C_y C_z + C_z C_x)/C_y.$$

$$C_3 = (C_x C_y + C_y C_z + C_z C_x)/C_z.$$

24.76: a) The force between the two parallel plates is:

$$F = qE = \frac{q\sigma}{2\varepsilon_0} = \frac{q^2}{2\varepsilon_0 A} = \frac{(CV)^2}{2\varepsilon_0 A} = \frac{\varepsilon_0{}^2 A^2}{z^2}\frac{V^2}{2\varepsilon_0 A} = \frac{\varepsilon_0 AV^2}{2z^2}.$$

b) When $V = 0,$ the separation is just z_0. So:

$$F_{4\,springs} = 4k(z_0 - z) = \frac{\varepsilon_0 AV^2}{2z^2} \Rightarrow 2z^3 - 2z^2 z_0 + \frac{\varepsilon_0 AV^2}{4k} = 0.$$

c) For $A = 0.300\ \text{m}^2,\ z_0 = 1.2\times10^{-3}\ \text{m},\ k = 25\ \text{N/m, and}\ V = 120\ \text{V},$

$2z^3 - (2.4\times10^{-3}\ \text{m})z^2 + 3.82\times10^{-10}\ \text{m}^3 = 0 \Rightarrow z = 0.537\ \text{mm}, 1.014\ \text{mm}.$

d) Stable equilibrium occurs if a slight displacement from equilibrium yields a force back toward the equilibrium point. If one evaluates the forces at small displacements from the equilibrium positions above, the 1.014 mm separation is seen to be stable, but not the 0.537 mm separation.

24.77: a) $C_0 = \frac{\varepsilon_0}{D}((L - x)\,L + xKL) = \frac{\varepsilon_0 L}{D}(L + (K-1)x).$

b) $\Delta U = \frac{1}{2}(\Delta C)V^2$ where $C = C_0 + \frac{\varepsilon_0 L}{D}(-dx + dxK)$

$$\Rightarrow \Delta U = \frac{1}{2}\left(\frac{\varepsilon_0 L\,dx}{D}(K-1)\right)V^2 = \frac{(K-1)\varepsilon_0 V^2 L}{2D}dx.$$

c) If the charge is kept constant on the plates, then:

$$Q = \frac{\varepsilon_0 LV}{D}(L + (K-1)x),\ \text{and}\ U = \frac{1}{2}CV^2 = \frac{1}{2}C_0 V^2\left(\frac{C}{C_0}\right)$$

$$\Rightarrow U \approx \frac{C_0 V^2}{2}\left(1 - \frac{\varepsilon_0 L}{DC_0}(K-1)dx\right) \Rightarrow \Delta U = U - U_0 = -\frac{(K-1)\varepsilon_0 V^2 L}{2D}dx.$$

d) Since $dU = -Fdx = -\frac{(K-1)\varepsilon_0 V^2 L}{2D}dx,$ then the force is in the opposite direction to the motion $dx,$ meaning that the slab feels a force pushing it out.

24.78: a) For a normal spherical capacitor: $C_0 = 4\pi\varepsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right)$. Here we have, in effect, two parallel capacitors, C_L and C_U.

$$C_L = \frac{KC_0}{2} = 2\pi K\varepsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right) \text{and } C_U = \frac{C_0}{2} = 2\pi\varepsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right).$$

b) Using a hemispherical Gaussian surface for each respective half:

$$E_L\frac{4\pi r^2}{2} = \frac{Q_L}{K\varepsilon_0} \Rightarrow E_L = \frac{Q_L}{2\pi K\varepsilon_0 r^2} \text{ and } E_U\frac{4\pi r^2}{2} = \frac{Q_U}{\varepsilon_0} \Rightarrow E_U = \frac{Q_U}{2\pi\varepsilon_0 r^2}.$$

But $Q_L = VC_L$ and $Q_U = VC_U$, $Q_L + Q_U = Q$.

So: $Q_L = \dfrac{VC_0 K}{2} = KQ_U \Rightarrow Q_U(1+K) = Q \Rightarrow Q_U = \dfrac{Q}{1+K}$ and $Q_L = \dfrac{KQ}{1+K}$.

$$\Rightarrow E_L = \frac{KQ}{1+K}\frac{1}{2\pi K\varepsilon_0 r^2} = \frac{2}{1+K}\frac{Q}{4\pi\varepsilon_0 r^2} \text{ and } E_U = \frac{Q}{1+K}\frac{1}{2\pi K\varepsilon_0 r^2} = \frac{2}{1+K}\frac{Q}{4\pi K\varepsilon_0 r^2}.$$

c) The free charge density on upper and lower hemispheres are:

$$(\sigma_{f_{r_a}})_U = \frac{Q_U}{4\pi r_a^2} = \frac{Q}{4\pi r_a^2(1+K)} \text{ and } (\sigma_{f_{r_a}})_U = \frac{Q_u}{4\pi r_b^2} = \frac{Q}{4\pi r_b^2(1+K)}.$$

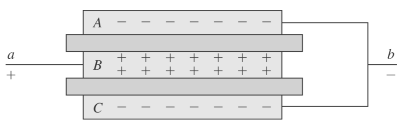
$$(\sigma_{f_{r_s}})_L = \frac{Q_L}{4\pi r_a^2} = \frac{KQ}{4\pi r_a^2(1+K)} \text{ and } (\sigma_{f_{r_a}})_L = \frac{Q_L}{4\pi r_b^2} = \frac{KQ}{4\pi r_b^2(1+K)}.$$

d) $$\sigma_{i_{r_a}} = \sigma_{f_{r_a}}(1 - 1/K) = \frac{(K-1)}{K}\frac{Q}{4\pi r_a^2}\frac{K}{K+1} = \frac{K-1}{K+1}\frac{Q}{4\pi r_a^2}.$$

$$\sigma_{i_{r_b}} = \sigma_{f_{r_b}}(1 - 1/K) = \frac{(K-1)}{K}\frac{Q}{4\pi r_a^2}\frac{K}{K+1} = \frac{K-1}{K+1}\frac{Q}{4\pi r_b^2}.$$

e) There is zero bound charge on the flat surface of the dielectric-air interface, or else that would imply a circumferential electric field, or that the electric field changed as we went around the sphere.

24.79: a)



b) $$C = 2\left(\frac{\varepsilon A}{d}\right) = \frac{2(4.2)\varepsilon_0(0.120\text{ m})^2}{4.5\times10^{-4}\text{ m}} \Rightarrow C = 2.38\times10^{-9}\text{ F}.$$

24.80: a) The capacitors are in parallel so:

$$C=\frac{\varepsilon_{eff}WL}{d}=\frac{\varepsilon_0 W(L-h)}{d}+\frac{K\varepsilon_0 Wh}{d}=\frac{\varepsilon_0 WL}{d}\left(1+\frac{Kh}{L}-\frac{h}{L}\right)\Rightarrow K_{eff}$$

$$=\left(1+\frac{Kh}{L}-\frac{h}{L}\right).$$

b) For gasoline, with $K=1.95$:

$\frac{1}{4}$ full: $K_{eff}\left(h=\frac{L}{4}\right)=1.24;$ $\frac{1}{2}$ full: $K_{eff}\left(h=\frac{L}{2}\right)=1.48;$

$\frac{3}{4}$ full: $K_{eff}\left(h=\frac{3L}{4}\right)=1.71.$

c) For methanol, with $K=33$:

$\frac{1}{4}$ full: $K_{eff}\left(h=\frac{L}{4}\right)=9;$ $\frac{1}{2}$ full: $K_{eff}\left(h=\frac{L}{2}\right)=17;$

$\frac{3}{4}$ full: $K_{eff}\left(h=\frac{3L}{4}\right)=25.$

d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

25.1: $Q = It = (3.6\text{ A})(3)(3600\text{ s}) = 3.89 \times 10^{4}\text{ C}.$

25.2: a) Current is given by $I = \frac{Q}{t} = \frac{420\text{ C}}{80(60\text{ s})} = 8.75 \times 10^{-2}\text{ A}.$

b) $I = nqv_d A$

$$\Rightarrow v_d = \frac{I}{nqA} = \frac{8.75 \times 10^{-2}\text{ A}}{(5.8 \times 10^{28})(1.6 \times 10^{-19}\text{ C})(\pi(1.3 \times 10^{-3}\text{ m})^2)}$$

$$= 1.78 \times 10^{-6}\text{ m/s}.$$

25.3: a) $v_d = \dfrac{I}{nqA} = \dfrac{4.85\text{ A}}{(8.5 \times 10^{28})(1.6 \times 10^{-19}\text{ C})(\pi/4)(2.05 \times 10^{-3}\text{ m})^2)}$

$$= 1.08 \times 10^{-4}\text{ m/s}$$

$\Rightarrow$ travel time $= \frac{d}{v_d} = \frac{0.71\text{ m}}{1.08 \times 10^{-4}\text{ m/s}} = 6574\text{ s} = 110\text{ min}$

b) If the diameter is now 4.12 mm, the time can be calculated using the formula above or comparing the ratio of the areas, and yields a time of 26542 s =442 min.

c) The drift velocity depends on the diameter of the wire as an inverse square relationship.

25.4: The cross-sectional area of the wire is

$$A = \pi r^2 = \pi(2.06 \times 10^{-3}\text{ m})^2 = 1.333 \times 10^{-5}\text{ m}^2.$$

The current density is

$$J = \frac{I}{A} = \frac{8.00\text{A}}{1.333 \times 10^{-5}\text{ m}^2} = 6.00 \times 10^{5}\text{ A/m}^2$$

We have $v_d = J/ne$; Therefore

$$n = \frac{J}{v_d e} = \frac{6.00 \times 10^{5}\text{ A/m}^2}{(5.40 \times 10^{-5}\text{ m/s})(1.60 \times 10^{-19}\text{ C/electron})} = 6.94 \times 10^{28}\ \frac{\text{electrons}}{\text{m}^3}$$

25.5: $J = n|q|v_{\text{d}}$, so J/v_{d} is constant.

$J_1/v_{d1} = J_2/v_{d2}$,

$$v_{d2} = v_{d1}(J_2/J_1) = v_{d1}(I_2/I_1) = (1.20 \times 10^{-4}\text{ m/s})(6.00/1.20) = 6.00 \times 10^{-4}\text{ m/s}$$

25.6: The atomic weight of copper is $63.55\ \text{g/mole}$, and its density is $8.96\ \text{g/cm}^3$. The number of copper atoms in $1.00\ \text{m}^3$ is thus

$$\frac{(8.96\ \text{g/cm}^3)(1.00\times10^6\ \text{cm}^3/\text{m}^3)(6.023\times10^{23}\ \text{atoms/mole})}{63.55\ \text{g/mole}}$$

$$= 8.49\times10^{28}\ \text{atoms/m}^3$$

Since there are the same number of free electrons$/\text{m}^3$ as there are atoms of copper$/\text{m}^3$ (see Ex. 25.1), The number of free electrons per copper atom is one.

25.7: Consider $1\ \text{m}^3$ of silver.

density $= 10.5\times10^3\ \text{kg/m}^3$, so $m = 10.5\times10^3\ \text{kg}$

$M = 107.868\times10^{-3}\ \text{kg/mol}$, so $n = m/M = 9.734\times10^4$ mol and

$N = nN_A = 5.86\times10^{28}\ \text{atoms/m}^3$

If there is one free electron per m^3, there are 5.86×10^{28} free electrons$/\text{m}^3$. This agrees with the value given in Exercise 25.2.

25.8: a) $Q_{total} = (n_{\text{Cl}} + n_{\text{Na}})e = (3.92\times10^{16} + 2.68\times10^{16})(1.60\times10^{-19}\ \text{C}) = 0.0106\ \text{C}$

$$\Rightarrow I = \frac{Q_{total}}{t} = \frac{0.0106\ \text{C}}{1.00\ \text{s}} = 0.0106\ \text{A} = 10.6\ \text{mA}.$$

b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

25.9: a) $Q = \int_0^8 I\,dt = \int_0^8 (55 - 0.65t^2)\,dt = 55t\Big|_0^8 + \frac{0.65}{3}t^3\Big|_0^8 = 329\ \text{C}.$

b) The same charge would flow in 10 seconds if there was a constant current of: $I = Q/t = (329\ \text{C})/(8\ \text{s}) = 41.1\ \text{A}.$

25.10: a) $J = \frac{I}{A} = \frac{3.6\ \text{A}}{(2.3\times10^{-3}\ \text{m})^2} = 6.81\times10^5\ \text{A/m}^2.$

b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\text{m})(6.81\times10^5\ \text{A/m}^2) = 0.012\ \text{V/m}.$

c) Time to travel the wire's length:

$$t = \frac{l}{v_d} = \frac{l\,nqA}{I} = \frac{(4.0\ \text{m})(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\ \text{C})(2.3\times10^{-3}\ \text{m})^2}{3.6\ \text{A}} = 8.0\times10^4\ \text{s}$$

$= 1333\ \text{min} \approx 22\ \text{hrs!}$

25.11: $R = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(24.0\ \text{m})}{(\pi/4)(2.05\times10^{-3}\ \text{m})^2} = 0.125\ \Omega.$

25.12: $R = \dfrac{\rho L}{A} \Rightarrow L = \dfrac{RA}{\rho} = \dfrac{(1.00\ \Omega)(\pi/4)(0.462\times 10^{-3}\ \text{m})^2}{1.72\times 10^{-8}\ \Omega\cdot\text{m}} = 9.75\ \text{m}.$

25.13: a) tungsten:

$$E = \rho J = \frac{\rho I}{A} = \frac{(5.25\times 10^{-8}\ \Omega/\text{m}^3)(0.820\ \text{A})}{(\pi/4)(3.26\times 10^{-3}\ \text{m})^2} = 5.16\times 10^{-3}\ \text{V/m}.$$

b) aluminum:

$$E = \rho J = \frac{\rho I}{A} = \frac{(2.75\times 10^{-8}\ \Omega/\text{m}^3)(0.820\ \text{A})}{(\pi/4)(3.26\times 10^{-3}\ \text{m})^2} = 2.70\times 10^{-3}\ \text{V/m}.$$

25.14: $R_{Al} = R_{Cu} \Rightarrow \dfrac{\rho_{Al} L}{A_{Al}} = \dfrac{\rho_{Cu} L}{A_{Cu}} \Rightarrow \dfrac{\pi d_{Al}{}^2}{4\rho_{Al}} = \dfrac{\pi d_{Cu}{}^2}{4\rho_{Cu}} \Rightarrow d_{Cu} = d_{Al}\sqrt{\dfrac{\rho_{Cu}}{\rho_{Al}}}$

$$\Rightarrow d_{Al} = (3.26\ \text{mm})\sqrt{\frac{1.72\times 10^{-8}\ \Omega\cdot\text{m}}{2.75\times 10^{-8}\ \Omega\cdot\text{m}}} = 2.6\ \text{mm}.$$

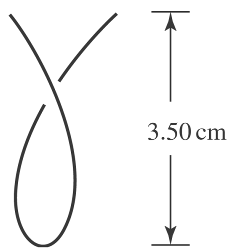
25.15: Find the volume of one of the wires:

$R = \dfrac{\rho L}{A}$ so $A = \dfrac{\rho L}{R}$ and

$\text{volume} = AL = \dfrac{\rho L^2}{R} = \dfrac{1.72\times 10^{-8}\ \text{Ohm}\cdot\text{m})(3.50\text{m})^2}{0.125\text{Ohm}} = 1.686\times 10^{-6}\text{mcb}$

$m = (\text{density})V = (8.9\times 10^3\ \text{kg/m}^3)(1.686\times 10^{-6}\ \text{m}^3) = 15\ \text{g}$

25.16:



$r_1 = \dfrac{3.5\ \text{cm}}{2} = 1.75\ \text{cm}$

$r_2 = \dfrac{3.25\ \text{mm}}{2} = 1.625\ \text{mm}$

25.17: a) From Example 25.1, an 18-gauge wire has $A = 8.17 \times 10^{-3}\ \text{cm}^2$

$$I = JA = (1.0 \times 10^5\ \text{A/cm}^2)(8.17 \times 10^{-3}\ \text{cm}^2) = 820\ \text{A}$$

b) $A = I/J = (1000\ \text{A})/(1.0 \times 10^6\ \text{A/cm}^2) = 1.0 \times 10^{-3}\ \text{cm}^2$

$$A = \pi r^2 \text{ so } r = \sqrt{A/\pi} = \sqrt{(1.0 \times 10^{-3}\ \text{cm}^2/\pi} = 0.0178\ \text{cm}$$

$$d = 2r = 0.36\ \text{mm}$$

25.18: Assuming linear variation of the resistivity with temperature:

$$\begin{aligned}\rho &= \rho_0[1 + \alpha(T - T_0)] \\ &= \rho_0[1 + (4.5 \times 10^{-3}/°\text{C})(320 - 20)°\text{C}] \\ &= 2.35\rho_0\end{aligned}$$

Since $\rho = E/J$, the electric field required to maintain a given current density is proportional to the resistivity. Thus $E = (2.35)(0.0560\ \text{V/m}) = 0.132\ \text{V/m}$

25.19: $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{L^2} = \dfrac{\rho}{L} = \dfrac{2.75 \times 10^{-8}\,\Omega \cdot \text{m}}{1.80\text{m}} = 1.53 \times 10^{-8}\,\Omega$

25.20: The ratio of the current at $20°\text{C}$ to that at the higher temperature is $(0.860\ \text{A})/(0.220\ \text{A}) = 3.909.$ Since the current density for a given field is inversely proportional to $\rho(\rho = E/J)$, The resistivity must be a factor of 3.909 higher at the higher temperature.

$$\frac{\rho}{\rho_0} = 1 + \alpha(T - T_0)$$

$$T = T_0 + \frac{\frac{\rho}{\rho_0} - 1}{\alpha} = 20°\text{C} + \frac{3.909 - 1}{4.5 \times 10^{-3}/°\text{C}} = 666°\text{C}$$

25.21: $R = \dfrac{V}{I} = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi r^2} \Rightarrow r = \sqrt{\dfrac{I\rho L}{\pi V}} = \sqrt{\dfrac{(6.00\ \text{A})(2.75 \times 10^{-8}\ \Omega \cdot \text{m})(1.20\ \text{m})}{\pi(1.50\ \text{V})}}$

$$= 2.05 \times 10^{-4}\ \text{m}.$$

25.22: $\rho = \dfrac{RA}{L} = \dfrac{VA}{IL} = \dfrac{(4.50\ \text{V})\pi(6.54 \times 10^{-4}\ \text{m})^2}{(17.6\ \text{A})(2.50\ \text{m})} = 1.37 \times 10^{-7}\ \Omega \cdot \text{m}.$

25.23: a) $I = JA = \dfrac{EA}{\rho} = \dfrac{(0.49\ \text{V/m})(\pi/4(0.84\times10^{-3}\ \text{m})^2)}{(2.44\times10^{-8}\ \Omega\cdot\text{m})} = 11.1\ \text{A}.$

b) $V = IR = \dfrac{I\rho L}{A} = \dfrac{(11.1\ \text{A})(2.44\times10^{-8}\ \Omega\cdot\text{m})(6.4\ \text{m})}{(\pi/4)(0.84\times10^{-3}\ \text{m})^2} = 3.13\ \text{V}.$

c) $R = \dfrac{V}{I} = \dfrac{3.13\ \text{V}}{11.1\text{A}} = 0.28\ \Omega.$

25.24: Because the density does not change, volume stays the same, so $LA = (2L)(A/2)$ and the area is halved. So the resistance becomes:

$$R = \frac{\rho(2L)}{A/2} = 4\frac{\rho L}{A} = 4R_0.$$

That is, four times the original resistance.

25.25: a) $E = \rho J = \dfrac{RAJ}{L} = \dfrac{RI}{L} = \dfrac{V}{L} = \dfrac{0.938\ \text{V}}{0.75\ \text{m}} = 1.25\ \text{V/m}.$

b) $\rho = \dfrac{RA}{L} = \dfrac{V}{JL} = \dfrac{0.938\ \text{V}}{(4.40\times10^{7}\ \text{A/m}^2)(0.75\ \text{m})} = 2.84\times10^{-8}\ \Omega\cdot\text{m}.$

25.26: $\dfrac{R - R_0}{R_0} = \alpha(T_f - T_i)$

$$\Rightarrow \alpha = \frac{R - R_0}{(T_f - T_i)R_0} = \frac{1.512\ \Omega - 1.484\ \Omega}{(34.0^\circ\text{C} - 20.0^\circ\text{C})(1.484\ \Omega)} = 1.35\times10^{-3}\ ^\circ\text{C}^{-1}.$$

25.27: a) $R_f - R_i = R_i\alpha(T_f - T_i) \Rightarrow R_f = 100\ \Omega - 100\ \Omega(0.0004^\circ\text{C}^{-1})(11.5^\circ\text{C}) = 99.54\ \Omega.$

b) $R_f - R_i = R_i\alpha(T_f - T_i) \Rightarrow R_f = 0.0160\ \Omega + 0.0160\ \Omega(-0.0005^\circ\text{C}^{-1})(25.8^\circ\text{C}) =$ $0.0158\ \Omega.$

25.28: $T_f - T_i = \dfrac{R_f - R_i}{\alpha R_i};\quad T_f = T_i + \dfrac{R_f - R_i}{\alpha R_i}$

$$= \frac{215.8\ \Omega - 217.3\ \Omega}{(-0.0005^\circ\text{C}^{-1})(217.3\ \Omega)} + 4^\circ\text{C} = 17.8^\circ\text{C}.$$

25.29: a) If 120 strands of wire are placed side by side, we are effectively increasing the area of the current carrier by 120. So the resistance is smaller by that factor:

$$R = 5.60 \times 10^{-6}\ \Omega/120 = 4.67 \times 10^{-8}\ \Omega.$$

b) If 120 strands of wire are placed end to end, we are effectively increasing the length of the wire by 120, and so $R = (5.60 \times 10^{-6}\ \Omega)120 = 6.72 \times 10^{-4}\ \Omega.$

25.30: With the $4.0\ \Omega$ load, where r = internal resistance

$$12.6\ \text{V} = (r + 4.0\ \Omega)I$$

Change in terminal voltage:

$$\Delta V_T = rI = 12.6\ \text{V} - 10.4\ \text{V} = 2.2\ \text{V}$$

$$I = \frac{2.2\ \text{V}}{r}$$

Substitute for I: $\quad 12.6\ \text{V} = (r + 4.0\ \Omega)\left(\dfrac{2.2\ \text{V}}{r}\right)$

Solve for r: $\quad r = 0.846\ \Omega$

25.31: a) $R = \dfrac{\rho L}{A} = \dfrac{1.72 \times 10^{-8}\ \Omega\text{m})(100 \times 10^{3}\,\text{m})}{\pi(0.050\text{m})^2} = 0.219\Omega$

$V = IR = (125\text{A})(0.219\Omega) = 27.4\text{V}$

b) $P = VI = (27.4\ \text{V})(125\ \text{A}) = 3422\ \text{W} = 3422\ \text{J/s}$

Energy $= Pt = (3422\ \text{J/s})(3600\ \text{s}) = 1.23 \times 10^{7}\ \text{J}$

25.32: a) $V_r = \mathcal{E} - V_{ab} = 24.0\ \text{V} - 21.2\ \text{V} = 2.8\ \text{V} \Rightarrow r = 2.8\ \text{V}/4.00\ \text{A} = 0.700\ \Omega$.

b) $V_R = 21.2\ \text{V} \Rightarrow R = 21.2\ \text{V}/4.00\ \text{A} = 5.30\ \Omega.$

25.33: a) An ideal voltmeter has infinite resistance, so there would be NO current through the $2.0\ \Omega$ resistor.

b) $V_{ab} = \mathcal{E} = 5.0\ \text{V};$ since there is no current there is no voltage lost over the internal resistance.

c) The voltmeter reading is therefore 5.0 V since with no current flowing, it measures the terminal voltage of the battery.

25.34: a) A voltmeter placed over the battery terminals reads the emf: $\mathcal{E} = 24.0\ \text{V}$.

b) There is no current flowing, so $V_r = 0$.

c) The voltage reading over the switch is that over the battery: $V_s = 24.0\ \text{V}$.

d) Having closed the switch:

$I = 24.0\ \text{V}/5.88\ \Omega = 4.08\ \text{A} \Rightarrow V_{ab} = 24.0\ \text{V} - (4.08\ \text{A})(0.28\ \Omega) = 22.9\ \text{V}$.

$V_r = IR = (4.08\ \text{A})(5.60\ \Omega) = 22.9\ \text{V}$.

$V_s = 0$, since all the voltage has been "used up" in the circuit. The resistance of the switch is zero so $V_s = IR = 0$.

25.35: a) When there is no current flowing, the voltmeter reading is simply the emf of the battery: $\mathcal{E} = 3.08\ \text{V}$.

b) The voltage over the internal resistance is:

$$V_r = 3.08\ \text{V} - 2.97\ \text{V} = 0.11\ \text{V} \Rightarrow r = \frac{V}{I} = \frac{0.11\ \text{V}}{1.65\ \text{A}} = 0.067\ \Omega.$$

c) $V_R = 2.97\ \text{V} = (1.65\ \text{A})R$

$$R = \frac{2.97\ \text{V}}{1.65\ \text{A}} = 1.8\ \Omega$$

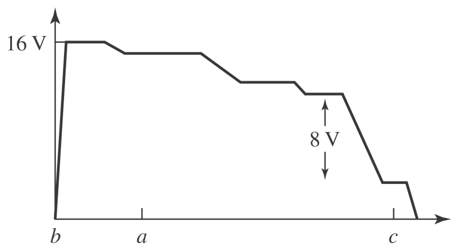
25.36: a) The current is counterclockwise, because the 16 V battery determines the direction of current flow. Its magnitude is given by:

$$I = \frac{\sum \mathcal{E}}{\sum R} = \frac{16.0\ \text{V} - 8.0\ \text{V}}{1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega} = 0.47\ \text{A}.$$

b) $V_{ab} = 16.0\ \text{V} - (1.6\ \Omega)(0.47\ \text{A}) = 15.2\ \text{V}$.

c) $V_{ac} = (5.0\ \Omega)(0.47\ \text{A}) + (1.4\ \Omega)(0.47\ \text{A}) + 8.0\ \text{V} = 11.0\ \text{V}$.

d)



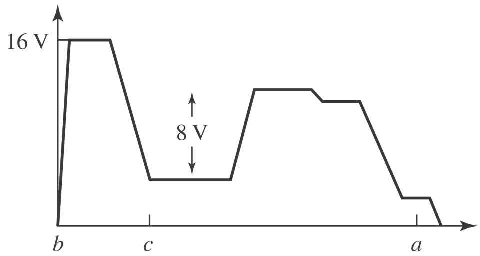
25.37: a) Now the current flows clockwise since both batteries point in that direction:

$$I = \frac{\sum \mathcal{E}}{\sum R} = \frac{16.0\ \text{V} + 8.0\ \text{V}}{1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega} = 1.41\ \text{A}.$$

b) $V_{ab} = -16.0\ \text{V} + (1.6\ \Omega)(1.41\ \text{A}) = -13.7\ \text{V}.$

c) $V_{ac} = -(5.0\ \Omega)(1.41\ \text{A}) - (1.4\ \Omega)(1.41\ \text{A}) + 8.0\ \text{V} = -1.0\ \text{V}.$

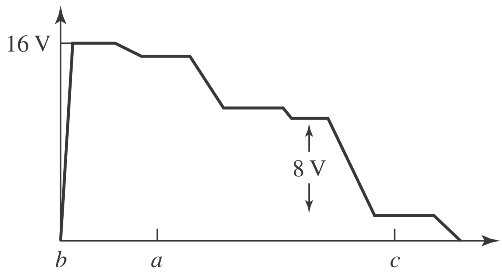
d)



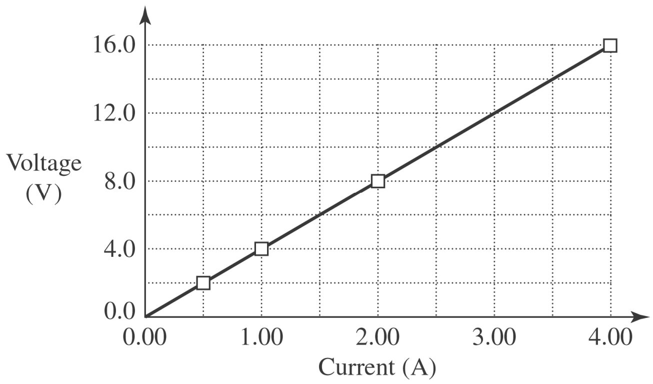
25.38: a) $V_{bc} = 1.9\ \text{V} \Rightarrow I = V_{bc}/R_{bc} = 1.9\ \text{V}/9.0\ \Omega = 0.21\ \text{A}.$

b) $\sum \mathcal{E} = \sum IR \Rightarrow 8.0\ \text{V} = ((1.6 + 9.0 + 1.4 + R)\Omega)(0.21\ \text{A}) \Rightarrow R = \dfrac{5.48}{0.21} = 26.1\ \Omega.$

c)



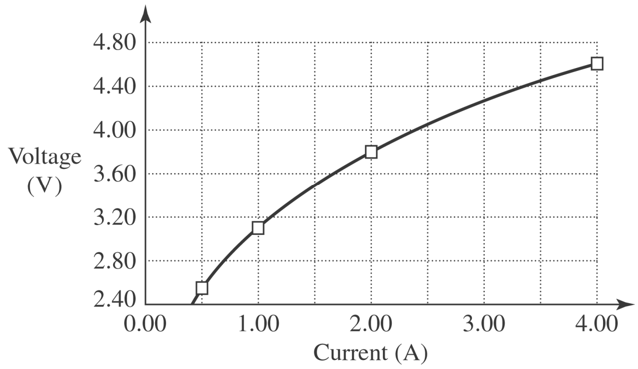
25.39: a) Nichrome wire:



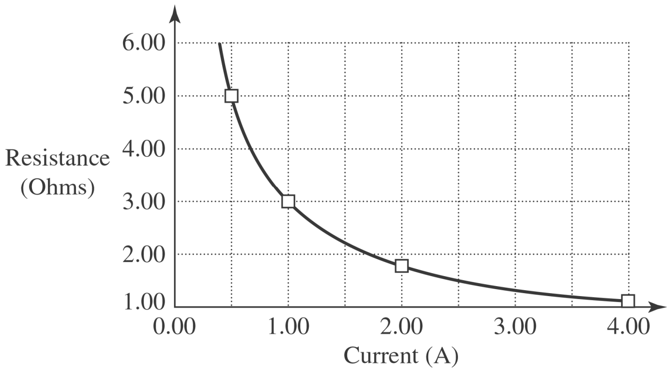
b) The Nichrome wire does obey Ohm's Law since it is a straight line.

c) The resistance is the voltage divided by current which is $3.88\ \Omega$.

25.40: a) Thyrite resistor:



b) The Thyrite is non-Ohmic since the plot is curved.
c) Calculating the resistance at each point by voltage divided by current:



25.41: a) $r = \mathcal{E}/I = 1.50\ \text{V}/14.8\ \text{A} = 0.101\,\Omega.$

b) $r = \mathcal{E}/I = 1.50\ \text{V}/6.8\ \text{A} = 0.22\ \Omega.$

c) $r = \mathcal{E}/I = 12.6\ \text{V}/1000\ \text{A} = 0.0126\,\Omega.$

25.42: a) $P = V^2/R \Rightarrow R = V^2/P = (15\ \text{V})^2/327\ \text{W} = 0.688\ \Omega.$

b) $V = IR \Rightarrow I = \dfrac{V}{R} = \dfrac{15\ \text{V}}{0.688\,\Omega} = 21.8\ \text{A}.$

25.43: $P = VI = (650\ \text{V})(0.80\ \text{A}) = 520\ \text{W}.$

25.44: $W = Pt = IVt = (0.13\ \text{A})(9\ \text{V})(1.5)(3600\,\text{s}) = 6318\ \text{J}.$

25.45: a) $P = I^2 R \Rightarrow p = \dfrac{P}{\text{vol}} = \dfrac{I^2 R}{AL} = \dfrac{J^2 A^2 R}{AL} = \dfrac{J^2 A(\rho L / A)}{L} = J^2 \rho \Rightarrow p = JE$ since $E = \rho J$.

b) From (a) $p = J^2 \rho.$

c) Since $J = E/\rho$, (a) becomes $p = E^2/\rho.$

25.46: a) $I = \sum \mathcal{E}/R_{total} = 8.0 \text{ V}/17\ \Omega = 0.47 \text{ A} \Rightarrow P_{5\Omega} = I^2 R = (0.47 \text{ A})^2 (5.0\ \Omega) =$ 1.1 W and $P_{9\Omega} = I^2 R = (0.47 \text{ A})^2 (9.0\ \Omega) = 2.0 \text{ W}.$

b) $P_{16V} = \mathcal{E}I - I^2 r = (16 \text{ V})(0.47 \text{ A}) - (0.47 \text{ A})^2 (1.6\ \Omega) = 7.2 \text{ W}.$

c) $P_{8V} = \mathcal{E}I + Ir^2 = (8.0 \text{ V})(0.47 \text{ A}) + (0.47 \text{ A})^2 (1.4\Omega) = 4.1 \text{ W}.$

d) (b) = (a) + (c)

25.47: a) $W = Pt = IVt = (60 \text{ A})(12 \text{ V})(3600 \text{ s}) = 2.59 \times 10^6 \text{ J}.$

b) To release this much energy we need a volume of gasoline given by:

$$m = \frac{2.59 \times 10^6 \text{ J}}{46{,}000 \text{ J/g}} = 56.0 \text{ g} \Rightarrow \text{vol} = \frac{m}{\rho} = \frac{0.056 \text{ kg}}{900 \text{ kg/m}^3} = 6.22 \times 10^{-5} \text{ m}^3 = 0.062 \text{ liters.}$$

c) To recharge the battery:

$$t = (Wh)/P = (720 \text{ Wh})/(450 \text{ W}) = 1.6 \text{ h}.$$

25.48: a) $I = \mathcal{E}/(R + r) = 12 \text{ V}/10\ \Omega = 1.2 \text{ A} \Rightarrow P = \mathcal{E}I = (12 \text{ V})(1.2 \text{ A}) = 14.4 \text{ W}.$ This is less than the previous value of 24 W.

b) The work dissipated in the battery is just: $P = I^2 r = (1.2\ A)^2 (2.0\ \Omega) = 2.9 \text{ W}.$ This is less than 8 W, the amount found in Example (25.9).

c) The net power output of the battery is $14.4 \text{ W} - 2.9 \text{ W} = 11.5 \text{ W}.$ This is less than 16 W, the amount found in Example (25.9).

25.49: a) $I = V/R = 12 \text{ V}/6\ \Omega = 2.0 \text{ A} \Rightarrow P = \mathcal{E}I = (12 \text{ V})\ (2.0 \text{ A}) = 24 \text{ W}.$

b) The power dissipated in the battery is $P = I^2 r = (2.0 \text{ A})^2 (1.0\ \Omega) = 4.0 \text{ W}.$

c) The power delivered is then $24 \text{ W} - 4 \text{ W} = 20 \text{ W}.$

25.50: a) $I = \sum \mathcal{E} / R = 3.0 \text{ V} / 17\ \Omega = 0.18 \text{ A} \Rightarrow P = I^2 R = 0.529 \text{ W}.$

b) $W = Pt = IVt = (0.18 \text{ A})(3.0 \text{ V})(5.0)(3600 \text{ s}) = 9530 \text{ J}.$

c) Now if the power to the bulb is 0.27 W,

$$P = I^2 R \Rightarrow 0.27 \text{ W} = \left(\frac{3.0 \text{ V}}{17\ \Omega + R}\right)^2 (17\ \Omega) \Rightarrow (17\ \Omega + R)^2 = 567\ \Omega^2 \Rightarrow R = 6.8\ \Omega.$$

25.51: a) $P = V^2/R \Rightarrow R = V^2/P = (120\text{ V})^2/540\text{ W} = 26.7\ \Omega.$

b) $I = V/R = 120\text{ V}/26.7\ \Omega = 4.5\text{ A}.$

c) If the voltage is just 110 V, then $I = 4.13\text{ A} \Rightarrow P = VI = 454\text{ W}.$

d) Greater. The resistance will be less so the current drawn will increase, increasing the power.

25.52: From Eq. (25.24), $\rho = \dfrac{m}{ne^2\tau}.$

$$\Rightarrow \tau = \frac{m}{ne^2\rho} = \frac{9.11\times10^{-31}\text{ kg}}{(1.0\times10^{16}\text{ m}^{-3})\,(1.60\times10^{-19}\text{C})^2\,(2300\ \Omega\cdot\text{m})} = 1.55\times10^{-12}\text{ s.}$$

b) The number of free electrons in copper $(8.5\times10^{28}\text{ m}^{-3})$ is much larger than in pure silicon $(1.0\times10^{16}\text{ m}^{-3}).$

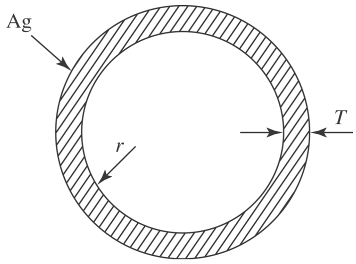
25.53: a) $\rho = \dfrac{RA}{L} = \dfrac{(0.104\ \Omega)\,(\pi/4)\,(2.50\times10^{-3}\text{m})^2}{14.0\text{ m}} = 3.65\times10^{-8}\ \Omega\cdot\text{m}.$

b) $I = JA = \dfrac{EA}{\rho} = \dfrac{(1.28\text{ V/m})\,(\pi/4)\,(2.50\times10^{-3}\text{ m})^2}{3.65\times10^{-8}\ \Omega\cdot\text{m}} = 172\text{ A}.$

c) $v_d = \dfrac{J}{nq} = \dfrac{E}{\rho nq} = \dfrac{1.28\text{ V/m}}{(3.65\times10^{-8}\ \Omega\cdot\text{m})\,(8.5\times10^{28}\text{ m}^{-3})\,(1.6\times10^{-19}\text{ C})}$

$= 2.58\times10^{-3}\text{ m/s}.$

25.54: r = 2.00 cm
T = 0.100 mm



$$I = \frac{V}{R} = \frac{V}{\rho l/A} = \frac{VA}{\rho l} = \frac{V(2\pi rT)}{\rho l}$$

$$= \frac{(12\text{ V})\,(2\pi)(2.00\times10^{-2}\text{ m})\,(0.100\times10^{-3}\text{ m})}{(1.47\times10^{-8}\ \Omega\cdot\text{m})\,(25.0\text{ m})}$$

$= 410\text{ A}$

25.55: With the voltmeter connected across the terminals of the battery there is no current through the battery and the voltmeter reading is the battery emf; $\varepsilon = 12.6\ \text{V}$.

With a wire of resistance R connected to the battery current I flows and
$\varepsilon - Ir - IR = 0$

Call the resistance of the 20.0-m piece R_1; then the resistance of the 40.0-m piece is $R_2 = 2R_1$.

$$\varepsilon - I_1 r - I_1 R_1 = 0; \qquad 12.6\ \text{V} - (7.00\ \text{A})r - (7.00\ \text{A})R_1 = 0$$

$$\varepsilon - I_2 r - I_2(2R_1) = 0; \quad 12.6\ \text{V} - (4.20\ \text{A})r - (4.20\ \text{A})(2R_1) = 0$$

Solving these two equations in two unknowns gives $R_1 = 1.20\Omega$. This is the resistance of 20.0 m, so the resistance of one meter is $[1.20\Omega/(20.0\text{m})]\,(1.00\text{m}) = 0.060\Omega$

25.56: a) $I = \dfrac{V}{R} = \dfrac{V}{R_{Cu} + R_{Ag}}$

and

$$R_{Cu} = \frac{\rho_{Cu} L_{Cu}}{A_{Cu}} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})\,(0.8\ \text{m})}{(\pi/4)\,(6.0 \times 10^{-4}\text{m})^2} = 0.049\ \Omega,$$

and

$$R_{Ag} = \frac{\rho_{Ag} L_{Ag}}{A_{Ag}} = \frac{(1.47 \times 10^{-8}\ \Omega \cdot \text{m})\,(1.2\ \text{m})}{(\pi/4)\,(6.0 \times 10^{-4}\ \text{m})^2} = 0.062\ \Omega$$

$$\Rightarrow I = \frac{5.0\ \text{V}}{0.049\ \Omega + 0.062\ \Omega} = 45\ \text{A}.$$

So the current in the copper wire is 45 A.

b) The current in the silver wire is 45 A, the same as that in the copper wire or else charge would build up at their interface.

c) $E_{Cu} = J\rho_{Cu} = \dfrac{IR_{Cu}}{L_{Cu}} = \dfrac{(45\ \text{A})\,(0.049\ \Omega)}{0.8\ \text{m}} = 2.76\ \text{V/m}.$

d) $E_{Ag} = J\rho_{Ag} = \dfrac{IR_{Ag}}{L_{Ag}} = \dfrac{(45\ \text{A})\,(0.062\ \Omega)}{1.2\ \text{m}} = 2.33\ \text{V/m}.$

e) $V_{Ag} = IR_{Ag} = (45\ \text{A})\,(0.062\ \Omega) = 2.79\ \text{V}.$

25.57: a) The current must be the same in both sections of the wire, so the current in the thin end is 2.5 mA.

b) $E_{1.6\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})\,(2.5\times10^{-3}\ \text{A})}{(\pi/4)\,(1.6\times10^{-3}\ \text{A})^2} = 2.14\ \times10^{-5}\ \text{V/m}.$

c) $E_{0.8\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\ \times10^{-8}\ \Omega\cdot\text{m})\,(2.5\times10^{-3}\ \text{A})}{(\pi/4)\,(0.80\times10^{-3}\ \text{A})^2}$

$= 8.55\times10^{-5}\ \text{V/m}\ (= 4E_{1.6\text{mm}}).$

d) $V = E_{1.6\text{ mm}} L_{1.6\text{ mm}} + E_{0.8\text{ mm}} L_{0.8\text{ mm}}$

$\Rightarrow V = (2.14\times10^{-5}\ \text{V/m})\,(1.20\ \text{m}) + (8.55\times10^{-5}\ \text{V/m})\,(1.80\ \text{m}) = 1.80\times10^{-4}\ \text{V}.$

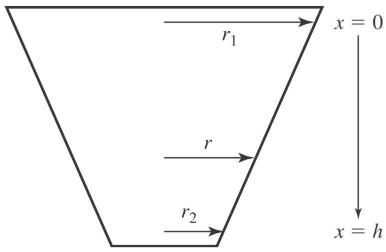
25.58: a) $\dfrac{K}{\text{volume}} = n\left(\dfrac{1}{2}mv_d{}^2\right)$

$\Rightarrow \dfrac{K}{\text{volume}} = \dfrac{1}{2}(8.5\times10^{28}\ \text{m}^{-3})\,(9.11\times10^{-31}\ \text{kg})\,(1.5\times10^{-4}\ \text{m/s})^2$

$= 8.7\times10^{-10}\ \text{J}/\text{m}^3.$

b) $U = qV = ne(\text{volume})V = (8.5\times10^{28}\text{m}^{-3})\,(1.6\times10^{-19}\text{C})\,(10^{-6}\ \text{m}^3)\,(1.0\ \text{V}) = 13600\ \text{J}.$ And the kinetic energy in 1.0 cm^3 is $K = (8.7\times10^{-10}\ \text{J/m}^3)\,(10^{-6}\text{m}) =$ 8.7×10^{-16} J. So $\dfrac{U}{K} = \dfrac{13600\ \text{J}}{8.7\times10^{-16}\ \text{J}} = 1.6\times10^{19}.$

25.59: a)



$$dR = \frac{\rho L}{A} = \frac{\rho dx}{\pi r^2} \text{ where } r = r_1 - \left(\frac{r_1 - r_2}{h}\right)x.$$

$$\Rightarrow R = \int_0^h \frac{\rho\, dx}{\pi\left(r_1 - \left(\frac{r_1 - r_2}{h}\right)x\right)^2} = -\frac{\rho h}{\pi(r_1 - r_2)}\int_{r_1}^{r_2} \frac{du}{u^2}$$

$$= \frac{\rho h}{\pi(r_1 - r_2)} \frac{1}{u}\bigg|_{r_1}^{r_2} \Rightarrow R = \frac{\rho h}{\pi}\left(\frac{1}{r_1 r_2}\right).$$

b) When $r_1 = r_2 = r,\ R = \dfrac{\rho h}{\pi r^2} = \dfrac{\rho L}{A}.$

25.60: a) $dR = \dfrac{\rho dr}{4\pi r^2} \Rightarrow R = \dfrac{\rho}{4\pi}\displaystyle\int_a^b \frac{dr}{r^2} = -\frac{\rho}{4\pi}\frac{1}{r}\bigg|_a^b = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right).$

b) $I = \dfrac{V_{ab}}{R} = \dfrac{V_{ab} 4\pi ab}{\rho(b-a)} \Rightarrow J = \dfrac{I}{A} = \dfrac{V_{ab} 4\pi ab}{\rho(b-a)4\pi r^2} = \dfrac{V_{ab} ab}{\rho(b-a)r^2}.$

c) If the thickness of the shells is small, we have the resistance given by:

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho(b-a)}{4\pi ab} \approx \frac{\rho L}{4\pi a^2} = \frac{\rho L}{A}, \text{ where } L = b - a.$$

25.61: $E = \rho J$ and $E = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{AK\varepsilon_0} \Rightarrow \rho J = \frac{Q}{AK\varepsilon_0} \Rightarrow AJ = I = \frac{Q}{K\varepsilon_0 \rho} =$ leakage current.

25.62: a) $I = \frac{V}{R} \Rightarrow J = \frac{I}{A} = \frac{V}{RA} = \frac{V}{(\rho L/A)A} = \frac{V}{\rho L}$. So to make the current density a maximum, we need the length between faces to be as small as possible, which means $L = d$. So the potential difference should be applied to those faces which are a distance d apart. This maximum current density is $J_{MAX} = \frac{V}{\rho d}$.

b) For a maximum current $I = \frac{V}{R} = \frac{VA}{\rho L} = JA$ must be a maximum. The maximum area is presented by the faces that are a distance d apart, and these two faces also have the greatest current density, so again, the potential should be placed over the faces a distance d apart. This maximum current is

$$I_{MAX} = 6\frac{Vd}{\rho}.$$

25.63: a) $R = \dfrac{\rho L}{A} = \dfrac{(9.5\times10^{-7}\ \Omega\cdot\text{m})\,(0.12\ \text{m})}{(\pi/4)\,(0.0016\ \text{m})^2} = 0.057\ \Omega.$

b) $\rho(T) = \rho_0(1+\alpha\Delta T) \Rightarrow \rho(60°\ \text{C}) = (9.5\times10^{-7}\ \Omega\cdot\text{m})\,(1+(0.00088(\text{C°})^{-1})\,(40°\text{C})$

$$\Rightarrow \rho(60°\text{C}) = 9.83\times10^{-7}\ \Omega\cdot\text{m} \Rightarrow \Delta\rho = 3.34\times10^{-8}\ \Omega\cdot\text{m}.$$

c) $\Delta V = \beta V_0 \Delta T \Rightarrow A\Delta L = A(\beta L_0 \Delta T) \Rightarrow \Delta L = \beta L_0 \Delta T = (18\times10^{-5}\ (\text{C°})^{-1})\times$ $(0.12\ \text{m})\,(40°\text{C}) \Rightarrow \Delta L = 8.64\times10^{-4}\ \text{m} = 0.86\ \text{mm}.$ The volume of the fluid remains constant. As the fluid expands the container, outward expansion "becomes" upward expansion due to surface effects.

d) $R = \dfrac{\rho L}{A} \Rightarrow \Delta R = \dfrac{\Delta\rho L}{A} + \dfrac{\rho \Delta L}{A}$

$$\Rightarrow \Delta R = \frac{(3.34\times10^{-8}\ \Omega\cdot\text{m})\,(0.12\ \text{m})}{(\pi/4)\,(0.0016\ \text{m})^2} + \frac{(95\times10^{-8}\ \Omega\cdot\text{m})\,(0.86\times10^{-3}\ \text{m})}{(\pi/4)\,(0.0016\ \text{m})^2}$$

$$= 2.40\times10^{-3}\ \Omega.$$

e) From Equation (25.12), $\alpha = \frac{1}{\Delta T}\left(\frac{R}{R_0} - 1\right) = \frac{1}{40°\text{C}}\left(\frac{(0.057\ \Omega + 2.40\times10^{-3}\ \Omega)}{0.057\ \Omega} - 1\right) =$ $1.1\times10^{-3}\ (\text{C°})^{-1}$. This value is greater than the temperature coefficient of resistivity and therefore is an important change caused by the length increase.

25.64: a) $I = \dfrac{\sum \mathcal{E}}{\sum R} = \dfrac{8.0\ \text{V} - 4.0\ \text{V}}{24.0\ \Omega} = 0.167\ \text{A}$

$\Rightarrow V_{ad} = 8.00\ \text{V} - (0.167\ \text{A})\ (8.50\ \Omega) = 6.58\ \text{V}.$

b) The terminal voltage is

$$V_{bc} = +\,4.00\ \text{V} + (0.167\ \text{A})\ (0.50\ \Omega) = +\,4.08\ \text{V}.$$

c) Adding another battery at point d in the opposite sense to the 8.0 V battery:

$$I = \frac{\sum \mathcal{E}}{\sum R} = \frac{10.3\ \text{V} - 8.0\ \text{V} + 4.0\ \text{V}}{24.5\ \Omega} = 0.257\ \text{A, and so}$$

$$\Rightarrow V_{bc} = 4.00\ \text{V} - (0.257\ \text{A})\ (0.50\ \Omega) = 3.87\ \text{V}.$$

25.65: a) $V_{ab} = \mathcal{E} - Ir \Rightarrow 8.4\ \text{V} = \mathcal{E} - (1.50\ \text{A})\ r$ and $9.4\ \text{V} = \mathcal{E} + (3.50\ \text{A})\ r$

$\Rightarrow 9.4\ \text{V} = (8.4\ \text{V} + (1.50\ \text{A})r) + (3.50\ \text{A})r$

$\Rightarrow r = \dfrac{9.4\ \text{V} - 8.4\ \text{V}}{5.00\ \text{A}} = 0.2\ \Omega.$

b) $\mathcal{E} = 8.4\ \text{V} + (1.50\ \text{A})\ (0.20\ \Omega) = 8.7\ \text{V}.$

25.66: a) $I = V / R = 14\ \text{kV} / (10\ \text{k}\Omega + 2\ \text{k}\Omega) = 1.17\ \text{A}.$

b) $P = I^2 R = (1.17\ \text{A})^2\ (10{,}000\ \Omega) = 13.7\ \text{kW}.$

c) If we want the current to be 1.0 mA, then the internal resistance must be:

$$R + r = \frac{14{,}000\ \text{V}}{0.001\ \text{A}} = 1.4 \times 10^7\ \Omega \Rightarrow R = 14\ \text{M}\Omega - 10\ \text{k}\Omega \approx 14\ \text{M}\Omega.$$

25.67: a) $R = \dfrac{\rho L}{A} = \dfrac{(5.0\ \Omega \cdot \text{m})\ (0.10\ \text{m})}{\pi (0.050\ \text{m})^2} = 1000\ \Omega.$

b) $V = IR = (100 \times 10^{-3}\ \text{A})\ (1000\ \Omega) = 100\ \text{V}.$

c) $P = VI = (100\text{V})\ (100 \times 10^{-3}\ \text{A}) = 10\ \text{W}.$

25.68: a) $V = 2.50I + 0.360I^2 = 4.0\ \text{V}.$ Solving the quadratic equation yields $I = 1.34\ \text{A or} -8.29\ \text{A},$ so the appropriate current through the semiconductor is $I = 1.34\ \text{A}.$

b) If the current $I = 2.68\ \text{A},$

$$\Rightarrow V = (2.50\ \text{V/A})\ (2.68\ \text{A}) + (0.36\ \text{V/A}^2)\ (2.68\ \text{A})^2 = 9.3\ \text{V}.$$

25.69: $V = IR + V(I) = IR + \alpha I + \beta I^2 = (\alpha + R)\ I + \beta I^2$

$$\Rightarrow \beta I^2 + (R + \alpha)\ I - V = 0$$

$$\Rightarrow (1.3)\ I^2 + (3.8 + 3.2)\ I - 12.6 = 0 \Rightarrow I = 1.42\ \text{A}.$$

25.70: a) $r = \dfrac{\mathcal{E}}{I} = \dfrac{7.86\ \text{V}}{9.25\ \text{A}} = 0.85\ \Omega \Rightarrow I = \dfrac{\mathcal{E}}{R + r} = \dfrac{7.86\ \text{V}}{0.85\ \Omega + 2.4\ \Omega} = 2.42\ \text{A}.$

b) $\beta I^2 + (\alpha + r)\ I - \mathcal{E} = 0 \Rightarrow 0.36I^2 + (2.50 + 0.85)\ I - 7.86 = 0$

$$\Rightarrow I = 1.94\ \text{A}$$

c) The terminal voltage at this current is

$$V_{ab} = \mathcal{E} - Ir = 7.86\ \text{V} - (1.94\ \text{A})\ (0.85\ \Omega) = 6.21\ \text{V}.$$

25.71: a) With an ammeter in the circuit:

$$I = \frac{\mathcal{E}}{r + R + R_A} \Rightarrow \mathcal{E} = I_A(r + R + R_A).$$

So with no ammeter:

$$I = \frac{\mathcal{E}}{r + R} = I_A \left(\frac{r + R + R_A}{r + R} \right) = I_A \left(1 + \frac{R_A}{r + R} \right).$$

b) We want:

$$\frac{I}{I_A} = \left(1 + \frac{R_A}{r + R} \right) \approx 1.01 \Rightarrow \frac{R_A}{r + R} \approx 0.01 \Rightarrow R_A\ (0.01)\ (0.45\ \Omega + 3.8\ \Omega)$$

$$= 0.0425\ \Omega.$$

c) This is a maximum value, since any larger resistance makes the current even less that it would be without it. That is, since the ammeter is in series, ANY resistance it has increases the circuit resistance and makes the reading less accurate.

25.72: a) With a voltmeter in the circuit:

$$I = \frac{\mathcal{E}}{r + R_V} \Rightarrow V_{ab} = \mathcal{E} - Ir = \mathcal{E}\left(1 - \frac{r}{r + R_V}\right).$$

b) We want:

$$\frac{V_{ab}}{\mathcal{E}} = \left(1 - \frac{r}{r + R_V}\right) \approx 0.99 \Rightarrow \frac{r}{r + R_V} \approx 0.01$$

$$\Rightarrow R_V \approx \frac{r - 0.01r}{0.01} = 99r = 99 \cdot 045\ \Omega = 44.6\ \Omega.$$

c) This is the minimum resistance necessary—any greater resistance leads to less current flow and hence less potential loss over the battery's internal resistance.

25.73: a) The line voltage, current to be drawn, and wire diameter are what must be considered in household wiring.

b) $P = VI \Rightarrow I = \dfrac{P}{V} = \dfrac{4200\ \text{W}}{120\ \text{V}} = 35\ \text{A},$ so the 8-gauge wire is necessary, since it can carry up to 40 A.

c) $P = I^2R = \dfrac{I^2 \rho L}{A} = \dfrac{(35\ \text{A})^2\ (1.72 \times 10^{-8}\ \Omega \cdot \text{m})\ (42.0\ \text{m})}{(\pi/4)\ (0.00326\ \text{m})^2} = 106\ \text{W}.$

d) If 6-gauge wire is used,

$$P = \frac{I^2 \rho L}{A} = \frac{(35\ \text{A})^2\ (1.72 \times 10^{-8}\ \Omega \cdot \text{m})\ (42\ \text{m})}{(\pi/4)\ )\ (0.00412\ \text{m})^2} = 66\ \text{W}$$

$$\Rightarrow \Delta E = \Delta Pt = (40\ \text{W})\ (365)\ (12\ \text{h}) = 175\ \text{kWh}$$

$$\Rightarrow \text{Savings} = (175\ \text{kWh})\ (\$0.11/\text{kWh}) = \$19.25.$$

25.74: Initially: $R_0 = V/I_0 = (120\ \text{V})/(1.35\ \text{A}) = 88.9\ \Omega.$

Finally: $R_f = V/I_f = (120\ \text{V})/(1.23\ \text{A}) = 97.6\ \Omega.$

And $\dfrac{R_f}{R_0} = 1 + \alpha\ (T_f - T_0) \Rightarrow (T_f - T_0) = \dfrac{1}{\alpha}\left(\dfrac{R_f}{R_0} - 1\right) = \dfrac{1}{4.5 \times 10^{-4}{}^\circ\text{C}^{-1}}\left(\dfrac{97.6\ \Omega}{88.9\ \Omega} - 1\right)$

$$\Rightarrow T_f - T_0 = 217^\circ\text{C} \Rightarrow T_f = 217^\circ\text{C} + 20^\circ\text{C} = 237^\circ\text{C}.$$

b) (i) $P_0 = VI_0 = (120\ \text{V})\ (1.35\ \text{A}) = 162\ \text{W}$

(ii) $P_f = VI_f = (120\ \text{V})\ (1.23\ \text{A}) = 148\ \text{W}$

25.75: a) $I = \dfrac{\Sigma\,\mathcal{E}}{\Sigma\,R} = \dfrac{12.0\ \text{V} - 8.0\ \text{V}}{10.0\ \Omega} = 0.40\ \text{A}.$

b) $P_{total} = I^2\, R_{total} = (0.40\ \text{A})^2\ (10\ \Omega) = 1.6\ \text{W}.$

c) Power generated in $\mathcal{E}_1$, $P = \mathcal{E}_1\ I = (12.0\ \text{V})\ (0.40\ \text{A}) = 4.8\ \text{W}.$

d) Rate of electrical energy transferred to chemical energy in $\mathcal{E}_2\ \ P = \mathcal{E}_2 I = (8.0\ \text{V}) \times (0.40\ \text{A}) = 3.2\ \text{W}.$

e) Note $(\text{c}) = (\text{b}) + (\text{d}),$ and so the rate of creation of electrical energy equals its rate of dissipation.

25.76: a) $R_{steel} = \dfrac{\rho L}{A} = \dfrac{(2.0 \times 10^{-7}\,\Omega \cdot \text{m})\ (2.0\ \text{m})}{(\pi/4)\ (0.018\ \text{m})^2} = 1.57 \times 10^{-3}\ \Omega$

$$R_{Cu} = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})\ (35\ \text{m})}{(\pi/4)\ (0.008\ \text{m})^2} = 0.012\,\Omega$$

$$\Rightarrow V = IR = I\ (R_{steel} + R_{Cu}) = (15000\ \text{A})\ (1.57 \times 10^{-3}\ \Omega + 0.012\,\Omega) = 204\ \text{V}.$$

b) $E = Pt = I^2 Rt = (15000\ \text{A})^2 (0.0136\,\Omega)\ (65 \times 10^{-6}\ \text{s}) = 199\ \text{J}.$

25.77: a) $\Sigma\, F = ma = |\,q\,|\,E \Rightarrow \dfrac{|\,q\,|}{m} = \dfrac{a}{E}.$

b) If the electric field is constant, $V_{bc} = EL \Rightarrow \dfrac{|\,q\,|}{m} = \dfrac{aL}{V_{bc}}.$

c) The free charges are “left behind” so the left end of the rod is negatively charged, while the right end is positively charged. Thus the right end is at the higher potential.

d) $a = \dfrac{V_{bc}\,|\,q\,|}{mL} = \dfrac{(1.0 \times 10^{-3}\ \text{V})\ (1.6 \times 10^{-19}\ \text{C})}{(9.11 \times 10^{-31}\ \text{kg})\ (0.50\ \text{m})} = 3.5 \times 10^8\ \text{m/s}^2.$

e) Performing the experiment in a rotational way enables one to keep the experimental apparatus in a localized area—whereas an acceleration like that obtained in (d), if linear, would quickly have the apparatus moving at high speeds and large distances.

25.78: a) We need to heat the water in 6 minutes, so the heat and power required are:

$$Q = mc_v \Delta T = (0.250\ \text{kg})\ (4190\ \text{J/kg°C})\ (80°\text{C}) = 83800\ \text{J}$$

$$\Rightarrow P = \frac{Q}{t} = \frac{83800\ \text{J}}{6(60\ \text{s})} = 233\ \text{W}.$$

$$\text{But } P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{233\ \text{W}} = 61.8\ \Omega.$$

b) $$R = \frac{\rho L}{A} = \frac{\rho L^2}{\text{vol}} \Rightarrow L = \sqrt{\frac{R \cdot \text{vol}}{\rho}} = \sqrt{\frac{(61.8\ \Omega)\ (2.5 \times 10^{-5}\ \text{m}^3)}{1.00 \times 10^{-6}\ \Omega \cdot \text{m}}} = 39\ \text{m}.$$

Now the radius of the wire can be calculated from the volume:

$$\text{vol} = L(\pi r^2) \Rightarrow r = \sqrt{\frac{\text{vol}}{\pi L}} = \sqrt{\frac{2.5 \times 10^{-5}\ \text{m}^3}{\pi(39\ \text{m})}} = 4.5 \times 10^{-4}\ \text{m}.$$

25.79: a) $V_{ab} = \mathcal{E} - Ir = 12.0\ \text{V} - (-10.0\ \text{A})\ (0.24\ \Omega) = 14.4\ \text{V}.$

b) $E = Pt = IVt = (10\ \text{A})\ (14.4\ \text{V})\ (5)\ (3600\ \text{s}) = 2.59 \times 10^6\ \text{J}.$

c) $E_{diss} = P_{diss} t = I^2 rt = (10\ \text{A})^2\ (0.24\ \Omega)\ (5)\ (3600\ \text{s}) = 4.32 \times 10^5\ \text{J}.$

d) Discharged at 10 A:

$$I = \frac{\mathcal{E}}{r + R} \Rightarrow R = \frac{\mathcal{E} - Ir}{I} = \frac{12.0\ \text{V} - (10\ \text{A})\ (0.24\ \Omega)}{10\ \text{A}} = 0.96\ \Omega.$$

e) $E = Pt = IVt = (10\ \text{A})\ (9.6\ \text{V})\ (5)\ (3600\ \text{s}) = 1.73 \times 10^6\ \text{J}.$

f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{diss} = 4.32 \times 10^5\ \text{J}.$

g) The energy originally supplied went into the battery and some was also lost over the internal resistance. So the stored energy was less than was needed to charge it. Then when discharging, even more energy is lost over the internal resistance, and what is left is dissipated over the external resistor.

25.80: a) $V_{ab} = \mathcal{E} - Ir = 12.0\text{ V} - (-30\text{ A})(0.24\ \Omega) = 19.2\text{ V}.$

b) $E = Pt = IVt = (30\text{ A})(19.2\text{ V})(1.7)(3600\text{ s}) = 3.53 \times 10^6\text{ J}.$

c) $E_{diss} = P_{diss}t = I^2Rt = (30\text{ A})^2\ (0.24\ \Omega)(1.7)(3600\text{ s}) = 1.32 \times 10^6\text{ J}.$

d) Discharged at 30 A:

$$I = \frac{\mathcal{E}}{r+R} \Rightarrow R = \frac{\mathcal{E} - Ir}{I} = \frac{12.0\text{ V} - (30\text{ A})(0.24\ \Omega)}{30\text{ A}} = 0.16\ \Omega.$$

e) $E = Pt = I^2Rt = (30\text{ A})^2\ (0.16\ \Omega)(1.7)(3600) = 8.81 \times 10^5\text{ J}.$

f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{diss} = 1.32 \times 10^6\text{ J}.$

g) Again, the energy originally supplied went into the battery and some was also lost over the internal resistance. So the stored energy was less than was needed to charge it. Then when discharging, even more energy is lost over the internal resistance, and what is left is dissipated over the external resistor. This time, at a higher current, much more energy is lost over the internal resistance.

25.81: a) $\alpha = \frac{1}{\rho}\left(\frac{d\rho}{dT}\right) = -\frac{n}{T} \Rightarrow \frac{ndT}{T} = \frac{d\rho}{\rho} \Rightarrow \ln(T^{-n}) = \ln(\rho) \Rightarrow \rho = \frac{a}{T^n}.$

b) $n = -\alpha T = -(-5 \times 10^{-4}\ (\text{K})^{-1})(293\text{ K}) = 0.15.$

$$\rho = \frac{a}{T^n} \Rightarrow a = \rho T^n = (3.5 \times 10^{-5}\ \Omega \cdot \text{m})(293\text{ K})^{0.15} = 8.0 \times 10^{-5}\ \Omega \cdot \text{m} \cdot \text{K}^{0.15}.$$

c) $T = -196°\text{C} = 77\text{ K}: \rho = \dfrac{8.0 \times 10^{-5}}{(77\text{ K})^{0.15}} = 4.3 \times 10^{-5}\ \Omega \cdot \text{m}.$

$$T = -300°\text{C} = 573\text{ K}: \rho = \frac{8.0 \times 10^{-5}}{(573\text{ K})^{0.15}} = 3.2 \times 10^{-5}\ \Omega \cdot \text{m}.$$

25.82: a) $\mathcal{E} = IR + IR_d \Rightarrow 2.00\text{ V} = I(1.0\ \Omega) + V \Rightarrow 2 = I_s[\exp(eV/kT) - 1] + V.$

b) $I_s = 1.50 \times 10^{-3}\text{ A}, T = 293\text{ K} \Rightarrow 1333 = \exp[39.6V - 667] + 667V.$

Trial and error shows that the right-hand side (rhs) above, for specific V values, equals 1333 V, when $V = 0.179\text{ V}.$ The current then is just

$$I = I_s \exp[39.6V - 1] = (1.5 \times 10^{-3}\text{ A}) \exp[39.6(0.179) - 1] = 1.80\text{ A}.$$

25.83: a) $R = \dfrac{\rho L}{A} \Rightarrow dR = \dfrac{\rho\, dx}{A} = \dfrac{\rho_0 \exp[-x/L]\, dx}{A} \Rightarrow R = \dfrac{\rho_0}{A}\displaystyle\int_0^L \exp[-x/L]\, dx$

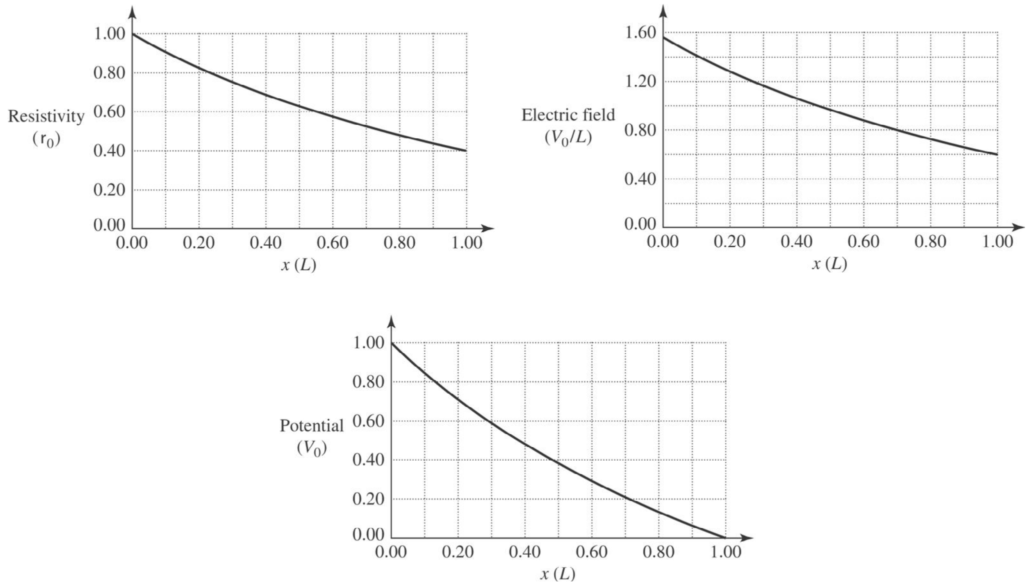
$$\Rightarrow R = \frac{\rho_0}{A}[-L\exp[-x/L]]_0^L = \frac{\rho_0 L}{A}(1-e^{-1}) \Rightarrow I = \frac{V_0}{R} = \frac{V_0 A}{\rho_0 L(1-e^{-1})}.$$

b) $E(x) = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial (IR)}{\partial x} = -\dfrac{\partial}{\partial x}\left(\dfrac{I\,\rho_0 L e^{-x/L}}{A}\right) = \dfrac{I\,\rho_0 e^{-x/L}}{A} = \dfrac{V_0 e^{-x/L}}{L(1-e^{-1})}.$

c) $V(x) = V_0\dfrac{e^{-x/L}}{(1-e^{-1})} + C \Rightarrow V(0) = V_0 = \dfrac{V_0}{(1-e^{-1})} + C \Rightarrow C = \dfrac{-V_0 e^{-1}}{L(1-e^{-1})}$

$$\Rightarrow V(x) = V_0\frac{(e^{-x/L} - e^{-1})}{(1-e^{-1})}.$$

d) Graphs of resistivity, electric field and potential from $x = 0$ to L.



25.84: a) $I = \dfrac{\mathcal{E}}{r+R} \Rightarrow P = \mathcal{E}I - I^2 r \Rightarrow \dfrac{dP}{dI} = \mathcal{E} - 2Ir = 0$ for maximum power output.

$$\Rightarrow I_{P\max} = \frac{1}{2}\frac{\mathcal{E}}{r} = \frac{1}{2} I_{\text{short circuit}}.$$

b) For the maximum power output of (a), $I = \dfrac{\mathcal{E}}{r+R} = \dfrac{1}{2}\dfrac{\mathcal{E}}{r} \Rightarrow +R = 2r \Rightarrow R = r.$

Then, $P = I^2 R = \left(\dfrac{\mathcal{E}}{2r}\right)^2 r = \dfrac{\mathcal{E}^2}{4r}.$

26.1: a) $R_{\text{eq}} = \left(\frac{1}{32} + \frac{1}{20}\right)^{-1} = 12.3\ \Omega.$

b) $I = \frac{V}{R_{\text{eq}}} = \frac{240\ \text{V}}{12.3\ \Omega} = 19.5\ \text{A}.$

c) $I_{32\Omega} = \frac{V}{R} = \frac{240\ \text{V}}{32\ \Omega} = 7.5\ \text{A};\ I_{20\Omega} = \frac{V}{R} = \frac{240\ \text{V}}{20\ \Omega} = 12\ \text{A}.$

26.2: $R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{R_1 + R_2}{R_1 R_2}\right)^{-1} \Rightarrow R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}.$

$\Rightarrow R_{\text{eq}} = R_1 \frac{R_2}{R_1 + R_2} < R_1$ and $R_{\text{eq}} = R_2 \frac{R_1}{R_1 + R_2} < R_2.$

26.3: For resistors in series, the currents are the same and the voltages add. a) true. b) false. c) $P = I^2 R$. i same, R different so P different; false. d) true. e) $V = IR$. I same, R different; false. f) Potential drops as move through each resistor in the direction of the current; false. g) Potential drops as move through each resistor in the direction of the current, so $V_b > V_c$; false. h) true.

26.4: a) False, current divides at junction *a*.

b) True by charge conservation.

c) True. $V_1 = V_2$, so $I \propto \frac{1}{R}$

d) False. $P = IV. V_1 = V_2$, but $I_1 \neq I_2$, so $P_1 \neq P_2$.

e) False. $P = IV = \frac{V^2}{R}$. Since $R_2 > R_1, P_2 < P_1$.

f) True. Potential is independent of path.

g) True. Charges lose potential energy (as heat) in R_1.

h) False. See answer to (g).

i) False. They are at the *same* potential.

26.5: a) $R_{eq} = \left(\frac{1}{2.4\,\Omega} + \frac{1}{1.6\,\Omega} + \frac{1}{4.8\,\Omega} \right)^{-1} = 0.8\,\Omega.$

b) $I_{2.4} = \varepsilon / R_{2.4} = (28\text{ V})/(2.4\,\Omega) = 11.67\text{ A}$; $I_{1.6} = \varepsilon / R_{1.6} = (28\text{ V})/(1.6\,\Omega) = 17.5\text{ A}$; $I_{4.8} = \varepsilon / R_{4.8} = (28\text{ V})/(4.8\,\Omega) = 5.83\text{ A}.$

c) $I_{total} = \varepsilon / R_{total} = (28\text{ V})/(0.8\,\Omega) = 35\text{ A}.$

d) When in parallel, all resistors have the same potential difference over them, so here all have V = 28 V.

e) $P_{2.4} = I^2 R_{2.4} = (11.67\text{ A})^2 (2.4\,\Omega) = 327\text{ W}$; $P_{1.6} = I^2 R_{1.6} = (17.5\text{ A})^2 (1.6\,\Omega) =$ 490 W; $P_{4.8} = I^2 R_{4.8} = (5.83\text{ A})^2 (4.8\,\Omega) = 163\text{ W}.$

f) For resistors in parallel, the most power is dissipated through the resistor with the least resistance since $P = I^2 R = \frac{V^2}{R}$, with V = constant.

26.6: a) $R_{eq} = \Sigma R_i = 2.4\,\Omega + 1.6\,\Omega + 4.8\,\Omega = 8.8\,\Omega.$

b) The current in each resistor is the same and is $I = \frac{\varepsilon}{R_{eq}} = \frac{28\text{ V}}{8.8\,\Omega} = 3.18\text{ A}.$

c) The current through the battery equals the current of (b), 3.18 A.

d) $V_{2.4} = IR_{2.4} = (3.18\text{ A})(2.4\,\Omega) = 7.64\text{ V}$; $V_{1.6} = IR_{1.6} = (3.18\text{ A})(1.6\,\Omega) =$ 5.09 V; $V_{4.8} = IR_{4.8} = (3.18\text{ A})(4.8\,\Omega) = 15.3\text{ V}.$

e) $P_{2.4} = I^2 R_{2.4} = (3.18\text{ A})^2 (2.4\,\Omega) = 24.3\text{ W}$; $P_{1.6} = I^2 R_{1.6} = (3.18\text{ A})^2 (1.6\,\Omega) =$ 16.2 W; $P_{4.8} = I^2 R_{4.8} = (3.18\text{ A})^2 (4.8\,\Omega) = 48.5\text{ W}.$

f) For resistors in series, the most power is dissipated by the resistor with the greatest resistance since $P = I^2 R$ with I constant.

26.7: a) $P = \frac{V^2}{R} \Rightarrow V = \sqrt{PR} = \sqrt{(5.0\text{ W})(15{,}000\,\Omega)} = 274\text{ V}.$

b) $P = \frac{V^2}{R} = \frac{(120\text{ V})^2}{9{,}000\,\Omega} = 1.6\text{ W}.$

26.8: $R_{eq} = \left(\left(\frac{1}{3.00\,\Omega} + \frac{1}{6.00\,\Omega} \right)^{-1} + \left(\frac{1}{12.0\,\Omega} + \frac{1}{4.00\,\Omega} \right)^{-1} \right) = 5.00\,\Omega$.

$I_{total} = \varepsilon / R_{total} = (6.00\text{ V})/(5.00\,\Omega) = 12.0\text{ A}$

$I_{12} = \frac{4}{12+4}(12.0) = 3.00\text{ A};\ I_4 = \frac{12}{12+4}(12.0) = 9.00\text{ A};$

$I_3 = \frac{6}{3+6}(12.0) = 8.00\text{ A};\ I_6 = \frac{3}{3+6}(12.0) = 4.00\text{ A}$.

26.9: $R_{eq} = \left(\frac{1}{3.00\,\Omega + 1.00\,\Omega} + \frac{1}{5.00\,\Omega + 7.00\,\Omega} \right)^{-1} = 3.00\,\Omega$.

$I_{total} = \varepsilon / R_{total} = (48.0\text{ V})/(3.00\,\Omega) = 16.0\text{ A}$.

$I_5 = I_7 = \frac{4}{4+12}(16.0) = 4.00\text{ A};\ I_1 = I_3 = \frac{12}{4+12}(16.0) = 12.0\text{ A}$.

26.10: a) The three resistors R_2, R_3 and R_4 are in parallel, so:

$$R_{234} = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)^{-1} = \left(\frac{1}{8.20\,\Omega} + \frac{1}{1.50\,\Omega} + \frac{1}{4.50\,\Omega} \right)^{-1} = 0.99\,\Omega$$

$\Rightarrow R_{eq} = R_1 + R_{234} = 3.50\,\Omega + 0.99\,\Omega = 4.49\,\Omega$.

b) $I_1 = \frac{\varepsilon}{R_{eq}} = \frac{6.0\text{ V}}{4.49\,\Omega} = 1.34\text{ A} \Rightarrow V_1 = I_1 R_1 = (1.34\text{ A})\,(3.50\,\Omega) = 4.69\text{ V}.$

$\Rightarrow V_{R_{234}} = I_1 R_{234} = (1.34\text{ A})\,(0.99\,\Omega) = 1.33\text{ V} \Rightarrow I_2 = \frac{V_{R_{234}}}{R_2} = \frac{1.33\,V}{8.20\,\Omega} = 0.162\text{ A},$

$I_3 = \frac{V_{R_{234}}}{R_3} = \frac{1.33\text{ V}}{1.50\,\Omega} = 0.887\text{ A}$ and $I_4 = \frac{V_{R_{234}}}{R_4} = \frac{1.33\text{ V}}{4.50\,\Omega} = 0.296\text{ A}.$

26.11: Using the same circuit as in Problem 27.10, with all resistances the same:

$$R_{\text{eq}} = R_1 + R_{234} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 4.50\ \Omega + \left(\frac{3}{4.50\ \Omega}\right)^{-1} = 6.00\ \Omega.$$

a) $I_1 = \dfrac{\varepsilon}{R_{\text{eq}}} = \dfrac{9.00\text{ V}}{6.00\ \Omega} = 1.50\text{ A},\ I_2 = I_3 = I_4 = \dfrac{1}{3}I_1 = 0.500\text{ A}.$

b) $P_1 = I_1^{\,2}R_1 = (1.50\text{ A})^2(4.50\ \Omega) = 10.13\text{ W},\ P_2 = P_3 = P_4 = \dfrac{1}{9}P_1 = 1.125\text{ W}.$

c) If there is a break at R_4, then the equivalent resistance increases:

$$R_{\text{eq}} = R_1 + R_{23} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 4.50\ \Omega + \left(\frac{2}{4.50\ \Omega}\right)^{-1} = 6.75\ \Omega.$$

And so:

$$I_1 = \frac{\varepsilon}{R_{\text{eq}}} = \frac{9.00\text{ V}}{6.75\ \Omega} = 1.33\text{ A},\ I_2 = I_3 = \frac{1}{2}I_1 = 0.667\text{ A}.$$

d) $P_1 = I_1^{\,2}R_1 = (1.33\text{ A})^2(4.50\ \Omega) = 7.96\text{ W},\ P_2 = P_3 = \dfrac{1}{4}P_1 = 1.99\text{ W}.$

e) So R_2 and R_3 are brighter than before, while R_1 is fainter. The amount of current flow is all that determines the power output of these bulbs since their resistances are equal.

26.12: From Ohm's law, the voltage drop across the 6.00 Ω resistor is $V = IR$ = $(4.00\text{ A})(6.00\ \Omega) = 24.0\text{ V}.$ The voltage drop across the 8.00 Ω resistor is the same, since these two resistors are wired in parallel. The current through the 8.00 Ω resistor is then $I = V/R = 24.0\text{ V}/8.00\ \Omega = 3.00\text{ A}.$ The current through the 25.0 Ω resistor is the sum of these two currents: 7.00 A. The voltage drop across the 25.0 Ω resistor is $V = IR$ = (7.00 A)(25.0 Ω) = 175 V, and total voltage drop across the top branch of the circuit is 175 + 24.0 = 199 V, which is also the voltage drop across the 20.0 Ω resistor. The current through the 20.0 Ω resistor is then $I = V/R = 199\text{ V}/20\ \Omega = 9.95\text{ A}.$

26.13: Current through 2.00-Ω resistor is 6.00 A. Current through 1.00-Ω resistor also is
6.00 A and the voltage is 6.00 V. Voltage across the 6.00-Ω resistor is 12.0 V + 6.0 V = 18.0 V. Current through the 6.00-Ω resistor is $(18.0\text{V})/(6.00\Omega) = 3.00\text{ A}.$ The battery voltage is 18.0 V.

26.14: a) The filaments must be connected such that the current can flow through each separately, and also through both in parallel, yielding three possible current flows. The parallel situation always has less resistance than any of the individual members, so it will give the highest power output of 180 W, while the other two must give power outputs of 60 W and
120 W.

$$60\ \text{W} = \frac{V^2}{R_1} \Rightarrow R_1 = \frac{(120\ \text{V})^2}{60\ \text{W}} = 240\ \Omega,\ \text{and}\ 120\ \text{W} = \frac{V^2}{R_2} \Rightarrow R_2 = \frac{(120\ \text{V})^2}{120\ \text{W}} = 120\ \Omega.$$

Check for parallel: $P = \dfrac{V^2}{(\frac{1}{R_1} + \frac{1}{R_2})^{-1}} = \dfrac{(120\ \text{V})^2}{(\frac{1}{240\ \Omega} + \frac{1}{120\ \Omega})^{-1}} = \dfrac{(120\ \text{V})^2}{80\ \Omega} = 180\ \text{W}.$

b) If R_1 burns out, the 120 W setting stays the same, the 60 W setting does not work and the 180 W setting goes to 120 W: brightnesses of zero, medium and medium.

c) If R_2 burns out, the 60 W setting stays the same, the 120 W setting does not work, and the 180 W setting is now 60 W: brightnesses of low, zero and low.

26.15: a) $I = \dfrac{\varepsilon}{R} = \dfrac{120\ \text{V}}{(400\ \Omega + 800\ \Omega)} = 0.100\ \text{A}.$

b) $P_{400} = I^2R = (0.100\ \text{A})^2(400\ \Omega) = 4.0\ \text{W}; P_{800} = I^2R = (0.100\ \text{A})^2(800\ \Omega) =$
$8.0\ \text{W} \Rightarrow P_{total} = 4\ \text{W} + 8\ \text{W} = 12\ \text{W}.$

c) When in parallel, the equivalent resistance becomes:

$$R_{\text{eq}} = \left(\frac{1}{400\ \Omega} + \frac{1}{800\ \Omega}\right)^{-1} = 267\ \Omega \Rightarrow I_{\text{total}} = \frac{\varepsilon}{R_{\text{eq}}} = \frac{120\ \text{V}}{267\ \Omega} = 0.449\ \text{A}.$$

$$I_{400} = \frac{800}{400 + 800}(0.449\ \text{A}) = 0.30\ \text{A}; I_{800} = \frac{400}{400 + 800}(0.449\ \text{A}) = 0.150\ \text{A}.$$

d) $P_{400} = I^2R = (0.30\ \text{A})^2(400\ \Omega) = 36\ \text{W}; P_{800} = I^2R = (0.15\ \text{A})^2(800\ \Omega) = 18\ \text{W}$

$$\Rightarrow P_{total} = 36\ \text{W} + 18\ \text{W} = 54\ \text{W}.$$

e) The 800 Ω resistor is brighter when the resistors are in series, and the 400 Ω is brighter when in parallel. The greatest total light output is when they are in parallel.

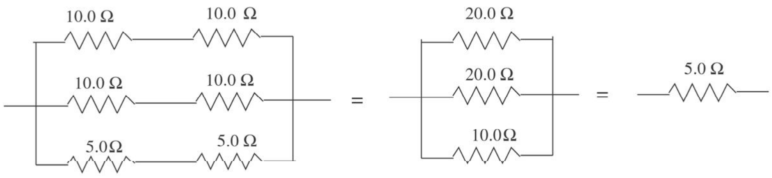
26.16: a) $R_{60\,\text{W}} = \dfrac{V^2}{P} = \dfrac{(120\ \text{V})^2}{60\ \text{W}} = 240\ \Omega;\ R_{200\,\text{W}} = \dfrac{V^2}{P} = \dfrac{(120\ \text{V})^2}{200\ \text{W}} = 72\ \Omega.$

$$\Rightarrow I_{60\,\text{W}} = I_{200\,\text{W}} = \frac{\varepsilon}{R} = \frac{240\ \text{V}}{(240\ \Omega + 72\ \Omega)} = 0.769\ \text{A}.$$

b)
$P_{60\,\text{W}} = I^2R = (0.769\ \text{A})^2(240\ \Omega) = 142\,\text{W};\ P_{200\,\text{W}} = I^2R = (0.769\ \text{A})^2(72\ \Omega) = 42.6\ \text{W}.$

c) The 60 W bulb burns out quickly because the power it delivers (142 W) is 2.4 times its rated value.

26.17:



$30.0\ \text{V} - I\ (20.0\ \Omega + 5.0\ \Omega + 5.0\ \Omega) = 0; \qquad I = 1.00\ \text{A}$

For the $20.0\text{-}\Omega$ resistor thermal energy is generated at the rate $P = I^2R = 20.0\ \text{W}.$

$Q = Pt$ and $Q = mc\Delta T$ gives

$$t = \frac{mc\Delta T}{P} = \frac{(0.100\ \text{kg})\ (4190\ \text{J/kg}\cdot\text{K})\ (40.0\ \text{C}°)}{20.0\ \text{W}} = 1.01\times10^3\,\text{s}$$

26.18: a) $$P_1 = I_1^2 R_1$$
$$20\text{ W} = (2\text{A})^2 R_1 \rightarrow R_1 = 5.00\Omega$$
R_1 and $10\ \Omega$ in parallel:
$$(10\ \Omega)I_{10} = (5\ \Omega)\ (2\text{ A})$$
$$I_{10} = 1\text{ A}$$
So $I_2 = 0.50$ A. R_1 and R_2 are in parallel, so
$$(0.50\text{ A})R_2 = (2\text{ A})\ (5\ \Omega)$$
$$R_2 = 20.0\ \Omega$$
b) $\varepsilon = V_1 = (2\text{A})(5\ \Omega) = 10.0\text{ V}$

c) From (a): $I_2 = 0.500\text{ A}, I_{10} = 1.00\text{ A}$

d) $$P_1 = 20.0\text{ W (given)}$$
$$P_2 = i_2^2\ R_2 = (0.50\text{ A})^2 (20\ \Omega) = 5.00\text{ W}$$
$$P_{10} = i_{10}^2\ R_{10} = (1.0\text{ A})^2 (10\ \Omega) = 10.0\text{ W}$$
$$P_{\text{Resist}} = 20\text{ W} + 5\text{ W} + 10\text{ W} = 35.0\text{ W}$$
$$P_{\text{Battery}} = I\,\varepsilon = (3.50\text{ A})\ (10.0\text{ V}) = 35.0\text{ W}$$
$$P_{\text{Resist}} = P_{\text{Battery,}}\quad \text{which agrees with the conservation of energy.}$$

26.19: a) $I_R = 6.00\text{ A} - 4.00\text{ A} = 2.00\text{ A}.$

b) Using a Kirchhoff loop around the outside of the circuit:
$$28.0\text{ V} - (6.00\text{ A})\ (3.00\ \Omega) - (2.00\text{ A})\ R = 0 \Rightarrow R = 5.00\ \Omega.$$
c) Using a counterclockwise loop in the bottom half of the circuit:
$$\varepsilon - (6.00\text{ A})\ (3.00\ \Omega) - (4.00\text{ A})\ (6.00\ \Omega) = 0 \Rightarrow \varepsilon = 42.0\text{ V}.$$
d) If the circuit is broken at point *x*, then the current in the 28 V battery is:
$$I = \frac{\sum \varepsilon}{\sum R} = \frac{28.0\text{ V}}{3.00\ \Omega + 5.00\ \Omega} = 3.50\text{ A}.$$

26.20: From the given currents in the diagram, the current through the middle branch of the circuit must be 1.00 A (the difference between 2.00 A and 1.00 A). We now use Kirchoff's Rules, passing counterclockwise around the top loop:
$$20.0\text{ V} - (1.00\text{ A})\left(6.00\ \Omega + 1.00\ \Omega\right) + \left(1.00\text{ A}\right)\left(4.00\ \Omega + 1.00\ \Omega\right) - \varepsilon_1 = 0 \Rightarrow \varepsilon_1 = 18.0\text{ V}.$$
Now traveling around the external loop of the circuit:
$$20.0\text{ V} - \left(1.00\text{ A}\right)\left(6.00\ \Omega + 1.00\ \Omega\right) - \left(2.00\text{ A}\right)\left(1.00\ \Omega + 2.00\ \Omega\right) - \varepsilon_2 = 0 \Rightarrow \varepsilon_2 = 7.0\text{ V}.$$
And
$$\text{V}_{ab} = -\left(1.00\text{ A}\right)\left(4.00\ \Omega + 1.00\ \Omega\right) + 18.0\text{ V} = +13.0\text{ V, so } V_{ba} = -13.0\text{ V}.$$

26.21: a) The sum of the currents that enter the junction below the $3\text{-}\Omega$ resistor equals 3.00 A + 5.00 A = 8.00 A.

b) Using the lower left loop:

$$\varepsilon_1 - (4.00\ \Omega)(3.00\ \text{A}) - (3.00\ \Omega)(8.00\ \text{A}) = 0$$
$$\Rightarrow \varepsilon_1 = 36.0\ \text{V}.$$

Using the lower right loop:

$$\varepsilon_2 - (6.00\ \Omega)(5.00\ \text{A}) - (3.00\ \Omega)(8.00\ \text{A}) = 0$$
$$\Rightarrow \varepsilon_2 = 54.0\ \text{V}.$$

c) Using the top loop:

$$54.0\ \text{V} - R(2.00\ \text{A}) - 36.0\ \text{V} = 0 \Rightarrow R = \frac{18.0\ \text{V}}{2.00\ \text{A}} = 9.00\ \Omega.$$

26.22: From the circuit in Fig. 26.42, we use Kirchhoff's Rules to find the currents, I_1 to the left through the 10 V battery, I_2 to the right through 5 V battery, and I_3 to the right through the $10\ \Omega$ resistor:

Upper loop:

$$10.0\ \text{V} - (2.00\ \Omega + 3.00\ \Omega)I_1 - (1.00\ \Omega + 4.00\ \Omega)I_2 - 5.00\ \text{V} = 0$$
$$\Rightarrow 5.0\ \text{V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0 \Rightarrow I_1 + I_2 = 1.00\ \text{A}.$$

Lower loop: $5.00\ \text{V} + (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0$

$$\Rightarrow 5.00\ \text{V} + (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0 \Rightarrow I_2 - 2I_3 = -1.00\ \text{A}$$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = 0.800\ \text{A}, I_2 = 0.200\ \text{A}, I_3 = 0.600\ \text{A}.$$

b) $V_{ab} = -(0.200\ \text{A})(4.00\ \Omega) - (0.800\ \text{A})(3.00\ \Omega) = -3.20\ \text{V}.$

26.23: After reversing the polarity of the 10-V battery in the circuit of Fig. 26.42, the only change in the equations from Problem 26.22 is the upper loop where the 10 V battery is:

Upper loop: $-10.0\ \text{V} - (2.00\ \Omega + 3.00\ \Omega)I_1 - (1.00\ \Omega + 4.00\ \Omega)I_2 - 5.00\ \text{V} = 0$

$$\Rightarrow -15.0\ \text{V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0 \Rightarrow I_1 + I_2 = -3.00\ \text{A}.$$

Lower loop: $5.00\ \text{V} + (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0$

$$\Rightarrow 5.00\ \text{V} + (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0 \Rightarrow I_2 - 2I_3 = -1.00\ \text{A}.$$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = -1.60\ \text{A}, I_2 = -1.40\ \text{A}, I_3 = -0.200\ \text{A}.$$

b) $V_{ab} = +(1.40\ \text{A})(4.00\ \Omega) + (1.60\ \text{A})(3.00\ \Omega) = 10.4\ \text{V}.$

26.24: After switching the 5-V battery for a 20-V battery in the circuit of Fig. 26.42, there is a change in the equations from Problem 26.22 in both the upper and lower loops:

Upper loop: $10.0\text{ V} - (2.00\ \Omega + 3.00\ \Omega)I_1 - (1.00\ \Omega + 4.00\ \Omega)I_2 - 20.00\text{ V} = 0$

$$\Rightarrow -10.0\text{ V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0 \Rightarrow I_1 + I_2 = -2.00\text{ A}.$$

Lower loop: $20.00\text{ V} + (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0$

$$\Rightarrow 20.00\text{ V} + (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0 \Rightarrow I_2 - 2I_3 = -4.00\text{ A}.$$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = -0.4\text{ A},\ I_2 = -1.6\text{ A},\ I_3 = +1.2\text{ A}.$$

b) $I_2(4\ \Omega) - I_1(3\ \Omega) = (1.6\text{ A})(4\ \Omega) + (0.4\text{ A})(3\ \Omega) = 7.6\text{ V}$

26.25: The total power dissipated in the four resistors of Fig. 26.10a is given by the sum of:

$$P_2 = I^2R_2 = (0.5\text{ A})^2(2\ \Omega) = 0.5\text{ W},\ P_3 = I^2\ R_3 = (0.5\text{ A})^2(3\ \Omega) = 0.75\text{ W},$$

$$P_4 = I^2R_4 = (0.5\text{ A})^2(4\ \Omega) = 1\text{ W},\ P_7 = I^2\ R_7 = (0.5\text{ A})^2(7\ \Omega) = 1.8\text{ W}.$$

$$\Rightarrow P_{\text{total}} = P_2 + P_3 + P_4 + P_7 = 4\text{ W}.$$

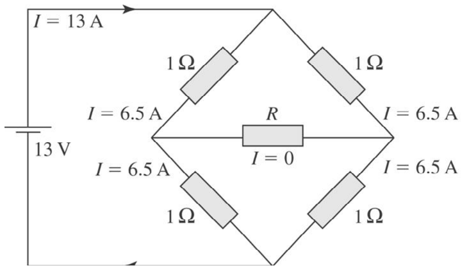
26.26: a) If the 12-V battery is removed and then replaced with the opposite polarity, the current will flow in the clockwise direction, with magnitude;

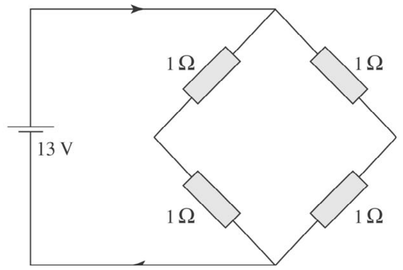
$$I = \frac{\sum \varepsilon}{\sum R} = \frac{12\text{ V} + 4\text{ V}}{16\ \Omega} = 1\text{ A}.$$

b) $V_{ab} = -(R_4 + R_7)I + \varepsilon_4 = -(4\ \Omega + 7\ \Omega)(1\text{ A}) + 4\text{ V} = -7\text{ V}.$

26.27: a) Since all the external resistors are equal, the current must be symmetrical through them. That is, there can be no current through the resistor R for that would imply an imbalance
in currents through the other resistors.

With no current going through R, the circuit is like that shown below at right.





So the equivalent resistance of the circuit is

$$R_{\text{eq}} = \left(\frac{1}{2\,\Omega} + \frac{1}{2\,\Omega}\right)^{-1} = 1\,\Omega \Rightarrow I_{total} = \frac{13\text{ V}}{1\,\Omega} = 13\text{ A}.$$

$$\Rightarrow I_{\text{each leg}} = \frac{1}{2} I_{total} = 6.5\text{ A},\ \text{and no current passes through } R.$$

b) As worked out above, $R_{\text{eq}} = 1\,\Omega$.

c) $V_{ab} = 0$, since no current flows.

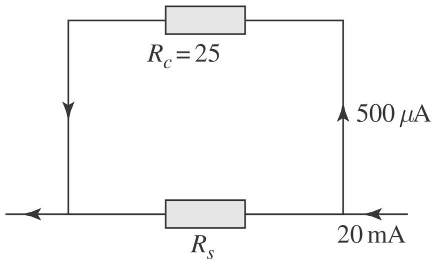
d) R does not show up since no current flows through it.

26.28: Given that the full-scale deflection current is 500 μA and the coil resistance is $25.0\ \Omega$:

a) For a 20-mA ammeter, the two resistances are in parallel:

$$V_c = V_s \Rightarrow I_c R_c = I_s R_s \Rightarrow (500 \times 10^{-6}\ \text{A})(25.0\ \Omega) = (20 \times 10^{-3}\ \text{A} - 500 \times 10^{-6}\ \text{A})R_s$$

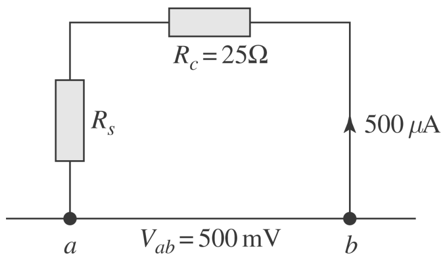
$$\Rightarrow R_s = 0.641\ \Omega$$



b) For a 500-m voltmeter, the resistances are in series:

$$V_{ab} = I(R_c + R_s) \Rightarrow R_s \Rightarrow \frac{V_{ab}}{I} - R_c$$

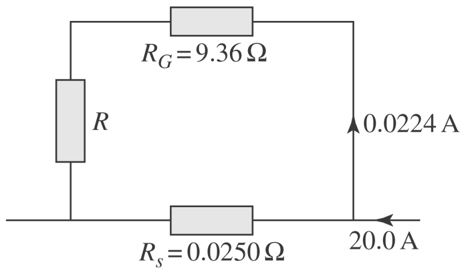
$$\Rightarrow R_s = \frac{500 \times 10^{-3}\ \text{V}}{500 \times 10^{-6}\ \text{A}} - 25.0\ \Omega = 975\ \Omega.$$



26.29: The full-scale deflection current is 0.0224 A, and we wish a full-scale reading for 20.0 A.

$$(0.0224\text{ A})(9.36\ \Omega + R) = (20.0\text{ A} - 0.0224\text{ A})(0.0250\ \Omega)$$

$$\Rightarrow R = \frac{0.499\ \Omega\text{A}}{0.0224\text{ A}} - 9.36\ \Omega = 12.9\ \Omega.$$



26.30: a) $I = \dfrac{\varepsilon}{R_{total}} = \dfrac{90\text{ V}}{(8.23\ \Omega + 425\ \Omega)} = 0.208\text{ A}$

$$\Rightarrow V = \varepsilon - Ir = 90\text{ V} - (0.208\text{ A})(8.23\ \Omega) = 88.3\ \Omega.$$

b) $V = \varepsilon - Ir = \varepsilon - \dfrac{\varepsilon r}{r + R_V} = \dfrac{\varepsilon R_V}{r + R_V} = \dfrac{\varepsilon}{(r/R_V) + 1} \Rightarrow \dfrac{r}{R_V} = \dfrac{\varepsilon}{V} - 1.$

Now if V is to be off by no more than 4% it requires: $\dfrac{r}{R_V} = \dfrac{90}{86.4} - 1 = 0.0416.$

26.31: a) When the galvanometer reading is zero:

$$\varepsilon_2 = IR_{cb} \text{ and } \varepsilon_1 = IR_{ab} \Rightarrow \varepsilon_2 = \varepsilon_1 \frac{R_{cb}}{R_{ab}} = \varepsilon_1 \frac{x}{l}.$$

b) The value of the galvanometer's resistance is unimportant since no current flows through it.

c) $\varepsilon_2 = \varepsilon_1 \dfrac{x}{l} = (9.15\text{ V})\dfrac{0.365\text{ m}}{1.000\text{ m}} = 3.34\text{ V}.$

26.32: Two voltmeters with different resistances are connected in series across a 120-V line. So the current flowing is $I = \dfrac{V}{R_{total}} = \dfrac{120 \text{ V}}{100 \times 10^3 \ \Omega} = 1.20 \times 10^{-3} \text{ A}.$ But the current required for full-scale deflection for each voltmeter is:

$$I_{fsd\,(10\ \text{k}\Omega)} = \frac{150 \text{ V}}{10{,}000\,\Omega} = 0.0150 \text{ A and } I_{fsd\,(90\ \text{k}\Omega)} = \frac{150 \text{ V}}{90{,}000\,\Omega} = 1.67 \times 10^{-3} \text{ A}.$$

So the readings are:

$$V_{10\,\text{k}\Omega} = 150 \text{ V}\left(\frac{1.20 \times 10^{-3} \text{ A}}{0.0150 \text{ A}}\right) = 12 \text{ V and } V_{90\ \text{k}\Omega} = 150 \text{ V}\left(\frac{1.20 \times 10^{-3} \text{ A}}{1.67 \times 10^{-3} \text{ A}}\right) = 108 \text{ V}.$$

26.33: A half-scale reading occurs with $R = 600\,\Omega.$ So the current through the galvanometer is half the full-scale current.

$$\Rightarrow \varepsilon = I\ R_{\text{total}} \Rightarrow 1.50 \text{ V} = \left(\frac{3.60 \times 10^{-3} \text{ A}}{2}\right)(15.0\,\Omega + 600\,\Omega + R_s) \Rightarrow R_s = 218\,\Omega.$$

26.34: a) When the wires are shorted, the full-scale deflection current is obtained:
$\varepsilon = IR_{total} \Rightarrow 1.52 \text{ V} = (2.50 \times 10^{-3} \text{ A})(65.0\,\Omega + R) \Rightarrow R = 543\,\Omega.$

b) If the resistance $R_x = 200\,\Omega : I = \dfrac{V}{R_{total}} = \dfrac{1.52 \text{ V}}{65.0\,\Omega + 543\,\Omega + R_x} = 1.88 \text{ mA}.$

c) $I_x = \dfrac{\varepsilon}{R_{total}} = \dfrac{1.52 \text{ V}}{65.0\,\Omega + 543\,\Omega + R_x} \Rightarrow R_x = \dfrac{1.52 \text{ V}}{I_x} - 608\,\Omega.$

$$\text{So: } I_x = \frac{1}{4} I_{fsd} = 6.25 \times 10^{-4} \text{ A} \Rightarrow R_x = \frac{1.52 \text{ V}}{6.25 \times 10^{-4} \text{ A}} - 608\,\Omega = 1824\,\Omega.$$

$$I_x = \frac{1}{2} I_{fsd} = 1.25 \times 10^{-3} \text{ A} \Rightarrow R_x = \frac{1.52 \text{ V}}{1.25 \times 10^{-3} \text{ A}} - 608\,\Omega = 608\,\Omega.$$

$$I_x = \frac{3}{4} I_{fsd} = 1.875 \times 10^{-3} \text{ A} \Rightarrow R_x = \frac{1.52 \text{ V}}{1.875 \times 10^{-3} \text{ A}} - 608\,\Omega = 203\,\Omega.$$

26.35: $[RC] = \left[\dfrac{V}{I}\dfrac{Q}{V}\right] = \left[\dfrac{Q}{I}\right] = \left[\dfrac{Q}{Q/t}\right] = [t]$

26.36: An uncharged capacitor is placed into a circuit.

a) At the instant the circuit is completed, there is no voltage over the capacitor, since it has no charge stored.

b) All the voltage of the battery is lost over the resistor, so $V_R = \varepsilon = 125\ \text{V}$.

c) There is no charge on the capacitor.

d) The current through the resistor is $i = \dfrac{\varepsilon}{R_{total}} = \dfrac{125\ \text{V}}{7500\ \Omega} = 0.0167\ \text{A}$.

e) After a long time has passed:

The voltage over the capacitor balances the emf: $V_c = 125\ \text{V}$.

The voltage over the resister is zero.

The capacitor's charge is $q = Cv_c = (4.60 \times 10^{-6}\ \text{F})\ (125\ \text{V}) = 5.75 \times 10^{-4}\ \text{C}$.

The current in the circuit is zero.

26.37: a) $i = \dfrac{q}{RC} = \dfrac{6.55 \times 10^{-8}\ \text{C}}{(1.28 \times 10^{6}\ \Omega)\ (4.55 \times 10^{-10}\ \text{F})} = 1.12 \times 10^{-4}\ \text{A}$.

b) $\tau = RC = (1.28 \times 10^{6}\ \Omega)\ (4.55 \times 10^{-10}\ \text{F}) = 5.82 \times 10^{-4}\ \text{s}$.

26.38:

$$v = v_0 e^{-\tau/RC} \Rightarrow C = \frac{\tau}{R \ln(v_0 / v)} = \frac{4.00\ \text{s}}{(3.40 \times 10^{6}\ \Omega)\ (\ln\ (12/3))} = 8.49 \times 10^{-7}\ \text{F}.$$

26.39: a) The time constant $RC = (0.895\times10^{6}\ \Omega)\,(12.4\times10^{-6}\ \text{F}) = 11.1\ \text{s}$. So at:

$t = 0\ \text{s}:\ q = C\varepsilon(1-e^{-t/RC}) = 0.$

$t = 5\ \text{s}:\ q = C\varepsilon(1-e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})\,(60.0\ \text{V})\,(1-e^{-(5.0\ \text{s})/(11.1\ \text{s})})$

$= 2.70\times10^{-4}\ \text{C}.$

$t = 10\ \text{s}: q = C\varepsilon(1-e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})\,(60.0\ \text{V})\,(1-e^{-(10.0\ \text{s})/(11.1\ \text{s})})$

$= 4.42\times10^{-4}\ \text{C}.$

$t = 20\ \text{s}: q = C\varepsilon(1-e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})\,(60.0\ \text{V})\,(1-e^{-(20.0\ \text{s})/(11.1\ \text{s})})$

$= 6.21\times10^{-4}\ \text{C}.$

$t = 100\ \text{s}: q = C\varepsilon(1-e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})\,(60.0\ \text{V})\,(1-e^{-(100\ \text{s})/(11.1\ \text{s})})$

$= 7.44\times10^{-4}\ \text{C}.$

b) The current at time t is given by: $i = \dfrac{\varepsilon}{R}e^{-t/RC}$. So at:

$$t = 0\ \text{s}: i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-0/11.1} = 6.70\times10^{-5}\ \text{A}.$$

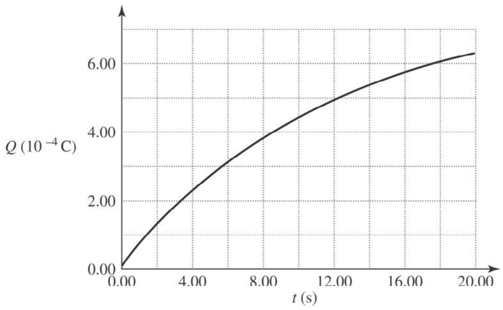
$$t = 5\ \text{s}: i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-5/11.1} = 4.27\times10^{-5}\ \text{A}.$$

$$t = 10\ \text{s}: i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-10/11.1} = 2.27\times10^{-5}\ \text{A}.$$

$$\text{t} = 20\ \text{s}: i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-20/11.1} = 1.11\times10^{-5}\ \text{A}.$$

$$t = 100\ \text{s}: i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-100/11.1} = 8.20\times10^{-9}\ \text{A}.$$

c) Charge against time:



Current against time:

26.40: a) Originally, $\tau = RC = 0.870\text{ s}.$ The combined capacitance of the two identical capacitors in series is given by

$$\frac{1}{C_{\text{tot}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C};\, C_{\text{tot}} = \frac{C}{2}$$

The new time constant is thus $R\,(\frac{C}{2}) = \frac{0.870\text{ s}}{2} = 0.435\text{ s}.$

b) With the two capacitors in parallel the new total capacitane is simply 2 C. Thus the time constant is $R(2C) = 2(0.870\text{ s}) = 1.74\text{ s}.$

26.41: $\varepsilon - V_R - V_C = 0$

$\varepsilon = 120\text{ V},\, V_R = IR = (0.900\text{ A})\,(80.0\,\Omega) = 72\text{V},$ so $V_C = 48\text{ V}$

$Q = CV = (4.00 \times 10^{-6}\text{F})\,(48\text{V}) = 192\ \mu\text{C})$

26.42: a) $Q = CV = (5.90 \times 10^{-6}\text{ F})\,(28.0\text{ V}) = 1.65 \times 10^{-4}\text{ C}.$

b) $q = Q(1 - e^{-t/RC}) \Rightarrow e^{-t/RC} = 1 - \dfrac{q}{Q} \Rightarrow R = \dfrac{-t}{C\,\ln(1 - q/Q)}.$

After $t = 3 \times 10^{-3}\text{ s} : R = \dfrac{-3 \times 10^{-3}\text{ s}}{(5.90 \times 10^{-6}\text{ F})\,(\ln(1 - 110/165))} = 463\,\Omega.$

c) If the charge is to be 99% of final value:

$$\frac{q}{Q} = (1 - e^{-t/RC}) \Rightarrow t = -\,RC\,\ln(1 - q/Q)$$

$$= -\,(463\,\Omega)\,(5.90 \times 10^{-6}\text{ F})\,\ln(0.01) = 0.0126\text{ s}.$$

26.43: a) The time constant $RC = (980\,\Omega)\,(1.50 \times 10^{-5}\text{ F}) = 0.0147\text{ s}.$

$t = 0.05\text{ s} : q = C\varepsilon(1 - e^{-t/RC}) = (1.50 \times 10^{-5}\text{ F})\,(18.0\text{ V})\,(1 - e^{-0.010/0.0147}) = 1.33 \times 10^{-4}\text{ C}.$

b) $i = \dfrac{\varepsilon}{R} e^{-t/RC} = \dfrac{18.0\text{ V}}{980\,\Omega} e^{-0.10/0.0147} = 9.30 \times 10^{-3}\text{ A}.$

$\Rightarrow V_R = IR = (9.30 \times 10^{-3}\text{ A})\,(980\,\Omega) = 9.11\text{ V and } V_C = 18.0\text{ V} - 9.11\text{ V} = 8.89\text{ V}.$

c) Once the switch is thrown, $V_R = V_C = 8.89\text{ V}.$

d) After $t = 0.01\text{ s} : q = Q_0 e^{-t/RC} = (1.50 \times 10^{-5}\text{ F})\,(8.89\text{ V})e^{-0.01/0.0147} = 6.75 \times 10^{-5}\text{ C}.$

26.44: a) $I = \frac{P}{V} = \frac{4100\ \text{W}}{240\ \text{V}} = 17.1\ \text{A}.$ So we need at lest 14-gauge wire (good up to 18 A). 12 gauge is ok (good up to 25 A).

b) $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(240\ \text{V})^2}{4100\ \text{W}} = 14\ \Omega$

c) At $11¢\ /\text{kWhr} \Rightarrow \text{in 1 hour, cost} = (11¢/\text{kWhr})\ (1\ \text{hr})(4.1\ \text{kW}) = 45¢.$

26.45: We want to trip a 20-A circuit breaker:

$$I = \frac{1500\ \text{W}}{120\ \text{V}} + \frac{P}{120\ \text{V}} \Rightarrow \text{With } P = 900\ \text{W} : I = \frac{1500\ \text{W}}{120\ \text{V}} + \frac{900\ \text{W}}{120\ \text{V}}\ 20\ \text{A}.$$

26.46: The current gets split evenly between all the parallel bulbs. A single bulb will draw $I = \frac{P}{V} = \frac{90\ \text{W}}{120\ \text{V}} = 0.75\ \text{A} \Rightarrow \text{Number of bulbs} \le \frac{20\ \text{A}}{0.75\ \text{A}} = 26.7.$ So you can attach 26 bulbs safely.

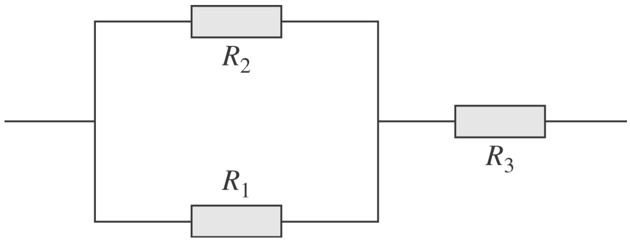
26.47: a) $I = \frac{V}{R} = \frac{120\ \text{V}}{20\ \Omega} = 6.0\ \text{A} \Rightarrow P = IV = (6.0\ \text{A})\ (120\ \text{V}) = 720\ \text{W}.$

b) At $T = 280°\text{C},\ R = R_0\ (1 + \alpha\Delta T) = 20\ \Omega\ (1 + (2.8 \times 10^{-3}\ (\text{C°})^{-1}\ (257°\text{C}))$

$$= 34.4\ \Omega.$$

$$\Rightarrow I = \frac{V}{R} = \frac{120\ \text{V}}{34.4\ \text{A}} = 3.49\ \text{A} \Rightarrow P = (3.49\ \text{A})\ (120\ \text{V}) = 419\ \text{W}.$$

26.48: a)

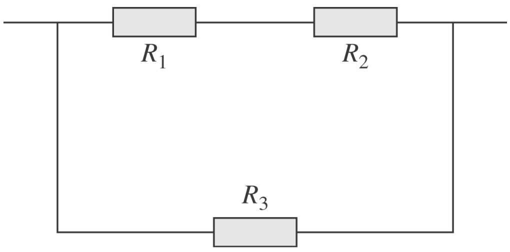


$$R_{eq} = R_3 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = R_3 + \left(\frac{R_1R_2}{R_1 + R_2}\right)$$

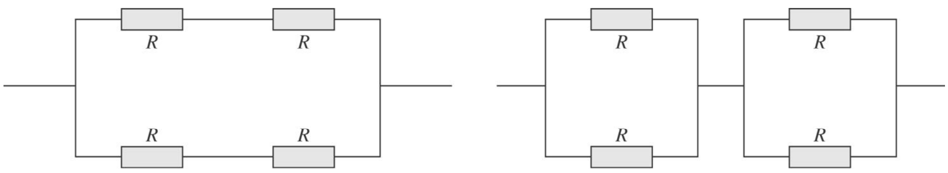
$$\text{If } R_{eq} = R_1 \Rightarrow R_3 = R_1 - \left(\frac{R_1R_2}{R_1 + R_2}\right) = \frac{R_1^2}{R_1 + R_2}.$$

b) $R_{eq} = \left(\dfrac{1}{R_1 + R_2} + \dfrac{1}{R_3}\right)^{-1} = \dfrac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$

If $R_{eq} = R_1 \Rightarrow R_1(R_1 + R_2 + R_3) = R_3(R_1 + R_2) \Rightarrow R_3 = R_1\ (R_1 + R_2)/R_2.$

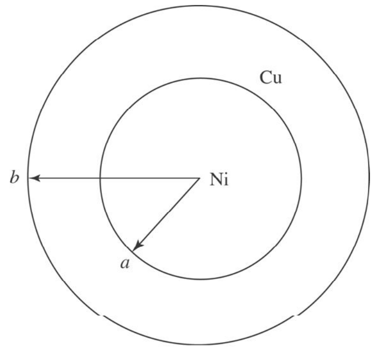


26.49: a) We wanted a total resistance of $400\ \Omega$ and power of $2.4\ \text{W}$ from a combination of individual resistors of $400\ \Omega$ and $1.2\ \text{W}$ power-rating.



b) The current is given by: $I = \sqrt{P/R} = \sqrt{2.4\ \text{W}/400\ \Omega} = 0.077\ \text{A}$. In each leg half the current flows, so the power in each resistor in each resistor in each combination is the same: $P = (I/2)^2 R = (0.039\ \text{A})^2(400\ \Omega) = 0.6\ \text{W}.$

26.50: a) First realize that the Cu and Ni cables are in parallel.



$$\frac{1}{R_{\text{Cable}}} = \frac{1}{R_{\text{Ni}}} + \frac{1}{R_{\text{Cu}}}$$

$$R_{\text{Ni}} = \rho_{\text{Ni}} L / A = \rho_{\text{Ni}} \frac{L}{\pi a^2}$$

$$R_{\text{Cu}} = \rho_{\text{Cu}}\ L/A = \rho_{\text{Cu}} \frac{L}{\pi(b^2 - a^2)}$$

So: $$\frac{1}{R_{\text{cable}}} = \frac{\pi a^2}{\rho_{\text{Ni}} L} + \frac{\pi(b^2 - a^2)}{\rho_{\text{Cu}} L}$$

$$= \frac{\pi}{L}\left(\frac{a^2}{\rho_{\text{Ni}}} + \frac{b^2 - a^2}{\rho_{\text{Cu}}}\right)$$

$$= \frac{\pi}{20\text{m}}\left[\frac{(0.050\text{ m})^2}{7.8 \times 10^{-8}\ \Omega\text{m}} + \frac{(0.100\text{ m})^2 - (0.050\text{ m})2}{1.72 \times 10^{-8}\ \Omega\text{m}}\right]$$

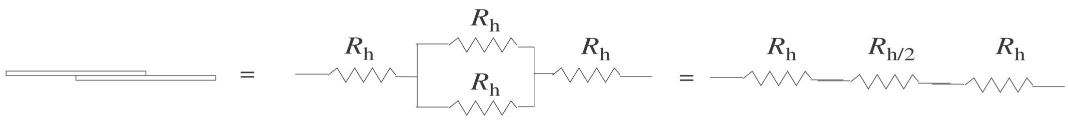
$$R_{\text{Cable}} = 13.6 \times 10^{-6}\ \Omega = 13.6\ \mu\Omega$$

b) $R = \rho_{\text{eff}} \dfrac{L}{A} = \rho_{\text{eff}} \dfrac{L}{\pi b^2}$

$$\rho_{\text{eff}} = \frac{\pi b^2\ R}{L} = \frac{\pi(0.10\text{ m})^2\ (13.6 \times 10^{-6}\ \Omega)}{20\text{ m}}$$

$$= 2.14 \times 10^{-8}\ \Omega\text{m}$$

26.51: Let $R = 1.00\ \Omega,$ the resistance of one wire. Each half of the wire has $R_{\mathrm{h}} = R/2.$



The equivalent resistance is $R_{\mathrm{h}} + R_{\mathrm{h}}/2 + R_{\mathrm{h}} = 5R_{\mathrm{h}}/2 = \tfrac{5}{2}(0.500\ \Omega)\ 1.25\ \Omega$

26.52: a) The equivalent resistance of the two bulbs is $1.0\ \Omega.$ So the current is:

$$I = \frac{V}{R_{total}} = \frac{8.0\ \mathrm{V}}{1.0\ \Omega + 0.80\ \Omega} = 4.4\ \mathrm{A} \Rightarrow \text{the current through each bulb is } 2.2\ \mathrm{A}.$$

$$V_{\mathrm{bulb}} = \varepsilon - Ir = 8.0\ \mathrm{V} - (4.4\ \mathrm{A})\ (0.80\ \Omega) = 4.4\ \mathrm{V} \Rightarrow P_{\mathrm{bulb}} = IV = (2.2\ \mathrm{A})\ (4.4\ \mathrm{V}) = 9.9\ \mathrm{W}$$

b) If one bulb burns out, then

$$I = \frac{V}{R_{total}} = \frac{8.0\ \mathrm{V}}{2.0\ \Omega + 0.80\ \Omega} = 2.9\ \mathrm{A} \Rightarrow P = I^2R = (2.9\ \mathrm{A})^2\ (2.0\ \Omega) = 16.3\ \mathrm{W},$$

so the remaining bulb is brighter than before.

26.53: The maximum allowed power is when the total current is the maximum allowed value of $I = \sqrt{P/R} = \sqrt{36\ \mathrm{W}/2.4\ \Omega} = 3.9\ \mathrm{A}.$ Then half the current flows through the parallel resistors and the maximum power is:

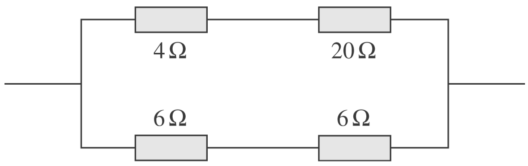
$$P_{\max} = (I/2)^2R + (I/2)^2R + I^2\ R = \frac{3}{2}I^2R = \frac{3}{2}(3.9\ \mathrm{A})^2\ (2.4\ \Omega) = 54\ \mathrm{W}.$$

26.54: a) $R_{\text{eq}}(8,16,16) = \left(\dfrac{1}{8\,\Omega} + \dfrac{1}{16\,\Omega} + \dfrac{1}{16\,\Omega}\right)^{-1} = 4.0\,\Omega;$

$$R_{\text{eq}}(9,18) = \left(\frac{1}{9\,\Omega} + \frac{1}{18\,\Omega}\right)^{-1} = 6.0\,\Omega.$$

So the circuit is equivalent to the one shown below. Thus:

$$R_{\text{eq}} = \left(\frac{1}{6\,\Omega + 6\,\Omega} + \frac{1}{20\,\Omega + 4\,\Omega}\right)^{-1} = 8.0\,\Omega$$



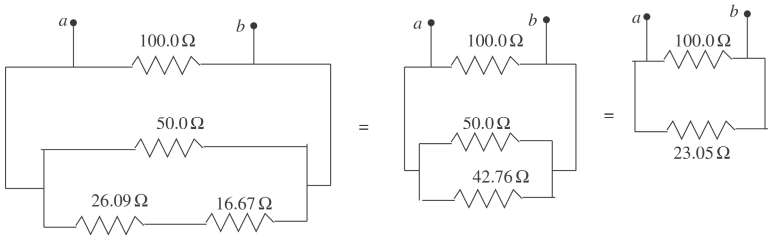
b) If the current through the $8\text{-}\Omega$ resistor is 2.4 A, then the top branch current is $I(8,16,16) = 2.4\text{ A} + \frac{1}{2}2.4\text{ A} + \frac{1}{2}2.4\text{ A} = 4.8\text{ A}.$ But the bottom branch current is twice that of the top, since its resistance is half. Therefore the potential of point a relative to point x is $V_{ax} = -IR_{\text{eq}}(9,18) = -(9.6\text{ A})(6.00\,\Omega) = -58\text{ V}.$

26.55: Circuit (a)

The $75.0\,\Omega$ and $40.0\,\Omega$ resistors are in parallel and have equivalent resistance 26.0

The $25.0\,\Omega$ and $50.0\,\Omega$ resistors are in parallel and have equivalent resistance 16.6

The network is equivalent to

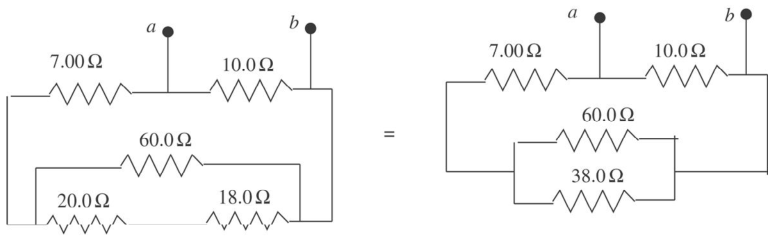


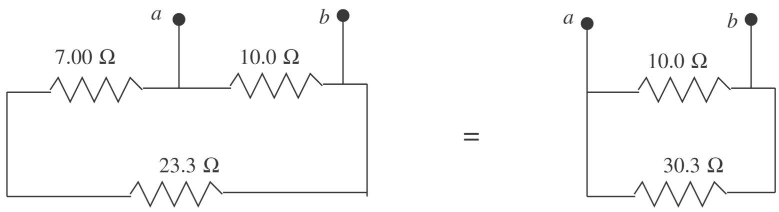
$$\frac{1}{R_{eq}} = \frac{1}{100.0\ \Omega} + \frac{1}{23.05\ \Omega} \text{ so } R_{eq} = 18.7\ \Omega$$

Circuit (b)

The
$30.0\ \Omega$ and $45.0\ \Omega$ resistors are in parallel and have equivalent resistance $18.0\ \Omega$.

The network is equivalent to





$$\frac{1}{R_{eq}} = \frac{1}{10.0\ \Omega} + \frac{1}{30.3\ \Omega} \text{ so } R_{eq} = 7.5\ \Omega$$

26.56: Recognize that the ohmmeter measures the equivalent parallel resistance, not just X.

$$\frac{1}{20.2\ \Omega} = \frac{1}{X} + \frac{1}{115\ \Omega} + \frac{1}{130\ \Omega} + \frac{1}{85\ \Omega}$$
$$X = 46.8\ \Omega$$

26.57: $\text{Top left loop}: \ 12 - 5(I_2 - I_3) - 1I_2 = 0 \Rightarrow 12 - 6I_2 + 5I_3 = 0.$

$\text{Top right loop}: 9 - 8(I_1 + I_3) - 1I_1 = 0 \Rightarrow 9 - 9I_1 - 8I_3 = 0.$
$\text{Bottom loop}: \ 12 - 10I_3 - 9 + 1I_1 - 1I_2 = 0 \Rightarrow 3 + I_1 - I_2 - 10I_3 = 0.$

Solving these three equations for the currents yields:

$I_1 = 0.848\ A,\ I_2 = 2.14\ \text{A, and}\ I_3 = 0.171\ \text{A.}$

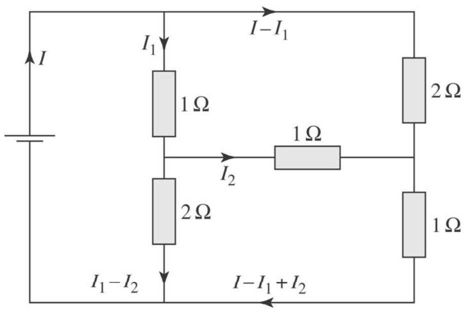
26.58: $\text{Outside loop}: 24 - 7(1.8) - 3(1.8 - I_\varepsilon) = 0 \Rightarrow I_\varepsilon = -2.0\ \text{A.}$
$\text{Right loop}: \ \varepsilon - 7(1.8) - 2(-2.0) = 0 \Rightarrow \varepsilon = 8.6\ \text{V.}$

26.59: $\text{Left loop}: \ 20 - 14 - 2I_1 + 4(I_2 - I_1) = 0 \Rightarrow 6 - 6I_1 + 4I_2 = 0.$
$\text{Right loop}: 36 - 5I_2 - 4(I_2 - I_1) = 0 \Rightarrow 36 + 4I_1 - 9I_2 = 0.$

Solving these two equations for the currents yields:

$I_1 = 5.21\ \text{A} = I_{2\Omega},\ I_2 = 6.32\ \text{A} = I_{5\Omega},\ \text{and}\ I_{4\Omega} = I_2 - I_1 = 1.11\ \text{A.}$

26.60: a) Using the currents as defined on the circuit diagram below we obtain three equations to solve for the currents:



$$\begin{aligned}
&\text{Left loop:} && 14 - I_1 - 2(I_1 - I_2) = 0 \\
& && \Rightarrow 3I_1 - 2I_2 = 14. \\
&\text{Top loop:} && -2(I - I_1) + I_2 + I_1 = 0 \\
& && \Rightarrow -2I + 3I_1 + I_2 = 0. \\
&\text{Bottom loop:} && -(I - I_1 + I_2) + 2(I_1 - I_2) - I_2 = 0 \\
& && \Rightarrow -I + 3I_1 - 4I_2 = 0.
\end{aligned}$$

Solving these equations for the currents we find:

$$I = I_{\text{battery}} = 10.0\text{ A};\ I_1 = I_{\text{R}_1} = 6.0\text{ A};\ I_2 = I_{R_3} = 2.0\text{ A}.$$

So the other currents are:

$$I_{R_2} = I - I_1 = 4.0\text{ A};\ I_{R_4} = I_1 - I_2 = 4.0\text{ A};\ I_{R_5} = I - I_1 + I_2 = 6.0\text{ A}.$$

b) $R_{\text{eq}} = \frac{V}{I} = \frac{14.0\text{ V}}{10.0\text{ A}} = 1.40\ \Omega.$

26.61: a) Going around the complete loop, we have:

$$\sum \varepsilon - \sum IR = 12.0 \text{ V} - 8.0 \text{ V} - I(9.0\ \Omega) = 0 \Rightarrow I = 0.44 \text{ A}.$$

$$\Rightarrow V_{ab} = \sum \varepsilon - \sum IR = 12.0 \text{ V} - 10.0 \text{ V} - (0.44 \text{ A})\,(2\,\Omega + 1\,\Omega + 1\,\Omega)$$

$$= +0.22 \text{ V}.$$

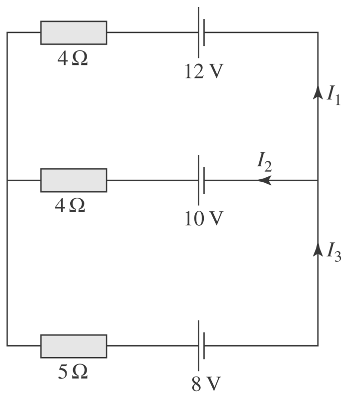
b) If now the points a and b are connected by a wire, the circuit becomes equivalent to the diagram shown below. The two loop equations for currents are (leaving out the units):

$$12 - 10 - 4I_1 + 4I_2 = 0 \Rightarrow I_2 = I_1 - 0.5$$

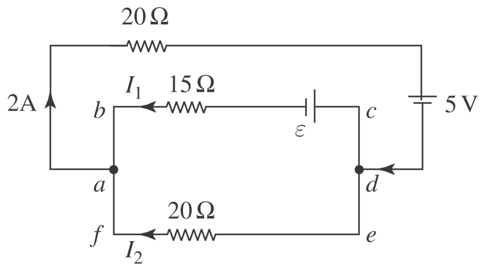
and

$$10 - 8 - 4I_2 - 5I_3 = 2 - 4I_2 - 5(I_1 + I_2) = 0$$
$$\Rightarrow 2 - (4I_1 - 2) - 5I_1 - 5I_1 + 2.5 = 0$$
$$\Rightarrow I_1 = 0.464 \text{ A}.$$

Thus the current through the 12-V battery is 0.464 A.



26.62: a) First do series/parallel reduction:



Now apply Kirchhoff's laws and solve for ε.

$$\Delta V_{\text{adefa}} = 0: -(20\ \Omega)(2\ \text{A}) - 5\ \text{V} - (20\ \Omega)I_2 = 0$$
$$I_2 = -2.25\ \text{A}$$
$$I_1 + I_2 = 2\ \text{A} \rightarrow I_1 = 2\ \text{A} - (-2.25\ \text{A}) = 4.25\ \text{A}$$

$$\Delta V_{\text{abcdefa}} = 0: (15\ \Omega)\ (4.25\ \text{A}) + \varepsilon - (20\ \Omega)\ (-2.25\ \text{A}) = 0$$
$$\varepsilon = -109\ \text{V; polarity should be reversed.}$$

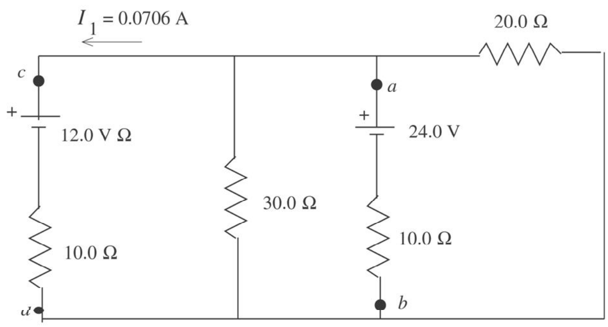
b) Parallel branch has a $10\ \Omega$ resistance.

$$\Delta V_{\text{par}} = RI = (10\ \Omega)\ (2\text{A}) = 20\ \text{V}$$

Current in upper part: $I = \frac{\Delta V}{R} = \frac{20\ \text{V}}{30\ \Omega} = \frac{2}{3}\ \text{A}$

$$Pt = U \rightarrow I^2\ Rt = U$$
$$\left(\frac{2}{3}\ \text{A}\right)^2 (10\ \Omega)t = 60\ \text{J}$$
$$t = 13.5\ \text{s}$$

26.63:



$$V_d + I_1(10.0\ \Omega) + 12.0\ \text{V} = V_c$$
$$V_c - V_d = 12.706\ \text{V};\quad V_a - V_b = V_c - V_d = 12.7\ \text{V}$$

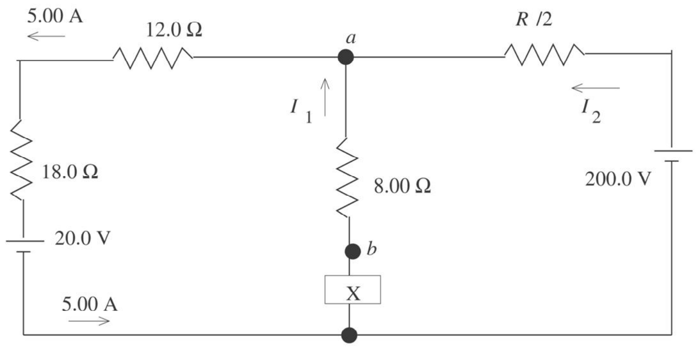
26.64: First recognize that if the 40 Ω resistor is safe, all the other resistors are also safe.

$$I^2R = P \rightarrow I^2(40\ \Omega) = 1\ \text{W}$$
$$\text{I} = 0.158\ \text{A}$$

Now use series / parallel reduction to simplify the circuit. The upper parallel branch is 6.38 Ω and the lower one is 25 Ω. The series sum is now 126 Ω. Ohm's law gives

$$\varepsilon = (126\ \Omega)(0.158\ \text{A}) = 19.9\ \text{V}$$

26.65: The 20.0-Ω and 30.0-Ω resistors are in parallel and have equivalent resistance 12.0 Ω. The two resistors *R* are in parallel and have equivalent resistance *R*/2. The circuit is equivalent to



26.66: For three identical resistors in series, $P_s = \dfrac{V^2}{3R}$. If they are now in parallel over the same voltage, $P_p = \dfrac{V^2}{R_{\text{eq}}} = \dfrac{V^2}{R/3} = \dfrac{9V^2}{3R} = 9P_s = 9(27\ \text{W}) = 243\ \text{W}.$

26.67: $P_1 = \varepsilon^2/R_1$ so $R_1 = \varepsilon^2/P_1$
$P_2 = \varepsilon^2/R_2$ so $R_2 = \varepsilon^2/P_2$

a) When the resistors are connected in parallel to the emf, the voltage across each resistor is ε and the power dissipated by each resistor is the same as if only the one resistor were connected. $P_{\text{tot}} = P_1 + P_2$

b) When the resistors are connected in series the equivalent resistance is $R_{\text{eq}} = R_1 + R_2$

$$p_{\text{tot}} = \frac{\varepsilon^2}{R_1 + R_2} = \frac{\varepsilon^2}{\varepsilon^2/P_1 + \varepsilon^2/P_2} = \frac{P_1 P_2}{P_1 + P_2}$$

26.68: a) Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is

$$\frac{1}{R_{eq}}=\frac{1}{6.00\ \Omega}+\frac{1}{3.00\ \Omega}=\frac{3}{6.00\ \Omega};\ R_{eq}=2.00\ \Omega$$

In the absence of the capacitor, the total current in the circuit (the current through the $8.00\ \Omega$ resistor) would be

$$i=\frac{\varepsilon}{R}=\frac{42.0\ \text{V}}{8.00\ \Omega+2.00\ \Omega}=4.20\ \text{A}$$

of which $2/3$, or 2.80 A, would go through the $3.00\ \Omega$ resistor and $1/3$, or 1.40 A, would go through the $6.00\ \Omega$ resistor. Since the current through the capacitor is given by

$$i=\frac{V}{R}e^{-t/RC},$$

at the instant $t=0$ the circuit behaves as through the capacitor were not present, so the currents through the various resistors are as calculated above.

b) Once the capacitor is fully charged, no current flows through that part of the circuit. The $8.00\ \Omega$ and the $6.00\ \Omega$ resistors are now in series, and the current through them is $i=\varepsilon/R=(42.0\ \text{V})/(8.00\ \Omega+6.00\ \Omega)=3.00\ \text{A}.$ The voltage drop across both the $6.00\ \Omega$ resistor and the capacitor is thus $V=iR=(3.00\ \text{A})(6.00\ \Omega)=18.0\ \text{V}.$ (There is no current through the $3.00\ \Omega$ resistor and so no voltage drop across it.) The change on the capacitor is

$$Q=CV=(4.00\times10^{-6}\ \text{farad})(18.0\ \text{V})=7.2\times10^{-5}\ \text{C}$$

26.69: a) When the switch is open, only the outer resistances have current through them. So the equivalent resistance of them is:

$$R_{eq}=\left(\frac{1}{6\ \Omega+3\ \Omega}+\frac{1}{3\ \Omega+6\ \Omega}\right)^{-1}=4.50\ \Omega\Rightarrow I=\frac{V}{R_{eq}}=\frac{36.0\ \text{V}}{4.50\ \Omega}=8.00\ \text{A}$$

$$\Rightarrow V_{ab}=\left(\frac{1}{2}8.00\ \text{A}\right)(3.00\ \Omega)-\left(\frac{1}{2}8.00\ \text{A}\right)(6.00\ \Omega)=-12.0\ \text{V}.$$

b) If the switch is closed, the circuit geometry and resistance ratios become identical to that of Problem 26.60 and the same analysis can be carried out. However, we can also use symmetry to infer the following:

$I_{6\Omega}=\frac{2}{3}I_{3\Omega}$, and $I_{\text{switch}}=\frac{1}{3}I_{3\Omega}$. From the left loop as in Problem 26.60:

$$36\ \text{V}-\left(\frac{2}{3}I_{3\Omega}\right)(6\ \Omega)-I_{3\Omega}(3\ \Omega)=0\Rightarrow I_{3\Omega}=5.14\ \text{A}\Rightarrow I_{\text{switch}}=\frac{1}{3}I_{3\Omega}=1.71\ \text{A}.$$

(c) $I_{\text{battery}}=\frac{2}{3}I_{3\Omega}+I_{3\Omega}=\frac{5}{3}I_{3\Omega}=8.57\ \text{A}\Rightarrow R_{eq}\ \frac{\varepsilon}{I_{\text{battery}}}=\frac{36.0\ \text{V}}{8.57\ \text{A}}=4.20\ \Omega.$

26.70: a) With an open switch: $V_{ab} = \varepsilon = 18.0\ \text{V},$ since equilibrium has been reached.

b) Point "a" is at a higher potential since it is directly connected to the positive terminal of the battery.

c) When the switch is closed:

$$18.0\ \text{V} = I(6.00\ \Omega + 3.00\ \Omega) \Rightarrow I = 2.00\ \text{A} \Rightarrow V_b = (2.00\ \text{A})(3.00\ \Omega) = 6.00\ \text{V}.$$

d) Initially the capacitor's charges were:

$$Q_3 = CV = (3.00 \times 10^{-6}\ \text{F})(18.0\ \text{V}) = 5.40 \times 10^{-5}\ \text{C}.$$

$$Q_6 = CV = (6.00 \times 10^{-6}\ \text{F})(18.0\ \text{V}) = 1.08 \times 10^{-4}\ \text{C}.$$

After the switch is closed:

$$Q_3 = CV = (3.00 \times 10^{-6}\ \text{F})(18.0\ \text{V} - 12.0\ \text{V}) = 1.80 \times 10^{-5}\ \text{C}.$$

$$Q_6 = CV = (6.00 \times 10^{-6}\ \text{F})(18.0\ \text{V} - 6.0\ \text{V}) = 7.20 \times 10^{-5}\ \text{C}.$$

So both capacitors lose $3.60 \times 10^{-5}\ \text{C}.$

26.71: a) With an open switch:

$$Q_3 = C_{\text{eq}}V = (2.00 \times 10^{-6}\ \text{F})(18.0\ \text{V}) = 3.60 \times 10^{-5}\ \text{C}.$$

Also, there is a current in the left branch:

$$I = \frac{18.0\ \text{V}}{6.00\ \Omega + 3.00\ \Omega} = 2.00\ \text{A}.$$

So, $V_{ab} = V_{6\mu F} - V_{6\Omega} = \dfrac{Q_{6\mu F}}{C} - IR_{6\Omega} = \dfrac{3.6 \times 10^{-5}\ \text{C}}{6.0 \times 10^{-6}\ \text{F}} - (2.0\ \text{A})(6.0\ \Omega) = -6.00\ \text{V}.$

b) Point "b" is at the higher potential.

c) If the switch is closed:

$$V_b = V_a = (2.00\ \text{A})(3.00\ \Omega) = 6.00\ \text{V}.$$

d) New charges are:

$$Q_3 = CV = (3.00 \times 10^{-6}\ \text{F})(6.0\ \text{V}) = 1.80 \times 10^{-5}\ \text{C}.$$

$$Q_6 = CV = (6.00 \times 10^{-6}\ \text{F})(-12.0\ \text{V}) = -7.20 \times 10^{-5}\ \text{C}.$$

$$\Rightarrow \Delta Q_3 = +3.60 \times 10^{-5}\ \text{C} - (1.80 \times 10^{-5}\ \text{C}) = +1.80 \times 10^{-5}\ \text{C}.$$

$$\Rightarrow \Delta Q_6 = -3.60 \times 10^{-5}\ \text{C} - (-7.20 \times 10^{-5}\ \text{C}) = +3.60 \times 10^{-5}\ \text{C}.$$

So the total charge flowing through the switch is $5.40 \times 10^{-5}\ \text{C}.$

26.72: The current for full-scale deflection is 0.02 A. From the circuit we can derive three equations:

(i) $(R_1 + R_2 + R_3)(0.100\text{ A} - 0.02\text{ A}) = 48.0\ \Omega(0.02\text{ A})$

$\Rightarrow R_1 + R_2 + R_3 = 12.0\ \Omega.$

(ii) $(R_1 + R_2)(1.00\text{ A} - 0.02\text{ A}) = (48.0\ \Omega + R_3)(0.02\text{ A})$

$\Rightarrow R_1 + R_2 - 0.0204 R_3 = 0.980\ \Omega.$

(iii) $R_1(10.0\text{ A} - 0.02\text{ A}) = (48.0\ \Omega + R_2 + R_3)(0.02\text{ A})$

$\Rightarrow R_1 - 0.002 R_2 - 0.002 R_3 = 0.096\ \Omega.$

From (i) and (ii) $\Rightarrow R_3 = 10.8\ \Omega.$

From (ii) and (iii) $R_2 = 1.08\ \Omega.$ And so $\Rightarrow R_1 = 0.12\ \Omega.$

26.73: From the 3-V range:

$$(1.00 \times 10^{-3}\text{ A})(40.0\ \Omega + R_1) = 3.00\text{ V} \Rightarrow R_1 = 2960\ \Omega \Rightarrow R_{overall} = 3000\ \Omega.$$

From the 15-V range:

$$(1.00 \times 10^{-3}\text{ A})(40.0\ \Omega + R_1 + R_2) = 15.0\text{ V} \Rightarrow R_2 = 12000\ \Omega \Rightarrow R_{overall} = 15000\ \Omega.$$

From the 150-V range:

$$(1.00 \times 10^{-3}\text{ A})(40.0\ \Omega + R_1 + R_2 + R_3) = 150\text{ V} \Rightarrow R_2 = 135{,}000\ \Omega$$

$$\Rightarrow R_{overall} = 150\text{ k}\Omega.$$

26.74: a) $R_{\text{eq}} = 100\text{ k}\Omega + \left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 140\text{ k}\Omega.$

$$\Rightarrow I = \frac{0.400\text{ kV}}{140\text{ k}\Omega} = 2.86 \times 10^{-3}\text{ A}.$$

$$\Rightarrow V_{200\text{k}\Omega} = I\ R = (2.86 \times 10^{-3}\text{ A})\left(\frac{1}{200\text{ k}\Omega} + \frac{1}{50\text{ k}\Omega}\right)^{-1} = 114.4\text{ V}.$$

b) *If* $V_R = 5.00 \times 10^6\ \Omega,$ then we carry out the same calculations as above to find $R_{\text{eq}} = 292\text{ k}\Omega \Rightarrow I = 1.37 \times 10^{-3}\text{ A} \Rightarrow V_{200\text{k}\Omega} = 263\text{ V}.$

c) If $V_R = \infty$, then we find $R_{\text{eq}} = 300\text{ k}\Omega \Rightarrow I = 1.33 \times 10^{-3}\text{ A} \Rightarrow V_{200\text{k}\Omega} = 266\text{ V}.$

26.75: $I = \dfrac{110\text{ V}}{(30\text{ k}\Omega + R)} \Rightarrow V = 100\text{ V} - \dfrac{(110\text{ V})30\text{ k}\Omega}{(30\text{ k}\Omega + R)} = 68\text{ V}.$

$$\Rightarrow (68\text{ V})(30\text{ k}\Omega + R) = (110\text{ V})30\text{ k}\Omega \Rightarrow R = 18.5\text{ k}\Omega.$$

26.76: a) $V = IR + IR_A \Rightarrow R = \frac{V}{I} - R_A$. The true resistance R is always less than the reading because in the circuit the ammeter's resistance causes the current to be less then it should. Thus the smaller current requires the resistance R to be calculated larger than it should be.

b) $I = \frac{V}{R} + \frac{V}{R_V} \Rightarrow R = \frac{VR_V}{IR_V - V} = \frac{V}{I - V/R_V}$. Now the current measured is greater than that through the resistor, so $R = V/I_R$ is always greater than V/I.

c) (a): $P = I^2R = I^2(V/I - R_A) = IV - I^2R_A$.

(b): $P = V^2/R = V(I - V/R_V) = IV - V^2/R_V$.

26.77: a) When the bridge is balanced, no current flows through the galvanometer:

$$I_G = 0 \Rightarrow V_N = V_P \Rightarrow NI_{NM} = PI_{PX} \Rightarrow N\frac{(P+X)}{(P+X+N+M)} = P\frac{(N+M)}{(P+X+N+M)}$$

$$\Rightarrow N(P+X) = P(N+M) \Rightarrow NX = PM \Rightarrow X = \frac{MP}{N}.$$

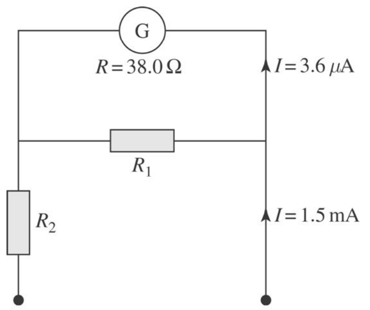
(b) $$X = \frac{(8.50\,\Omega)(33.48\,\Omega)}{15.00\,\Omega} = 1897\,\Omega.$$

26.78: In order for the second galvanometer to give the same full-scale deflection and to have the same resistance as the first, we need two additional resistances as shown below. So:

$$(3.6\ \mu\text{A})(38.0\ \Omega) = (1.496\ \text{mA})R_1 \Rightarrow R_1 = 91.4\ \text{m}\Omega.$$

And for the total resistance to be 65 Ω:

$$65 = R_2 + \left(\frac{1}{38.0\ \Omega} + \frac{1}{0.0914\ \Omega}\right)^{-1} \Rightarrow R_2 = 64.9\ \Omega.$$



26.79: a) $I = \dfrac{90V}{(224\,\Omega + 589\,\Omega)} = 0.111\text{ A.}$

$$\Rightarrow V_{224\Omega} = (0.111\text{ A})(224\,\Omega) = 24.9\text{ V.}$$

$$\Rightarrow V_{589\Omega} = (0.111\text{ A})(589\,\Omega) = 65.4\text{ V.}$$

b) $I = \dfrac{90\text{ V}}{589\,\Omega + (\frac{1}{R_V} + \frac{1}{224\,\Omega})^{-1}}$ and $V_{224\Omega} = \varepsilon - IR_{589\Omega}$

$$\Rightarrow 23.8\text{ V} = 90\text{ V} - \frac{(90\text{ V})(589\,\Omega)}{589\,\Omega + \left(\frac{1}{R_V} + \frac{1}{224\,\Omega}\right)^{-1}}$$

$$\Rightarrow \left(\frac{1}{R_V} + \frac{1}{224\,\Omega}\right)^{-1} = 211.8\,\Omega \Rightarrow R_V = 3874\,\Omega.$$

c) If the voltmeter is connected over the $589\text{-}\Omega$ resistor, then:

$$R_{\text{eq}} = 224\,\Omega + \left(\frac{1}{3874} + \frac{1}{589\,\Omega}\right)^{-1} = 735\,\Omega$$

$$\Rightarrow I = \frac{90\text{ V}}{735\,\Omega} = 0.122\text{ A} = I_V + I_{589\Omega} \text{ also } 3874 I_V = 589 I_{589\Omega}$$

$$\Rightarrow I_{589\Omega} = \frac{0.122\text{ A}}{(1 + \frac{589}{3875})} = 0.106\text{ A} \Rightarrow V_{589\Omega} = I_{589\Omega}\, R = (0.106\text{ A})(589\Omega) = 62.4\text{ V.}$$

d) No. From the equation in part (b) one can see that any voltmeter with finite resistance R_V placed in parallel with any other resistance will always decrease the measured voltage.

26.80: a) (i) $P_R = \dfrac{V^2}{R} = \dfrac{(120\text{ V})^2}{4.26\,\Omega} = 3380\text{ W}$ (ii) $P_C = \dfrac{dU}{dt} = \dfrac{1}{2C}\dfrac{d(q^2)}{dt} = \dfrac{iq}{C} = 0.$

(iii) $P_\varepsilon = \varepsilon I = (120\text{ V})\dfrac{120\text{ V}}{4.26\,\Omega} = 3380\text{ W.}$

b) After a long time, $i = 0 \Rightarrow P_R = 0,\ P_C = 0,\ P_\varepsilon = 0.$

26.81: a) If the given capacitor was fully charged for the given emf, $Q_{max} = CV =$ $(3.4 \times 10^{-6}\ \text{F})(180\ \text{V}) = 6.12 \times 10^{-4}\ \text{C}$. Since it has more charge than this *after* it was connected, this tells us the capacitor is discharging and so the current must be flowing toward the negative plate. The capacitor started with more charge than was "allowed" for the given emf. Let

$Q(t=0) = Q_0$ and $Q(t=\infty) = Q_f$. For all t, $Q(t) = (Q_0 - Q_f)e^{-t/RC} + Q_f$

We are given Q at some time $t = T; Q(t=T) = 8.15 \times 10^{-4}\,\text{C}$ and from above $Q_f = 6.12 \times 10^{-4}\,\text{C}$. The current $I(t) = \frac{dQ(t)}{dt} = \frac{(Q_0 - Q_f)}{RC} e^{-t/RC}$. At $t = T$, $Q(T) =$ $(Q_0 - Q_f)e^{-T/RC} + Q_f$. So the current at $t = T$ *is* $I(T) = \frac{-(Q_0 - Q_f)}{RC}(-e^{T/RC}) = \frac{-(Q(T) - Q_f)}{RC}$.

Thus $I(T) = \frac{-8.15 \times 10^{-4}\ \text{C} + 6.12 \times 10^{-4}\ \text{C}}{(7.25 \times 10^{3}\,\Omega)(3.40 \times 10^{-6}\ F)} = -8.24 \times 10^{-3}\,\text{A}$ (toward the negative plate).

b) As time goes on, the capacitor will discharge to $6.12 \times 10^{-4}\ \text{C}$ as calculated above.

26.82: For a charged capacitor, connected into a circuit:

$$I_0 = \frac{Q_0}{RC} \Rightarrow Q_0 = I_0 RC = (0.620\ \text{A})(5.88\ \text{k}\Omega)(8.55 \times 10^{-10}\ \text{F}) = 3.12 \times 10^{-6}\ \text{C}.$$

26.83: $\varepsilon = I_0 R \Rightarrow R = \dfrac{\varepsilon}{I_0} = \dfrac{110\ \text{V}}{6.5 \times 10^{-5}\ \text{A}} = 1.69 \times 10^{6}\ \Omega \Rightarrow$

$$C = \frac{\tau}{R} = \frac{6.2\ \text{s}}{1.69 \times 10^{6}\ \Omega} = 3.67 \times 10^{-6}\ \text{F}.$$

26.84: a) $U_0 = \dfrac{Q_0^{\,2}}{2C} = \dfrac{(0.0081\,\text{C})^2}{2(4.62 \times 10^{-6}\ \text{F})} = 7.10\ \text{J}.$

b) $P_0 = I_0^{\,2} R = \left(\dfrac{Q_0}{RC}\right)^2 R = \dfrac{(0.0081\,\text{C})^2}{(850\,\Omega)(4.62 \times 10^{-6}\ \text{F})^2} = 3616\ \text{W}.$

c) When $U = \dfrac{1}{2}U_0 = \dfrac{1}{2}\dfrac{Q_0^{\,2}}{2C}$

$$\Rightarrow Q = \frac{Q_0}{\sqrt{2}} \Rightarrow P = \left(\frac{Q}{RC}\right)^2 R = \frac{1}{2}\left(\frac{Q_0}{RC}\right)^2 R = \frac{1}{2}P_0 = 1808\ \text{W}.$$

26.85: a) We will say that a capacitor is discharged if its charge is less than that of one electron, The time this takes is then given by:

$$q = Q_0 e^{-t/RC} \Rightarrow t = RC \ln (Q_0 / e)$$

$$\Rightarrow t = (6.7 \times 10^5\ \Omega)(9.2 \times 10^{-7}\ \text{F}) \ln (7.0 \times 10^{-6}\ \text{C} / 1.6 \times 10^{-19}\ \text{C}) = 19.36\ \text{s},$$

or 31.4 time constants.

b) As shown in (a), $t = \tau \ln (Q_0 / q)$, and so the number of time constants required to discharge the capacitor is independent of R and C, and depends only on the initial charge.

26.86: a) The equivalent capacitance and time constant are:

$$C_{\text{eq}} = \left(\frac{1}{3\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}} \right)^{-1} = 2.00 \mu\text{F} \Rightarrow \tau = R_{total} C_{\text{eq}} = (6.00\ \Omega)(2.00\ \mu\text{F}) = 1.20 \times 10^{-5}\ \text{s}.$$

b) After $t = 1.20 \times 10^{-5}$ s, $q = Q_f (1 - e^{-t/RC_{\text{eq}}}) = C_{\text{eq}} \varepsilon (1 - e^{-t/RC_{\text{eq}}})$

$$\Rightarrow V_{3\mu\text{F}} = \frac{q}{C_{3\mu\text{F}}} = \frac{C_{\text{eq}} \varepsilon}{C_{3\mu\text{F}}} (1 - e^{-t/RC_{\text{eq}}}) = \frac{(2.0\ \mu\text{F})(12\ \text{V})}{3.0\ \mu\text{F}} (1 - e^{-1}) = 5.06\ \text{V}.$$

26.87: a) $E_{total} = \int_0^\infty P_\varepsilon \, dt = \int_0^\infty \varepsilon I \, dt = \frac{\varepsilon^2}{R} \int e^{-t/RC} \, dt = \varepsilon^2 C(1) = \varepsilon^2 C.$

b) $E_R = \int_0^\infty P_R \, dt = \int_0^\infty i^2 R \, dt = \frac{\varepsilon^2}{R} \int_0^\infty e^{-2t/RC} \, dt = \frac{1}{2} \varepsilon^2 C.$

c) $U = \frac{Q_0^2}{2C} = \frac{V^2 C}{2} = \frac{1}{2} \varepsilon^2 C = E_{total} - E_R.$

d) One half of the energy is stored in the capacitor, regardless of the sizes of the resistor.

26.88: $i = -\frac{Q_0}{RC} e^{-t/RC} \Rightarrow P = i^2 R = \frac{Q_0^2}{RC^2} e^{-2t/RC} \Rightarrow E = \frac{Q_0^2}{RC^2} \int_0^\infty e^{-2t/RC} \, dt$

$$= \frac{Q_0^2}{RC^2} \frac{RC}{2} = \frac{Q_0^2}{2C} = U_0.$$

26.89: a) Using Kirchhoff's Rules on the circuit we find:

Left loop: $92 - 140I_1 - 210I_2 + 55 = 0 \Rightarrow 147 - 140I_1 - 210I_2 = 0.$

Right loop: $57 - 35I_3 - 210I_2 + 55 = 0 \Rightarrow 112 - 210I_2 - 35I_3 = 0.$

Currents: $\Rightarrow I_1 - I_2 + I_3 = 0.$

Solving for the three currents we have:

$$I_1 = 0.300 \text{ A}, \quad I_2 = 0.500 \text{ A}, \quad I_3 = 0.200 \text{ A}.$$

b) Leaving only the 92-V battery in the circuit:

Left loop: $92 - 140I_1 - 210I_2 = 0.$

Right loop: $-35I_3 - 210I_2 = 0.$

Currents: $I_1 - I_2 + I_3 = 0.$

Solving for the three currents:

$$I_1 = 0.541 \text{ A}, \quad I_2 = 0.077 \text{ A}, \quad I_3 = -0.464 \text{ A}.$$

c) Leaving only the 57-V battery in the circuit:

Left loop: $140I_1 + 210I_2 = 0.$

Right loop: $57 - 35I_3 - 210I_2 = 0.$

Currents: $I_1 - I_2 + I_3 = 0.$

Solving for the three currents:

$$I_1 = -0.287 \text{ A}, \quad I_2 = 0.192 \text{ A}, \quad I_3 = 0.480 \text{ A}.$$

d) Leaving only the 55-V battery in the circuit:

Left loop: $55 - 140I_1 - 210I_2 = 0.$

Right loop: $55 - 35I_3 - 210I_2 = 0.$

Currents: $I_1 - I_2 + I_3 = 0.$

Solving for the three currents:

$$I_1 = 0.046 \text{ A}, \quad I_2 = 0.231 \text{ A}, \quad I_3 = 0.185 \text{ A}.$$

e) If we sum the currents from the previous three parts we find:

$I_1 = 0.300 \text{ A}, \quad I_2 = 0.500 \text{ A}, \quad I_3 = 0.200 \text{ A},$ just as in part (a).

f) Changing the 57-V battery for an 80-V battery just affects the calculation in part (c). It changes to:

Left loop: $140I_1 + 210I_2 = 0.$

Right loop: $80 - 35I_3 - 210I_2 = 0.$

Currents: $I_1 - I_2 + I_3 = 0.$

Solving for the three currents:

$$I_1 = -0.403 \text{ A}, \quad I_2 = 0.269 \text{ A}, \quad I_3 = 0.672 \text{ A}.$$

So the total current for the full circuit is the sum of (b), (d) and (f) above:

$$I_1 = 0.184 \text{ A}, \quad I_2 = 0.576 \text{ A}, \quad I_3 = 0.392 \text{ A}.$$

26.90: a) Fully charged:

$$Q = CV = (10.0 \times 10^{-12}\ \text{F})(1000\ \text{V}) = 1.00 \times 10^{-8}\ \text{C}.$$

b) $i_0 = \dfrac{\varepsilon - V_{C'}}{R} = \dfrac{\varepsilon}{R} - \dfrac{q}{RC'} \Rightarrow i(t) = \left(\dfrac{\varepsilon}{R} - \dfrac{q}{RC'}\right)e^{-t/RC'}$, where $C' = 1.1C.$

c) We need a resistance such that the current will be greater than $1\ \mu\text{A}$ for longer than $200\ \mu\text{s}.$

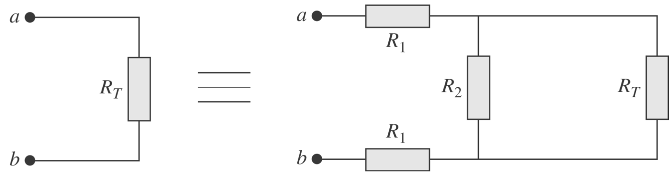
$$\Rightarrow i(200\ \mu s) = 1.0 \times 10^{-6}\ \text{A} = \frac{1}{R}\left(1000\ \text{V} - \frac{1.0 \times 10^{-8}\ \text{C}}{1.1(1.0 \times 10^{-11}\ \text{F})}\right)e^{-\frac{2.0 \times 10^{-4}\ \text{s}}{R(11 \times 10^{-12}\ \text{F})}}$$

$$\Rightarrow 1.0 \times 10^{-6}\ \text{A} = \frac{1}{R}(90.9)e^{-(1.8 \times 10^{7}\ \Omega)/R} \Rightarrow 18.3R - R\ln R - 1.8 \times 10^{7} = 0.$$

Solving for R numerically we find $7.15 \times 10^{6}\ \Omega \le R \le 7.01 \times 10^{7}\ \Omega.$

If the resistance is too small, then the capacitor discharges too quickly, and if the resistance is too large, the current is not large enough.

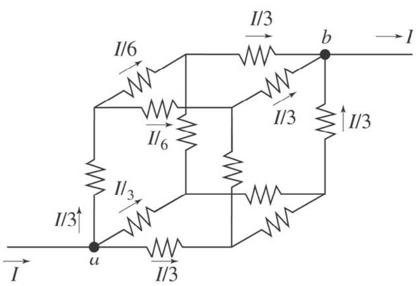
26.91: We can re-draw the circuit as shown below:



$$\Rightarrow R_T = 2R_1 + \left(\frac{1}{R_2} + \frac{1}{R_T}\right)^{-1} = 2R_1 + \frac{R_2 R_T}{R_2 + R_T} \Rightarrow R_T{}^2 - 2R_1R_T - 2R_1R_2 = 0.$$

$$\Rightarrow R_T = R_1 \pm \sqrt{R_1{}^2 + 2R_1R_2} \text{ but } R_T > 0 \Rightarrow R_T = R_1 + \sqrt{R_1{}^2 + 2R_1R_2}.$$

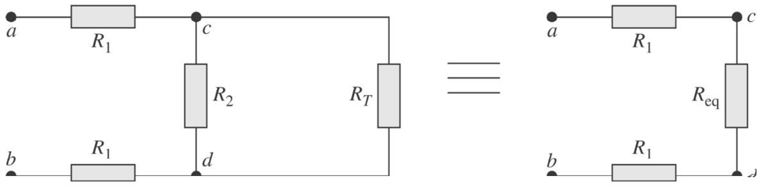
26.92:



Let current I enter at a and exit at b. At a there are three equivalent branches, so current is $I/3$ in each. At the next junction point there are two equivalent branches so each gets current $I/6$. Then at b there are three equivalent branches with current $I/3$ in each. The voltage drop from a to b then is $V = \left(\frac{I}{3}\right)R + \left(\frac{I}{6}\right)R + \left(\frac{I}{3}\right)R = \frac{5}{6}IR.$

This must be the same as $V = IR_{\text{eq}}$, so $R_{\text{eq}} = \dfrac{5}{6}R.$

26.93: a) The circuit can be re-drawn as follows:



Then $V_{cd} = V_{ab}\dfrac{R_{\rm eq}}{2R_1 + R_{\rm eq}} = V_{ab}\dfrac{1}{2R_1/R_{\rm eq}+1}$ and $R_{\rm eq} = \dfrac{R_2 R_T}{R_2 + R_T}$.

But $\beta = \dfrac{2R_1(R_T + R_2)}{R_T R_2} = \dfrac{2R_1}{R_{\rm eq}} \Rightarrow V_{cd} = V_{ab}\dfrac{1}{1+\beta}$.

b) Recall $V_1 = \dfrac{V_0}{(1+\beta)} \Rightarrow V_2 = \dfrac{V_1}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^2} \Rightarrow V_n = \dfrac{V_{n-1}}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^n}$.

If $R_1 = R_2 \Rightarrow R_T = R_1 + \sqrt{R_1^{\;2} + 2R_1R_1} = R_1(1+\sqrt{3})$ and $\beta = \dfrac{2(2+\sqrt{3})}{1+\sqrt{3}} = 2.73$. So, for the nth segment to have 1% of the original voltage, we need:

$$\frac{1}{(1+\beta)^n} = \frac{1}{(1+2.73)^n} \le 0.01 \Rightarrow n = 4 : V_4 = 0.005V_0.$$

c) $R_T = R_1 + \sqrt{R_1^{\;2} + 2R_1R_2}$

$$\Rightarrow R_T = 6400\,\Omega + \sqrt{(6400\,\Omega)^2 + 2(6400\,\Omega)\,(8.0\times10^8\ \Omega)} = 3.2\times10^6\ \Omega.$$

$$\Rightarrow \beta = \frac{2(6400\,\Omega)\,(3.2\times10^6\ \Omega + 8.0\times10^8\ \Omega)}{(3.2\times10^6\ \Omega)\,(8.0\times10^8\ \Omega)} = 4.0\times10^{-3}.$$

d) Along a length of 2.0 mm of axon, there are 2000 segments each 1.0 μm long. The voltage therefore attenuates by:

$$V_{2000} = \frac{V_0}{(1+\beta)^{2000}} \Rightarrow \frac{V_{2000}}{V_0} = \frac{1}{(1+4.0\times10^{-3})^{2000}} = 3.4\times10^{-4}.$$

e) If $R_2 = 3.3\times10^{12}\ \Omega \Rightarrow R_T = 2.1\times10^8\ \Omega$ and $\beta = 6.2\times10^{-5}$.

$$\Rightarrow \frac{V_{2000}}{V_0} = \frac{1}{(1+6.2\times10^{-5})^{2000}} = 0.88.$$

27.1: a) $\vec{F} = q\vec{v} \times \vec{B}(-1.24 \times 10^{-8}\ \text{C})(-3.85 \times 10^{4}\ \text{m/s})(1.40\ \text{T})(\hat{j} \times \hat{i})$

$\Rightarrow \vec{F} = -(6.68 \times 10^{-4}\ \text{N})\hat{k}.$

b) $\vec{F} = q\vec{v} \times \vec{B}$

$\Rightarrow \vec{F} = (-1.24 \times 10^{-8}\ \text{C})(1.40\ \text{T})[(-3.85 \times 10^{4}\ \text{m/s})(\hat{j} \times \hat{k}) + (4.19 \times 10^{4}\ \text{m/s})(\hat{i} \times$

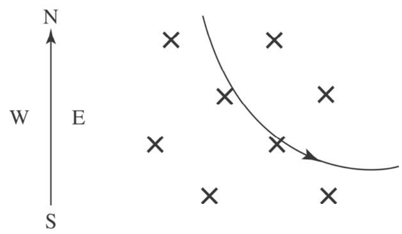
$\Rightarrow \vec{F} = (6.68 \times 10^{-4}\ \text{N})\ \hat{i} + (7.27 \times 10^{-4}\ \text{N})\ \hat{j}.$

27.2: Need a force from the magnetic field to balance the downward gravitational force. Its magnitude is:

$$qvB = mg \Rightarrow B = \frac{mg}{qv} = \frac{(1.95 \times 10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{(2.50 \times 10^{-8}\ \text{C})(4.00 \times 10^{4}\ \text{m/s})} = 1.91\ \text{T}.$$

The right-hand rule requires the magnetic field to be to the east, since the velocity is northward, the charge is negative, and the force is upwards.

27.3: By the right-hand rule, the charge is positive.

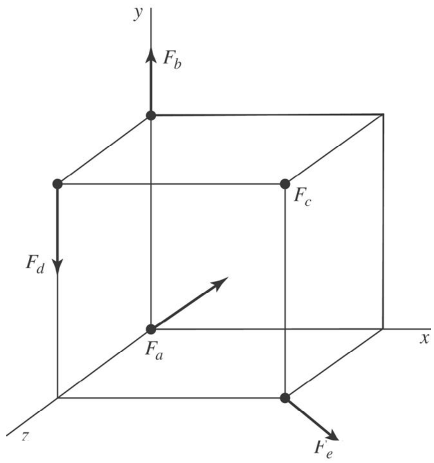


27.4: $\vec{F} = m\vec{a} = q\vec{v} \times \vec{B} \Rightarrow \vec{a} = \dfrac{q\vec{v} \times \vec{B}}{m}$

$$\Rightarrow \vec{a} = \frac{(1.22 \times 10^{-8}\ \text{C})(3.0 \times 10^{4}\ \text{m/s})(1.63\ \text{T})(\hat{j} \times \hat{i})}{1.81 \times 10^{-3}\ \text{kg}} = -(0.330\ \text{m/s}^2)\hat{k}.$$

27.5: See figure on next page. Let $F_0 = qvB$, then:

$$F_a = F_0 \text{ in the } -\hat{\boldsymbol{k}} \text{ direction}$$
$$F_b = F_0 \text{ in the } +\hat{\boldsymbol{j}} \text{ direction}$$
$$F_c = 0, \text{ since } B \text{ and velocity are parallel}$$
$$F_d = F_0 \sin 45^{\circ} \text{ in the } -\hat{\boldsymbol{j}} \text{ direction}$$
$$F_e = F_0 \text{ in the } -(\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}) \text{ direction}$$



27.6: a) The smallest possible acceleration is zero, when the motion is parallel to the magnetic field. The greatest acceleration is when the velocity and magnetic field are at right angles:

$$a = \frac{qvB}{m} = \frac{(1.6\times10^{-19}\ \text{C})(2.50\times10^{6}\ \text{m/s})(7.4\times10^{-2}\ \text{T})}{(9.11\times10^{-31}\ \text{kg})} = 3.25\times10^{16}\ \text{m/s}^2.$$

b) If $a = \frac{1}{4}(3.25\times10^{16}\ \text{m/s}^2) = \frac{qvB\sin\phi}{m} \Rightarrow \sin\phi = 0.25 \Rightarrow \phi = 14.5^{\circ}.$

27.7: $F = |q| v\, B \sin\phi \Rightarrow v = \dfrac{F}{|q|\, B\sin\phi} = \dfrac{4.60\times10^{-15}\ \text{N}}{(1.6\times10^{-19}\ \text{C})(3.5\times10^{-3}\ \text{T})\sin 60^{\circ}}$

$$= 9.49\times10^{6}\ \text{m/s}.$$

27.8: a) $\vec{F} = q\vec{v} \times \vec{B} = qB_z[v_x(\hat{\boldsymbol{i}} \times \hat{\boldsymbol{k}}) + v_y(\hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}}) + v_z(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}})] = qB_z[v_x\ (-\hat{\boldsymbol{j}}) + v_y(\hat{\boldsymbol{i}})].$

Set this equal to the given value of $\vec{F}$ to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40 \times 10^{-7}\ \text{N})}{-(-5.60 \times 10^{-9}\ \text{C})(-1.25\ \text{T})} = -106\ \text{m/s}$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40 \times 10^{-7}\ \text{N})}{(-5.60 \times 10^{-9}\ \text{C})(-1.25\ \text{T})} = -48.6\ \text{m/s}.$$

b) The value of v_z is indeterminate.

c) $\vec{v} \cdot \vec{F} = v_x F_x + v_y F_y + v_z F_z = \dfrac{F_y}{-qB_z} F_x + \dfrac{F_x}{qB_z} F_y = 0; \theta = 90^\circ.$

27.9: $\vec{F} = q\vec{v} \times \vec{B}, \vec{v} = v_y \hat{\boldsymbol{j}}\ with\ v_y = -3.80 \times 10^3 \text{m/s}$

$F_x = +7.60 \times 10^{-3}$ N, $F_y = 0$, and $F_z = -5.20 \times 10^{-3}$ N

$F_x = q(v_y B_z - v_z B_y) = qv_y B_z$

$B_z = F_x/qv_y = (7.60 \times 10^{-3}\ \text{N})/([7.80 \times 10^{-6}\ \text{C})(-3.80 \times 10^3 \text{m/s})] = -0.256\ \text{T}$

$F_y = q(v_z B_x - v_x B_z) = 0,$ which is consistent with $\vec{F}$ as given in the problem. No force component along the direction of the velocity.

$F_z = q(v_x B_y - v_y B_x) = -qv_y B_x$

$B_x = -F_z/qv_y = -0.175\ \text{T}$

b) B_y is not determined. No force due to this component of $\vec{B}$ along $\vec{v}$; measurement of the force tells us nothing about B_y.

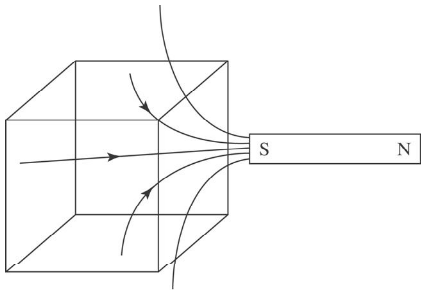
c) $\vec{B} \cdot \vec{F} = B_x F_x + B_y F_y + B_z F_z = (-0.175\ \text{T})(+7.60 \times 10^{-3}\ \text{N}) +$
$(-0.256\ \text{T})(-5.20 \times 10^{-3}\ \text{N})$

$\vec{B} \cdot \vec{F} = 0$; $\vec{B}$ and $\vec{F}$ are perpendicular (angle is 90°)

27.10: a) The total flux must be zero, so the flux through the remaining surfaces must be $-0.120\,\text{Wb}$.

b) The shape of the surface is unimportant, just that it is closed.

c)



27.11: a) $\Phi_B = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = (0.230\text{ T})\pi(0.065\text{ m})^2 = 3.05 \times 10^{-3}\ \text{Wb}.$

b) $\Phi_B = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = (0.230\text{ T})\pi(0.065\text{ m})^2 \cos 53.1° = 1.83 \times 10^{-3}\ \text{Wb}.$

c) $\Phi_B = 0$ since $\vec{\boldsymbol{B}} \perp \vec{\boldsymbol{A}}$.

27.12: a) $\Phi_B(abcd) = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = 0.$

b) $\Phi_B(befc) = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = -(0.128\text{T})(0.300\text{ m})(0.300\text{ m}) = -0.0115\ \text{Wb}.$

c) $\Phi_B(aefd) = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = BA\cos\phi = \frac{3}{5}(0.128\text{ T})(0.500\text{ m})(0.300\text{ m}) = +0.0115\ \text{Wb}.$

d) The net flux through the rest of the surfaces is zero since they are parallel to the x-axis so the total flux is the sum of all parts above, which is zero.

27.13: a) $\vec{\boldsymbol{B}} = [(\beta - \gamma y^2)]\hat{\boldsymbol{j}}$ and we can calculate the flux through each surface. Note that there is no flux through any surfaces parallel to the y-axis. Thus, the total flux through the closed surface is:

$$\Phi_B(abe) = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = ([-(0.300\text{ T} - 0)] + [0.300\text{ T} - (2.00\text{ T/m}^2)(0.300\text{ m})^2])$$
$$\times \frac{1}{2}(0.400\text{ m})(0.300\text{ m})$$
$$= -0.0108\ \text{Wb}.$$

b) The student's claim is implausible since it would require the existence of a magnetic monopole to result in a net non-zero flux through the closed surface.

27.14: a) $p = mv = m\left(\dfrac{RqB}{m}\right) = RqB = (4.68\times10^{-3}\text{ m})(6.4\times10^{-19}\text{ C})(1.65\text{ T})$

$= 4.94\times10^{-21}\text{ kg m/s}$.

b) $L = Rp = R^2qB = (4.68\times10^{-3}\text{ m})^2(6.4\times10^{-19}\text{ C})(1.65\text{ T}) = 2.31\times10^{-23}\text{ kg m}^2\text{/s}$.

27.15: a) $B = \dfrac{mv}{|q|R} = \dfrac{(9.11\times10^{-31}\text{ kg})(1.41\times10^6\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.0500\text{ m})} = 1.61\times10^{-4}\text{ T}.$

The direction of the magnetic field is into the page (the charge is negative).

b) The time to complete half a circle is just the distance traveled divided by the velocity:

$$t = \frac{D}{v} = \frac{\pi R}{v} = \frac{\pi(0.0500\text{ m})}{1.41\times10^6\text{ m/s}} = 1.11\times10^{-7}\text{ s}.$$

27.16: a) $B = \dfrac{mv}{qR} = \dfrac{(1.67\times10^{-27}\text{ kg})(1.41\times10^6\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.0500\text{ m})} = 0.294\text{ T}$

The direction of the magnetic field is out of the page (the charge is positive).

b) The time to complete half a circle is unchanged:

$$t = 1.11\times10^{-7}\text{ s}.$$

27.17: $K_1 + U_1 = K_2 + U_2$

$U_1 = K_2 = 0,$ so $K_1 = U_2;\ \ \dfrac{1}{2}mv^2 = ke^2/r$

$$v = e\sqrt{\frac{2k}{mr}} = (1.602\times10^{-19}\text{ C})\sqrt{\frac{2k}{(3.34\times10^{-27}\text{ kg})(1.0\times10^{-15}\text{ m})}} = 1.2\times10^7\text{ m/s}$$

b) $\sum\vec{F} = m\vec{a}$ gives $qvB = mv^2/r$

$$B = \frac{mv}{qr} = \frac{(3.34\times10^{-27}\text{ kg})(1.2\times10^7\text{m/s})}{(1.602\times10^{-19}\text{ C})(2.50\text{ m})} = 0.10\text{ T}$$

27.18: a) $F = qvB\sin\theta$

$$B = \frac{F}{qv\sin\theta} = \frac{0.00320\times10^{-9}\ \text{N}}{8(1.60\times10^{-19}\ \text{C})(500,000\ \text{m/s})\sin 90°}$$

$B = 5.00\ \text{T}$. If the angle θ is less than $90°$, a larger field is needed to produce the same force. The direction of the field must be toward the south so that $\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$ can be downward.

b) $F = qvB\sin\theta$

$$v = \frac{F}{qB\sin\theta} = \frac{4.60\times10^{-12}\ \text{N}}{(1.60\times10^{-19}\ \text{C})(2.10\ \text{T})\sin 90°}$$

$v = 1.37\times10^{7}\ \text{m/s}$. If θ is less than $90°$, the speed would have to be larger to have the same force. The force is upward, so $\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$ must be downward since the electron is negative, so the velocity must be toward the south.

27.19: $q = (4.00\times10^{8})(-1.602\times10^{-19}\ \text{C}) = 6.408\times10^{-11}\ \text{C}$

speed at bottom of shaft: $\frac{1}{2}mv^2 = mgy;\ v = \sqrt{2gy} = 49.5\ \text{m/s}$

$\vec{\boldsymbol{v}}$ is downward and $\vec{\boldsymbol{B}}$ is west, so $\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$ is north. Since $q<0$, $\vec{\boldsymbol{F}}$ is south.

$F = qvB\sin\theta = (6.408\times10^{-11}\ \text{C})(49.5\ \text{m/s})(0.250\ \text{T})\sin 90° = 7.93\times10^{-10}\ \text{N}$

27.20: (a) $R = \dfrac{mv}{qB}$

$$v = \frac{qBR}{m} = \frac{3(1.60\times10^{-19}\ \text{C})(0.250\ \text{T})(\frac{0.950}{2}\ \text{m})}{12(1.67\times10^{-27}\ \text{kg})}$$

$v = 2.84\times10^{6}\ \text{m/s}$

Since $\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$ is to the left but the charges are bent to the right, they must be negative.

b) $F_{\text{grav}} = mg = 12(1.67\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2) = 1.96\times10^{-25}\ \text{N}$

$F_{\text{magnetic}} = qvB = 3(1.6\times10^{-19}\ \text{C})(2.84\times10^{6}\ \text{m/s})(0.250\ \text{T})$

$= 3.41\times10^{-13}\ \text{N}$

Since $F_{\text{magn}} \approx 10^{12}\times F_{\text{grav,}}$ we can safely neglect gravity.

c) The speed does not change since the magnetic force is perpendicular to the velocity and therefore does not do work on the particles.

27.21: a) $v = \dfrac{qRB}{m} = \dfrac{(1.60\times10^{-19}\ \text{C})(6.96\times10^{-3}\ \text{m})(2.50\ \text{T})}{(3.34\times10^{-27}\ \text{kg})} = 8.34\times10^{5}\,\text{m/s}.$

b) $t = \dfrac{D}{v} = \dfrac{\pi R}{v} = \dfrac{\pi(6.96\times10^{-3}\text{m})}{8.34\times10^{5}\,\text{m/s}} = 2.62\times10^{-8}\ \text{s}.$

c) $\dfrac{1}{2}mv^{2} = qV \Rightarrow V = \dfrac{mv^{2}}{2q} = \dfrac{(3.34\times10^{-27}\ \text{kg})(8.34\times10^{5}\,\text{m/s})^{2}}{2(1.60\times10^{-19}\ \text{C})} = 7260\ \text{V}.$

27.22: $R = \dfrac{mv}{qB} = \dfrac{(9.11\times10^{-31}\ \text{kg})(2.8\times10^{6}\,\text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.0877\ \text{T})} = 1.82\times10^{-4}\text{m}.$

27.23: a) $B = \dfrac{m2\pi f}{|q|} = \dfrac{(9.11\times10^{-31}\ \text{kg})2\pi(3.00\mp\times10^{12}\ \text{Hz})}{(1.60\times10^{-19}\ \text{C})} = 107\ \text{T}.$

This is about 2.4 times the greatest magnitude yet obtained on earth.

b) Protons have a greater mass than the electrons, so a greater magnetic field would be required to accelerate them with the same frequency, so there would be no advantage in using them.

27.24: The initial velocity is all in the y-direction, and we want the pitch to equal the radius of curvature

$$\Rightarrow d_{x} = v_{x}T = \frac{mv_{y}}{qB} = R.$$

But $$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}.$$

$$\Rightarrow \frac{2\pi m v_{x}}{qB} = \frac{mv_{y}}{qB} \Rightarrow \frac{v_{y}}{v_{x}} = 2\pi = \tan\theta \Rightarrow \theta = 81.0^{\circ}.$$

27.25: a) The radius of the path is unaffected, but the pitch of the helix varies with time as the proton is accelerated in the x-direction.

b) $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{qB} = \dfrac{2\pi(1.67\times10^{-27}\ \text{kg})}{(1.60\times10^{-19}\ \text{C})(0.500\ \text{T})} = 1.31\times10^{-7}\ \text{s}, t = T/2,$ and

$$a_{x} = \frac{F}{m} = \frac{qE}{m} = \frac{(1.6\times10^{-19}\ \text{C})(2.00\times10^{4}\ \text{V/m})}{1.67\times10^{-27}\ \text{kg}} = 1.92\times10^{12}\,\text{m/s}^{2}.$$

$$d_{x} = v_{0x}t + \frac{1}{2}a_{x}t^{2} = (1.5\times10^{5}\,\text{m/s})(6.56\times10^{-8}\ \text{s}) + \frac{(1.92\times10^{12}\,\text{m/s}^{2})(6.56\times10^{-8}}{2}$$

$$\Rightarrow d_{x} = 0.014\ \text{m}.$$

27.26: $\frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(1.6\times10^{-19}\text{ C})(220\text{ V})}{(1.16\times10^{-26}\text{ kg})}} = 7.79\times10^4\text{ m/s}.$

$$\Rightarrow R = \frac{mv}{qB} = \frac{(1.16\times10^{-26}\text{ kg})(7.79\times10^4\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.723\text{ T})} = 7.81\times10^{-3}\text{ m}.$$

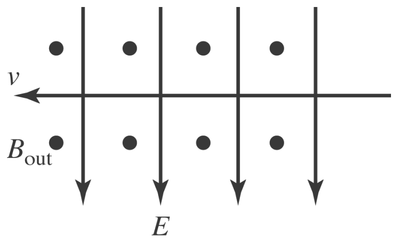
27.27: $\frac{1}{2}mv^2 = |q|\Delta V \Rightarrow v = \sqrt{\frac{2|q|\Delta V}{m}} = \sqrt{\frac{2(1.6\times10^{-19}\text{ C})\,(2.0\times10^3\text{ V})}{(9.11\times10^{-31}\text{ kg})}}$

$$= 2.65\times10^7\text{ m/s}.$$

$$\Rightarrow B = \frac{mv}{|q|R} = \frac{(9.11\times10^{-31}\text{ kg})(2.65\times10^7\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.180\text{ m})} = 8.38\times10^{-4}\text{ T}.$$

27.28: a) $v = E/B = (1.56\times10^4\text{ V/m})/(4.62\times10^{-3}\text{ T}) = 3.38\times10^6\text{ m/s}.$

b)



c) $R = \frac{mv}{|q|B} = \frac{(9.11\times10^{-31}\text{ kg})(3.38\times10^6\text{ m/s})}{(1.60\times10^{-19}\text{ C})(4.62\times10^{-3}\text{ T})}$

$$\Rightarrow R = 4.17\times10^{-3}\text{ m}.$$

$$T = \frac{2\pi m}{|q|B} = \frac{2\pi R}{v} = \frac{2\pi(4.17\times10^{-3}\text{ m})}{(3.38\times10^6\text{ m/s})} = 7.74\times10^{-9}\text{ s}.$$

27.29: a) $F_B = F_E$ so $|q|vB = |q|E;\quad B = E/v = 0.10\text{ T}$

Forces balance for either sign of q.

b) $E = V/d$ so $v = E/B = V/dB$

smallest v:

largest V, smallest B, $v_{\min} = \frac{120\text{ V}}{(0.0325\text{ m})(0.180\text{ T})} = 2.1\times10^4\text{ m/s}$

largest v:

smallest V, largest B, $v_{\min} = \frac{560\text{ V}}{(0.0325\text{ m})(0.054\text{ T})} = 3.2\times10^5\text{ m/s}$

27.30: To pass undeflected in both cases, $E = vB = (5.85\times10^3\ \text{m/s})(1.35\ \text{T}) = 7898\ \text{N/C}$.

a) If $q = 0.640\times10^{-9}$ C, the electric field direction is given by $-(\hat{\boldsymbol{j}}\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the opposite direction to the magnetic force.

b) If $q = -0.320\times10^{-9}$ C, the electric field direction is given by $((-\hat{\boldsymbol{j}})\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the same direction as the magnetic force, which has swapped from part (a). The electric force will now point opposite to the magnetic force for this negative charge using $\vec{\boldsymbol{F}}_e = q\vec{\boldsymbol{E}}$.

27.31: $R = \dfrac{mv}{qB} = \dfrac{mE}{qB^2} \Rightarrow m = \dfrac{RqB^2}{E} = \dfrac{(0.310\ \text{m})(1.60\times10^{-19}\ \text{C})(0.540\ \text{T})^2}{(1.12\times10^5\ \text{V/m})}$
$= 1.29\times10^{-25}$ kg

$$\Rightarrow m(\text{amu}) = \frac{1.29\times10^{-25}\ \text{kg}}{1.66\times10^{-27}\ \text{kg}} = 78\ \text{atomic mass units.}$$

27.32: a) $E = vB = (1.82\times10^6\ \text{m/s})(0.650\ \text{T}) = 1.18\times10^6\ \text{V/m}$.

b) $E = V/d \Rightarrow V = Ed = (1.18\times10^6\ \text{V/m})(5.20\times10^{-3}\ \text{m}) = 6.14\ \text{kV}$.

27.33: a) For minimum magnitude, the angle should be adjusted so that $(\vec{\boldsymbol{B}})$ is parallel to the ground, thus perpendicular to the current. To counter gravity, $ILB = mg$, so $B = \frac{mg}{IL}$.

b) We want the magnetic force to point up. With a northward current, a westward B field will accomplish this.

27.34: a) $F = Ilb = (1.20\ \text{A})\,(0.0100\ \text{m})\,(0.588\ \text{T}) = 7.06\times10^{-3}\ \text{N}$, and by the righthand rule, the easterly magnetic field results in a southerly force.

b) If the field is southerly, then the force is to the west, and of the same magnitude as part (a), $F = 7.06\times10^{-3}$ N.

c) If the field is $30°$ south of west, the force is $30°$ west of north ($90°$ counterclockwise from the field) and still of the same magnitude, $F = 7.60\times10^{-6}$ N.

27.35: $I = \dfrac{F}{lB} = \dfrac{0.13\ \text{N}}{(0.200\ \text{m})\,(0.067\ \text{T})} = 9.7\ \text{A}.$

27.36: $F = IlB = (10.8\ \text{A})(0.050\ \text{m})(0.550\ \text{T}) = 0.297\ \text{N}.$

27.37: The wire lies on the x-axis and the force on 1 cm of it is

a) $\vec{F} = I\,\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})(-0.65\ \text{T})(\hat{i} \times \hat{j}) = +(0.023\ \text{N})\ \hat{k}.$

b) $\vec{F} = I\,\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})(+0.56\ \text{T})(\hat{i} \times \hat{k}) = +(0.020\ \text{N})\ \hat{j}.$

c) $\vec{F} = I\,\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})(-0.31\ \text{T})(\hat{i} \times \hat{i}) = 0.$

d) $\vec{F} = I\,\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})(-0.28\ \text{T})(\hat{i} \times \hat{k}) = (-9.8 \times 10^{-3}\ \text{N})\ \hat{j}.$

e) $\vec{F} = I\,\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})[0.74\ \text{T}\ (\hat{i} \times \hat{j}) - 0.36\ \text{T}\ (\hat{i} \times \hat{k})]$

$= -(0.026\ \text{N})\hat{k} - (0.013\ \text{N})\hat{j}.$

27.38: $$\vec{F} = I\,\vec{l} \times \vec{B}$$

Between the poles of the magnet, the magnetic field points to the right. Using the fingertips of your right hand, rotate the current vector by $90°$ into the direction of the magnetic field vector. Your thumb points downward–which is the direction of the magnetic force.

27.39: a) $F_I = mg$ when bar is just ready to levitate.

$$IlB = mg,\ I = \frac{mg}{lB} = \frac{(0.750\ \text{kg})(9.80\text{m/s}^2)}{(0.500\ \text{m})(0.450\ \text{T})} = 32.67\ \text{A}$$

$$\varepsilon = IR = (32.67\ \text{A})(25.0\ \Omega) = 817\ \text{V}$$

b) $R = 2.0\ \Omega,\ I = \varepsilon/R = (816.7\ \text{V})/(2.0\ \Omega) = 408\ \text{A}$

$F_I = IlB = 92\ \text{N}$

$a = (F_I - mg)/a = 113\ \text{m/s}^2$

27.40: (a) The magnetic force on the bar must be upward so the current through it must be to the right. Therefore *a* must be the positive terminal.

(b) For balance, $F_{\text{magn}} = mg$

$$IlB\sin\theta = mg$$

$$m = \frac{IlB\sin\theta}{g}$$

$$I = \varepsilon/R = 175\ \text{V}/5.00\ \Omega = 35.0\ \text{A}$$

$$m = \frac{(35.0\ \text{A})(0.600\ \text{m})(1.50\ \text{T})}{9.80\ \text{m/s}^2} = 3.21\ \text{kg}$$

27.41: a) The force on the straight section along the $-x$-axis is zero.

For the half of the semicircle at negative x the force is out of the page. For the half of the semicircle at positive x the force is into the page. The net force on the semicircular section is zero.

The force on the straight section that is perpendicular to the plane of the figure is in the $-y$-direction and has magnitude $F = ILB$.

The total magnetic force on the conductor is ILB, in the $-y$-direction.

b) If the semicircular section is replaced by a straight section along the x-axis, then the magnetic force on that straight section would be zero, the same as it is for the semicircle.

27.42: a) $\tau = IBA = (6.2\ \text{A})(0.19\ \text{T})(0.050\ \text{m})(0.080\ \text{m}) = 4.71\times10^{-3}\ \text{N}\cdot\text{m}.$

b) $\mu = IA = (6.2\ \text{A})(0.050\ \text{m})(0.080\ \text{m}) = 0.025\ \text{A}\cdot\text{m}^2.$

c) Maximum torque will occur when the area is largest, which means a circle:

$$2\pi R = 2(0.050\ \text{m} + 0.080\ \text{m}) \Rightarrow R = 0.041\ \text{m}.$$

$$\Rightarrow \tau_{\max} = IBA = (6.2\ \text{A})(0.19\ \text{T})\pi(0.04041\ \text{m})^2 = 6.22\times10^{-3}\ \text{N}\cdot\text{m}.$$

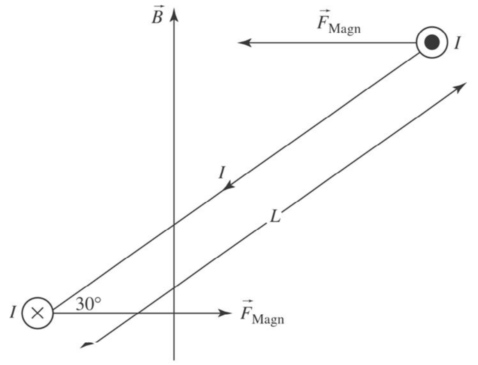
27.43: a) The torque is maximum when the plane of loop is parallel to B.

$\tau = NIBA\sin\phi \Rightarrow \tau_{\max} = (15)(2.7\ \text{A})(0.56\ \text{T})\pi(0.08866\ \text{m}/2)^2\ \sin 90° = 0.132\ \text{N}\cdot\text{m}.$

b) The torque on the loop is 71% of the maximum when $\sin\phi = 0.71 \Rightarrow \phi = 45°.$

27.44: (a) The force on each segment of the coil is toward the center of the coil, as the net force and net torque are both zero.

(b) As viewed from above:



As in (a), the forces cancel.

$$\begin{aligned}\sum \tau &= 2F_{\text{magn}} \frac{L}{2} \sin\theta \\ &= IlBL \sin\theta \\ &= (1.40\ \text{A})(0.220\ \text{m})(1.50\ \text{T})(0.350\ \text{m}) \sin 30° \\ &= 8.09 \times 10^{-2}\ \text{N} \cdot \text{m}\ \textit{counterclockwise}\end{aligned}$$

27.45: a) $T = 2\pi r/v = 1.5 \times 10^{-16}$ s

b) $I = Q/t = e/t = 1.1\ \text{mA}$

c) $\mu = IA = I\pi r^2 = 9.3 \times 10^{-24}\ \text{A} \cdot \text{m}^2$

27.46: a) $\phi = 90° : \tau = NIAB \sin(90°) = NIAB$, direction: $\hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}} = -\hat{\boldsymbol{i}}, U = -N\mu B \cos\phi =$

b) $\phi = 0 : \tau = NIAB \sin(0) = 0$, no direction, $U = -N\mu B \cos\phi = -NIAB.$

c) $\phi = 90° : \tau = NIAB \sin(90°) = NIAB$, direction: $-\hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{i}}, U = -N\mu B \cos\phi = 0.$

d) $\phi = 180° : \tau = NIAB \sin(180°) = 0$, no direction, $U = -N\mu B \cos(180°) = NIAB.$

27.47: $\Delta U = U_f - U_i = -\mu B \cos 0° + \mu B \cos 180° = -2\mu B$

$= -2(1.45\ \text{A} \cdot \text{m}^2)(0.835\ \text{T}) = -2.42\ \text{J}.$

27.48: a) $V_{ab} = \varepsilon + Ir \Rightarrow I = \frac{V_{ab} - \varepsilon}{r} = \frac{120\text{ V} - 105\text{ V}}{3.2\ \Omega} = 4.7\text{ A}.$

b) $P_{\text{supplied}} = IV_{ab} = (4.7\text{ A})(120\text{ V}) = 564\text{ W}.$

c) $P_{\text{mech}} = IV_{ab} - I^2 r = 564\text{ W} - (4.7\text{ A})^2(3.2\ \Omega) = 493\text{ W}.$

27.49: a) $I_f = \frac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}.$

b) $I_r = I_{\text{total}} - I_f = 4.82\ A - 1.13\ A = 3.69\ A.$

c) $V = \varepsilon + I_r R_r \Rightarrow \varepsilon = V - I_r R_r = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}.$

d) $P_{\text{mech}} = \varepsilon I_r = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W}.$

27.50: a) Field current $I_f = \frac{120\text{ V}}{218\ \Omega} = 0.550\text{ A}.$

b) Rotor current $I_r = I_{\text{total}} - I_f = 4.82\text{ A} - 0.550\text{ A} = 4.27\text{ A}.$

c) $V = \varepsilon + I_r R_r \Rightarrow \varepsilon = V - I_r R_r = 120\text{ V} - (4.27\text{ A})(5.9\ \Omega) = 94.8\text{ V}.$

d) $P_f = I_f^2 R_f = (0.550\text{ A})^2(218\ \Omega) = 65.9\text{ W}.$

e) $P_r = I_r^2 R_r = (4.27\text{ A})^2(5.9\ \Omega) = 108\text{ W}.$

f) *Power input* = (120 V) (4.82 A) = 578 W.

g) *Efficiency* = $\frac{P_{\text{output}}}{P_{\text{input}}} = \frac{(578\text{ W} - 65.9\text{ W} - 108\text{ W} - 45\text{ W})}{578\text{ W}} = \frac{359\text{ W}}{578\text{ W}} = 0.621.$

27.51: a) $v_d = \frac{J}{n|q|} = \frac{I}{An|q|}$

$$= \frac{120\text{ A}}{(0.0118\text{ m})(2.3\times10^{-4}\text{ m})(5.85\times10^{28}\text{ m}^{-3})(1.6\times10^{-19}\text{ C})}$$

$$\Rightarrow v_d = 4.72\times10^{-3}\text{ m/s}.$$

b) $E_z = v_d B_y = (4.72\times10^{-3}\text{ m/s})(0.95\text{ T}) = 4.48\times10^{-3}\text{ N/C}$, in the $+z$-direction (negative charge).

c) $V_{\text{Hall}} = zE_z = (0.0118\text{ m})(4.48\times10^{-3}\text{ N/C}) = 5.29\times10^{-5}\text{ V}.$

27.52: $$n = \frac{J_x B_y}{|q|E_z} = \frac{IB_y}{A|q|E_z} = \frac{IB_y z_1}{A|q|\varepsilon_z} = \frac{IB_y}{y_1|q|\varepsilon}$$

$$= \frac{(78.0\ \text{A})(2.29\ \text{T})}{(2.3\times10^{-4}\ \text{m})(1.6\times10^{-19}\ \text{C})(1.31\times10^{-4}\ \text{V})}$$

$$\Rightarrow n = 3.7\times10^{28}\ \text{electrons per cubic meter.}$$

27.53: a) By inspection, using $\vec{F} = q\vec{v}\times\vec{B}$, $\vec{B} = -B\,\hat{j}$ will provide the correct direction for each force. Using either force, say F_2, $B = \dfrac{F_2}{q|v_2|}$.

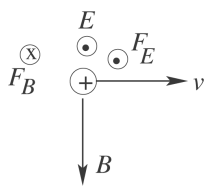
b) $F_1 = q|v_1|B\sin 45° = \dfrac{q|v_2|B}{\sqrt{2}} = \dfrac{F_2}{\sqrt{2}}$ (since $|v_1| = |v_2|$).

27.54: a) $\vec{F} = q\vec{v}\times\vec{B} = -qV[B_x(\hat{j}\times\hat{i}) + B_z(\hat{j}\times\hat{k})] = qVB_x\hat{k} - qVB_z\hat{i}$

b) $B_x > 0$, $B_z < 0$, sign of B_y doesn't matter.

c) $\vec{F} = |q|VB_x\hat{i} - |q|VB_x\hat{k}$, $\left|\vec{F}\right| = \sqrt{2}|q|vB_x$.

27.55: The direction of $\vec{E}$ is horizontal and perpendicular to $\vec{v}$, as shown in the sketch:



$F_B = qvB,\ F_E = qE$

$F_B = F_E$ for no deflection, so $qvB = qE$

$E = vB = (14.0\ \text{m/s})(0.500\ \text{T}) = 7.00\ \text{V/m}$

We ignored the gravity force. If the target is 5.0 m from the rifle, it takes the bullet 0.36 s to reach the target and during this time the bullet moves downward $y - y_0 = \frac{1}{2}a_y t^2 = 0.62$ m. The magnetic and electric forces we considered are horizontal. A vertical electric field of $E = mg/q = 0.038$ V/m would be required to cancel the gravity force. Air resistance has also been neglected.

27.56: a) Motion is circular:

$$x^2+y^2=R^2 \Rightarrow x=D \Rightarrow y_1=\sqrt{R^2-D^2} \text{ (path of deflected particle)}$$

$$y_2=R \text{ (equation for tangent to the circle, path of undeflected particle)}$$

$$d=y_2-y_1=R-\sqrt{R^2-D^2}=R-R\sqrt{1-\frac{D^2}{R^2}}=R\left[1-\sqrt{1-\frac{D^2}{R^2}}\right]$$

$$\text{If } R>>D \Rightarrow d\approx R\left[1-\left(1-\frac{1}{2}\frac{D^2}{R^2}\right)\right]=\frac{D^2}{2R}.$$

For a particle moving in a magnetic field, $R=\frac{mv}{qB}$.

But $\frac{1}{2}mv^2=qV$, so $R=\frac{1}{B}\sqrt{\frac{2mV}{q}}$.

Thus, the deflection $d\approx\frac{D^2B}{2}\sqrt{\frac{q}{2mV}}=\frac{D^2B}{2}\sqrt{\frac{e}{2mV}}$.

b) $d=\frac{(0.50\text{ m})^2(5.0\times10^{-5}\text{ T})}{2}\sqrt{\frac{(1.6\times10^{-19}\text{ C})}{2(9.11\times10^{-31}\text{ kg})(750\text{ V})}}=0.067\text{ m}=6.7\text{ cm}.$

$d\approx 13\%$ of D, which is fairly significant.

27.57: a) $v_{max}=\frac{qBR}{m}=\frac{(1.6\times10^{-19}\text{ C})\,(0.85\text{ T})\,(0.40\text{ m})}{1.67\times10^{-27}\text{ kg}}=3.3\times10^{7}\text{ m/s}.$

$$\Rightarrow \mathrm{E}_{max}=\frac{1}{2}m\,v^2{}_{max}=\frac{(1.67\times10^{-27}\text{ kg})\,(3.3\times10^{7}\text{ m/s}^2)}{2}=8.9\times10^{-13}\text{ J}=5.5\text{ MeV}.$$

b) $T=\frac{2\pi R}{v}=\frac{2\pi(0.4\text{ m})}{3.3\times10^{7}\text{ m/s}}=7.6\times10^{-8}\text{ s}.$

c) If the energy was to be doubled, then the speed would have to be increased by $\sqrt{2}$, as would the magnetic field. Therefore the new magnetic field would be $B_{new}=\sqrt{2}B_0=1.2\text{ T}$.

d)For alpha particles,

$$E_{max}(\alpha)=E_{max}(p)\frac{m_p}{m_\alpha}\frac{q_\alpha{}^2}{q_p{}^2}=E_{max}(p)\frac{m_p}{(4m_p)}\frac{(2q_p)^2}{q_p{}^2}=E_{max}(p).$$

27.58: a) $\vec{F} = q\vec{v} \times \vec{B} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & v \\ B_x & B_y & B_z \end{vmatrix} = -qvB_y\hat{i} + qvB_x\hat{j}.$

But $\vec{F} = 3F_0\hat{i} + 4F_0\hat{j}$, so $3F_0 = -qvB_y$ and $4F_0 = qvB_x$

$\Rightarrow B_y = -\dfrac{3F_0}{qv},\ B_x = \dfrac{4F_0}{qv},\ B_z$ is arbitrary.

b) $B = \dfrac{6F_0}{qv} = \sqrt{B_x^{\ 2} + B_y^{\ 2} + B_z^{\ 2}} = \dfrac{F_0}{qv}\sqrt{9 + 16 + B_z^{\ 2}} = \dfrac{F_0}{qv}\sqrt{25 + B_z^{\ 2}}$

$\Rightarrow B_z = \pm\dfrac{11F_0}{qv}\cdot$

27.59: $f = \dfrac{\omega}{2\pi} = \dfrac{qB}{2\pi m} \Rightarrow \dfrac{f_e}{f_{Li}} = \dfrac{q_e B/2\pi m_e}{q_{Li}B/2\pi m_{Li}} = \dfrac{em_{Li}}{3em_e} = \left(\dfrac{1}{3}\right)\dfrac{1.16\times10^{-26}\ \text{kg}}{9.11\times10^{-31}\ \text{kg}} = 4244.$

27.60: a) $K = 2.7\ \text{MeV}\ (2.7\times10^6\text{eV})\ (1.6\times10^{-19}\ \text{J/eV}) = 4.32\times10^{-13}\ \text{J}.$

$\Rightarrow v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(4.32\times10^{-13}\ \text{J})}{1.67\times10^{-27}\ \text{kg}}} = 2.27\times10^7\ \text{m/s}.$

$\Rightarrow R = \dfrac{mv}{qB} = \dfrac{(1.67\times10^{-27}\ \text{kg})\ (2.27\times10^7\ \text{m/s})}{(1.6\times10^{-19}\ \text{C})\ (3.5\ \text{T})} = 0.068\ \text{m}.$

Also, $\omega = \dfrac{v}{R} = \dfrac{2.27\times10^7\ \text{m/s}}{0.068\ \text{m}} = 3.34\times10^8\ \text{rad/s}.$

b) If the energy reaches the final value of 5.4 MeV, the velocity increases by $\sqrt{2}$, as does the radius, to 0.096 m. The angular frequency is unchanged from part (a) at 3.34×10^8 rad / s.

27.61: a) $\vec{F} = q\vec{v}\times\vec{B} = q\left[(v_y B_z)\hat{i} - (v_x B_z)\hat{j}\right] \Rightarrow F^2 = q^2\left[(v_y B_z)^2 - (v_x B_z)^2\right]$

$$\Rightarrow q^2 = \frac{F^2}{B_z^{\ 2}}\frac{1}{(v_y)^2 + (v_x)^2}$$

$$\Rightarrow q = \frac{-1.25\ \text{N}}{0.120\ \text{T}}\sqrt{\frac{1}{\left[4(1.05\times10^6\ \text{m/s})\right]^2 + \left[-3(1.05\times10^6\ \text{m/s})\right]^2}}$$

$$= -1.98\times10^{-6}\ \text{C}.$$

b) $$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{v}\times\vec{B}}{m} = \frac{q}{m}\left[(v_y B_z)\hat{i} - (v_x B_z)\hat{j}\right]$$

$$\Rightarrow \vec{a} = \frac{-1.98\times10^{-6}\ \text{C}}{2.58\times10^{-15}\ \text{kg}}(1.05\times10^6\ \text{m/s})\ (-0.120\ \text{T})\left[4\hat{i} + 3\hat{j}\right]$$

$$\Rightarrow \vec{a} = 9.67\times10^{13}\,\text{m/s}^2\left[4\hat{i} + 3\hat{j}\right].$$

c) The motion is helical since the force is in the *xy*-plane but the velocity has a *z*-component. The radius of the circular part of the motion is:

$$R = \frac{mv}{qB} = \frac{(2.58\times10^{-15}\ \text{kg})\ (5)\ (1.05\times10^6\ \text{m/s})}{(1.98\times10^{-6}\ \text{C})\ (0.120\ \text{T})} = 0.057\ \text{m}.$$

d) $$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} = \frac{(1.98\times10^{-6}\ \text{C})\ (0.120\ \text{T})}{2\pi(2.58\times10^{-15}\ \text{kg})} = 14.7\ \text{MHz}.$$

e) After two complete cycles, the *x* and *y* values are back to their original values, $x = R$ and $y = 0$, but *z* has changed.

$$z = 2Tv_z = \frac{2v_z}{f} = \frac{2(+12)\ (1.05\times10^6\ \text{m/s})}{1.47\times10^7\ \text{Hz}} = 1.71\,\text{m}.$$

27.62: a) $\dfrac{mv^2}{R} = qE \Rightarrow v\sqrt{\dfrac{qER}{m}} = \sqrt{\dfrac{qV_{ab}}{m\ln(b/a)}} = \sqrt{\dfrac{(1.6\times10^{-19}\ \text{C})\,(120\ \text{V})}{(9.11\times10^{-31}\ \text{kg})\ln(5.00/0.100)}}$

$\Rightarrow v = 2.32\times10^{6}\ \text{m/s}.$

b) $\dfrac{mv^2}{R} = q(E + vB) \Rightarrow \left(\dfrac{m}{R}\right)v^2 - (qB)v - qE = 0$

$\Rightarrow (2.28\times10^{-29})\,v^2 - (2.08\times10^{-23})v - (1.23\times10^{-16}) = 0$

$\Rightarrow v = 2.82\times10^{6}\ \text{m/s or } -1.91\times10^{6}\ \text{m/s},$

but we need the positive velocity to get the correct force, so $v = 2.82\times10^{6}\ \text{m/s}.$

c) If the direction of the magnetic field is reversed, then there is a smaller net force and a smaller velocity, and the value is the second root found in part (b), $\Rightarrow v = 3.19\times10^{6}\ \text{m/s}.$

27.63: $v = \dfrac{E}{B} = \dfrac{1.88\times10^{4}\ \text{N/C}}{0.701\,\text{T}} = 2.68\times10^{4}\ \text{m/s, and } R = \dfrac{mv}{qB}\text{, so:}$

$$R_{82} = \frac{82(1.66\times10^{-27}\ \text{kg})\,(2.68\times10^{4}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})\,(0.701\,\text{T})} = 0.0325\,\text{m}.$$

$$R_{84} = \frac{84(1.66\times10^{-27}\ \text{kg})(2.68\times10^{4}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})\,(0.701\,\text{T})} = 0.0333\,\text{m}.$$

$$R_{86} = \frac{86(1.66\times10^{-27}\ \text{kg})\,(2.68\times10^{4}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})\,(0.701\,\text{T})} = 0.0341\,\text{m}.$$

So the distance between two adjacent lines is $2R = 1.6$ mm.

27.64: $F_x = q(v_yB_z - v_zB_y) = 0.$

$F_y = q(v_zB_x - v_xB_z) = (9.45\times10^{-8}\ \text{C})\,(5.85\times10^{4}\ \text{m/s})\,(0.450\,\text{T})$

$= 2.49\times10^{-3}\ \text{N}.$

$F_z = q(v_xB_y - v_yB_x) = -(9.45\times10^{-8}\ \text{C})\,(-3.11\times10^{4}\,\text{m/s})\,(0.450\,\text{T})$

$= 1.32\times10^{-3}\ \text{N}.$

27.65: a) $l_{ab}: \vec{F} = I\vec{l}_{ab} \times \vec{B} = I(l_{ab}B)\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} = -(6.58\text{ A})(0.750\text{ m})(0.860\text{ T})\hat{\boldsymbol{k}}$
$= (-4.24\text{ N})\hat{\boldsymbol{k}}.$

$$l_{bc}: \vec{F} = I\vec{l}_{bc} \times \vec{B} = I(l_{bc}B)\left[\frac{(\hat{\boldsymbol{i}} - \hat{\boldsymbol{k}})}{\sqrt{2}} \times \hat{\boldsymbol{i}}\right] = -(6.58\text{ A})(0.750\text{ m})(0.860\text{ T})\hat{\boldsymbol{j}}$$

$$= (-4.24\text{ N})\hat{\boldsymbol{j}}.$$

$$l_{cd}: \vec{F} = I\vec{l}_{cd} \times \vec{B} = I(l_{cd}B)\left[\frac{(\hat{\boldsymbol{k}} - \hat{\boldsymbol{j}})}{\sqrt{2}} \times \hat{\boldsymbol{i}}\right] = -(6.58\text{ A})(0.750\text{ m})(0.860\text{ T})\left[\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right]$$

$$\Rightarrow \vec{F} = (4.24\text{ N})\left[\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right]$$

$$l_{de}: \vec{F} = I\vec{l}_{de} \times \vec{B} = Il_{de}B\left[-\hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}}\right] = -(6.58\text{ A})(0.750\text{ m})(0.860\text{ T})\hat{\boldsymbol{j}} = (-4.24\text{ N})\hat{\boldsymbol{j}}$$

$$l_{ef}: \vec{F} = I\vec{l}_{ef} \times \vec{B} = I(l_{ef}B)(-\hat{\boldsymbol{i}}) \times \hat{\boldsymbol{i}} = 0.$$

b) Summing all the forces in part (a) we have $\vec{F}_{\text{total}} = (-4.24\text{ N})\hat{\boldsymbol{j}}.$

27.66: a) $F = ILB$, to the right.

b) $v^2 = 2ad \Rightarrow d = \dfrac{v^2}{2a} = \dfrac{v^2 m}{2ILB}.$

c) $d = \dfrac{(1.12 \times 10^4\text{ m/s})^2(25\text{ kg})}{2(2000\text{ A})(0.50\text{ m})(0.50\text{ T})} = 3.14 \times 10^6\text{ m} = 3140\text{ km!}$

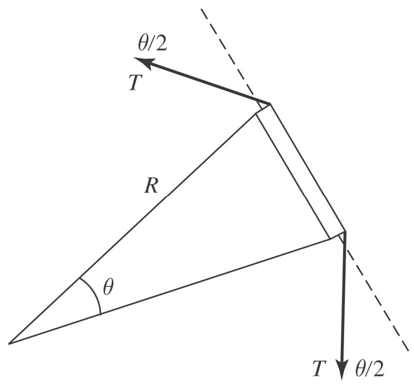
27.67: The current is to the left, so the force is into the plane.

$$\sum F_y = N\cos\theta - \text{M}g = 0 \text{ and } \sum F_x = N\sin\theta - F_B = 0.$$

$$\Rightarrow F_B = \text{M}g\tan\theta = ILB \Rightarrow I = \frac{\text{M}g\tan\theta}{LB}$$

27.68: a) By examining a small piece of the wire (shown below) we find:

$$F_B = ILB = 2T\ \sin(\theta/2)$$
$$\Rightarrow ILB \approx \frac{2T\theta}{2} = \frac{2T\,L/R}{2} \Rightarrow \frac{T}{I\,B} = R.$$



b) For a particle:

$$qvB = \frac{mv^2}{R} \Rightarrow B = \frac{mv}{Rq} = \frac{mvIB}{Tq} \Rightarrow v = \frac{Tq}{mI}.$$

27.69: a) $\frac{1}{2}mv_x{}^2 = qV \Rightarrow v_x = \sqrt{\frac{2qV}{m}}$. Also $a = \frac{qv_xB}{m}$, and $t = \frac{x}{v_x}$.

$$\Rightarrow y = \frac{1}{2}at^2 = \frac{1}{2}a\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qv_xB}{m}\right)\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qBx^2}{m}\right)\left(\frac{m}{2qV}\right)^{1/2}$$

$$\Rightarrow y = Bx^2\left(\frac{q}{8mV}\right)^{\frac{1}{2}}.$$

b) This can be used for isotope separation since the mass in the denominator leads to different locations for different isotopes.

27.70: (a) During acceleration of the ions:

$$qV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

In the magnetic field:

$$R = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB}$$

$$m = \frac{qB^2R^2}{2V}$$

(b) $V \dfrac{qB^2R^2}{2m} = \dfrac{(1.60\times10^{-19}\ \text{C})(0.150\ \text{T})^2(0.500\ \text{m})^2}{2(12)(1.66\times10^{-27}\ \text{kg})}$

$V = 2.26\times10^4$ volts

(c) The ions are separated by the differences in their diameters.

$$D = 2R = 2\sqrt{\frac{2Vm}{qB^2}}$$

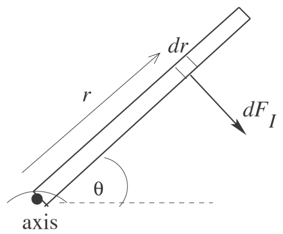
$$\Delta D = D_{14} - D_{12} = 2\left.\sqrt{\frac{2Vm}{qB^2}}\right|_{14} - 2\left.\sqrt{\frac{2Vm}{qB^2}}\right|_{12}$$

$$= 2\sqrt{\frac{2V(1\ \text{amu})}{qB^2}}\left(\sqrt{14} - \sqrt{12}\right)$$

$$= 2\sqrt{\frac{2(2.26\times10^4\ \text{V})(1.66\times10^{-27}\ \text{kg})}{(1.6\times10^{-19}\ \text{C})(0.150\ \text{T})^2}}\left(\sqrt{14} - \sqrt{12}\right)$$

$= 8.01\times10^{-2}$ m ≈ 8 cm – easily distinguishible.

27.71: a)



Divide the rod into infinitesimal sections of length *dr*.

The magnetic force on this section is $dF_I = IB\,dr$ and is perpendicular to the rod. The torque $d\tau$ due to the force on this section is $d\tau = r dF_I = IBr\,dr.$ The total torque is $\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442\text{ N/m}$, clockwise. This is the same torque calculated from a force diagram in which the total magnetic force $F_I = IlB$ acts at the center of the rod.

b) F_I produces a clockwise torque so the spring force must produce a counterclockwise torque. The spring force must be to the left, the spring is stretched.

Find *x*, the amount the spring is stretched:

$\sum \tau = 0,$ axis at hinge, counterclockwise torques positive

$(kx)l\sin 53° - \frac{1}{2}Il^2B = 0$

$$x = \frac{IlB}{2k\sin 53.0°} = \frac{(6.50\text{ A})(0.200\text{ m})(0.340\text{ T})}{2(4.80\text{ N/m})\sin 53.0°} = 0.05765\text{ m}$$

$U = \frac{1}{2}kx^2 = 7.98\times10^{-3}\text{ J}$

27.72: a) $\vec{F} = I\vec{l} \times \vec{B} \Rightarrow F_{PQ} = (5.00\text{ A})\,(0.600\text{ m})\,(3.00\text{ T})\sin(0°) = 0\text{ N},\ F_{RP}$ $= (5.00\text{ A})\,(0.800\text{ m})\,(3.00\text{ T})\sin(90°) = 12.0\text{ N(into the page)},\ F_{QR} = (5.00\text{ A})\,(1.00\text{ m})\,(3$ $\left(\frac{0.800}{1.00}\right) = 12.0\text{ N (out of the page)}.$

b) The net force on the triangular loop of wire is zero.

c) For calculating torque on a uniform wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the *PR*-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the *x*-axis.

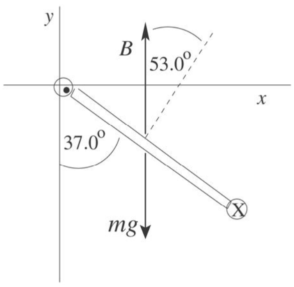
$$\left|\vec{\tau}\right| = \left|\vec{r} \times \vec{F}\right| = rF\sin(\theta) \Rightarrow \tau_{PQ} = r(0\text{ N}) = 0,\ \tau_{RP} = (0\text{ m})\,F\sin\theta = 0,\ \tau_{QR} =$$

$$(0.300\text{ m})\,(12.0\text{ N})\sin(90°) = 3.60\text{ N}\cdot\text{m (pointing to the right and parallel to } PR)$$

d) According to Eqn. 27.28, $\tau = NIAB\sin\phi = (1)\,(5.00\text{ A})\left(\frac{1}{2}\right)(0.600\text{ m})\,(0.800\text{ m})$ $(3.00\text{ T})\sin(90°) = 3.60\text{ N}\cdot\text{m}$, which agrees with part (c).

e) The point *Q* will be rotated out of the plane of the figure.

27.73:



$$\sum\tau = 0,$$

counterclockwise torques positive

$$mg(l/2)\sin 37.0° - IAB\sin 53.0°,\text{ with } A = l^2$$

$$I = \frac{mg\sin 37°}{2lB\sin 53°} = \frac{mg\tan 37°}{2lB} = 10.0\text{ A}$$

27.74: a) $\vec{F} = I\vec{l} \times \vec{B} = I(l\hat{\boldsymbol{k}}) \times \vec{B} = Il\left[(-B_y)\hat{\boldsymbol{i}} + (B_x)\hat{\boldsymbol{j}}\right]$

$$\Rightarrow F_x = -IlB_y = -(9.00\text{ A})\,(0.250\text{ m})\,(-0.985\text{ T}) = 2.22\text{ N}.$$

$$F_y = IlB_x = (9.00\text{ A})\,(0.250\text{ m})\,(-0.242\text{ T}) = -0.545\text{ N}$$

$$\Rightarrow F_z = 0,\text{ since the wire is in the z - direction.}$$

b) $F = \sqrt{F_x^{\ 2} + F_y^{\ 2}} = \sqrt{(2.22\text{ N})^2 + (0.545\text{ N})^2} = 2.29\text{ N}.$

27.75: Summing the torques on the wire from gravity and the magnetic field will enable us to find the magnetic field value.

$$\tau_B = IAB \sin 60° = B(8.2\text{ A})\,(0.060\text{ m})\,(0.080\text{ m}) \sin 60° = (0.0341\,\text{N}\cdot\text{m/T})B.$$

There are three sides to consider for the gravitational torque, leading to:

$$\tau_g = m_6 g l_6 \sin\phi + 2m_8 g l_8 \sin\phi,$$

where l_6 is the moment arm from the pivot to the far 6 cm leg and l_8 is the moment arm from the pivot to the centers of mass of the 8 cm legs.

$$\Rightarrow \tau_g = (9.8\text{ m/s}^2)\sin 30°[(0.00015\,\text{kg/cm})\,(6\text{ cm})\,(0.080\text{ m}) + 2(0.00015\,\text{kg/cm})\,(8\text{ cm})\,(0.040\text{ m})]$$

$$\Rightarrow \tau_g = 8.23\times 10^{-4}\ \text{N}\cdot\text{m} \Rightarrow B = \frac{8.23\times 10^{-4}\ \text{N}\cdot\text{m}}{0.0341\,\text{N}\cdot\text{m/T}} = 0.024\,\text{T, in the } y\text{-direction.}$$

27.76: a) $\tau = IAB \sin 60° = (15.0\text{ A})(0.060\text{ m})\,(0.080\text{ m})\,(0.48\text{ T}) \sin 60° = 0.030\ \text{N}\cdot\text{m}$ in the $-\hat{\boldsymbol{j}}$ direction. To keep the loop in place, you must provide a torque in the $+\hat{\boldsymbol{j}}$ direction.

b) $\tau = IAB \sin 30° = (15.0\text{ A})(0.60\text{ m})\,(0.080\text{ m})\,(0.48\text{ T}) \sin 30° = 0.017\ \text{N}\cdot\text{m},$ in the $+\hat{\boldsymbol{j}}$ direction you must provide a torque in the $-\hat{\boldsymbol{j}}$ direction to keep the loop in place.

c) If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.77:

$$|\vec{\boldsymbol{\tau}}| = I_s|\vec{\alpha}| = I_s \frac{d|\vec{\omega}|}{dt} = -I_s \frac{d^2\phi}{dt^2}$$

$$\text{but } |\vec{\boldsymbol{\tau}}| = \mu B \sin\phi \cong NIAB\,\phi \ (\sin\phi \approx \phi) \text{ if } \phi \text{ is small)}$$

$$\Rightarrow \frac{d^2\phi}{dt^2} = -\frac{NIAB}{I_s}\phi.$$

This describes simple harmonic motion with

$$\omega^2 = \frac{NIAB}{I_s} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_s}{NIAB}}.$$

27.78: $$|\vec{\tau}| = \mu B\sin\phi = IAB\sin\phi.$$

$$\phi = 90°,\ I = qf = \frac{q\omega}{2\pi},\ A = \pi r^2 = \pi\left(\frac{L}{2\pi}\right)^2 \Rightarrow \tau = \left(\frac{q\omega}{2\pi}\right)\left(\frac{\pi L^2}{4\pi^2}\right)B = \frac{q\omega\omega^2 B}{8\pi^2}.$$

27.79: The y-components of the magnetic field provide forces which cancel as you go around the loop. The x-components of the magnetic field, however, provide a net force in the $-y$- direction.

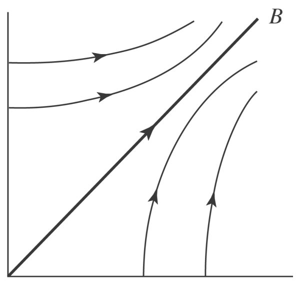
$$F = \int NIB\,dl\sin 60° = NIB\sin 60°\int_0^{2\pi R} dl = 2\pi\ RNIB\sin 60°$$

$$\Rightarrow F = 2\pi(0.0156/2\text{ m})\ (50)\ (0.950\text{ A})\ (0.220\text{ T})\sin 60° = 0.444\text{ N}.$$

27.80: $\sum\vec{\boldsymbol{\tau}}_i = \sum\vec{\boldsymbol{r}}_i\times\vec{\boldsymbol{F}}_i = \sum\vec{\boldsymbol{r}}_i\times\vec{\boldsymbol{F}}_i - \vec{\boldsymbol{r}}_p\times\sum\vec{\boldsymbol{F}}_i = \sum(\vec{\boldsymbol{r}}_i - \vec{\boldsymbol{r}}_p)\times\vec{\boldsymbol{F}}_i = \sum\vec{\boldsymbol{\tau}}_i(P).$

Note that we added a term after the second equals sign that was zero because the body is in translational equilibrium.

27.81: a)



b) Side 1: $\vec{\boldsymbol{F}} = \int_0^L Id\,\vec{\boldsymbol{l}}\times\vec{\boldsymbol{B}} = I\int_0^L \frac{B_0 y\,dy}{L}\hat{\boldsymbol{k}} = \frac{1}{2}B_0 LI\hat{\boldsymbol{i}}.$

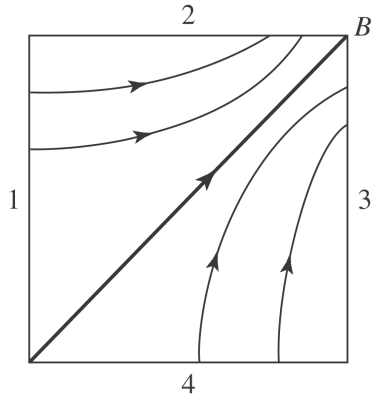
Side 2: $\vec{\boldsymbol{F}} = \int_{0,y=L}^{L} Id\,\vec{\boldsymbol{l}}\times\vec{\boldsymbol{B}} = I\int_{0,y=L}^{L}\frac{B_0 y\,dx}{L}\hat{\boldsymbol{j}} = -IB_0 L\hat{\boldsymbol{j}}.$

Side 3: $\vec{\boldsymbol{F}} = \int_{L,x=L}^{L} Id\,\vec{\boldsymbol{l}}\times\vec{\boldsymbol{B}} = I\int_{L,x=L}^{0}\frac{B_0 y\,dy}{L}(-\hat{\boldsymbol{i}}) = -\frac{1}{2}IB_0 L\hat{\boldsymbol{i}}.$

Side 4: $\vec{\boldsymbol{F}} = \int_{L,y=0}^{0} Id\,\vec{\boldsymbol{l}}\times\vec{\boldsymbol{B}} = I\int_{L,y=0}^{0}\frac{B_0 y\,dx}{L}\hat{\boldsymbol{j}} = 0.$

c) The sum of all forces is $\vec{\boldsymbol{F}}_{\text{total}} = -IB_0 L\hat{\boldsymbol{j}}.$

27.82: a)



b) Side 1: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 y\, dy}{L}(-\hat{k}) = -\frac{1}{2}B_0 LI\hat{k}.$

Side 2: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 x\, dx}{L}\hat{k} = \frac{1}{2}IB_0 L\hat{k}.$

Side 3: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 y dy}{L}\hat{k} = +\frac{1}{2}IB_0 L\hat{k}.$

Side 4: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 x dx}{L}(-\hat{k}) = -\frac{1}{2}IB_0 L\hat{k}.$

c) If free to rotate about the x-axis $\Rightarrow \vec{\tau} = \vec{L} \times \vec{F} = \frac{IB_0 L^2}{2}\hat{i} = \frac{1}{2}IAB_0\hat{i}.$

d) If free to rotate about the y-axis $\Rightarrow \vec{\tau} = \vec{L} \times \vec{F} = \frac{IB_0 L^2}{2}\hat{j} = -\frac{1}{2}IAB_0\hat{j}.$

e) The form of the torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ is not appropriate, since the magnetic field is not constant.

27.83: a) $\Delta y = 0.350\text{ m} - 0.025\text{ m} = 0.325\text{ m},$ we must subtract off the amount immersed since the bar is accelerating until it leaves the pools and thus hasn't reached v_0 yet.

$$v^2 = 0 = v_0{}^2 - 2g\Delta y \Rightarrow v_0 = \sqrt{2g\Delta y.}$$

$$\Rightarrow v_0 = \sqrt{2(9.8\text{ m/s}^2)(0.325)} = 2.52\text{ m/s.}$$

b) In a distance of 0.025 m the wire's speed increases from zero to 2.52 m/s.

$\Rightarrow a = \dfrac{v^2}{2\Delta y} = \dfrac{(2.52\text{ m/s})^2}{2(0.025\text{ m})} = 127\text{ m/s}^2$. But

$$F = ILB - mg = ma \Rightarrow I = \frac{m(g+a)}{LB} = \frac{(5.40\times 10^{-5}\text{ kg})\,((127+9.8)\text{ m/s}^2)}{(0.15\text{ m})\,(0.00650\text{ T})} = 7.58\text{ A.}$$

c) $V = IR \Rightarrow R = \dfrac{V}{I} = \dfrac{1.50\,V}{7.58\,A} = 0.20\,\Omega.$

27.84: a) $I_u = \dfrac{dq}{dt} = \dfrac{\Delta q}{\Delta t} = \dfrac{q_u v}{2\pi r} \Rightarrow I_u = \dfrac{ev}{3\pi r}.$

b) $\mu_u = I_u A = \dfrac{ev}{3\pi r}\pi r^2 = \dfrac{evr}{3}.$

c) Since there are two down quarks, each of half the charge of the up quark,

$$\mu_d = \mu_{\text{u}} = \frac{evr}{3} \Rightarrow \mu_{\text{total}} = \frac{2evr}{3}.$$

d) $v = \dfrac{3\mu}{2er} = \dfrac{3(9.66\times 10^{-27}\text{ A}\cdot\text{m}^2}{2(1.60\times 10^{-19}\text{ C})(1.20\times 10^{-15}\text{ m})} = 7.55\times 10^7\text{ m/s.}$

27.85: a) $\vec{\boldsymbol{\mu}} = IA\,\hat{\boldsymbol{n}} = -IA\hat{\boldsymbol{k}}$ using the right-hand rule.

b) $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}} = \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ 0 & 0 & -IA \\ B_x & B_y & \mathrm{B}_z \end{vmatrix} = \hat{\boldsymbol{i}}(IAB_y) - \hat{\boldsymbol{j}}(IAB_x).$

But $\vec{\boldsymbol{\tau}} = 4D\hat{\boldsymbol{i}} - 3D\hat{\boldsymbol{j}}$, so $IAB_y = 4D, -IAB_x = -3D$

$$\Rightarrow B_x = \frac{3D}{IA},\ \ B_y = \frac{4D}{IA}.$$

But $B_0 = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2} = \dfrac{13D}{IA}$, so $\sqrt{\dfrac{25D^2}{I^2A^2} + B_z{}^2} = \dfrac{13D}{IA}$

$\Rightarrow B_z = \pm\dfrac{12D}{IA}$, but $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} < 0,$ so take $B_z = -\dfrac{12D}{IA}.$

27.86: a) $d\vec{l} = dl\hat{t} = Rd\theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]$. Note that this implies that when $\theta = 0$, the line element points in the + y-direction, and when the angle is $90°$, the line element points in the – x-direction. This is in agreement with the diagram.

$$d\vec{F} = Id\vec{l} \times \vec{B} = IRd\theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]\times(B_x\hat{i}) \Rightarrow d\vec{F} = IB_xRd\theta\left[-\cos\theta\hat{k}\right].$$

b) $\vec{F} = \int_0^{2\pi} -\cos\theta IB_xR\, d\theta\hat{k} = -IB_xR\int_0^{2\pi}\cos\theta d\theta\hat{k} = 0.$

c) $d\vec{\tau} = \vec{r} \times d\vec{F} = R(\cos\theta\hat{i} + \sin\theta\hat{j}) \times (IB_xR\, d\theta\left[-\cos\theta\hat{k}\right])$

$\Rightarrow d\vec{\tau} = -R^2IB_xd\theta\ (\sin\theta\cos\theta\hat{i} - \cos^2\theta\hat{j}).$

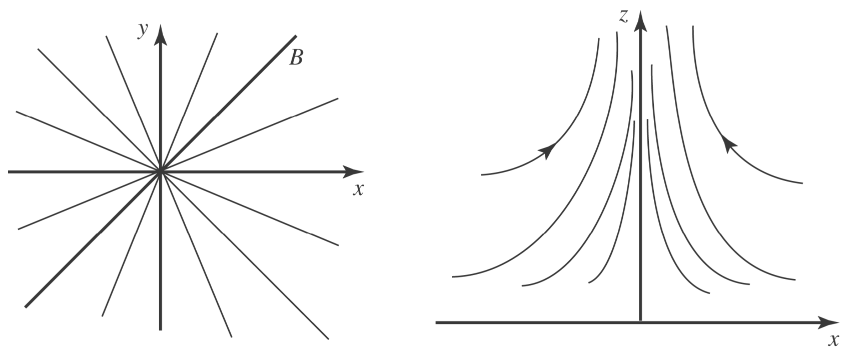
d) $\vec{\tau} = \int d\vec{\tau} = -R^2IB_x\left(\int_0^{2\pi}\sin\theta\cos\theta d\theta\hat{i} - \int_0^{2\pi}\cos^2\theta d\theta\hat{j}\right) = IR^2B_x\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)_0^{2\pi}\hat{j}$

$\Rightarrow \vec{\tau} = IR^2B_x\pi\hat{j} = I\pi R^2B_x\hat{j} = IA\hat{k} \times B_x\hat{i} \Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$

27.87: a) $\oint\vec{B}\cdot d\vec{A} = \int_{\text{top}} B_z dA + \int_{\text{barrel}} B_r dA = \int_{\text{top}}(\beta L)dA + \int_{\text{barrel}} B_r dA = 0.$

$\Rightarrow 0 = \beta L\pi r^2 + B_r 2\pi rL \Rightarrow B_r(r) = -\frac{\beta r}{2}.$

b) The two diagrams show views of the field lines from the top and side:



27.88: a) $\Delta U = -(\vec{\mu}_f\cdot\vec{B} - \vec{\mu}_i\cdot\vec{B})$

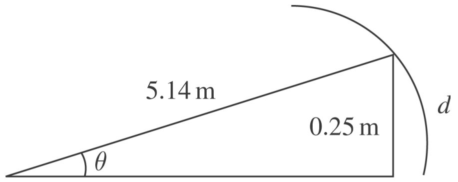
$= -(\vec{\mu}_f - \vec{\mu}_i)\cdot\vec{B} = \left[-\mu(-\hat{k} - (-0.8\hat{i} + 0.6\hat{j}))\right]\cdot\left[B_0(12\hat{i} + 3\hat{j} - 4\hat{k})\right]$

$\Rightarrow \Delta U = IAB_0[(-0.8)(+12) + (0.6)(+3) + (+1)(-4)]$

$\Rightarrow \Delta U = (12.5\text{ A})(4.45\times10^{-4}\text{ m}^2)(0.0115\text{ T})(-11.8) = -7.55\times10^{-4}\text{ J}.$

b) $\Delta K = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{2\Delta K}{I}} = \sqrt{\frac{2(7.55\times10^{-4}\text{ J})}{8.50\times10^{-7}\text{ kg}\cdot\text{m}^2}} = 42.1\text{ rad/s}.$

27.89: a) $R = \dfrac{mv}{qB} = \dfrac{(3.20 \times 10^{-11}\ \text{kg})(1.45 \times 10^{5}\ \text{m/s})}{(2.15 \times 10^{-6}\ \text{C})(0.420\ \text{T})} = 5.14\ \text{m}.$



b) The distance along the curve, d, is given by

$$d = R\theta = (5.14\ \text{m})\sin^{-1}(0.25/5.14) = 0.25\ \text{m}.$$

And $t = \dfrac{d}{v} = \dfrac{0.25\ \text{m}}{1.45 \times 10^{5}\ \text{m/s}} = 1.72 \times 10^{-6}\,\text{s}.$

c) $\Delta x_1 = d\tan(\theta/2) = (0.25\ \text{m})\tan(2.79°/2) = 6.08 \times 10^{-3}\ \text{m}.$

d) $\Delta x = \Delta x_1 + \Delta x_2 = 6.08 \times 10^{-3}\ \text{m} + (0.50\ \text{m})\tan(2.79°) = 0.0304\ \text{m}.$

27.90: a) $\Delta p = FA = IlBA = JlB.$

b) $J = \dfrac{\Delta p}{lB} = \dfrac{(1.00\ \text{atm})(1.013 \times 10^{5}\ \text{Pa/atm})}{(0.0350\ \text{m})(2.20\ \text{T})} = 1.32 \times 10^{6}\ \text{A/m}^2.$

27.91: a) The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0,$ the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

b) $W = Fd = qEd = qEy = \dfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2qEy}{m}}.$

c) At the top,

$$F_y = qE - qvB = -\frac{mv^2}{R} = -\frac{m}{2y}\frac{2qEy}{m}$$

$$= -qE \Rightarrow 2qE = qvB \Rightarrow v = \frac{2E}{B}.$$

28.1: For a charge with velocity $\vec{v} = (8.00\times10^6\ \text{m/s})\hat{j}$, the magnetic field produced at a position r away from the particle is $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$. So for the cases below:

a) $\vec{r} = (+0.500\ \text{m})\hat{i} \Rightarrow \hat{v}\times\hat{r} = -\hat{k},\ r_0^2 = \frac{1}{4}$

$$\Rightarrow \vec{B} = -\frac{\mu_0}{4\pi}\frac{qv}{r_0^2}\hat{k} = -\frac{\mu_0}{4\pi}\frac{(6.0\times10^{-6}\ \text{C})(8.0\times10^6\,\text{m/s})}{(0.50\ \text{m})^2}\hat{k} = -(1.92\times10^{-5}\ \text{T})\hat{k} \equiv -B_0\hat{k}.$$

b) $\vec{r} = (-0.500\ \text{m})\hat{j} \Rightarrow \hat{v}\times\hat{r} = 0 \Rightarrow \vec{B} = 0.$

c) $\vec{r} = (0.500\ \text{m})\hat{k} \Rightarrow \hat{v}\times\hat{r} = +\hat{i},\ r_0^2 = \dfrac{1}{4}.$

$$\Rightarrow \vec{B} = +\frac{\mu_0}{4\pi}\frac{qv}{r_0^2}\hat{i} = B_0\hat{i}.$$

d) $\vec{r} = -(0.500\ \text{m})\hat{j} + (0.500\ \text{m})\hat{k} \Rightarrow \hat{v}\times\hat{r} = -\hat{i},\ r^2 = \dfrac{1}{2} = 2r_0$

$$\Rightarrow \vec{B} = +\frac{\mu_0}{4\pi}\frac{qv}{r^2}\frac{\hat{i}}{\sqrt{2}} = +\frac{B_0}{2}\frac{\hat{i}}{\sqrt{2}} = +\frac{B_0\hat{i}}{2\sqrt{2}}$$

28.2: $B_{\text{total}} = B + B' = \dfrac{\mu_0}{4\pi}\left(\dfrac{qv}{d^2} + \dfrac{q'v'}{d^2}\right)$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\left(\frac{(8.0\times10^{-6}\ \text{C})(4.5\times10^6\,\text{m/s})}{(0.120\ \text{m})^2} + \frac{(3.0\times10^{-6}\ \text{C})(9.0\times10^6\,\text{m/s})}{(0.120\ \text{m})^2}\right)$$

$\Rightarrow B = 4.38\times10^{-4}\ \text{T}$, into the page.

28.3: $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\vec{r}}{r^3}$

a) $\vec{v} = v\hat{i},\ \vec{r} = r\hat{i};\quad \vec{v}\times\vec{r} = \mathbf{0},\ B = 0$

b) $\vec{v} = v\hat{i},\ \vec{r} = r\hat{j};\quad \vec{v}\times\vec{r} = vr\hat{k},\ r = 0.500\ \text{m}$

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{1\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^{5}\,\text{m/s})}{(0.500\,\text{m})^2} = 1.31\times10^{-6}\ \text{T}$$

q is negative, so $\vec{B} = -(1.31\times 0^{-6}\ \text{T})\hat{k}$

c) $\vec{v} = v\hat{i},\ \vec{r} = (0.500\ \text{m})(\hat{i} + \hat{j});\quad \vec{v}\times\vec{r} = (0.500\,\text{m})v\hat{k},\ r = 0.7071\,\text{m}$

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(|q||\vec{v}\times\vec{r}|/r^3\right) = \frac{1\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(0.500\,\text{m})(6.80\times10^{5}\,\text{m/s})}{(0.7071\,\text{m})^3}$$

$B = 4.62\times10^{-7}\ \text{T};\quad \vec{B} = -(4.62\times10^{-7}\ \text{T})\hat{k}$

d) $\vec{v} = v\hat{i},\ \vec{r} = r\hat{k};\ \vec{v}\times\vec{r} = -vr\hat{j},\ r = 0.500\ \text{m}$

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{1\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^{5}\ \text{m/s})}{(0.500\ \text{m})^2}$$

$B = 1.31\times10^{-6}\ \text{T};\quad \vec{B} = (1.31\times10^{-6}\ \text{T})\hat{j}$

28.4: a) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi}\frac{qq'vv'}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(8.00\times10^{-6}\ \text{C})(3.00\times10^{-6}\ \text{C})(4.50\times10^{6}\text{m/s})(9.00\times10^{6}\text{m}}{(0.240\ \text{m})^2}$$

$= 1.69\times10^{-3}$ N (the force on the upper charge points up and the force on the lower char points down).

The Coulomb force between the charges is

$F = k\frac{q_1q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ \frac{(8.00)(3.00)\times10^{-12}\,\text{C}^2}{(0.240\ \text{m})^2} = 3.75\ \text{N}$ (the force on the upper charge points up and the force on the lower charge points down).

The ratio of the Coulomb force to the magnetic force is $\frac{3.75\ \text{N}}{1.69\times10^{-3}\ \text{N}} = 2.22\times10^3 = \frac{c^2}{v_1v_2}$.

b) The magnetic forces are reversed when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

28.5: The magnetic field is into the page at the origin, and the magnitude is

$$B = B + B' = \frac{\mu_0}{4\pi}\left(\frac{qv}{r^2} + \frac{q'v'}{r'^2}\right)$$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\left(\frac{(4.0\times10^{-6}\ \text{C})(2.0\times10^5\,\text{m/s})}{(0.300\,\text{m})^2} + \frac{(1.5\times10^{-6}\ \text{C})(8.0\times10^5\,\text{m/s})}{(0.400\,\text{m})^2}\right)$$

$\Rightarrow B = 1.64\times10^{-6}$ T, into the page.

28.6: a) $q' = -q;\ B_q = \dfrac{\mu_0 qv}{4\pi d^2}$ into the page; $B_{q'} = \dfrac{\mu_0 qv'}{4\pi d^2}$ out of the page.

(i) $v' = \dfrac{v}{2} \Rightarrow B = \dfrac{\mu_0 qv}{4\pi(2d^2)}$ into the page.

(ii) $v' = v \Rightarrow B = 0.$

(iii) $v' = 2v \Rightarrow B = \dfrac{\mu_0 qv}{4\pi d^2}$ out of the page.

b) $\vec{F} = q'\vec{v}' \times \vec{B}_q \Rightarrow \dfrac{\mu_0 q^2 v'v}{4\pi(2d)^2}$ and is attractive.

c) $F_B = \dfrac{\mu_0 q^2 vv'}{4\pi(2d)^2},\ F_C = \dfrac{q^2}{4\pi\varepsilon_0(2d)^2} \Rightarrow \dfrac{F_B}{F_C} = \mu_0\varepsilon_0 vv' = \mu_0\varepsilon_0(3.00\times10^5\,\text{m/s})^2$

$= 1.00\times10^{-6}.$

28.7: a) $\hat{\boldsymbol{r}} = \cos\theta\hat{\boldsymbol{i}} + \sin\theta\hat{\boldsymbol{j}} = \cos(150°)\hat{\boldsymbol{i}} + \sin(150°)\hat{\boldsymbol{j}} = -(0.866)\hat{\boldsymbol{i}} + (0.500)\hat{\boldsymbol{j}}.$

b) $d\vec{l} \times \hat{\boldsymbol{r}} = (-dl\,\hat{\boldsymbol{i}}) \times (-(0.866)\hat{\boldsymbol{i}} + (0.500)\hat{\boldsymbol{j}}) = -dl(0.500)\hat{\boldsymbol{k}} = -(5.00\times10^{-3}\,\text{m})\hat{\boldsymbol{k}}$

c) $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\vec{l}\times\hat{\boldsymbol{r}}}{r^2} = -\dfrac{\mu_0}{4\pi}\dfrac{I\ dl\ (0.500\,\text{m})}{r^2}\hat{\boldsymbol{k}} = -\dfrac{\mu_0}{4\pi}\dfrac{(125\,\text{A})(0.010\,\text{m})(0.500\,\text{m})}{(1.20\,\text{m})^2}\hat{\boldsymbol{k}}$

$\Rightarrow d\vec{B} = -(4.3\times10^{-8}\ \text{T})\hat{\boldsymbol{k}}.$

28.8: The magnetic field at the given points is:

$$dB_a = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\text{ A})\,(0.000100\text{ m})}{(0.100\text{ m})^2} = 2.00\times10^{-6}\text{ T}.$$

$$dB_b = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\text{ A})\,(0.000100\text{ m})\sin 45^\circ}{2(0.100\text{ m})^2} = 0.705\times10^{-6}\text{ T}.$$

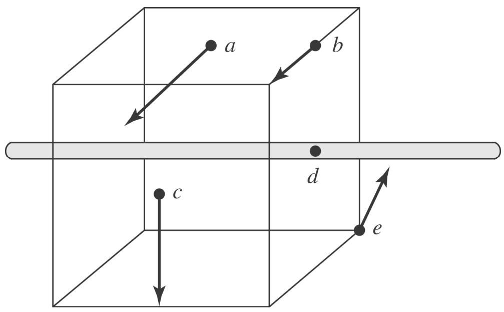
$$dB_c = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\text{ A})\,(0.000100\text{ m})}{(0.100\text{ m})^2} = 2.00\times10^{-6}\text{ T}.$$

$$dB_d = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin(0^\circ)}{r^2} = 0.$$

$$dB_e = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$\Rightarrow dB_e = \frac{\mu_0}{4\pi}\frac{(200\text{ A})\,(0.00100\text{ m})}{3(0.100\text{ m})^2}\frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow dB_e = 0.545\times10^{-6}\text{ T}.$$



28.9: The wire carries current in the z-direction. The magnetic field of a small piece of wire $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$ at different locations is therefore:

a) $\vec{r} = (2.00\text{ m})\hat{i} \Rightarrow \hat{l} \times \hat{r} = \hat{j}$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}\hat{j} = \frac{\mu_0}{4\pi}\frac{(4.00\text{ A})(5\times10^{-4}\text{ m})\sin 90°}{(2.00\text{ m})^2} = 5.00\times10^{-11}\text{ T}\hat{j}.$$

b) $\vec{r} = (2.00\text{ m})\hat{j} \Rightarrow \hat{l} \times \hat{r} = -\hat{i}.$

$$\Rightarrow d\vec{B} = \frac{-\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}\hat{i} = \frac{-\mu_0}{4\pi}\frac{(4.00\text{ A})(5\times10^{-4}\text{ m})\sin(90°)}{(2.00\text{ m})^2}\hat{i}$$

$$= -5.00\times10^{-11}\text{ T}\hat{i}.$$

c) $\vec{r} = (2.00\text{ m})\hat{i} + (2.00\text{ m})\hat{j} \Rightarrow \hat{l} \times \hat{r} = \frac{1}{\sqrt{2}}(\hat{j} - \hat{i})$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}\frac{1}{\sqrt{2}}(\hat{j} - \hat{i}) = \frac{\mu_0}{4\pi}\frac{(4.00\text{ A})(5.0\times10^{-4}\text{ m})}{(2.00\text{ m})^2 + (2.00\text{ m})^2}\frac{1}{\sqrt{2}}(\hat{j} - \hat{i})$$

$$= 1.77\times10^{-11}\text{ T}(\hat{j} - \hat{i})$$

d) $\vec{r} = (2.00\text{ m})\hat{k} \Rightarrow \hat{l} \times \hat{r} = 0$

28.10: a) At $x = \frac{d}{2}: B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{d/2} + \frac{1}{3d/2}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{8}{3d}\right) = \frac{4\mu_0 I}{3\pi d}$, in the $\hat{j}$ direction.

b) The position $x = -\frac{d}{2}$ is symmetrical with that of part (a), so the magnetic field there is $B = \frac{4\mu_0 I}{3\pi d}$, in the $\hat{j}$ direction.

28.11: a) At the point exactly midway between the wires, the two magnetic fields are in opposite directions and cancel.

b) At a distance a above the top wire, the magnetic fields are in the same direction and add up: $\vec{B} = \frac{\mu_0 I}{2\pi r_1}\hat{k} + \frac{\mu_0 I}{2\pi r_2}\hat{k} = \frac{\mu_0 I}{2\pi a}\hat{k} + \frac{\mu_0 I}{2\pi(3a)}\hat{k} = \frac{2\mu_0 I}{3\pi a}\hat{k}$.

c) At the same distance as part (b), but below the lower wire, yields the same magnitude magnetic field but in the opposite direction: $\vec{B} = -\frac{2\mu_0 I}{3\pi a}\hat{k}$.

28.12: The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field. So:

a) At (0, 0, 1 m):

$$\vec{B} = \vec{B}_0 - \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{i}} = (1.50\times10^{-6}\,\text{T})\hat{\boldsymbol{i}} - \frac{\mu_0(8.00\text{ A})}{2\pi(1.00\text{ m})}\hat{\boldsymbol{i}} = -(1.0\times10^{-7}\,\text{T})\hat{\boldsymbol{i}}.$$

b) At (1 m, 0, 0):

$$\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{k}} = (1.50\times10^{-6}\,\text{T})\hat{\boldsymbol{i}} + \frac{\mu_0(8.00\text{ A})}{2\pi(1.00\text{ m})}\hat{\boldsymbol{k}}$$

$$\Rightarrow \vec{B} = (1.50\times10^{-6}\,\text{T})\hat{\boldsymbol{i}} + (1.6\times10^{-6}\text{ T})\hat{\boldsymbol{k}} = 2.19\times10^{-6}\text{ T},\ \text{at } \theta = 46.8^\circ$$

from x to z.

c) At (0, 0, – 0.25 m): $\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{i}} = (1.50\times10^{-6}\,\text{T})\hat{\boldsymbol{i}} + \frac{\mu_0(8.00\text{ A})}{2\pi(0.25\text{ m})}\hat{\boldsymbol{i}}$

$$= (7.9\times10^{-6}\text{ T})\hat{\boldsymbol{i}}.$$

28.13: $B = \dfrac{\mu_0 I}{4\pi}\displaystyle\int_{-a}^{a}\frac{x\,dy}{(x^2+y^2)^{3/2}} = \frac{\mu_0 I x}{4\pi}\,\frac{y}{x^2(x^2+y^2)^{1/2}}\bigg|_{-a}^{a} = \frac{\mu_0 I}{4\pi}\,\frac{2a}{x(x^2+a^2)^{1/2}}.$

28.14: a) $B_0 = \dfrac{\mu_0 I}{2\pi r} \Rightarrow I = \dfrac{2\pi r B_0}{\mu_0} = \dfrac{2\pi(0.040\text{ m})\,(5.50\times10^{-4}\text{ T})}{\mu_0} = 110\text{ A}.$

b) $B = \dfrac{\mu_0 I}{2\pi r}$, so $B(r = 0.080\text{ m}) = \dfrac{B_0}{2} = 2.75\times10^{-4}\text{ T},$

$$B(r = 0.160\text{ m}) = \frac{B_0}{4} = 1.375\times10^{-4}\text{ T}.$$

28.15: a) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\mu_0(800\text{ A})}{2\pi(5.50\text{ m})} = 2.90\times10^{-5}\text{ T},$ to the east.

b) Since the magnitude of the earth's magnetic filed is 5.00×10^{-5} T, to the north, the total magnetic field is now 30° east of north with a magnitude of 5.78×10^{-5} T. This could be a problem!

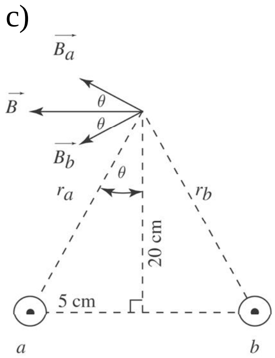
28.16: a) $B = 0$ since the fields are in opposite directions.

b) $$B = B_a + B_b = \frac{\mu_0 I}{2\pi r_a} + \frac{\mu_0 I}{2\pi r_b} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_a} + \frac{1}{r_b}\right)$$

$$= \frac{(4\pi \times 10^{-7}\ \text{Tm/A})\ (4.0\ \text{A})}{2\pi}\left(\frac{1}{0.3\ \text{m}} + \frac{1}{0.2\ \text{m}}\right)$$

$$= 6.67 \times 10^{-6}\ \text{T} = 6.67\ \mu\text{T}$$

c)



Note that $\vec{\boldsymbol{B}}_a \perp r_a$ and $\vec{\boldsymbol{B}}_b \perp r_b$

$$B = B_a \cos\theta + B_b \cos\theta$$
$$= 2B_a \cos\theta$$

$$\tan\theta = \frac{5}{20} \rightarrow \theta = 14.04^\circ : r_a = \sqrt{(0.20\ \text{m})^2 + (0.05\ \text{m})^2}$$

$$B = 2\frac{\mu_0 I}{2\pi r_a}\cos\theta$$

$$= 2\frac{(4\pi + 10^{-7}\ \text{Tm/A})\ (4.0\ \text{A})}{2\pi\ \sqrt{(0.20\ \text{m})^2 + (0.05\ \text{m})^2}}\cos 14.04^\circ$$

$$= 7.53 \times 10^{-6}\ \text{T} = 7.53\ \mu\text{T},\ \text{to the left.}$$

28.17: The only place where the magnetic fields of the two wires are in opposite directions is between the wires, in the plane of the wires.

Consider a point a distance x from the wire carrying $I_2 = 75.0$ A. B_{tot} will be zero where $B_1 = B_2$.

$$\frac{\mu_0 I_1}{2\pi(0.400\ \text{m} - x)} = \frac{\mu_0 I_2}{2\pi x}$$

$$I_2(0.400\ \text{m} - x) = I_1 x;\ \ I_1 = 25.0\ \text{A},\ I_2 = 75.0\ \text{A}$$

$x = 0.300$ m; $B_{\text{tot}} = 0$ along a line 0.300 m from the wire carrying 75.0 A amd 0.100 m from the wire carrying current 25.0 A.

b) Let the wire with $I_1 = 25.0$ A be 0.400 m above the wire with $I_2 = 75.0$ A. The magnetic fields of the two wires are in opposite directions in the plane of the wires and at points above both wires or below both wires. But to have $B_1 = B_2$ must be closer to wire #1 since $I_1 < I_2$, so can have $B_{\text{tot}} = 0$ only at points above both wires.

Consider a point a distance x from the wire carrying $I_1 = 25.0$ A. B_{tot} will be zero where $B_1 = B_2$.

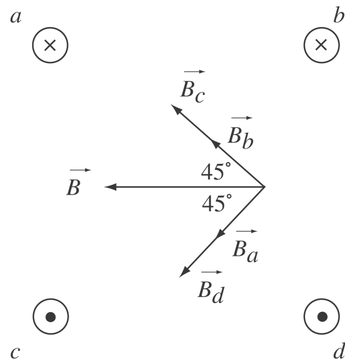
$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi\ (0.400\ \text{m} + x)}$$

$$I_2 x = I_1(0.400\ \text{m} + x);\ \ x = 0.200\ \text{m}$$

$B_{\text{tot}} = 0$ along a line 0.200 m from the wire carrying 25.0 A and 0.600 m from the wire carrying current $I_2 = 75.0$ A.

28.18: (a) and (b) $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c)



$$B = B_a \cos 45° + B_b \cos 45° + B_c \cos 45° + B_d \cos 45°$$

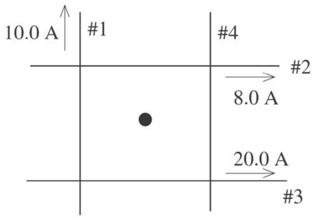
$$= 4B_a \cos 45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45°$$

$$r = \sqrt{(10\text{ cm})^2 + (10\text{ cm})^2} = 10\sqrt{2}\text{ cm} = 0.10\sqrt{2}\text{ m}$$

$$B = 4\frac{(4\pi \times 10^{-7}\text{ Tm/A})\,(100\text{ A})}{2\pi(0.10\sqrt{2}\text{ m})}\cos 45°$$

$$= 4.0 \times 10^{-4}\text{ T, to the left.}$$

28.19:



$$\vec{B}_1 \otimes, \vec{B}_2 \otimes, \vec{B}_3 \odot$$

$$B = \frac{\mu_0 I}{2\pi r};\ r = 0.200 \text{ m for each wire}$$

$B_1 = 1.00 \times 10^{-5}$ T, $B_2 = 0.80 \times 10^{-5}$ T , $B_3 = 2.00 \times 10^{-5}$ T

Let $\odot$ be the positive z-direction. $I_1 = 10.0$ A, $I_2 = 8.0$ A, $I_3 = 20.0$ A

$B_{1z} = -1.00 \times 10^{-5}$ T, $B_{2z} = -0.80 \times 10^{-5}$ T, $B_{3z} = +2.00 \times 10^{-5}$ T

$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0$

$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0 \times 10^{-6}$ T

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200 \text{ m})\,(2.0 \times 10^{-6} \text{ T})}{(2 \times 10^{-7} \text{ T} \cdot \text{m/A})} = 2.0 \text{ A}$$

28.20: On the top wire: $\dfrac{F}{L} = \dfrac{\mu_0 I^2}{2\pi}\left(\dfrac{1}{d} - \dfrac{1}{2d}\right) = \dfrac{\mu_0 I^2}{4\pi d}$, upward.

On the middle wire, the magnetic fields cancel so the force is zero.

On the bottom wire: $\dfrac{F}{L} = \dfrac{\mu_0 I^2}{2\pi}\left(-\dfrac{1}{d} + \dfrac{1}{2d}\right) = \dfrac{\mu_0 I^2}{4\pi d}$, downward.

28.21: We need the magnetic and gravitational forces to cancel:

$$\Rightarrow \lambda L g = \frac{\mu_0 I^2 L}{2\pi h} \Rightarrow h = \frac{\mu_0 I^2}{2\pi \lambda g}$$

28.22: a) $F = \frac{\mu_0 I_1 I_2 L}{2\pi r} = \frac{\mu_0 (5.00\text{ A})(2.00\text{ A})(1.20\text{ m})}{2\pi(0.400\text{ m})} = 6.00 \times 10^{-6}\text{ N},$ and the force is repulsive since the currents are in opposite directions.

b) Doubling the currents makes the force increase by a factor of four to $F = 2.40 \times 10^{-5}\text{ N}.$

28.23: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} \Rightarrow I_2 = \frac{F}{L}\frac{2\pi r}{\mu_0 I_1} = (4.0 \times 10^{-5}\text{ N/m})\frac{2\pi(0.0250\text{ m})}{\mu_0(0.60\text{ A})} = 8.33\text{ A}.$

b) The two wires repel so the currents are in opposite directions.

28.24: There is no magnetic field at the center of the loop from the straight sections. The magnetic field from the semicircle is just half that of a complete loop:

$$B = \frac{1}{2}B_{\text{loop}} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right) = \frac{\mu_0 I}{4R},$$

into the page.

28.25: As in Exercise 28.24, there is no contribution from the straight wires, and now we have two oppositely oriented contributions from the two semicircles:

$$B = (B_1 - B_2) = \frac{1}{2}\left(\frac{\mu_0}{2R}\right)\left|I_1 - I_2\right|,$$

into the page. Note that if the two currents are equal, the magnetic field goes to zero at the center of the loop.

28.26: a) The field still points along the positive *x*-axis, and thus points into the loop from this location.

b) If the current is reversed, the magnetic field is reversed. At point P the field would then point into the loop.

c) Point the thumb of your right hand in the direction of the magnetic moment, under the given circumstances, the current would appear to flow in the direction that your fingers curl (*i.e.*, clockwise).

28.27: a) $B_x = \mu_0 NI/2a$, so $I = \dfrac{2aB_x}{\mu_0 N} = \dfrac{2(0.024\text{ m})\,(0.0580\text{ T})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})\,(800)} = 2.77\text{ A}$

b) At the center, $B_c = \mu_0 NI/2a$. At a distance x from the center,

$$B_x = \frac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}} = \left(\frac{\mu_0 NI}{2a}\right)\left(\frac{a^3}{(x^2+a^2)^{3/2}}\right) = B_c\left(\frac{a^3}{(x^2+a^2)^{3/2}}\right)$$

$$B_x = \tfrac{1}{2}B_c \text{ means } \frac{a^3}{(x^2+a^2)^{3/2}} = \frac{1}{2}$$

$(x^2+a^2)^3 = 4a^6$, with $a = 0.024$ m, so $x = 0.0184$ m

28.28: a) From Eq. (29-17), $B_{\text{center}} = \dfrac{\mu_0 NI}{2a} = \dfrac{\mu_0(600)\,(0.500\text{ A})}{2(0.020\text{ m})} = 9.42\times10^{-3}$ T.

b) From Eq. (29-16),

$$B(x) = \frac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}} \Rightarrow B(0.08\text{ m}) = \frac{\mu_0(600)\,(0.500\text{ A})\,(0.020\text{ m})^2}{2((0.080\text{ m})^2+(0.020\text{ m})^2)^{3/2}} = 1.34\times10^{-4}\text{ T.}$$

28.29: $B(x) = \dfrac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}} \Rightarrow N = \dfrac{2B(x)\,(x^2+a^2)^{3/2}}{\mu_0 Ia^2}$

$$= \frac{2(6.39\times10^{-4}\text{ T})\left[(0.06\text{ m})^2+(0.06\text{ m})^2\right]^{3/2}}{\mu_0(2.50\text{ A})\,(0.06\text{ m})^2} = 69$$

28.30: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83\times10^{-4}\text{ T}\cdot\text{m} \Rightarrow I_{\text{encl}} = 305$ A.

b) -3.83×10^{-4} since $d\vec{l}$ points opposite to $\vec{B}$ everywhere.

28.31: We will travel around the loops in the counterclockwise direction.

a) $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{l} = 0.$

b) $I_{\text{encl}} = -I_1 = -4.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = -\mu_0(4.0\text{ A}) = -5.03\times10^{-6}\text{T}\cdot\text{m}.$

c) $I_{\text{encl}} = -I_1 + I_2 = -4.0\text{ A} + 6.0\text{ A} = 2.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0(2.0\text{ A})$

$= 2.51\times10^{-6}\text{ T}\cdot\text{m}.$

d) $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = +\mu_0(4.0\text{ A}) = 5.03\times10^{-6}\text{ T}\cdot\text{m}.$

Using Ampere's Law in each case, the sign of the line integral was determined by using the right-hand rule. This determines the sign of the integral for a counterclockwise path.

28.32: Consider a coaxial cable where the currents run in OPPOSITE directions.

a) For $a < r < b, I_{encl} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}.$

b) For $r > c,$ the enclosed current is zero, so the magnetic field is also zero.

28.33: Consider a coaxial cable where the currents run in the SAME direction.

a) For $a < r < b, I_{\text{encl}} = I_1 \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I_1 \Rightarrow B2\pi r = \mu_0 I_1 \Rightarrow B = \dfrac{\mu_0 I_1}{2\pi r}.$

b) For $r > c, I_{\text{encl}} = I_1 + I_2 \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 (I_1 + I_2) \Rightarrow B2\pi r = \mu_0 (I_1 + I_2)$

$$\Rightarrow B = \frac{\mu_0 (I_1 + I_2)}{2\pi r}.$$

28.34: Using the formula for the magnetic field of a solenoid:

$$B = \mu_0 n I = \frac{\mu_0 N I}{L} = \frac{\mu_0 (600)\,(8.00\text{ A})}{(0.150\text{ m})} = 0.0402\text{ T}.$$

28.35: a) $B = \dfrac{\mu_0 N I}{L} \Rightarrow N = \dfrac{BL}{\mu_0 I} = \dfrac{(0.0270\text{ T})\,(0.400\text{ m})}{\mu_0 (12.0\text{ A})} = 716\text{ turns}$

$$\Rightarrow n = \frac{N}{L} = \frac{716\text{ turns}}{0.400\text{ m}} = 1790\text{ turns/m}.$$

b) The length of wire required is $2\pi r N = 2\pi (0.0140\text{ m})(116) = 63\text{ m}.$

28.36:

$$B = \mu_0 I \frac{N}{L}$$

$$I = \frac{BL}{\mu_0 N}$$

$$= \frac{(0.150\text{ T})\,(1.40\text{ m})}{(4\pi \times 10^{-7}\text{ Tm/A})(4000)}$$

$$= 41.8\text{ A}$$

28.37: a) $B = \dfrac{\mu_0 I}{2\pi r}$, so $I = \dfrac{Br}{(\mu_0/2\pi)} = 3.72\times10^6$ A

b) $B_x = \dfrac{\mu_0 NI}{2a}$, so $I = \dfrac{2aB_x}{\mu_0 N} = 2.49\times10^5$ A

c) $B = \mu_0 nI = \mu_0 (N/L)I$, so $I = BL/\mu_0 N = 237$ A

28.38: Outside a toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B = \dfrac{\mu_0 NI}{2\pi r}$.

a) $r = 0.12$ m, which is outside the toroid, so $B = 0$.

b) $r = 0.16 \text{ m} \Rightarrow B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0 (250)\ (8.50\text{ A})}{2\pi (0.160\text{ m})} = 2.66\times10^{-3}$ T.

c) $r = 0.20$ m, which is outside the toroid, so $B = 0$

28.39: $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0 (600)\ (0.650\text{ A})}{2\pi\ (0.070\text{ m})} = 1.11\times10^{-3}$ T.

28.40: a) $B = \dfrac{\mu\ NI}{2\pi r} = \dfrac{K_m \mu_0 NI}{2\pi r} = \dfrac{\mu_0 (80)\ (400)\ (0.25\text{ A})}{2\pi\ (0.060\text{ m})} = 0.0267$ T.

b) The fraction due to atomic currents is $B' = \frac{79}{80} B = \frac{79}{80}(0.0267\text{ T}) = 0.0263\text{ T}$.

28.41: a) If $K_m = 1400 \Rightarrow B = \dfrac{K_m \mu_0 NI}{2\pi r} \Rightarrow I = \dfrac{2\pi r\ B}{K_m \mu_0 N} = \dfrac{2\pi (0.0290\text{ m})\ (0.350\text{ T})}{\mu_0 (1400)(500)} =$ 0.0725 A.

b) If $K_{\text{m}} = 5200 \Rightarrow I = \dfrac{1400}{5200} I_{\text{part(a)}} = 0.0195$ A.

28.42: a) $B = \dfrac{K_m \mu_0 NI}{2\pi r} \Rightarrow K_m = \dfrac{2\pi rB}{\mu_0 NI} = \dfrac{2\pi (0.2500\text{ m})\ (1.940\text{ T})}{\mu_0 (500)\ (\ 2.400\text{ A})} = 2021.$

b) $\mathrm{X}_m = K_m - 1 = 2020.$

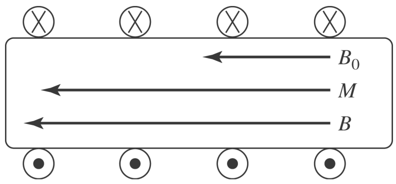
28.43: a) The magnetic field from the solenoid alone is:

(i) $B_0 = \mu_0 nI = \mu_0(6000\ \text{m}^{-1})\ (0.15\ \text{A}) \Rightarrow B_0 = 1.13\times10^{-3}\ \text{T}.$

(ii) But $M = \dfrac{K_m - 1}{\mu_0}B_0 = \dfrac{5199}{\mu_0}(1.13\times10^{-3}\ \text{T}) \Rightarrow M = 4.68\times10^{6}\ \text{A/m}.$

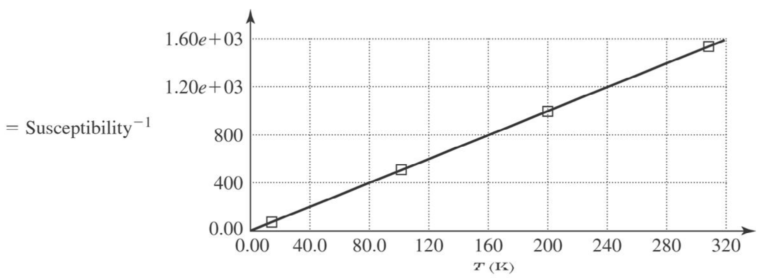
(iii) $B = K_m B_0 = (5200)(1.13\times10^{-3}\ \text{T}) = 5.88\ \text{T}.$

b)



28.44: $\left[\dfrac{\text{J}}{\text{T}}\right] = \left[\dfrac{\text{N}\cdot\text{m}}{\text{N}\cdot\text{s/C}\cdot\text{m}}\right] = \left[\dfrac{\text{C}\cdot\text{m}^2}{\text{s}}\right] = \left[\text{A}\cdot\text{m}^2\right].$

28.45:



The material does obey Curie's Law because we have a straight line for temperature against one over the magnetic susceptibility. The Curie constant from the graph is

$$C = \frac{1}{\mu_0(\text{slope})} = \frac{1}{\mu_0(5.13)} = 1.55\times10^{5}\ \text{K}\cdot\text{A/T}\cdot\text{m}.$$

28.46: The magnetic field of charge q' at the location of charge q is into the page.

$$\vec{\boldsymbol{F}} = q\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}' = (qv)\hat{\boldsymbol{i}}\times\frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}'\times\hat{\boldsymbol{r}}}{r^2} = (qv)\hat{\boldsymbol{i}}\times\left(\frac{\mu_0}{4\pi}\frac{qv'\sin\theta}{r^2}\right)(-\hat{\boldsymbol{k}}) = \left(\frac{\mu_0}{4\pi}\frac{qq'\sin\theta}{r^2}\right)\hat{\boldsymbol{j}}$$

where θ is the angle between v' and $\hat{\boldsymbol{r}}'$.

$$\Rightarrow \vec{\boldsymbol{F}} = \left(\frac{\mu_0}{4\pi}\frac{(8.00\times10^{-6}\ \text{C})(5.00\times10^{-6}\text{C})(9.00\times10^{4}\ \text{m/s})(6.50\times10^{4}\ \text{m/s})}{(0.500\ \text{m})^2}\left(\frac{0.4}{0.5}\right)\right)\hat{\boldsymbol{j}}$$

$$\Rightarrow \vec{\boldsymbol{F}} = (7.49\times10^{-8}\ \text{N})\hat{\boldsymbol{j}}.$$

28.47: $F = qvB = qv\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0}{2\pi}\frac{(1.60\times10^{-19}\,\text{C})(6.00\times10^{4}\ \text{m/s})(2.50\ \text{A})}{(0.045\ \text{m})}$

$= 1.07\times10^{-19}\ \text{N}.$

Let the current run left to right, the electron moves in the opposite direction, below the wire, then the magnetic field at the electron is into the page, and the electron feels a force upward, toward the wire, by the right-hand rule (remember the electron is negative).

28.48: (a) $a = \frac{F}{m} = \frac{qvB\sin\theta}{m} = \frac{ev}{m}\left(\frac{\mu_0 I}{2\pi r}\right)$

$$a = \frac{(1.6\times10^{-17}\,\text{C})(250{,}000\,\text{m/s})(4\pi\times10^{-7}\ \text{Tm/A})(25\ \text{A})}{(9.11\times10^{-31}\,\text{kg})(2\pi)(0.020\ \text{m})}$$

$= 1.1\times10^{13}\ \text{m/s}^2,$ away from the wire.

b) The electric force must balance the magnetic force.

$eE = eVB$

$E = vB = v\frac{\mu_0 i}{2\pi r}$

$= \frac{(250,000\ \text{m/s})(4\pi\times10^{-7}\ \text{Tm/A})(25\ \text{A})}{2\pi(0.020\ \text{m})}$

$= 62.5\ \text{N/C},$ away from the wire.

(c) $mg = (9.11\times10^{-31}\ \text{kg})(9.8\ \text{m/s}^2) \approx 10^{-29}\ \text{N}$

$F_{\text{el}} = eE = (1.6\times10^{-19}\ \text{C})(62.5\ \text{N/C}) \approx 10^{-17}\ \text{N}$

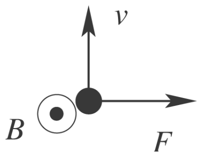
$F_{\text{el}} \approx 10^{12}\ F_{\text{grav}},$ so we can neglect gravity.

28.49: Let the wire connected to the 25.0 Ω resistor be #2 and the wire connected to the 10.0 Ω resistor be #1. Both I_1 and I_2 are directed toward the right in the figure, so at the location of the proton I_2 *is* $\otimes$ and $I_1 = \odot$

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \text{ and } B_2 = \frac{\mu_0 I_2}{2\pi r}, \text{ with r} = 0.0250\,\text{m}.$$

$$B_1 = 8.00\times10^{-5}\ \text{T}, B_2 = 3.20\times10^{-5}\ \text{T and } B = B_1 - B_2 = 4.80\times10^{-5}\ \text{T}$$

and in the direction $\odot$.



Force is to the right.

$$F = qvB = (1.602\times10^{-19}\text{C})(650\times10^{3}\ \text{m/s})(4.80\times10^{-5}\ \text{T}) = 5.00\times10^{-18}\ \text{N}$$

28.50: The fields add

$$B = B_1 + B_2 = 2B_1 = 2\left[\frac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}\right]$$

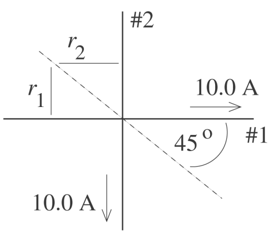
$$= \frac{(4\pi\times10^{-7}\text{Tm/A})(1.50\ \text{A})(0.20\ \text{m})^2}{[(0.20\ \text{m})^2 + (0.125\ \text{m})^2]^{3/2}} = 5.75\times10^{-6}\ \text{T}$$

$$F = qv\,B\sin\theta$$

$$= (1.6\times10^{-19}\ \text{C})(2400\ \text{m/s})(5.75\times10^{-6}\ \text{T})\sin 90°$$

$= 2.21\times10^{-21}$ N, perpendicular to the line *ab* and to the velocity.

28.51: a)



Along the dashed line, $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions. If the line has slope -1.00 then $r_1 = r_2$ and $B_1 = B_2$, so $B_{tot} = 0$.

28.52: a) $\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\overrightarrow{\boldsymbol{v}_0} \times \hat{\boldsymbol{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ v_{0x} & v_{0y} & v_{0z} \\ 1 & 0 & 0 \end{vmatrix}$

$$= \frac{\mu_0}{4\pi}\frac{q}{r^2}\left(v_{0z}\hat{\boldsymbol{j}} - v_{0y}\hat{\boldsymbol{k}}\right) = (6.00 \times 10^{-6}\ \text{T})\hat{\boldsymbol{j}}$$

$$\Rightarrow \frac{\mu_0}{4\pi}\frac{q}{r^2}v_{0y} = 0 \Rightarrow v_{0y} = 0 \text{ and } -\frac{\mu_0}{4\pi}\frac{|q|}{r^2}v_{0z} = 6.00 \times 10^{-6}\ \text{T}$$

$$\Rightarrow v_{0z} = -\frac{4\pi(6.00 \times 10^{-6}\ \text{T})(0.25\ \text{m})^2}{\mu_0(-7.20 \times 10^{-3}\ \text{C})} = -521\ \text{m/s}.$$

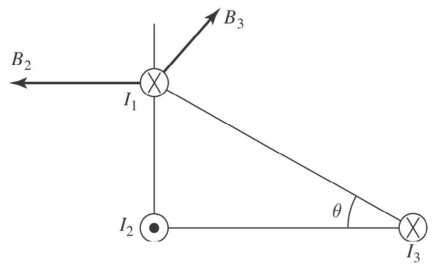
And $v_{0x} = \pm\sqrt{v_0^2 - v_{0y}{}^2 - v_{0z}{}^2} = \pm\sqrt{(800\ \text{m/s})^2 - (-521\ \text{m/s})^2} = \pm 607\ \text{m/s}.$

b) $\vec{\boldsymbol{B}}(0,\ 0.250\ \text{m},\ 0) = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}_0 \times \hat{\boldsymbol{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ v_{0x} & v_{0y} & v_{0z} \\ 0 & 1 & 0 \end{vmatrix} = +\frac{\mu_0}{4\pi}\frac{q}{r^2}\left(v_{0x}\hat{\boldsymbol{k}} - v_{0z}\hat{\boldsymbol{i}}\right)$

$$\Rightarrow B(0,\ 0.250\ \text{m},\ 0) = \frac{\mu_0}{4\pi}\frac{|q|}{r^2}v_0 = \frac{\mu_0}{4\pi}\frac{(-7.20 \times 10^{-3}\ \text{C})}{(0.250\ \text{m})^2}800\ \text{m/s} = 9.2 \times 10^{-6}\ \text{T}.$$

28.53: Choose a cube of edge length L, with one face on the *y*-*z* plane. Then:
$0 = \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = \int\int_{x=L} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = \int\int_{x=L} \frac{B_0 x}{a}\hat{\boldsymbol{i}} \cdot d\vec{\boldsymbol{A}} = \frac{B_0 L}{a}\int\int_{x=L} dA = \frac{B_0 L^3}{a} \Rightarrow B_0 = 0,$
so the only possible field is a zero field.

28.54: a)



b) $\vec{\boldsymbol{B}}_2 = -\left(\dfrac{\mu_0 I_2}{2\pi\, r^2}\right)\hat{i}$ $\qquad$ $\vec{\boldsymbol{B}}_3 = \left(\dfrac{\mu_0 I_3}{2\pi r_3}\right)(\sin\theta\hat{\boldsymbol{i}} + \cos\theta\hat{\boldsymbol{j}})$

And so

$$\vec{\boldsymbol{B}} = \left(\frac{\mu_0}{2\pi}\right)\left(\left(-\frac{I_2}{r_2} + \frac{I_3}{r_3}\sin\theta\right)\hat{\boldsymbol{i}} + \frac{I_3}{r_3}\cos\theta\ \hat{\boldsymbol{j}}\right)$$

$$\Rightarrow \vec{\boldsymbol{B}} = \left(\frac{\mu_0}{2\pi}\right)\left(\left(-\frac{I_2}{(0.030\text{ m})} + \frac{I_3}{(0.050\text{ m})}(0.6)\right)\hat{\boldsymbol{i}} + \frac{I_3}{(0.050\text{ m})}(0.8)\hat{\boldsymbol{j}}\right)$$

$$\Rightarrow \vec{\boldsymbol{B}} = \left(\frac{\mu_0}{2\pi}\right)\left((12I_3 - 33.3I_2)\ \hat{\boldsymbol{i}} + (16I_3)\ \hat{\boldsymbol{j}}\right)$$

$$= \frac{\mu_0}{2\pi}\left((12)(4.00\text{ A}) - (33.3)(2.00\text{ A}))\ \hat{\boldsymbol{i}} + (16)(4.00\text{ A})\hat{\boldsymbol{j}}\right)$$

$$= -3.72\times10^{-6}\text{ T}\hat{\boldsymbol{i}} + 1.28\times10^{-5}\text{ T}\hat{\boldsymbol{j}}.$$

c) $\vec{\boldsymbol{F}} = I_1\vec{\boldsymbol{l}}\times\vec{\boldsymbol{B}} = I_1 l B_x\hat{\boldsymbol{j}} + I_1 l B_y\hat{\boldsymbol{i}}$

$$\Rightarrow (1.00\text{ A})(0.010\text{ m})[(3.72\times10^{-6}\text{ T})\hat{\boldsymbol{j}} + (1.28\times10^{-5}\text{ T})\hat{\boldsymbol{i}}]$$

$= 3.72\times10^{-8}\text{ T}\hat{\boldsymbol{j}} + 1.28\times10^{-7}\text{ T}\hat{\boldsymbol{i}};\ F = 1.33\times10^{-7}\text{ N}, 16.2°$ counterclockwise from $+x$-axis.

28.55: a) If the magnetic field at point P is zero, then from Figure (28.46) the current I_2 must be out of the page, in order to cancel the field from I_1. Also:

$$B_1 = B_2 \Rightarrow \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0 I_2}{2\pi r_2} \Rightarrow I_2 = I_1 \frac{r_2}{r_1} = (6.00\text{ A})\frac{(0.500\text{ m})}{(1.50\text{ m})} = 2.00\text{ A}.$$

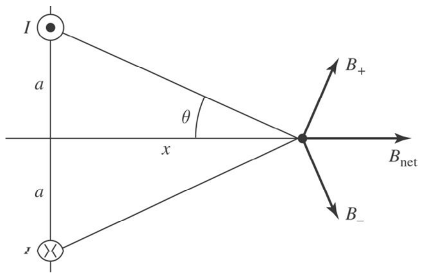
b) Given the currents, the field at Q points to the right and has magnitude

$$B_Q = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = \frac{\mu_0}{2\pi}\left(\frac{6.00\text{ A}}{0.500\text{ m}} - \frac{2.00\text{ A}}{1.50\text{ m}}\right) = 2.13\times10^{-6}\text{ T}.$$

c) The magnitude of the field at S is given by the sum of the squares of the two fields because they are at right angles. So:

$$B_S = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi}\sqrt{\left(\frac{I_1}{r_1}\right)^2 + \left(\frac{I_2}{r_2}\right)^2} = \frac{\mu_0}{2\pi}\sqrt{\left(\frac{6.00\text{ A}}{0.60\text{ m}}\right)^2 + \left(\frac{2.00\text{ A}}{0.80\text{ m}}\right)^2} = 2.1\times10^{-6}\text{ T}.$$

28.56: a)



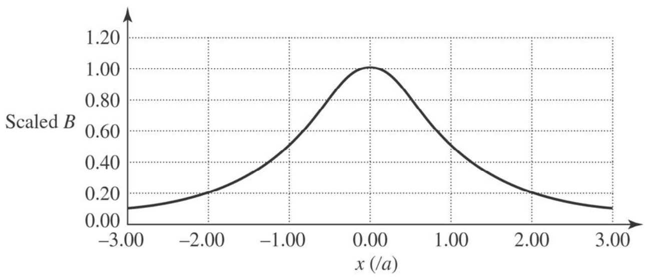
b) At a position on the x-axis:

$$B_{\text{net}} = 2\frac{\mu_0 I}{2\pi r}\sin\theta = \frac{\mu_0 I}{\pi\sqrt{x^2 + a^2}}\frac{a}{\sqrt{x^2 + a^2}}$$

$$\Rightarrow B_{net} = \frac{\mu_0 I a}{\pi(x^2 + a^2)},$$

in the positive x-direction , as shown at left.

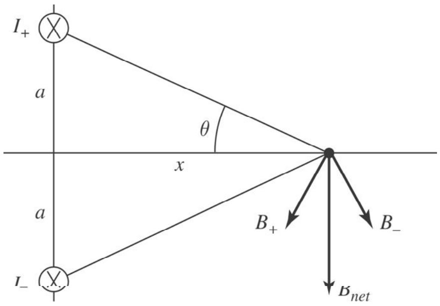
c)



d) The magnetic field is a maximum at the origin, $x = 0$.

e) When $x >> a$, $B \approx \dfrac{\mu_0 I a}{\pi x^2}$.

28.57: a)



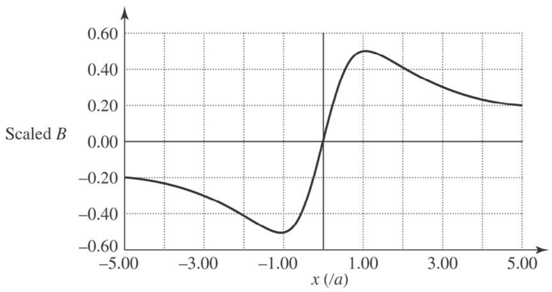
b) At a position on the *x*-axis:

$$B_{\text{net}} = 2\frac{\mu_0 I}{2\pi r}\cos\theta = \frac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\frac{x}{\sqrt{x^2+a^2}}$$

$$\Rightarrow B_{\text{net}} = \frac{\mu_0 I x}{\pi\left(x^2+a^2\right)},$$

in the negative *y*-direction, as shown at left.

c)



d) The magnetic field is a maximum when:

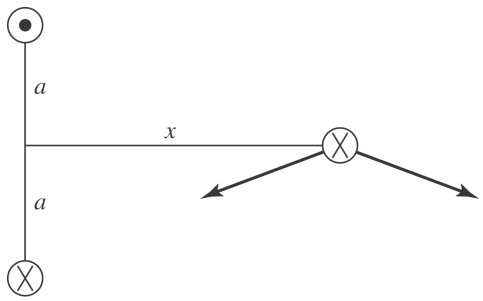
$$\frac{dB}{dx} = 0 = \frac{C}{x^2+a^2} - \frac{2Cx^2}{\left(x^2+a^2\right)^2} \Rightarrow \left(x^2+a^2\right) = 2x^2 \Rightarrow x = \pm a$$

e) When $x >> a,\ B \approx \frac{\mu_0 I}{\pi x}$, which is just like a wire carrying current 2I.

28.58: a) Wire carrying current into the page, so it feels a force downward from the other wires, as shown at right.

$$\frac{F}{L} = IB = I\left(\frac{\mu_0 Ia}{\pi(x^2 + a^2)}\right)$$

$$\Rightarrow \frac{F}{L} = \frac{\mu_0 (6.00\ A)^2 (0.400\ m)}{\pi\left((0.600\ m)^2 + (0.400\ m)^2\right)} = 1.11 \times 10^{-5}\ \text{N/m}.$$



b) If the wire carries current out of the page then the forces felt will be the opposite of part (a) . Thus the force will be $1.11 \times 10^{-5}\ \text{N/m}$, upward.

28.59: The current in the wires is $I = \varepsilon/\text{R} = (45.0\ \text{V})/(0.500\ \Omega) = 90.0\ \text{A}$. The currents in the wires are in opposite directions, so the wires repel. The force each wire exerts on the other is

$$F = \frac{\mu_0 II'L}{2\pi r} = \frac{(2 \times 10^{-7}\ \text{N/A}^2)(90.0\ \text{A})^2 (3.50\ \text{m})}{(0.0150\ \text{m})} = 0.378\ \text{N}$$

To hold the wires at rest, each spring exerts a force of 0.189 N on each wire.

$$F = kx \ \text{ so } \ k = F/x = (0.189\ \text{N})/(0.0050\ \text{m}) = 37.8\ \text{N/m}$$

28.60: a) Note that the Earth's magnetic field exerts no force on wire B, since the current in wire B is parallel to the Earth's magnetic field. Thus, for equilibrium, the remaining two forces that act on wire B must cancel. Assuming that the length of wire B is L and that wire A carries a current I we obtain

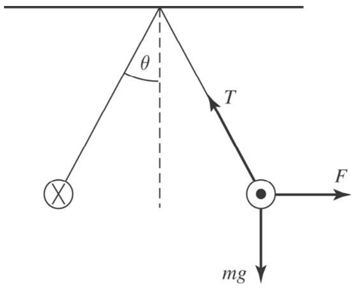
$$-\frac{\mu_0 I(1.0\ A)L}{2\pi(0.050\ m)} + \frac{\mu_0 (1.0\ \text{A})(3.0\ \text{A})L}{2\pi(0.100\ \text{m})} = 0$$

So

$$I = (3.0\ \text{A}) \cdot \frac{0.050\ \text{m}}{0.100\ \text{m}} = 1.5\ \text{A}$$

b) Note that the force on wire B that is generated by wire C is to the right. Thus, if the current in wire C is increased, wire B will slide to the right.

28.61:



The wires are in equilibrium, so:

$$x: F = T\sin\theta \text{ and } y: T\cos\theta = mg$$

$$\Rightarrow F = IlB = T\sin\theta = mg\tan\theta \Rightarrow I = \frac{mg\tan\theta}{lB}$$

But $B = \dfrac{\mu_0 I}{2\pi r} \Rightarrow I = \dfrac{2\pi r\, mg\tan\theta}{l\mu_0 I} \Rightarrow I = \sqrt{\dfrac{2\pi r\, mg\tan\theta}{l\mu_0}}$

And $r = [2(0.0400\text{ m})\sin(6.00°)] = 8.36\times10^{-3}\text{ m}.$

$$\Rightarrow I = \sqrt{\frac{2\pi(8.36\times10^{-3}\text{ m})(0.0125\text{ kg/m})(9.80\text{ m/s}^2)\tan(6.00°)}{\mu_0}} = 23.2\text{ A}.$$

28.62: The forces on the top and bottom segments cancel, leaving the left and right sides:

$$\vec{F} = \vec{F}_l + \vec{F}_r = -(IlB_l)\hat{i} + (IlB_r)\hat{i} = Il\left(-\frac{\mu_0 I_{\text{wire}}}{2\pi r_l} + \frac{\mu_0 I_{\text{wire}}}{2\pi r_r}\right)\hat{i} = \frac{\mu_0 IlI_{\text{wire}}}{2\pi}\left(\frac{1}{r_r} - \frac{1}{r_l}\right)\hat{i}$$

$$\Rightarrow \vec{F} = \frac{\mu_0(5.00\text{ A})(0.200\text{ m})(14.0\text{ A})}{2\pi}\left(\frac{1}{0.100\text{ m}} - \frac{1}{0.026\text{ m}}\right)\hat{i} = -(7.97\times10^{-5}\text{ N})\hat{i}.$$

28.63: a) $x >> a \Rightarrow B = \dfrac{N\mu_0 Ia^2}{2(x^2+a^2)^{3/2}} \approx \dfrac{N\mu_0 Ia^2}{2x^3}$ and $|\vec{\tau}| = |\vec{\mu}\times\vec{B}| = \mu B\sin\theta$

$$\Rightarrow \tau = (N'I'A')\left(\frac{N\mu_0 Ia^2}{2x^3}\right)\sin\theta = \frac{NN'\mu_0\pi I\, I'a^2a'^2\sin\theta}{2x^3}$$

b) $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta = -(N'I'\pi a'^2)\left(\dfrac{N\mu_0 Ia^2}{2x^3}\right)\cos\theta = -\dfrac{NN'\mu_0\pi II'a^2a'^2\cos\theta}{2x^3}.$

c) Having $x >> a$ allows us to simplify the form of the magnetic field, whereas assuming $x >> a'$ means we can assume that the magnetic field from the first loop is constant over the second loop.

28.64: $B = B_a - B_b = \frac{1}{2}\left(\frac{\mu_0 I}{2}\right)\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_0 I}{4a}\left(1 - \frac{a}{b}\right)$, out of the page.

28.65: a) Recall for a single loop: $B = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$. Here we have two loops, each of N turns, and measuring the field along the x-axis from between them means that the "x" in the formula is different for each case:

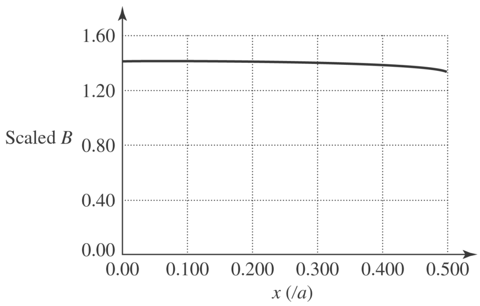
Left coil: $$x \to x + \frac{a}{2} \Rightarrow B_l = \frac{\mu_0 N I a^2}{2((x+a/2)^2 + a^2)^{3/2}}.$$

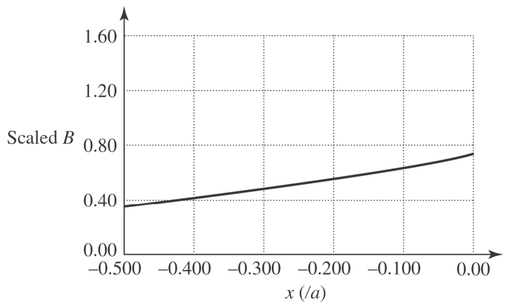
Right coil: $$x \to x - \frac{a}{2} \Rightarrow B_r = \frac{\mu_0 N I a^2}{2((x-a/2)^2 + a^2)^{3/2}}.$$

So the total field at a point x from the point between them is:

$$B = \frac{\mu_0 N I a^2}{2}\left(\frac{1}{((x+a/2)^2 + a^2)^{3/2}} + \frac{1}{((x-a/2)^2 + a^2)^{3/2}}\right).$$

b) Below left: Total magnetic field. Below right: Magnetic field from right coil.





c) At point P, $x = 0 \Rightarrow B = \frac{\mu_0 N I a^2}{2}\left(\frac{1}{((a/2)^2 + a^2)^{3/2}} + \frac{1}{((-a/2)^2 + a^2)^{3/2}}\right)$

$$\Rightarrow B = \frac{\mu_0 N I a^2}{(5a^2/4)^{3/2}} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{a}.$$

d) $B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{a} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 (300)(6.00\text{ A})}{(0.080\text{ m})} = 0.0202\text{ T}.$

e) $$\frac{dB}{dx} = \frac{\mu_0 N I a^2}{2}\left(\frac{-3(x+a/2)}{((x+a/2)^2 + a^2)^{5/2}} + \frac{-3(x-a/2)}{((x-a/2)^2 + a^2)^{5/2}}\right)$$

$$\Rightarrow \left.\frac{dB}{dx}\right|_{x=0} = \frac{\mu_0 N I a^2}{2}\left(\frac{-3(a/2)}{((a/2)^2 + a^2)^{5/2}} + \frac{-3(-a/2)}{((-a/2)^2 + a^2)^{5/2}}\right) = 0.$$

$$\frac{d^2B}{dx^2} = \frac{\mu_0 N I a^2}{2}\left(\frac{-3}{((x+a/2)^2 + a^2)^{5/2}} + \frac{6(x+a/2)^2(5/2)}{((x+a/2)^2 + a^2)^{7/2}}\right.$$

$$\left. + \frac{-3}{((x-a/2)^2 + a^2)^{5/2}} + \frac{6(x-a/2)^2(5/2)}{((x-a/2)^2 + a^2)^{7/2}}\right)$$

28.66: A wire of length l produces a field $B = \frac{\mu_0 I}{4\pi}\frac{l}{x\sqrt{x^2+(l/2)^2}}$. Here all edges produce a field into the page so we can just add them up:

$$x = a/2 \text{ and } l = b \Rightarrow B_{\text{left}} = \frac{\mu_0 I}{4\pi}\frac{b}{(a/2)\sqrt{(a/2)^2+(b/2)^2}} = \frac{\mu_0 I}{\pi}\left(\frac{b}{a}\right)\frac{1}{\sqrt{a^2+b^2}}.$$

$$x = b/2 \text{ and } l = a \Rightarrow B_{\text{top}} = \frac{\mu_0 I}{4\pi}\frac{a}{(b/2)\sqrt{(b/2)^2+(a/2)^2}} = \frac{\mu_0 I}{\pi}\left(\frac{a}{b}\right)\frac{1}{\sqrt{a^2+b^2}}.$$

And the right and bottom edges just produce the same contribution as the left and top, respectively. Thus the total magnetic field is:

$$B = \frac{2\mu_0 I}{\pi}\left(\frac{b}{a}+\frac{a}{b}\right)\frac{1}{\sqrt{a^2+b^2}} = \frac{2\mu_0 I}{\pi ab}\sqrt{a^2+b^2}.$$

28.67: The contributions from the straight segments is zero since $d\vec{l} \times \vec{r} = 0$. The magnetic field from the curved wire is just one quarter of a full loop:

$$\Rightarrow B = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right),$$

and is out of the page.

28.68: The horizontal wire yields zero magnetic field since $d\vec{l} \times \vec{r} = 0$. The vertical current provides the magnetic field of HALF of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

$$\Rightarrow B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi R}\right),$$

and is out of the page.

28.69: a) $I = \int_s J dA = \int_s \alpha r r dr d\theta = \alpha 2\pi \int_0^R r^2 dr = \frac{2\pi\alpha R^3}{3} \Rightarrow \alpha = \frac{3I}{2\pi R^3}.$

b) $(i)\ r \le R \Rightarrow I_{\text{encl}} = \frac{3I}{2\pi R^3}\int_s r^2 dr d\theta = \frac{3I}{2\pi R^3}2\pi\int_0^r r^2 dr = I\frac{r^3}{R^3}.$

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0\left(I\frac{r^3}{R^3}\right) \Rightarrow B = \frac{\mu_0 I r^2}{2\pi R^3}.$$

$$(ii)\ r \ge R \Rightarrow I_{\text{encl}} = I \Rightarrow \oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}.$$

28.70: $a) r < a \Rightarrow I_{\text{encl}} = I\left(\frac{A_r}{A_a}\right) = I\left(\frac{r^2}{a^2}\right) \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I\left(\frac{r^2}{a^2}\right)$

$\Rightarrow B = \frac{\mu_0 I r}{2\pi a^2}.$

When $r = a$, $B = \frac{\mu_0 I}{2\pi a}$ which is just what was found from Exercise 28.32, part (a).

$b)\ b < r < c \Rightarrow I_{\text{encl}} = I - I\left(\frac{A_{b\to r}}{A_{b\to c}}\right) = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$

$\Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = \mu_0 I\left(\frac{c^2 - r^2}{c^2 - b^2}\right) \Rightarrow B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right).$

When $r = b$, $B = \frac{\mu_0 I}{2\pi b}$, just as in Ex. Exercise 28.32, part (a), and at $r = c$, $B = 0$, just as in Ex
Exercise 28.32, part (b).

28.71: If there is a magnetic field component in the z-direction, it must be constant because of the symmetry of the wire. Therefore the contribution to a surface integral over a closed cylinder, encompassing a long straight wire will be zero: no flux through the barrel of the cylinder, and equal but opposite flux through the ends. The radial field will have no contribution through the ends, but through the barrel:

$$0 = \oint_s \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = \oint_s \vec{\boldsymbol{B}}_r \cdot d\vec{\boldsymbol{A}} = \int_{\text{barrel}} \vec{\boldsymbol{B}}_r \cdot d\vec{\boldsymbol{A}} = \int_{\text{barrel}} B_r dA = B_r A_{\text{barrel}} = 0 \Rightarrow B_r = 0.$$

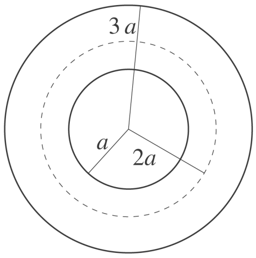
28.72: a) $r < a \Rightarrow I_{\text{encl}} = 0 \Rightarrow B = 0.$

b) $a < r < b \Rightarrow I_{\text{encl}} = I\left(\frac{A_{a\to r}}{A_{a\to b}}\right) = I\left(\frac{\pi(r^2 - a^2)}{\pi(b^2 - a^2)}\right) = I\frac{(r^2 - a^2)}{(b^2 - a^2)}$

$\Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I\frac{(r^2 - a^2)}{(b^2 - a^2)} \Rightarrow B = \frac{\mu_0 I}{2\pi r}\frac{(r^2 - a^2)}{(b^2 - a^2)}.$

c) $r > b \Rightarrow I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}.$

28.73:



Apply Ampere's law to a circular path of radius $2a$.

$B(2\pi r) = \mu_0 I_{\text{encl}}$

$$I_{\text{encl}} = I\left(\frac{(2a)^2 - a^2}{(3a)^2 - a^2}\right) = 3I/8$$

$B = \dfrac{3}{16}\dfrac{\mu_0 I}{2\pi a}$; this is the magnetic field inside the metal at a distance of $2a$ from the cylinder axis.

Outside the cylinder, $B = \dfrac{\mu_0 I}{2\pi r}$. The value of r where these two fields are equal is given by $1/r = 3/(16a)$ and $r = 16a/3$.

28.74: At the center of the circular loop the current I_2 generates a magnetic field that is into the page–so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude

$$\frac{\mu_0 I_1}{2\pi D} = \frac{\mu_0 I_2}{2R}$$

Thus, $I_1 = \frac{\pi D}{R} I_2$

28.75: a) $I = \int_s \vec{J} \cdot d\vec{\mathbf{A}} = \frac{2I_0}{\pi a^2} \int_s \left(1 - \frac{r^2}{a^2}\right) r dr d\theta = \frac{2I_0}{\pi a^2} 2\pi \int_0^a \left(r - \frac{r^3}{a^2}\right) dr =$

$\frac{4I_0}{a^2}\left(\frac{r^2}{2} - \frac{r^4}{4a^2}\right)\Bigg|_0^a \Rightarrow I = \frac{4I_0}{a^2}\left(\frac{a^2}{2} - \frac{a^4}{4a^2}\right) = I_0.$

b) For $r \geq a \Rightarrow \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = B2\pi r = \mu_0 I_{\text{encl}} \Rightarrow \mu_0 I_0 \Rightarrow B = \frac{\mu_0 I_0}{2\pi r}.$

c) For $r \leq a \Rightarrow I_{\text{encl}} = \oint_s \vec{J} \cdot d\vec{\mathbf{A}} = \frac{2I_0}{\pi a^2} \int_s \left(1 - \frac{r'^2}{a^2}\right) r' dr' d\theta = \frac{2I_0}{\pi a^2} 2\pi \int_0^r \left(r' - \frac{r'^3}{a^2}\right) dr'$

$$\Rightarrow I_{\text{encl}} = \frac{4I_0}{a^2}\left(\frac{r'^2}{2} - \frac{r'^4}{4a^2}\right)\Bigg|_0^r = 2I_0 \frac{r^2}{a^2}\left(1 - \frac{r^2}{2a^2}\right).$$

d) For $r \leq a \Rightarrow \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = B2\pi r = \mu_0 I_{\text{encl}} = 2\mu_0 I_0 \frac{r^2}{a^2}\left(1 - \frac{r^2}{2a^2}\right)$

$$\Rightarrow B = \frac{\mu_0 I_0 r}{\pi a^2}\left(1 - \frac{r^2}{2a^2}\right).$$

At $r = a$, $B = \frac{\mu_0 I_0}{2\pi a}$ for both parts (b) and (d).

28.76: a) $I_0 = \int_s \vec{J} \cdot d\vec{A} = \int_s \left(\frac{b}{r} e^{(r-a)/\delta}\right) r dr d\theta = 2\pi b \int_0^a e^{(r-a)/\delta} dr = 2\pi b\delta \, e^{(r-a)/\delta}\Big|_0^a =$

$2\pi b\delta(1 - e^{-a/\delta}) \Rightarrow I_0 = 2\pi(600 \text{ A/m})\,(0.025 \text{ m})\,(1 - e^{(0.050/0.025)}) = 81.5 \text{ A}.$

b) For $r \geq a \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \Rightarrow B = \dfrac{\mu_0 I_0}{2\pi r}.$

c) $r \leq a \Rightarrow I(r) = \int_s \vec{J} \cdot d\vec{A} = \int_s \left(\frac{b}{r'} e^{(r'-a)/\delta}\right) r' dr' d\theta$

$$= 2\pi b \int_0^r e^{(r-a)/\delta} dr = 2\pi b\delta e^{(r'-a)/\delta}\Big|_0^r$$

$$\Rightarrow I(r) = 2\pi b\delta(e^{(r'-a)/\delta} - e^{-a/\delta}) = 2\pi b\delta e^{-a/\delta}(e^{r/\delta} - 1) \Rightarrow I(r) = I_0 \frac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}.$$

d) For $r \leq a \Rightarrow \oint \vec{B} \cdot d\vec{l} = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \dfrac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)} \Rightarrow B = \dfrac{\mu_0 I_0 (e^{r/\delta} - 1)}{2\pi r(e^{a/\delta} - 1)}.$

e) At $r = \delta = 0.025 \text{ m} \Rightarrow B = \dfrac{\mu_0 I_0 (e - 1)}{2\pi\delta(e^{a/\delta} - 1)} = \dfrac{\mu_0 (81.5 \text{ A})}{2\pi(0.025 \text{ m})} \dfrac{(e - 1)}{(e^{0.050/0.025} - 1)}$

$= 1.75 \times 10^{-4} \text{ T}.$

At $r = a = 0.050 \text{ m} \Rightarrow B = \dfrac{\mu_0 I_0}{2\pi a} \dfrac{(e^{a/\delta} - 1)}{(e^{a/\delta} - 1)} = \dfrac{\mu_0 (81.5 \text{ A})}{2\pi(0.050 \text{ m})} = 3.26 \times 10^{-4} \text{ T}.$

At $r = 2a = 0.100 \text{ m} \Rightarrow B = \dfrac{\mu_0 I_0}{2\pi r} = \dfrac{\mu_0 (81.5 \text{ A})}{2\pi(0.100 \text{ m})} = 1.63 \times 10^{-4} \text{ T}.$

28.77: $\int_{-\infty}^{\infty} B_x dx = \int_{-\infty}^{\infty} \dfrac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} dx = \dfrac{\mu_0 I}{2} \int_{-\infty}^{\infty} \dfrac{1}{((x/a)^2 + 1)^{3/2}} d(x/a)$

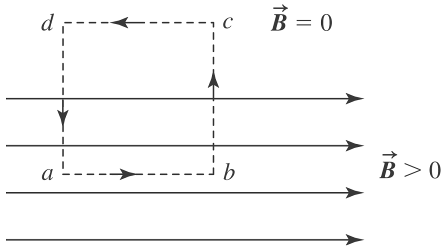
$$= \frac{\mu_0 I}{2} \int_{-\infty}^{\infty} \frac{dz}{(z^2 + 1)^{3/2}} \Rightarrow \int_{-\infty}^{\infty} B_x dx = \frac{\mu_0 I}{2} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \frac{\mu_0 I}{2} (\sin\theta)\Big|_{-\pi/2}^{\pi/2} = \mu_0 I,$$

where we used the substitution $z = \tan\theta$ to go from the first to second line. This is just what Ampere's Law tells us to expect if we imagine the loop runs along the *x*-axis closing on itself at infinity: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I.$

28.78: $\oint \vec{B} \cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $B = B_0\hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$.

$$\int_{abcde} \vec{B} \cdot d\vec{l} = B_{ab}L - B_{cd}L = 0,$$

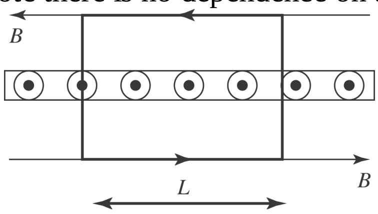
but $B_{cd} = 0$. $B_{ab}L = 0$, but $B_{ab} \neq 0$. This is a contradiction and violates Ampere's Law. See the figure on the next page.



28.79: a) Below the sheet, all the magnetic field contributions from different wires add up to produce a magnetic field that points in the positive *x*-direction. (Components in the *z*-direction cancel.) Using Ampere's Law, where we use the fact that the field is anti-symmetrical above and below the current sheet, and that the legs of the path perpendicular provide nothing to the integral: So, at a distance a beneath the sheet the magnetic field is:

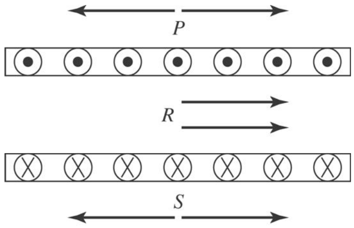
$$I_{\text{encl}} = nLI \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2L = \mu_0 nLI \Rightarrow B = \frac{\mu_0 nI}{2},$$

in the positive *x*-direction. (Note there is no dependence on *a*.)



b) The field has the same magnitude above the sheet, but points in the negative *x*-direction.

28.80: Two infinite sheets, as in Problem 28.79, are placed one above the other, with their currents opposite.



a) Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

28.81: $M_{Fe} = (\mu_{\text{atom of } Fe})(\# Fe \text{ atoms/m}^3) = (\mu_{\text{atom of } Fe})N_A(\# Fe \text{ moles/m}^3)$

$$\Rightarrow M_{Fe} = (\mu_{\text{atom of } Fe})N_A \frac{\rho_{Fe}}{m_{\text{mol}}(\text{Fe})} \Rightarrow \mu_{\text{atom of } Fe} = \frac{M_{Fe} m_{\text{mol}}(\text{Fe})}{N_A \rho_{Fe}}$$

$$\Rightarrow \mu_{\text{atom of } Fe} = \frac{(6.50 \times 10^4 \text{A/m})(0.0558 \text{ kg/mol})}{(6.02 \times 10^{23} \text{ atoms/mol})(7.8 \times 10^3 \text{ kg/m}^3)}$$

$$= 7.72 \times 10^{-25} \text{ A} \cdot \text{m}^2.$$

$$\Rightarrow \mu_{\text{atom of } Fe} = \frac{7.72 \times 10^{-25} \text{ A} \cdot \text{m}^2}{9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2} \mu_B = 0.0833 \mu_B.$$

28.82: The microscopic magnetic moments of an initially unmagnetized ferromagnetic material experience torques from a magnet that aligns the magnetic domains with the external field, so they are attracted to the magnet. For a paramagnetic material, the same attraction occurs because the magnetic moments align themselves parallel to the external field.

For a diamagnetic material, the magnetic moments align anti-parallel to the external field so it is like two magnets repelling each other.

b) The magnet can just pick up the iron cube so the force it exerts is:

$$F_{Fe} = m_{Fe}g = \rho_{Fe}a^3g = (7.8\times10^3\ \mathrm{kg/m^3})(0.020\,\mathrm{m})^3(9.8\,\mathrm{m/s^2}) = 0.612\ \mathrm{N}.$$

But $F_{Fe} = IaB = \dfrac{\mu_{Fe}B}{a} = 0.612\ \mathrm{N} \Rightarrow \dfrac{B}{a} = \dfrac{0.612\ \mathrm{N}}{\mu_{Fe}}$.

So if the magnet tries to lift the aluminum cube of the same dimensions as the iron block, then the upward force felt by the cube is:

$$F_{Al} = \frac{\mu_{Al}B}{a} = \frac{\mu_{Al}}{\mu_{Fe}}0.612\ \mathrm{N} = \frac{K_{Al}}{H_{Fe}}0.612\ \mathrm{N} = \frac{1.000022}{1400}0.612\ \mathrm{N} = 4.37\times10^{-4}\ \mathrm{N}.$$

But the weight of the aluminum cube is:

$$W = m_{Al}g = \rho_{Al}a^3g = (2.7\times10^3\ \mathrm{kg/m^3})(0.020\,\mathrm{m})^3(9.8\,\mathrm{m/s^2}) = 0.212\ \mathrm{N}.$$

So the ratio of the magnetic force on the aluminum cube to the weight of the cube is $\frac{4.37\times10^{-4}\ \mathrm{N}}{0.212\ \mathrm{N}} = 2.1\times10^{-3}$, and the magnet cannot lift it.

c) If the magnet tries to lift a silver cube of the same dimensions as the iron block, then the DOWNWARD force felt by the cube is:

$$F_{Al} = \frac{\mu_{Ag}B}{a} = \frac{\mu_{Ag}}{\mu_{Fe}}0.612\ \mathrm{N} = \frac{K_{Ag}}{K_{Fe}}0.612\ \mathrm{N} = \frac{(1.00 - 2.6\times10^{-5})}{1400}0.612\ \mathrm{N}$$
$$= 4.37\times10^{-4}\ \mathrm{N}.$$

But the weight of the silver cube is:

$$W = m_{Ag}g = \rho_{Ag}a^3g = (10.5\times10^3\ \mathrm{kg/m^3})(0.020\ \mathrm{m})^3(9.8\,\mathrm{m/s^2}) = 0.823\ \mathrm{N}.$$

So the ratio of the magnetic force on the silver cube to the weight of the cube is $\frac{4.37\times10^{-4}\ \mathrm{N}}{0.823\ \mathrm{N}} = 5.3\times10^{-4}$, and the magnet's effect would not be noticeable.

28.83: a) The magnetic force per unit length between two parallel, long wires is:

$$\frac{F}{L} = IB = \frac{\mu_0}{2\pi d} I^2 = \frac{\mu_0}{2\pi d}\left(\frac{I_0}{\sqrt{2}}\right)^2 = \frac{\mu_0}{4\pi d}\left(\frac{V}{R}\right)^2 = \frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2,$$

where $\frac{I_0}{\sqrt{2}}$ is the rms current over the short discharge time.

$$\frac{F}{L} = \frac{m}{L}a = \lambda a = \frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2 \Rightarrow a = \frac{\mu_0 Q_0^{\,2}}{4\pi\lambda d R^2 C^2} \Rightarrow v_0 = at = aRC = \frac{\mu_0 Q_0^{\,2}}{4\pi\lambda dRC}.$$

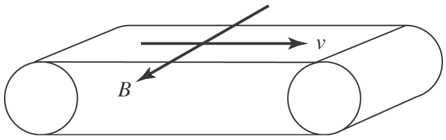
b) $$v_0 = \frac{\mu_0 (CV)^2}{4\pi\lambda dRC} = \frac{\mu_0 CV^2}{4\pi\lambda dR} = \frac{\mu_0 (2.50\times 10^{-6}\ \text{F})\,(3000\ \text{V})^2}{4\pi(4.50\times 10^{-3}\ \text{kg/m})(0.03\ \text{m})(0.048\ \Omega)} = 0.347\ \text{m/s}.$$

c) Height that the wire reaches above the original height:

$$\frac{1}{2}mv_0^{\,2} = mgh \Rightarrow h = \frac{v_0^{\,2}}{2g} = \frac{(0.347\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 6.14\times 10^{-3}\ \text{m}.$$

28.84: The amount of charge on a length Δx of the belt is:

$$\Delta Q = L\Delta x\sigma \Rightarrow I = \frac{\Delta Q}{\Delta t} = L\frac{\Delta x}{\Delta t}\sigma = Lv\sigma.$$



Approximating the belt as an infinite sheet:

$$B = \frac{\mu_0 I}{2L} = \frac{\mu_0 v\sigma}{2},$$

out of the page, as shown at left.

28.85: The charge on a ring of radius r is $q = \sigma A = \sigma 2\pi r dr = \dfrac{2Qrdr}{a^2}$. If the disk rotates at n turns per second, then the current from that ring is:

$$I = \frac{\Delta q}{\Delta t} = nq = \frac{2Qnrdr}{a^2} \Rightarrow dB = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\frac{2Qnrdr}{a^2} = \frac{\mu_0 nQdr}{a^2}.$$

So we integrate out from the center to the edge of the disk to find:

$$B = \int_0^a dB = \int_0^a \frac{\mu_0 nQdr}{a^2} = \frac{\mu_0 nQ}{a}.$$

28.86: There are two parts to the magnetic field: that from the half loop and that from the straight wire segment running from $-a$ to a.

$$B_x(ring) = \frac{1}{2}B_{\text{loop}} = -\frac{\mu_0 Ia^2}{4(x^2+a^2)^{3/2}}$$

$$dB_y(ring) = dB\sin\theta\sin\phi = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{x}{(x^2+a^2)^{1/2}}\sin\phi = \frac{\mu_0 Iax\sin\phi\, d\phi}{4\pi(x^2+a^2)^{3/2}}$$

$$\Rightarrow B_y(ring) = \int_0^{\pi} dB_y(ring) = \int_0^{\pi}\frac{\mu_0 Iax\sin\phi d\phi}{4\pi(x^2+a^2)^{3/2}} = \frac{\mu_0 Iax}{4\pi(x^2+a^2)^{3/2}}\cos\phi\Big|_0^{\pi}$$

$$= -\frac{\mu_0 Iax}{2\pi(x^2+a^2)^{3/2}}.$$

$B_y(rod) = \frac{\mu_0 Ia}{2\pi x(x^2+a^2)^{1/2}}$, using Eq. (28.8). So the total field components are:

$$B_x = -\frac{\mu_0 Ia^2}{4(x^2+a^2)^{3/2}}$$

and

$$B_y = \frac{\mu_0 Ia}{2\pi x(x^2+a^2)^{1/2}}\left(1-\frac{x^2}{x^2+a^2}\right) = \frac{\mu_0 Ia^3}{2\pi x(x^2+a^2)^{3/2}}.$$

29.1: $\Phi_{B_f} = NBA$, and $\Phi_{B_i} = NBA\cos 37.0° \Rightarrow \Delta\Phi_B = NBA(1-\cos 37.0°)$

$$\Rightarrow \varepsilon = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{NBA(1-\cos 37.0°)}{\Delta t}$$

$$= -\frac{(80)(1.10\text{ T})(0.400\text{ m})(0.25\text{ m})(1-\cos 37.0°)}{0.0600\text{ s}}$$

$$\Rightarrow |\varepsilon| = 29.5\text{ V}.$$

29.2: a) Before: $\Phi_B = NBA = (200)(6.0\times 10^{-5}\text{ T})(12\times 10^{-4}\text{ m}^2)$

$$= 1.44\times 10^{-5}\text{ T}\cdot\text{m}^2;\text{ after}: 0$$

b) $$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = \frac{NBA}{\Delta t} = \frac{(200)(6.0\times 10^{-5}\text{ T})(1.2\times 10^{-3}\text{ m}^2)}{0.040\text{ s}} = 3.6\times 10^{-4}\text{ V}.$$

29.3: a) $$\varepsilon = \frac{\Delta\Phi_B}{\Delta t} = \frac{NBA}{\Delta t} = IR = \left(\frac{Q}{\Delta t}\right)R \Rightarrow QR = NBA \Rightarrow Q = \frac{NBA}{R}.$$

b) A credit card reader *is* a search coil.

c) Data is stored in the charge measured so it is independent of time.

29.4: From Exercise (29.3),

$$Q = \frac{NBA}{R} = \frac{(90)(2.05\text{ T})(2.20\times 10^{-4}\text{ m}^2)}{6.80\ \Omega + 12.0\ \Omega} = 2.16\times 10^{-3}\text{ C}.$$

29.5: From Exercise (29.3),

$$Q = \frac{NBA}{R} \Rightarrow B = \frac{QR}{NA} = \frac{(3.56\times 10^{-5}\text{ C})(60.0\ \Omega + 45.0\ \Omega)}{(120)(3.20\times 10^{-4}\text{ m}^2)} = 0.0973\text{ T}.$$

29.6: a) $$\varepsilon = \frac{Nd\Phi_B}{dt} = NA\frac{d}{dt}(B) = NA\frac{d}{dt}\Big((0.012\text{ T/s})t + (3.00\times 10^{-5}\text{ T/s}^4)t^4\Big)$$

$$\Rightarrow \varepsilon = NA\Big((0.012\text{ T/s}) + (1.2\times 10^{-4}\text{ T/s}^4)t^3\Big)$$

$$= 0.0302\text{ V} + (3.02\times 10^{-4}\text{ V/s}^3)t^3.$$

b) At $t = 5.00\text{ s} \Rightarrow \varepsilon = 0.0302\text{ V} + (3.02\times 10^{-4}\text{ V/s}^2)(5.00\text{ s})^3 = +0.0680\text{ V}$.

$$\Rightarrow I = \frac{\varepsilon}{R} = \frac{0.0680\text{ V}}{600\ \Omega} = 1.13\times 10^{-4}\text{ A}$$

29.7: a) $\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[NAB_0\left(1-\cos\left(\frac{2\pi t}{T}\right)\right)\right] = -\frac{2\pi NAB_0}{T}\sin\left(\frac{2\pi t}{T}\right)$ for $0 < t < T$; zero otherwise..

b) $\varepsilon = 0$ at $t = \frac{T}{2}$

c) $\varepsilon_{\max} = \frac{2\pi NAB_0}{T}$ occurs at $t = \frac{T}{4}$ and $t = \frac{3T}{4}$.

d) From $0 < t < \frac{T}{2}$, B is getting larger and points in the $+z$ direction. This gives a clockwise current looking down the $-z$ axis. From $\frac{T}{2} t < T$, B is getting smaller but still points in the $+z$ direction. This gives a counterclockwise current.

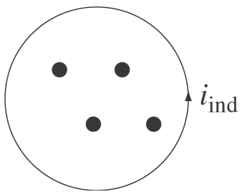
29.8: a) $\left|\varepsilon_{\text{ind}}\right| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}(B_1 A)$

$$\left|\varepsilon_{\text{ind}}\right| = A\sin 60°\frac{dB}{dt} = A\sin 60°\frac{d}{dt}\left((1.4\text{ T})e^{-0.057\text{s}^{-1}\text{t}}\right)$$
$$= (\pi r^2)(\sin 60°)(1.4\text{ T})(0.057\text{s}^{-1})e^{-0.057\text{s}^{-1}\text{t}}$$
$$= \pi(0.75\text{ m})^2(\sin 60°)(1.4\text{ T})(0.057\text{ s}^{-1})e^{-0.057s^{-1}t}$$
$$= 0.12\text{ V } e^{-0.057\, s^{-1}t}$$

b) $\varepsilon = \frac{1}{10}\varepsilon_0 = \frac{1}{10}(0.12\ \text{V})$

$$\frac{1}{10}(0.12\text{ V}) = 0.12\text{ V } e^{-0.057\, s^{-1}t}$$
$$\ln(1/10) = -0.057\text{ s}^{-1}t \rightarrow t = 40.4\text{s}$$

c) B is getting weaker, so the flux is decreasing. By Lenz's law, the induced current must cause an upward magnetic field to oppose the loss of flux. Therefore the induced current must flow *counterclockwise* as viewed from above.



29.9: a) $c = 2\pi r$ and $A = \pi r^2$ so $A = c^2/4\pi$

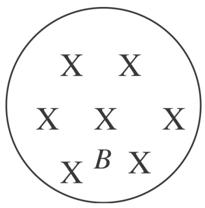
$\Phi_B = BA = (B/4\pi)c^2$

$|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right| = \left(\dfrac{B}{2\pi}\right) c \left|\dfrac{dc}{dt}\right|$

At $t = 9.0\ \text{s}$, $c = 1.650\ \text{m} - (9.0\ \text{s})(0.120\ \text{s}) = 0.570\ \text{m}$

$|\varepsilon| = (0.500\ \text{T})(1/2\pi)(0.570\ \text{m})(0.120\ \text{m/s}) = 5.44\ \text{mV}$

b)



Flux $\otimes$ is decreasing so the flux of the induced current Φ_{ind} is $\otimes$ and I is clockwise.

29.10: According to Faraday's law (assuming that the area vector points in the positive z-direction)

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = -\frac{0 - (1.5\ \text{T})\pi(0.120\ \text{m})^2}{2.0\times10^{-3}\ \text{s}} = +34\ \text{V(counterclockwise)}$$

29.11: $\Phi_B = BA\cos\phi;\ \phi$ is the angle between the normal to the loop and $\vec{\boldsymbol{B}}$, so $\phi = 53°$

$|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right| = (A\cos\phi)(dB/dt) = (0.100\ \text{m})^2 \cos 53°(1.00\times10^{-3}\ \text{T/s}) = 6.02\times10^{-6}\ \text{V}$

29.12: a)

$|\varepsilon| = \dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}(NBA\cos\omega t) = NBA\,\omega\sin\omega t$ and $1200\ \text{rev/min} = 20\ \text{rev/s}$, so :

$\Rightarrow \varepsilon_{\max} = NBA\omega = (150)(0.060\ \text{T})\pi(0.025\ \text{m})^2(440\ \text{rev/min})(1\ \text{min}/60\text{sec})(2\pi\ \text{rad/rev})$:

b) Average $\varepsilon = \dfrac{2}{\pi}\varepsilon_{\max} = \dfrac{2}{\pi}0.814\ \text{V} = 0.518\ \text{V}.$

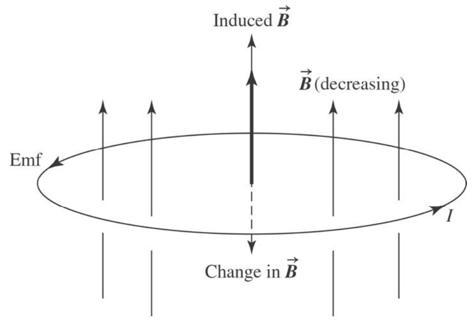
29.13: From Example 29.5,

$$\varepsilon_{\text{av}} = \frac{2N\omega BA}{\pi} = \frac{2(500)(56\ \text{rev/s})(2\pi\ \text{rad/rev})(0.20\ \text{T})(0.10\ \text{m})^2}{\pi} = 224\ \text{V}$$

29.14: $\varepsilon = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}(NBA\cos\omega t) = NBA\omega\sin\omega t \Rightarrow \varepsilon_{max} = NBA\omega$

$$\Rightarrow \omega = \frac{\varepsilon_{max}}{NBA} = \frac{2.40\times10^{-2}\ \text{V}}{(120)(0.0750\ \text{T})(0.016\ \text{m})^2} = 10.4\ \text{rad/s}.$$

29.15:



29.16: a) If the magnetic field is increasing into the page, the induced magnetic field must oppose that change and point opposite the external field's direction, thus requiring a counterclockwise current in the loop.

b) If the magnetic field is decreasing into the page, the induced magnetic field must oppose that change and point in the external field's direction, thus requiring a clockwise current in the loop.

c) If the magnetic field is constant, there is no changing flux, and therefore no induced current in the loop.

29.17: a) When the switch is opened, the magnetic field to the right decreases. Therefore the second coil's induced current produces its own field to the right. That means that the current must pass through the resistor from point *a* to point *b*.

b) If coil *B* is moved closer to coil *A*, more flux passes through it toward the right. Therefore the induced current must produce its own magnetic field to the left to oppose the increased flux. That means that the current must pass through the resistor from point *b* to point *a*.

c) If the variable resistor *R* is decreased, then more current flows through coil *A*, and so a stronger magnetic field is produced, leading to more flux to the right through coil *B*. Therefore the induced current must produce its own magnetic field to the left to oppose the increased flux. That means that the current must pass through the resistor from point *b* to point *a*.

29.18: a) With current passing from $a \to b$ and is increasing the magnetic, field becomes stronger to the left, so the induced field points right, and the induced current must flow from right to left through the resistor.

b) If the current passes from $b \to a$, and is decreasing, then there is less magnetic field pointing right, so the induced field points right, and the induced current must flow from right to left through the resistor.

c) If the current passes from $b \to a,$ and is increasing, then there is more magnetic field pointing right, so the induced field points left, and the induced current must flow from left to right through the resistor.

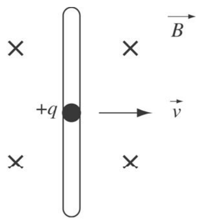
29.19: a) Φ_B is $\odot$ and increasing so the flux Φ_{ind} of the induced current is clockwise.

b) The current reaches a constant value so Φ_B is constant. $d\Phi_B / dt = 0$ and there is no induced current.

c) Φ_B is $\odot$ and decreasing, so Φ_{ind} is $\odot$ and current is counterclockwise.

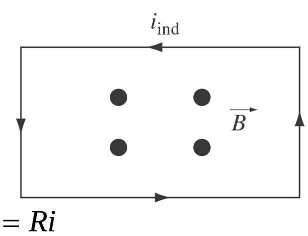
29.20: a) $\varepsilon = vBl = (5.0 \text{ m/s})(0.750 \text{ T})(1.50 \text{ m})$
$= 5.6 \text{ V}$

b) (i)



Let q be a positive charge in the moving bar. The magnetic force on this charge $\vec{F} = q\vec{v} \times \vec{B},$ which points *upward*. This force pushes the current in a *counterclockwise* direction through the circuit.

(ii) The flux through the circuit is increasing, so the induced current must cause a magnetic field out of the paper to oppose this increase. Hence this current must flow in a *counterclockwise* sense.



c) $\varepsilon = Ri$

$$i = \frac{\varepsilon}{R} = \frac{5.6 \text{ V}}{25 \,\Omega} = 0.22 \text{ A}$$

29.21: $[vBL] = \left[\dfrac{\text{m}}{\text{s}}\text{Tm}\right] = \left[\dfrac{\text{m}}{\text{s}}\dfrac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\text{m}\right] = \left[\dfrac{\text{N}\cdot\text{m}}{\text{C}}\right] = \left[\dfrac{\text{J}}{\text{C}}\right] = [\text{V}].$

29.22: a) $\varepsilon = vBL = (5.00\text{ m/s})(0.450\text{ T})(0.300\text{ m}) = 0.675\text{ V}.$

b) The potential difference between the ends of the rod is just the motional emf $V = 0.675\text{ V}.$

c) The positive charges are moved to end *b*, so *b* is at the higher potential.

d) $E = \dfrac{V}{L} = \dfrac{0.675\text{ V}}{0.300\text{ m}} = 2.25\dfrac{\text{V}}{\text{m}}.$

e) b

29.23: a) $\varepsilon = vBL \Rightarrow v = \dfrac{\varepsilon}{BL} = \dfrac{0.620\text{ V}}{(0.850\text{ T})(0.850\text{ m})} = 0.858\text{ m/s}.$

b) $I = \dfrac{\varepsilon}{R} = \dfrac{0.620\text{ V}}{0.750\ \Omega} = 0.827\text{ A}.$

c) $F = ILB = (0.827\text{ A})(0.850\text{ m})(0.850\text{ T}) = 0.598\text{ N}$, to the left, since you must pull it to get the current to flow.

29.24: a) $\varepsilon = vBL = (7.50\text{ m/s})(0.800\text{ T})(0.500\text{ m}) = 3.00\text{ V}.$

b) The current flows counterclockwise since its magnetic field must oppose the increasing flux through the loop.

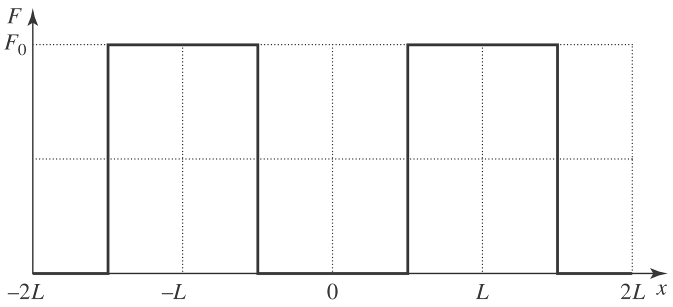
c) $F = ILB = \dfrac{\varepsilon LB}{R} = \dfrac{(3.00\text{ V})(0.500\text{ m})(0.800\text{ T})}{1.50\ \Omega} = 0.800\text{ N},$ to the right.

d) $P_{\text{mech}} = Fv = (0.800\text{ N})(7.50\text{ m/s}) = 6.00\text{ W}.$

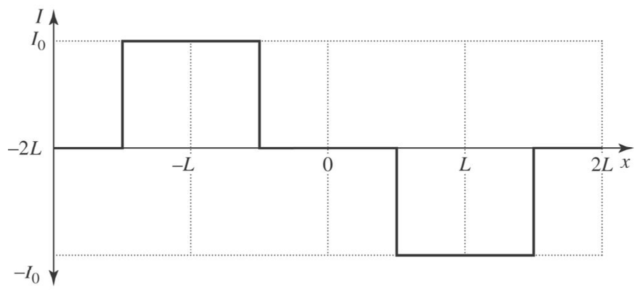
$P_{\text{elec}} = \dfrac{\varepsilon^2}{R} = \dfrac{(3.00\text{ V})^2}{1.50\ \Omega} = 6.00$ W. So both rates are equal.

29.25: For the loop pulled through the region of magnetic field,

a)



b)



Where $\varepsilon = vBL = IR \Rightarrow I_0 = \dfrac{vBL}{R}$ and $F_0 = ILB = \dfrac{vB^2L^2}{R}$.

29.26: a) Using Equation (29.6): $\varepsilon = vBL \Rightarrow B = \dfrac{\varepsilon}{vL} = \dfrac{0.450 \text{ V}}{(4.50 \text{ m/s})(0.120 \text{ m})} = 0.833 \text{ T}.$

b) Point a is at a higher potential than point b, because there are more positive charges there.

29.27: $\varepsilon = \dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}(BA) = \dfrac{d}{dt}(\mu_0 nIA) = \mu_0 nA\dfrac{dI}{dt}$ and $\oint \vec{E} \cdot d\vec{l} = \varepsilon \Rightarrow$

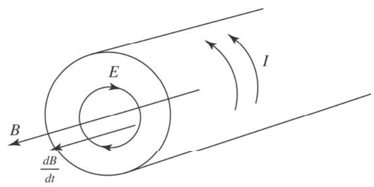
$$E = \frac{\varepsilon}{2\pi r} = \frac{\mu_0 nA}{2\pi r}\frac{dI}{dt} = \frac{\mu_0 nr}{2}\frac{dI}{dt}.$$

a) $r = 0.50 \text{ cm} \Rightarrow E = \dfrac{\mu_0 (900 \text{ m}^{-1})(0.0050 \text{ m})}{2}(60 \text{ A/s}) = 1.70 \times 10^{-4} \text{ V/m}.$

b) $r = 1.00 \text{ cm} \Rightarrow E = 3.39 \times 10^{-4} \text{ V/m}.$

29.28: a) $\dfrac{d\Phi_B}{dt} = A\dfrac{dB}{dt} = \pi r_1^{\,2}\dfrac{dB}{dt}.$

b) $E = \dfrac{1}{2\pi r_1}\dfrac{d\Phi_B}{dt} = \dfrac{\pi r_1^{\,2}}{2\pi r_1}\dfrac{dB}{dt} = \dfrac{r_1}{2}\dfrac{dB}{dt}.$



c) All the flux is within $r < R$, so outside the solenoid

$$E = \frac{1}{2\pi r_2}\frac{d\Phi_B}{dt} = \frac{\pi R^2}{2\pi r_2}\frac{dB}{dt} = \frac{R^2}{2r_2}\frac{dB}{dt}.$$

29.29: a) The induced electric field lines are concentric circles since they cause the current to flow in circles.

b) $E = \dfrac{1}{2\pi r}\varepsilon = \dfrac{1}{2\pi r}\dfrac{d\Phi_B}{dt} = \dfrac{1}{2\pi r}A\dfrac{dB}{dt} = \dfrac{r}{2}\dfrac{dB}{dt} = \dfrac{0.100\text{ m}}{2}(0.0350\text{ T/s})$

$\Rightarrow E = 1.75\times10^{-3}\text{ V/m},$ in the clockwise direction, since the induced magnetic field must reinforce the decreasing external magnetic field.

c) $I = \dfrac{\varepsilon}{R} = \dfrac{\pi r^2}{R}\dfrac{dB}{dt} = \dfrac{\pi(0.100\text{ m})^2}{4.00\ \Omega}(0.0350\text{ T/s}) = 2.75\times10^{-4}\text{ A}.$

d) $\varepsilon = IR = IR_{TOT}/2 = \dfrac{(2.75\times10^{-4}\text{ A})(4.00\ \Omega)}{2} = 5.50\times10^{-4}\text{ V}.$

e) If the ring was cut and the ends separated slightly, then there would be a potential difference between the ends equal to the induced emf:

$$\varepsilon = \pi r^2\frac{dB}{dt} = \pi(0.100\text{ m})^2(0.0350\text{ T/s}) = 1.10\times10^{-3}\text{ V}.$$

29.30: $\varepsilon = \dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}(BA) = \dfrac{d}{dt}(\mu_0 nIA) = \mu_0 nA\dfrac{dI}{dt} \Rightarrow \dfrac{dI}{dt} = \dfrac{E\cdot 2\pi r}{\mu_0 nA}$

$$\Rightarrow \frac{dI}{dt} = \frac{(8.00\times10^{-6}\text{ V/m})2\pi(0.0350)}{\mu_0(400\text{ m}^{-1})\pi(0.0110\text{ m})^2} = 9.21\text{ A}/s.$$

29.31: a)

$$W = \int \vec{F} \cdot d\vec{l} = qE2\pi R = (6.50 \times 10^{-6}\ \text{C})(8.00 \times 10^{-6}\ \text{V/m})2\pi(0.0350\ \text{m}) = 1.14 \times 10^{-11}\ \text{J}.$$

(b) For a conservative field, the work done for a closed path would be zero.

(c) $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt} \Rightarrow EL = BA\dfrac{di}{dt}$. A is the area of the solenoid.

For a circular path:

$E2\pi r = BA\dfrac{di}{dt} =$ constant for all circular paths that enclose the solenoid.

So $W = qE2\pi r =$ constant for all paths outside the solenoid.

$W = 1.14 \times 10^{-11}$ J if $r = 7.00$ cm.

29.32: $\varepsilon = -\dfrac{N\Delta\Phi_B}{\Delta t} = -\dfrac{NA(B_f - B_i)}{\Delta t} = \dfrac{NA\mu_o nI}{\Delta t}$

$$= \frac{\mu_o(12)(8.00 \times 10^{-4}\ \text{m}^2)(9000\ \text{m}^{-1})(0.350\ \text{A})}{0.0400\ \text{s}}$$

$\Rightarrow \varepsilon = 9.50 \times 10^{-4}$ V.

29.33: $i_D = \varepsilon\dfrac{d\Phi_E}{dt} = (3.5 \times 10^{-11}\ \text{F/m})(24.0 \times 10^{3}\ \text{V}\cdot\text{m/s}^3)t^2$

$i_D = 21 \times 10^{-6}$ A gives $t = 5.0$ s

29.34: According to Eqn.29.14 $\varepsilon = \dfrac{i_D}{\left(\dfrac{d\Phi_E}{dt}\right)} = \dfrac{12.9 \times 10^{-12}\ \text{A}}{4(8.76 \times 10^{3}\ \text{V}\cdot\text{m/s}^4)(26.1 \times 10^{-3}\ \text{s})^3} =$

2.07×10^{-11} F/m. Thus, the dielectric constant is $K = \frac{\varepsilon}{\varepsilon_0} = 2.34$.

29.35: a) $j_D = \varepsilon_0 \dfrac{dE}{dt} = \varepsilon_0 \dfrac{i_c}{\varepsilon_0 A} = \dfrac{i_c}{A} = \dfrac{0.280\text{ A}}{\pi(0.0400\text{ m})^2} = 55.7\text{ A}/\text{m}^2.$

b) $\dfrac{dE}{dt} = \dfrac{j_D}{\varepsilon_0} = \dfrac{55.7\text{ A}/\text{m}^2}{\varepsilon_0} = 6.29 \times 10^{12}\text{ V}/\text{m}\cdot\text{s}.$

c) Using Ampere's Law

$$r < R : B = \frac{\mu_0}{2\pi}\frac{r}{R^2} i_D = \frac{\mu_0}{2\pi}\frac{0.0200\text{ m}}{(0.0400\text{ m})^2}(0.280\text{ A}) = 7.0 \times 10^{-7}\text{ T}.$$

d) Using Ampere's Law

$$r < R : B = \frac{\mu_0 r}{2\pi R^2} i_D = \frac{\mu_0}{2\pi}\frac{(0.0100\text{ m})}{(0.0400\text{ m})^2}(0.280) = 3.5 \times 10^{-7}\text{ T}.$$

29.36: a) $Q = CV = \left(\dfrac{\varepsilon A}{d}\right)V = \dfrac{(4.70)\varepsilon_0(3.00 \times 10^{-4}\text{ m}^2)(120\text{ V})}{2.50 \times 10^{-3}\text{ m}} = 5.99 \times 10^{-10}\text{ C}.$

b) $\dfrac{dQ}{dt} = i_c = 6.00 \times 10^{-3}\text{ A}.$

c) $j_D = \varepsilon \dfrac{dE}{dt} = K\varepsilon_0 \dfrac{i_c}{K\varepsilon_0 A} = \dfrac{i_c}{A} = j_c \Rightarrow i_D = i_c = 6.00 \times 10^{-3}\text{ A}.$

29.37: a) $q = i_c t = (1.80 \times 10^{-3}\text{ A})\ (0.500 \times 10^{-6}\text{ s}) = 0.900 \times 10^{-9}\text{ C}$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\text{A}\varepsilon_0} = \frac{0.900 \times 10^{-9}\text{ C}}{(5.00 \times 10^{-4}\text{ m}^2)\varepsilon_0} = 2.03 \times 10^5\text{ V/m}.$$

$$\Rightarrow V = Ed = (2.03 \times 10^5\text{ V/m})\ (2.00 \times 10^{-3}\text{ m}) = 406\text{ V}.$$

b) $\dfrac{dE}{dt} = \dfrac{i_c}{A\varepsilon_0} = \dfrac{1.80 \times 10^{-3}\text{ A}}{(5.00 \times 10^{-4}\text{ m}^2)\varepsilon_0} = 4.07 \times 10^{11}\text{ V/m}\cdot\text{s},$ and is constant in time.

c) $j_D = \varepsilon_0 \dfrac{dE}{dt} = \varepsilon_0(4.07 \times 10^{11}\text{ V/m}\cdot\text{s}) = 3.60\text{ A/m}^2$

$\Rightarrow i_D = j_D A = (3.60\text{ A/m}^2)\ (5.00 \times 10^{-4}\text{ m}^2) = 1.80 \times 10^{-3}\text{ A},$ which is the same as i_c.

29.38: a) $E = \rho J = \dfrac{\rho I}{A} = \dfrac{(2.0\times10^{-8}\ \Omega\text{m})(16\ \text{A})}{2.1\times10^{-6}\ \text{m}^2} = 0.15\ \text{V/m}.$

b) $\dfrac{dE}{dt} = \dfrac{d}{dt}\left(\dfrac{\rho I}{A}\right) = \dfrac{\rho}{A}\dfrac{dI}{dt} = \dfrac{2.0\times10^{-8}\ \Omega\text{m}}{2.1\times10^{-6}\ \text{m}^2}(4000\ A/s) = 38\,V/\text{m}\cdot\text{s}.$

c) $j_D = \varepsilon_0 \dfrac{dE}{dt} = \varepsilon_0(38\ \text{V/s}\cdot\text{s}) = 3.4\times10^{-10}\ A/\text{m}^2.$

d) $i_D = j_D A = (3.4\times10^{-10}\ \text{A/m}^2)\,(2.1\times10^{-6}\ \text{m}^2) = 7.14\times10^{-16}\ \text{A}$

$\Rightarrow B_D = \dfrac{\mu_0 I_D}{2\pi r} = \dfrac{\mu_0(7.14\times10^{-16}\ \text{A})}{2\pi(0.060\ \text{m})} = 2.38\times10^{-21}$ T, and this is a negligible contribution. $B_c = \dfrac{\mu_0 I_c}{2\pi r} = \dfrac{\mu_0}{2\pi}\dfrac{(16\ \text{A})}{(0.060\ \text{m})} = 5.33\ \times10^{-5}$ T.

29.39: In a superconductor there is no internal magnetic field, and so there is no changing flux and no induced emf, and no induced electric field.

$$0 = \oint_{\substack{\text{Inside}\\\text{material}}} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{l}} = \mu_0 I_{\text{encl}} = \mu_0(I_c + I_D) = \mu_0 I_c \Rightarrow I_c = 0,$$

and so there is no current inside the material. Therefore, it must all be at the surface of the cylinder.

29.40: Unless some of the regions with resistance completely fill a cross-sectional area of a long type-II superconducting wire, there will still be no total resistance. The regions of no resistance provide the path for the current. Indeed, it will be like two resistors in parellel, where one has zero resistance and the other is non-zero. The equivalent resistance is still zero.

29.41: a) For magnetic fields less than the critical field, there is no internal magnetic field, so:

Inside the superconductor: $\vec{\boldsymbol{B}} = 0,\ \vec{\boldsymbol{M}} = -\dfrac{\vec{B}_0}{\mu_0} = -\dfrac{(0.130\,T)\hat{\boldsymbol{i}}}{\mu_0} = -(1.03\times10^5\ \text{A/m})\hat{\boldsymbol{i}}.$

Outside the superconductor: $\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_0 = (0.130T)\hat{\boldsymbol{i}},\ \vec{\boldsymbol{M}} = 0.$

b) For magnetic fields greater than the critical field, $\chi = 0 \Rightarrow \vec{\boldsymbol{M}} = 0$ both inside and outside the superconductor, and $\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_0 = (0.260\ \text{T})\hat{\boldsymbol{i}},$ both inside and outside the superconductor.

29.42: a) Just under $\vec{B}_{c1}$ (threshold of superconducting phase), the magnetic field in the material must be zero, and $\vec{M} = -\dfrac{\vec{B}_{c1}}{\mu_0} = -\dfrac{55\times10^{-3}\ \text{T}\hat{\boldsymbol{i}}}{\mu_0} = -(4.38\times10^{4}\ \text{A/m})\hat{\boldsymbol{i}}$.

b) Just over $\vec{B}_{c2}$ (threshold of normal phase), there is zero magnetization, and $\vec{B} = \vec{B}_{c2} = (15.0\ \text{T})\hat{\boldsymbol{i}}$.

29.43: a) The angle ϕ between the normal to the coil and the direction of $\vec{B}$ is 30.0°.

$|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right| = (N\pi r^2)\ dB/dt$ and $I = |\varepsilon|/R$.

For $t < 0$ and $t > 1.00$ s, $dB/dt = 0, |\varepsilon| = 0$ and $I = 0$

For $0 \le t \le 1.00$ s, $dB/dt = (0.120\ \text{T})\pi \sin \pi t$

$|\varepsilon| = (N\pi r^2)\pi(0.120\ \text{T})\sin \pi t = (0.9475\ \text{V}) \sin \pi t$

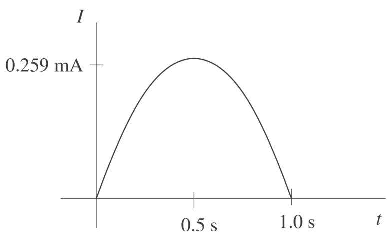
R for wire : $R_\text{w} = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi r^2}$; $\rho = 1.72\times10^{-8}\,\Omega\cdot\text{m}$, $r = 0.0150\times10^{-3}\,\text{m}$

$L = Nc = N2\pi r = (500)\ (2\pi)\ (0.0400\ \text{m}) = 125.7\ \text{m}$

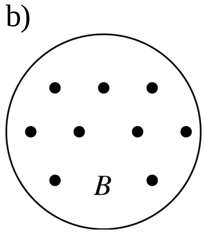
$R_\text{w} = 3058\,\Omega$ and the total resistance of the circuit is

$R = 3058\,\Omega + 600\,\Omega = 3658\,\Omega$

$I = |\varepsilon|/R = (0.259\ \text{mA})\sin \pi t$



b)



B increasing so Φ_B is $\odot$ and increasing

Φ_ind is $\otimes$ so I is clockwise

29.44: a) The large circuit is an RC circuit with a time constant of $\tau = RC = (10\ \Omega)\ (20 \times 10^{-6}\ \text{F}) = 200\mu\text{s}.$ Thus, the current as a function of time is

$$i = \left(\frac{100\ \text{V}}{10\ \Omega}\right) e^{-\frac{t}{200\mu\text{s}}}$$

At $t = 200\mu\text{s},$ we obtain $i = (10\ \text{A})\ (e^{-1}) = 3.7\ \text{A}.$

b) Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop and referring to the solution of Problem 29.54 we obtain

$$\Phi_B = \int_c^{c+a} \frac{\mu_0 ib}{2\pi r} dr = \frac{\mu_0 ib}{2\pi} \ln\,(1 + \frac{a}{c})$$

So the emf induced in the small loop at $t = 200\mu\text{s}$ is

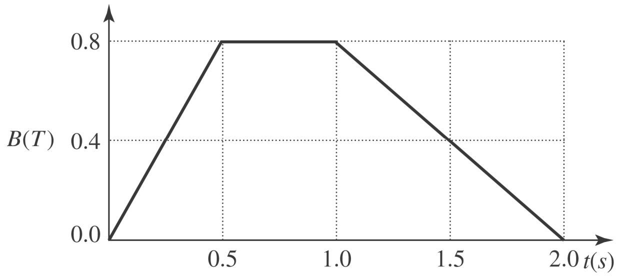
$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{\mu_0 b}{2\pi} \ln\left(1 + \frac{a}{c}\right)\frac{di}{dt} = -\frac{(4\pi\ \times 10^{-7}\ \frac{\text{Wb}}{\text{A}\cdot\text{m}^2})\ (0.200\ \text{m})}{2\pi} \ln\,(3.0)\ (-\frac{3.7\ \text{A}}{200 \times 10^{-6}\text{s}}) = +$$

Thus, the induced current in the small loop is $i' = \frac{\varepsilon}{R} = \frac{0.81\ \text{mV}}{25(0.600\ \text{m})\ (1.0\ \Omega/\text{m})} = 54\mu\text{A}.$

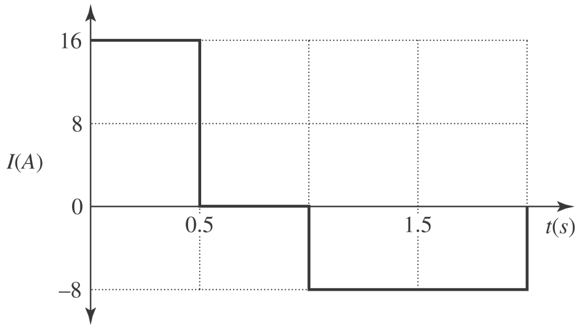
c) The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

d) Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop–as can be seen by comparing the distance c to the dimensions of the large loop.

29.45: a)



b)



c) $E_{\max} = \dfrac{\varepsilon}{N2\pi r} = \dfrac{1}{2\pi} NA\dfrac{dB}{dt} = \dfrac{1}{2\pi r}\pi r^2 \dfrac{dB}{dt} = \dfrac{r}{2}\dfrac{dB}{dt} = \dfrac{(0.50\text{ m})}{2}\dfrac{0.80\text{ T}}{0.50\text{ s}} = 0.4\text{ V/m}.$

29.46: a) $I = \dfrac{\varepsilon}{R} = \dfrac{1}{R}\dfrac{d\Phi_B}{dt} = \dfrac{1}{R}\dfrac{d(BA\cos\omega t)}{dt} = \dfrac{BA\,\omega\sin\omega t}{R}.$

b) $P = I^2R = \dfrac{B^2A^2\omega^2\sin^2\omega t}{R}.$

c) $\mu = IA = \dfrac{BA^2\omega\sin\omega t}{\mathrm{R}}.$

d) $\tau = \mu B\sin\phi = \mu \mathrm{B}\sin\omega t = \dfrac{\mathrm{B}^2A^2\omega\sin^2\omega t}{R}.$

e) $P = \tau\omega = \dfrac{B^2A^2\omega^2\sin^2\omega t}{R}$, which is the same as part (b).

29.47: a) $\Phi_B = BA = \dfrac{\mu_0 i}{2a}\pi a^2 = \dfrac{\mu_0 i\pi a}{2}.$

b) $\varepsilon = -\dfrac{d\Phi_B}{dt} = iR \Rightarrow -\dfrac{d}{dt}\left(\dfrac{\mu_0 i\pi a}{2}\right) = -\dfrac{\mu_0 \pi a}{2}\dfrac{di}{dt} = iR \Rightarrow \dfrac{di}{dt} = -i\dfrac{2R}{\mu_0 \pi a}$

c) Solving $\dfrac{di}{i} = -dt\dfrac{2R}{\mu_0 \pi a}$ for *i(t)* yields $i(t) = i_0 e^{-t(2R/\mu_0 \pi a)}$.

d) We want $i(t) = i_0(0.010) = i_0 e^{-t(2R/\mu_0 \pi a)} \Rightarrow \ln(0.010) = -t(2R/\mu_0 \pi a)$

$$\Rightarrow t = -\frac{\mu_0 \pi a}{2R}\ln(0.010) = -\frac{\mu_0 \pi (0.50\text{ m})}{2(0.10\,\Omega)}\ln(0.010) = 4.55\times 10^{-5}\text{ s}.$$

e) We can ignore the self-induced currents because it takes only a very short time for them to die out.

29.48: a) Choose the area vector to point out of the page. Since the area and its orientation to the magnetic field are fixed, we can write the induced emf in the 10 cm radius loop as

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A_z\frac{dB_z}{dt} = -\pi(0.10\text{ m})^2\frac{dB_z}{dt} = 10^{-4}[(20.0\text{ V}) - (4.00\text{ V/s})t]$$

After solving for $\frac{dB_z}{dt}$ and integrating we obtain

$$B_z(t = 2.00\text{s}) - B_z(t = 0\text{ s}) = -\frac{10^{-4}}{\pi(0.10\text{ m})^2}\int_0^2 [(20.0\text{ V}) - (4.00\text{ V/s})t]dt.$$

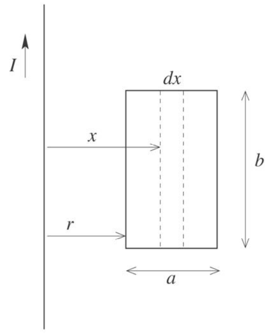
Thus,

$$B_z = (-0.800\text{ T}) - \frac{10^{-2}\text{ m}^{-2}}{\pi}\left[(20.0\text{ V})(2.00\text{ s}) - (2.00\text{ V/s})(2.00\text{ s})^2\right] = -0.902\text{ T}$$

b) Repeat part (a) but set $\varepsilon = -(2.00\times 10^{-3}\text{ V}) + (4.00\times 10^{-4}\text{ V/s})\,t$ to obtain $B_z = -0.698\text{ T}$

c) In part (a) the flux has decreased (*i.e.*, it has become more negative) and in part (b) the flux has increased. Both results agree with the expectations of Lenz's law.

29.49: a) (i) $|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right|$



Consider a narrow strip of width dx and a distance x from the long wire.
The magnetic field of the wire at the strip is $B = \mu_0 I/2\pi x$.
The flux through the strip is

$$d\Phi_B = Bbdx = (\mu_0 Ib/2\pi)\,(dx/x)$$

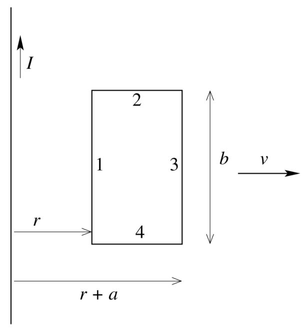
The total flux through the loop is $\Phi_B = \int d\Phi_B = \left(\dfrac{\mu_0 Ib}{2\pi}\right)\int_r^{r+a}\dfrac{dx}{x}$

$$\Phi_B = \left(\frac{\mu_0 lb}{2\pi}\right)\ln\left(\frac{r + \mathrm{a}}{r}\right)$$

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dt}\frac{dr}{dt} = \frac{\mu_0 Ib}{2\pi}\left(-\frac{a}{r(r+a)}\right) v$$

$$|\varepsilon| = \frac{\mu_0 Iabv}{2\pi r(r+a)}$$

(ii) $\varepsilon = Bvl$ for a bar of length l moving at speed v perpendicular to magnetic field B.



The emf in each side of the loop is

29.50: a) Rotating about the $y-$axis:

$$\varepsilon_{\max} = \frac{d\Phi_B}{dt} = \omega BA = (35.0 \text{ rad/s})\,(0.450 \text{ T})\,(6.00 \times 10^{-2}\text{ m}) = 0.945 \text{ V}.$$

b) Rotating about the $x-$axis: $\frac{d\Phi_B}{dt} = 0 \Rightarrow \varepsilon = 0.$

c) Rotating about the $z-$axis:

$$\varepsilon_{\max} = \frac{d\Phi_B}{dt} = \omega BA = (35.0 \text{ rad/s})\,(0.450 \text{ T})\,(6.00 \times 10^{-2}\text{ m}) = 0.945 \text{ V}.$$

29.51: From Example 29.4, $\varepsilon = \omega\, BA \sin \omega t;\ \varepsilon_{\max} = \omega BA$

For N loops, $\varepsilon_{\max} = N\omega BA$

$N = 400,\ B = 1.5\text{ T},\ A = (0.100\text{ m})^2,\ \varepsilon_{\max} = 120\text{ V}$

$\omega = \varepsilon_{\max}/NBA = (20\text{ rad/s})\,(1\text{ rev}/2\pi\text{ rad})\,(60\text{ s}/1\text{min}) = 190\text{ rpm}$

29.52: a) The flux through the coil is given by $NBA\cos(\omega t)$, where N is the number of turns, B is the strength of the Earth's magnetic field, and ω is the angular velocity of the rotating coil. Thus, $\varepsilon = \omega\ NBA\sin(\omega t)$, which has a peak amplitude of $\varepsilon_0 = \omega\ NBA.$ Solving for A we obtain

$$A = \frac{\varepsilon_0}{\omega NB} = \frac{9.0\text{ V}}{(30\text{ rev/min})\,(1\text{ min}/60\text{s})\,(2\pi\text{ rad/rev})\,(2000\text{ turns})\,(8.0\times 10^{-5}\text{T})} = 18\text{ m}^2$$

b) Assuming a point on the coil at maximum distance from the axis of rotation we have

$$v = r\omega = \sqrt{\frac{A}{\pi}}\omega = \sqrt{\frac{18\text{ m}^2}{\pi}}\,(30\text{ rev/min})\,(1\text{ min}/60\text{ s})\,(2\pi\text{ rad/rev}) = 7.5\text{ m/s}.$$

29.53: a) $\varepsilon = -\frac{\Delta\Phi_B}{\Delta t} = -B\frac{\Delta A}{\Delta t} = -B\frac{-\pi r^2}{\Delta t} = (0.950\text{ T})\frac{\pi(0.0650/2\text{ m})^2}{0.250\text{ s}} = 0.0126\text{ V}.$

b) Since the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore the current flows from point a through the resistor to point b.

29.54: a) When $I = i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}$, into the page.

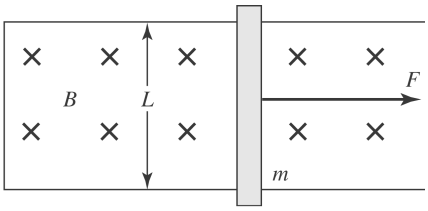
b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r} Ldr.$

c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 iL}{2\pi} \int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 iL}{2\pi} \ln(b/a).$

d) $\varepsilon = \dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 L}{2\pi} \ln(b/a) \dfrac{di}{dt}.$

e) $\varepsilon = \dfrac{\mu_0 (0.240 \text{ m})}{2\pi} \ln(0.360/0.120)\,(9.60 \text{ A/s}) = 5.06 \times 10^{-7} \text{ V}.$

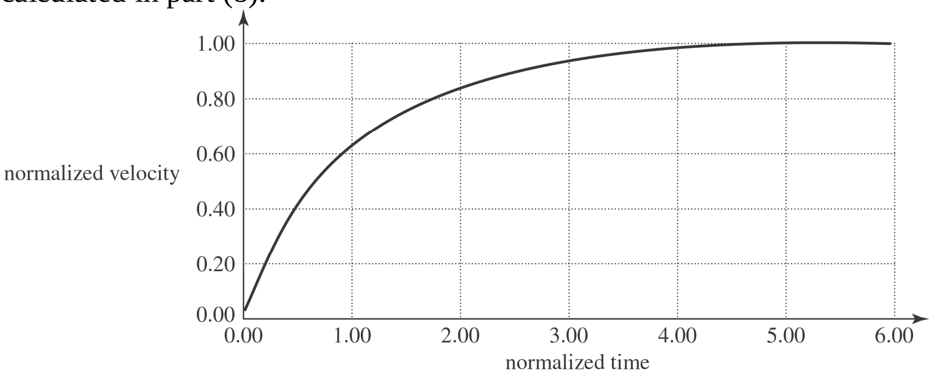
29.55: a)



$\varepsilon = vBL = IR \Rightarrow I = \dfrac{vBL}{R}$, and $F - F_B = F - ILB = ma$

$\Rightarrow a = \left(\dfrac{F - ILB}{m}\right) = \dfrac{F}{m} - \dfrac{vB^2L^2}{mR}.$

$\Rightarrow \dfrac{dv}{dt} = \dfrac{F}{m} - \dfrac{vB^2L^2}{mR} \Rightarrow v(t) = v_t\left(1 - e^{-t(B^2L^2/mR)}\right)$, where v_t is the terminal velocity calculated in part (b).



b) The terminal speed v_t occurs when the pulling force is equaled by the magnetic force: $F_B = ILB = \left(\dfrac{v_t LB}{R}\right) LB = \dfrac{v_t L^2 B^2}{R} = F \Rightarrow v_t = \dfrac{FR}{L^2 B^2}.$

29.56: The bar will experience a magnetic force due to the induced current in the loop. According to Example 29.6, the induced voltage in the loop has a magnitude BLv, which opposes the voltage of the battery, ε. Thus, the net current in the loop is $I = \frac{\varepsilon - BLv}{R}$. The acceleration of the bar is $a = \frac{F}{m} = \frac{ILB\sin(90°)}{m} = \frac{(\varepsilon - BLv)\,LB}{mR}$.

a) To find $v(t)$, set $\frac{dv}{dt} = a = \frac{(\varepsilon - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$$\int_0^v \frac{dv}{(\varepsilon - BLv)} = \int_0^t \frac{LB}{mR}dt \rightarrow v = \frac{\varepsilon}{BL}(1 - e^{-\frac{B^2L^2}{mR}t}) = (10 \text{ m/s})\ (1 - e^{-\frac{t}{3.1\text{ s}}}).$$

Note that the graph of this function is similar in appearance to that of a charging capacitor.

b) $I = \varepsilon/R = 2.4$ A; $F = ILB = 2.88$ N; $a = F/m = 3.2 \text{ m/s}^2$

c) When

$$v = 2.0 \text{ m/s},\ a = \frac{[12 \text{ V} - (1.5 \text{ T})\,(0.8 \text{ m})\,(2.0 \text{ m/s})]\,(0.8 \text{ m})\,(1.5 \text{ T})}{(0.90 \text{ kg})\,(5.0\Omega)} = 2.6 \text{ m/s}^2$$

d) Note that as the velocity increases, the acceleration decreases. The velocity will asymptotically approach the terminal velocity $\frac{\varepsilon}{BL} = \frac{12 \text{ V}}{(1.5 \text{ T})\,(0.8 \text{ m})} = 10 \text{ m/s}$, which makes the acceleration zero.

29.57: $\varepsilon = Bvl;\ B = 8.0 \times 10^{-5}$ T, $L = 2.0$ m

Use $\sum \vec{F} = m\vec{a}$ applied to the satellite motion to find the speed v of the satellite.

$$G\frac{mm_E}{r^2} = m\frac{v^2}{r};\ r = 400 \times 10^3 \text{ m} + R_E$$

$$v = \sqrt{\frac{Gm_E}{r}} = 7.665 \times 10^3 \text{ m/s}$$

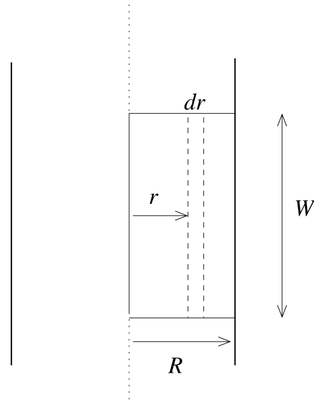
Using this v gives $\varepsilon = 1.2$ V

29.58: a) According to Example 29.6 the induced emf is $\varepsilon = BLv = (8 \times 10^{-5}\text{T})$ $(0.004 \text{ m})\,(300\ m/s) = 96\mu \text{ V} \approx 0.1 \text{ mV}$. Note that L is the size of the bar measured in a direction that is perpendicular to both the magnetic field and the velocity of the bar. Since a positive charge moving to the east would be deflected upward, the top of the bullet will be at a higher potential.

b) For a bullet that travels south, the induced emf is zero.

c) In the direction parallel to the velocity the induced emf is zero.

29.59: From Ampere's law (Example 28.9), the magnetic field inside the wire, a distance r from the axis, is $B(r) = \mu_0 Ir/2\pi R^2$.



Consider a small strip of length W and width dr that is a distance r from the axis of the wire.
The flux through the strip is

$$d\Phi_B = B(r)W\ dr = \frac{\mu_0 IW}{2\pi R^2} r\ dr$$

The total flux through the rectangle is

$$\Phi_B = \int d\Phi_B = \left(\frac{\mu_0 IW}{2\pi R^2}\right)\int_0^R r\ dr = \frac{\mu_0 IW}{4\pi}$$

Note that the result is independent of the radius R of the wire.

29.60: a) $\Phi_B = BA = B_0 \pi r_0^{\ 2}(1 - 3(t/t_0)^2 + 2(t/t_0)^3)$.

b) $\varepsilon = -\dfrac{d\Phi_B}{dt} = -B_0 \pi r_0^{\ 2} \dfrac{d}{dt}(1 - 3(t/t_0)^2 + 2(t/t_0)^3) = -\dfrac{B_0 \pi r_0^{\ 2}}{t_0}(-6(t/t_0) + 6(t/t_0)^2)$

$\Rightarrow \varepsilon = -\dfrac{6\ B_0 \pi r_0^{\ 2}}{t_0} = \left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right)$ so at $t = 5.0 \times 10^{-3}$s,,

$\varepsilon = -\dfrac{6 B_0 \pi (0.0420\text{ m})^2}{0.010\text{ s}}\left(\left(\dfrac{5.0 \times 10^{-3}\text{ s}}{0.010\text{ s}}\right)^2 - \left(\dfrac{5.0 \times 10^{-3}\text{ s}}{0.010\text{ s}}\right)\right) = 0.0665\text{ V}$, counterclockwise

c) $i = \dfrac{\varepsilon}{R_{\text{total}}} \Rightarrow R_{\text{total}} = r + R = \dfrac{\varepsilon}{i} \Rightarrow r = \dfrac{0.0655\text{ V}}{3.0 \times 10^{-3}\text{ A}} - 12\ \Omega = 10..2\ \Omega.$

d) Evaluating the emf at $t = 1.21 \times 10^{-2}$ s, using the equations of part (b): $\varepsilon = -0.0676$ V, and the current flows clockwise, from *b* to *a* through the resistor.

e) $\varepsilon = 0 \Rightarrow 0 = \left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right) \Rightarrow 1 = \dfrac{t}{t_0} \Rightarrow t = t_0 = 0.010\text{ s}.$

29.61: a) $d\varepsilon = (\vec{v} \times \vec{B}) \cdot d\vec{r} = vBdr = \dfrac{\mu_0 I v}{2\pi r}dr \Rightarrow \varepsilon \dfrac{\mu_0 I v}{2\pi}\int_d^{d+L}\dfrac{dr}{r} = \dfrac{\mu_0 I v}{2\pi}\ln\left(\dfrac{d+L}{d}\right).$

b) The magnetic force is strongest at the top end, closest to the current carrying wire. Therefore, the top end, point *a*, is the higher potential since the force on positive charges is greatest there, leading to more positives gathering at that end.

c) If the single bar was replaced by a rectangular loop, the edges parallel to the wire would have no emf induced, but the edges perpendicular to the wire will have an emf induced, just as in part (b). However, no current will flow because each edge will have its highest potential closest to the current carrying wire. It would be like having two batteries of opposite polarity connected in a loop.

29.62: Wire A : $\vec{v} \times \vec{B} = 0 \Rightarrow \varepsilon = 0.$

Wire C: $\varepsilon = vBL \sin\phi = (0.350\text{ m/s})(0.120\text{ T})(0.500\text{ m}) \sin 45° = 0.0148\text{ V}.$

Wire D: $\varepsilon = vBL \sin\phi = (0.350\text{ m/s})(0.120\text{ T})\sqrt{2}(0.500\text{ m}) \sin 45° = 0.0210\text{ V}.$

29.63: a) $d\varepsilon = (\vec{v} \times \vec{B}) \cdot d\vec{r} = \omega r\ Bdr \Rightarrow \varepsilon = \int_0^L \omega rBdr = \frac{1}{2}\omega L^2 B$

$= \frac{(8.80\ \text{rad/sec})(0.24\ \text{m})^2(0.650\ \text{T})}{2} = 0.164\ \text{V}.$

b) The potential difference between its ends is the same as the induced emf.

c) Zero, since the force acting on each end points toward the center.

$$\Delta V_{\text{center}} = \frac{\varepsilon_{\text{part(a)}}}{4} = 0.0410\ \text{V}.$$

29.64: a) From Example 29.7, the power required to keep the bar moving at a constant velocity is $P = \frac{(BLv)^2}{R} \Rightarrow R = \frac{(BLv)^2}{P} = \frac{[(0.25\ \text{T})(3.00\ \text{m/s})]^2}{25\ \text{W}} = 0.090\ \Omega.$

b) For a 50 W power dissipation we would require that the resistance be decreased to half the previous value.

c) Using the resistance from part (a) and a bar length of 0.20 m

$$P = \frac{(BLv)^2}{R} = \frac{[(0.25\ \text{T})(0.20\ \text{m})(2.0\ \text{m/s})]^2}{0.090\ \Omega} = 0.11\ \text{W}$$

29.65: a) $I = \frac{\varepsilon}{R} = \frac{vBa}{R} \Rightarrow F = IaB = \frac{vB^2a^2}{R}.$

b)

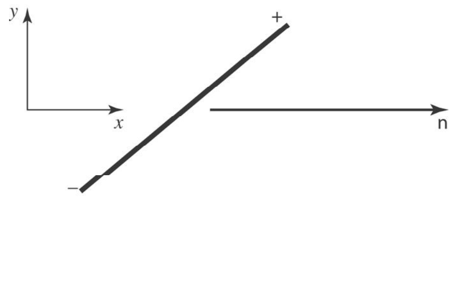
$$F = ma = m\frac{dv}{dt} = \frac{vB^2a^2}{R} \Rightarrow \int_{v_0}^{v} \frac{dv'}{v'} = \frac{B^2a^2}{mR}\int_0^t dt' \Rightarrow v = v_0 e^{-t(B^2a^2/mR)} = \frac{dx}{dt} \Rightarrow$$

$$\int_0^x dx' = v_0 \int_0^\infty e^{-t'(B^2a^2/mR)}dt' \Rightarrow x = -\frac{mRv_0}{B^2a^2} = e^{-t'(B^2a^2/mR)}\Big|_0^\infty = \frac{mRv_0}{B^2a^2}.$$

29.66: a) $\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{L} = (4.20\ \text{m/s})\hat{\boldsymbol{i}} \times ((0.120\ \text{T})\hat{\boldsymbol{i}}$ - $(0.220\ \text{T})\hat{\boldsymbol{j}}$ - $(0.0900\ \text{T})\hat{\boldsymbol{k}}) \cdot \vec{L}$

$\Rightarrow \varepsilon = \left((0.378\ \text{V/m})\hat{\boldsymbol{j}}\right.$ - $\left.(0.924\ \text{V/m})\hat{\boldsymbol{k}}\right) \cdot \left((0.250\ \text{m})(\cos 36.9°\hat{\boldsymbol{i}} + \sin 36.9°\hat{\boldsymbol{j}})\right)$

$\Rightarrow \varepsilon = (0.378)(0.250)\sin 36.9° = 0.0567\ \text{V}.$



29.67: At point $a: \varepsilon = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2 \frac{dB}{dt}$ and $F = qE = q\frac{\varepsilon}{2\pi r} = \frac{qr}{2}\frac{dB}{dt}$, to the left. At point b, the field is the same magnitude as at a since they are the same distance from the center. So $F = \frac{qr}{2}\frac{dB}{dt}$, but upward.

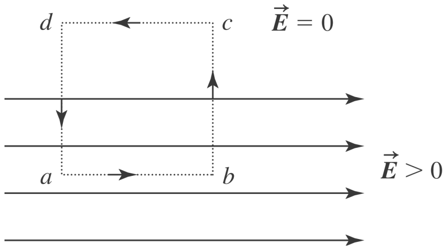
At point c, there is no force by symmetry arguments: one cannot have one direction picked out over any other, so the force must be zero.

29.68: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$.

If $\vec{B}$ = constant then $\frac{d\Phi_B}{dt} = 0$, *so* $\oint \vec{E} \cdot d\vec{l} = 0$.

$\int_{abcda} \vec{E} \cdot d\vec{l} = E_{ab}L - E_{da}L = 0$, but $E_{da} = 0$ so $E_{da}L = 0$.

But since we assumed $E_{ab} \neq 0$, this contradicts Faraday's law. Thus, we can't have a uniform electric field abruptly drop to zero in a region in which the magnetic field is constant.



29.69: At the terminal speed, the upward force F_B exerted on the loop due to the induced current equals the downward force of gravity: $F_B = mg$

$\varepsilon = Bvs$, $I = Bvs/R$ and $F_B = IsB = B^2s^2v/R$

$\frac{B^2s^2v_\tau}{R} = mg$ and $v_T = \frac{mgR}{B^2s^2}$

$m = \rho_m V = \rho_m(4s)\pi(d/2)^2 = \rho_m \pi s d^2$

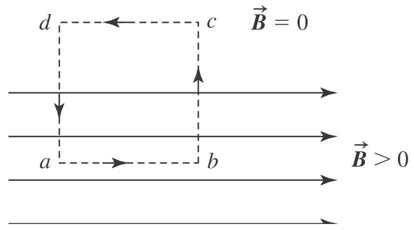
$R = \frac{\rho L}{A} = \frac{\rho_R 4s}{\frac{1}{4}\pi d^2} = \frac{16\rho_R s}{\pi d^2}$

Using these expressions for m and R gives $v_T = 16\rho_m\rho_R g/B^2$

29.70: $\oint \vec{B} \cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $B = B_0\hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$.

$$\int_{abcde} \vec{B} \cdot d\vec{l} = B_{ab}L - B_{cd}L = 0,$$

but $B_{cd} = 0.\ B_{ab}L = 0$, but $B_{ab} \neq 0.$ This is a contradiction and violates Ampere's Law. See the figure on the next page.



29.71: a) $j_c = \dfrac{I}{A} = \dfrac{V}{AR} = \dfrac{VA}{Ad\rho} = \dfrac{\mathrm{V}}{d\rho} = \dfrac{q}{Cd\rho} = \dfrac{qd}{K\varepsilon_0 Ad\rho}$

and

$$RC = \frac{\rho d}{A}\frac{K\varepsilon_0 A}{d} = K\varepsilon_0\rho.$$

$$\Rightarrow j_c(t) = \frac{q}{K\varepsilon_0 A\rho} = \frac{Q_0}{K\varepsilon_0 A\rho}e^{-t/RC} = \frac{Q_0}{K\varepsilon_0 A\rho}e^{-t/K\varepsilon_0\rho}$$

b) $j_D(t) = K\varepsilon_0 \dfrac{dE}{dt} = K\varepsilon_0 \dfrac{d(\rho j_c)}{dt} = K\varepsilon_0\rho \dfrac{Q_0}{K\varepsilon_0 A\rho}\dfrac{d(e^{-t/K\varepsilon_0\rho})}{dt}$

$$= -\frac{Q_0}{K\varepsilon_0 A\rho}e^{-t/K\varepsilon_0\rho} = -j_c(t).$$

29.72: a) $j_c(\max) = \dfrac{E_0}{\rho} = \dfrac{0.450\ \text{V/m}}{2300\ \Omega\ \text{m}} = 1.96 \times 10^{-4}\ \text{A/m}^2$.

b) $j_D(\max) = \varepsilon_0 \dfrac{dE}{dt} = \varepsilon_0 \omega E_0 = 2\pi\varepsilon_0 f E_0 = 2\pi\varepsilon_0 (120\ \text{Hz})(0.450\ \text{V/m})$

$$\Rightarrow j_D(\max) = 3.00 \times 10^{-9}\ \text{A/m}^2.$$

c) If $j_c = j_D \Rightarrow \dfrac{E_0}{\rho} = \omega\varepsilon_0 E_0 \Rightarrow \omega = \dfrac{1}{\rho\varepsilon_0} = 4.91 \times 10^7\ \text{rad/s}$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{4.91 \times 10^7\ \text{rad/s}}{2\pi} = 7.82 \times 10^6 \text{Hz}.$$

d) The two current densities are out of phase by $90°$ because one has a sine function and the other has a cosine, so the displacement current leads the conduction current by $90°$.

29.73: a) $\vec{\tau}_G = \sum \vec{r}_{cm} \times m\vec{g}$, summed over each leg,

$$= (0)\left(\frac{m}{4}\right) g \sin(90-\phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right) g \sin(90-\phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right) g \sin(90-\phi)$$

$$+ (L)\left(\frac{m}{4}\right) g \sin(90-\phi)$$

$$= \frac{mgL}{2}\cos\phi \text{ (clockwise)}.$$

$$\vec{\tau}_B = \vec{\mu} \times \vec{B} = IAB\sin\phi \text{ (counterclockwise)}.$$

$I = \dfrac{\varepsilon}{R} = \dfrac{BA}{R}\dfrac{d}{dt}\cos\phi = -\dfrac{BA}{R}\dfrac{d\phi}{dt}\sin\phi = \dfrac{BA\omega}{R}\sin\phi.$ The current is going counterclockwise looking to the $-\hat{k}$ direction.

$$\Rightarrow \tau_B = \frac{B^2A^2\omega}{R}\sin^2\phi = \frac{B^2L^4\omega}{R}\sin^2\phi,$$

so $\tau = \dfrac{mgL}{2}\cos\phi - \dfrac{B^2L^4\omega}{R}\sin^2\phi$, opposite to the direction of the rotation.

b) $\tau = I\alpha$ (I being the moment of inertia).

About this axis $I = \dfrac{5}{12}mL^2$.

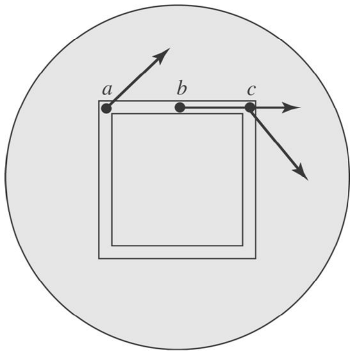
$$\text{So } \alpha = \frac{12}{5}\frac{1}{mL^2}\left[\frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi\right]$$

$$= \frac{6g}{5L}\cos\phi - \frac{12B^2L^2\omega}{5m\,R}\sin^2\phi.$$

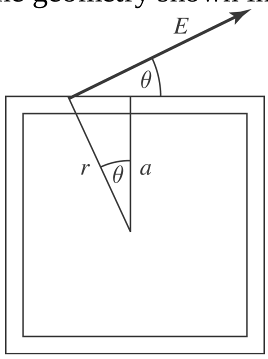
c) The magnetic torque slows down the fall (since it opposes the gravitational torque).

d) Some energy is lost through heat from the resistance of the loop.

29.74: a) For clarity, figure is rotated so B comes out of the page.



b) To work out the amount of the electric field that is in the direction of the loop at a general position, we will use the geometry shown in the diagram below.



$$E_{\text{loop}} = E\cos\theta \text{ but } E = \frac{\varepsilon}{2\pi r} = \frac{\varepsilon}{2\pi(a/\cos\theta)} = \frac{\varepsilon\cos\theta}{2\pi a}$$

$$\Rightarrow E_{\text{loop}} = \frac{\varepsilon\cos^2\theta}{2\pi a} \text{ but } \varepsilon = \frac{d\Phi_B}{dt} = A\frac{dB}{dt}\pi r^2\frac{dB}{dt} = \frac{\pi a^2}{\cos^2\theta}\frac{dB}{dt}$$

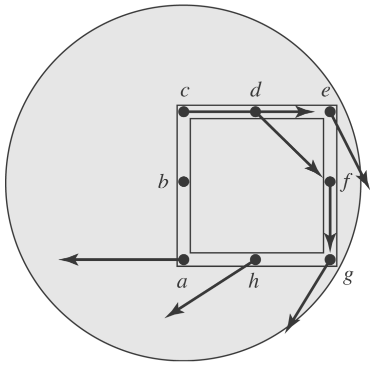
$$\Rightarrow E_{\text{loop}} = \frac{\pi a^2}{2\pi a}\frac{dB}{dt} = \frac{a}{2}\frac{dB}{dt},$$ which is exactly the value for a ring, obtained in Exercise 29.29, and has no dependence on the part of the loop we pick.

c) $$I = \frac{\varepsilon}{R} = \frac{A}{R}\frac{dB}{dt} = \frac{L^2}{R}\frac{dB}{dt} = \frac{(0.20\,\text{m})^2(0.0350\,\text{T/s})}{1.90\,\Omega} = 7.37\times10^{-4}\text{ A.}$$

d) $$\varepsilon_{ab} = \frac{1}{8}\varepsilon = \frac{1}{8}L^2\frac{dB}{dt} = \frac{(0.20\,\text{m})^2(0.0350\ \text{T/s})}{8} = 1.75\times10^{-4}\text{ V.}$$

But there is potential drop $V = IR = -1.75\times10^{-4}$ V, so the potential difference is zero.

29.75: a)



b) The induced emf on the side ac is zero, because the electric field is always perpendicular to the line ac.

c) To calculate the total emf in the loop, $\varepsilon = \dfrac{d\Phi_B}{dt} = A\dfrac{dB}{dt} = L^2\dfrac{dB}{dt}$

$\Rightarrow \varepsilon = (0.20\text{m})^2(0.035\ \text{T/s}) = 1.40 \times 10^{-3}\ \text{V}.$

d) $I = \dfrac{\varepsilon}{R} = \dfrac{1.40 \times 10^{-3}\ \text{V}}{1.90\ \Omega} = 7.37 \times 10^{-4}\ \text{A}.$

e) Since the loop is uniform, the resistance in length ac is one quarter of the total resistance. Therefore the potential difference between a and c is:
$V_{ac} = IR_{ac} = (7.37 \times 10^{-4}\ \text{A})(1.90\ \Omega/4) = 3.50 \times 10^{-4}\ \text{V},$ and the point a is at a higher potential since the current is flowing from a to c.

29.76: a) As the bar starts to slide, the flux is decreasing, so the current flows to increase the flux, which means it flows from a to b.

b) The magnetic force on the bar must eventually equal that of gravity.

$$F_B = iLB = \frac{LB}{R}\varepsilon = \frac{LB}{R}\frac{d\Phi_B}{dt} = \frac{LB}{R}B\frac{dA}{dt} = \frac{LB^2}{R}(vL\cos\phi) = \frac{vL^2B^2}{R}\cos\phi$$

$$\Rightarrow F_g = mg\tan\phi = \frac{v_t L^2 B^2}{R}\cos\phi \Rightarrow v_t = \frac{Rmg\tan\phi}{L^2B^2\cos\phi}.$$

c) $i = \dfrac{\varepsilon}{R} = \dfrac{1}{R}\dfrac{d\Phi_B}{dt} = \dfrac{1}{R}B\dfrac{dA}{dt} = \dfrac{B}{R}(vL\cos\phi) = \dfrac{vL\,B\cos\phi}{R} = \dfrac{mg\tan\phi}{LB}.$

d) $P = i^2R = \dfrac{Rm^2g^2\tan^2\phi}{L^2B^2}.$

e) $P_g = Fv\cos(90° - \phi) = mg\left(\dfrac{Rmg\tan\phi}{L^2B^2\cos\phi}\right)\sin\phi \Rightarrow P_g = \dfrac{Rm^2g^2\tan^2\phi}{L^2B^2},$ which is the same as found in part (d).

29.77: The primary assumption throughout the problem is that the square patch is small enough so that the velocity is constant over its whole areas, that is, $v = \omega r \approx \omega d$.

a) $\omega \rightarrow$ clockwise, $B \rightarrow$ into page:

$\varepsilon = vBL = \omega d\ BL$

$\Rightarrow I = \dfrac{\varepsilon}{R} = \dfrac{\varepsilon A}{\rho L} = \dfrac{\omega dBA}{\rho}$. Since $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$ points outward, A is just the cross-sectional area tL.

$\Rightarrow I = \dfrac{\omega dBLt}{\rho}$ flowing radially outward since $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$ points outward.

b) $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{d}} \times \vec{\boldsymbol{F}}_b$; $\vec{\boldsymbol{F}}_B = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}} = ILB$ pointing counterclockwise.

So $\tau = \dfrac{\omega d^2 B^2 L^2 t}{\rho}$ pointing out of the page (a counterclockwise torque opposing the clockwise rotation).

c) If $\omega \rightarrow$ counterclockwise and $B \rightarrow$ into page,

$\Rightarrow I \rightarrow$ flow inward radially since $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$ points inward.

$\tau \rightarrow$ clockwise (again opposing the motion);

If $\omega \rightarrow$ counterclockwise and $B \rightarrow$ out of the page

$\Rightarrow I \rightarrow$ radially outward

$\tau \rightarrow$ clockwise (opposing the motion)

The magnitudes of I and τ are the same as in part (a).

30.1: a) $\varepsilon_2 = M(di_1/dt) = (3.25\times10^{-4}\,\text{H})\,(830\ \text{A/s}) = 0.270\ \text{V}$, and is constant.

b) If the second coil has the same changing current, then the induced voltage is the same and $\varepsilon_1 = 0.270\ \text{V}$.

30.2: For a toroidal solenoid, $M = N_2\Phi_{B_2}/i_1$, and $\Phi_{B_2} = \mu_0 N_1 i_1 A/2\pi r$.
So, $M = \mu_0 A N_1 N_2/2\pi r$.

30.3: a) $M = N_2\Phi_{B_2}/i_1 = (400)\,(0.0320\ \text{Wb})/(6.52\ \text{A}) = 1.96\ \text{H}$.

b) When $i_2 = 2.54\ \text{A}$, $\Phi_{B_1} = i_2 M/N_1 = (2.54\ \text{A})\,(1.96\ \text{H})/(700) = 7.11\times10^{-3}\,\text{Wb}$.

30.4: a) $M = \varepsilon_2/(di/dt) = 1.65\times10^{-3}\ \text{V}/\,(-0.242\ \text{A/s}) = 6.82\times10^{-3}\text{H}$.

b) $N_2 = 25,\ i_1 = 1.20\ \text{A}$,

$$\Rightarrow \Phi_{B_2} = i_1 M/N_2 = (1.20\ \text{A})\,(6.82\times10^{-3}\text{H})/25$$
$$= 3.27\times10^{-4}\ \text{Wb}.$$

c) $di_2/dt = 0.360\ \text{A/s}$ and $\varepsilon_1 = M di_2/dt = (6.82\times10^{-3}\ \text{H})\,(0.360\ \text{A/s}) = 2.45\ \text{mV}$.

30.5: $1\ \text{H} = 1\ \text{Wb/A} = 1\ \text{Tm}^2/\text{A} = 1\ \text{Nm/A}^2 = 1\ \text{J/A}^2 = 1\,(\text{J/AC})\text{s} = 1\,(\text{V/A})\text{s} = 1\,\Omega\text{s}$.

30.6: For a toroidal solenoid, $L = N\Phi_B/i = \varepsilon/(di/dt)$. So solving for N we have:

$$N = \varepsilon i/\Phi_B(di/dt) = \frac{(12.6\times10^{-3}\text{V})\,(1.40\ \text{A})}{(0.00285\ \text{Wb})\,(0.0260\ \text{A/s})} = 238\ \text{turns}.$$

30.7: a) $|\varepsilon| = L(di_1/dt) = (0.260\ \text{H})\,(0.0180\ \text{A/s}) = 4.68\times10^{-3}\text{V}$.

b) Terminal a is at a higher potential since the coil pushes current through from b to a and if replaced by a battery it would have the $+$ terminal at a.

30.8: a) $L_{K_m} = K_m\mu_0 N^2 A/2\pi r = \dfrac{(500\mu_0)\,(1800)^2\ (4.80\times10^{-5}\ \text{m}^2)}{2\pi(0.120\ \text{m})} = 0.130\ \text{H}$.

b) Without the material, $L = \dfrac{1}{K_m}L_{K_m} = \dfrac{1}{500}(0.130\ \text{H}) = 2.60\times10^{-4}\ \text{H}$.

30.9: For a long, straight solenoid:
$L = N\Phi_B / i$ and $\Phi_B = \mu_0 NiA/l \Rightarrow L = \mu_0 N^2 A/l.$

30.10: a) Note that points a and b are reversed from that of figure 30.6. Thus, according to Equation 30.8, $\frac{di}{dt} = \frac{V_b - V_a}{L} = \frac{-1.04 \text{ V}}{0.260 \text{ H}} = -4.00 \text{ A/s}.$ Thus, the current is decreasing.

b) From above we have that $di = (-4.00 \text{ A/s})dt.$ After integrating both sides of this expression with respect to t, we obtain

$$\Delta i = (-4.00 \text{ A/s})\Delta t \Rightarrow i = (12.0 \text{ A}) - (4.00 \text{ A/s})\,(2.00 \text{ s}) = 4.00 \text{ A}.$$

30.11: a) $L = \varepsilon/(di/dt) = (0.0160 \text{ V})/(0.0640 \text{ A/s}) = 0.250 \text{ H}.$

b) $\Phi_B = iL/N = (0.720 \text{ A})\,(0.250 \text{ H})/(400) = 4.50 \times 10^{-4} \text{ Wb}.$

30.12: a) $U = \frac{1}{2}LI^2 = (12.0 \text{ H})\,(0.300 \text{ A})^2/2 = 0.540 \text{ J}.$

b) $P = I^2 R = (0.300 \text{ A})^2\,(180\,\Omega) = 16.2 \text{ W}.$

c) No. Magnetic energy and thermal energy are independent. As long as the current is constant, $U =$ constant.

30.13: $U = \frac{1}{2}LI^2 = \frac{\mu_0 N^2 A l^2}{4\pi r}$

$$\Rightarrow N = \sqrt{\frac{4\pi r U}{\mu_0 A I^2}} = \sqrt{\frac{4\pi(0.150 \text{ m})\,(0.390 \text{ J})}{\mu_0 (5.00 \times 10^{-4} \text{m}^2)\,(12.0 \text{ A})^2}} = 2850 \text{ turns.}$$

30.14: a) $U = Pt = (200 \text{ W})\,(24 \text{ h/day} \times 3600 \text{ s/h}) = 1.73 \times 10^7 \text{J}.$

b) $U = \frac{1}{2}LI^2 \Rightarrow L = \frac{2U}{I^2} = \frac{2(1.73 \times 10^7 \text{ J})}{(80.0 \text{ A})^2} = 5406 \text{ H}.$

30.15: Starting with Eq. (30.9), follow exactly the same steps as in the text except that the magnetic permeability μ is used in place of μ_0.

30.16: a) free space: $U = u\mathrm{V} = \dfrac{B^2}{2\mu_0}\mathrm{V} = \dfrac{(0.560\ \mathrm{T})^2}{2\mu_0}(0.0290\ \mathrm{m}^3) = 3619\ \mathrm{J}.$

b) material with $K_\mathrm{m} = 450 \Rightarrow U = u\mathrm{V} = \dfrac{B^2}{2K_\mathrm{m}\mu_0}\mathrm{V} = \dfrac{(0.560\ \mathrm{T})^2}{2(450)\mu_0}(0.0290\ \mathrm{m}^3) = 8.04\ \mathrm{J}.$

30.17: a) $u = \dfrac{U}{Vol} = \dfrac{B^2}{2\mu_0} \Rightarrow \mathrm{Volume} = \dfrac{2\mu_0 U}{B^2} = \dfrac{2\mu_0(3.60\times 10^6\ \mathrm{J})}{(0.600\ \mathrm{T})^2} = 25.1\ \mathrm{m}^3.$

b) $B^2 = \dfrac{2\mu_0 U}{Vol} = \dfrac{2\mu_0(3.60\times 10^6\ \mathrm{J})}{(0.400\ \mathrm{m})^3} = 141.4\ \mathrm{T}^2 \Rightarrow B = 11.9\ \mathrm{T}.$

30.18: a) $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0(600)\ (2.50\ \mathrm{A})}{2\pi\ (0.0690\ \mathrm{m})} = 4.35\ \mathrm{mT}.$

b) From Eq. (30.10), $u = \dfrac{B^2}{2\mu_0} = \dfrac{(4.35\times 10^{-3}\ \mathrm{T})^2}{2\mu_0} = 7.53\ \mathrm{J/m^3}.$

c) Volume $\mathrm{V} = 2\pi r A = 2\pi\ (0.0690\ \mathrm{m})\ (3.50\times 10^{-6}\ \mathrm{m}^2) = 1.52\times 10^{-6}\ \mathrm{m}^3.$

d) $U = uV = (7.53\ \mathrm{J/m^3})\ (1.52\times 10^{-6}\mathrm{m}^3) = 1.14\times 10^{-5}\ \mathrm{J}.$

e) $L = \dfrac{\mu_0 N^2 A}{2\pi r} = \dfrac{\mu_0 (600)^2\ (3.50\times 10^{-6}\ \mathrm{m}^2)}{2\pi\ (0.0690\ \mathrm{m})} = 3.65\times 10^{-6}\ \mathrm{H}.$

$U = \dfrac{1}{2}LI^2 = \dfrac{1}{2}(3.65\times 10^{-6}\ \mathrm{H})\ (2.50\ \mathrm{A})^2 = 1.14\times 10^{-5}\ \mathrm{J}$ same as (d).

30.19: a) $\dfrac{di}{dt} = \dfrac{\varepsilon - iR}{L}$. When $i = 0 \Rightarrow \dfrac{di}{dt} = \dfrac{6.00\ \mathrm{V}}{2.50\ \mathrm{H}} = 2.40\ \mathrm{A/s}.$

b) When $i = 1.00\ \mathrm{A} \Rightarrow \dfrac{di}{dt} = \dfrac{6.00\ \mathrm{V} - (0.500\ \mathrm{A})\ (8.00\ \Omega)}{2.50\ \mathrm{H}} = 0.800\ \mathrm{A/s}.$

c) At $t = 0.200\ \mathrm{s} \Rightarrow i = \dfrac{\varepsilon}{R}(1 - e^{-(R/L)t}) = \dfrac{6.00\ \mathrm{V}}{8.00\ \Omega}(1 - e^{-(8.00\ \Omega/2.50\ \mathrm{H})\ (0.250\ \mathrm{s})}) = 0.413\ \mathrm{A}.$

d) As $t \to \infty \Rightarrow i \to \dfrac{\varepsilon}{R} = \dfrac{6.00\ \mathrm{V}}{8.00\ \Omega} = 0.750\ \mathrm{A}.$

30.20: (a) $i_{\max} = \frac{30\ \text{V}}{1000\ \Omega} = 0.030\ \text{A} = 30\ \text{mA},$ long after closing the switch.

b)

$$i = i_{\max}(1 - \mathrm{e}^{-t/(L/R)}) = 0.030\ \text{A}\left(1 - \mathrm{e}^{-\frac{20\ \mu s}{10\ \mu s}}\right)$$

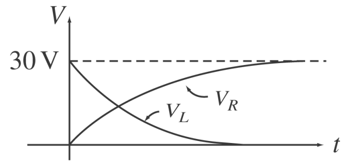
$$= 0.0259\ \text{A}$$

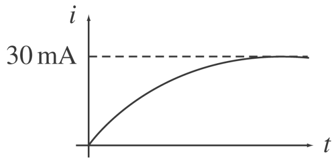
$$V_{\text{R}} = Ri = (1000\ \Omega)\ (0.0259\ \text{A}) = 26\ \text{V}$$

$$V_{\text{L}} = \varepsilon_{\text{Battery}} - V_{\text{R}} = 30\ \text{V} - 26\ \text{V} = 4.0\ \text{V}$$

(or, could use $V_L = L\frac{di}{dt}$ at $t = 20\ \mu$s)

c)





30.21: a) $i = \varepsilon/R(1 - e^{-t/\tau}),\ \tau = L/R$

$i_{\max} = \varepsilon/R$ so $i = i_{\max}/2$ when $(1 - e^{-t/\tau}) = \frac{1}{2}$, and $e^{-t/\tau} = \frac{1}{2}$

$$-t/\tau = \ln\left(\tfrac{1}{2}\right) \text{ and } t = \frac{L \ln 2}{\text{R}} = \frac{(\ln 2)\ (1.25 \times 10^{-3}\,\text{H})}{50.0\ \Omega} = 17.3\ \mu s$$

b) $U = \frac{1}{2}Li^2; U_{\max} = \frac{1}{2}Li^2{}_{\max}$

$U = \frac{1}{2}U_{\max}$ when $i = i_{\max}/\sqrt{2}$

$1 - e^{-t/\tau} = 1\sqrt{2}$ so $\mathrm{e}^{-t/\tau} = 1 - 1/\sqrt{2} = 0.2929$

$t = -L\ \ln(0.2929)/\text{R} = 30.7\ \mu s$

30.22: a) $U = \dfrac{1}{2}LI^2 \Rightarrow I = \sqrt{\dfrac{2U}{L}} = \sqrt{\dfrac{2(0.260\text{ J})}{0.115\text{ H}}} = 2.13\text{ A}$

$$\Rightarrow \varepsilon = IR = (2.13\text{ A})\,(120\,\Omega) = 256\text{ V}.$$

b) $i = Ie^{-(R/L)t}$ and $U = \dfrac{1}{2}Li^2 = \dfrac{1}{2}Li^2 e^{-2(R/L)t} = \dfrac{1}{2}U_0 = \dfrac{1}{2}\left(\dfrac{1}{2}LI^2\right)$

$$\Rightarrow e^{-2(R/L)t} = \frac{1}{2}$$

$$\Rightarrow t = -\frac{L}{2R}\ln\left(\frac{1}{2}\right) = -\frac{0.115\text{ H}}{2(120\,\Omega)}\ln\left(\frac{1}{2}\right) = 3.32\times10^{-4}\text{s}.$$

30.23: a) $I_0 = \dfrac{\varepsilon}{R} = \dfrac{60\text{ V}}{240\,\Omega} = 0.250\text{ A}.$

b) $i = I_0 e^{-(R/L)t} = (0.250\text{ A})\,e^{-(240\,\Omega/0.160\text{ H})\,(4.00\times10^{-4}\text{s})} = 0.137\text{ A}.$

c) $V_{cb} = V_{ab} = iR = (0.137\text{ A})\,(240\,\Omega) = 32.9\text{ V},$ and c is at the higher potential.

d) $\dfrac{i}{I_0} = \dfrac{1}{2} = e^{-(R/L)t_{1/2}} \Rightarrow t_{1/2} = -\dfrac{L}{R}\ln\left(\dfrac{1}{2}\right) = -\dfrac{(0.160\text{ H})}{(240\,\Omega)}\ln\left(\dfrac{1}{2}\right) = 4.62\times10^{-4}$ s.

30.24: a) At $t = 0 \Rightarrow v_{ab} = 0$ and $v_{bc} = 60$ V.

b) As $t \to \infty \Rightarrow v_{ab} \to 60$ V and $v_{bc} \to 0$.

c) When $i = 0.150\text{ A} \Rightarrow v_{ab} = iR = 36.0$ V and $v_{bc} = 60.0\text{ V} - 36.0\text{ V} = 24.0$ V.

30.25: a) $P = \varepsilon i = \varepsilon I_0\,(1 - e^{-(R/L)t}) = \dfrac{\varepsilon^2}{R}(1 - e^{-(R/L)t}) = \dfrac{(6.00\text{ V})^2}{8.00\,\Omega}(1 - e^{-(8.00\,\Omega/2.50\text{ H})t})$

$$\Rightarrow P = (4.50\text{ W})\,(1 - e^{-(3.20\text{ s}^{-1})t}).$$

b) $P_R = i^2R = \dfrac{\varepsilon^2}{R}(1 - e^{-(R/L)t})^2 = \dfrac{(6.00\text{ V})^2}{8.00\,\Omega}(1 - e^{-(8.00\,\Omega/2.50\text{ H})t})^2$

$$\Rightarrow P_R = (4.50\text{ W})\,(1 - e^{-(3.20\text{ s}^{-1})t})^2.$$

c) $P_L = iL\dfrac{di}{dt} = \dfrac{\varepsilon}{R}(1 - e^{-(R/L)t})\,L\left(\dfrac{\varepsilon}{L}e^{-(R/L)t}\right) = \dfrac{\varepsilon^2}{R}(e^{-(R/L)t} - e^{-2(R/L)t})$

$$\Rightarrow P_L = (4.50\text{ W})\,(e^{-(3.20\text{ s}^{-1})t} - e^{-(6.40\text{ s}^{-1})t}).$$

d) Note that if we expand the exponential in part (b), then parts (b) and (c) add to give part (a), and the total power delivered is dissipated in the resistor and inductor.

30.26: When switch 1 is closed and switch 2 is open:

$$L\frac{di}{dt} + iR = 0 \Rightarrow \frac{di}{dt} = -i\frac{R}{L} \Rightarrow \int_{I_0}^{i}\frac{di'}{i'} = -\frac{R}{L}\int_0^t dt'$$

$$\Rightarrow \ln(i/I_0) = -\frac{R}{L}t \Rightarrow i = I_0 e^{-t(R/L)}.$$

30.27: Units of $L/R = \mathrm{H}/\Omega = (\Omega\ \mathrm{s})/\Omega = \mathrm{s} =$ units of time.

30.28: a) $\omega = \dfrac{1}{\sqrt{LC}} = 2\pi f$

$$\Rightarrow L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(1.6\times10^6)^2(4.18\times10^{-12}\,\mathrm{F})} = 2.37\times10^{-3}\,\mathrm{H}.$$

b) $$C_{\max} = \frac{1}{4\pi^2 f^2{}_{\min} L} = \frac{1}{4\pi^2(5.40\times10^5)^2(2.37\times10^{-3}\,\mathrm{H})} = 3.67\times10^{-11}\,\mathrm{F}.$$

30.29: a) $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{LC} = 2\pi\sqrt{(1.50\text{ H})(6.00\times10^{-5}\text{F})}$

$= 0.0596\text{ s}, \ \omega = 105\text{ rad/s}.$

b) $Q = CV = (6.00\times10^{-5}\text{F})(12.0\text{ V}) = 7.20\times10^{-4}\text{C}.$

c) $U_0 = \dfrac{1}{2}CV^2 = \dfrac{1}{2}(6.00\times10^{-5}\text{F})(12.0\text{ V})^2 = 4.32\times10^{-3}\text{J}.$

d) At $t = 0$, $q = Q = Q\cos(\omega t + \phi) \Rightarrow \phi = 0.$

$$t = 0.0230\text{ s}, \ q = Q\cos(\omega t) = (7.20\times10^{-4}\text{C})\ \cos\left(\frac{0.0230\text{ s}}{\sqrt{(1.50\text{ H})(6.00\times10^{-5}\text{F})}}\right)$$

$= -5.43\times10^{-4}\text{C}.$ Signs on plates are opposite to those at $t = 0.$

e) $t = 0.0230\ \text{s}, \ i = \dfrac{dq}{dt} = -\omega Q\ \sin(\omega t)$

$$\Rightarrow i = -\frac{7.20\times10^{-4}\text{C}}{\sqrt{(1.50\text{H})(6.00\times10^{-5}\text{H})}}\sin\left(\frac{0.0230\text{s}}{\sqrt{(1.50\text{ H})(6.00\times10^{-5}\text{H})}}\right) = -0.0499\text{A}.$$

Positive charge flowing away from plate which had positive charge at $t = 0.$

f) Capacitor: $U_C = \dfrac{q^2}{2C} = \dfrac{(5.43\times10^{-4}\text{C})^2}{2(6.00\times10^{-5}\text{F})} = 2.46\times10^{-3}\text{J}.$

Inductor: $U_L = \dfrac{1}{2}Li^2 = \dfrac{1}{2}(1.50\text{H})(0.0499\text{A})^2 = 1.87\times10^{-3}\text{J}.$

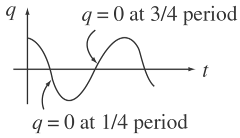
30.30: (a) Energy conservation says $U_L(\max) = U_C(\max)$

$$\frac{1}{2}Li_{\max}^2 = \frac{1}{2}\mathrm{CV}^2$$

$$i_{\max} = V\sqrt{C/L} = (22.5\mathrm{V})\sqrt{\frac{18\times10^{-6}\,\mathrm{F}}{12\times10^{-3}\,\mathrm{H}}} = 0.871\mathrm{A}$$

The charge on the capacitor is zero because all the energy is in the inductor.

(b)



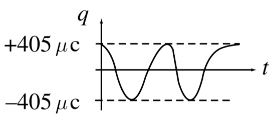
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$

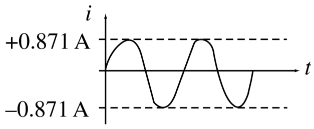
at 1/4 period: $\dfrac{1}{4}T = \dfrac{1}{4}(2\pi\sqrt{LC}) = \dfrac{\pi}{2}\sqrt{(12\times10^{-3}\,\mathrm{H})(18\times10^{-6}\,\mathrm{F})}$

$$= 7.30\times10^{-4}\,\mathrm{s}$$

at 3/4 period: $\dfrac{3}{4}T = 3(7.30\times10^{-4}\,\mathrm{s}) = 2.19\times10^{-3}\,\mathrm{s}$

(c) $q_0 = CV = (18\mu\mathrm{F})(22.5\mathrm{V}) = 405\mu C$





30.31: $C = \dfrac{Q}{V} = \dfrac{150\times10^{-9}\,\mathrm{C}}{4.29\times10^{-3}\,\mathrm{V}} = 30.0\ \mu\mathrm{F}$

For an L-C circuit, $\omega = \sqrt{1/LC}$ and $T = 2\pi/\omega = 2\pi\sqrt{LC}$

$$L = \frac{(T/2\pi)^2}{C} = 0.601\ \mathrm{mH}$$

30.32: $\omega = \dfrac{1}{\sqrt{(0.0850\,\text{H})(3.20\times10^{-6}\,\text{F})}} = 1917\ \text{rad/s}$

a) $i_{\max} = \omega Q_{\max} \Rightarrow Q_{\max} = \dfrac{i_{\max}}{\omega} = \dfrac{8.50\times10^{-4}\,\text{A}}{1917\ \text{rad/s}} = 4.43\times10^{-7}\,\text{C}$

b) From Eq. 31.26 $q = \sqrt{Q^2 - LCi^2} = \sqrt{(4.43\times10^{-7}\,\text{C})^2 - \left(\dfrac{5.00\times10^{-4}\,\text{A}}{1917\,\text{s}^{-1}}\right)^2}$

$= 3.58\times10^{-7}\,\text{C}.$

30.33: a) $\dfrac{d^2q}{dt^2} + \dfrac{1}{LC}q = 0 \Rightarrow q = LC\dfrac{di}{dt} = (0.640\ \text{H})(3.60\times10^{-6}\,\text{F})(2.80\ \text{A/s})$

$= 6.45\times10^{-6}\,\text{C}.$

b) $\varepsilon = \dfrac{q}{C} = \dfrac{8.50\times10^{-6}\,\text{C}}{3.60\times10^{-6}\,\text{F}} = 2.36\,\text{V}.$

30.34: a) $i_{\max} = \omega Q_{\max} \Rightarrow Q_{\max} = \dfrac{i_{\max}}{\omega} = i_{\max}\sqrt{LC}.$

$\Rightarrow Q_{\max} = (1.50\,\text{A})\sqrt{(0.400\text{H})(2.50\times10^{-10}\,\text{F})} = 1.50\times10^{-5}\,\text{C}.$

$\Rightarrow U_{\max} = \dfrac{Q^2{}_{\max}}{2C} = \dfrac{(1.50\times10^{-5}\,\text{C})^2}{2(2.50\times10^{-10}\,\text{F})} = 0.450\ \text{J}$

b) $2f = \dfrac{2\omega}{2\pi} = \dfrac{1}{\pi\sqrt{LC}} = \dfrac{1}{\pi\sqrt{(0.400\ \text{H})(2.50\times10^{-10}\ \text{F})}} = 3.18\times10^{4}\,\text{s}^{-1}$

(must double the frequency since it takes the required value twice per period).

30.35: $[LC] = \text{H}\cdot\text{F} = \text{H}\cdot\dfrac{\text{C}}{\text{V}} = \Omega\cdot\text{s}\cdot\dfrac{\text{C}}{\text{V}} = \dfrac{\Omega}{\text{V}}\cdot\dfrac{\text{C}}{\text{s}}\cdot\text{s}^2 = \dfrac{1}{\text{A}}\cdot\text{A}\cdot\text{s}^2 = \text{s}^2 \Rightarrow \left[\sqrt{LC}\right] = s.$

30.36: Equation (30.20) is $\dfrac{d^2q}{dt^2} + \dfrac{1}{LC}q = 0.$ We will solve the equation using:

$q = Q\cos(\omega t + \phi) \Rightarrow \dfrac{dq}{dt} = -\omega Q\ \sin(\omega t + \phi) \Rightarrow \dfrac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t + \phi).$

$\Rightarrow \dfrac{d^2q}{dt^2} + \dfrac{1}{LC}q = -\omega^2 Q\ \cos(\omega t + \phi) + \dfrac{Q}{LC}\cos(\omega t + \phi) = 0 \Rightarrow \omega^2 = \dfrac{1}{LC} \Rightarrow \omega = \dfrac{1}{\sqrt{LC}}.$

30.37: a) $U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{Q^2\cos^2(\omega t+\phi)}{C}$.

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}L\omega^2 Q^2\sin^2(\omega t+\phi) = \frac{1}{2}\frac{Q^2\sin^2(\omega t+\phi)}{C}, \text{ since } \omega^2 = \frac{1}{\sqrt{LC}}.$$

b) $$U_{\text{Total}} = U_C + U_L = \frac{1}{2}\frac{Q^2}{C}\cos^2(\omega t+\phi) + \frac{1}{2}L\omega^2 Q^2\sin^2(\omega t+\phi)$$

$$= \frac{1}{2}\frac{Q^2}{C}\cos^2(\omega t+\phi) + \frac{1}{2}L\left(\frac{1}{LC}\right)Q^2\sin^2(\omega t+\phi)$$

$$= \frac{1}{2}\frac{Q^2}{C}(\cos^2(\omega t+\phi) + \sin^2(\omega t+\phi))$$

$$= \frac{1}{2}\frac{Q^2}{C} \Rightarrow U_{\text{Total}} \text{ is a constant.}$$

30.38: a) $q = Ae^{-(R/2L)t}\cos(\omega' t+\phi)$

$$\Rightarrow \frac{dq}{dt} = -A\frac{R}{2L}e^{-(R/2L)t}\cos(\omega' t+\phi) - \omega' Ae^{-(R/2L)t}\sin(\omega' t+\phi).$$

$$\Rightarrow \frac{d^2q}{dt^2} = A\left(\frac{R}{2L}\right)^2 e^{-(R/2L)t}\cos(\omega' t+\phi) + 2\omega' A\frac{R}{2L}e^{-(R/2L)t}\sin(\omega' t+\phi)$$

$$-\omega'^2 Ae^{-(R/2L)t}\cos(\omega' t+\phi).$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = q\left(\left(\frac{R}{2L}\right)^2 - \omega'^2 - \frac{R^2}{2L^2} + \frac{1}{LC}\right) = 0$$

$$\Rightarrow \omega'^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$$

b) $At\ t = 0,\ q = Q,\ i = \frac{dq}{dt} = 0:$

$$\Rightarrow q = A\cos\phi = Q \text{ and } \frac{dq}{dt} = -\frac{R}{2L}A\cos\phi - \omega' A\sin\phi = 0$$

$$\Rightarrow A = \frac{Q}{\cos\phi} \text{ and } -\frac{QR}{2L} - \omega' Q\tan\phi = -\frac{R}{2L\omega'}$$

$$= -\frac{R}{2L\sqrt{1/LC - R^2/4L^2}}.$$

30.39: Subbing $x \to q, m \to L, b \to R, k \to \dfrac{1}{C}$, we find:

a) Eq. (13.41): $\dfrac{d^2x}{dt^2} + \dfrac{b}{m}\dfrac{dx}{dt} + \dfrac{kx}{m} = 0 \to \text{Eq.}(30.27): \dfrac{d^2q}{dt^2} + \dfrac{R}{L}\dfrac{dq}{dt} + \dfrac{q}{LC} = 0.$

b) Eq. (13.43): $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}} \to \text{Eq.}(30.28): \omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}.$

c) Eq. (13.42): $x = Ae^{-(b/2m)t}\cos(\omega' t + \phi) \to \text{Eq.}(30.28): q = Ae^{-(R/2L)t}\cos(\omega' t + \phi).$

30.40: $\left[\dfrac{L}{C}\right] = \dfrac{\text{H}}{\text{F}} = \dfrac{\Omega \cdot \text{s}}{\text{C/V}} = \dfrac{\Omega \cdot \text{V}}{\text{A}} = \Omega^2 \Rightarrow \left[\sqrt{\dfrac{L}{C}}\right] = \Omega.$

30.41: $\omega'^2 = \dfrac{1}{LC} - \dfrac{R^2}{4L^2} = \dfrac{1}{6LC} \Rightarrow R^2 = 4L^2\left(\dfrac{1}{LC} - \dfrac{1}{6LC}\right) \Rightarrow R = 2L\sqrt{\dfrac{1}{LC} - \dfrac{1}{6LC}}$

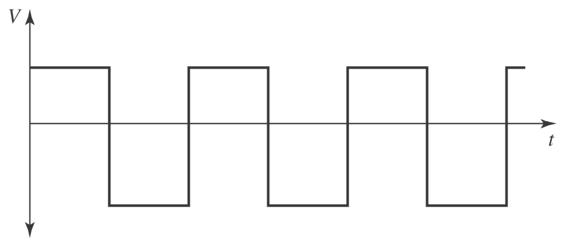
$$\Rightarrow R = 2(0.285\ \text{H})\sqrt{\frac{1}{(0.285\ \text{H})\,(4.60 \times 10^{-4}\ \text{F})} - \frac{1}{6(0.285\ \text{H})\,(4.60 \times 10^{-4}\ \text{F})}} = 45.4\ \ \Omega.$$

30.42: a) When $R = 0$, $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.450\ \text{H})\,(2.50 \times 10^{-5}\ \text{F})}} = 298\ \text{rad/s}.$

b) We want $\dfrac{\omega}{\omega_0} = 0.95 \Rightarrow \dfrac{(1/LC - R^2/4L^2)}{1/LC} = 1 - \dfrac{R^2C}{4L} = (0.95)^2$

$$\Rightarrow R = \sqrt{\frac{4L}{C}(1 - (0.95)^2)} = \sqrt{\frac{4(0.450\ \text{H})\ \ (0.0975)}{(2.50 \times 10^{-5}\ \text{F})}} = 83.8\ \Omega.$$

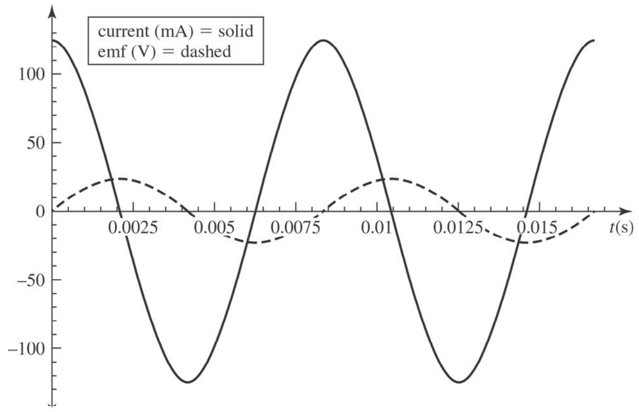
30.43: a)



b) Since the voltage is determined by the derivative of the current, the V versus t graph is indeed proportional to the derivative of the current graph.

30.44: a) $\varepsilon = -L\dfrac{di}{dt} = -L\dfrac{d}{dt}((0.124\ \text{A})\cos[(240\,\pi/\text{s})t]$

$\Rightarrow \varepsilon = +(0.250\ \text{H})\ (0.124\ \text{A})\ (240\,\pi)\sin((240\pi/\text{s})t) = +(23.4\ \text{V})\ \sin\,((240\pi/\text{s})t).$



b) $\varepsilon_{\max} = 23.4\ \text{V}; i = 0,$ since the emf and current are $90°$ out of phase.

c) $i_{\max} = 0.124\ \text{A}; \varepsilon = 0,$ since the emf and current are $90°$ out of phase.

30.45: a) $\Phi_B = \displaystyle\int_a^b B(hdr) = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right)(hdr) = \frac{\mu_0 Nih}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 Nih}{2\pi}\ln(b/a).$

b) $L = \dfrac{N\Phi_B}{i} = \dfrac{\mu_0 N^2 h}{2\pi}\ln(b/a).$

c) $\ln(b/a) = \ln(1-(b-a)/a) \approx \dfrac{b-a}{a} + \dfrac{(b-a)^2}{2a^2} + \cdots \Rightarrow \text{L} \approx \dfrac{\mu_0 N^2 h}{2\pi}\left(\dfrac{b-a}{a}\right).$

30.46: a) $M = \dfrac{N_2}{I}\Phi_{B_2} = \dfrac{N_2}{I}\dfrac{A_2}{A_1}\Phi_{B_1} = \dfrac{N_2 A_2}{IA_1}\dfrac{\mu_0 N_1 IA_1}{l_1} = \dfrac{\mu_0 N_1 N_2 A_2}{l_1} = \dfrac{\mu_0 N_1 N_2 \pi r_2^{\,2}}{l_1}.$

b) $|\varepsilon_2| = N_2\dfrac{d\Phi_{B_2}}{dt} = N_2\dfrac{\mu_0 N_1 A_2}{l_1}\dfrac{di_1}{dt} = \dfrac{\mu_0 N_1 N_2 \pi r_2^{\,2}}{l_1}\dfrac{di_1}{dt}.$

c) $|\varepsilon_1| = M_{12}\dfrac{di_2}{dt} = M\dfrac{di_2}{dt} = \dfrac{\mu_0 N_1 N_2 \pi r_2^{\,2}}{l_1}\dfrac{di_2}{dt}.$

30.47: a) $\varepsilon = -L\dfrac{di}{dt} \Rightarrow L = \varepsilon/(di/dt) = (30.0\ \text{V})/(4.00\ \text{A}/s) = 7.5\ \text{H}.$

b) $\varepsilon = \dfrac{d\Phi}{dt} \Rightarrow \Phi_f - \Phi_i = \varepsilon\Delta t \Rightarrow \Phi_f = (30.0\ \text{V})(12.0\ \text{s}) = 360\ \text{Wb}.$

c) $P_L = Li\dfrac{di}{dt} = (7.50\ \text{H})(48.0\ \text{A})(4.00\ \text{A/s}) = 1440\ \text{W}.$

$P_\text{R} = i^2R = (48.0\ \text{A})^2(60.0\ \Omega) = 138240\ \text{W} \Rightarrow \dfrac{P_L}{P_R} = 0.0104.$

30.48: a) $\varepsilon = L\dfrac{di}{dt} = (3.50\times10^{-3}\text{H})\dfrac{d}{dt}((0.680\ \text{A})\cos(\pi t/0.0250\ \text{s}))$

$$\Rightarrow \varepsilon_\text{max} = (3.50\times10^{-3}\ \text{H})(0.680\ \text{A})\frac{\pi}{0.0250\ \text{s}} = 0.299\ \text{V}.$$

b) $\Phi_{B\text{max}} = \dfrac{Li_\text{max}}{N} = \dfrac{(3.50\times10^{-3}\text{H})(0.680\ \text{A})}{400} = 5.95\times10^{-6}\,\text{Wb}.$

c) $\varepsilon(t) = -L\dfrac{di}{dt} = -(3.50\times10^{-3}\text{H})(0.680\ \text{A})(\pi/0.0250\ \text{s})\sin(\pi t/0.0250\ \text{s}).$

$\Rightarrow \varepsilon(t) = -(0.299\ \text{V})\sin((125.6\ \text{s}^{-1})t)$
$\Rightarrow \varepsilon(0.0180\ \text{s}) = -(0.299\ \text{V})\sin((125.6\ \text{s}^{-1})(0.0180\ \text{s}))$
$\Rightarrow \varepsilon(t) = 0.230\ \text{V}$

30.49: a) Series: $L_1\dfrac{di_1}{dt} + L_2\dfrac{di_2}{dt} = L_\text{eq}\dfrac{di}{dt},$

but $i_1 = i_2 = i$ for series components so $\dfrac{di_1}{dt} = \dfrac{di_2}{dt} = \dfrac{di}{dt}$, thus $L_1 + L_2 = L_\text{eq}$

b) Parallel: Now $L_1\dfrac{di_1}{dt} = L_2\dfrac{di_2}{dt} = L_\text{eq}\dfrac{di}{dt}$, where $i = i_1 + i_2$.

So $\dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$. But $\dfrac{di_1}{dt} = \dfrac{L_\text{eq}}{L_2}\dfrac{di}{dt}$ and $\dfrac{di_2}{dt} = \dfrac{L_\text{eq}}{L_2}\dfrac{di}{dt}$

$$\Rightarrow \frac{di}{dt} = \frac{L_\text{eq}}{L_1}\frac{di}{dt} + \frac{L_\text{eq}}{L_2}\frac{di}{dt} \Rightarrow L_\text{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}.$$

30.50: a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B 2\pi r = \mu_0 i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}.$

b) $d\Phi_B = B dA = \dfrac{\mu_0 i}{2\pi r} l dr.$

c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 i l}{2\pi} \int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 i l}{2\pi} \ln(b/a).$

d) $L = \dfrac{N\Phi_B}{i} = l \dfrac{\mu_0}{2\pi} \ln(b/a).$

e) $U = \dfrac{1}{2} L i^2 = \dfrac{1}{2} l \dfrac{\mu_0}{2\pi} \ln(b/a) i^2 = \dfrac{\mu_0 l i^2}{4\pi} \ln(b/a).$

30.51: a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B 2\pi r = \mu_0 i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}.$

b) $u = \dfrac{B^2}{2\mu_0} \Rightarrow dU = u dV = u(l 2\pi r dr) = \dfrac{1}{2\mu_0}\left(\dfrac{\mu_0 i}{2\pi r}\right)^2 (l 2\pi r dr) = \dfrac{\mu_0 i^2 l}{4\pi r} dr.$

c) $U = \int_a^b dU = \dfrac{\mu_0 i^2 l}{4\pi} \int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 i^2 l}{4\pi} \ln(b/a).$

d) $U = \dfrac{1}{2} L i^2 \Rightarrow L = \dfrac{2U}{i^2} = l \dfrac{\mu_0}{2\pi} \ln(b/a),$ which is the same as in Problem 30.50.

30.52: a) $L_1 = \dfrac{N_1 \Phi_{B_1}}{i_1} = \dfrac{N_1 A}{i_1}\left(\dfrac{\mu_0 N_1 i_1}{2\pi r}\right) = \dfrac{\mu_0 N_1{}^2 A}{2\pi r},$

$L_2 = \dfrac{N_2 \Phi_{B_2}}{i_2} = \dfrac{N_2 A}{i_2}\left(\dfrac{\mu_0 N_2 i_2}{2\pi r}\right) = \dfrac{\mu_0 N_2{}^2 A}{2\pi r}$

b) $M^2 = \left(\dfrac{\mu_0 N_1 N_2 A}{2\pi r}\right)^2 = \dfrac{\mu_0 N_1{}^2 A}{2\pi r} \dfrac{\mu_0 N_2{}^2 A}{2\pi r} = L_1 L_2.$

30.53: $u_B = u_E \Rightarrow \dfrac{\varepsilon_0 E^2}{2} = \dfrac{B^2}{2\mu_0} \Rightarrow B = \sqrt{\varepsilon_0 \mu_0 E^2} = \sqrt{\mu_0 \varepsilon_0} E$

$= \sqrt{\varepsilon_0 \mu_0}(650 \text{ V/m}) = 2.17 \times 10^{-6} \text{T}.$

30.54: a) $R = \dfrac{V}{i_f} = \dfrac{12.0\,V}{6.45\times10^{-3}\text{ A}} = 1860\,\Omega.$

b) $i = i_f(1 - e^{-(R/L)t}) \Rightarrow \dfrac{Rt}{L} = -\ln(1 - i/i_f) \Rightarrow L = \dfrac{-Rt}{\ln(1 - i/i_f)}$

$$\Rightarrow L = \frac{-(1860\,\Omega)(7.25\times10^{-4}\text{s})}{\ln(1-(4.86/6.45))} = 0.963\text{ H}.$$

30.55: a) After one time constant has passed:

$$i = \frac{\varepsilon}{R}(1 - e^{-1}) = \frac{6.00\text{ V}}{8.00\,\Omega}(1 - e^{-1}) = 0.474\text{ A}$$

$$\Rightarrow U = \frac{1}{2}Li^2 = \frac{1}{2}(2.50\text{ H})(0.474\text{ A})^2 = 0.281\text{ J}.$$

Or, using Problem (30.25(c)):

$$U = \int P_L dt = (4.50\,W)\int_0^{3/7}(e^{-(3.20)t} - e^{-(6.40)t})dt.$$

$$= (4.50\text{ W})\left(\frac{(1-e^{-1})}{3.20} - \frac{(1-e^{-2})}{6.40}\right) = 0.281\text{ J}$$

b) $U_{\text{tot}} = (4.50\text{ W})\displaystyle\int_0^{L/R}(1 - e^{-(R/L)t})dt = (4.50\text{ W})\left(\frac{L}{R} + \frac{L}{R}(e^{-1} - 1)\right)$

$$\Rightarrow U_{\text{tot}} = (4.50\text{ W})\frac{2.50\text{ H}}{8.00\,\Omega}e^{-1} = 0.517\text{ J}$$

c) $U_R = (4.50\text{ W})\displaystyle\int_0^{L/R}(1 - 2e^{-(R/L)t} + e^{-2(R/L)t})dt$

$$= (4.50\text{ W})\left(\frac{L}{R} + \frac{2L}{R}(e^{-1} - 1) - \frac{L}{2R}(e^{-2} - 1)\right)$$

$$\Rightarrow U_R = (4.50\text{ W})\frac{2.50\text{ H}}{8.00\,\Omega}(0.168) = 0.236\text{ J}.$$

The energy dissipated over the inductor (part (a)), plus the energy lost over the resistor (part (c)), sums to the total energy output (part (b)).

30.56: a) $U = \frac{1}{2}Li_0^{\;2} = \frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2 = \frac{1}{2}(0.160\text{ H})\left(\frac{60\text{ V}}{240\ \Omega}\right)^2 = 5.00\times10^{-3}\text{ J}.$

b) $$i = \frac{\varepsilon}{R}e^{-(R/L)t} \Rightarrow \frac{di}{dt} = -\frac{R}{L}i \Rightarrow \frac{dU_L}{dt} = iL\frac{di}{dt} = -Ri^2 = \frac{\varepsilon^2}{R}e^{-2(R/L)t}$$

$$\Rightarrow \frac{dU_L}{dt} = -\frac{(60\,V)^2}{240\ \Omega}e^{-2(240/0.160)(4.00\times10^{-4})} = -4.52\text{ W}.$$

c) In the resistor:

$$\frac{dU_R}{dt} = i^2R = \frac{\varepsilon^2}{R}e^{-2(R/L)t} = \frac{(60\text{ V})^2}{240\ \Omega}e^{-2(240/0.160)(4.00\times10^{-4}).} = 4.52\text{ W}.$$

d) $$P_R(t) = i^2R = \frac{\varepsilon^2}{R}e^{-2(R/L)t}$$

$$\Rightarrow U_R = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2(R/L)t} = \frac{\varepsilon^2}{R}\frac{L}{2R} = \frac{(60\text{ V})^2(0.160\text{ H})}{2(240\ \Omega)^2} = 5.00\times10^{-3}\text{J},$$

which is the same as part (a).

30.57: Multiplying Eq. (30.27) by *i*, yields:

$$i^2\text{R} + Li\frac{di}{dt} - \frac{q}{C}i = i^2R + Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = i^2R + \frac{d}{dt}\left(\frac{1}{2}Li^2\right) + \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right)$$
$$= P_R + P_L + P_C = 0.$$

That is, the rate of energy dissipation throughout the circuit must balance over all of the circuit elements.

30.58: a) If $t = \frac{3T}{8} \Rightarrow q = Q\cos(\omega t) = Q\cos\left(\frac{2\pi}{T}\frac{3T}{8}\right) = Q\cos\left(\frac{3\pi}{4}\right) = \frac{Q}{\sqrt{2}}$

$$\Rightarrow i = \frac{1}{\sqrt{LC}}(\sqrt{Q^2 - q^2}) = \frac{1}{\sqrt{LC}}(\sqrt{Q^2 - Q^2/2}) = \sqrt{\frac{Q^2}{2LC}}$$

$$\Rightarrow U_E = \frac{1}{2}Li^2 = \frac{1}{2}L\frac{Q^2}{2LC} = \frac{1}{2}\frac{Q^2}{2C} = \frac{q^2}{2C} = U_B.$$

b) The two energies are next equal when $q = \frac{Q}{\sqrt{2}} \Rightarrow \omega t = \frac{5\pi}{8} \Rightarrow t = \frac{5T}{8}$.

30.59: $V_C = 12.0\text{ V}; U_C = \frac{1}{2}CV_C^2$ so $C = 2U_C/V_C^2 = 2(0.0160\text{ J})/(12.0\text{ V})^2 = 222\ \mu F$

$f = \frac{1}{2\pi\sqrt{LC}}$ so $L = \frac{1}{(2\pi f)^2 C}$

$f = 3500\text{ Hz}$ gives $L = 9.31\mu\text{H}$

30.60: a) $V_{\max} = \frac{Q}{C} = \frac{6.00\times10^{-6}\text{ C}}{2.50\times10^{-4}\text{ F}} = 0.0240\text{ V}.$

b) $\frac{1}{2}Li_{\max}^2 = \frac{Q^2}{2C} \Rightarrow i_{\max} = \frac{Q}{\sqrt{LC}} = \frac{6.00\times10^{-6}}{\sqrt{(0.0600\text{ H})(2.50\times10^{-4}\text{ F})}} = 1.55\times10^{-3}\text{ A}$

c) $U_{\max} = \frac{1}{2}Li_{\max}^2 = \frac{1}{2}(0.0600\text{ H})(1.55\times10^{-3}\text{ A})^2 = 7.21\times10^{-8}\text{ J}.$

d) If $i = \frac{1}{2}i_{\max} \Rightarrow U_L = \frac{1}{4}U_{\max} = 1.80\times10^{-8}\text{ J} \Rightarrow U_C = \frac{3}{4}U_{\max} = \frac{\left(\sqrt{\frac{3}{4}}Q\right)^2}{2C} = \frac{q^2}{2C}$

$$\Rightarrow q = \sqrt{\frac{3}{4}}Q = 5.20\times10^{-6}\text{ C}.$$

$U_{\max} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$ for all times.

30.61: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366\times10^4\,\text{J}/\text{m}^3$.

The total energy stored in the sunspot is $U_B = u_B V$.

The mass of the material in the sunspot is $m = \rho V$.

$$K = U_B \text{ so } \frac{1}{2}mv^2 = U_B; \quad \frac{1}{2}\rho V v^2 = u_B V$$

The volume divides out, and $v = \sqrt{2u_B/\rho} = 2\times10^4\,\text{m/s}$

30.62: (a) The voltage behaves the same as the current. Since $V_R \propto i$, the scope must be across the 150 Ω resistor.

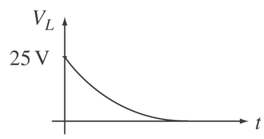
(b) From the graph, as $t\to\infty, V_R \to 25\text{V}$, so there is no voltage drop across the inductor, so its internal resistance must be zero.

$$V_R = V_{\max}(1-e^{-t/r})$$

when $t=\tau, V_R = V_{\max}(1-\frac{1}{e}) \approx 0.63 V_{\max}$. From the graph, when $V = 0.63\ V_{\max} = 16\text{V},\ t \approx 0.5\text{ ms} = \tau$

$$L/R = 0.5\text{ ms} \to L = (0.5\text{ ms})(150\Omega) = 0.075\text{ H}$$

(c) Scope across the inductor:



30.63: a) In the *R-L* circuit the voltage across the resistor starts at zero and increases to the battery voltage. The voltage across the solenoid (inductor) starts at the battery voltage and decreases to zero. In the graph, the voltage drops, so the oscilloscope is across the solenoid.

b) At $t \to \infty$ the current in the circuit approaches its final, constant value. The voltage doesn't go to zero because the solenoid has some resistance R_L. The final voltage across the solenoid is IR_L, where *I* is the final current in the circuit.

c) The emf of the battery is the initial voltage across the inductor, 50 V. Just after the switch is closed, the current is zero and there is no voltage drop across any of the resistance in the circuit.

d) As $t \to \infty$, $\varepsilon - IR - IR_L = 0$

$\varepsilon = 50\text{ V}$ and from the graph $I\ R_L = 15\text{ V}$ (the final voltage across the inductor), so

$I\ R = 35\text{ V}$ and $I = (35\text{ V})/\text{R} = 3.5\text{ A}$

e) $I\ R_L = 15\text{ V}$, so $R_{\text{L}} = (15\text{ V})/(3.5\text{ A}) = 4.3\Omega$

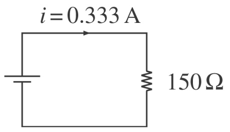
$\varepsilon - V_L - iR = 0$, where V_L includes the voltage across the resistance of the solenoid.

$$V_L = \varepsilon - iR,\ i = \frac{\varepsilon}{R_{\text{tot}}}(1 - e^{-t/\tau}),\ \text{so}\ V_L = \varepsilon[1 - \frac{R}{R_{\text{tot}}}(1 - e^{-t/\tau})]$$

$\varepsilon = 50\text{ V},\ R = 10\ \Omega,\ R_{\text{tot}} = 14.3\ \Omega$, so when $t = \tau, V_l = 27.9\text{ V}$

From the graph, V_L has this value when *t* = *3.0* ms (read approximately from the
ɔh), so $\tau = L/R_{\text{tot}} = 3.0\text{ ms}$. Then $L = (3.0\text{ ms})(14.3\ \Omega) = 43\text{ mH}$.

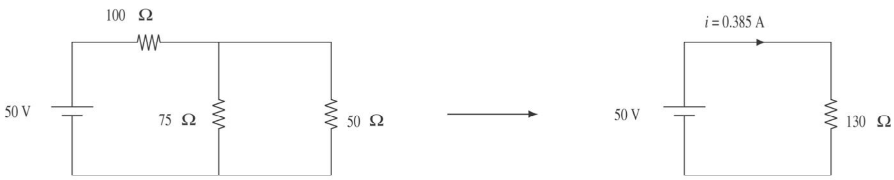
54: (a) Initially the inductor blocks current through it, so the simplified equivalent circuit is



$$i = \frac{\varepsilon}{R} = \frac{50\text{ V}}{150\ \Omega} = 0.333\text{ A}$$

$V_1 = (100\ \Omega)(0.333\text{ A}) = 33.3\text{ V}$
$V_4 = (50\ \Omega)(0.333\text{ A}) = 16.7\text{ V}$
$V_3 = 0$ since no current flows through it.
$V_2 = V_4 = 16.7\text{ V}$ (inductor in parallel with 50 Ω resistor)
$A_1 = A_3 = 0.333\text{ A},\ A_2 = 0$

(b) Long after S is closed, steady state is reached, so the inductor has no potential drop across it. Simplified circuit becomes



$$i = \varepsilon / R = \frac{50\text{ V}}{130\ \Omega} = 0.385\text{ A}$$

$V_1 = (100\ \Omega)(0.385\text{ A}) = 38.5\text{ V}\ \ ; V_2 = 0$

$V_3 = V_4 = 50\text{ V} - 38.5\text{ V} = 11.5\text{ V}$

$$i_1 = 0.385\text{ A},\ i_2 = \frac{11.5\text{ V}}{75\ \Omega} = 0.153\text{ A}$$

$$i_3 = \frac{11.5\text{ V}}{50\ \Omega} = 0.230\text{ A}$$

55: a) Just after the switch is closed the voltage V_5 across the capacitor is zero and there lso no current through the inductor, so $V_3 = 0. V_2 + V_3 = V_4 = V_5$, and since = 0 and $V_3 = 0, V_4$ and V_2 are also zero. $V_4 = 0$ means V_3 reads zero.

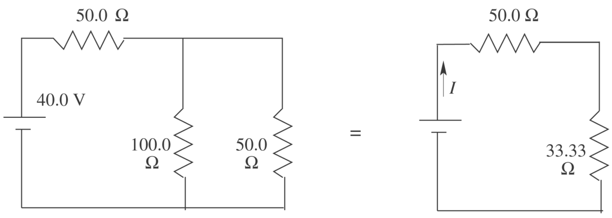
V_1 then must equal 40.0 V, and this means the current read by A_1 is .0 V)/(50.0 Ω) = 0.800 A.

$A_2 + A_3 + A_4 = A_1$, but $A_2 = A_3 = 0$ so $A_4 = A_1 = 0.800$ A.

$A_1 = A_4 = 0.800$ A; all other ammeters read zero.

$V_1 = 40.0$ V and all other voltmeters read zero.

b) After a long time the capacitor is fully charged so $A_4 = 0$. The current through the inductor isn't changing, so $V_2 = 0$. The currents can be calculated from the equivalent circuit that replaces the inductor by a short-circuit.:



$I = (40.0 \text{ V})/(83.33\,\Omega) = 0.480$ A; A_1 reads 0.480 A
$V_1 = I(50.0\,\Omega) = 24.0$ V

The voltage across each parallel branch is $40.0 \text{ V} - 24.0 \text{ V} = 16.0 \text{ V}$

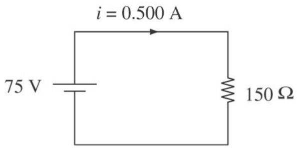
$V_2 = 0, V_3 = V_4 = V_5 = 16.0$ V

$V_3 = 16.0$ V means A_2 reads 0.160 A. $V_4 = 16.0$ V means A_3 reads 0.320 A. A_4 reads zero. Note that $A_2 + A_3 = A_1$.

c) $V_5 = 16.0$ V so $Q = CV = (12.0\ \mu\text{F})(16.0 \text{ V}) = 192\ \mu\text{C}$

d) At $t = 0$ and $t \to \infty, V_2 = 0$. As the current in this branch increases from zero to 0.160 A the voltage V_2 reflects the rate of change of current.

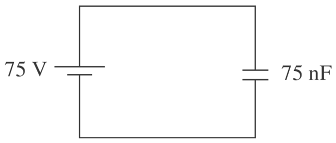
30.66: (a) Initially the capacitor behaves like a short circuit and the inductor like an open circuit. The simplified circuit becomes



$$i = \frac{\varepsilon}{R} = \frac{75\text{ V}}{150\ \Omega} = 0.500\text{ A}$$

$$V_1 = Ri = (50\ \Omega)(0.50\text{ A}) = 25.0\text{ V}$$
$$V_3 = 0,\ V_4 = (100\ \Omega)(0.50\text{ A}) = 50.0\text{ V}$$
$$V_2 = V_4\ \text{(in parallel)} = 50.0\text{ V}$$
$$A_1 = A_3 = 0.500\text{ A},\ A_2 = 0$$

(b) Long after S is closed, capacitor stops all current. Circuit becomes

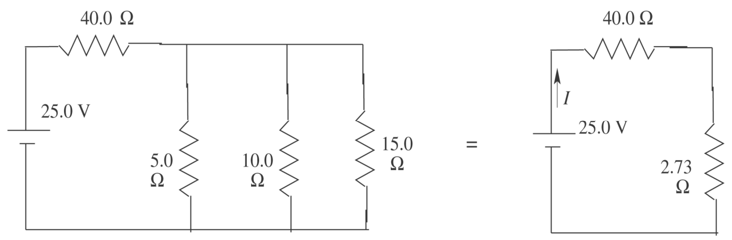


$V_3 = 75.0\text{ V}$ and all other meters read zero.

(c) $q = CV = (75\text{ nF})(75\text{ V}) = 5630\text{ nC},$ long after S is closed.

30.67: a) Just after the switch is closed there is no current through either inductor and they act like breaks in the circuit. The current is the same through the $40.0\ \Omega$ and $15.0\ \Omega$ resistors and is equal to $(25.0\text{ V})/(40.0\ \Omega + 15.0\ \Omega) = 0.455\text{ A}$. $A_1 = A_4 = 0.455\text{ A}$; $A_2 = A_3 = 0$.

b) After a long time the currents are constant, there is no voltage across either inductor, and each inductor can be treated as a short-circuit . The circuit is equivalent to:



$I = (25.0\text{ V})/(42.73\ \Omega) = 0.585\text{ A}$

A_1 reads 0.585 A. The voltage across each parallel branch is $25.0\text{ V} - (0.585\text{ A})(40.0\ \Omega) =$

1.60 V. A_2 reads $(1.60\text{ V})/(5.0\ \Omega) = 0.320\text{ A}$. A_3 reads $(1.60\text{ V})/10.0\ \Omega) = 0.160\text{ A}$. A_4 reads $(1.60\text{ V})/(15.0\ \Omega) = 0.107\text{ A}$.

30.68: (a) $\tau = L/R = \frac{10\text{ mH}}{25\ \Omega} = 0.40\text{ ms}$ since $0.50\text{ s} >> \tau$, steady state has been reached, for all practical purposes.

$$i = \varepsilon/R = 50\text{ V}/25\ \Omega = 2.00\text{ A}$$

The upper limit of the energy that the capacitor can get is the energy stored in the inductor initially.

$$U_C = U_L \rightarrow \frac{Q_{\text{max}}^2}{2C} = \frac{1}{2}Li_0^2 \rightarrow Q_{\text{max}} = i_0\sqrt{LC}$$

$$Q_{\text{max}} = (2.00\text{ A})\sqrt{(10\times10^{-3}\text{H})\ (20\times10^{-6}\text{ F})} = 0.90\times10^{-3}\text{ C}$$

(b) Eventually *all* the energy in the inductor is dissipated as heat in the resistor.

$$U_R = U_L = \frac{1}{2}Li_0^2 = \frac{1}{2}(10\times10^{-3}\text{H})\ (2.00\text{ A})^2$$
$$= 2.0\times10^{-2}\text{ J}$$

30.69: a) At $t = 0$, all the current passes through the resistor R_1, so the voltage v_{ab} is the total voltage of 60.0 V.

b) Point a is at a higher potential than point b. c) $v_{cd} = 60.0\text{ V}$ since there is no current through R_2.

d) Point c is at a higher potential than point b.

e) After a long time, the switch is opened, and the inductor initially maintains the current of $i_{R_2} = \dfrac{\varepsilon}{R_2} = \dfrac{60.0\text{ V}}{25.0\ \Omega} = 2.40\text{ A}$. Therefore the potential between a and b is $v_{ab} = -i\text{R}_1 = -(2.40\text{ A})\,(40.0\ \Omega) = -96.0\text{ V}$.

f) Point b is at a higher potential than point a.

g) $v_{cd} = -i(R_1 + R_2) = -(2.40\text{ A})\,(40\ \Omega + 25\ \Omega) = -156\text{ V}$

h) Point d is at a higher potential than point c.

30.70: a) Switch is closed, then at some later time:

$$\frac{di}{dt} = 50.0\text{ A/s} \Rightarrow v_{cd} = L\frac{di}{dt} = (0.300\text{ H})\,(50.0\text{ A/s}) = 15.0\text{ V}.$$

The top circuit loop: $60.0\text{ V} = i_1 R_1 \Rightarrow i_1 = \dfrac{60.0\text{ V}}{40.0\ \Omega} = 1.50\text{ A}$.

The bottom loop: $60\text{ V} - i_2 R_2 - 15.0\text{ V} = 0 \Rightarrow i_2 = \dfrac{45.0\text{ V}}{25.0\ \Omega} = 1.80\text{ A}$.

b) After a long time: $i_2 = \dfrac{60.0\text{ V}}{25.0\ \Omega} = 2.40\text{ A}$, and immediately when the switch is opened, the inductor maintains this current, so $i_1 = i_2 = 2.40\text{ A}$.

30.71: a) Immediately after S_1 is closed, $i_0 = 0, v_{ac} = 0$, and $v_{cb} = 36.0\text{ V}$, since the inductor stops the current flow.

b) After a long time, $i_0 = \dfrac{\varepsilon}{R_0 + R} = \dfrac{36.0\text{ V}}{50\,\Omega + 150\,\Omega} = 0.180\text{ A}$,

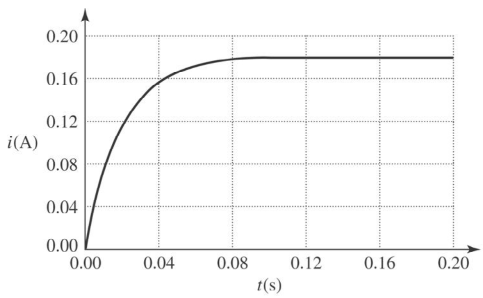
$v_{ac} = i_0 R_0 = (0.18\text{ A})\,(50\,\Omega) = 9.00\text{ V}$, and $v_{cb} = 36.0\text{ V} - 9.00\text{ V} = 27.0\text{ V}$.

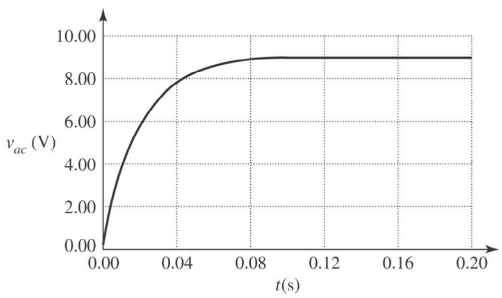
c) $i(t) = \dfrac{\varepsilon}{R_{\text{total}}}(1 - e^{-(R_{\text{total}}/L)t}) \Rightarrow i(t) = (0.180\text{ A})\,(1 - e^{-(50\text{ s}^{-1})t})$,

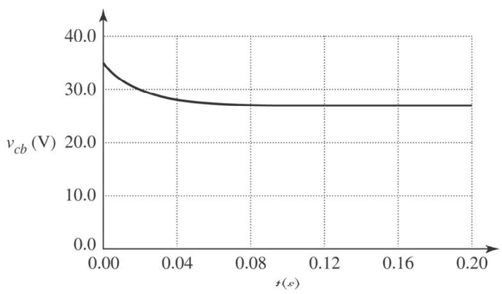
$v_{ac}(t) = i(t)\,R_0 = (9.00\text{ V})\left(1 - e^{-(50\text{ s}^{-1})t}\right)$ and

$\text{v}_{cb}(t) = \varepsilon - i(t)\,R_0 = 36.0\text{ V} - (9.00\text{ V})\left(1 - e^{-(50\,s^{-1})t}\right) = (9.00\text{ V})\left(3 + e^{-(50\,s^{-1})t}\right).$

Below are the graphs of current and voltage found above.







30.72: a) Immediately after S_2 is closed, the inductor maintains the current $i = 0.180$ A through R. The Kirchoff's Rules around the outside of the circuit yield:

$$\varepsilon + \varepsilon_L - iR - i_0R_0 = 36.0 \text{ V} + (0.18)(150) - (0.18)(150) - i_0(50) = 0$$

$$\Rightarrow i_0 = \frac{36 \text{ V}}{50\ \Omega} = 0.720 \text{ A},\ v_{ac} = (0.72 \text{ A})(50 \text{ V}) = 36.0 \text{ V and } v_{cb} = 0.$$

b) After a long time, $v_{ac} = 36.0$ V, and $v_{cb} = 0$. Thus

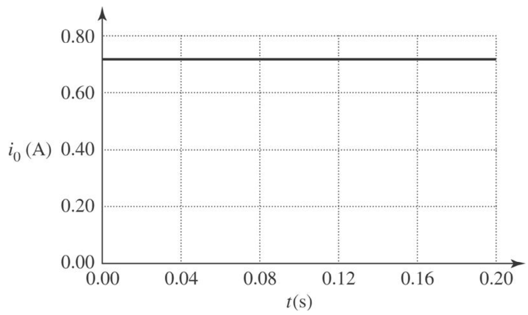
$$i_0 = \frac{\varepsilon}{R_0} = \frac{36.0 \text{ V}}{50\ \Omega} = 0.720 \text{ A},$$

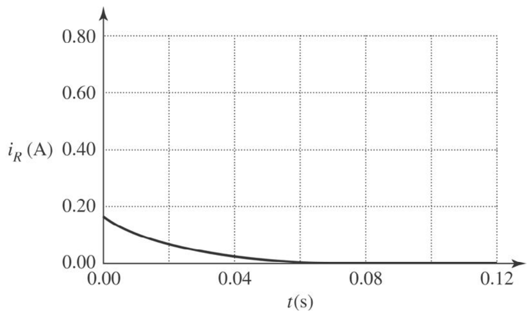
$$i_R = 0, \text{ and } i_{s_2} = 0.720 \text{ A}$$

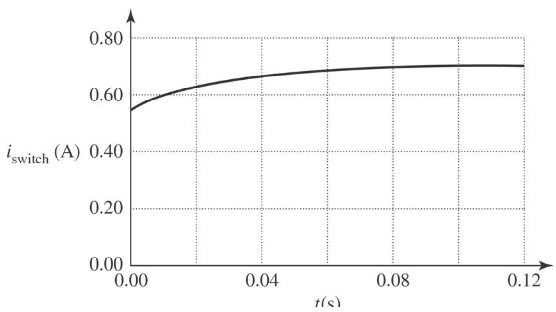
c) $i_0 = 0.720 \text{ A},\ i_R(t) = \dfrac{\varepsilon}{R_{\text{total}}} e^{-(R/L)t} \Rightarrow i_R(t) = (0.180 \text{ A})e^{-(12.5\,\text{s}^{-1})t}$, and

$$i_{s_2}(t) = (0.720 \text{ A}) - (0.180\ A)e^{-(12.5\,\text{s}^{-1})t} = (0.180 \text{ A})\left(4 - e^{-(12.5\,\text{s}^{-1})t}\right)$$

Below are the graphs of currents found above.

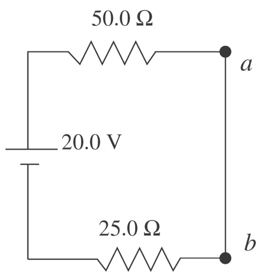






30.73: a) Just after the switch is closed there is no current in the inductors. There is no current in the resistors so there is no voltage drop across either resistor. A reads zero and V reads 20.0 V.

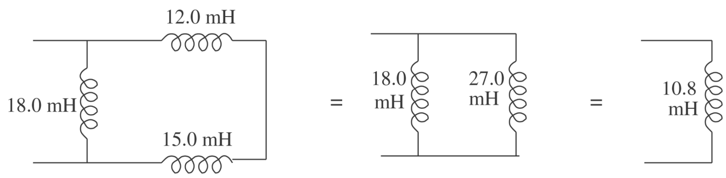
b) After a long time the currents are no longer changing, there is no voltage across the inductors, and the inductors can be replaced by short-circuits. The circuit becomes equivalent to



$I^a = (20.0\text{ V})/(75.0\ \Omega) = 0.267\text{ A}$

The voltage between points a and b is zero, so the voltmeter reads zero.

c) Use the results of problem 30.49 to combine the inductor network into its equivalent:



$R = 75.0\ \Omega$ is the equivalent resistance.

Eq.(30.14) says $i = (\varepsilon/R)(1 - e^{-t/\tau})$, with $\tau = L/R = (10.8\text{ mH})/(75.0\ \Omega) = 0.144\text{ ms}$

$\varepsilon = 20.0\,V$, $R = 75.0\ \Omega$, $t = 0.115\text{ ms}$, so $i = 0.147\text{ A}$

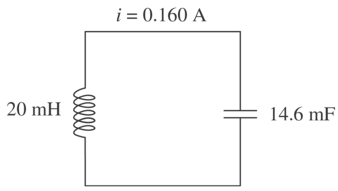
$V_R = iR = (0.147\text{ A})(75.0\ \Omega) = 11.0\text{ V}$

$20.0\text{ V} - V_R - V_L = 0$ so $V_L = 20.0\text{ V} - \text{V}_R = 9.0\text{ V}$

30.74: (a) Steady state: $i = \frac{\varepsilon}{R} = \frac{75.0\text{ V}}{125\,\Omega} = 0.600\text{ A}$

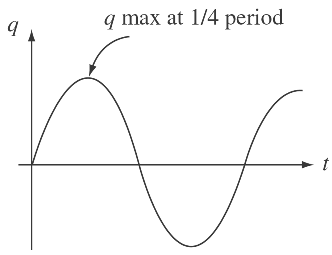
(b) Equivalent circuit:

$$\frac{1}{C_s} = \frac{1}{25\ \mu\text{F}} + \frac{1}{35\mu\text{F}}$$
$$C_s = 14.6\mu\text{F}$$



Energy conservation: $\frac{q^2}{2C} = \frac{1}{2}Li_0^2$

$$q = i_0\sqrt{LC} = (0.600\text{ A})\sqrt{(20\times10^{-3}\text{H})(14.6\times10^{-6}\text{F})}$$
$$= 3.24\times10^{-4}C$$



$$t = \frac{1}{4}T = \frac{1}{4}(2\pi\sqrt{LC}) = \frac{\pi}{2}\sqrt{LC}$$
$$t = \frac{\pi}{2}\sqrt{(20\times10^{-3}\text{H})(14.6\times10^{-6}\text{F})} = 8.49\times10^{-4}\text{s}$$

30.75: a) Using Kirchhoff's Rules: $\varepsilon - i_1 R_1 = 0 \Rightarrow i_1 = \dfrac{\varepsilon}{R_1}$, and

$$\varepsilon - L\frac{di_2}{dt} - i_2 R_2 = 0 \Rightarrow i_2 = \frac{\varepsilon}{R_2}(1 - e^{-(R_2/L)t}).$$

b) After a long time, $i_1 = \dfrac{\varepsilon}{R_1}$ still, and $i_2 = \dfrac{\varepsilon}{R_2}$.

c) After the switch is opened, $i_1 = i_2 = \dfrac{\varepsilon}{R_2} e^{-((R_1 + R_2)/L)t}$, and the current drops off.

d) A 40-W light bulb implies $R = \dfrac{V^2}{P} = \dfrac{(120\ \text{V})^2}{40\ \text{W}} = 360\ \Omega$. If the switch is opened, and the current is to fall from 0.600A to 0.150 A in 0.0800 s,

then: $i_2 = (0.600\ \text{A})e^{-((R_1 + R_2)/L)t} \Rightarrow 0.150\ \text{A} = (0.600\ \text{A})e^{-((360\ \Omega + R_2)/22.0\ \text{H})(0.0800\ \text{s})}$

$$\Rightarrow \frac{22.0\ \text{H}}{0.0800\ s}\ln(4.00) = 360\ \Omega + R_2 \Rightarrow R_2 = 21.0\ \Omega$$

$$\Rightarrow \varepsilon = i_2 R_2 = (0.600\ \text{A})(21.2\ \Omega) = 12.7\ \text{V}.$$

e) Before the switch is opened, $i_0 = \dfrac{\varepsilon}{R_1} = \dfrac{12.7\ \text{V}}{360\ \Omega} = 0.0354\ \text{A}$

30.76: Series: $L_1 \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} + M_{21} \dfrac{di_1}{dt} + M_{12} \dfrac{di_2}{dt} \equiv L_{\text{eq}} \dfrac{di}{dt}$.

But $i = i_1 + i_2 \Rightarrow \dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$ and $M_{12} = M_{21} \equiv M$.

So $(L_1 + L_2 + 2M)\dfrac{di}{dt} = L_{\text{eq}} \dfrac{di}{dt}$,

or $L_{\text{eq}} = L_1 + L_2 + 2M$.

Parallel: We have $L_1 \dfrac{di_1}{dt} + M_{12} \dfrac{di_2}{dt} = L_{\text{eq}} \dfrac{di}{dt}$

and $L_2 \dfrac{di_2}{dt} + M_{21} \dfrac{di_1}{dt} = L_{eq} \dfrac{di}{dt}$,

with $\dfrac{di_1}{dt} + \dfrac{di_2}{dt} = \dfrac{di}{dt}$ and $M_{12} = M_{21} \equiv M$.

To simplify the algebra let $A = \dfrac{di_1}{dt}, B = \dfrac{di_2}{dt}$, and $C = \dfrac{di}{dt}$.

So $L_1 A + MB = L_{\text{eq}} C, L_2 B + MA = L_{\text{eq}} C,\ A + B = C.$

Now solve for A and B in terms of C.

$$\Rightarrow (L_1 - M)A + (M - L_2)B = 0 \text{ using } A = C - B.$$

$$\Rightarrow (L_1 - M)(C - B) + (M - L_2)B = 0$$

$$\Rightarrow (L_1 - M)C - (L_1 - M)B + (M - L_2)B = 0$$

$$\Rightarrow (2M - L_1 - L_2)B = (M - L_1)C \Rightarrow B = \frac{(M - L_1)}{(2M - L_1 - L_2)}C.$$

But $A = C - B = C - \dfrac{(M - L_1)C}{(2M - L_1 - L_2)} = \dfrac{(2M - L_1 - L_2) - M + L_1}{(2M - L_1 - L_2)}C$,

or $A = \dfrac{M - L_2}{2M - L_1 - L_2}C$. Substitute A in B back into original equation.

So $\dfrac{L_1(M - L_2)C}{2M - L_1 - L_2} + \dfrac{M(M - L_1)}{(2M - L_1 - L_2)}C = L_{\text{eq}}C$

$$\Rightarrow \frac{M^2 - L_1 L_2}{2M - L_1 - L_2}C = L_{\text{eq}}C.$$

Finally, $L_{\text{eq}} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

30.77: a) Using Kirchhoff's Rules on the top and bottom branches of the circuit:

$$\varepsilon - i_1 R_1 - L\frac{di_1}{dt} = 0 \Rightarrow i_1 = \frac{\varepsilon}{R_1}(1 - e^{-(R_1/L)t}).$$

$$\varepsilon - i_2 R_2 - \frac{q_2}{C} = 0 \Rightarrow -\frac{di_2}{dt}R_2 - \frac{i_2}{C} = 0 \Rightarrow i_2 = \frac{\varepsilon}{R_2}e^{-(1/R_2C)t})$$

$$\Rightarrow q_2 = \int_0^t i_2\, dt' = -\frac{\varepsilon}{R_2}R_2 C e^{-(1/R_2C)t'}\Big|_0^t = \varepsilon C(1 - e^{-(1/R_2C)t}).$$

b) $i_1(0) = \dfrac{\varepsilon}{R_1}(1 - e^0) = 0, i_2 = \dfrac{\varepsilon}{R_2}e^0 = \dfrac{48.0\text{ V}}{5000\,\Omega} = 9.60\times10^{-3}\text{ A}.$

c) As $t \to \infty : i_1(\infty) = \dfrac{\varepsilon}{R_1}(1 - e^{-\infty}) = \dfrac{\varepsilon}{R_1} = \dfrac{48.0\text{ V}}{25.0\,\Omega} = 1.92\text{ A}, i_2 = \dfrac{\varepsilon}{R_2}e^{-\infty} = 0.$

A good definition of a "long time" is many time constants later.

d) $i_1 = i_2 \Rightarrow \dfrac{\varepsilon}{R_1}(1 - e^{-(R_1/L)t}) = \dfrac{\varepsilon}{R_2}e^{-(1_1/R_2C)t} \Rightarrow (1 - e^{-(R_1/L)t}) = \dfrac{R_1}{R_2}e^{-(1/R_2C)t}.$

Expanding the exponentials like $e^x = 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \cdots$, we find :

$$\frac{R_1}{L}t - \frac{1}{2}\left(\frac{R_1}{L}\right)^2 t^2 + \cdots = \frac{R_1}{R_2}\left(1 - \frac{t}{RC} + \frac{t^2}{2R^2C^2} - \cdots\right)$$

$\Rightarrow t\left(\dfrac{R_1}{L} + \dfrac{R_1}{R_2{}^2 C}\right) + O(t^2) + \cdots = \dfrac{R_1}{R_2}$, if we have assumed that $t << 1$. Therefore:

$$\Rightarrow t \approx \frac{1}{R_2}\left(\frac{1}{(1/L) + (1/R_2{}^2C)}\right) = \left(\frac{LR_2C}{L + R_2{}^2C}\right)$$

$$\Rightarrow t = \left(\frac{(8.0\text{ H})(5000\,\Omega)(2.0\times10^{-5}\text{ F})}{8.0\text{ H} + (5000\,\Omega)^2(2.0\times10^{-5}\text{ F})}\right) = 1.6\times10^{-3}\text{s}.$$

e) At $t = 1.57\times10^{-3}\text{s} : i_1 = \dfrac{\varepsilon}{R_1}(1 - e^{-(R_1/L)t}) = \dfrac{48\text{ V}}{25\,\Omega}(1 - e^{-(25/8)t}) = 9.4\times10^{-3}\text{ A}.$

f) We want to know when the current is half its final value. We note that the current i_2 is very small to begin with, and just gets smaller, so we ignore it and find:

$$i_{1/2} = 0.960\text{ A} = i_1 = \frac{\varepsilon}{R_1}(1 - e^{-(R_1/L)t}) = (1.92\text{ A})(1 - e^{-(R_1/L)t}).$$

$$\Rightarrow e^{-(R_1/L)t} = 0.500 \Rightarrow t = \frac{L}{R_1}\ln(0.5) = \frac{8.0\text{ H}}{25\,\Omega}\ln(0.5) = 0.22\text{ s}$$

30.78: a) Using Kirchoff's Rules on the left and right branches:

Left: $\varepsilon - (i_1 + i_2)\,R - L\dfrac{di_1}{dt} = 0 \Rightarrow R(i_1 + i_2) + L\dfrac{di_1}{dt} = \varepsilon.$

Right: $\varepsilon - (i_1 + i_2)\,R - \dfrac{q_2}{C} = 0 \Rightarrow R(i_1 + i_2) + \dfrac{q_2}{C} = \varepsilon.$

b) Initially, with the switch just closed, $i_1 = 0, i_2 = \dfrac{\varepsilon}{R}$ and $q_2 = 0.$

c) The substitution of the solutions into the circuit equations to show that they satisfy the equations is a somewhat tedious exercise in bookkeeping that is left to the reader.

We will show that the initial conditions are satisfied:

At $t = 0, q_2 = \dfrac{\varepsilon}{\omega R} e^{-\beta t} \sin(\omega t) = \dfrac{\varepsilon}{\omega R}\sin(0) = 0$

$i_1(t) = \dfrac{\varepsilon}{R}(1 - e^{-\beta t}[(2\omega RC)^{-1}\sin(\omega t) + \cos(\omega t)] \Rightarrow i_1(0) = \dfrac{\varepsilon}{R}(1 - [\cos(0)]) = 0.$

d) When does i_2 first equal zero? $\omega = \sqrt{\dfrac{1}{LC} - \dfrac{1}{(2RC)^2}} = 625\text{ rad/s}$

$i_2(t) = 0 = \dfrac{\varepsilon}{R} e^{-bt}[-(2\omega RC)^{-1}\sin(\omega t) + \cos(\omega t)] \Rightarrow -(2\omega RC)^{-1}\tan(\omega t) + 1 = 0$

$\Rightarrow \tan(\omega t) = +2\omega RC = +2(625\text{ rad/s})(400\ \Omega)(2.00 \times 10^{-6}\text{ F}) = +1.00.$

$\Rightarrow \omega t = \arctan(+1.00) = +0.785 \Rightarrow t = \dfrac{0.785}{625\text{ rad/s}} = 1.256 \times 10^{-3}\text{ s}.$

30.79: a) $\Phi_B = BA = B_L A_L + B_{\text{Air}} A_{\text{Air}} = \dfrac{\mu_0 Ni}{W}((D-d)W) + \dfrac{K\mu_0 Ni}{W}(dW) =$
$\mu_0 Ni[(D-d) + Kd]$

$$\Rightarrow L = \frac{N\Phi_B}{i} = \mu_0 N^2[(D-d) + Kd] = L_0 - L_0 \frac{d}{D} + L_f \frac{d}{D} = L_0 + \left(\frac{L_f - L_0}{D}\right)d$$

$$\Rightarrow d = \left(\frac{L - L_0}{L_f - L_0}\right)D, \text{ where } L_0 = \mu_0 N^2 D, \text{ and } L_f = K\mu_0 N^2 D.$$

b) Using $K = \chi_m + 1$ we can find the inductance for any height $L = L_0\left(1 + \chi_m \dfrac{d}{D}\right)$.

Height of Fluid	Inductance of Liquid Oxygen	Inductance of Mercury
$d = D/4$	0.63024 H	0.63000 H
$d = D/2$	0.63048 H	0.62999 H
$d = 3D/4$	0.63072 H	0.62999 H
$d = D$	0.63096 H	0.62998 H

Where are used the values $\chi_m(O_2) = 1.52 \times 10^{-3}$ and $\chi_m(\text{Hg}) = -2.9 \times 10^{-5}$.

d) The volume gauge is much better for the liquid oxygen than the mercury because there is an easily detectable spread of values for the liquid oxygen, but not for the mercury.

31.1: a) $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{45.0\ \text{V}}{\sqrt{2}} = 31.8\ \text{V}.$

b) Since the voltage is sinusoidal, the average is zero.

31.2: a) $I = \sqrt{2} I_{\text{rms}} = \sqrt{2}(2.10\ \text{A}) = 2.97\ \text{A}.$

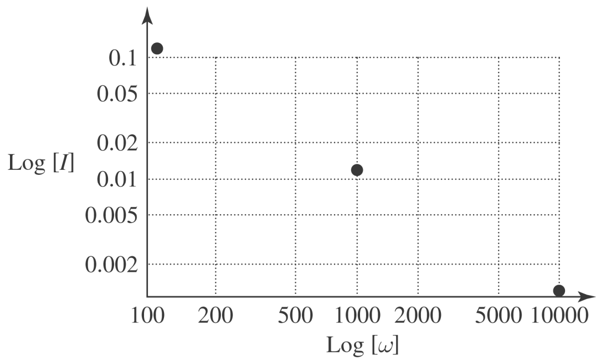
b) $I_{\text{rav}} = \dfrac{2}{\pi} I = \dfrac{2}{\pi}(2.97\ \text{A}) = 1.89\ \text{A}.$

c) The root-mean-square voltage is always greater than the rectified average, because squaring the current before averaging, then square-rooting to get the root-mean-square value will always give a larger value than just averaging.

31.3: a) $V = IX_L = I\omega L \Rightarrow I = \dfrac{V}{\omega L} = \dfrac{60.0\ \text{V}}{(100\ \text{rad/s})\ (5.00\ \text{H})} = 0.120\ \text{A}.$

b) $I = \dfrac{V}{\omega L} = \dfrac{60.0\ \text{V}}{(1000\ \text{rad/s})\ (5.00\ \text{H})} = 0.0120\ \text{A}.$

c) $I = \dfrac{V}{\omega L} = \dfrac{60.0\ \text{V}}{(10{,}000\ \text{rad/s})\ (5.00\ \text{H})} = 0.00120\ \text{A}.$

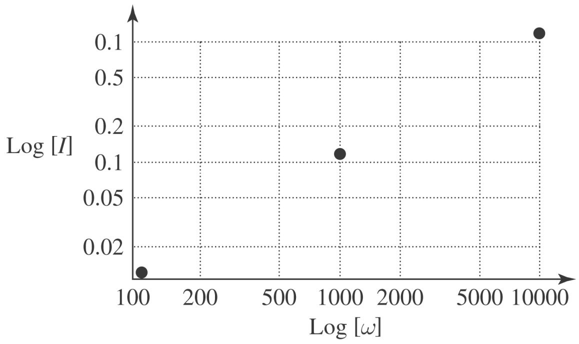


31.4: a) $V = IX_C = \dfrac{I}{\omega C} \Rightarrow I = V\omega C = (60.0\text{ V})\,(100\text{ rad/s})\,(2.20\times10^{-6}\text{ F}) = 0.0132\text{ A}.$

b) $I = V\omega C = (60.0\text{ V})\,(10000\text{ rad/s})\,(2.20\times10^{-6}\text{ F}) = 0.132\text{ A}.$

c) $I = V\omega C = (60.0\text{ V})\,(10{,}000\text{ rad/s})\,(2.20\times10^{-6}\text{ F}) = 1.32\text{ A}.$

d)



31.5: a) $X_L = \omega L = 2\pi fL = 2\pi(80\text{ Hz})\,(3.00\text{ H}) = 1508\,\Omega.$

b) $X_L = \omega L = 2\pi fL \Rightarrow L = \dfrac{X_L}{2\pi f} = \dfrac{120\,\Omega}{2\pi(80\text{ Hz})} = 0.239\text{ H}.$

c) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(80\text{ Hz})\,(4.0\times10^{-6}\text{ F})} = 497\,\Omega.$

d) $X_C = \dfrac{1}{2\pi fC} \Rightarrow C = \dfrac{1}{2\pi fX_C} = \dfrac{1}{2\pi(80\text{ Hz})\,(120\,\Omega)} = 1.66\times10^{-5}\text{ F}.$

31.6: a) $X_L = \omega L = 2\pi fL = 2\pi(60\text{ Hz})(0.450\text{ H}) = 170\Omega.$ If $f = 600\text{ Hz},\ X_L = 1700\Omega.$

b) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60\text{ Hz})\,(2.50\times10^{-6}\text{ F})} = 1061\,\Omega.$ If $f = 600\text{ Hz},\ X_C =$ $106.1\,\Omega.$

c) $X_C = X_L \Rightarrow \dfrac{1}{\omega C} = \omega L \Rightarrow \omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.450\text{ H})\,(2.50\times10^{-6}\text{ Hz})}} = 943\text{ rad/s},$

so $f = 150\text{ Hz}.$

31.7: $V_C = \dfrac{I}{\omega C} \Rightarrow C = \dfrac{I}{\omega V_C} = \dfrac{(0.850\text{ A})}{2\pi(60\text{ Hz})\,(170\text{ V})} = 1.32\times10^{-5}\text{ F}.$

31.8: $V_L = I\omega L \Rightarrow f = \dfrac{V_L}{2\pi IL} = \dfrac{(12.0\text{ V})}{2\pi(2.60\times10^{-3}\text{ A})\,(4.50\times10^{-4}\text{ H})} = 1.63\times10^{6}\text{ Hz.}$

31.9: a) $i = \dfrac{v}{R} = \dfrac{(3.80\text{ V})\cos((720\text{ rad/s})t)}{150\,\Omega} = (0.0253\text{ A})\cos((720\text{ rad/s})t).$

b) $X_L = \omega L = (720\text{ rad/s})\,(0.250\text{ H}) = 180\,\Omega.$

c) $v_L = L\dfrac{di}{dt} = -(\omega L)\,(0.0253\text{ A})\sin((720\text{ rad/s})t) = -(4.55\text{ V})\sin((720\text{ rad/s})t).$

31.10: a) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(120\text{ rad/s})\,(4.80\times10^{-6}\text{ F})} = 1736\,\Omega.$

b) To find the voltage across the resistor we need to know the current, which can be found from the capacitor (remembering that it is out of phase by 90° from the capacitor's voltage).

$$i = \frac{v_C}{X_C} = \frac{v\cos(\omega t)}{X_C} = \frac{(7.60\text{ V})\cos((120\text{ rad/s})t)}{1736\,\Omega} = (4.38\times10^{-3}\text{ A})\cos((120\text{ rad/s})t)$$

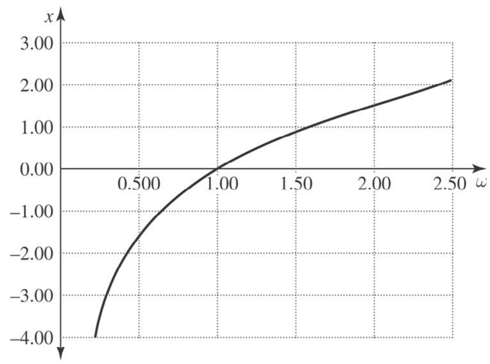
$$\Rightarrow v_R = iR = (4.38\times10^{-3}\text{ A})\,(250\,\Omega)\cos((120\text{ rad/s})t) = (1.10\text{ V})\cos((120\text{ rad/s})t).$$

31.11: a) If $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}} \Rightarrow X = \omega L - \dfrac{1}{\omega C} \Rightarrow X = \dfrac{L}{\sqrt{LC}} - \dfrac{1}{C/\sqrt{LC}} = 0.$

b) When $\omega > \omega_0 \Rightarrow X > 0.$

c) When $\omega > \omega_0 \Rightarrow X < 0.$

d) The graph of X against ω is on the following page.



31.12: a) $Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{(200\ \Omega)^2 + ((250\ \text{rad/s})\ (0.400\ \text{H}))^2} = 224\ \Omega.$

b) $I = \dfrac{V}{Z} = \dfrac{30.0\ \text{V}}{224\ \Omega} = 0.134\ \text{A}$

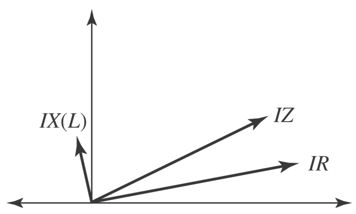
c) $V_R = IR = (0.134\ \text{A})\ (200\ \Omega) = 26.8\ \text{V};$

$V_\text{L} = I\omega L = (0.134\ \text{A})\ (250\ \text{rad/s})\ (0.400\ \text{H})$

$\Rightarrow V_L = 13.4\ \text{V}.$

d) $\phi = \arctan\left(\dfrac{v_L}{v_R}\right) = \arctan\left(\dfrac{13.4\ \text{V}}{26.8\ \text{V}}\right) = 26.6^\circ,$ and the voltage leads the current.

e)



31.13: a) $Z = \sqrt{R^2 + (1/\omega C)^2} = \sqrt{(200\ \Omega)^2 + 1/((250\ \text{rad/s})\ (6.00 \times 10^{-6}\ \text{F}))^2}$

$= 696\ \Omega.$

b) $I = \dfrac{V}{Z} = \dfrac{30.0\ \text{V}}{696\ \Omega} = 0.0431\ \text{A}.$

$V_R = IR = (0.0431\ \text{A})\ (200\ \Omega) = 8.62\ \text{V};$

c) $V_\text{C} = \dfrac{I}{\omega C} = \dfrac{(0.0431\ \text{A})}{(250\ \text{rad/s})\ (6.00 \times 10^{-6}\ \text{F})} = 28.7\ \text{V}.$

d) $\phi = \arctan\left(\dfrac{V_C}{V_R}\right) = \arctan\left(\dfrac{28.7\ \text{V}}{8.62\ \text{V}}\right) = -73.3^\circ,$ and the voltage lags the current.

31.14: a)

$$Z = (\omega L - 1/\omega C) = (250 \text{ rad/s})\,(0.400 \text{ H}) - \frac{1}{(250 \text{ ad/s})\,(6.00 \times 10^{-6} \text{ F})} = 567\ \Omega.$$

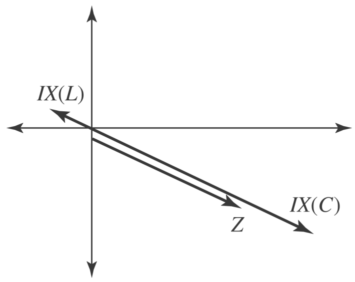
b) $I = \dfrac{V}{Z} = \dfrac{30.0 \text{ V}}{567\ \Omega} = 0.0529 \text{ A}.$

c) $V_C = I\omega L = (0.0529)\,(250 \text{ rad/s})\,(0.400 \text{ H}) = 5.29 \text{ V}$

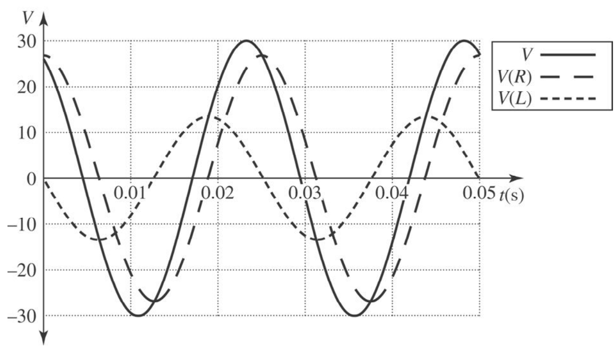
$V_C = \dfrac{I}{\omega C} = \dfrac{(0.0529 \text{ A})}{(250 \text{ rad/s})\,(6.00 \times 10^{-6} \text{ F})} = 35.3 \text{ V}.$

d) $\phi = \arctan\left(\dfrac{V_L - V_C}{V_R}\right) = \arctan(-\infty) = -90.0°,$ and the voltage lags the current.

e)



31.15: a)



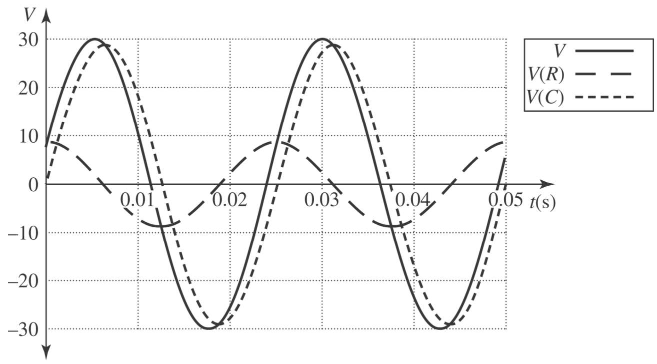
b) The different voltages are:

$v = (30.0 \text{ V})\cos(250t + 26.6°)$, $v_R = (26.8 \text{ V})\cos(250t)$, $v_L = (13.4 \text{ V})\cos(250t + 9$

At $t = 20$ ms: $v = 20.5$ V, $v_R = 7.60$ V, $v_L = 12.85$ V. Note $v_R + v_L = v$.

c) At $t = 40$ ms: $v = -15.2$ V, $v \ = -22.49$ V, $v_L = 7.29$ V. Note $v_R + v_L = v$. Be careful with radians vs. degrees in above expressions!

31.16: a)



b) The different voltage are:

$v = (30.0\text{ V})\cos(250t - 73.3°)$, $v_R = (8.62\text{ V})\cos(250t)$, $v_C = (28.7\text{ V})\cos(250t - 90°)$

At $t = 20\text{ ms}: v = -25.1\text{ V}, v_R = 2.45\text{ V}, v_C = -27.5\text{ V}$. Note $v_R + v_C = v$.

c) At $t = 40\text{ ms}: v = -22.9\text{ V}, v_R = -7.23\text{ V}, v_C = -15.6\text{ V}$. Note $v_R + v_C = v$. Careful with radians vs. degrees!

31.17: a) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$$\Rightarrow Z = \sqrt{(200\,\Omega)^2 + ((250\text{ rad/s})\,(0.0400\text{ H}) - 1/\,((250\text{ rad/s})\,(6.00\times10^{-6}\text{ F})))^2}$$
$$= 601\,\Omega.$$

b) $I = \dfrac{V}{Z} = \dfrac{30\text{ V}}{601\,\Omega} = 0.0499\text{ A}.$

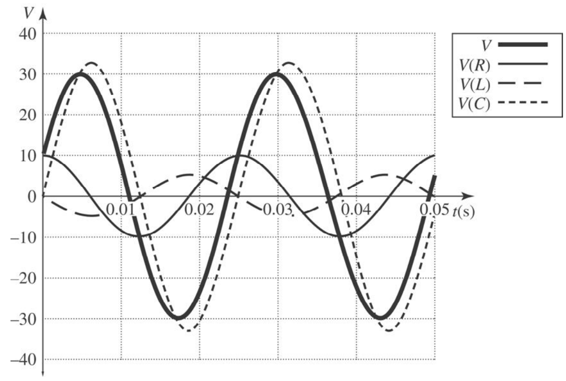
c) $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{100\,\Omega - 667\,\Omega}{200\,\Omega}\right) = -70.6^\circ$, and the voltage lags the current.

d) $V_R = IR = (0.0499\text{ A})\,(200\,\Omega) = 9.98\text{ V};$

$$V_L = I\omega L = (0.0499\text{ A})\,(250\text{ rad/s})(0.400\text{ H}) = 4.99\text{ V};$$
$$V_C = \frac{I}{\omega C} = \frac{(0.0499\text{ A})}{(250\text{ rad/s})\,(6.00\times10^{-6}\text{ F})} = 33.3\text{ V}.$$

e) Because of the charge-storing nature of the capacitor, its voltage will tag the source voltage. That is, the capacitor’s voltage will peak after the source voltage.

31.18: a)



The different voltages plotted above are:
$v = (30 \text{ V}) \cos(250t - 70.6°)$, $v_R = (9.98 \text{ V}) \cos(250t)$,
$v_L = (4.99 \text{ V}) \cos(250t + 90°) v_C = (33.3 \text{ V}) \cos(250t - 90°)$.

b) At $t = 20 \text{ ms}: v = -24.3 \text{ V}, v_R = 2.83 \text{ V}, v_L = 4.79 \text{ V}, v_C = -31.9 \text{ V}.$

c) At $t = 40 \text{ ms}: v = -23.8 \text{ V}, v_R = -8.37 \text{ V}, v_L = 2.71 \text{ V}, v_C = -18.1 \text{ V}.$

In both parts (b) and (c), note that the voltage equals the sum of the other voltages at the given instant. Be careful with degrees vs. radians!

31.19: a) Current largest at the resonance frequency
$f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 113 \text{ Hz}$. At resonance, $X_L = X_C$ and $Z = R$. $I = V/R = 15.0 \text{ mA}$

b) $X_C = 1/\omega C = 500\ \Omega$; $X_L = \omega L = 160\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (160\ \Omega - 500\ \Omega)^2} = 394.5\ \Omega$

$I = V/Z = 7.61 \text{ mA}$

$X_C > X_L$ so source voltage lags the current.

31.20: Using $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ and $\phi = \arctan\left(\dfrac{\omega L - 1/(\omega C)}{R}\right)$, along with the values $R = 200\ \Omega$, $L = 0.400 \text{ H}$, and $C = 6.00 \times 10^{-6} \text{ F}$:

a) $\omega = 1000 \text{ rad/s}: Z = 307\ \Omega, \phi = 49.4°;$
$\omega = 600 \text{ rad/s}: Z = 204\ \Omega, \phi = -10.7°;$
$\omega = 200 \text{ rad/s}: Z = 779\ \Omega, \phi = -75.1°.$

b) The current increases at first, then decreases again since $I = \dfrac{V}{Z}$.

c) The phase angle was calculated in part (a) for all frequencies.

31.21: $V^2 = V_R^2 + (V_L - V_C)^2$

$$V = \sqrt{(30.0\ \text{V})^2 + (50.0\ \text{V} - 90.0\ \text{V})^2} = 50.0\ \text{V}$$

31.22: a) First, let us find the phase angle between the voltage and the current

$$\tan(\phi) = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{2\pi(1.25\times 10^3\ \text{Hz})\,(20.0\times 10^{-3}\ \text{H}) - \frac{1}{2\pi(1.25\times 10^3\ \text{Hz})\,(140\times 10^{-9}\ \text{C})}}{350\ \Omega} \Rightarrow \phi = -65$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{(350\ \Omega)^2 = +(-752\ \Omega)^2} = 830\ \Omega.$$

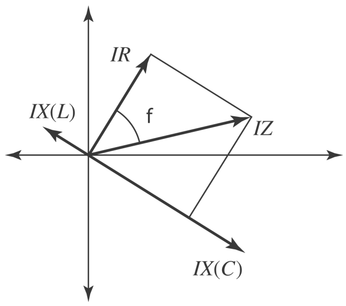
The average power provided by the supply is then

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\phi) = \frac{V_{\text{rms}}^2}{Z}\cos(\phi) = \frac{(120\ \text{V})^2}{830\ \Omega}\cos(-65.1°) = 7.32\ \text{W}$$

b) The average power dissipated by the resistor is $P_R = I_{\text{rms}}^2 R = \left(\frac{120\ \text{V}}{830\ \Omega}\right)^2 (350\ \Omega) = 7.32\ \text{W}$

31.23: a) Using the phasor diagram at right we can see:

$$\cos\phi = \frac{IR}{I\sqrt{R^2 + X_L{}^2 - X_C{}^2}} = \frac{R}{Z}.$$



b) $P_{av} = \frac{1}{2}\frac{V^2}{Z}\cos\phi = \frac{V_{\text{rms}}^2}{Z}\cos\phi$

$$\Rightarrow P_{av} = \frac{V_{\text{rms}}^2}{Z}\frac{R}{Z} = I_{\text{rms}}^2 R.$$

31.24: $$P_{av}=\frac{V_{\text{rms}}^2}{Z}\cos\phi=\frac{V_{\text{rms}}^2}{Z}\frac{R}{Z}$$
$$=\frac{V_{\text{rms}}^2}{Z^2}R=\frac{(80.0\text{ V})^2}{(105\,\Omega)^2}(75.0\,\Omega)=43.5\text{ W}.$$

31.25: a) $$\cos\phi=\frac{R}{Z}=\frac{R}{\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}}$$

$$=\frac{240\,\Omega}{\sqrt{(240\,\Omega)^2+\left(2\pi(400\text{ Hz})\,(0.120\text{ H})-\dfrac{1}{2\pi(400\text{ Hz})\,(7.30\times10^{-6}\text{ F})}\right)^2}}$$
$$=\frac{240\,\Omega}{344\,\Omega}=0.698$$
$$\Rightarrow\phi=\cos^{-1}(0.698)=45.8°.$$

b) From (a), $Z=344\,\Omega.$

c) $V_{\text{rms}}=I_{\text{rms}}Z=(0.450\text{ A})\,(344\,\Omega)=155\text{ V}.$

d) $P_{av}=V_{\text{rms}}I_{\text{rms}}\cos\phi=(155\text{ V})\,(0.450\text{ A})\,(0.698)=48.7\text{ W}.$

e) $P_R=P_{av}=48.7\text{ W}.$

f) Zero.

g) Zero.

For pure capacitors and inductors there is no average energy flow.

31.26: a) The power factor equals:
$$\cos\phi=\frac{R}{Z}=\frac{R}{\sqrt{R^2+(\omega L)^2}}=\frac{(360\,\Omega)}{\sqrt{(360\,\Omega)^2+(((2\pi)60\text{ rad/s})\,(5.20\text{ H}))^2}}=0.181.$$

b)
$$P_{av}=\frac{1}{2}\frac{V^2}{Z}\cos\phi=\frac{1}{2}\frac{(240\text{ V})^2}{\sqrt{(360\,\Omega)^2+(((2\pi)60\text{ rad/s})\,(5.20\text{ H}))^2}}(0.181)=2.62\text{ W}.$$

31.27: a) At the resonance frequency, $Z = R.$

$V = IZ = IR = (0.500\ \text{A})\ (300\ \Omega) = 150\ \text{V}$

b) $V_R = IR = 150\ \text{V}$

$X_L = \omega L = L(1/\sqrt{LC} = \sqrt{L/C} = 2582\ \Omega; V_L = IX_L = 1290\ \text{V}$

$X_C = 1/(\omega C) = \sqrt{L/C} = 2582\ \Omega;$

$V_C = IX_C = 1290\ \text{V}$

c) $P_{av} = \frac{1}{2} V\ I\ \cos\phi = \frac{1}{2} I^2\ R,$ since $V = IR$ and $\cos\phi = 1$ at resonance.

$P_{av} = \frac{1}{2}(0.500\ \text{A})^2\ (300\ \Omega) = 37.5\ \text{W}$

31.28: a) The amplitude of the current is given by

$$I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Thus, the current will have a maximum amplitude when

$\omega L = \frac{1}{\omega^2 C} \Rightarrow C = \frac{1}{\omega^2 L} = \frac{1}{(50.0\ \text{rad/s})^2\ (9.00\ \text{H})} = 44.4\ \mu\text{F}.$

b) With the capacitance calculated above we find that $Z = R$, and the amplitude of the current is $I = \frac{V}{R} = \frac{120\ V}{400\ \Omega} = 0.300\ \text{A}.$ Thus, the amplitude of the voltage across the inductor is $V = I(\omega L) = (0.300\ \text{A})\ (50.0\ \text{rad/s})\ (9.00\ \text{H}) = 135\ \text{V}.$

31.29: a) At resonance, the power factor is equal to one, because the impedance of the circuit is exactly equal to the resistance, so $\dfrac{R}{Z} = 1.$

b) Average power: $P_{av} = \dfrac{V^2{}_{\text{rms}}}{R} = \dfrac{1}{2}\dfrac{(150\ \text{V})^2}{150\ \Omega} = 75\ \text{W}\ .$

c) If the capacitor is changed, and then resonance is again attained, the power factor again equals one. The average power still has no dependence on the capacitor, so $P_{av} = 75\ \text{W}$ again.

31.30: a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.350\ \text{H})(1.20 \times 10^{-8}\ \text{F})}} = 15.4 \times 10^3\ \text{rad/s}\ .$

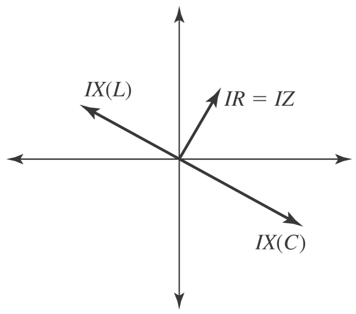
b) $V_C = \dfrac{I}{\omega C} \Rightarrow I = V_C \omega C = (550\ \text{V})(15.4 \times 10^3\ \text{rad/s})(1.20 \times 10^{-8}\ \text{F}) = 0.102\ \text{A}$

$\Rightarrow V_{\max(source)} = IR = (0.102\ \text{A})(400\ \Omega) = 40.8\ \text{V}.$

31.31: a) At resonance:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.400\ \text{H})(6.00\times10^{-6}\ \text{F})}}$$
$$\Rightarrow \omega_0 = 645.5\ \text{rad/s} \Rightarrow 103\ \text{Hz}\,.$$

b)



c) $V_1 = V_{\text{rms}(source)} = \frac{V}{\sqrt{2}} = \frac{30.0\ \text{V}}{\sqrt{2}} = 21.2\ \text{V},\ I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} = \frac{21.2\ \text{V}}{200\ \Omega}$

$= 0.106\ \text{A}$

$V_2 = I_{\text{rms}}\omega_0 L = (0.106\ \text{A})(645.5\ \text{rad/s})(0.400\ \text{H}) = 27.4\ \text{V}.$

$V_3 = \frac{I_{\text{rms}}}{\omega_0 C} = \frac{(0.106\ \text{A})}{(645.5\ \text{rad/s})(6.00\times10^{-6}\ \text{F})} = 27.4\ \text{V} = V_2,$

$V_4 = 0$, since the capacitor and inductor's voltages cancel each other.

$V_5 = V_{\text{rms}(source)} = \frac{V}{\sqrt{2}} = \frac{30\ \text{V}}{\sqrt{2}} 21.2\ \text{V}\,.$

d) If the resistance is changed, that has no affect upon the resonance frequency:
$\omega_0 = 645.5\ \text{rad/s} \Rightarrow 103\ \text{Hz}$

e) $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} = \frac{21.2\ \text{V}}{100\ \Omega} = 0.212\ \text{A}\,.$

31.32: a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.280\ \text{H})(4.00\times10^{-6}\ \text{F})}} = 945\ \text{rad/s}\,.$

b) I = 1.20 A at resonance, so: $R = Z = \frac{V}{I} = \frac{120\ \text{V}}{1.70\ \text{A}} = 70.6\ \Omega$

c) At resonance:
$V_{\text{peak}}(R) = 120\ \text{V},\ V_{\text{peak}}(L) = V_{\text{peak}}(C) = I\omega L = (1.70\ \text{A})(945\ \text{rad/s})(0.280\ \text{H})$
$= 450\ \text{V}.$

31.33: a) $\dfrac{N_1}{N_2} = \dfrac{120}{12} = 10$.

b) $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{12.0\ \text{V}}{5.00\ \Omega} = 2.40\ \text{A}$

c) $P_{av} = I_{\text{rms}} V_{\text{rms}} = (2.40\ \text{A})(12.0\ \text{V}) = 28.8\ \text{W}$.

d) $R = \dfrac{V^2_{\text{rms}}}{P} = \dfrac{(120\ \text{V})^2}{28.8\ \text{W}} = 500\ \Omega$, and note that this is the same as

$$(5.00\ \Omega)\left(\frac{N_1}{N_2}\right)^2 = (5.00\ \Omega)\left(\frac{120}{12.0}\right)^2 = 500\ \Omega.$$

31.34: a) $\dfrac{N_2}{N_1} = \dfrac{13000}{120} = 108.$

b) $P = I_2 V_2 = (0.00850\ \text{A})(13000\ \text{V}) = 110.5\ \text{W}$.

c) $I_1 = I_2 \dfrac{N_2}{N_1} = (0.00850\ \text{A})(108) = 0.918\ \text{A}$.

31.35: a) $R_1 = R_2 \left(\dfrac{N_1}{N_2}\right)^2 \Rightarrow \dfrac{N_1}{N_2} = \sqrt{\dfrac{R_1}{R_2}} = \sqrt{\dfrac{12.8\times 10^3\ \Omega}{8.00\ \Omega}} = 40.$

b) $V_2 = V_1\left(\dfrac{N_2}{N_1}\right) = (60.0\ \text{V})\dfrac{1}{40} = 1.50\ \text{V}$

31.36: a) $Z_{\text{tweeter}} = \sqrt{R^2 + (1/\omega C)^2}$

b) $Z_{\text{woofer}} = \sqrt{R^2 + (\omega L)^2}$

c) If $Z_{\text{tweeter}} = Z_{\text{woofer}}$, then the current splits evenly through each branch.

d) At the crossover point, where currents are equal:

$$R^2 + \left(1/\omega C^2\right) = R^2 + (\omega L)^2 \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

31.37: $\phi = \arctan\left(\dfrac{\omega L}{R}\right) \Rightarrow L = \dfrac{R}{\omega}\tan\phi = \dfrac{R}{2\pi f}\tan\phi$

$$= \left(\frac{48.0\ \Omega}{2\pi(80\ \text{Hz})}\right)\tan(52.3°) = 0.124\ \text{H}.$$

31.38: a) If $\omega = 200\ \text{rad/s} : Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$$\Rightarrow Z = \sqrt{(200\ \Omega)^2 + ((200\ \text{rad/s})(0.400\ \text{H}) - 1/((200\ \text{rad/s})(6.00\times10^{-6}\ \text{F})))^2} = 779\ \Omega.$$

$$\Rightarrow I = \frac{V}{Z} = \frac{30\ \text{V}}{779\ \Omega} = 0.0385\ \text{A} \Rightarrow I_{\text{rms}} = \frac{1}{\sqrt{2}} = 0.0272\ \text{A}.$$

So, $V_1 = I_{\text{rms}}R = (0.0272\ \text{A})(200\ \Omega) = 5.44\ \text{V}$,

$V_2 = I_{\text{rms}}X_L = I_{\text{rms}}\omega L = (0.0272\ \text{A})(200\ \text{rad/s})(0.400\ \text{H}) = 2.18\ \text{V}$,

$V_3 = I_{\text{rms}}X_C = \dfrac{I_{\text{rms}}}{\omega C} = \dfrac{(0.0272\ \text{A})}{(200\ \text{rad/s})(6.00\times10^{-6}\ \text{F})} = 22.7\ \text{V}$,

$V_4 = V_3 - V_2 = 22.7\ \text{V} - 2.18\ \text{V} = 20.5\ \text{V}$, and $V_5 = \varepsilon_{\text{rms}} = \frac{30.0}{\sqrt{2}}\ \text{V} = 21.2\ \text{V}$.

b) If $\omega = 1000\ \text{rad/s}$, using the same steps as above in part (a): $Z = 307\ \Omega, V_1 = 13.8\ \text{V}, V_2 = 27.6\ \text{V}, V_3 = 11.5\ \text{V}, V_4 = 16.1\ \text{V}, V_5 = 21.2\ \text{V}$.

31.39: a) $I_{\text{rav}} = 0$ when $\omega t = (n + 1/2)\pi \Rightarrow t_1 = \dfrac{\pi}{2\omega}, t_2 = \dfrac{3\pi}{2\omega} \Rightarrow t_2 - t_1 = \dfrac{\pi}{\omega}$.

b) $\displaystyle\int_{t_1}^{t_2} i\,dt = \int_{t_1}^{t_2} I\cos(\omega t)dt = \frac{I}{\omega}\sin(\omega t)\Big|_{t_1}^{t_2} = \frac{I}{\omega}[\sin(3\pi/2) - \sin(\pi/2)] = -\frac{2I}{\omega} = \frac{2I}{\omega}$, since it is rectified.

c) So, $I_{\text{rav}}(t_2 - t_1) = \dfrac{2I}{\omega} \Rightarrow I_{\text{rav}} = \dfrac{\omega}{\pi}\dfrac{2I}{\omega} = \dfrac{2I}{\pi}$.

31.40: a) $X_L = \omega L \Rightarrow L = \dfrac{XL}{\omega} = \dfrac{250\ \Omega}{2\pi(120\ \text{Hz})} = 0.332\ \Omega$

b) $Z = \sqrt{R^2 + X_L{}^2} = \sqrt{(400\ \Omega)^2 + (250\ \Omega)^2} = 472\ \Omega, \cos\phi = \dfrac{R}{Z}$.

$$P_{av} = \frac{V^2{}_{\text{rms}}}{Z}\frac{R}{Z} \Rightarrow V_{\text{rms}} = Z\sqrt{\frac{P_{av}}{R}} = (472\ \Omega)\sqrt{\frac{800\ \text{W}}{400\ \Omega}} = 668\ \text{V}.$$

31.41: a) If the original voltage was lagging the circuit current, the addition of an inductor will help it "catch up," since a pure LR circuit would have the voltage leading. This will increase the power factor, because it is largest when the current and voltage are in phase.

b) Since the voltage is lagging, the impedance is dominated by a capacitive element so we need an inductor such that $X_L = X_0$, where X_0 is the original capacitively dominated reactance (this could include inductors, but the capacitors "win").

$$R = 0.720\,Z = 0.720(60.0\ \Omega) = 43.2\ \Omega$$

$$\Rightarrow Z = \sqrt{R^2 + X_C^2} \Rightarrow X_0 = \sqrt{Z^2 - R^2} = \sqrt{(60\ \Omega)^2 - (43.2\ \Omega)^2} = 41.6\ \Omega.$$

$$X_L = X_C = 41.6\ \Omega = \omega L \Rightarrow L = \frac{X_C}{\omega} = \frac{41.6\ \Omega}{2\pi(50\ \text{Hz})} = 0.132\ \text{H}$$

31.42: $Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{240\ \text{V}}{3.00\ \text{A}} = 80.0\ \Omega = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (50.0\ \Omega)^2}$. Thus,

$R = \sqrt{(80.0\ \Omega)^2 - (50.0\ \Omega)^2} = 62.4\ \Omega.$. The average power supplied to this circuit is equal to the power dissipated by the resistor, which is

$$P = I^2_{\text{rms}} R = (3.00\ \text{A})^2\ (62.4\ \Omega) = 562\ \text{W}$$

31.43: a) $\omega_0 = 1/\sqrt{LC} = 3162\ \text{rad/s};\ \omega = 2\omega_0 = 6324\ \text{rad/s}$

$X_L = \omega L = 31.62\ \Omega$; $\quad X_C = 1/(\omega C) = 7.906\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = X_L - X_C = 23.71\ \Omega$

$I = V/Z = (5.00\times 10^{-3}\,\text{V})/(23.71\ \Omega) = 2.108\times 10^{-4}\ \text{A}$

$V_C = IX_C = 1.667\times 10^{-3}\ \text{V}$; this is the maximum voltage across the capacitor.

$Q = CV_C = (20.0\times 10^{-6}\text{F})(1.667\times 10^{-3}\ \text{V}) = 33.34\ \text{nC}$

b) In part (a) we found $I = 0.211$ mA

c) $X_L > X_C$ and $R = 0$ gives that the source and inductor voltages are in phase; the voltage across the capacitor lags the source and inductor voltages by 180°.

31.44: a) $X_{L_2} = \omega_2 L = 2\omega_1 L = 2\left(\frac{1}{\omega_1 C}\right) = 2\left(\frac{2}{\omega_2 C}\right) = 4\text{X}_{C_2} \Rightarrow \frac{X_{L_2}}{X_{C_2}} = 4$, and so the inductor's reactance is greater than that of the capacitor.

b) $X_{L_3} = \omega_3 L = \frac{\omega_1 L}{3} = \left(\frac{1}{3\omega_1 C}\right) = \left(\frac{1}{9\omega_3 C}\right) = \frac{1}{9} X_{C_3} \Rightarrow \frac{X_{L_2}}{X_{C_2}} = \frac{1}{9}$, and so the capacitor's reactance is greater than that of the inductor.

c) Since $X_L = X_C$ at ω_1, that is the resonance frequency.

31.45: $V_{\text{out}} = \sqrt{V_R^2 + V_L^2} = I\sqrt{R^2 + (\omega L)^2} = \dfrac{V_s}{Z}\sqrt{R^2 + (\omega L)^2}$

$$\Rightarrow \frac{V_{\text{out}}}{V_s} = \frac{\sqrt{R^2 + (\omega L)^2}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$$

It ω is small: $\dfrac{V_{\text{out}}}{V_s} \approx \dfrac{R}{\sqrt{R^2 + (1/\omega C)^2}} = \dfrac{\omega R}{\sqrt{\omega^2 R^2 + (1/C)^2}} \approx \omega RC.$

If ω is large: $\dfrac{V_{\text{out}}}{V_s} \approx \dfrac{\sqrt{(\omega L)^2}}{\sqrt{(\omega L)^2}} = 1.$

31.46: $V_{\text{out}} = V_C = \dfrac{I}{\omega C} \Rightarrow \dfrac{V_{\text{out}}}{V_s} = \dfrac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

If ω is large: $\dfrac{V_{\text{out}}}{V_s} = \dfrac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \dfrac{1}{\omega C\sqrt{(\omega L)^2}} = \dfrac{1}{(LC)\omega^2}.$

If ω is small: $\dfrac{V_{\text{out}}}{V_s} \approx \dfrac{1}{\omega C\sqrt{(1/\omega C)^2}} = \dfrac{\omega C}{\omega C} = 1.$

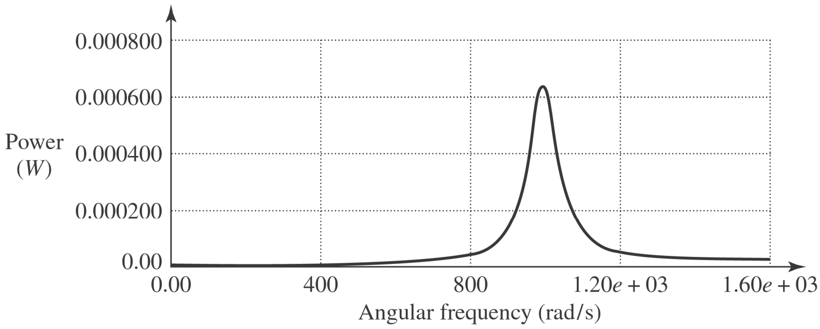
31.47: a) $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

b) $P_{av} = \dfrac{1}{2} I^2 R = \dfrac{1}{2}\left(\dfrac{V}{Z}\right)^2 R = \dfrac{V^2 R/2}{R^2 + (\omega L - 1/\omega C)^2}$.

c) The average power and the current amplitude are both greatest when the denominator is smallest, which occurs for $\omega_0 L = \dfrac{1}{\omega_0 C} \Rightarrow \omega_0 = \dfrac{1}{\sqrt{LC}}$.

d) $P_{av} = \dfrac{(100\ \text{V})^2 (200\ \Omega)/2}{(200\ \Omega)^2 + \left(\omega(2.00\ \text{H}) - 1/\omega(5.00 \times 10^{-6}\ \text{F})\right)^2}$.

$\Rightarrow P_{av} = \dfrac{25\omega^2}{40{,}000\omega^2 + (2\omega^2 - 2{,}000{,}000)^2}$.

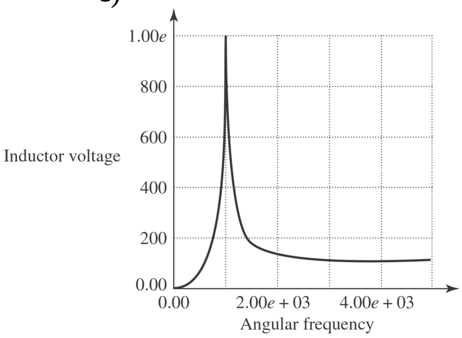


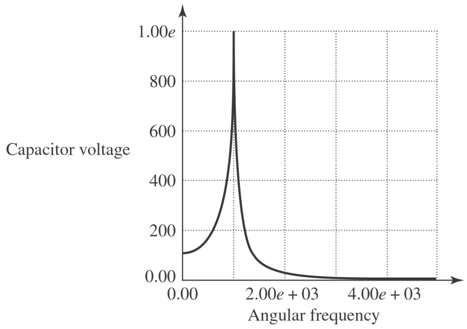
Note that as the angular frequency goes to zero, the power and current are zero, just as they are when the angular frequency goes to infinity. This graph exhibits the same strongly peaked nature as the light red curve in Fig. (31.15).

31.48: a) $V_L = I\omega\omega = \dfrac{V\omega L}{Z} = \dfrac{V\omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

b) $V_C = \dfrac{I}{\omega C} = \dfrac{I}{\omega CZ} = \dfrac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

c)





d) When the angular frequency is zero, the inductor has zero voltage while the capacitor has voltage of 100 V (equal to the total source voltage). At very high frequencies, the capacitor voltage goes to zero, while the inductor's voltage goes to 100 V. At resonance, $\omega_0 = \dfrac{1}{\sqrt{LC}} = 1000 \text{ rad/s}$, the two voltages are equal, and are a maximum, 1000 V.

31.49: a) $U_B = \frac{1}{2}Li^2 \Rightarrow \langle U_B \rangle = \frac{1}{2}L\langle i^2 \rangle = \frac{1}{2}LI_{\mathrm{rms}}{}^2 = \frac{1}{2}L\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4}LI^2.$

$$U_E = \frac{1}{2}Cv^2 \Rightarrow \langle U_E \rangle = \frac{1}{2}C\langle v^2 \rangle = \frac{1}{2}CV_{\mathrm{rms}}{}^2 = \frac{1}{2}C\left(\frac{V}{\sqrt{2}}\right)^2 = \frac{1}{4}CV^2.$$

b) Using Problem (31.47a):

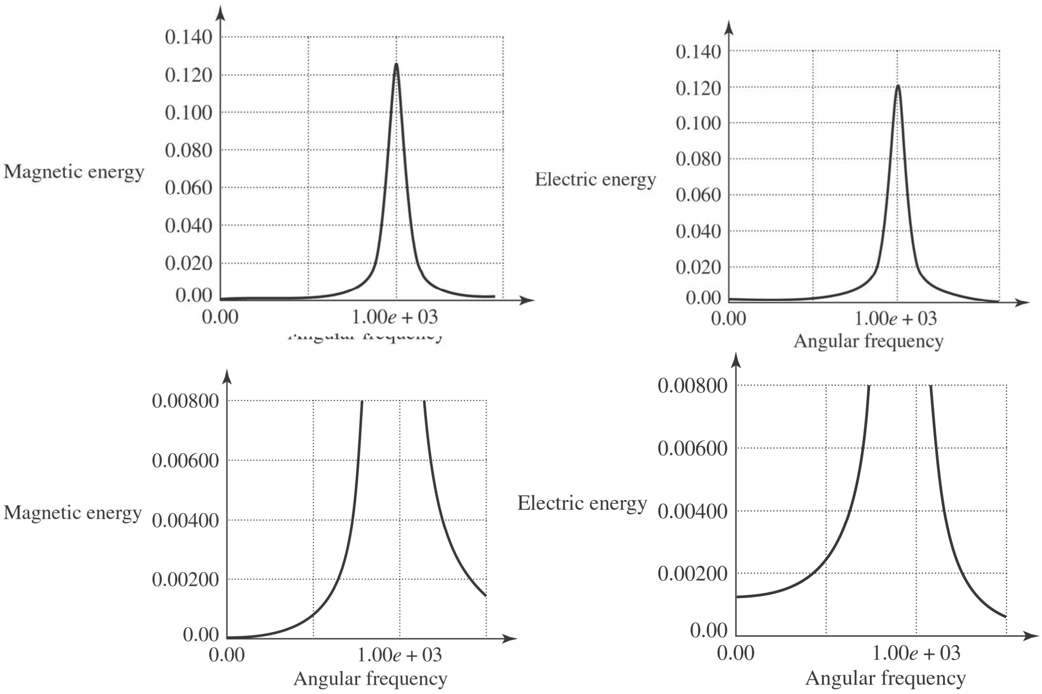
$$\langle U_B \rangle = \frac{1}{4}LI^2 = \frac{1}{4}L\left(\frac{V^2}{\sqrt{R^2+(\omega L - 1/\omega C)^2}}\right)^2 = \frac{LV^2}{4\left(R^2+(\omega L - 1/\omega C)^2\right)}.$$

Using Problem (31.47b):

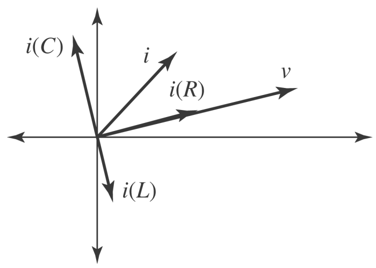
$$\langle U_E \rangle = \frac{1}{4}CV_C{}^2 = \frac{1}{4}C\frac{V^2}{\omega^2C^2\left(R^2+(\omega L - 1/\omega C)^2\right)} = \frac{V^2}{4\omega^2C\left(R^2+(\omega L - 1/\omega C)^2\right)}.$$

c) Below are the graphs of the magnetic and electric energies, the top two showing the general features, while the bottom two show the details close to angular frequency equal to zero.

d) When the angular frequency is zero, the magnetic energy stored in the inductor is zero, while the electric energy in the capacitor is $U_E = CV^2/4$. As the frequency goes to infinity, the energy noted in both inductor and capacitor go to zero. The energies equal each other at the resonant frequency where $\omega_0 = \frac{1}{\sqrt{LC}}$ and $U_B = U_E = \frac{LV^2}{4R^2}$.



31.50: a) Since the voltage drop between any two points must always be equal, the parallel LRC circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v$. Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.



b) $i_R = \frac{v}{R}$ is always in phase with the voltage. $i_L = \frac{v}{\omega L}$ lags the voltage by $90°$, and $i_C = v\omega C$ leads the voltage by $90°$.

c) From the diagram,

$$I^2 = I_R{}^2 + (I_C - I_L)^2 = \left(\frac{V}{R}\right)^2 + \left(V\omega C - \frac{V}{\omega L}\right)^2$$

d) From (c): $I = V\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}\cdot$ But

$$I = \frac{V}{Z} \Rightarrow \frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}.$$

31.51: a) At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 C = \frac{1}{\omega_0 L} \Rightarrow I_C = V\omega_0 C = \frac{V}{\omega_0 L} = I_L$ so $I = I_R$ and I is a minimum.

b) $P_{av} = \frac{V_{rms}^2}{Z}\cos\phi = \frac{V^2}{R}$ at resonance where $R < Z$ so power is a maximum.

c) At $\omega = \omega_0$, I and V are in phase, so the phase angle is zero, which is the same as a series resonance.

31.52: a) $V = \sqrt{2}V_{\text{rms}} = 311\,V$; $I_R = \dfrac{V}{R} = \dfrac{311\text{ V}}{400\ \Omega} = 0.778\text{ A}.$

b) $I_C = V\omega C = (311\text{ V})(360\text{ rad/s})(6.00\times10^{-6}\text{ F}) = 0.672\text{ A}$.

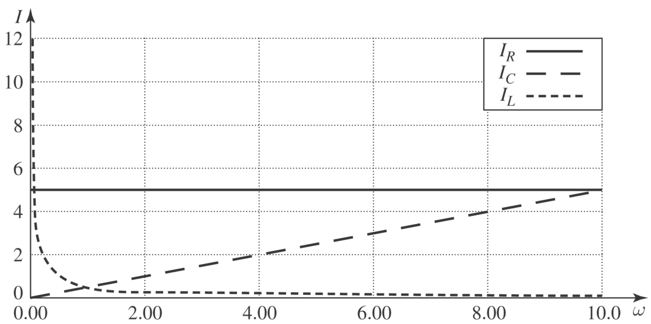
c) $\phi = \arctan\left(\dfrac{I_C}{I_R}\right) = \arctan\left(\dfrac{0.672\text{ A}}{0.778\text{ A}}\right) = 40.8°$, leading the voltage.

d) $I = \sqrt{I_R{}^2 + I_C{}^2} = \sqrt{(0.778\text{ A})^2 + (0.672\text{ A})^2} = 1.03\text{ A}$.

e) Leads since $\phi > 0$.

31.53: a) $I_R = \dfrac{V}{R}$; $I_C = V\omega C$; $I_L = \dfrac{V}{\omega L}$.

b)

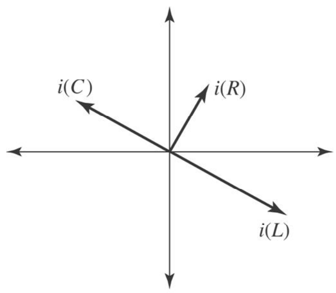


c) $\omega \to 0: I_C \to 0; I_L \to \infty.\ \omega \to \infty: I_C \to \infty; I_L \to 0$.

At low frequencies, the current is not changing much so the inductor's back-emf doesn't "resist." This allows the current to pass fairly freely. However, the current in the capacitor goes to zero because it tends to "fill up" over the slow period, making it less effective at passing charge.

At high frequency, the induced emf in the inductor resists the violent changes and passes little current. The capacitor never gets a chance to fill up so passes charge freely.

d) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(2.0\text{ H})(0.50\times10^{-6}\ f)}} = 1000\text{ rad/sec} \Rightarrow f = 159\text{ Hz}$



e) $I = \sqrt{\left(\dfrac{V}{R}\right)^2 + (V\omega C - \dfrac{v}{\omega L})^2}$

$= \sqrt{\left(\dfrac{100\text{V}}{200\ \Omega}\right)^2 + \left((100\text{ V})(1000\text{ s}^{-1})(0.50\times10^{-6}\text{ F}) - \dfrac{100\text{V}}{(1000\text{s}^{-1})(2.0\text{H})}\right)^2} = 0.50\text{A}$

f) At resonance $I_L = I_C = V\omega C = (100\text{ v})(1000\text{ s}^{-1})(0.50\times10^{-6}\text{ F}) = 0.05\text{ A}$ and $I_R = \dfrac{V}{R} = \dfrac{100\text{ V}}{200\,\Omega} = 0.50\text{ A}$.

31.54: a) Note that as $\omega \to \infty, \omega L \to \infty$ and $\frac{1}{\omega C} \to 0.$ Thus, at high frequencies the current through R_1 is nearly zero and the power dissipated by the circuit is

$$P = \frac{V_{\text{rms}}^2}{R_2} = \frac{(240\text{ V})^2}{40.0\ \Omega} = 1.44\text{ kW}.$$

b) Now we let $\omega \to 0$, and so $\omega L \to 0$ and $\frac{1}{\omega C} \to \infty.$ Thus, at low frequencies the current through R_2 is nearly zero and the power dissipated by the circuit is

$$P = \frac{V_{\text{rms}}^2}{R_1} = \frac{(240\text{ V})^2}{60.0\ \Omega} = 0.960\text{ kW}.$$

31.55: Connect the source, capacitor, resistor, and inductor in series.

31.56: a) $$P_{av} = \frac{V_{\text{rms}}^2}{Z}\cos\phi \Rightarrow Z = \frac{V_{\text{rms}}^2 \cos\phi}{P_{av}} = \frac{(120\text{ V})^2(0.560)}{(220\text{ W})} = 36.7\ \Omega$$

$$\Rightarrow R = Z\cos\phi = (36.7\ \Omega)(0.560) = 20.6\ \Omega.$$

b) $Z = \sqrt{R^2 + X_L{}^2} = X_L = \sqrt{Z^2 - R^2} = \sqrt{(36.7\ \Omega)^2 - (20.6\ \Omega)^2} = 30.4\ \Omega.$ But at $\phi = 0$ this is resonance, so the inductive and capacitive reactances equal each other. So:

$$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50.0\text{ Hz})(30.4\ \Omega)} = 1.05 \times 10^{-4}\ F.$$

c) At resonance, $P = \frac{V^2}{R} = \frac{(120\text{ V})^2}{20.6\ \Omega} = 699\text{ W}.$

31.57: a) $$\tan\phi = \frac{X_L - X_C}{R} \Rightarrow X_L = X_C + R\tan\phi.$$

$$= 350\ \Omega + (180\ \Omega)\tan(-54°) = 102\ \Omega.$$

b) $P_{av} = I_{\text{rms}}^2 R \Rightarrow I_{\text{rms}} = \sqrt{\frac{P_{av}}{R}} = \sqrt{\frac{(140\text{ W})}{(180\ \Omega)}} = 0.882\text{ A}.$

c) $V_{\text{rms}} = I_{\text{rms}} Z = I_{\text{rms}}\sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow V_{\text{rms}} = (0.882\text{ A})\sqrt{(180\ \Omega)^2 + (102\ \Omega - 350\ \Omega)^2} = 270\text{ V}.$$

31.58: a) For $\omega = 800\,\text{rad}/\text{s}$, $Z = \sqrt{R^2 + (\omega\text{L} - 1/\omega\text{C})^2}$

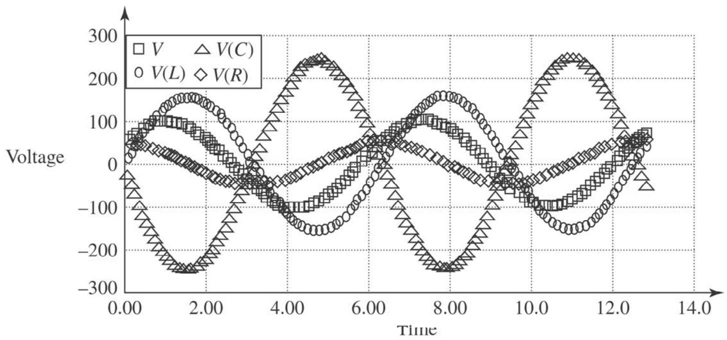
$$\Rightarrow Z = \sqrt{(500\,\Omega)^2 + ((800\text{ rad/s})(2.0\text{ H}) - 1/((800\,\text{rad/s})(5.0\times10^{-7}\text{ F})))^2} = 1030$$

$$\Rightarrow I = \frac{V}{Z} = \frac{100\text{ V}}{1030\,\Omega} = 0.0971\text{ A} \Rightarrow \text{V}_R = IR = (0.0971\text{ A})(500\,\Omega) = 48.6\text{ V}.$$

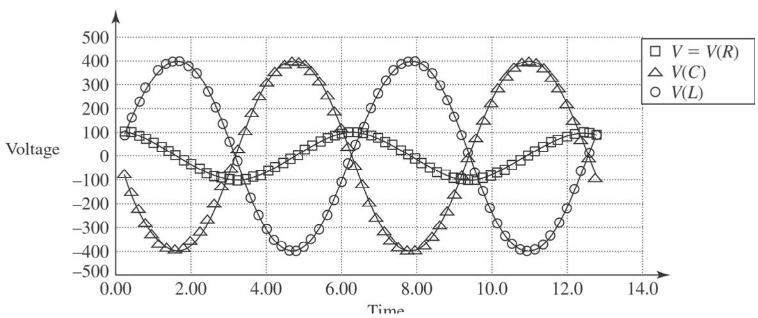
$$\text{V}_C = \frac{1}{\omega C} = \frac{0.0971\text{ A}}{(800\text{ rad}/\text{s})(5.0\times10^{-7}\text{ F})} = 243\text{ V}.$$

$$\text{V}_L = I\omega L = (0.0971\text{ A})(800\text{ rad}/\text{s})(2.00\text{ H}) = 155\text{ V}.$$

Also note $\phi = \arctan\left(\dfrac{\omega L - 1/(\omega C)}{R}\right) = -60.9°.$



b) Repeating exactly the same calculations as above for
$\omega = 1000\text{ rad}/\text{s}: \text{Z} = R = 500\,\Omega; \phi = 0.; I = 0.200\text{ A}; V_\text{R} = V = 100\text{ V}; V_\text{C} = V_L = 400\text{ V}.$



c) Repeating exactly the same calculations as part (a) for
$\omega = 1250\text{ rad}/\text{s}: Z = R = 1030\,\Omega; \phi = +60.9°; I = 0.0971\,A; V_R = 48.6\,V; V_C = 155\,V; V_L =$

31.59: a) $V_C = IX_C \Rightarrow I = \dfrac{V_C}{X_C} = \dfrac{360\text{ V}}{480\,\Omega} = 0.75\text{ A}.$

b) $Z = \dfrac{V}{I} = \dfrac{120\text{ V}}{0.75\text{ A}} = 160\,\Omega.$

c) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow X_L = X_C \pm \sqrt{Z^2 - R^2} = 480\,\Omega \pm \sqrt{(160\,\Omega)^2 - (80\,\Omega)^2}$$

$$\Rightarrow X_L = 619\,\Omega \text{ or } 341\,\Omega$$

d) If $\omega < \omega_0$ then $X_C = \dfrac{1}{\omega C} > X_L = \omega L.$ For us, $X_L = 341\,\Omega$ if $\omega < \omega_0$.

31.60: We want $P_{av}(\omega_1) = \text{maximum}$, $\text{P}_{av}(\omega_2) = 0.01\,P_{av}(\omega_1)$. Maximum power implies

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega_0^2} = \frac{1}{(1.0\times10^{-6}\text{ H})[2\pi(94.1\times10^6\text{ Hz})]^2} = 2.86\times10^{-12}\text{ F}.$$

$$P_{av}(\omega_2) = 0.01P_{av}(\omega_1) \Rightarrow \frac{V^2\,R/2}{R^2 + (\omega L - 1/\omega C)^2} = \frac{1}{100}\left(\frac{V^2}{2R}\right)$$

$$\Rightarrow 100\,R^2 = R^2 + (\omega L - 1/\omega C)^2 \Rightarrow R = \sqrt{\frac{(\omega L - 1/\omega C)^2}{99}} = \frac{(\omega L - 1/\omega C)}{\sqrt{99}}$$

$$\Rightarrow R = \frac{1}{\sqrt{99}}\left(2\pi(94.0\times10^6\text{ Hz})(1.00\times10^{-6}\text{ H}) - \frac{1}{2\pi(94.0\times10^6\text{ Hz})(2.8}\right.$$

$$\Rightarrow R = 0.126\,\Omega.$$

This answer is very sensitive to the capacitance so you may have to carry the first part of the problem out to more significant figures.

31.61: The average current is zero because the current is symmetrical above and below the axis. We must calculate the rms-current:

$$I(t) = \frac{2I_0 t}{\tau} \Rightarrow I^2(t) = \frac{4I_0^2 t^2}{\tau^2} \Rightarrow \int_0^{\tau/2} I^2(t)dt = \frac{4I_0^2}{\tau^2}\left[\frac{t^3}{3}\right]_0^{\tau/2} = \frac{I_0^2\tau}{6}.$$

$$\Rightarrow \langle I^2 \rangle = \left(\frac{I_0^2\tau}{6}\right)\Big/\left(\frac{\tau}{2}\right) = \frac{I_0^2}{3} \Rightarrow I_{rms} = \sqrt{\frac{I_0^2}{3}} = \frac{I_0}{\sqrt{3}}.$$

31.62: a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(1.80\ \text{H})(9.00\times10^{-7}\ \text{F})}} = 786\ \text{rad/s}.$

b) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$\Rightarrow Z = \sqrt{(300\ \Omega)^2 + ((786\ \text{rad/s})(1.80\ \text{H}) - 1/((786\ \text{rad/s})(9.00\times10^{-7}\ \text{F}))^2} = 300\ \Omega.$

$\Rightarrow I_{\text{rms}_0} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{60\ \text{V}}{300\ \Omega} = 0.200\ \text{A}.$

c) We want

$$I = \frac{1}{2} I_{\text{rms}_0} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \Rightarrow R^2 + (\omega L - 1/\omega C)^2 = \frac{4V_{\text{rms}}^2}{I_{\text{rms}_0}^2}$$

$$\Rightarrow \omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} + R^2 - \frac{4V_{\text{rms}}^2}{I_{\text{rms}_0}^2} = 0$$

$$\Rightarrow (\omega^2)^2\ L^2 + \omega^2 \left(R^2 - \frac{2L}{C} - \frac{4V_{\text{rms}}^2}{I_{\text{rms}_0}^2} \right) + \frac{1}{C^2} = 0.$$

Substituting in the values for this problem, the equation becomes: $(\omega^2)^2 (3.24) + \omega^2 (-4.27\times10^6) + 1.23\times10^{12} = 0.$

Solving this quadratic equation in ω^2 we find $\omega^2 = 8.90\times10^5\ \text{rad}^2/\text{s}^2$ or $4.28\times 10^5\ \text{rad}^2/\text{s}^2 \Rightarrow \omega = 943\ \text{rad/s}$ or $654\ \text{rad/s}.$

d) (i) $R = 300\ \Omega,\ I_{\text{rms}_0} = 0.200,\ |\omega_1 - \omega_2| = 289\ \text{rad/sec.}$ ii) $R = 30\ \Omega,\ I_{\text{rms}_0} = 2\text{A},$ $|\omega_1 - \omega_2| = 28\ \text{rad/sec.}$ (iii) $R = 3\ \Omega,\ I_{\text{rms}_0} = 20\text{A},\ |\omega_1 - \omega_2| = 2.88.$

Width gets smaller as R gets smaller; I_{rms_0} gets larger as R gets smaller.

31.63: a) $I_0 = \frac{V}{Z} = \frac{V}{R}$ at resonance since $X_L = X_C$.

b) $\omega = \omega_0 + \Delta\omega$ is small compared to ω_0.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{1}{\omega^2 C^2}(\omega^2 LC - 1)^2.$$

$$\omega_0^2 = \frac{1}{LC} \text{ so } C^2 = \frac{1}{L^2\omega_0^4}. \text{ Thus } \frac{1}{\omega^2 C^2} = \frac{L^2\omega_0^4}{(\omega_0^2 + 2\omega_0\Delta\omega + \Delta\omega^2)}$$

but $\Delta\omega^2$ is very small so

$$\frac{1}{\omega^2 C^2} \approx \frac{L^2\omega_0^4}{(\omega_0^2 + 2\omega_0\Delta\omega)} = \frac{L^2\omega_0^2}{\left(1 + \frac{2\Delta\omega}{\omega_0}\right)} \approx L^2\omega_0^2\left(1 - \frac{2\Delta\omega}{\omega_0}\right).$$

$$\omega^2 LC - 1 = (\omega_0^2 + 2\omega_0\Delta\omega + \Delta\omega^2)\left(\frac{1}{\omega_0^2}\right) - 1 = 1 + 2\frac{\Delta\omega}{\omega_0} + \frac{\Delta\omega^2}{\omega_0^2} - 1 = \frac{2\Delta\omega}{\omega_0} + \frac{\Delta\omega^2}{{\omega_0}^2}.$$

Again, $\Delta\omega^3$ is very small compared to ${\omega_0}^2$, so $\omega^2 LC - 1 \approx \frac{2\Delta\omega}{\omega_0}$.

Putting this together gives

$$\left(\omega L - \frac{1}{\omega C}\right)^2 \cong L^2{\omega_0}^2\left(1 - \frac{2\Delta\omega}{\omega_0}\right)\left(\frac{2\Delta\omega}{\omega_0}\right)^2 = 4L^2\Delta\omega^2 - \frac{8L^2\Delta\omega^3}{\omega_0}.$$

But $\Delta\omega^3$ is *much* smaller than ω_0. Finally

$$\left(\omega L - \frac{1}{\omega_c}\right)^2 \approx 4L^2\Delta\omega^2, \text{ so } Z \cong \sqrt{R^2 + 4L^2\Delta\omega^2}.$$

c) $I = \frac{1}{2}I_0 \Rightarrow \frac{V}{Z} = \frac{1}{2}\frac{V}{R}$ or $Z^2 = (2R)^2$.

$$R^2 + 4L^2\Delta\omega^2 = 4R^2 \Rightarrow \Delta\omega = \pm\sqrt{\frac{3R^2}{4L^2}} = \pm\sqrt{\frac{3}{4}}\frac{R}{L}.$$

$$\omega = \omega_0 \pm \sqrt{\frac{3}{4}}\frac{R}{L} \text{ but } \sqrt{\frac{3}{4}}\frac{R}{L} << \frac{1}{\sqrt{LC}} \Rightarrow R << \sqrt{\frac{4L}{3C}}.$$

d) $|\omega_1 - \omega_2| = 2\Delta\omega = \sqrt{3}\frac{R}{L}$. As R increases so does the width.

e) (i) $I_0 = \frac{120\,V}{15\,\Omega} = 8\text{ A}; \omega_0 = \frac{1}{\sqrt{(2.50\text{ H})(0.400\times10^{-6}\text{ F})}} = 1000\text{ rad/sec};$

$|\omega_1 - \omega_2| = \sqrt{3}\frac{15\,\Omega}{2.50\text{ H}} = 10.4\text{ rad/sec.}$ (ii) $I_0 = 80\text{A}$, $\omega_0 = 1000\text{ rad/s}$, $|\omega_1 - \omega_2| = 1.04\text{ rad/se}$

31.64: a) $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega c}\right)^2}}$ at resonance $\omega L = \dfrac{1}{\omega C}$. So $I_{\max} = \dfrac{V}{R}\cdot$

b) $V_{C_{\max}} = I_{\max} X_C = \dfrac{V}{R\omega_0 C} = \dfrac{V}{R}\sqrt{\dfrac{L}{C}}.$

c) $V_{L_{\max}} = I_{\max} X_L = \dfrac{V}{R}\omega_0 L = \dfrac{V}{R}\sqrt{\dfrac{L}{C}}.$

d) $U_{C_{\max}} = \dfrac{1}{2} C V_{C_{\max}}^2 = \dfrac{1}{2} C \dfrac{V^2}{R^2}\dfrac{L}{C} = \dfrac{1}{2} L \dfrac{V^2}{R^2}.$

e) $U_{L_{\max}} = \dfrac{1}{2} L I_{\max}^2 = \dfrac{1}{2} L \dfrac{V^2}{R^2}.$

31.65: $\omega = \dfrac{\omega_0}{2}.$

a) $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + \left(\dfrac{\omega_0 L}{2} - \dfrac{2}{\omega_0 C}\right)^2}} = \dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}.$

b) $V_{C_{\max}} = I X_C = \dfrac{2}{\omega_0 C}\dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}} = \sqrt{\dfrac{L}{C}}\dfrac{2V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}.$

c) $V_{L_{\max}} = I X_L = \dfrac{\omega_0 L}{2}\dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}} = \sqrt{\dfrac{L}{C}}\dfrac{V/2}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}.$

d) $U_{C_{\max}} = \dfrac{1}{2} C V_{C_{\max}}^2 = \dfrac{2LV^2}{R^2 + \dfrac{9}{4}\dfrac{L}{C}}.$

e) $U_{L_{\max}} = \dfrac{1}{2} L I^2 = \dfrac{1}{2}\dfrac{LV^2}{R^2 + \dfrac{9}{2}\dfrac{L}{C}}.$

31.66: $\omega = 2\omega_0$.

a) $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (2\omega_0 L - \dfrac{1}{2\omega_0 C})^2}} = \dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}.$

b) $V_{C_{\max}} = IX_C = \dfrac{1}{2\omega_0 C}\dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}} = \sqrt{\dfrac{L}{C}}\dfrac{V/2}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}.$

c) $V_{L_{\max}} = IX_L = 2\omega_0 L\dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}} = \sqrt{\dfrac{L}{C}}\dfrac{2V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}.$

d) $U_{C_{\max}} = \dfrac{1}{2}CV_{C_{\max}}^2 = \dfrac{LV^2}{8\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}.$

e) $U_{L_{\max}} = \dfrac{1}{2}LI^2 = \dfrac{LV^2}{2\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}.$

31.67: a) $p_R = i^2 R = I^2\cos^2(\omega t)R = V_R I\cos^2(\omega t)) = \dfrac{1}{2}V_R I(1+\cos(2\omega t))$

$$\Rightarrow P_{av}(R) = \frac{1}{T}\int_0^T p_R dt = \frac{V_R I}{2T}\int_0^T (1+\cos(2\omega t))dt = \frac{V_R I}{2T}[t]_0^T = \frac{1}{2}V_R I.$$

b) $p_L = Li\dfrac{di}{dt} = -\omega LI^2\cos(\omega t)\sin(\omega t) = -\dfrac{1}{2}V_L I\sin(2\omega t).$

But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{av}(L) = 0.$

c) $p_c = \dfrac{dU}{dt} = \dfrac{d}{dt}\left(\dfrac{q^2}{2C}\right) = \dfrac{q}{C}i = v_C i = V_C I\sin(\omega t)\cos(\omega t) = \dfrac{1}{2}V_C I\sin(2\omega t).$

But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{av}(C) = 0.$

d) $p = p_R + p_L + p_c = V_R I\cos^2(\omega t) - \dfrac{1}{2}V_L I\sin(2\omega t) + \dfrac{1}{2}V_C I\sin(2\omega t)$

$$\Rightarrow P = I\cos(\omega t)(V_R\cos(\omega t) - V_L\sin(\omega t) + V_C\sin(\omega t)).$$

But $\cos\phi = \dfrac{V_R}{V}$ and $\sin\phi = \dfrac{V_L - V_C}{V} \Rightarrow p = VI\cos(\omega t)(\cos\phi\cos(\omega t) - \sin\phi\sin(\omega t)),$ at any instant of time.

31.68: a) $V_R =$ maximum when $V_C = V_L \Rightarrow \omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$.

b) From Problem (31.48a), $V_L =$ maximum when $\dfrac{dV_L}{d\omega} = 0.$ Therefore:

$$\frac{dV_L}{d\omega} = 0 = \frac{d}{d\omega}\left(\frac{V\omega L}{\sqrt{R^2 + \omega L - 1/\omega C)^2}}\right)$$

$$\Rightarrow 0 = \frac{VL}{\sqrt{R^2+(\omega L - 1/\omega C)^2}} - \frac{V\omega^2 L(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}$$

$$\Rightarrow R^2 + (\omega L - 1/\omega C)^2 = \omega^2 (L^2 - 1/\omega^4 C^2)$$

$$\Rightarrow R^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} = -\frac{1}{\omega^2 C^2} \Rightarrow \frac{1}{\omega^2} = LC - \frac{R^2 C^2}{2} \Rightarrow \omega = \frac{1}{\sqrt{LC - R^2 C^2/2}}.$$

c) From Problem (31.48b), $V_C =$ maximum when $\dfrac{dV_C}{d\omega} = 0.$ Therefore:

$$\frac{dV_C}{d\omega} = 0 = \frac{d}{d\omega}\left(\frac{V}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)$$

$$\Rightarrow 0 = -\frac{V}{\omega^2 C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{C(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}$$

$$\Rightarrow R^2 + (\omega L - 1/\omega C)^2 = -\omega^2 (L^2 - 1/\omega^4 C^2)$$

$$\Rightarrow R^2 + \omega^2 L^2 - \frac{2L}{C} = -\omega^2 L^2 \Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}.$$

31.69: a) From the current phasors we know that

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\Rightarrow Z = \sqrt{(400\ \Omega)^2 + \left((1000\ \text{rad/s})(0.50\ \text{H}) - \frac{1}{(1000\ \text{rad/s})(1.25\times10^{-6}\ \text{F})}\right)^2} = 500\ \Omega.$$

$$\Rightarrow I = \frac{V}{Z} = \frac{200\ \text{V}}{500\ \Omega} = 0.400\ \text{A}.$$

b) $$\phi = \arctan\left(\frac{\omega L - 1/(\omega C)}{R}\right)$$

$$\Rightarrow \phi = \arctan\left(\frac{(1000\ \text{rad/s})(0.500\ \text{H}) - 1/(1000\ \text{rad/s})(1.25\times10^{-6}\ \text{F})}{400\ \Omega}\right) = +36.9°$$

c) $$Z_{\text{cpx}} = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

$$\Rightarrow Z_{\text{cpx}} = 400\ \Omega - i\left((1000\ \text{rad/s})(0.50\ \text{H}) - \frac{1}{(1000\ \text{rad/s})(1.25\times10^{-6}\ \text{F})}\right)$$

$$= 400\ \Omega - 300\ \Omega i$$

$$\Rightarrow Z = \sqrt{(400\ \Omega)^2 + (-300\ \Omega)^2} = 500\ \Omega.$$

d) $$I_{\text{cpx}} = \frac{V}{Z_{\text{cpx}}} = \frac{200\ \text{V}}{(400 - 300i)\ \Omega} = \left(\frac{8+6i}{25}\right)\text{A}$$

$$\Rightarrow I = \sqrt{\left(\frac{8+6i}{25}\right)\left(\frac{8-6i}{25}\right)} = 0.400\ \text{A}.$$

e) $$\tan\phi = \frac{\text{Im}(I_{\text{cpx}})}{\text{Re}(I_{(\text{cpx})}} = \frac{6/25}{8/25} = 0.75 \Rightarrow \phi = +36.9°.$$

f) $$V_{R_{\text{cpx}}} = I_{\text{cpx}} R = \left(\frac{8+6i}{25}\right)(400\ \Omega) = (128 + 96i)\text{V}.$$

$$V_{L_{\text{cpx}}} = iI_{\text{cpx}}\omega L = i\left(\frac{8+6i}{25}\right)(1000\ \text{rad/s})(0.500\ \text{H}) = (-120 + 160i)\text{V}.$$

$$\text{V}_{Lcpx} = i\frac{I_{\text{cpx}}}{\omega C} = i\left(\frac{8+6i}{25}\right)\frac{1}{(1000\ \text{rad/s})(1.25\times10^{-6}\ \text{F})} = (+192 - 256i)\text{V}\cdot$$

g) $$V_{\text{cpx}} = V_{R_{\text{cpx}}} + V_{L_{\text{cpx}}} + V_{L_{\text{cpx}}} = (128 + 96i)V + (-120 + 160i)\text{V}$$
$$+ (192 - 256i)\text{V} = 200\ \text{V}.$$

32.1: a) $t = \dfrac{d}{c} = \dfrac{3.84 \times 10^{8}\ \text{m}}{3.00 \times 10^{8}\ \text{m/s}} = 1.28\ \text{s}.$

b) Light travel time is:

$$8.61\ \text{years} = (8.61\ \text{years})\ \frac{(365\ \text{days})}{(1\ \text{year})}\ \frac{(24\ \text{hours})}{(1\ \text{day})}\ \frac{(3600\ \text{s})}{(1\ \text{hour})} = 2.72 \times 10^{8}\ \text{s}$$

$$d = ct = (3.0 \times 10^{8}\ \text{m/s})\ (2.72 \times 10^{8}\ \text{s}) = 8.16 \times 10^{16}\ \text{m} = 8.16 \times 10^{13}\ \text{km}.$$

32.2: $d = c\Delta t = (3.0 \times 10^{8}\ \text{m/s})\ (6.0 \times 10^{-7}\ \text{s}) = 180\ \text{m}.$

32.3: $\vec{\boldsymbol{B}}(z,t) = B_{\max} \cos(kz - \omega t)\hat{\boldsymbol{j}} = B_{\max} \cos\left(2\pi f\left(\dfrac{z}{c} - t\right)\right)\hat{\boldsymbol{j}}$

$$\Rightarrow \vec{\boldsymbol{B}}(z,t) = (5.80 \times 10^{-4}\ \text{T})\cos\left(2\pi(6.10 \times 10^{14}\ \text{Hz})\left(\frac{z}{(3.00 \times 10^{8}\ \text{m/s})} - t\right)\right)\hat{\boldsymbol{j}}$$

$$\Rightarrow \vec{\boldsymbol{B}}(z,t) = (5.80 \times 10^{-4}\ \text{T})\cos((1.28 \times 10^{7}\ \text{m}^{-1})z - (3.83 \times 10^{15}\ \text{rad/s})t)\hat{\boldsymbol{j}}.$$

$$\vec{\boldsymbol{E}}(z,t) = (B_y(z,t)\hat{\boldsymbol{j}}) \times (c\hat{\boldsymbol{k}})$$

$$\Rightarrow \vec{\boldsymbol{E}}(z,t) = (1.74 \times 10^{5}\ \text{V/m})\cos((1.28 \times 10^{7}\ \text{m}^{-1})z - (3.83 \times 10^{15}\ \text{rad/s})t)\hat{\boldsymbol{i}}.$$

32.4: a) $f = \dfrac{c}{\lambda} = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{4.35 \times 10^{-7}\ \text{m}} = 6.90 \times 10^{14}\ \text{Hz}.$

b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{2.70 \times 10^{-3}\ \text{V/m}}{3.00 \times 10^{8}\ \text{m/s}} = 9.00 \times 10^{-12}\ \text{T}.$

c) The electric field is in the x-direction, and the wave is propagating in the $-z$-direction. So the magnetic field is in the $-y$-direction, since $\vec{\boldsymbol{S}} \propto \vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}$. Thus:

$$\vec{\boldsymbol{E}}(z,t) = E_{\max}\cos(kz + \omega t)\hat{\boldsymbol{i}} = E_{\max}\cos\left(2\pi f\left(\frac{z}{c} + t\right)\right)\hat{\boldsymbol{i}}$$

$$\Rightarrow \vec{\boldsymbol{E}}(z,t) = (2.70 \times 10^{-3}\ \text{V/m})\cos\left(2\pi(6.90 \times 10^{14}\ \text{Hz})\left(t + \frac{z}{3.00 \times 10^{8}\ \text{m/s}}\right)\right)\hat{\boldsymbol{i}}$$

$$\Rightarrow \vec{\boldsymbol{E}}(z,t) = (2.70 \times 10^{-3}\ \text{V/m})\cos((1.45 \times 10^{7}\ \text{m}^{-1})z + (4.34 \times 10^{15}\ \text{rad/s})t)\hat{\boldsymbol{i}}.$$

And $\vec{\boldsymbol{B}}(z,t) = \dfrac{-E(z,t)}{c}\hat{\boldsymbol{j}} = -(9.00 \times 10^{-12}\ \text{T})\cos((1.45 \times 10^{7}\ \text{m}^{-1})z + (4.34 \times 10^{15}\text{rad/s})t)\hat{\boldsymbol{j}}.$

32.5: a) $+y$ direction.

b) $\omega = 2\pi f = \dfrac{2\pi c}{\lambda} \Rightarrow \lambda = \dfrac{2\pi c}{\omega} = \dfrac{2\pi(3.00\times10^{8}\ \text{m/s})}{(2.65\times10^{12}\ \text{rad/s})} = 7.11\times10^{-4}\ \text{m}.$

c) Since the electric field is in the $-z$-direction, and the wave is propagating in the $+y$-direction, then the magnetic field is in the $-x$-direction $(\vec{\boldsymbol{S}} \propto \vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}})$. So:

$$\vec{\boldsymbol{B}}(y,t) = \frac{-E(y,t)}{c}\hat{\boldsymbol{i}} = \frac{-E_0}{c}\sin(ky-\omega t)\hat{\boldsymbol{i}} = \frac{-E_0}{c}\sin(\frac{\omega}{c}y-\omega t)\hat{\boldsymbol{i}}$$

$$\Rightarrow \vec{\boldsymbol{B}}(y,t) = -\left(\frac{3.10\times10^{5}\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}}\right)\sin\left(\frac{(2.65\times10^{12}\ \text{rad/s})}{(3.00\times10^{8}\ \text{m/s})}y-(2.65\times10^{12}\ \text{rad/s})t\right)\hat{\boldsymbol{i}}$$

$$\Rightarrow \vec{\boldsymbol{B}}(y,t) = -(1.03\times10^{-3}\ \text{T})\sin((8.83\times10^{3}\ \text{m})y-(2.65\times10^{12}\ \text{rad/s})t)\hat{\boldsymbol{i}}.$$

32.6: a) $-x$ direction.

b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c} \Rightarrow f = \dfrac{kc}{2\pi} = \dfrac{(1.38\times10^{4}\ \text{rad/m})(3.0\times10^{8}\ \text{m/s})}{2\pi} = 6.59\times10^{11}\ \text{Hz}.$

c) Since the magnetic field is in the $+y$-direction, and the wave is propagating in the $-x$-direction, then the electric field is in the $+z$-direction $(\vec{\boldsymbol{S}} \propto \vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}})$. So:

$$\vec{\boldsymbol{E}}(x,t) = +cB(x,t)\hat{\boldsymbol{k}} = +cB_0\ \sin(kx+2\pi\ f)\hat{\boldsymbol{k}}$$

$$\Rightarrow \vec{\boldsymbol{E}}(x,t) = +(c(3.25\times10^{-9}\ \text{T}))\sin((1.38\times10^{4}\ \text{rad/m})x+(4.14\times10^{12}\ \text{rad/s})t)\hat{\boldsymbol{k}}$$

$$\Rightarrow \vec{\boldsymbol{E}}(y,t) = +(2.48\ \text{V/m})\sin((1.38\times10^{4}\ \text{rad/m})x+(4.14\times10^{12}\ \text{rad/s})t)\hat{\boldsymbol{k}}.$$

32.7: a) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{8.30\times10^{5}\ \text{Hz}} = 361\,\text{m}.$

b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{361\,\text{m}} = 0.0174\,\text{m}^{-1}$

c) $\omega = 2\pi f = 2\pi(8.30\times10^{5}\ \text{Hz}) = 5.21\times10^{6}\ \text{rad/s}.$

$$E_{\max} = cB_{\max} = (3.00\times10^{8}\ \text{m/s})\,(4.82\times10^{-11}\ \text{T}) = 0.0145\ \text{V/m}.$$

32.8: $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{3.85\times10^{-3}\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}} = 1.28\times10^{-11}\ \text{T}.$

So $\dfrac{B_{\max}}{B_{\text{earth}}} = \dfrac{1.28\times10^{-11}\ \text{T}}{5\times10^{-5}\ \text{T}} = 2.56\times10^{-7}$, and thus $B_{\max}$ is much weaker than B_{earth}.

32.9: $E = vB = \dfrac{B}{\sqrt{\varepsilon\mu}} = \dfrac{B}{\sqrt{K_E \varepsilon_0 K_B \mu_0}} = \dfrac{cB}{\sqrt{K_E K_B}}$

$$\Rightarrow E = \frac{(3.00\times 10^8 \text{ m/s})\,(3.80\times 10^{-9} \text{ T})}{\sqrt{(1.74)\,(1.23)}} = 0.779 \text{ V/m}.$$

32.10: a) $v = \dfrac{c}{\sqrt{K_E K_B}} = \dfrac{(3.00\times 10^8 \text{m/s})}{\sqrt{(3.64)\,(5.18)}} = 6.91\times 10^7 \text{ m/s}.$

b) $\lambda = \dfrac{v}{f} = \dfrac{6.91\times 10^7 \text{ m/s}}{65.0 \text{ Hz}} = 1.06\times 10^6 \text{ m}.$

c) $B = \dfrac{E}{v} = \dfrac{7.20\times 10^{-3} \text{ V/m}}{6.91\times 10^7 \text{ m/s}} = 1.04\times 10^{-10} \text{ T}.$

d) $I = \dfrac{EB}{2K_B\mu_0} = \dfrac{(7.20\times 10^{-3} \text{ V/m})(1.04\times 10^{-10} \text{ T})}{2(5.18)\mu_0} = 5.75\times 10^{-8} \text{ W/m}^2.$

32.11: a) $\lambda = \dfrac{v}{f} = \dfrac{2.17\times 10^8 \text{ m/s}}{5.70\times 10^{14} \text{ Hz}} = 3.81\times 10^{-7} \text{ m}.$

b) $\lambda_0 = \dfrac{c}{f} = \dfrac{3.00\times 10^8 \text{ m/s}}{5.70\times 10^{14} \text{ Hz}} = 5.26\times 10^{-7} \text{ m}.$

c) $n = \dfrac{c}{v} = \dfrac{3.00\times 10^8 \text{ m/s}}{2.17\times 10^8 \text{ m/s}} = 1.38.$

d) $v = \dfrac{c}{\sqrt{K_E}} \Rightarrow K_E = \dfrac{c^2}{v^2} = n^2 = (1.38)^2 = 1.90.$

32.12: a) $v = f\lambda = (3.80\times 10^7 \text{ Hz})(6.15 \text{ m}) = 2.34\times 10^8 \text{ m/s}.$

b) $K_E = \dfrac{c^2}{v^2} = \dfrac{(3.00\times 10^8 \text{ m/s})^2}{(2.34\times 10^8 \text{ m/s})^2} = 1.64.$

32.13: a) $I = \dfrac{1}{2}\varepsilon_0 c E_{\max}^2;\ E_{\max} = 0.090 \text{ V/m, so } I = 1.1\times 10^{-5} \text{ W/m}^2$

b) $E_{\max} = cB_{\max}$ so $B_{\max} = E_{\max}/c = 3.0\times 10^{-10}$ T

c) $P_{\text{av}} = I(4\pi r^2) = (1.075\times 10^{-5} \text{ W/m}^2)(4\pi)(2.5\times 10^3 \text{ m})^2 = 840 \text{ W}$

d) Calculation in part (c) assumes that the transmitter emits uniformly in all directions.

32.14: The intensity of the electromagnetic wave is given by Eqn. 32.29: $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2 = \varepsilon_0 c E_{\text{rms}}^2$. Thus the total energy passing through a window of area A during a time t is

$\varepsilon_0 c E_{\text{rms}}^2 At = (8.85\times10^{-12}\text{F/m})\ (3.00\times10^8\ \text{m/s})\ (0.0200\ \text{V/m})^2(0.500\ \text{m}^2)(30.0\ \text{s}) = 15.9\,\mu\text{J}$

32.15: $P_{\text{av}} = I(4\pi r^2) = (5.0\times10^3\ \text{W/m}^2)(4\pi)(2.0\times10^{10}\text{m})^2 = 2.5\times10^{25}\ \text{J}$

32.16: a) The average power from the beam is

$$P = IA = (0.800\ \text{W/m}^2)(3.0\times10^{-4}\ \text{m}^2) = 2.4\times10^{-4}\,\text{W}$$

b) We have, using Eq. 32.29, $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2 = \varepsilon_0 c E_{\text{rms}}^2$. Thus,

$$E_{\text{rms}} = \sqrt{\frac{I}{\varepsilon_0 c}} = \sqrt{\frac{0.800\ \text{W/m}^2}{(8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})}} = 17.4\ \text{V/m}$$

32.17: $p_{\text{rad}} = I/c$ so $I = cp_{\text{rad}} = 2.70\times10^3\ \text{W/m}^2$

Then $P_{av} = I(4\pi r^2) = (2.70\times10^3\ \text{W/m}^2)\ (4\pi)\ (5.0\ \text{m})^2 = 8.5\times10^5\ \text{W}$

32.18: a) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\ \text{m/s}}{0.354\ \text{m}} = 8.47\times10^8\ \text{Hz.}$

b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{0.0540\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 1.80\times10^{-10}\,\text{T.}$

c) $I = S_{av} = \dfrac{EB}{2\mu_0} = \dfrac{(0.0540\ \text{V/m})\ (1.80\times10^{-10}\,\text{T})}{2\mu_0} = 3.87\times10^{-6}\ \text{W/m}^2.$

32.19: $P = S_{av}A = \dfrac{E_{\max}^2}{2c\mu_0}\cdot(4\pi r^2) \Rightarrow E_{\max} = \sqrt{\dfrac{Pc\mu_0}{2\pi r^2}}$

$$\Rightarrow E_{\max} = \sqrt{\frac{(60.0\ \text{W})\ (3.00\times10^8\ \text{m/s})\mu_0}{2\pi(5.00\ \text{m})^2}} = 12.0\ \text{V/m.}$$

$$\Rightarrow B_{\max} = \frac{E_{\max}}{c} = \frac{12.0\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 4.00\times10^{-8}\,\text{T.}$$

32.20: a) The electric field is in the $-y$-direction, and the magnetic filed is in the $+z$-direction, so $\hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = (-\hat{\boldsymbol{j}}) \times \hat{\boldsymbol{k}} = -\hat{\boldsymbol{i}}$. That is, the Poynting vector is in the $-x$-direction.

b) $S(x,t) = \dfrac{E(x,t)B(x,t)}{\mu_0} = -\dfrac{E_{\max} B_{\max}}{\mu_0}\cos(kx + \omega t)$

$$= -\frac{E_{\max} B_{\max}}{2\mu_0}(1 + \cos(2(\omega t + kx))).$$

But over one period, the cosine function averages to zero, so we have:

$$|S_{av}| = \frac{E_{\max} B_{\max}}{2\mu_0}.$$

32.21: a) The momentum density $\dfrac{dp}{dV} = \dfrac{S_{av}}{c^2} = \dfrac{780\ \text{W/m}^2}{(3.0 \times 10^8\ \text{m/s})^2} = 8.7 \times 10^{-15}\ \text{kg/m}^2 \cdot \text{s}.$

b) The momentum flow rate $\dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{S_{av}}{c} = \dfrac{780\ \text{W/m}^2}{3.0 \times 10^8\ \text{m/s}} = 2.6 \times 10^{-6}\,\text{Pa}.$

32.22: a) Absorbed light: $p_{\text{rad}} = \dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{S_{av}}{c} = \dfrac{2500\ \text{W/m}^2}{3.0 \times 10^8\ \text{m/s}} = 8.33 \times 10^{-6}\ \text{Pa}.$

$$\Rightarrow p_{\text{rad}} = \frac{8.33 \times 10^{-6}\ \text{Pa}}{1.013 \times 10^5\ \text{Pa/atm}} = 8.23 \times 10^{-11}\,\text{atm}.$$

b) Reflecting light: $p_{\text{rad}} = \dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{2S_{av}}{c} = \dfrac{2(2500\ \text{W/m}^2)}{3.0 \times 10^8\ \text{m/s}} = 1.67 \times 10^{-5}\ \text{Pa}.$

$\Rightarrow p_{\text{rad}} = \dfrac{1.67 \times 10^{-5}\ \text{Pa}}{1.013 \times 10^5\ \text{Pa/atm}} = 1.65 \times 10^{-10}\,\text{atm}.$ The factor of 2 arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

c) The momentum density $\dfrac{dp}{dV} = \dfrac{S_{av}}{c^2} = \dfrac{2500\ \text{W/m}^2}{(3.0 \times 10^8\ \text{m/s})^2} = 2.78 \times 10^{-14}\ \text{kg/m}^2 \cdot \text{s}.$

32.23: $S = \dfrac{\varepsilon_0}{\sqrt{\varepsilon_0 \mu_0}}E^2 = \sqrt{\dfrac{\varepsilon_0}{\mu_0}}E^2 = \sqrt{\dfrac{\varepsilon_0}{\mu_0}}Ec\dfrac{E}{c} = c\sqrt{\dfrac{\varepsilon_0}{\mu_0}}EB = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}\sqrt{\dfrac{\varepsilon_0}{\mu_0}}EB =$
$\dfrac{EB}{\mu_0} = \dfrac{E^2}{\mu_0 c} = \varepsilon_0 c E^2.$

32.24: Recall that $\vec{S} \propto \vec{E} \times \vec{B}$, so :

a) $\hat{S} = \hat{i} \times (-\hat{j}) = -\hat{k}$.

b) $\hat{S} = \hat{j} \times \hat{i} = -\hat{k}$.

c) $\hat{S} = (-\hat{k}) \times (-\hat{i}) = \hat{j}$.

d) $\hat{S} = \hat{i} \times (-\hat{k}) = \hat{j}$.

32.25: $B_{\text{max}} = E_{\text{max}} / c = 1.33 \times 10^{-8}$ T

$\vec{E} \times \vec{B}$ is in the direction of propagation. For $\vec{E}$ in the $+x$-direction, $\vec{E} \times \vec{B}$ is in the $+z$-direction when $\vec{B}$ is in the $+y$-direction.

32.26: a) $\Delta x = \dfrac{\lambda}{2} = \dfrac{c}{2f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{2\,(75.0 \times 10^6 \text{ Hz})} = 2.00 \text{ m}.$

b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength $= \dfrac{\lambda}{4} = \dfrac{\Delta x}{2} = \dfrac{2.00 \text{ m}}{2} = 1.00 \text{ m}.$

32.27: a) The node-antinode distance $= \dfrac{\lambda}{4} = \dfrac{v}{4f} = \dfrac{2.10 \times 10^8 \text{ m/s}}{4(1.20 \times 10^{10} \text{ Hz})} = 4.38 \times 10^{-3} \text{m}.$

b) The distance between the electric and magnetic antinodes is one-quarter of a wavelength $= \dfrac{\lambda}{4} = \dfrac{v}{4f} = \dfrac{2.10 \times 10^8 \text{ m/s}}{4\,(1.20 \times 10^{10} \text{ Hz})} = 4.38 \times 10^{-3} \text{ m}.$

c) The distance between the electric and magnetic nodes is also one-quarter of a wavelength $=$

$$\frac{\lambda}{4} = \frac{v}{4f} = \frac{2.10 \times 10^8 \text{ m/s}}{4(1.20 \times 10^{10} \text{ Hz})} = 4.38 \times 10^{-3} \text{ m}.$$

32.28: $\Delta x_{\text{nodes}} = \dfrac{\lambda}{2} = \dfrac{c}{2f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{2(7.50 \times 10^8 \text{ Hz})} = 0.200 \text{ m} = 20.0 \text{ cm}.$ There must be nodes at the planes, which are 80.0 cm apart, and there are two nodes between the planes, each 20.0 cm from a plane. It is at 20 cm, 40 cm, and 60 cm that a point charge will remain at rest, since the electric fields there are zero.

32.29: a) $\Delta x = \dfrac{\lambda}{2} \Rightarrow \lambda = 2\Delta x = 2(3.55 \text{ mm}) = 7.10 \text{ mm}.$

b) $\Delta x_E = \Delta x_B = 3.55 \text{ mm}.$

c) $v = f\lambda = (2.20 \times 10^{10} \text{ Hz})(7.10 \times 10^{-3} \text{ m}) = 1.56 \times 10^8 \text{ m/s}.$

32.30: a) $\dfrac{\partial^2 E_y(x,t)}{\partial x^2} = \dfrac{\partial^2}{\partial x^2}(-2E_{\max}\sin kx \sin \omega t) = \dfrac{\partial}{\partial x}(-2kE_{\max}\cos kx \sin \omega t)$

$$\Rightarrow \frac{\partial^2 E_y(x,t)}{\partial x^2} = 2k^2 E_{\max}\sin kx \sin \omega t = \frac{\omega^2}{c^2}2E_{\max}\sin kx \sin \omega t = \varepsilon_0\mu_0\frac{\partial^2 E_y(x,t)}{\partial t^2}.$$

Similarly: $\dfrac{\partial^2 B_z(x,t)}{\partial x^2} = \dfrac{\partial^2}{\partial x^2}(-2B_{\max}\cos kx \cos \omega t) = \dfrac{\partial}{\partial x}(+2kB_{\max}\sin kx \cos \omega t)$

$$\Rightarrow \frac{\partial^2 B_z(x,t)}{\partial x^2} = 2k^2 B_{\max}\cos kx \cos \omega t = \frac{\omega^2}{c^2}2B_{\max}\cos kx \cos \omega t = \varepsilon_0\mu_0\frac{\partial^2 B_z(x,t)}{\partial t^2}.$$

b) $\dfrac{\partial E_y(x,t)}{\partial x} = \dfrac{\partial}{\partial x}(-2E_{\max}\sin kx \sin \omega t) = -2kE_{\max}\cos kx \sin \omega t$

$$\Rightarrow \frac{\partial E_y(x,t)}{\partial x} = -\frac{\omega}{c}2E_{\max}\cos kx \sin \omega t = -\omega 2\frac{E_{\max}}{c}\cos kx \sin \omega t = -\omega 2B_{\max}\cos kx \sin \omega t.$$

$$\Rightarrow \frac{\partial E_y(x,t)}{\partial x} = +\frac{\partial}{\partial t}(2B_{\max}\cos kx \cos \omega t) = -\frac{\partial B_z(x,t)}{\partial t}.$$

Similarly: $-\dfrac{\partial B_z(x,t)}{\partial x} = \dfrac{\partial}{\partial x}(+2B_{\max}\cos kx \cos \omega t) = -2kB_{\max}\sin kx \cos \omega t$

$$\Rightarrow -\frac{\partial B_z(x,t)}{\partial x} = -\frac{\omega}{c}2B_{\max}\sin kx \cos \omega t = -\frac{\omega}{c^2}2cB_{\max}\sin kx \cos \omega t$$

$$\Rightarrow -\frac{\partial B_z(x,t)}{\partial x} = -\varepsilon_0\mu_0\omega 2E_{\max}\sin kx \cos \omega t = \varepsilon_0\mu_0\frac{\partial}{\partial t}(-2E_{\max}\sin kx \sin \omega t) = \varepsilon_0\mu_0\frac{\partial E_y(x,t}{\partial t}$$

32.31: a) Gamma rays: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^8\ \text{m/s}}{6.50\times 10^{21}\ \text{Hz}} = 4.62\times 10^{-14}\ \text{m} = 4.62\times 10^{-5}\ \text{nm}.$

b) Green light: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^8\ \text{m/s}}{5.75\times 10^{14}\,\text{Hz}} = 5.22\times 10^{-7}\ \text{m} = 522\ \text{nm}.$

32.32: a) $f = \dfrac{c}{\lambda} = \dfrac{3.0\times 10^8\ \text{m/s}}{5000\ \text{m}} = 6.0\times 10^4\ \text{Hz}.$

b) $f = \dfrac{c}{\lambda} = \dfrac{3.0\times 10^8\ \text{m/s}}{5.0\ \text{m}} = 6.0\times 10^7\,\text{Hz}.$

c) $f = \dfrac{c}{\lambda} = \dfrac{3.0\times 10^8\ \text{m/s}}{5.0\times 10^{-6}\ \text{m}} = 6.0\times 10^{13}\,\text{Hz}.$

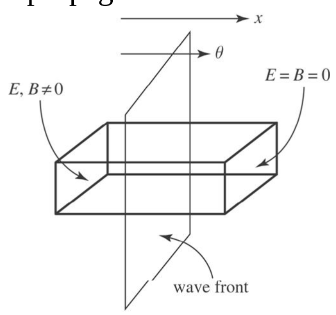
d) $f = \dfrac{c}{\lambda} = \dfrac{3.0\times 10^8\ \text{m/s}}{5.0\times 10^{-9}\ \text{m}} = 6.0\times 10^{16}\ \text{Hz}.$

32.33: Using a Gaussian surface such that the front surface is ahead of the wave front (no electric or magnetic fields) and the back face is behind the wave front (as shown at right), we have:

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = E_x \boldsymbol{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} = 0 \Rightarrow E_x = 0.$$

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = B_x A = 0 \Rightarrow B_x = 0.$$

So the wave must be transverse, since there are no components of the electric or magnetic field in the direction of propagation.



32.34: Assume $\vec{\boldsymbol{E}} = E_{\max} \hat{\boldsymbol{j}} \sin(kx - \omega t)$ *and* $\vec{\boldsymbol{B}} = B_{\max} \hat{\boldsymbol{k}} \sin(kx - \omega t + \phi)$, with $-\pi < \phi < \pi$.

Then Eq. (32.12) implies:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} x \Rightarrow +kE_{\max} \cos(kx - \omega t) = +\omega B_{\max} \cos(kx - \omega t + \phi) \Rightarrow \phi = 0.$$

$$\Rightarrow kE_{\max} = \omega B_{\max} \Rightarrow E_{\max} = \frac{\omega}{k} B_{\max} = \frac{2\pi f}{2\pi / \lambda} B_{\max} = f \lambda B_{\max} = cB_{\max}.$$

Similarly for Eq.(32.14)

$$-\frac{\partial B_z}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t} \Rightarrow -kB_{\max} \cos(kx - \omega t + \phi) = -\varepsilon_0 \mu_0 \omega E_{\max} \cos(kx - \omega t) \Rightarrow \phi = 0.$$

$$\Rightarrow kB_{\max} = \varepsilon_0 \mu_0 \omega E_{\max} \Rightarrow B_{\max} = \frac{\varepsilon_0 \mu_0 \omega}{k} E_{\max} = \frac{2\pi f}{c^2 2\pi / \lambda} E_{\max} = \frac{f\lambda}{c^2} E_{\max} = \frac{1}{c} E_{\max}.$$

32.35: From Eq. (32.12): $\frac{\partial}{\partial t}\left(\frac{\partial E_y(x,t)}{\partial x}\right)=\frac{\partial}{\partial t}\left(-\frac{\partial B_z(x,t)}{\partial t}\right)=-\frac{\partial^2 B_z(x,t)}{\partial t^2}$

But also from Eq. (32.14): $-\frac{\partial}{\partial x}\left(\frac{\partial B_z(x,t)}{\partial x}\right)=\frac{\partial}{\partial x}\left(\varepsilon_0\mu_0\frac{\partial E_y(x,t)}{\partial t}\right)=$

$-\varepsilon_0\mu_0\frac{\partial^2 B_z(x,t)}{\partial t^2}$

$\Rightarrow \frac{\partial^2 B_z(x,t)}{\partial x^2}=\varepsilon_0\mu_0\frac{\partial^2 B_z(x,t)}{\partial t^2}.$

32.36: $E_y(x,t)=E_{\max}\cos(kx-\omega t)\Rightarrow u_E=\frac{1}{2}\varepsilon_0 E^2=\frac{1}{2}\varepsilon_0 E_{\max}^2\cos(kx-\omega t)$

$$\Rightarrow u_E=\frac{\varepsilon_0 c^2}{2}\left(\frac{E_{\max}}{c}\right)^2\cos(kx-\omega t)=\frac{1}{2\mu_0}B_{\max}^2\cos(kx-\omega t)=\frac{B_z^2}{2\mu_0}=u_B$$

32.37: a) The energy incident on the mirror is $Pt=IAt=\frac{1}{2}\varepsilon_0 cE^2 At$

$\Rightarrow E=\frac{1}{2}\varepsilon_0(3.00\times10^8\text{ m/s})(0.028\text{ V/m})^2(5.00\times10^{-4}\text{ m}^2)(1.00\text{ s})=5.20\times10^{-10}\text{J}.$

b) The radiation pressure $p_{\text{rad}}=\frac{2I}{c}=\varepsilon_0 E^2=\varepsilon_0(0.0280\text{ V/m})^2=6.94\times10^{-15}\text{ Pa}.$

c) Power $P=I\cdot 4\pi R^2=cp_{\text{rad}}2\pi R^2$

$\Rightarrow P=2\pi(3.00\times10^8\text{ m/s})(6.94\times10^{-15}\text{ Pa})(3.20\text{ m})^2=1.34\times10^{-4}\text{ W}.$

32.38: a) $f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{0.0384\ \text{m}} = 7.81\times10^{9}\,\text{Hz}.$

b) $B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{1.35\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}} = 4.50\times10^{-9}\ \text{T}.$

c) $I = \frac{1}{2}\varepsilon_0 c E_{\text{max}}^2 = \frac{1}{2}\varepsilon_0(3.00\times10^{8}\ \text{m/s})(1.35\ \text{V/m})^2 = 2.42\times10^{-3}\ \text{W/m}^2.$

d) $F = pA = \frac{IA}{c} = \frac{EBA}{2\mu_0 c} = \frac{(1.35\ \text{V/m})(4.50\times10^{-9}\ \text{T})(0.240\ \text{m}^2)}{2\mu_0(3.00\times10^{8}\ \text{m/s})} = 1.93\times10^{-12}\ \text{N}.$

32.39: **a)** The laser intensity $I = \frac{P}{A} = \frac{4P}{\pi D^2} = \frac{4(3.20\times10^{-3}\ \text{W})}{\pi(2.50\times10^{-3}\ \text{m})^2} = 652\ \text{W/m}^2.$

But $I = \frac{1}{2}\varepsilon_0 c E^2 \Rightarrow E = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(652\ \text{W/m}^2)}{\varepsilon_0(3.00\times10^{8}\ \text{m/s})}} = 701\ \text{V/m}.$

And $B = \frac{E}{c} = \frac{701\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}} = 2.34\times10^{-6}\ \text{T}.$

b) $u_{B_{av}} = u_{E_{av}} = \frac{1}{4}\varepsilon_0 E_{\text{max}}^2 = \frac{1}{4}\varepsilon_0(701\ \text{V/m})^2 = 1.09\times10^{-6}\ \text{J/m}^3$. Note the extra factor of $\frac{1}{2}$ since we are averaging.

c) In one meter of the laser beam, the total energy is:

$$E_{\text{tot}} = u_{\text{tot}}\text{Vol} = 2u_E(AL) = 2u_E\pi D^2\,L/4$$

$$\Rightarrow E_{\text{tot}} = 2(1.09\times10^{-6}\ \text{J/m}^3)\pi\ (2.50\times10^{-3}\text{m})^2\ (1.00\ \text{m})/4 = 1.07\times10^{-11}\,\text{J}.$$

32.40: a) The change in the momentum vector determines p_{rad}. If W is the fraction absorbed, $\Delta \vec{\boldsymbol{P}} = \vec{\boldsymbol{P}}_{\text{out}} - \vec{\boldsymbol{P}}_{\text{in}} = (1-W)p - (-p) = (2-W)p.$ Here, $(1-W)$ is the fraction reflected. The positive direction was chosen in the direction of reflection. p is the magnitude of the incoming momentum. With Eq. 32.31, and taking the average, we get $p_{\text{rad}} = (2-W)\frac{I}{C}$. Be careful not to confuse p, the momentum of the incoming wave, with p_{rad}, the radiation pressure.

b) (i) totally absorbing $W = 1$ so $p_{\text{rad}} = \dfrac{I}{C}$

(ii) totally reflecting $W = 0$ so $p_{\text{rad}} = \dfrac{2}{C}$

These are just equations 32.32 and 32.33.

c) $W = 0.9,\ I = 1.40\times10^{2}\ \text{W/m}^2 \Rightarrow p_{\text{rad}} = \dfrac{(2-0.9)(1.40\times10^{3}\ \text{W/m}^2)}{3.00\times10^{8}\ \frac{\text{m}}{\text{s}}} = 5.13\times10^{-6}\ \text{Pa}$

$W = 0.1,\ I = 1.40\times10^{3}\ \text{W/m}^2 \Rightarrow p_{\text{rad}} = \dfrac{(2-0.1)(1.40\times10^{2}\ \text{W/m}^2)}{3.00\times10^{8}\ \frac{\text{m}}{\text{s}}} = 8.87\times10^{-6}\ \text{Pa}$

32.41: a) At the sun's surface:

$$P = IA \Rightarrow I = \frac{P}{A} = \frac{P}{4\pi R^2} = \frac{3.9\times10^{26}\ \text{W}}{4\pi(6.96\times10^{8}\ \text{m})^2} = 6.4\times10^{7}\ \text{W/m}^2$$

$$\Rightarrow p_{\text{rad}} = \frac{I}{c} = \frac{6.4\times10^{7}\ \text{W/m}^2}{3.00\times10^{8}\ \text{m/s}} = 0.21\ \text{Pa}.$$

Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $p_{\text{rad}}(R_{\text{sun}}/2) = 0.85$ Pa.

At the top of the earth's atmosphere, the measured sunlight intensity is $1400\ \text{W/m}^2 =$

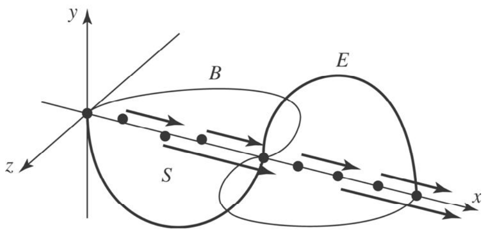
5×10^{-6} Pa, which is about 100,000 times less than the values above.

b) The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about 6×10^{13} times greater than the radiation, pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.42: a) $\vec{S}(x,t) = \dfrac{E_{\max} B_{\max}}{2\mu_0}(1-\cos 2\,(kx-\omega t))\hat{\boldsymbol{i}} \Rightarrow S(x,t) < 0 \Rightarrow \cos 2(kx-\omega t) > 1,$

which never happens. So the Poynting vector is always positive, which makes sense since the direction of wave propagation by definition is the direction of energy flow.

b)



32.43: a) $B = \mu_0 ni \Rightarrow \dfrac{dB}{dt} = \mu_0 n \dfrac{di}{dt} \Rightarrow \dfrac{d\Phi_B}{dt} = \dfrac{dB}{dt} A = \mu_0 nA \dfrac{di}{dt}.$

So, $\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\dfrac{d\Phi_B}{dt} \Rightarrow E 2\pi r = -\,\mu_0 nA \dfrac{di}{dt} = -\,\mu_0 n \pi r^2 \dfrac{di}{dt}$

$$\Rightarrow E = -\frac{\mu_0 nr}{2}\frac{di}{dt}.$$

b) The direction of the Poynting vector is radially inward, since the magnetic field is along the solenoid's axis and the electric filed is circumferential. It's magnitude $S = \dfrac{EB}{\mu_0} = \dfrac{\mu_0 n^2 ri}{2}\dfrac{di}{dt}.$

c) $u = \dfrac{B^2}{2\mu_0} = \dfrac{(\mu_0 ni)^2}{2\mu_0} = \dfrac{\mu_0 n^2 i^2}{2} \Rightarrow U = u(lA) = ul\pi a^2 = \dfrac{\mu_0 \pi n^2 i^2 l a^2}{2}.$

But also $U = \dfrac{Li^2}{2} \Rightarrow Li = \dfrac{2U}{i} = \dfrac{\mu_0 \pi n^2 i^2 la^2}{i} = \mu_0 \pi n^2 ila^2,$ and so the rate of energy increase due to the increasing current is given by $P = Li\dfrac{di}{dt} = \mu_0 \pi n^2 ila^2 \dfrac{di}{dt}.$

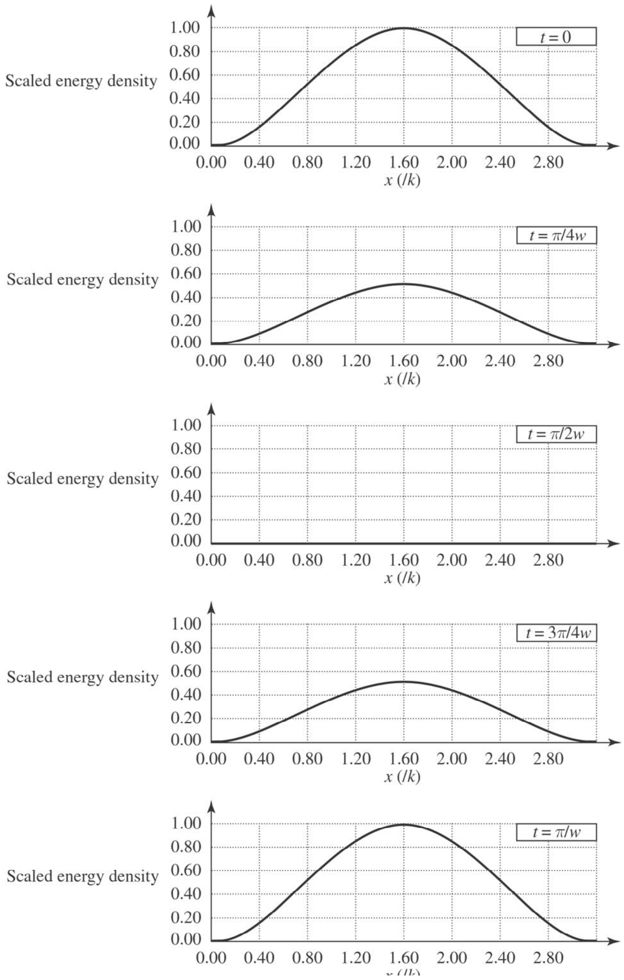
d) The in-flow of electromagnetic energy through a cylindrical surface located at the solenoid coils is $\int\int \vec{\boldsymbol{S}} \cdot d\vec{\boldsymbol{A}} = S 2\pi al = \dfrac{\mu_0 n^2 ai}{2}\dfrac{di}{dt} \cdot 2\pi al = \mu_0 \pi n^2 ila^2 \dfrac{di}{dt}.$

e) The values from parts (c) and (d) are identical for the flow of energy, and hence we can consider the energy stored in a current carrying solenoid as having entered through its cylindrical walls while the current was attaining its steady-state value.

32.44: a) The energy density, as a function of x, for the equations for the electrical and magnetic fields of Eqs. (32.34) and (32.35) is given by:
$u = \varepsilon_0 E^2 = 4\varepsilon_0 E_{\max}^2 \ \sin^2 kx \sin \omega t$

b) At $t = \dfrac{\pi}{4\omega}$, $\cos \ \omega t = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ and $\sin \omega t = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$.

For $0 < x < \dfrac{\pi}{2k}$, $\sin \ kx > 0$, $\cos \ \ kx > 0 \Rightarrow \hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = -\,\hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = -\hat{\boldsymbol{i}}$.



And for $\dfrac{\pi}{2k} < x < \dfrac{\pi}{k}$, $\sin \ kx > 0$, $\cos kx < 0 \Rightarrow \hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = -\,\hat{\boldsymbol{j}} \times \ \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}}$.

At $t = \dfrac{3\pi}{4\omega}, \cos \omega t = \cos\dfrac{3\pi}{4} = -\dfrac{1}{\sqrt{2}}$ and $\sin \omega t = \sin\dfrac{3\pi}{4} = \dfrac{1}{\sqrt{2}}$.

For 0 $< x < \dfrac{\pi}{2k}$, $\sin \ kx > 0$, $\cos \ kx > 0 \Rightarrow \hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}}$.

And for $\dfrac{\pi}{2k} < x < \dfrac{\pi}{k}$, $\sin \ kx > 0$, $\cos kx < 0 \Rightarrow \hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = \hat{\boldsymbol{j}} \times -\hat{\boldsymbol{k}} = \ -\hat{\boldsymbol{i}}$.

c) the plots from part (a) can be interpreted as two waves passing through each other in opposite directions, adding constructively at certain times, and destructively at others.

32.45: a) $E = \rho J = \dfrac{\rho I}{A} = \dfrac{\rho I}{\pi a^2}$, in the direction of the current.

b) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi a}$, counterclockwise when looking into the current.

c) The direction of the Poynting vector $\hat{S} = \hat{E} \times \hat{B} = \hat{k} \times \hat{\phi} = -\hat{\rho}$, where we have used cylindrical coordinates, with the current in the z-direction.

Its magnitude is $S = \dfrac{EB}{\mu_0} = \dfrac{1}{\mu_0} \dfrac{\rho I}{\pi a^2} \dfrac{\mu_0 I}{2\pi a} = \dfrac{\rho I^2}{2\pi^2 a^3}$.

d) Over a length l, the rate of energy flowing in is $SA = \dfrac{\rho I^2}{2\pi^2 a^3} 2\pi a l = \dfrac{\rho l I^2}{\pi a^2}$.

The thermal power loss is $I^2 R = I^2 \dfrac{\rho l}{A} = \dfrac{\rho l I^2}{\pi a^2}$, which exactly equals the flow of electromagnetic energy.

32.46: $B = \dfrac{\mu_0 i}{2\pi r}$, and $\oint_S \vec{E} \cdot d\vec{A} = EA = \dfrac{q}{\varepsilon_0} \Rightarrow E = \dfrac{q}{\pi \varepsilon_0 r^2}$, so the magnitude of the Poynting vector is $S = \dfrac{EB}{\mu_0} = \dfrac{qi}{2\varepsilon_0 \pi^2 r^3} = \dfrac{q}{2\varepsilon_0 \pi^2 r^3} \dfrac{dq}{dt}$.

Now, the rate of energy flow into the region between the plates is:

$$\int\int \vec{S} \cdot d\vec{A} = S(2\pi r l) = \frac{lq}{\varepsilon_0 \pi r^2} \frac{dq}{dt} = \frac{1}{2} \frac{l}{\varepsilon_0 \pi r^2} \frac{d(q^2)}{dt} = \frac{d}{dt}\left(\frac{1}{2} \frac{l}{\varepsilon_0 A} q^2\right) = \frac{d}{dt}\left(\frac{q^2}{2C}\right) = \frac{dU}{dt}.$$

This is just rate of increase in electrostatic energy U stored in the capacitor.

32.47: The power from the antenna is $P = IA = \dfrac{cB_{\text{max}}^2}{2\mu_0} 4\pi r^2$. So

$$\Rightarrow B_{\text{max}} = \sqrt{\frac{2\mu_0 P}{4\pi r^2 c}} = \sqrt{\frac{2\mu_0 (5.50 \times 10^4 \text{ W})}{4\pi (2500 \text{ m})^2 (3.00 \times 10^8 \text{ m/s})}} = 2.42 \times 10^{-9} \text{ T}$$

$$\Rightarrow \frac{dB}{dt} = \omega B_{\text{max}} = 2\pi f B_{\text{max}} = 2\pi (9.50 \times 10^7 \text{ Hz})\,(2.42 \times 10^{-9} \text{ T}) = 1.44 \text{ T/s}$$

$$\Rightarrow \varepsilon = -\frac{d\Phi}{dt} = -A\frac{dB}{dt} = \frac{\pi D^2}{4} \frac{dB}{dt} = \frac{\pi (0.180 \text{ m})^2 (1.44 \text{ T/s})}{4} = 0.0366 \text{ V}.$$

32.48: $I = \dfrac{P}{A} = \dfrac{1}{2} \varepsilon_0 c E^2 \Rightarrow E = \sqrt{\dfrac{2I}{\varepsilon_0 c}} = \sqrt{\dfrac{2(2.80 \times 10^3 \text{ W/36 m}^2)}{\varepsilon_0 (3.00 \times 10^8 \text{ m/s})}} = 242 \text{ V/m}.$

32.49: a) Find the force on you due to the momentum carried off by the light: $p_{\text{rad}} = I/c$ and $F = p_{\text{rad}}\ A$ gives $F = I\,A/c = P_{\text{av}}/c$

$a_x = F/m = P_{\text{av}}/(mc) = (200\ \text{W})/[(150\ \text{kg})(3.00\times10^{8}\ \text{m/s})] = 4.44\times10^{-9}\ \text{m/s}^2$

Then $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $t = \sqrt{2(x-x_0)/a_x} = \sqrt{2(16.0\ \text{m})/(4.44\times10^{-9}\ \text{m/s}^2)} =$ $8.49\times10^{4}\,\text{s} = 23.6\ \text{h}$

The radiation force is very small. In the calculation we have ignored any other forces on you.

b) You could throw the flashlight in the direction away from the ship. By conservation of linear momentum you would move toward the ship with the same magnitude of momentum as you gave the flashlight.

32.50: $P = IA \Rightarrow I = \dfrac{P}{A} = \dfrac{1}{2}\varepsilon_0 cE^2 \Rightarrow E = \sqrt{\dfrac{2P}{A\varepsilon_0 c}} = \sqrt{\dfrac{2Vi}{A\varepsilon_0 c}}$

$$\Rightarrow E = \sqrt{\frac{2Vi}{A\varepsilon_0 c}} = \sqrt{\frac{2(5.00\times10^{5}\ \text{V})\ (1000\ \text{A})}{(100\ \text{m}^2)\varepsilon_0(3.00\times10^{8}\ \text{m/s})}} = 6.14\times10^{4}\ \text{V/m}.$$

And

$$B = \frac{E}{c} = \frac{6.14\times10^{4}\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}} = 2.05\times10^{4}\ \text{T}.$$

32.51: a) $$F_G = \frac{GM_S m}{r^2} = \frac{GM_S}{r^2}\cdot\frac{4\pi R^3\rho}{3} = \frac{4\pi GM_S R^3\rho}{3r^2}.$$

b) Assuming that the sun's radiation is intercepted by the particle's cross-section, we can write the force on the particle as:

$$F = \frac{IA}{c} = \frac{L}{4\pi r^2}\cdot\frac{\pi R^2}{c} = \frac{LR^2}{4cr^2}.$$

c) So if the force of gravity and the force from the radiation pressure on a particle from the sun are equal, we can solve for the particle's radius:

$$F_G = F \Rightarrow \frac{4\pi GM_S R^3\rho}{3r^2} = \frac{LR^2}{4cr^2} \Rightarrow R = \frac{3L}{16\pi GM_S\rho c}.$$

$$\Rightarrow R = \frac{3(3.9\times10^{26}\ \text{W})}{16\pi\,(6.7\times10^{-11}\ \ \text{N}\cdot\text{m}^2/\text{kg}^2)\,(2.0\times10^{30}\ \text{kg})\,(3000\ \text{kg/m}^3)\,(3.0\times10^{8}\ \text{m/s})}$$

$$\Rightarrow R = 1.9\times10^{-7}\ \text{m}.$$

d) If the particle has a radius smaller than that found in part (c), then the radiation pressure overcomes the gravitational force and results in an acceleration away from the sun, thus removing all such particles from the solar system.

32.52: a) The momentum transfer is always greatest when reflecting surfaces are used (consider a ball colliding with a wall—the wall exerts a greater force if the ball rebounds rather than sticks). So in solar sailing one would want to use a reflecting sail.

b) The equation for repulsion comes from balancing the gravitational force and the force from the radiation pressure. As seen in Problem 32.51, the latter is:

$$F_{\text{rad}} = \frac{2LA}{4\pi r^2 c}.\ \text{Thus}: F_G = F_{\text{rad}} \Rightarrow \frac{GM_S m}{r^2} = \frac{2LA}{4\pi r^2 c} \Rightarrow A = \frac{4\pi GM_S mc}{2L}$$

$$\Rightarrow A = \frac{4\pi(6.7\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\,(2.0\times10^{30}\ \text{kg})\,(10000\,\text{kg})\,(3.0\times10^{8}\ \text{m/s})}{(2)3.9\times10^{26}\ \text{W}}$$

$$\Rightarrow A = 6.48\times10^{6}\ \text{m}^2 = 6.48\ \text{km}^2 = \frac{6.48\ \text{km}^2}{(1.6\ \text{km/mile})^2} = 2.53\ \text{mi}^2$$

c) This answer is independent of the distance from the sun since both the gravitational force and the radiation pressure go down like one over the distance squared, and thus the distance cancels out of the problem.

32.53: a) $\left[\dfrac{q^2a^2}{6\pi\varepsilon_0 c^3}\right] = \dfrac{C^2(\text{m/s}^2)^2}{(C^2/\text{N}\cdot\text{m}^2)\,(\text{m/s})^3} = \dfrac{\text{Nm}}{\text{s}} = \dfrac{\text{J}}{\text{s}} = \text{W} = \left[\dfrac{dE}{dt}\right].$

b) For a proton moving in a circle, the acceleration can be rewritten:

$$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(6.00\times10^{6}\ \text{eV})\,(1.6\times10^{-19}\ \text{J/eV})}{(1.67\times10^{-27}\ \text{kg})\,(0.75\,\text{m})} = 1.53\times10^{15}\ \text{m/s}^2.$$

The rate at which it emits energy because of its acceleration is:

$$\frac{dE}{dt} = \frac{q^2a^2}{6\pi\varepsilon_0 c^3} = \frac{(1.6\times10^{-19}\,\text{C})^2(1.53\times10^{15}\ \text{m/s}^2)^2}{6\pi\varepsilon_0(3.0\times10^{8}\ \text{m/s})^3} = 1.33\times10^{-23}\ \text{J/s}$$

$$= 8.32\times10^{-5}\ \text{eV/s}.$$

So the fraction of its energy that it radiates every second is:

$$\frac{(dE/dt)(1\,\text{s})}{E} = \frac{8.32\times10^{-5}\ \text{eV}}{6.00\times10^{6}\ \text{eV}} = 1.39\times10^{-11}.$$

c) Carrying out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses:

$$E_e = E_p\frac{m_e}{m_p} = (6.00\times10^{6}\ \text{eV})\frac{(9.11\times10^{-31}\ \text{kg})}{(1.67\times10^{-27}\ \text{kg})} = 3273\,\text{eV}.$$

So the fraction of its energy that it radiates every second is:

$$\frac{(dE/dt)(1\,\text{s})}{E} = \frac{8.32\times10^{-5}\ \text{eV}}{3273\,\text{eV}} = 2.54\times10^{-8}.$$

32.54: For the electron in the classical hydrogen atom, its acceleration is:

$$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(13.6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(9.11\times10^{-31}\ \text{kg})(5.29\times10^{-11}\ \text{m})} = 9.03\times10^{22}\ \text{m/s}^2.$$

Then using the formula for the rate of energy emission given in Pr. (33-49):

$$\frac{dE}{dt} = \frac{q^2a^2}{6\pi\varepsilon_0 c^3} = \frac{(1.60\times10^{-19}\,\text{C})^2(9.03\times10^{22}\ \text{m/s}^2)^2}{6\pi\varepsilon_0(3.00\times10^8\ \text{m/s})^3}$$

$\Rightarrow \dfrac{dE}{dt} = 4.64\times10^{-8}\ \text{J/s} = 2.89\times10^{11}\ \text{eV/s},$ which means that the electron would almost immediately lose all its energy!

32.55: a) $E_y(x,t) = E_{\max}\,\text{e}^{-k_c x}\sin(k_c x - \omega t).$

$$\frac{\partial E_y}{\partial x} = E_{\max}\ (-k_c)\text{e}^{-k_c x}\sin(k_c x - \omega t) + E_{\max}\ (+k_c)\text{e}^{-k_c x}\cos(k_c x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max}\ (+k_c^2)\text{e}^{-k_c x}\sin(k_c x - \omega t) + E_{\max}\ (-k_c^2)\text{e}^{-k_c x}\cos(k_c x - \omega t)$$

$$+ E_{\max}\ (-k_c^2)\text{e}^{-k_c x}\cos(k_c x - \omega t) + E_{\max}\ (-k_c^2)\text{e}^{-k_c x}\sin(k_c x - \omega t)$$

$$= -2E_{\max}k_c^2\text{e}^{-k_c x}\cos(k_c x - \omega t).$$

$$\frac{\partial E_y}{\partial t} = E_{\max}\ \text{e}^{-k_c x}\ \omega\cos(k_c x - \omega t).$$

Set $\dfrac{\partial^2 E_y}{\partial x^2} = \dfrac{\mu\partial E_y}{\rho\partial t} \Rightarrow 2E_{\max}k_c^2\text{e}^{-k_c x}\cos(k_c x - \omega t) = E_{\max}\text{e}^{-k_c x}\ \omega\cos(k_c x - \omega t).$ This will only be true if $\dfrac{2k_c^2}{\omega} = \dfrac{\mu}{\rho}$ or $k_c = \sqrt{\dfrac{\omega\mu}{2\rho}}.$

b) The hint basically answers the question.

c) $E_y = \dfrac{E_{y0}}{\text{e}} \Rightarrow k_c x = 1,\ x = \dfrac{1}{k_c} = \sqrt{\dfrac{2\rho}{\omega\mu}} = \sqrt{\dfrac{2(1.72\times10^{-8}\ \Omega\text{m})}{2\pi(1.0\times10^6\ \text{Hz})\mu_0}} = 6.60\times10^{-5}\,\text{m}.$

33.1: a) $v = \dfrac{c}{n} = \dfrac{3.00 \times 10^{8} \text{ m/s}}{1.47} = 2.04 \times 10^{8} \text{ m/s}$.

b) $\lambda = \dfrac{\lambda_0}{n} = \dfrac{(6.50 \times 10^{-7} \text{ m})}{1.47} = 4.42 \times 10^{-7} \text{ m}.$

33.2: a) $\lambda_{\text{vacuum}} = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8} \text{ m/s}}{5.80 \times 10^{14} \text{ Hz}} = 5.17 \times 10^{-7} \text{ m}.$

b) $\lambda_{\text{glass}} = \dfrac{c}{fn} = \dfrac{3.00 \times 10^{8} \text{ m/s}}{(5.80 \times 10^{14} \text{ Hz})(1.52)} = 3.40 \times 10^{-7} \text{ m}.$

33.3: a) $n = \dfrac{c}{v} = \dfrac{3.00 \times 10^{8} \text{ m/s}}{1.94 \times 10^{8} \text{ m/s}} = 1.54.$

b) $\lambda_0 = n\lambda = (1.54)(3.55 \times 10^{-7} \text{ m}) = 5.47 \times 10^{-7} \text{ m}.$

33.4: $\lambda_{\text{water}} n_{\text{water}} = \lambda_{\text{Benzene}} n_{\text{Benzene}} \Rightarrow \lambda_{CS_2} = \dfrac{\lambda_{\text{water}} n_{\text{water}}}{n_{\text{Benzene}}} = \dfrac{(4.38 \times 10^{-7} \text{ m})(1.333)}{1.501}$

33.5 : a) Incident and reflected angles are always equal $\Rightarrow \theta_r' = \theta_a' = 47.5°.$

b) $\theta_b' = \dfrac{\pi}{2} - \theta_b = \dfrac{\pi}{2} - \arcsin\left(\dfrac{n_a}{n_b} \sin \theta_a\right) = \dfrac{\pi}{2} - \arcsin\left(\dfrac{1.00}{1.66} \sin 42.5°\right) = 66.0°.$

33.6: $v = \dfrac{d}{t} = \dfrac{2.50 \text{ m}}{11.5 \times 10^{-9} \text{s}} = 2.17 \times 10^{8} \text{ m/s}$

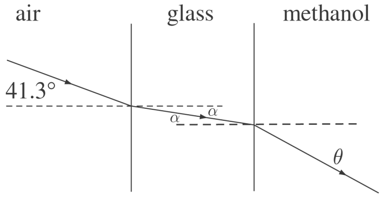
$n = \dfrac{c}{v} = \dfrac{3.00 \times 10^{8} \text{ m/s}}{2.17 \times 10^{8} \text{ m/s}} = 1.38$

33.7: $n_a \sin \theta_a = n_b \sin \theta_b$

$n_b = n_a \left(\dfrac{\sin \theta_a}{\sin \theta_b}\right) = 1.00 \left(\dfrac{\sin 62.7°}{\sin 48.1°}\right) = 1.194$

$n = c/v$ so $v = c/n = (3.00 \times 10^{8} \text{ m/s})/1.194 = 2.51 \times 10^{8} \text{ m/s}$

33.8 (a)



Apply Snell's law at both interfaces.

33.9: a) Let the light initially be in the material with refractive index n_a and let the third and final slab have refractive index n_b Let the middle slab have refractive index n_1

$$\text{1st interface}: n_a \sin\theta_a = n_1 \sin\theta_1$$

$$\text{2nd interface}: n_1 \sin\theta_1 = n_b \sin\theta_b$$

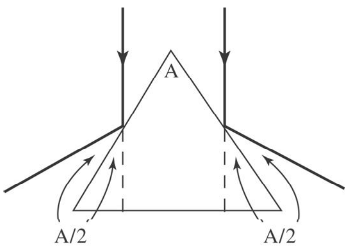
$$\text{Combining the two equations gives } n_a \sin\theta_a = n_b \sin\theta_b.$$

b) For N slabs, where the first slab has refractive index n_a and the final slab has refractive index n_b, $n_a \sin\theta_a = n_1 \sin\theta_1$, $n_1 \sin\theta_1 = n_2 \sin\theta_2, \ldots,\ n_{N-2} \sin\theta_{N-2} = n_b \sin\theta_b$. This gives $n_a \sin\theta_a = n_b \sin\theta_b$. The final direction of travel depends on the angle of incidence in the first slab and the indicies of the first and last slabs.

33.10: a) $\theta_{\text{water}} = \arcsin\left(\dfrac{n_{\text{air}}}{n_{\text{water}}} \sin\theta_{\text{air}}\right) = \arcsin\left(\dfrac{1.00}{1.33} \sin 35.0°\right) = 25.5°.$

b) This calculation has no dependence on the glass because we can omit that step in the chain: $n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{glass}} \sin\theta_{\text{glass}} = n_{\text{water}} \sin\theta_{\text{water}}$.

33.11: As shown below, the angle between the beams and the prism is A/2 and the angle between the beams and the vertical is A, so the total angle between the two beams is 2A.



33.12: Rotating a mirror by an angle θ while keeping the incoming beam constant leads to an increase in the incident angle ϕ by θ. Therefore the angle between incoming and outgoing beams becomes $2\theta + 2\phi$ where an additional deflection of 2θ arose from the mirror rotation.

33.13: $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.70}{1.58}\sin 62.0°\right) = 71.8°.$

33.14: $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.33}{1.52}\sin 45.0°\right) = 38.2.$ But this is the angle from the normal to the surface, so the angle from the vertical is an additional $15°$ because of the tilt of the surface. Therefore the angle is $53.2°$.

33.15: a) Going from the liquid into air:

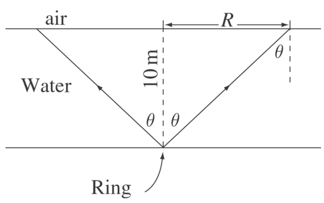
$$\frac{n_b}{n_a} = \sin\theta_{\text{crit}} \Rightarrow n_a = \frac{1.00}{\sin 42.5°} = 1.48.$$

$$\text{So}: \theta_{\text{b}} = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.48}{1.00}\sin 35.0°\right) = 58.1°.$$

b) Going from air into the liquid:

$$\theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.00}{1.48}\sin 35.0°\right) = 22.8°.$$

33.16:



If $\theta >$ critical angle, no light escapes,

so for the largest circle, $\theta = \theta_c$

$$n_{\text{w}} \sin\theta_{\text{c}} = n_{\text{air}} \sin 90° = (1.00)(1.00) = 1.00$$

$$\theta_{\text{c}} = \sin^{-1}(1/n_{\text{w}}) = \sin^{-1}\frac{1}{1.333} = 48.6°$$

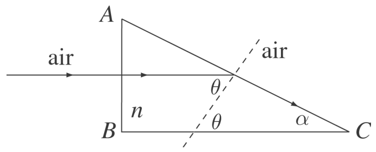
$$\tan\theta_{\text{c}} = R/10.0\text{ m} \rightarrow R = (10.0\text{ m})\tan 48.6° = 11.3\text{ m}$$

$$A = \pi R^2 = \pi\,(13.3\text{ m})^2 = 401\text{ m}^2$$

33.17: For glass $\rightarrow$ water, $\theta_{\text{crit}} = 48.7°$

$$n_a \sin\theta_{\text{crit}} = n_b \sin 90°, \text{ so } n_a = \frac{n_b}{\sin\theta_{\text{crit}}} = \frac{1.333}{\sin 48.7°} = 1.77$$

33.18: (a)



Total internal reflection occurs at AC: $n \sin\theta = (1.00)\sin 90° = 1.00$

$$(1.52)\sin\theta = 1.00$$

$$\theta = 41.1°$$

$$\alpha + \theta = 90° \rightarrow \alpha = 90° - 41.1° = 48.9°$$

If α is larger, θ is smaller and thus less than the critical angle, so this answer is the *largest* that α can be.

(b) Same approach as in (a), except AC is now a glass-water boundary.

$$n \sin\theta = n_{\text{w}} \sin 90° = 1.333$$

$$1.52 \sin\theta = 1.333$$

$$\theta = 61.3°$$

$$\alpha = 90° - 61.3° = 28.7°$$

33.19: a) The slower the speed of the wave, the larger the index of refraction—so air has a larger index of refraction than water.

b) $\theta_{\text{crit}} = \arcsin\left(\dfrac{n_b}{n_a}\right) = \arcsin\left(\dfrac{v_{\text{air}}}{v_{\text{water}}}\right) = \arcsin\left(\dfrac{344 \text{ m/s}}{1320 \text{ m/s}}\right) = 15.1°.$

c) Air. For total internal reflection, the wave must go from higher to lower index of refraction—in this case, from air to water.

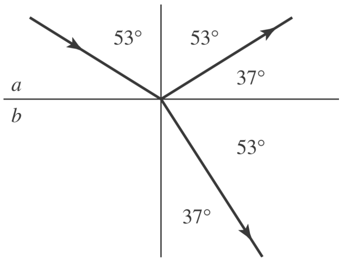
33.20 : $\theta_{\text{crit}} = \arcsin\left(\dfrac{n_b}{n_a}\right) = \arcsin\left(\dfrac{1.00}{2.42}\right) = 24.4°.$

33.21 : a) $\tan\theta_p = \dfrac{n_b}{n_a} = \tan 54.5° = 1.40 \Rightarrow n_b = 1.40.$

b) $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.00}{1.40}\sin 54.5°\right) = 35.6°.$

33.22 : From the picture on the next page, $\theta_r = 37.0°$, and so :

$$n_b = n_a \frac{\sin\theta_a}{\sin\theta_b} = 1.33 \frac{\sin 53°}{\sin 37°} = 1.77.$$



33.23 : a) $\tan\theta_p = \dfrac{n_b}{n_a} \Rightarrow n_a = \dfrac{n_b}{\tan\theta_p} = \dfrac{1.00}{\tan 31.2°} = 1.65.$

b) $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.65}{1.00}\sin 31.2°\right) = 58.7°.$

33.24 : a) In air $\theta_p = \arctan\left(\dfrac{n_b}{n_a}\right) = \arctan\left(\dfrac{1.66}{1.00}\right) = 58.9°.$

b) In water $\theta_p = \arctan\left(\dfrac{n_b}{n_a}\right) = \arctan\left(\dfrac{1.66}{1.33}\right) = 51.3°.$

33.25 : a) Through the first filter : $I_1 = \dfrac{1}{2} I_0.$

The second filter : $I_2 = \dfrac{1}{2} I_0 \cos^2(41.0°) = 0.285 I_0.$

b) The light is linearly polarized.

33.26 : a) $I = I_{\max}\cos^2\phi \Rightarrow I = I_{\max}\cos^2(22.5°) = 0.854 I_{\max}.$

b) $I = I_{\max}\cos^2\phi \Rightarrow I = I_{\max}\cos^2(45.0°) = 0.500 I_{\max}.$

c) $I = I_{\max}\cos^2\phi \Rightarrow I = I_{\max}\cos^2(67.5°) = 0.146 I_{\max}.$

33.27 : After the first filter the intensity is $\frac{1}{2} I_0 = 10.0 \text{ W/m}^2$ and the light is polarized along the axis of the first filter. The intensity after the second filter is $I = I_0 \cos^2\phi$, where $I_0 = 10.0 \text{ W/m}^2$ and $\omega = 62.0° - 25.0° = 37.0°$. Thus, $I = 6.38 \text{ W/m}^2$.

33.28: Let the intensity of the light that exits the first polarizer be I_1, then, according to repeated application of Malus' law, the intensity of light that exits the third polarizer is

$$75.0\ \text{W/cm}^2 = I_1 \cos^2(23.0°)\cos^2(62.0° - 23.0°).$$

So we see that $I_1 = \dfrac{75.0\ \text{W/cm}^2}{\cos^2(23.0°)\cos^2(62.0° - 23.0°)}$, which is also the intensity incident on the third polarizer after the second polarizer is removed. Thus, the intensity that exits the third polarizer after the second polarizer is removed is

$$\frac{75.0\ \text{W/cm}^2\ \cos^2(62.0°)}{\cos^2(23.0°)\cos^2(62.0° - 23.0°)} = 32.3\ \text{W/cm}^2.$$

33.29 : a) $I_1 = \frac{1}{2}I_0,\ I_2 = \frac{1}{2}I_0\cos^2(45.0°) = 0.250 I_0,\ I_3 = I_2\cos^2(45.0°) = 0.125 I_0.$

b) $I_1 = \frac{1}{2}I_0,\ I_2 = \frac{1}{2}I_0\cos^2(90.0°) = 0.$

33.30: a) All the electric field is in the plane perpendicular to the propagation direction, and maximum intensity through the filters is at 90° to the filter orientation for the case of minimum intensity. Therefore rotating the second filter by 90° when the situation originally showed the maximum intensity means one ends with a dark cell.

b) If filter P_1 is rotated by 90°, then the electric field oscillates in the direction pointing toward the P_2 filter, and hence no intensity passes through the second filter: see a dark cell.

c) Even if P_2 is rotated back to its original position, the new plane of oscillation of the electric field, determined by the first filter, allows zero intensity to pass through the second filter.

33.31: Consider three mirrors, M_1 in the (x,y)-plane, M_2 in the (y,z)-plane, and M_3 in the (x,z)-plane. A light ray bouncing from M_1 changes the sign of the z-component of the velocity, bouncing from M_2 changes the x-component, and from M_3 changes the y-component. Thus the velocity, and hence also the path, of the light beam flips by 180°

33.32 : a) $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{v_b}{v_a}\sin\theta_a\right) = \arcsin\left(\dfrac{1480}{344}\sin 9.73°\right) = 46.6°.$

b) $\theta_{\text{crit}} = \arcsin\left(\dfrac{v_a}{v_b}\right) = \arcsin\left(\dfrac{344}{1480}\right) = 13.4°.$

33.33: a) $n_1\sin\theta_1 = n_2\sin\theta_2$ and $n_2\sin\theta_2 = n_3\sin\theta_3$, so $n_1\sin\theta_1 = n_3\sin\theta_3$
$\sin\theta_3 = (n_1\sin\theta_1)/n_3$ b) $n_3\sin\theta_3 = n_2\sin\theta_2$ and $n_2\sin\theta_2 = n_1\sin\theta_1$, so $n_1\sin\theta_1 =$ $n_3\sin\theta_3$ and the light makes the same angle with respect to the noral in the material with n_1 as it did in part (a).

c) For reflection, $\theta_r = \theta_a$. These angles are still equal if θ_r becomes the incident angle; reflected rays are also reversible.

33.34: It takes the light an additional 4.2 ns to travel 0.840 m after the glass slab is inserted into the beam. Thus,

$$\frac{0.840\ \text{m}}{c/n} - \frac{0.840\ \text{m}}{c} = (n-1)\frac{0.840\ \text{m}}{c} = 4.2\ \text{ns}.$$

We can now solve for the index of refraction:

$$n = \frac{(4.2\times 10^{-9}\ \text{s})\,(3.00\times 10^{8}\ \text{m/s})}{0.840\ \text{m}} + 1 = 2.50.$$

The wavelength inside of the glass is

$$\lambda' = \frac{490\ \text{nm}}{2.50} = 196\ \text{nm} \approx 200\ \text{nm}.$$

33.35: $\theta_b = 90° - \arcsin\left(\frac{n_a}{n_b}\right) = 90° - \arcsin\left(\frac{1.00}{1.38}\right) = 43.6°.$

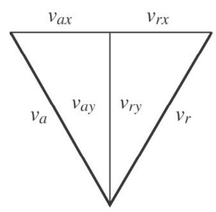
But $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow \theta_a = \arcsin\left(\frac{n_b \sin\theta_b}{n_a}\right) = \arcsin\left(\frac{1.38\sin(43.6°)}{1.00}\right) = 72.1°.$

33.36: $n_a \sin\theta_a = n_b \sin\theta_b = n_b \sin\left(\frac{\theta_a}{2}\right)$

$$\Rightarrow (1.00)\sin\theta_a = \sin 2\left(\frac{\theta_a}{2}\right) = 2\sin\left(\frac{\theta_a}{2}\right)\cos\left(\frac{\theta_a}{2}\right) = (1.80)\sin\left(\frac{\theta_a}{2}\right)$$

$$\Rightarrow 2\cos\left(\frac{\theta_a}{2}\right) = (1.80) \Rightarrow \theta_a = 2\arccos\left(\frac{1.80}{2}\right) = 51.7°.$$

33.37: The velocity vector "maps out" the path of the light beam, so the geometry as shown below leads to:



$v_a = v_r$ and $\theta_a = \theta_r \Rightarrow \arccos\left(\frac{v_{a_y}}{v_a}\right) = \arccos\left(\frac{v_{r_y}}{v_r}\right) \Rightarrow v_{a_y} = -v_{r_y}$, with the minus sign chosen by inspection. Similarly, $\Rightarrow \arcsin\left(\frac{v_{a_x}}{v_a}\right) = \arcsin\left(\frac{v_{r_x}}{v_r}\right) \Rightarrow v_{a_x} = v_{r_x}$.

33.38: $\#\lambda = (\#\lambda)_{\text{air}} + (\#\lambda)_{\text{glass}} = \dfrac{d_{\text{air}}}{\lambda} + \dfrac{d_{\text{glass}}}{\lambda} n = \dfrac{(0.0180\text{ m} - 0.00250\text{ m})}{5.40\times10^{-7}\text{ m}} + \dfrac{0.00250\text{ m}}{5.40\times10^{-7}\text{ m}} \times$ $(1.40) = 3.52\times10^{4}.$

33.39: $\theta_{\text{crit}} = \arctan\left(\dfrac{(0.00534\text{ m})/2}{0.00310\text{ m}}\right) = 40.7° = \arcsin\left(\dfrac{n_b}{n_a}\right) = \arcsin\left(\dfrac{1.0}{n}\right) \Rightarrow n = \dfrac{1}{\sin(40.7}$

Note: The radius is reduced by a factor of two since the beam must be incident at θ_{crit}, the on the glass-air interface to create the ring.

33.40: $\theta_a = \arctan\left(\dfrac{1.5\text{ m}}{1.2\text{ m}}\right) = 51°$

$$\Rightarrow \theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.00}{1.33}\sin 51°\right) = 36°.$$

So the distance along the bottom of the pool from directly below where the light enters to where it hits the bottom is:

$$x = (4.0\text{ m})\tan\theta_b = (4.0\text{ m})\tan 36° = 2.9\text{ m}.$$

$$\Rightarrow x_{\text{total}} = 1.5\text{ m} + x = 1.5\text{ m} + 2.9\text{ m} = 4.4\text{ m}.$$

33.41 $\theta_a = \arctan\left(\dfrac{8.0\text{ cm}}{16.0\text{ cm}}\right) = 27°$ and $\theta_b = \arctan\left(\dfrac{4.0\text{ cm}}{16.0\text{ cm}}\right) = 14°.$

So, $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow n_b = \left(\dfrac{n_a \sin\theta_a}{\sin\theta_b}\right) = \left(\dfrac{1.00\sin 27°}{\sin 14°}\right) = 1.8.$

33.42: The beam of light will emerge at the same angle as it entered the fluid as seen by following what happens via Snell's Law at each of the interfaces. That is, the emergent beam is at $42.5°$ from the normal.

33.43: a) $\theta_i = \arcsin\left(\dfrac{n_a \sin 90°}{n_w}\right) = \arcsin\left(\dfrac{1.000}{1.333}\right) = 48.61°.$

The ice does not come into the calculation since $n_{\text{air}} \sin 90° = n_{\text{ice}} \sin\theta_c = n_w \sin\theta_i$.

b) Same as part (a).

33.44: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow n_a = \left(\dfrac{n_b \sin\theta_b}{\sin\theta_a}\right) = \left(\dfrac{1.33\sin 90°}{\sin 45°}\right) = 1.9.$

33.45: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow \theta_b = \arcsin\left(\frac{n_a \sin\theta_a}{n_b}\right)$

$= \arcsin\left(\frac{1.66\sin(25.0°)}{1.00}\right) = 44.6°.$

So the angle below the horizontal is $\theta_b - 25.0° = 44.6° - 25.0° = 19.6°$, and thus the angle between the two emerging beams is $39.2°$.

33.46: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow n_a = \left(\frac{n_b \sin\theta_b}{\sin\theta_a}\right) = \left(\frac{1.62\sin 60°}{\sin 90°}\right) = 1.40.$

33.47: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow n_a = \left(\frac{n_b \sin\theta_b}{\sin\theta_a}\right) = \left(\frac{1.52\sin 57.2°}{\sin 90°}\right) = 1.28.$

33.48: a) For light in air incident on a parallel-faced plate, Snell's Law yields:
$n\sin\theta_a = n'\sin\theta_b' = n'\sin\theta_b = n\sin\theta_a' \Rightarrow \sin\theta_a = \sin\theta_a' \Rightarrow \theta_a = \theta_a'.$

b) Adding more plates just adds extra steps in the middle of the above equation that always cancel out. The requirement of parallel faces ensures that the angle $\theta_n' = \theta_{n'}$ and the chain of equations can continue.

c) The lateral displacement of the beam can be calculated using geometry:

$$d = L\sin(\theta_a - \theta_b') \text{ and } L = \frac{t}{\cos\theta_b'} \Rightarrow d = \frac{t\sin(\theta_a - \theta_b')}{\cos\theta_b'}.$$

d) $\theta_b' = \arcsin\left(\frac{n\sin\theta_a}{n'}\right) = \arcsin\left(\frac{\sin 66.0°}{1.80}\right) = 30.5°$

$$\Rightarrow d = \frac{(2.40\text{ cm})\sin(66.0° - 30.5°)}{\cos 30.5°} = 1.62\text{ cm}.$$

33.49: a) For sunlight entering the earth's atmosphere from the sun BELOW the horizon, we can calculate the angle δ as follows:

$n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow (1.00)\sin\theta_a = n\sin\theta_b$, where $n_b = n$ is the atmosphere's index of refraction. But the geometry of the situation tells us:

$$\sin\theta_b = \frac{R}{R+h} \Rightarrow \sin\theta_a = \frac{nR}{R+h} \Rightarrow \delta = \theta_a - \theta_b = \arcsin\left(\frac{nR}{R+h}\right) - \arcsin\left(\frac{R}{R+h}\right).$$

b) $\delta = \arcsin\left(\frac{(1.0003)(6.4\times10^6\text{m})}{6.4\times10^6\text{ m} + 2.0\times10^4\text{ m})}\right) - \arcsin\left(\frac{6.4\times10^6\text{ m}}{64.\times10^6\text{ m} + 2.0\times10^4\text{ m}}\right) \Rightarrow$

$\delta = 0.22°$. This is about the same as the angular radius of the sun, $0.25°$.

33.50: A quarter-wave plate shifts the phase of the light by $\theta = 90°$. Circularly polarized light is out of phase by $90°$, so the use of a quarter-wave plate will bring it back into phase, resulting in linearly polarized light.

33.51: a) $I = \frac{1}{2} I_0 \cos^2\theta \cos^2(90° - \theta) = \frac{1}{2} I_0 (\cos\theta \sin\theta)^2 = \frac{1}{8} I_0 \sin^2 2\theta.$

b) For maximum transmission, we need $2\theta = 90°$, so $\theta = 45°.$

33.52: a) The distance traveled by the light ray is the sum of the two diagonal segments: $d = \left(x^2 + y_1^2\right)^{1/2} + \left((l - x)^2 + y_2^2\right)^{1/2}.$

Then the time taken to travel that distance is just:

$$t = \frac{d}{c} = \frac{(x^2 + y_1^2)^{1/2} + \left((l - x)^2 + y_2^2\right)^{1/2}}{c}$$

b) Taking the derivative with respect to x of the time and setting it to zero yields:

$$\frac{dt}{dx} = \frac{1}{c}\frac{d}{dt}\left[(x^2 + y_1^2)^{1/2} + \left((l - x)^2 + y_2^2\right)^{1/2}\right]$$

$$\Rightarrow \frac{dt}{dx} = \frac{1}{c}\left[x(x^2 + y_1^2)^{-1/2} - (l - x)\left((l - x)^2 + y_2^2\right)^{-1/2}\right] = 0$$

$$\Rightarrow \frac{x}{\sqrt{x^2 + y_1^2}} = \frac{(l - x)}{\sqrt{(l - x)^2 + y_2^2}} \Rightarrow \sin\theta_1 = \sin\theta_2 \Rightarrow \theta_1 = \theta_2.$$

33.53: a) The time taken to travel from point A to point B is just:

$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l - x)^2}}{v_2}.$$

Taking the derivative with respect to x of the time and setting it to zero yields:

$$\frac{dt}{dx} = 0 = \frac{d}{dt}\left[\frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l - x)^2}}{v_2}\right] = \frac{x}{v_1\sqrt{h_1^2 + x^2}} - \frac{(l - x)}{v_2\sqrt{h_2^2 + (l - x)^2}}.$$

But $v_1 = \frac{c}{n_1}$ and $v_2 = \frac{c}{n_2} \Rightarrow \frac{n_1 x}{\sqrt{h_1^2 + x^2}} = \frac{n_2(l - x)}{\sqrt{h_2^2 + (l - x)^2}} \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2.$

33.54: a) n decreases with increasing λ, so n is smaller for red than for blue. So beam a is the red one.

b) The separation of the emerging beams is given by some elementary geometry.

$x = x_r - x_v = d\tan\theta_r - d\tan\theta_v \Rightarrow d = \dfrac{x}{\tan\theta_r - \tan\theta_v}$, where x is the vertical beam separation as they emerge from the glass $x = \dfrac{1.00\text{ mm}}{\sin 20°} = 2.92\text{ mm}$. From the ray geometry, we also have

$$\theta_r = \arcsin\left(\frac{\sin 70°}{1.61}\right) = 35.7° \text{ and } \theta_v = \arcsin\left(\frac{\sin 70°}{1.66}\right) = 34.5°\text{, so :}$$

$$d = \frac{x}{\tan\theta_r - \tan\theta_v} = \frac{2.92\text{ mm}}{\tan 35.7° - \tan 34.5°} = 9\text{ cm.}$$

33.55: a) $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow \sin\theta_a = n_b \sin\dfrac{A}{2}.$

But $\theta_a = \dfrac{A}{2} + \alpha \Rightarrow \sin\left(\dfrac{A}{2} + \alpha\right) = \sin\dfrac{A + 2\alpha}{2} = n\sin\dfrac{A}{2}.$

At each face of the prism the deviation is α, so $2\alpha = \delta \Rightarrow \sin\dfrac{A+\delta}{2} = n\sin\dfrac{A}{2}.$

b) From part (a), $\delta = 2\arcsin\left(n\sin\dfrac{A}{2}\right) - A$

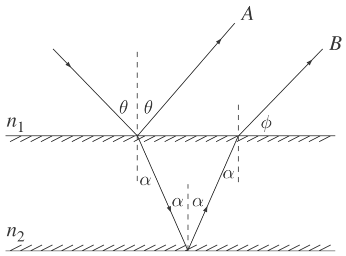
$$\Rightarrow \delta = 2\arcsin\left((1.52)\sin\frac{60.0°}{2}\right) - 60.0° = 38.9.$$

c) If two colors have different indices of refraction for the glass, then the deflection angles for them will differ:

$$\delta_{\text{red}} = 2\arcsin\left((1.61)\sin\frac{60.0°}{2}\right) - 60.0° = 47.2°$$

$$\delta_{\text{violet}} = 2\arcsin\left((1.66)\sin\frac{60.0°}{2}\right) - 60.0° = 52.2° \Rightarrow \Delta\delta = 52.2° - 47.2° = 5.0°.$$

33.56:



Direction of ray A: θ by law of reflection.

Direction of ray B:

At upper surface: $n_1 \sin\theta = n_2 \sin\alpha$

The lower surface reflects at α. Ray B returns to upper surface at angle of incidence $\alpha : n_2 \sin\alpha = n_1 \sin\phi$

Thus

$$n_1 \sin\theta = n_1 \sin\phi$$
$$\phi = \theta$$

Therefore rays A and B are parallel.

33.57: Both *l*-leucine and *d*-glutamic acid exhibit linear relationships between concentration and rotation angle. The dependence for *l*-leucine is:

Rotation angle $(°) = (-0.11° 100\,\text{ml/g})C(\text{g/100 ml})$, and for d-glutamic acid is:

Rotation angle $(°) = (0.124° 100\,\text{ml/g})C(\text{g/100 ml})$.

33.58: a) A birefringent material has different speeds (or equivalently, wavelengths) in two different directions, so:

$$\lambda_1 = \frac{\lambda_0}{n_1} \text{ and } \lambda_2 = \frac{\lambda_0}{n_2} \Rightarrow \frac{D}{\lambda_1} = \frac{D}{\lambda_2} + \frac{1}{4} \Rightarrow \frac{n_1 D}{\lambda_0} = \frac{n_2 D}{\lambda_0} + \frac{1}{4} \Rightarrow D = \frac{\lambda_0}{4(n_1 - n_2)}.$$

b) $D = \dfrac{\lambda_0}{4(n_1 - n_2)} = \dfrac{5.89 \times 10^{-7}\ \text{m}}{4(1.875 - 1.635)} = 6.14 \times 10^{-7}\,\text{m}.$

33.59: a) The maximum intensity from the table is at $\theta = 35°,$ so the polarized component of the wave is in that direction (or else we would not have maximum intensity at that angle).

b) At $\theta = 40° : I = 24.8 \text{ W/m}^2 = \frac{1}{2} I_0 + I_p \cos^2(40° - 35°)$

$\Rightarrow 24.8 \text{ W/m}^2 = 0.500\, I_0 + 0.996 I_p$ (1).

At $\theta = 120° : I = 5.2 \text{ W/m}^2 = \frac{1}{2} I_0 + I_p \cos^2(120° - 35°)$

$\Rightarrow 5.2 \text{ W/m}^2 = 0.500 I_0 + 7.60 \times 10^{-3} I_p$ (2).

Solving equations (1) and (2) we find:

$\Rightarrow 19.6 \text{ W/m}^2 = 0.989 I_p \Rightarrow I_p = 19.8 \text{ W/m}^2.$

Then if one subs this back into equation (1), we find:

$5.049 = 0.500 I_0 \Rightarrow I_0 = 10.1 \text{ W/m}^2.$

33.60: a) To let the most light possible through N polarizers, with a total rotation of $90°$, we need as little shift from one polarizer to the next. That is, the angle between successive polarizers should be constant and equal to $\frac{\pi}{2N}$. Then:

$$I_1 = I_0 \cos^2\left(\frac{\pi}{2N}\right), I_2 = I_0 \cos^4\left(\frac{\pi}{2N}\right), \cdots \Rightarrow I = I_N = I_0 \cos^{2N}\left(\frac{\pi}{2N}\right).$$

b) If $n >> 1,\ \cos^n \theta = \left(1 - \frac{\theta^2}{2} + \cdots\right)^n = 1 - \frac{n}{2}\theta^2 + \cdots$

$$\Rightarrow \cos^{2N}\left(\frac{\pi}{2N}\right) \approx 1 - \frac{(2N)}{2}\left(\frac{\pi}{2N}\right)^2 = 1 - \frac{\pi^2}{4N} \approx 1, \text{ for large } N.$$

33.61: a) Multiplying Eq. (1) by $\sin\beta$ and Eq. (2) by $\sin\alpha$ yields:

$$(1): \frac{x}{a}\sin\beta = \sin\omega t\cos\alpha\sin\beta - \cos\omega t\sin\alpha\sin\beta$$

$$(2): \frac{y}{a}\sin\alpha = \sin\omega t\cos\beta\sin\alpha - \cos\omega t\sin\beta\sin\alpha$$

Subtracting yields: $\dfrac{x\sin\beta - y\sin\alpha}{a} = \sin\omega t(\cos\alpha\sin\beta - \cos\beta\sin\alpha).$

b) Multiplying Eq. (1) by $\cos\beta$ and Eq. (2) by $\cos\alpha$ yields:

$$(1): \frac{x}{a}\cos\beta = \sin\omega t\cos\alpha\cos\beta - \cos\omega t\sin\alpha\cos\beta$$

$$(2): \frac{y}{a}\cos\alpha = \sin\omega t\cos\beta\cos\alpha - \cos\omega t\sin\beta\cos\alpha$$

Subtracting yields: $\dfrac{x\cos\beta - y\cos\alpha}{a} = -\cos\omega t(\sin\alpha\cos\beta - \sin\beta\cos\alpha).$

(c) Squaring and adding the results of parts (a) and (b) yields:

$$(x\sin\beta - y\sin\alpha)^2 + (x\cos\beta - y\cos\alpha)^2 = a^2(\sin\alpha\ \cos\beta - \sin\beta\cos\alpha)^2$$

(d) Expanding the left-hand side, we have:

$$x^2(\sin^2\beta + \cos^2\beta) + y^2(\sin^2\alpha + \cos^2\alpha) - 2xy\,(\sin\alpha\sin\beta + \cos\alpha\cos\beta)$$
$$= x^2 + y^2 - 2xy\,(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = x^2 + y^2 - 2xy\cos(\alpha - \beta).$$

The right-hand side can be rewritten:

$$a^2(\sin\alpha\cos\beta - \sin\beta\cos\alpha)^2 = a^2\sin^2(\alpha - \beta).$$

Therefore: $x^2 + y^2 - 2xy\cos(\alpha - \beta) = a^2\sin^2(\alpha - \beta).$

Or: $x^2 + y^2 - 2xy\cos\delta = a^2\sin^2\delta$, where $\delta = \alpha - \beta$.

(e) $\delta = 0: x^2 + y^2 - 2xy = (x - y)^2 = 0 \Rightarrow x = y$, which is a straight diagonal line.

$\delta = \dfrac{\pi}{4}: x^2 + y^2 - \sqrt{2}xy = \dfrac{a^2}{2}$, which is an ellipse.

$\delta = \dfrac{\pi}{2}: x^2 + y^2 = a^2$, which is a circle.

This pattern repeats for the remaining phase differences.

33.62: a) By the symmetry of the triangles, $\theta_b^A = \theta_a^B$, and $\theta_a^C = \theta_r^B = \theta_a^B = \theta_b^A$.

Therefore, $\sin\theta_b^C = n\sin\theta_a^C = n\sin\theta_b^A = \sin\theta_a^A = \theta_b^C = \theta_a^A$.

b) The total angular deflection of the ray is:

$\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_a^B + \theta_b^C - \theta_a^C = 2\theta_a^A - 4\theta_b^A + \pi.$

c) From Snell's Law, $\sin\theta_a^A = n\sin\theta_b^A \Rightarrow \theta_b^A = \arcsin\left(\frac{1}{n}\sin\theta_a^A\right)$

$$\Rightarrow \Delta = 2\theta_a^A - 4\theta_b^A + \pi = 2\theta_a^A - 4\arcsin\left(\frac{1}{n}\sin\theta_a^A\right) + \pi.$$

d) $\dfrac{d\Delta}{d\theta_a^A} = 0 = 2 - 4\dfrac{d}{d\theta_a^A}\left(\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \dfrac{4}{\sqrt{1-\sin^2\theta_1/n^2}}\cdot\left(\dfrac{\cos\theta_1}{n}\right)$

$$\Rightarrow 4\left(1-\frac{\sin^2\theta_1}{n^2}\right) = \left(\frac{16\cos^2\theta_1}{n^2}\right) \Rightarrow 4\cos^2\theta_1 = n^2 - 1 + \cos^2\theta_1$$

$$\Rightarrow 3\cos^2\theta_1 = n^2 - 1 \Rightarrow \cos^2\theta_1 = \frac{1}{3}(n^2-1).$$

e) For violet: $\theta_1 = \arccos\left(\sqrt{\frac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{3}(1.342^2-1)}\right) = 58.89°$

$$\Rightarrow \Delta_{\text{violet}} = 139.2° \Rightarrow \theta_{\text{violet}} = 40.8°.$$

For red: $\theta_1 = \arccos\left(\sqrt{\frac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{3}(1.330^2-1)}\right) = 59.58°$

$$\Rightarrow \Delta_{\text{red}} = 137.5° \Rightarrow \theta_{\text{red}} = 42.5°.$$

Therefore the color that appears higher is red.

33.63: a) For the secondary rainbow, we will follow similar steps to Pr. (34-51). The total angular deflection of the ray is:

$\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_b^A + \pi - 2\theta_b^A + \theta_a^A - \theta_b^A = 2\theta_a^A - 6\theta_b^A + 2\pi$, where we have used the fact from the previous problem that all the internal angles are equal and the two external equals are equal. Also using the Snell's Law relationship, we have:

$$\theta_b^A = \arcsin\left(\frac{1}{n}\sin\theta_a^A\right).$$

$$\Rightarrow \Delta = 2\theta_a^A - 6\theta_b^A + 2\pi = 2\theta_a^A - 6\arcsin\left(\frac{1}{n}\sin\theta_a^A\right) + 2\pi.$$

b) $\dfrac{d\Delta}{d\theta_a^A} = 0 = 2 - 6\dfrac{d}{d\theta_a^A}\left(\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \dfrac{6}{\sqrt{1-\sin^2\theta_2/n^2}}\cdot\left(\dfrac{\cos\theta_2}{n}\right)$

$$\Rightarrow n^2(1-\sin^2\theta_2/n^2) = (n^2 - 1 + \cos^2\theta_2) = 9\cos^2\theta_2 \Rightarrow \cos^2\theta_2 = \frac{1}{8}(n^2-1).$$

c) For violet: $\theta_2 = \arccos\left(\sqrt{\frac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{8}(1.342^2-1)}\right) = 71.55°$

$$\Rightarrow \Delta_{\text{violet}} = 233.2° \Rightarrow \theta_{\text{violet}} = 53.2°.$$

For red: $\theta_2 = \arccos\left(\sqrt{\frac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{8}(1.330^2-1)}\right) = 71.94°.$

$$\Rightarrow \Delta_{\text{red}} = 230.1° \Rightarrow \theta_{\text{red}} = 50.1°.$$

Therefore the color that appears higher is violet.

34.1: If up is the $+y$-direction and right is the $+x$-direction, then the object is at $(-x_0, -y_0)$, P_2' is at $(x_0, -y_0)$, and mirror 1 flips the y-values, so the image is at (x_0, y_0) which is P_3'.

34.2: Using similar triangles,

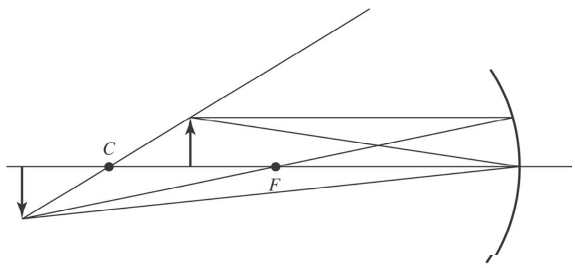
$$\frac{h_{\text{tree}}}{h_{\text{mirror}}} = \frac{d_{\text{tree}}}{d_{\text{mirror}}} \Rightarrow h_{\text{tree}} = h_{\text{mirror}} \frac{d_{\text{tree}}}{d_{\text{mirror}}} = 0.040 \text{ m} \frac{28.0 \text{ m} + 0.350 \text{ m}}{0.350 \text{ m}} = 3.24 \text{ m}.$$

34.3: A plane mirror does not change the height of the object in the image, nor does the distance from the mirror change. So, the image is 39.2 cm to the right of the mirror, and its height is 4.85 cm.

34.4: a) $f = \frac{R}{2} = \frac{34.0 \text{ cm}}{2} = 17.0 \text{ cm}.$

b) If the spherical mirror is immersed in water, its focal length is unchanged—it just depends upon the physical geometry of the mirror.

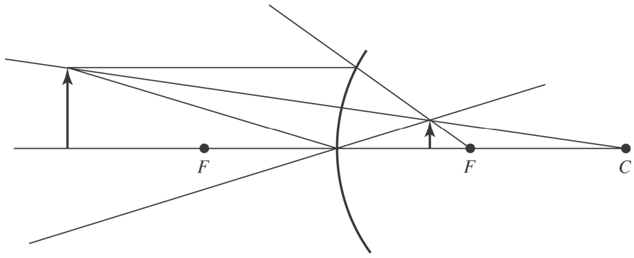
34.5: a)



b) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{2}{22.0 \text{ cm}} - \frac{1}{16.5 \text{ cm}} \Rightarrow s' = 33.0 \text{ cm},$ to the left of the mirror.

$$y' = -y\frac{s'}{s} = -(0.600 \text{ cm})\frac{33.0 \text{ cm}}{16.5 \text{ cm}} = -1.20 \text{ cm},$$ and the image is inverted and real.

34.6: a)



b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=-\frac{2}{22.0\text{ cm}}-\frac{1}{16.5\text{ cm}}\Rightarrow s'=-6.60\text{ cm},$ to the right of the mirror.

$y'=-y\frac{s'}{s}=-(0.600\text{ cm})\frac{(-6.60\text{ cm})}{16.5\text{ cm}}=0.240\text{ cm},$ and the image is upright and virtual.

34.7: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{1}{1.75\text{ m}}-\frac{1}{5.58\times10^{10}\text{ m}}\Rightarrow s'=-1.75\text{ m}.$

$$\Rightarrow m=-\frac{1.75}{5.58\times10^{10}}=3.14\times10^{-11}\Rightarrow y'=my$$
$$=(3.14\times10^{-11})(6794\times10^{3}\text{ m})=2.13\times10^{-4}\text{ m}.$$

34.8: $R=-3.00\text{ cm},\ \frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=-\frac{2}{3.00\text{ cm}}-\frac{1}{21.0\text{ cm}}\Rightarrow s'=-1.40\text{ cm}$ (in the ball). The magnification is $m=-\frac{s'}{s}=-\frac{-1.40\text{ cm}}{21.0\text{ cm}}=0.0667.$

34.9: a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{fs}\Rightarrow s'=\frac{sf}{s-f}$. Also $m=-\frac{s'}{s}=\frac{f}{f-s}$.

b) For $f>0,\ s>f\Rightarrow s'>0,$ so the image is always on the outgoing side and is real. The magnification is $m=\frac{f}{f-s}<0$, since $f<s$.

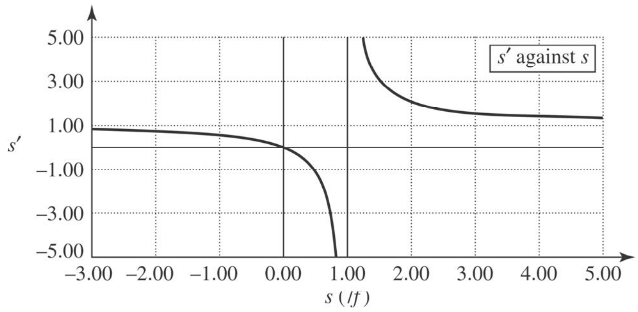
c) For $s\geq 2f\Rightarrow |m|<\left|\frac{f}{-f}\right|=1,$ which means the image is always smaller and inverted since the magnification is negative.

For $f<s<2f\Rightarrow 0<s-f<f\Rightarrow |m|>\frac{f}{f}=1.$

d) Concave mirror: $0<s<f\Rightarrow s'<0,$ and we have a virtual image to the right of the mirror. $|m|>\frac{f}{f}=1,$ so the image is upright and larger than the object.

34.10: For a convex mirror, $f<0\Rightarrow s'=\frac{sf}{s-f}=-\frac{s|f|}{s+|f|}<0.$ Therefore the image is always virtual. Also $m=\frac{f}{f-s}=\frac{-|f|}{-|f|-s}=\frac{|f|}{|f|+s}>0,$ so the image is erect, and $m<1$ since $|f|+s>|f|,$ so the image is smaller.

34.11: a)



b) $s' > 0$ for $s > f$, $s < 0$.

c) $s' < 0$ for $0 < s < f$.

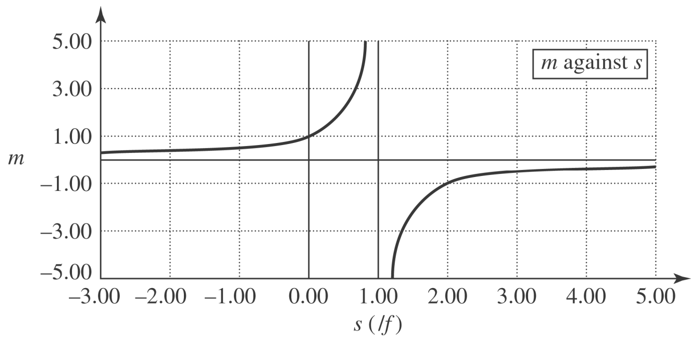
d) If the object is just outside the focal point, then the image position approaches positive infinity.

e) If the object is just inside the focal point, the image is at negative infinity, "behind" the mirror.

f) If the object is at infinity, then the image is at the focal point.

g) If the object is next to the mirror, then the image is also at the mirror.

h)



i) The image is erect if $s < f$.

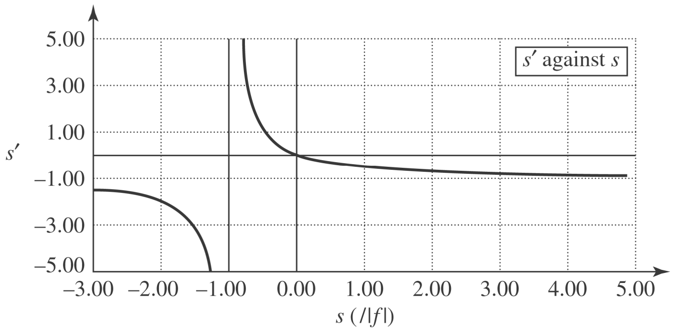
j) The image is inverted if $s > f$.

k) The image is larger if $0 < s < 2f$.

l) The image is smaller if $s > 2f$ or $s < 0$.

m) As the object is moved closer and closer to the focal point, the magnification INCREASES to infinite values.

34.12: a)



a) $s' > 0$ for $-|f| < s < 0$.

b) $s' < 0$ for s $< -|f|$ and $s < 0$.

c) If the object is at infinity, the image is at the outward going focal point.

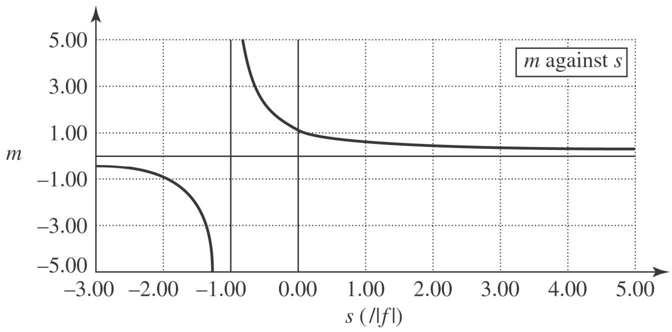
d) If the object is next to the mirror, then the image is also at the mirror. For the answers to (e), (f), (g), and (h), refer to the graph on the next page.

e) The image is erect (magnification greater than zero) for $s > -|f|$.

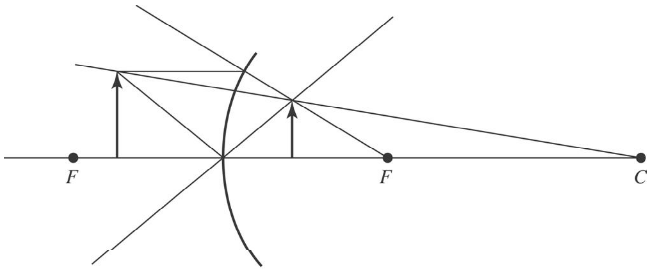
f) The image is inverted (magnification less than zero) for $s < -|f|$.

g) The image is larger than the object (magnification greater than one) for $-2|f| < s < 0$.

h) The image is smaller than the object (magnification less than one) for $s > 0$ and $s < -2|f|$.



34.13: a)



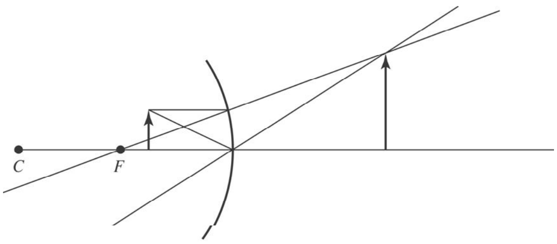
b) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s'}=-\dfrac{2}{20.0\text{ cm}}-\dfrac{1}{12.0\text{ cm}}\Rightarrow s'=-5.45\text{ cm},$ to the right of the mirror.

$y'=-y\dfrac{s'}{s}=-(0.9\text{ cm})\dfrac{-5.45\text{ cm}}{12.0\text{ cm}}=0.409\text{ cm},$ and the image is upright and virtual.

34.14: a) $m=-\dfrac{s'}{s}=-\dfrac{-48.0}{12.0}=4.00,$ where s' comes from part (b).

b) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s'}=\dfrac{2}{32.0\text{ cm}}-\dfrac{1}{12.0\text{ cm}}\Rightarrow s'=-48.0\text{ cm}.$ Since s' is negative, the image is virtual.

c)



34.15: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=0\Rightarrow\dfrac{1.309}{3.50\text{ cm}}+\dfrac{1.00}{s'}=0\Rightarrow s'=-2.67\text{ cm}.$

34.16: a) $\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{1.33}{7.00\text{ cm}}+\frac{1.00}{s'}=0 \Rightarrow s'=-5.26\text{cm}$, so the fish appears 5.26 cm below the surface.

b) $\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{1.33}{33.0\text{ cm}}+\frac{1.00}{s'}=0 \Rightarrow s'=-24.8\text{cm}$, so the image of the fish appears 24.8 cm below the surface.

34.17: a) For $R>0$ and $n_a>n_b$, with $\theta_a=\alpha+\phi$ and $\theta_b=\phi+\beta$, we have:

$$n_b\theta_b=n_a\theta_b \Rightarrow \theta_b=\phi+\beta=\frac{n_a}{n_b}(\alpha+\phi) \Rightarrow n_a\alpha-n_b\beta=(n_b-n_a)\phi.$$

But $\alpha=\frac{h}{s}$, $\beta=\frac{h}{-s'}$, and $\phi=\frac{h}{R}$, so subbing them in one finds:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{(n_b-n_a)}{R}.$$

Also, the magnification calculation yields:

$$\tan\theta_a\frac{y}{s'} \text{ and } \tan\theta_b\frac{y'}{-s'} \Rightarrow \frac{n_a y}{s'}=\frac{n_b y'}{s'} \Rightarrow m=\frac{y'}{y}=-\frac{n_a s'}{n_b s}.$$

b) For $R<0$ and $n_a<n_b$, with $\theta_a=\alpha-\phi$ and $\theta_b=\beta-\phi$, we have: $n_b\beta-n_a\alpha=(n_b-n_a)\phi$. But $\alpha=\frac{h}{s}$, $\beta=\frac{h}{-s}$, and $\phi=\frac{h}{-R} \Rightarrow -\frac{n_a}{s}-\frac{n_b}{s'}=-\frac{(n_b-n_a)}{R}$, so subbing them in one finds: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{(n_b-n_a)}{R}$. Also, the magnification calculation yields:

$$n_a\tan\theta_a \approx n_b\tan\theta_b \Rightarrow \frac{n_a y}{s}=-\frac{n_b y'}{s'} \Rightarrow m=\frac{y'}{y}=-\frac{n_a s'}{n_a s}.$$

34.18: a) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{\infty}+\frac{1.60}{s'}=\frac{0.60}{3.00\text{ cm}} \Rightarrow s'=8.00\text{ cm}.$

b) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{12.0\text{cm}}+\frac{1.60}{s'}=\frac{0.60}{3.00\text{ cm}} \Rightarrow s'=13.7\text{ cm}.$

c) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{2.00\text{ cm}}+\frac{1.60}{s'}=\frac{0.60}{3.00\text{ cm}} \Rightarrow s'=-5.33\text{ cm}.$

34.19: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow n_a=\frac{sR}{(s+R)}\frac{(s'-R)}{s'R}n_b=\frac{s}{s'}\frac{(s'-R)}{(s+R)}n_b.$

$$\Rightarrow n_a=\frac{90.0\text{ cm}}{160\text{ cm}}\frac{(160\text{ cm}-3.00\text{ cm})}{(90.0\text{ cm}+3.00\text{ cm})}(1.60)=1.52.$$

34.20: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{24.0\text{ cm}}+\frac{1.60}{s'}=\frac{0.60}{4.00\text{ cm}}\Rightarrow s'=14.8\text{ cm}.$

$y'=\left(\frac{-n_a s'}{n_b s}\right)y=\left(\frac{-14.8\text{ cm}}{(1.60)(24.0\text{ cm})}\right)1.50\text{ mm}=-0.578\text{ mm}$, so the image height is 0.578 mm, and is inverted.

34.21: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{24.0\text{ cm}}+\frac{1.60}{s'}=\frac{-0.60}{4.00\text{ cm}}\Rightarrow s'=-8.35\text{ cm}.$

$y'=\left(\frac{-n_a s'}{n_b s}\right)y=\left(\frac{-(-8.35\text{ cm})}{(1.60)(24.0\text{ cm})}\right)1.50\text{ mm}=0.326\text{ mm}$, so the image height is 0.326 mm, and is erect.

34.22: a) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.33}{14.0\text{ cm}}+\frac{1.00}{s'}=\frac{-0.33}{-14.0\text{ cm}}\Rightarrow s'=-14.0\text{ cm}$, so the fish appears to be at the center of the bowl.

$$m=\left(\frac{-n_a s'}{n_b s}\right)=\left(\frac{-(1.33)(-17.0\text{ cm})}{(1.00)(17.0\text{ cm})}\right)=+1.33.$$

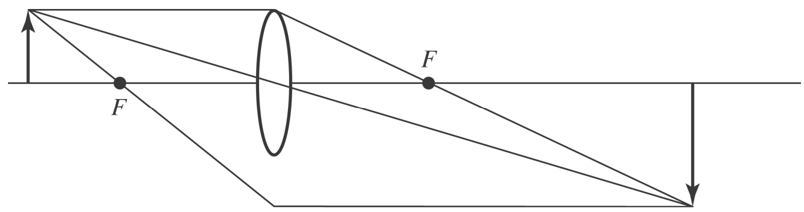
b) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.00}{\infty}+\frac{1.33}{s'}=\frac{0.33}{+14.0\text{ cm}}\Rightarrow s'=56.4\text{ cm}$, which is outside the bowl.

34.23: For $s = 18 \text{ cm}$:

a) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s'} = \dfrac{1}{14.0 \text{ cm}} - \dfrac{1}{18.0 \text{ cm}} \Rightarrow s' = 63.0 \text{ cm}.$

b) $m = -\dfrac{s'}{s} = -\dfrac{63.0}{18.0} = -3.50.$

c) and d) From the magnification, we see that the image is real and inverted.

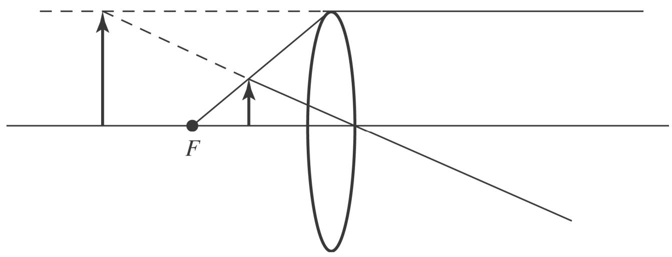


For $s = 7.00 \text{ cm}$:

a) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s'} = \dfrac{1}{14.0 \text{ cm}} - \dfrac{1}{7.00 \text{ cm}} \Rightarrow s' = -14.0 \text{ cm}.$

b) $m = -\dfrac{s'}{s} = -\dfrac{-14.0}{7.00} = 2.00.$

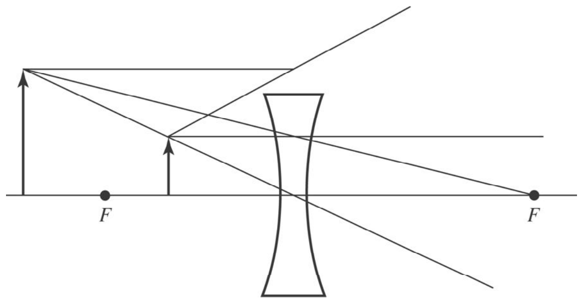
c) and d) From the magnification, we see that the image is virtual and erect.



34.24: a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{f}=\frac{1}{16.0\text{ cm}}+\frac{1}{-12.0\text{ cm}} \Rightarrow f=-48.0\text{ cm},$ and the lens is diverging.

b) $y'=y\left(-\frac{s'}{s}\right)=(0.850\text{ cm})\left(-\frac{(-12.0)}{16.0}\right)=0.638\text{ cm},$ and is erect.

c)



34.25: $m=\frac{y'}{y}=\frac{1.30}{0.400}=3.25=-\frac{s'}{s}.$ Also:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s}=\frac{1}{7.00\text{ cm}}-\frac{1}{s'} \Rightarrow \frac{s'}{s}=\frac{s'}{7.00\text{ cm}}-1=-3.25 \Rightarrow s'=-15.75\text{ cm}$$

(to the left).

$\Rightarrow \frac{1}{s}=\frac{1}{7.0\text{ cm}}-\frac{1}{-15.75\text{ cm}} \Rightarrow s=4.85\text{ cm},$ and the image is virtual (since $s'<0$).

34.26: $m=\frac{y'}{y}=-\frac{4.50}{3.20}=-\frac{s'}{s} \Rightarrow s=0.711s'.$ Also:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow =\frac{1}{0.711\,s'}+\frac{1}{s'}=\frac{1}{90.0} \Rightarrow s'=217\text{ cm (to the right).}$$

$\Rightarrow s=0.711(217\text{ cm})=154\text{ cm},$ and the image is real (since $s'>0$).

34.27: $\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{1}{18.0\text{ cm}}+\frac{1}{s'}=(0.48)\left(\frac{1}{5.00\text{ cm}}-\frac{1}{3.50\text{ cm}}\right)$

$\Rightarrow s'=-10.3\text{ cm}$ (to the left of the lens).

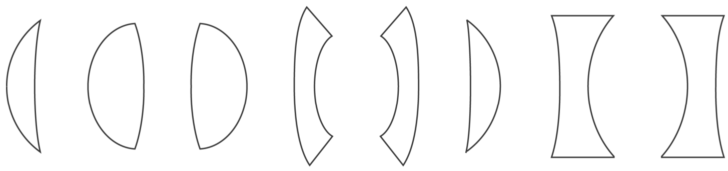
34.28: a) Given $s' = 80.0s$, and $s + s' = 6.00 \text{ m} \Rightarrow 81.00s = 6.00 \text{ m} \Rightarrow s = 0.0741 \text{ m}$ and $s' = 5.93 \text{ m}$.

b) The image is inverted since both the image and object are real $(s' > 0,\ s > 0)$.

c) $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{0.0741 \text{ m}} + \dfrac{1}{5.93 \text{ m}} \Rightarrow f = 0.0732 \text{ m}$, and the lens is converging.

34.29: $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = (0.60)\left(\dfrac{1}{\pm 4.00 \text{ cm}} - \dfrac{1}{\pm 8.00 \text{ cm}}\right)$

$\Rightarrow f = \pm 4.44 \text{ cm}, \pm 13.3 \text{ cm}.$



$f_1 = +13.3 \text{ cm};\ f_2 = +4.44 \text{ cm};\ f_3 = 4.44 \text{ cm};\ f_4 = -13.3 \text{ cm};\ f_5 = -13.3 \text{ cm};\ f_6 = +13.3$ cı
$f_7 = -4.44 \text{ cm};\ f_8 = -4.44 \text{ cm}.$

34.30: We have a converging lens if the focal length is positive, which requires:
$\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) > 0 \Rightarrow \left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) > 0.$ This can occur in one of three ways:

(i) $\{R_1 < R_2\} \cup \{R_1, R_2 > 0\}$ (ii) $R_1 > 0,\ R_2 < 0$

(iii) $\{|R_1| > |R_2|\} \cup \{R_1, R_2 < 0\}$. Hence the three lenses in Fig. (35.29a).

We have a diverging lens if the focal length is negative, which requires:

$\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) < 0 \Rightarrow \left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) < 0.$ This can occur in one of three ways:

(i) $\{R_1 > R_2\} \cup \{R_1, R_2 > 0\}$ (ii) $R_1 > R_2 > 0$ (iii) $R_1 < 0,\ R_2 > 0.$

Hence the three lenses in Fig. (34.29b).

34.31: a) The lens equation is the same for both thin lenses and spherical mirrors, so the derivation of the equations in Ex. (34.9) is identical and one gets:

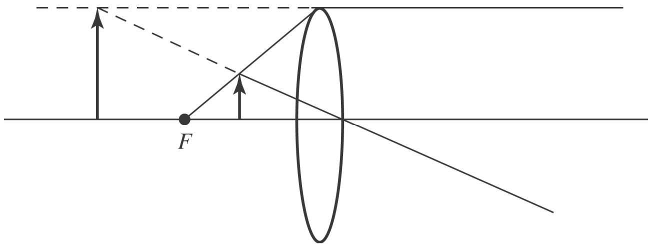
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{fs}\Rightarrow s'=\frac{sf}{s-f}, \text{ and also } m=-\frac{s'}{s}=\frac{f}{f-s}.$$

b) Again, one gets exactly the same equations for a converging lens rather than a concave mirror because the equations are identical. The difference lies in the interpretation of the results. For a lens, the outgoing side is *not* that on which the object lies, unlike for a mirror. So for an object on the left side of the lens, a positive image distance means that the image is on the right of the lens, and a negative image distance means that the image is on the left side of the lens.

c) Again, for Ex. (34.10) and (34.12), the change from a convex mirror to a diverging lens changes nothing in the exercises, except for the interpretation of the location of the images, as explained in part (b) above.

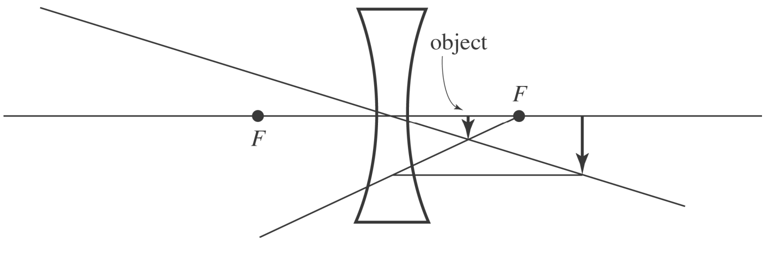
34.32: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{1}{12.0\text{ cm}}-\frac{1}{-17.0\text{ cm}}\Rightarrow s=7.0\text{ cm}.$

$m=-\frac{s'}{s}=-\frac{(-17.0)}{7.2}=+2.4\Rightarrow y=\frac{y'}{m}=\frac{0.800\text{ cm}}{+2.4}=+0.34\text{ cm}$, so the object is 0.34 cm tall, erect, same side.



34.33: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{1}{-48.0\text{ cm}}+\frac{1}{17.0\text{ cm}}\Rightarrow s=+26.3\text{ cm}.$

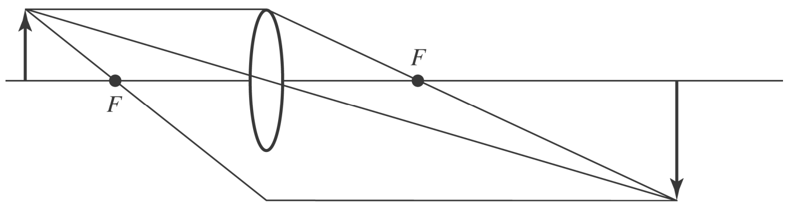
$m=-\frac{s'}{s}=-\frac{-17.0}{+26.3}=0.646\Rightarrow y=\frac{y'}{m}=\frac{0.800\text{ cm}}{0.646}=1.24\text{ cm}$ tall, erect, same side.



34.34: a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{f}=\frac{1}{16.0\text{ cm}}+\frac{1}{36.0\text{ cm}} \Rightarrow f=11.1\text{ cm}$, converging.

b) $y=y'\left(-\frac{s'}{s}\right)=(0.80\text{ cm})\left(-\frac{36}{16}\right)=-1.8\text{ cm}$, so the image is inverted.

c)



34.35: a) $|m|=\left|\frac{y'}{y}\right|=\left|\frac{s'}{s}\right| \Rightarrow s'=s\left|\frac{y'}{y}\right|=600\text{ m}\left(\frac{0.024\text{ m}}{240\text{ m}}\right)=0.0600\text{ m}=60\text{ mm}.$

$\Rightarrow \frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{6.0\times10^{5}\text{ mm}}+\frac{1}{60\text{ mm}} \Rightarrow f=60\text{ mm}.$ So one should use the 85-mm lens.

b) $s'=s\left|\frac{y'}{y}\right|=40.0\text{ m}\left(\frac{0.036\text{ m}}{9.6\text{ m}}\right)=0.15\text{ m}=150\text{ mm}.$

$\Rightarrow \frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{40\times10^{3}\text{ mm}}+\frac{1}{150\text{ mm}} \Rightarrow f=149\text{ mm}.$ So one should use the 135-mm lens.

34.36: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{3.90\text{ m}}+\frac{1}{s'}=\frac{1}{0.085\text{ m}} \Rightarrow s'=0.0869\text{ m}.$

$y'=-\frac{s'}{s}y=-\frac{0.0869}{3.90}1750\text{ mm}=39.0\text{ mm}$, so it will not fit on the 24-mm$\times$ 36-mm film.

34.37: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s}+\frac{1}{20.4\text{ cm}}=\frac{1}{20.0\text{ cm}} \Rightarrow s=1020\text{ cm}.$

34.38: $y'=-\frac{s'}{s}y \cong -\frac{f}{s}y=\frac{5.00\text{ m}}{9.50\times10^{3}\text{ m}}(70.7\text{ m})=0.0372\text{ m}=37.2\text{ mm}.$

34.39: a) $|m| = \frac{s'}{s} \approx \frac{f}{s} \Rightarrow |m| = \frac{28\text{ mm}}{200{,}000\text{ mm}} = 1.4 \times 10^{-4}.$

b) $|m| = \frac{s'}{s} \approx \frac{f}{s} \Rightarrow |m| = \frac{105\text{ mm}}{200{,}000\text{ mm}} = 5.3 \times 10^{-4}.$

c) $|m| = \frac{s'}{s} \approx \frac{f}{s} \Rightarrow |m| = \frac{300\text{ mm}}{200{,}000\text{ mm}} = 1.5 \times 10^{-3}.$

34.40: a) $s_1 = \infty \Rightarrow s'_1 = f_1 = 12\text{ cm}.$

b) $s_2 = 4.0\text{ cm} - 12\text{ cm} = -8\text{ cm}.$

c) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-8\text{ cm}} + \frac{1}{s'_2} = \frac{1}{-12\text{ cm}} \Rightarrow s'_2 = 24\text{ cm}$, to the right.

d) $s_1 = \infty \Rightarrow s'_1 = f_1 = 12\text{ cm}.$

$s_2 = 8.0\text{ cm} - 12\text{ cm} = -4\text{ cm}.$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-4\text{ cm}} + \frac{1}{s'_2} = \frac{1}{-12\text{ cm}} \Rightarrow s'_2 = 6\text{ cm}.$

34.41: a) $f/4 \Rightarrow 4 = \frac{f}{D} \Rightarrow D = \frac{f}{4} = \frac{300\text{ mm}}{4} = 75\text{ mm}.$

b) $f/8 \Rightarrow D = \frac{f}{8}$, so the diameter is 0.5 times smaller, and the area is 0.25 times smaller. Therefore only a quarter of the light entered the aperture, and the film must be exposed four times as long for the correct exposure.

34.42: The square of the aperture diameter (~ the area) is proportional to the length of the exposure time required. $\left(\frac{1}{30}s\right)\left(\frac{8\text{ mm}}{23.1\text{ mm}}\right)^2 \cong \left(\frac{1}{250}s\right).$

34.43: a) A real image is formed at the film, so the lens must be convex.

b) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ so $\frac{1}{s'} = \frac{s-f}{sf}$ and $s' = \frac{sf}{s-f}$, with $f = +50.0.0\text{ mm}$

For $s = 45\text{ cm} = 450\text{ mm}, s' = 56\text{ mm}.$

For $s = \infty, s' = f = 50\text{ mm}.$

The range of distances between the lens and film is $50\text{ mm to }56\text{ mm}.$

34.44: a) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s}+\dfrac{1}{9.00\text{ m}}=\dfrac{1}{0.150\text{ m}}\Rightarrow s=0.153\text{ m}=15.3\text{ cm}.$

b) $|m|=\dfrac{s'}{s}=\dfrac{9.00}{0.153}=58.8\Rightarrow$ dimensions are $(24\text{ mm}\times 36\text{ mm})m=(1.41\text{ m}\times 2.12\text{ m}).$

34.45: a) $f=\dfrac{1}{\text{power}}=\dfrac{1}{2.75\text{ m}^{-1}}=0.364\text{ m}=36.4\text{ cm}.$ The near-point is normally at $25\text{ cm}:\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{25\text{ cm}}+\dfrac{1}{s'}=\dfrac{1}{36.4\text{ cm}}\Rightarrow s'=-80\text{ cm},$ in front of the eye.

b) $f=\dfrac{1}{\text{power}}=\dfrac{1}{-1.30\text{ m}^{-1}}=-0.769\text{ m}=-76.9\text{ cm}.$ The far point is ideally at infinity, so: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{\infty}+\dfrac{1}{s'}=\dfrac{1}{-76.9\text{ cm}}\Rightarrow s'=-76.9\text{ cm}.$

34.46: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{1}{40.0\text{ cm}}+\dfrac{1.40}{2.60\text{ cm}}=\dfrac{0.40}{R}\Rightarrow R=0.710\text{ cm}.$

34.47: a) $\dfrac{1}{f}=\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{0.25\text{ m}}+\dfrac{1}{-0.600\text{ m}}\Rightarrow\text{power}=\dfrac{1}{f}=+2.33$ diopters.

b) $\dfrac{1}{f}=\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{\infty}+\dfrac{1}{-0.600\text{ m}}\Rightarrow\text{power}=\dfrac{1}{f}=-1.67$ diopters.

34.48: a) Angular magnification $M=\dfrac{25.0\text{ cm}}{f}=\dfrac{25.0\text{ cm}}{6.00\text{ cm}}=4.17.$

b) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s}+\dfrac{1}{-25.0\text{ cm}}=\dfrac{1}{6.00\text{ cm}}\Rightarrow s=4.84\text{ cm}.$

34.49: a) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s}+\dfrac{1}{-25.0\text{ m}}=\dfrac{1}{8.00\text{ cm}}\Rightarrow s=6.06\text{ cm}.$

b) $|m|=\dfrac{s'}{s}=\dfrac{25.0\text{ cm}}{6.06\text{ cm}}=4.13\Rightarrow y'=ym=(1.00\text{ mm})(4.13)=4.13\text{ mm}$

34.50: $\theta = \dfrac{y}{f} \Rightarrow f = \dfrac{y}{\theta} = \dfrac{2.00\text{ mm}}{0.025\text{ rad}} = 80.0\text{ mm} = 8.00\text{ cm}.$

34.51: $m = -\dfrac{s'}{s} = 6.50 \Rightarrow s' = -6.50s \Rightarrow \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{s} + \dfrac{1}{-6.50s} = \dfrac{1}{4.00\text{ cm}}$

$\Rightarrow \dfrac{1}{s}\left(1 - \dfrac{1}{6.50}\right) = \dfrac{1}{4.00} \Rightarrow s = 3.38\text{ cm},\ s' = -6.50s = -22.0\text{ cm}.$

34.52: a) $M = \dfrac{(250\text{ mm})s_1'}{f_1 f_2} = \dfrac{(250\text{ mm})(160\text{ mm} + 5.0\text{ mm})}{(5.00\text{ mm})(26.0\text{ mm})} = 317.$

b) $m = \dfrac{y'}{y} \Rightarrow y = \dfrac{y'}{m} = \dfrac{0.10\text{ mm}}{317} = 3.15 \times 10^{-4}\text{ mm}.$

34.53: a) The image from the objective is at the focal point of the eyepiece, so $s_1' = d_{oe} - f_2 = 19.7\text{ cm} - 1.80\text{ cm} = 17.9\text{ cm}$

$\Rightarrow \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} + \dfrac{1}{17.9\text{ cm}} = \dfrac{1}{0.800\text{ cm}} \Rightarrow s = 0.837\text{ cm}.$

b) $|m_1| = \dfrac{s'}{s} = \dfrac{17.9\text{ cm}}{0.837\text{ cm}} = 21.4.$

c) The overall magnification is $|M| = |m_1|\dfrac{25.0\text{ cm}}{f_2} = (21.4)\dfrac{25.0\text{ cm}}{1.80\text{ cm}} = 297.$

34.54: Using the approximation $s_1 \approx f$, and then $|m_1| = \dfrac{s_1'}{f_1}$, we have:

$$f = 16 \text{ mm}: s' = 120 \text{ mm} + 16 \text{ mm} = 136 \text{ mm}; s = 16 \text{ mm}$$

$$\Rightarrow |m_1| = \frac{s'}{s} = \frac{136 \text{ mm}}{16 \text{ mm}} = 8.5.$$

$$f = 4 \text{ mm}: s' = 120 \text{ mm} + 4 \text{ mm} = 124 \text{ mm}; s = 4 \text{ mm} \Rightarrow |m_1| = \frac{s'}{s} = \frac{124 \text{ mm}}{4 \text{ mm}} = 31.$$

$$f = 1.9 \text{ mm}: s' = 120 \text{ mm} + 1.9 \text{ mm} = 122 \text{ mm}; s = 1.9 \text{ mm}$$

$$\Rightarrow |m_1| = \frac{s'}{s} = \frac{122 \text{ mm}}{1.9 \text{ mm}} = 64.$$

The eyepiece magnifies by either 5 or 10, so:

a) The maximum magnification occurs for the 1.9-mm objective and 10x eyepiece:

$$\Rightarrow M = |m_1| m_e = (64)(10) = 640.$$

b) The minimum magnification occurs for the 16-mm objective and 5x eyepiece:

$$\Rightarrow M = |m_1| m_e = (8.5)(5) = 43.$$

34.55: a) $M = -\dfrac{f_1}{f_2} = -\dfrac{95.0 \text{ cm}}{15.0 \text{ cm}} = -6.33.$

b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{3000 \text{ m}} + \dfrac{1}{s'} = \dfrac{1}{0.950 \text{ m}} \Rightarrow s' = 0.950 \text{ m},$ so the height of an image of a building is $|y'| = \dfrac{s'}{s} y = \dfrac{0.950}{3000}(60.0 \text{ m}) = 0.019 \text{ m}.$

c) $\theta' = M\theta = 6.33 \arctan(60.0/3000) \approx 6.33(60.0)/(3000) = 0.127 \text{ rad}.$

34.56: $f_1 + f_2 = d_{ss'} \Rightarrow f_1 = d_{ss'} - f_2 = 1.80 \text{ m} - 0.0900 \text{ m} = 1.71 \text{ m}$

$$\Rightarrow M = -\frac{f_1}{f_2} = -\frac{171}{9.00} = -19.0.$$

34.57: $\dfrac{f}{D} = 19.0 \Rightarrow f = (19.0)D = (19.0)(1.02 \text{ m}) = 19.4 \text{ m}.$

34.58: $|y'| = y\dfrac{s'}{s} = y\dfrac{f}{s} = \theta f = (0.014°)\left(\dfrac{\pi}{180°}\right)(18 \text{ m}) = 4.40 \times 10^{-3} \text{ m}.$

34.59: a) $f_1 = \frac{R}{2} = 0.650 \text{ m} \Rightarrow d = f_1 + f_2 = 0.661 \text{ m}.$

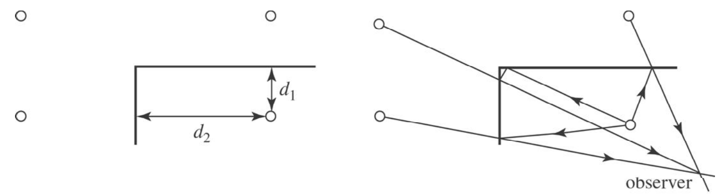
b) $|M| = \frac{f_1}{f_2} = \frac{0.650 \text{ m}}{0.011 \text{ m}} = 59.1.$

34.60: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{0.75 \text{ m} - 1.3 \text{ m}} + \frac{1}{0.75 \text{ m} + 0.12 \text{ m}} = \frac{1}{f} \Rightarrow f = -1.50 \text{ m} \Rightarrow R =$ $2f = -3.0 \text{ m}.$

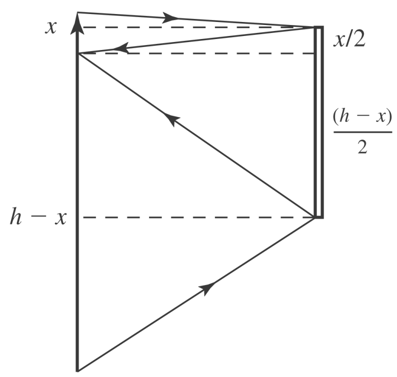
So the smaller mirror must be convex (negative focal length) and have a radius of curvature equal to 3.0 m.

34.61: If you move away from the mirror at 2.40 m/s, then your image moves away from the mirror at the same speed, but in the opposite direction. Therefore you see the image receding at 4.80 m/s, the sum of your speed and that of the image in the mirror.

34.62: a) There are three images formed.



34.63: The minimum length mirror for a woman to see her full height *h*, is $h/2$, as shown in the figure below.



34.64: $|m| = 2.25 = \frac{s'}{s} = \frac{s + 4.00 \text{ m}}{s} \Rightarrow 1.25s = 4.00 \text{ m} \Rightarrow s = 3.2 \text{ m}.$ So the mirror is 7.20 m from the wall. Also:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{2}{R} = \frac{1}{3.2 \text{ m}} + \frac{1}{7.20 \text{ m}} \Rightarrow R = 4.43 \text{ m}.$$

34.65: a) $|m| = \frac{y'}{y} = \frac{360}{6.00} = 60.0 = \frac{s'}{s} \Rightarrow s = \frac{8.00 \text{ m}}{60.0} = 0.133$ m is where the filament should be placed.

b) $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{2}{R} = \frac{1}{0.133 \text{ m}} + \frac{1}{8.00 \text{ m}} \Rightarrow R = 0.261 \text{ m}.$

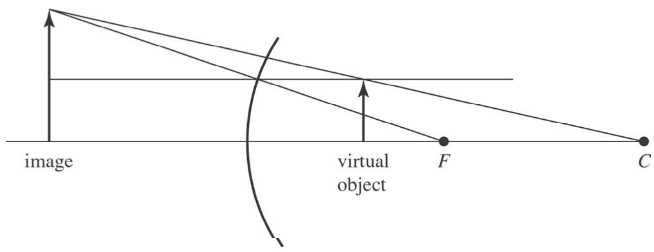
34.66: $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{13.0 \text{ m}} + \frac{1}{s'} = \frac{2}{-0.180 \text{ m}} \Rightarrow s' = -0.0894 \text{ m}.$

$$y' = y\left(-\frac{s'}{s}\right) = (1.50 \text{ m})\left(-\frac{-0.0894}{13.0}\right) = 0.0103 \text{ m}.$$

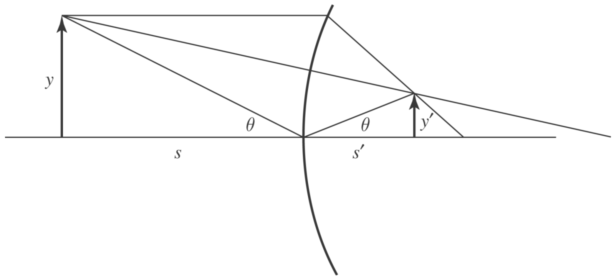
b) The height of the image is less then 1% of the true height of the car, and is less than the image would appear in a plane mirror at the same location. This gives the illusion that the car is further away then "expected."

34.67: a) $R < 0$ and $s < 0$, so a real image $(s' > 0)$ is produced for virtual object positions between the focal point and vertex of the mirror. So for a 24.0 cm radius mirror, the virtual object positions must be between the vertex and 12.0 cm to the right of the mirror. b) The image orientation is erect, since $m = -\frac{s'}{s} = -\frac{s'}{-|s|} > 0.$

c)



34.68: The derivations of Eqs. (34.6) and (34.7) are identical for convex mirrors, as long as one recalls that R and s' are negative. Consider the diagram below:



We have: $\frac{1}{\infty}+\frac{1}{s'}=\frac{2}{R} \Rightarrow s'=f=\frac{R}{2} \Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{2}{R}=\frac{1}{f}$ and $m=\frac{y'}{y}=-\frac{s'}{s}$, since s' is not on the outgoing side of the mirror.

34.69: a) $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R} \Rightarrow \frac{1}{8.0\text{ cm}}+\frac{1}{s'}=\frac{2}{19.4\text{ cm}} \Rightarrow s'=-46\text{ cm},$ so the image is virtual.

b) $m=-\frac{s'}{s}=-\frac{-46}{8.0}=5.8,$ so the image is erect, and its height is: $y'=(5.8)y=(5.8)(5.0\text{ mm})=29\text{ mm}.$

c) When the filament is 8 cm from the mirror, there is no place where a real image can be formed.

34.70: $m=\frac{5}{2}=\frac{-s'}{s} \Rightarrow s=\frac{-2}{5}s' \Rightarrow$ since $m>0,\ s'<0,\ \frac{5}{2s'}-\frac{1}{|s'|}=\frac{2}{R} \Rightarrow \frac{3}{4}R=|s'|$

$\Rightarrow s=\frac{3}{10}R$ and $s'=-\frac{3}{4}R.$

34.71: a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and taking its derivative with respect to s we have

$$0=\frac{d}{ds}\left(\frac{1}{s}+\frac{1}{s'}-\frac{1}{f}\right)=-\frac{1}{s^2}-\frac{1}{s'^2}\frac{ds'}{ds}\Rightarrow\frac{ds'}{ds}=-\frac{s'^2}{s^2}=-m^2.$$ But $\frac{ds'}{ds}=m'\Rightarrow m'=$ $-m^2$. Images are always inverted longitudinally.

b) (i) Front face: $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{1}{200.000\text{ cm}}+\frac{1}{s'}=\frac{2}{150.000\text{ cm}}\Rightarrow s'=120.000\text{ cm}.$

Rear face: $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{1}{200.100\text{ cm}}+\frac{1}{s'}=\frac{2}{150.000\text{ cm}}\Rightarrow s'=119.964\text{ cm}.$

(ii) Front face: $m=-\frac{s'}{s}=-\frac{120.000}{200.000}=-0.600000, m'=-m^2=-(-0.600000)^2=$ $-0.360.$

Rear face: $m=-\frac{s'}{s}=-\frac{119.964}{200.100}=-0.599520, m'=-m^2=-(-0.599520)^2=$ $-0.359425.$

(iii) So the front legs are magnified by 0.600000, the back legs by 0.599520, and the side legs by 0.359425, the average of the front and back longitudinal magnifications.

34.72: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ and taking its derivative with respect to s we have:

$$0=\frac{d}{ds}\left(\frac{n_a}{s}+\frac{n_b}{s'}-\frac{n_b-n_a}{R}\right)=-\frac{n_a}{s^2}-\frac{n_b}{s'^2}\frac{ds'}{ds}$$

$$\Rightarrow\frac{ds'}{ds}=-\frac{s'^2}{s^2}\frac{n_a}{n_b}=-\left(\frac{s'^2}{s^2}\frac{n_a^2}{n_b^2}\right)\frac{n_b}{n_a}=-m^2\frac{n_b}{n_a}.$$

But $\frac{ds'}{ds}=m'\Rightarrow m'=-m^2\frac{n_b}{n_a}.$

34.73: a) $R < 0$ for convex so $\frac{1}{s}+\frac{1}{s'}=\frac{2}{-R} \Rightarrow s'=\frac{-sR}{2s+R} \Rightarrow v'=\frac{ds'}{dt}=\frac{ds'}{ds}\frac{ds}{dt}=$ $v\left(\frac{-R}{2s+R}+\frac{2sR}{(2s+R)^2}\right)$

$$\Rightarrow v'=-v\frac{R^2}{(2s+R)^2}=-(-2.50\text{ m/s})\frac{(1.25\text{ m})^2}{(2(10.0\text{ m})+1.25\text{ m})^2}=+8.65\times10^{-3}\text{ m/s}.$$

b) $v'=-v\frac{R^2}{(2s+R)^2}=-(-2.50\text{ m/s})\frac{(1.25\text{ m})^2}{(2(2.0\text{ m})+1.25\text{ m})^2}=+0.142\text{ m/s}.$

Note: The signs are somewhat confusing. If a *real object* is moving with $v > 0$, this implies it is moving *away* from the mirror. However, if a *virtual image* is moving with $v > 0$, this implies it is moving from "behind" the mirror *toward* the vertex.

34.74: In this context, the microscope just takes an image and makes it visible. The real optics are at the glass surfaces.

$$\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{n}{s}+\frac{1}{s'}=0 \Rightarrow n=-\frac{s}{s'}=-\frac{2.50\text{ mm}+0.780\text{ mm}}{-2.50\text{ mm}}=1.31.$$

Note that the object and image are measured from the front surface of the second plate, making the image virtual.

34.75: a) Reflection from the front face of the glass means that the image is just h below the glass surface, like a normal mirror.

b) The reflection from the mirrored surface behind the glass will not be affected because of the intervening glass. The light travels through a distance $2d$ of glass, so the path through the glass appears to be $\frac{2d}{n}$, and the image appears to be $h+\frac{2d}{n}$ behind the front surface of the glass.

c) The distance between the two images is just $\frac{2d}{n}$.

34.76: a) The image from the left end acts as the object for the right end of the rod.

b) $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{1}{23.0\text{ cm}}+\dfrac{1.60}{s'}=\dfrac{0.60}{6.0\text{ cm}}\Rightarrow s'=28.3\text{ cm}.$

So the second object distance is $s_2=40.0\text{ cm}-28.3\text{ cm}=11.7\text{ cm}.$

Also: $m_1=-\dfrac{n_a s'}{n_b s}=-\dfrac{28.3}{(1.60)(23.0)}=-0.769.$

c) The object is real and inverted.

d) $\dfrac{n_a}{s_2}+\dfrac{n_b}{s_2'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{1.60}{11.7\text{ cm}}+\dfrac{1}{s_2'}=\dfrac{-0.60}{-12.0\text{ cm}}\Rightarrow s'=-11.5\text{ cm}.$

Also: $m_2=-\dfrac{n_a s'}{n_b s}=-\dfrac{(1.60)(-11.5)}{11.7}=1.57\Rightarrow m=m_1m_2=(-0.769)(1.57)=-1.21.$

e) So the final image is virtual, and inverted.

f) $y'=(1.50\text{ mm})(-1.21)=-1.82\text{ mm}.$

34.77: a) $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{1}{23.0\text{ cm}}+\dfrac{1.60}{s'}=\dfrac{0.60}{6.0\text{ cm}}\Rightarrow s'=28.3\text{ cm}.$

So the second object distance is $s_2=25.0\text{ cm}-28.3\text{ cm}=-3.3\text{ cm}.$

Also: $m_1=-\dfrac{n_a s'}{n_b s}=-\dfrac{28.3}{(1.60)(23.0)}=-0.769.$

b) The object is virtual.

c) $\dfrac{n_a}{s_2}+\dfrac{n_b}{s_2'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{1.60}{-3.3\text{ cm}}+\dfrac{1}{s_2'}=\dfrac{-0.60}{-12.0\text{ cm}}\Rightarrow s_2'=1.87\text{ cm}.$

Also:

$$m_2=-\frac{n_a s_2'}{n_b s_2}=-\frac{(1.60)(1.87)}{-3.3}=0.901\Rightarrow m=m_1m_2=(-0.769)(0.901)=-0.693.$$

d) So the final image is real and inverted.

e) $y'=ym=(1.50\text{ mm})(-0.693)=-1.04\text{ mm}.$

34.78: For the water-benzene interface to get the apparent water depth:

$$\frac{n_a}{s}+\frac{n_b}{s'}=0\Rightarrow\frac{1.33}{6.50\text{ cm}}+\frac{1.50}{s'}=0\Rightarrow s'=-7.33\text{ cm}.$$

For the benzene-air interface, to get the total apparent distance to the bottom:

$$\frac{n_a}{s}+\frac{n_b}{s'}=0\Rightarrow\frac{1.50}{(7.33\text{ cm}+2.60\text{ cm})}+\frac{1}{s'}=0\Rightarrow s'=-6.62\text{ cm}.$$

34.79: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}$, but $s=\infty$, $s'=2R \Rightarrow \dfrac{1}{\infty}+\dfrac{n}{2R}=\dfrac{n-1}{R}\Rightarrow \dfrac{n}{2}=1\Rightarrow n=2.00.$

34.80: a) $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow \dfrac{1}{12.0\text{ cm}}+\dfrac{1.60}{s'}=\dfrac{0.60}{15.0\text{ cm}}\Rightarrow s'=-36.9\text{ cm}.$ So the object distance for the far end of the rod is $50.0\text{ cm}-(-36.9\text{ cm})=86.9\text{ cm}.$

$$\Rightarrow \frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow \frac{1.60}{86.9\text{ cm}}+\frac{1}{s'}=0\Rightarrow s'=-54.3\text{ cm}.$$

b) The magnification is the product of the two magnifications:

$$m_1=-\frac{n_a s'}{n_b s}=-\frac{-36.9}{(1.60)(12.0)}=1.92,\ m_2=1.00\Rightarrow m=m_1m_2=1.92.$$

34.81: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow \dfrac{1}{\infty}+\dfrac{1.80}{s'}=\dfrac{0.80}{4.00\text{ cm}}\Rightarrow s'=9.00\text{ cm}.$ So the object distance for the far side of the ball is $8.00\text{ cm}-9.00\text{ cm}=-1.00\text{ cm}.$

$$\Rightarrow \frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow \frac{1.80}{-1.00\text{ cm}}+\frac{1}{s'}=\frac{-0.80}{-4.00\text{ cm}}\Rightarrow s'=0.50\text{ cm},$$ which is 4.50 cm from the center of the sphere.

34.82: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=0\Rightarrow \dfrac{n}{15.0\text{ cm}}+\dfrac{1}{-9.50\text{ cm}}=0\Rightarrow n=\dfrac{15.0}{9.50}=1.58.$

When viewed from the curved end of the rod:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow \frac{n}{s}+\frac{1}{s'}=\frac{1-n}{\text{R}}\Rightarrow \frac{1.58}{15.0\text{ cm}}+\frac{1}{s'}=\frac{-0.58}{-10.0\text{ cm}}\Rightarrow s'=-21.1\text{ cm},$$

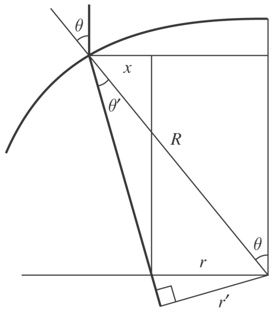
so the image is 21.1 cm within the rod from the curved end.

34.83: a) From the diagram:

$$\sin\theta = \frac{0.190}{R} = 1.50\,\sin\theta'. \text{ But } \sin\theta' = \frac{r'}{R} \approx \frac{r}{R} = \frac{0.190}{R(1.50)}$$

$$\Rightarrow r = \frac{0.190\text{ cm}}{1.50} = 0.127\text{ cm.}$$

So the diameter of the light hitting the surface is $2r = 0.254$ cm.



b) There is no dependence on the radius of the glass sphere in the calculation above.

34.84: a) Treating each of the goblet surfaces as spherical surfaces, we have to pass, from left to right, through four interfaces. For the empty goblet:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{\infty}+\frac{1.50}{s_1'}=\frac{0.50}{4.00\text{ cm}}\Rightarrow s_1'=12\text{ cm}$$

$$\Rightarrow s_2=0.60\text{ cm}-12\text{ cm}=-11.4\text{ cm}\Rightarrow\frac{1.50}{-11.4\text{ cm}}+\frac{1}{s_2'}=\frac{-0.50}{3.40\text{ cm}}\Rightarrow s_2'=-64.6\text{ c}$$

$$\Rightarrow s_3=64.6\text{ cm}+6.80\text{ cm}=71.4\text{ cm}\Rightarrow\frac{1}{71.4\text{ cm}}+\frac{1.50}{s_3'}=\frac{0.50}{-3.40\text{ cm}}\Rightarrow s_3'=-9.3$$

$$\Rightarrow s_4=9.31\text{ cm}+0.60\text{ cm}=9.91\text{ cm}\Rightarrow\frac{1.50}{9.91\text{ cm}}+\frac{1}{s_4'}=\frac{-0.50}{-4.00\text{ cm}}\Rightarrow s_4'=-37.9\text{ c}$$

So the image is $37.9\text{ cm}-2(4.0\text{ cm})=29.9\text{ cm}$ to the left of the goblet.

b) For the wine-filled goblet:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{\infty}+\frac{1.50}{s_1'}=\frac{0.50}{4.00\text{ cm}}\Rightarrow s_1'=12\text{ cm}$$

$$\Rightarrow s_2=0.60\text{ cm}-12\text{ cm}=-11.4\text{ cm}\Rightarrow\frac{1.50}{-11.4\text{ cm}}+\frac{1.37}{s_2'}=\frac{-0.13}{3.40\text{ cm}}\Rightarrow s_2'=14.7\text{ c}$$

$$\Rightarrow s_3=6.80\text{ cm}-14.7\text{ cm}=-7.9\text{ cm}\Rightarrow\frac{1.37}{-7.9\text{ cm}}+\frac{1.50}{s_3'}=\frac{0.13}{-3.40\text{ cm}}\Rightarrow s_3'=11.1$$

$$\Rightarrow s_4=0.60\text{ cm}-11.1\text{ cm}=-10.5\text{ cm}\Rightarrow\frac{1.50}{-10.5\text{ cm}}+\frac{1}{s_4'}=\frac{-0.50}{-4.00\text{ cm}}\Rightarrow s_4'=3.73\text{ cı}$$

to the right of the goblet.

34.85: Entering the sphere: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{3R}+\frac{4}{3s'}=\frac{1}{3R}\Rightarrow s'=\infty.$

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.33}{\infty}+\frac{1}{s'}=\frac{1}{3R}\Rightarrow s'=3R.$$

So the final image is a distance $3R$ from the right-hand side of the sphere, or $4R$ to the right of the center of the globe.

34.86: a) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{n_a}{f}+\frac{n_b}{\infty}=\frac{n_b-n_a}{R}$ and $\frac{n_a}{\infty}+\frac{n_b}{f'}=\frac{n_b-n_a}{R}$.

$\Rightarrow\frac{n_a}{f}=\frac{n_b-n_a}{R}$ and $\frac{n_b}{f'}=\frac{n_b-n_a}{R}\Rightarrow\frac{n_a}{f}=\frac{n_b}{f'}\Rightarrow n_a=n_b\frac{f}{f'}$.

b) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{n_b f}{sf'}+\frac{n_b}{s'}=\frac{n_b(1-f/f')}{R}\Rightarrow\frac{f}{s}+\frac{f'}{s'}=\frac{f'(1-f/f')}{R}=$
$\frac{f'-f}{R}=1.$

Note that the first two equations on the second line can be rewritten as $\frac{n_a}{n_b-n_a}=\frac{f}{R}$ and $\frac{n_b}{n_b-n_a}=\frac{f'}{R}$ so we can write $\frac{f'-f}{R}=1$.

34.87: Below, x is the distance from object to the screen's original position.

$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow\frac{1}{x-30\text{ cm}}+\frac{1}{30\text{ cm}}=\frac{1}{f}$ and $\frac{1}{x-26\text{ cm}}+\frac{1}{22\text{ cm}}=\frac{1}{f}$

$\Rightarrow x^2-56x+450\text{ cm}^2=0\Rightarrow x=46.3\text{ cm}, 9.72\text{ cm}.$ But the object must be to the left of the lens, so $s=46.3\text{ cm}-30\text{ cm}=16.3\text{ cm}$. The corresponding focal length is 10.56 cm.

34.88: We have images formed from both ends. From the first:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{25.0\text{ cm}}+\frac{1.55}{s'}=\frac{0.55}{6.00\text{ cm}}\Rightarrow s'=30.0\text{ cm}.$$

This image becomes the object for the second end:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.55}{d-30.0\text{ cm}}+\frac{1}{65.0\text{ cm}}=\frac{-0.55}{-6.00\text{ cm}}$$

$$\Rightarrow d-30.0\text{ cm}=20.3\text{ cm}\Rightarrow d=50.3\text{ cm}.$$

34.89: a) For the first lens: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{20.0\text{ cm}}+\frac{1}{s'}=\frac{1}{12.0\text{ cm}} \Rightarrow s'=30.0\text{ cm}.$

So $m_1=-\frac{30.0}{20.0}=-1.50.$

For the second lens: $s=9.00\text{ cm}-30.0\text{ cm}=-21.0\text{ cm}.$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{-21.0\text{ cm}}+\frac{1}{s'}=-\frac{1}{12.0\text{ cm}} \Rightarrow s'=-28.0\text{ cm}, m_2=-\frac{-28.0}{-21.0}=-1.33$$

So the image is 28.0 cm to the left of the second lens, and is therefore 19.0 cm to the left of the first lens.

b) The final image is virtual.

c) Since the magnification is $m=m_1m_2=(-1.50)(-1.33)=2.00,$ the final image is erect and has a height $y'=(2.00)(2.50\text{ mm})=5.00\text{ mm}.$

34.90: a) $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.60)\left(\frac{1}{12.0\text{ cm}}-\frac{1}{28.0\text{ cm}}\right) \Rightarrow f=35.0\text{ cm}.$

$$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{45\text{ cm}}+\frac{1}{s'}=\frac{1}{35\text{ cm}} \Rightarrow s'=158\text{ cm, and}$$

$$y'=y\left(-\frac{s'}{s}\right)=(0.50\text{ cm})\left(-\frac{158}{45}\right)=-1.76\text{ cm}.$$

b) Adding a second identical lens 315 cm to the right of the first means that the first lens's image becomes an object for the second, a distance of 157 cm from that second lens.

$$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{157\text{ cm}}+\frac{1}{s'}=\frac{1}{35\text{ cm}}$$

$$\Rightarrow s'=45.0\text{ cm}, y'=(-1.76\text{ cm})\left(-\frac{45}{157}\right)=0.5\text{ cm},$$

and the image is erect.

c) Putting an identical lens just 45 cm from the first means that the first lens's image becomes an object for the second, a distance of 113 cm to the right of the second lens.

$$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{-113\text{ cm}}+\frac{1}{s'}=\frac{1}{35\text{ cm}} \Rightarrow s'=26.7\text{ cm, and } y'=(-1.76\text{ cm})\times\left(\frac{26.7}{113}\right)=$$

and the image is inverted.

34.91: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{80.0\text{ cm}}+\frac{1}{s'}=\frac{1}{40.0\text{ cm}} \Rightarrow s'=80.0\text{ cm}.$

So the object distance for the second lens is 52.0 cm – (8.00 cm) = – 28.0 cm.

$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{-28.0\text{ cm}}+\frac{1}{s'}=\frac{1}{40.0\text{ cm}} \Rightarrow s'=16.47\text{ cm}.$

So the object distance for the third lens in 52.0 cm – (16.47 cm) = 35.53 cm.

$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{35.53\text{ cm}}+\frac{1}{s'}=\frac{1}{40.0\text{ cm}} \Rightarrow s'=-318\text{ cm},$ so the final image is virtual and 318 cm to the left of the third mirror, or equivalently 214 cm to the left of the first mirror.

34.92: a) $s+s'=18.0\text{ cm}$ and $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{18.0\text{ cm}-s'}+\frac{1}{s'}=\frac{1}{3.00\text{ cm}}$

$\Rightarrow (s')^2-(18.0\text{ cm})s'+54.0\text{ cm}^2=0 \Rightarrow s'=14.2\text{ cm}, 3.80\text{ cm}=s.$ So the screen must either be 3.80 cm or 14.2 cm from the object.

b) $s=3.80\text{ cm}: m=-\frac{s'}{s}=-\frac{3.80}{14.2}=-0.268.$

$s=14.2\text{ cm}: m=-\frac{s'}{s}=-\frac{14.2}{3.80}=-3.74.$

34.93: a) Bouncing first off the convex mirror, then the concave mirror:

$$\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{1}{0.600\text{ m}-x}+\frac{1}{s'}=\frac{2}{-0.360\text{ m}}\Rightarrow\frac{1}{s'}=-5.56\text{ m}^{-1}-\frac{1}{x-0.600\text{ m}}$$

$$\Rightarrow s'=\frac{x-0.600\text{ m}}{-5.56\text{ m}^{-1}x+4.33}.$$

But the object distance for the concave mirror is just

$$s=0.600\text{ m}-s'=\frac{4.33x+3.20\text{ m}}{5.56\text{ m}^{-1}x-4.33}.$$

So for the concave mirror: $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{5.56\text{ m}^{-1}x-4.33}{4.33x+3.20\text{ m}}+\frac{1}{x}=\frac{2}{0.360}$

$\Rightarrow 18.5x^2-17.8x+3.20=0\Rightarrow x=0.72\text{ m},\ 0.24\text{ m}.$

But the object position must be between the mirrors, so the distance must be the smaller of the two above, 0.24 m, from the concave mirror.

b) Now having the light bounce first from the concave mirror, and then the convex mirror, we have:

$$\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{1}{x}+\frac{1}{s'}=\frac{2}{0.360}\Rightarrow\frac{1}{s'}=5.56\text{ m}^{-1}-\frac{1}{x}\Rightarrow s'=\frac{x}{5.56\text{ m}^{-1}x-1.00}.$$

But the object distance for the convex mirror is just

$$s=0.600\text{ m}-s'=\frac{2.33x-0.600\text{ m}}{5.56\text{ m}^{-1}x-1}.$$

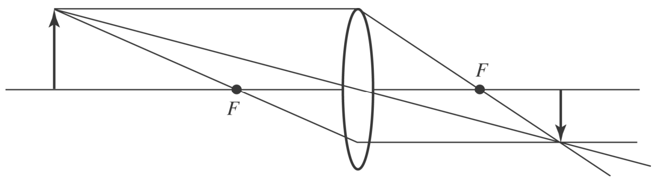
So for the convex mirror: $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{5.56\text{ m}^{-1}x-1}{2.33x-0.600\text{ m}}+\frac{1}{x}=\frac{2}{-0.360}$

$\Rightarrow 18.5x^2-2.00x-0.600=0\Rightarrow x=-0.13\text{ m},\ 0.24\text{m}.$

But the object position must be between the mirrors, so the distance must be 0.24 m from the concave mirror.

34.94: Light passing straight through the lens:

a)



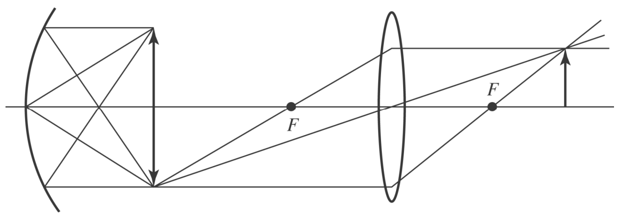
b) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow \dfrac{1}{85.0\text{ cm}}+\dfrac{1}{s'}=\dfrac{1}{32.0\text{ cm}}\Rightarrow s'=51.3\text{ cm},$ to the right of the lens.

c) The image is real.

d) The image is inverted.

For light reflecting off the mirror, and then passing through the lens:

a)



$\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow \dfrac{1}{20.0\text{ cm}}+\dfrac{1}{s'}=\dfrac{1}{10.0\text{ cm}}\Rightarrow s'=20.0\text{ cm},$ so the image from the mirror, which becomes the new object for the lens, is at the same location as the object. So the final image position is 51.3 cm to the right of the lens, as in the first case above.

c) The image is real

d) The image is erect.

34.95: Parallel light coming in from the left is focused 12.0 cm from the left lens, which is 8.00 cm to right of the second lens. Therefore:

$\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow \dfrac{1}{-8.00\text{ cm}}+\dfrac{1}{s'}=\dfrac{1}{12.0\text{ cm}}\Rightarrow s'=4.80\text{ cm},$ to the right of the second lens, and this is where the first focal point of the eyepiece is located. The second focal point is obtained by sending in parallel light from the right, and the symmetry of the lens set-up enables us to immediately state that the second focal point is 4.80 cm to the left of the first lens.

34.96: a) With two lenses of different focal length in contact, the image distance from the first lens becomes exactly minus the object distance for the second lens. So we have:

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow\frac{1}{s_1'}=\frac{1}{f_1}-\frac{1}{s_1}\text{ and }\frac{1}{s_2}+\frac{1}{s_2'}$$

$$=\frac{1}{-s_1'}+\frac{1}{s_2'}=\left(\frac{1}{s_1}-\frac{1}{f_1}\right)+\frac{1}{s_2'}=\frac{1}{f_2}.$$

But overall for the lens system, $\dfrac{1}{s_1}+\dfrac{1}{s_2'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{f}=\dfrac{1}{f_2}+\dfrac{1}{f_1}.$

b) With carbon tetrachloride sitting in a meniscus lens, we have two lenses in contact. All we need in order to calculate the system's focal length is calculate the individual focal lengths, and then use the formula from part (a).

For the meniscus: $\dfrac{1}{f_m}=(n_b-n_a)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)=$

$$(0.55)\left(\frac{1}{4.50\text{ cm}}-\frac{1}{9.00\text{ cm}}\right)=0.061.$$

For the CCl_4 : $\dfrac{1}{f_w}=(n_b-n_a)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)=(0.46)\left(\dfrac{1}{9.00\text{ cm}}-\dfrac{1}{\infty}\right)=0.051.$

$$\Rightarrow\frac{1}{f}=\frac{1}{f_2}+\frac{1}{f_1}=0.112\Rightarrow f=8.93\text{ cm}.$$

34.97: At the first surface, $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=0\Rightarrow s'=-\dfrac{n_b}{n_a}s=-\dfrac{1.60}{1.00}(-14.4\text{ cm})=23.04\text{ cm}.$

At the second surface,

$$s'=14.7\text{ cm}-t=-\frac{n_b}{n_a}s=+\frac{1.00}{1.60}(23.0\text{ cm}-t)\Rightarrow 23.52-1.60t=23.04-t$$

$$\Rightarrow 0.60t=0.48\Rightarrow t=0.80\text{ cm}.$$

(Note, as many significant figures as possible should be kept during the calculation, since numbers comparable in size are subtracted.)

34.98: a) Starting with the two equations:

$$\frac{n_a}{s_1}+\frac{n_b}{s_1'}=\frac{n_b-n_a}{R_1} \text{ and } \frac{n_b}{s_2}+\frac{n_c}{s_2'}=\frac{n_c-n_b}{R_2}, \text{ and using } n_a=n_{\text{liq}}=n_c, n_b=n, \text{ and}$$

$$s_1'=-s_2, \text{ we get}: \frac{n_{\text{liq}}}{s_1}+\frac{n}{s_1'}=\frac{n-n_{\text{liq}}}{R_1} \text{ and } \frac{n}{-s_1'}+\frac{n_{\text{liq}}}{s_2'}=\frac{n_{\text{liq}}-n}{R_2}.$$

$$\Rightarrow \frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{f'}=(n/n_{\text{liq}}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

b) Comparing the equations for focal length in and out of air we have:

$$f(n-1)=f'(n/n_{\text{liq}}-1)=f'\left(\frac{n-n_{\text{liq}}}{n_{\text{liq}}}\right)\Rightarrow f'=\left[\frac{n_{\text{liq}}(n-1)}{n-n_{\text{liq}}}\right]f.$$

34.99: The image formed by the converging lens is 30.0 cm from the converging lens, and becomes a virtual object for the diverging lens at a position 15.0 cm to the right of the diverging lens. The final image is projected $15+19.2=34.2\text{ cm}$ from the diverging lens.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow \frac{1}{-15.0\text{ cm}}+\frac{1}{34.2\text{ cm}}=\frac{1}{f}\Rightarrow f=-26.7\text{ cm}.$$

34.100: The first image formed by the spherical mirror is the one where the light immediately strikes its surface, without bouncing from the plane mirror.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow \frac{1}{10.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}}\Rightarrow s'=-7.06\text{ cm}, \text{and the image height:}$$

$$y'=-\frac{s'}{s}y=-\frac{-7.06}{10.0}(0.250\text{ cm})=0.177\text{ cm}.$$

The second image is of the plane mirror image, located $(20.0\text{ cm}+10.0\text{ cm})$ from the vertex of the spherical mirror. So:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow \frac{1}{30.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}}\Rightarrow s'=-13.3\text{ cm} \text{ and the image height.}$$

$$y'=-\frac{s'}{s}y=-\frac{-13.3}{30.0}(0.250\text{ cm})=0.111\text{ cm}.$$

34.101: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=0\Rightarrow \dfrac{1}{6.00\text{ cm}}+\dfrac{1.55}{s_1'}=0\Rightarrow s_1'=-9.30\text{ cm}$

$$\Rightarrow s_2=3.50\text{ cm}+9.30\text{ cm}=12.80\text{ cm}\Rightarrow \frac{1.55}{12.8\text{ cm}}+\frac{1}{s_1'}=0\Rightarrow s_1'=-8.26\text{ cm}.$$

So the image is 8.26 cm below the top glass surface, or 1.24 cm above the page.

34.102: a) $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)\Rightarrow\dfrac{1}{40\text{ cm}}=0.52\left(\dfrac{2}{R}\right)\Rightarrow R=41.6\text{ cm}.$

At the air–lens interface: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{1}{70.0\text{ cm}}+\dfrac{1.52}{s_1'}=\dfrac{0.52}{41.6\text{ cm}}$

$\Rightarrow s_1'=-851\,\text{cm}=s_2.$

At the lens–water interface: $\Rightarrow\dfrac{1.52}{851\,\text{cm}}+\dfrac{1.33}{s_2'}=\dfrac{-0.187}{-41.6\text{ cm}}\Rightarrow s_2'=491\,\text{cm}.$

The mirror reflects the image back (since there is just 90 cm between the lens and mirror.) So, the position of the image is 401 cm to the left of the mirror, or 311 cm to the left of the lens. So:

At the water–lens interface: $\Rightarrow\dfrac{1.33}{-311\,\text{cm}}+\dfrac{1.52}{s_3'}=\dfrac{0.187}{41.6\text{ cm}}\Rightarrow s_3'=+173\,\text{cm}.$

At the lens–air interface: $\Rightarrow\dfrac{1.52}{-173\,\text{cm}}+\dfrac{1}{s_4'}=\dfrac{-0.52}{-41.6\text{ cm}}\Rightarrow s_4'=+47.0\text{ cm},$ to the left of lens.

$$m=m_1m_2m_3m_4=\left(\frac{n_{a1}s_1'}{n_{b1}s_1}\right)\left(\frac{n_{a2}s_2'}{n_{b2}s_2}\right)\left(\frac{n_{a3}s_3'}{n_{b3}s_3}\right)\left(\frac{n_{a4}s_4'}{n_{b4}s_4}\right)$$

$$=\left(\frac{-851}{70}\right)\left(\frac{491}{-851}\right)\left(\frac{+173}{-311}\right)\left(\frac{+47.0}{-173}\right)=-1.06.$$

(Note all the indices of refraction cancel out.)

b) The image is real.

c) The image is inverted.

d) The final height is $y'=my=(1.06)(4.00\text{ mm})=4.24\text{ mm}.$

34.103: a) $m=-\dfrac{s'}{s}=\dfrac{y'}{y}=\dfrac{3}{4}\dfrac{(0.0360\text{ m})}{(22.7\text{ m})}\Rightarrow s'=(1.19\times10^{-3})\,s$

$$\Rightarrow\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{(1.19\times10^{-3})s}=\frac{1}{s}\left(1+\frac{1}{1.19\times10^{-3}}\right)=\frac{1}{f}=\frac{1}{0.035\text{ m}}\Rightarrow s=29.4\text{ m}.$$

b) To just fill the frame, the magnification must be 1.59×10^{-3} so:

$$\Rightarrow\frac{1}{s}\left(1+\frac{1}{1.59\times10^{-3}}\right)=\frac{1}{f}=\frac{1}{0.035\text{ m}}\Rightarrow s=22.0\text{ m}.$$

34.104: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{25{,}000\text{ mm}}+\frac{1}{s'}=\frac{1}{35.0\text{ mm}} \Rightarrow s'=35.05\text{ mm}.$

The resolution of 120 lines per millimeter means that the image line width is 0.0083 mm between lines. That is $y'=0.0083\text{ mm}.$ But:

$\left|\frac{y'}{y}\right|=\left|\frac{s'}{s}\right| \Rightarrow y=|y'|\cdot\frac{s}{s'}=(0.0083\text{ mm})\frac{25{,}000\text{ mm}}{35.05\text{ mm}}=5.92\text{ mm},$ which is the minimum separation between two lines 25.0 m away from the camera.

34.105: a) From the diagram below, we see that $|m|=\frac{d}{W}=\frac{s'}{s} \Rightarrow \frac{1}{s'}=\frac{W}{sd}.$

$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{W}{sd}=\frac{1}{s}\left(1+\frac{W}{d}\right)=\frac{d+W}{sd}=\frac{1}{f} \Rightarrow f=\frac{sd}{d+W}.$ But when the object is much larger than the image we have the approximation:

$s'\approx f$ and $d+W\approx W \Rightarrow m=\frac{d}{W}\approx\frac{f}{s} \Rightarrow \tan\frac{\theta}{2}=\frac{W}{2}\cdot\frac{1}{s}=\frac{d}{2f} \Rightarrow \theta=$ $2\arctan\left(\frac{d}{2f}\right).$

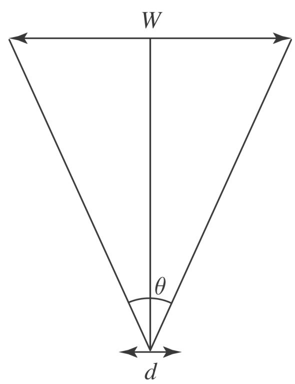
b) The film is $24\text{ mm}\times 36\text{ mm},$ so the diagonal length is just:

$d=\left(\sqrt{24^2+36^2}\right)\text{mm}=43.3\text{ mm}.$ So:

$f=28\text{ mm}: \theta=2\arctan\left(\frac{43.3\text{ mm}}{2(28\text{ mm})}\right)=75°.$

$f=105\text{ mm}: \theta=2\arctan\left(\frac{43.3\text{ mm}}{2(105\text{ mm})}\right)=23°.$

$f=300\text{ mm}: \theta=2\arctan\left(\frac{43.3\text{ mm}}{2(300\text{ mm})}\right)=8.2°.$



34.106: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{1300\text{ mm}}+\frac{1}{s'}=\frac{1}{90\text{ mm}} \Rightarrow s'=96.7\text{ mm.}$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{6500\text{ mm}}+\frac{1}{s'}=\frac{1}{90\text{ mm}} \Rightarrow s'=91.3\text{ mm.}$

$\Rightarrow \Delta s'=96.7\text{ mm}-91.3\text{ mm}=5.4\text{ mm toward the film}$

34.107: a) At age $10: f_n=7\text{ cm}: M=2.0=\frac{7\text{ cm}}{f} \Rightarrow f=3.5\text{ cm.}$

b) At age $30: f_n=14\text{ cm}: M=2.0=\frac{14\text{ cm}}{f} \Rightarrow f=7.0\text{ cm.}$

c) At age $60: f_n=200\text{ cm}: M=2.0=\frac{200\text{ cm}}{f} \Rightarrow f=100\text{ cm.}$

d) If the 2.8 cm focal length lens is used by the 60-year old, then $M=\frac{200\text{ cm}}{f}=\frac{200\text{ cm}}{3.5\text{ cm}}=57.1.$

e) This does not mean that the older viewer sees a more magnified image. The object is over 28 times further away from the 60-year old, which is exactly the ratio needed to result in the magnification of 2.0 as seen by the 10-year old.

34.108: a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s}+\frac{1}{-25\text{ cm}}=\frac{1}{f} \Rightarrow s=\frac{f(25\text{ cm})}{f+25\text{ cm}}.$

b) Height $=y \Rightarrow \theta'=\arctan\left(\frac{y}{s}\right)=\arctan\left(\frac{y(f+25\text{ cm})}{f(25\text{ cm})}\right)\approx\frac{y(f+25\text{cm})}{f(25\text{ cm})}.$

c) $\text{M}=\frac{\theta'}{\theta}=\frac{y(f+25\text{ cm})}{f(25\text{ cm})}.\frac{1}{y/25\text{ cm}}=\frac{f+25\text{ cm}}{f}.$

d) If $f=10\text{ cm} \Rightarrow M=\frac{10\text{ cm}+25\text{ cm}}{10\text{ cm}}=3.5.$ This is 1.4 times greater than the magnification obtained if the image if formed at infinity $(M_\infty=\frac{25\text{ cm}}{f}=2.5).$

e) Having the first image form just within the focal length puts one in the situation described above, where it acts as a source that yields an enlarged virtual image. If the first image fell just outside the second focal point, then the image would be real and diminished.

34.109: The near point is at infinity, so that is where the image must be found for any objects that are close. So:

$$P=\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{24\text{ cm}}+\frac{1}{-\infty}=\frac{1}{0.24\text{ m}}=4.17\ \text{ diopters.}$$

34.110: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{36.0\text{ cm}}+\frac{1.40}{s'}=\frac{0.40}{0.75\text{ cm}}\Rightarrow s'=2.77\text{ cm}.$ This distance is greater than the normal eye, which has a cornea vertex to retina distance of about 2.6 cm.

34.111: a) From Figure 34.56, we define

$$x=r_0-r_0'\Rightarrow r_0'=r_0-x=r_0-r_0\left(\frac{d}{f_1}\right)=\frac{r_0(f_1-d)}{f_1}.$$

b) $s_2=d-f_1\Rightarrow\frac{1}{d-f_1}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow\frac{1}{s_2'}=\frac{d-f_1-f_2}{f_2(d-f_1)}=\frac{|f_2|-f_1+d}{|f_2|(f_1-d)}$

$$\Rightarrow s_2'=\frac{|f_2|(f_1-d)}{|f_2|-f_1+d}$$

. c) $\frac{r_0'}{s_2'}=\frac{r_0}{f}\Rightarrow f=\frac{r_0}{r_0'}s_2'=\frac{f_1}{f_1-d}.\frac{|f_2|(f_1-d)}{|f_2|-f_1+d}\Rightarrow f=\frac{f_1|f_2|}{|f_2|-f_1+d}.$

d) $f_{\max}=\frac{f_1|f_2|}{|f_2|-f_1+d}=\frac{(12.0\text{ cm})(18.0\text{ cm})}{(18.0\text{ cm}-12.0\text{ cm}+0.0\text{ cm})}=36\text{ cm}.$

$$f_{\min}=\frac{f_1|f_2|}{|f_2|-f_1+d}=\frac{(12.0\text{ cm})(18.0\text{ cm})}{(18.0\text{ cm}-12.0\text{ cm}+4.0\text{ cm})}=21.6\text{ cm}.$$

If the effective focal length is 30 cm, then the separation can be calculated:

$$f=\frac{f_1|f_2|}{|f_2|-f_1+d}\Rightarrow 30\text{ cm}=\frac{(12.0\text{ cm})(18.0\text{ cm})}{(18.0\text{ cm}-12.0\text{ cm}+d)}$$

$$\Rightarrow 18.0\text{ cm}-12.0\text{ cm}+d=7.2\text{ cm}\Rightarrow d=1.2\text{ cm}.$$

34.112: First recall that $|M| = \dfrac{\theta'}{\theta}$, and that $\theta = \left|\dfrac{y_1'}{f_1}\right|$ and $\theta' = \left|\dfrac{y_2'}{s_2'}\right| \Rightarrow |M| = \left|\dfrac{y_2'}{s_2'} \cdot \dfrac{f_1}{y_1'}\right|$.

But since the image formed by the objective is used as the object for the eyepiece, $y_1' = y_2$. So $|M| = \left|\dfrac{y_2'}{s_2'} \cdot \dfrac{f_1}{y_2}\right| = \left|\dfrac{y_2'}{y_2} \cdot \dfrac{f_1}{s_2'}\right| = \left|\dfrac{s_2'}{s_2} \cdot \dfrac{f_1}{s_2'}\right| = \left|\dfrac{f_1}{s_2}\right|$.

Therefore, $s_2 = \dfrac{f_1}{|M|} = \dfrac{48.0 \text{ cm}}{36} = 1.33 \text{ cm}$, and this is just outside the eyepiece focal point.

Now the distance from the mirror vertex to the lens is $f_1 + s_2 = 49.3$ cm, and so $\dfrac{1}{s_2} + \dfrac{1}{s_2'} = \dfrac{1}{f_2} \Rightarrow s_2' = \left(\dfrac{1}{1.20 \text{ cm}} - \dfrac{1}{1.33 \text{ cm}}\right)^{-1} = 12.3 \text{ cm}$. Thus we have a final image which is real and 12.3 cm from the eyepiece. (Take care to carry plenty of figures in the calculation because two close numbers are subtracted.)

34.113: a) $\dfrac{1}{s_1} + \dfrac{1}{s_1'} = \dfrac{1}{f_1} \Rightarrow \dfrac{1}{s_1} + \dfrac{1}{18.0 \text{ cm}} = \dfrac{1}{0.800 \text{ cm}} \Rightarrow s_1 = 0.837 \text{ cm}$.

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow \frac{1}{s_2} + \frac{1}{200 \text{ cm}} = \frac{1}{7.50 \text{ cm}} \Rightarrow s_2 = 7.79 \text{ cm}.$$

Also $m_1 = -\dfrac{s_1'}{s_1} = -\dfrac{18.0 \text{ cm}}{0.837 \text{ cm}} = -21.5$ and $m_2 = -\dfrac{s_2'}{s_2} = -\dfrac{200 \text{ cm}}{7.79 \text{ cm}} = -25.7$.

$\Rightarrow m_{\text{total}} = m_1 m_2 = (-21.5)(-25.7) = 553.$

b) $d = s_1' + s_2 = 18.0 \text{ cm} + 7.79 \text{ cm} = 25.8 \text{ cm}.$

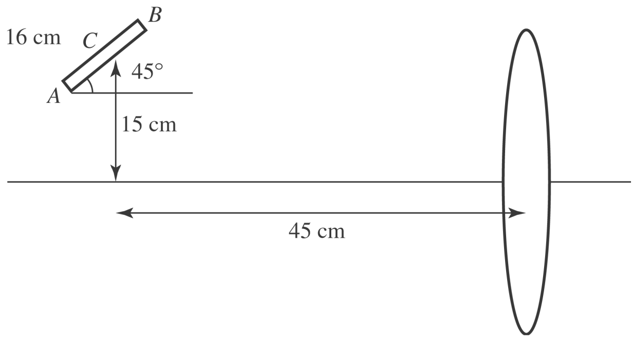
34.114: a) From the figure, $u = \dfrac{y}{f_1}$ and $u' = \dfrac{y}{|f_2|} = -\dfrac{y}{f_2}$.

So the angular magnification is: $M = \dfrac{u'}{u} = -\dfrac{f_1}{f_2}$.

b) $M = -\dfrac{f_1}{f_2} \Rightarrow f_2 = -\dfrac{f_1}{M} = -\dfrac{95.0 \text{ cm}}{6.33} = -15.0 \text{ cm}.$

c) The length of the telescope is $95.0 \text{ cm} - 15.0 \text{ cm} = 80.0 \text{ cm}$, compared to the length of 110 cm for the telescope in Ex. 34.55.

34.115:



a) For point $C: \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{45.0\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}} \Rightarrow s'=36.0\text{ cm}.$

$y'=-\frac{s'}{s}y=-\frac{36.0}{45.0}(15.0\text{ cm})=-12.0\text{ cm}$, so the image of point C is 36.0 cm to the right of the lens, and 12.0 cm below the axis.

For point *A:* $s=45.0\text{ cm}+8.00\text{ cm}(\cos 45°)=50.7\text{ cm}$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{50.7\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}} \Rightarrow s'=33.0\text{ cm}.$

$y'=-\frac{s'}{s}y=-\frac{33.0}{45.0}(15.0\text{ cm}-8.00\text{ cm}(\sin 45°))=-6.10\text{ cm}$, so the image of point A is 33.0 cm to the right of the lens, and 6.10 cm below the axis. For point *B:* $s=45.0\text{ cm}-8.00\text{ cm}(\cos 45°)=39.3\text{ cm}$

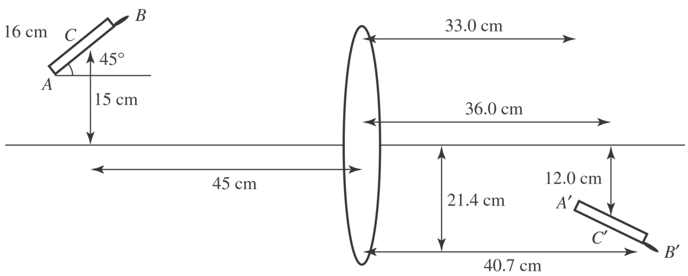
$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{39.3\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}} \Rightarrow s'=40.7\text{ cm}.$

$y'=-\frac{s'}{s}y=-\frac{40.7}{39.3}(15.0\text{ cm}+8.00\text{ cm}(\sin 45°))=-21.4\text{ cm}$, so the image of point B is 40.7 cm to the right of the lens, and 21.4 cm below the axis.

b) The length of the pencil is the distance from point A to *B:*

$L=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}=\sqrt{(33.0\text{ cm}-40.7\text{ cm})^2+(6.10\text{ cm}-21.4\text{ cm})^2}$

$\Rightarrow L=17.1\text{ cm}.$



34.116: a) Using the diagram and law of sines

$$\frac{\sin\theta}{(R-f)} = \frac{\sin\alpha}{g} \text{ but } \sin\theta = \frac{h}{R} = \sin\alpha \text{ (Reflection Rule).}$$

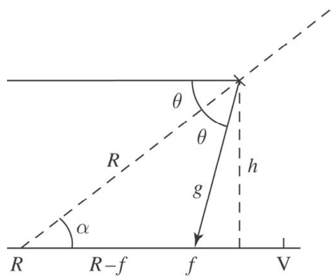
So $g = (R-f)$.

$$\text{Bisecting the triangle: } \cos\theta = \frac{R/2}{(R-f)} \Rightarrow R\cos\theta - f\cos\theta = \frac{R}{2}$$

$$\Rightarrow f = \frac{R}{2}\left[2 - \frac{1}{\cos\theta}\right] = f_0\left[2 - \frac{1}{\cos\theta}\right] = f_0 = \frac{R}{2} (\theta \text{ near } 0).$$

b) $\dfrac{f - f_0}{f_0} = -0.02 \Rightarrow \dfrac{f}{f_0} = 0.98 \text{ so } 2 - \dfrac{1}{\cos\theta} = 0.98 \Rightarrow \cos\theta = \dfrac{1}{2 - 0.98} = 0.98$

$\Rightarrow \theta = 11.4°.$



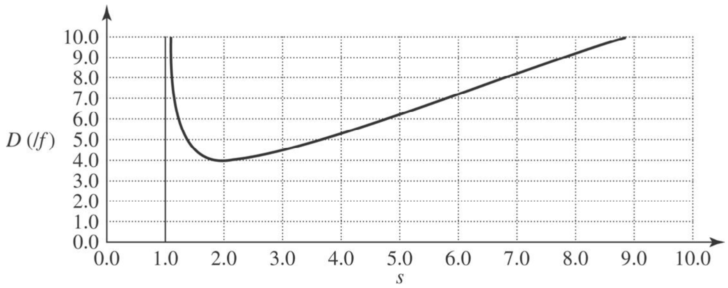
34.117: a) The distance between image and object can be calculated by taking the derivative of the separation distance and minimizing it.

$$D = s + s' \text{ but } s' = \frac{sf}{s-f} \Rightarrow D = s + \frac{sf}{s-f} = \frac{s^2}{s-f}$$

$$\Rightarrow \frac{dD}{ds} = \frac{d}{ds}\left(\frac{s^2}{s-f}\right) = \frac{2s}{s-f} - \frac{s^2}{(s-f)^2} = \frac{s^2-2sf}{(s-f)^2} = 0$$

$\Rightarrow s^2 - 2sf = 0 \Rightarrow s(s-2f) = 0 \Rightarrow s = 0,\, 2f = s'$, so for a real image, the minimum separation between object and image is $4f$.

b)

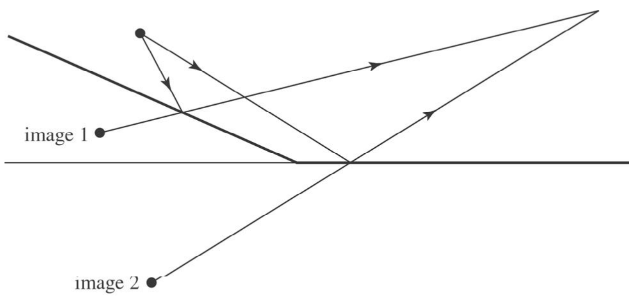


Note that the minimum does occur for $D=4f$.

34.118: a) By the symmetry of image production, any image must be the same distance *D* as the object from the mirror intersection point. But if the images and the object are equal distances from the mirror intersection, they lie on a circle with radius equal to *D*.

b) The center of the circle lies at the mirror intersection as discussed above.

c)



34.119: a) People with normal vision cannot focus on distant objects under water because the image is unable to be focused in a short enough distance to form on the retina. Equivalently, the radius of curvature of the normal eye is about five or six times too great for focusing at the retina to occur.

b) When introducing glasses, let's first consider what happens at the eye:

$\frac{n_a}{s_2} + \frac{n_b}{s_2'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.33}{s_2} + \frac{1.40}{2.6 \text{ cm}} = \frac{0.07}{0.74 \text{ cm}} \Rightarrow s_2 = -3.00 \text{ cm}.$ That is, the object for the cornea must be 3.00 cm behind the cornea. Now, assume the glasses are 2.00 cm in front of the eye, then:

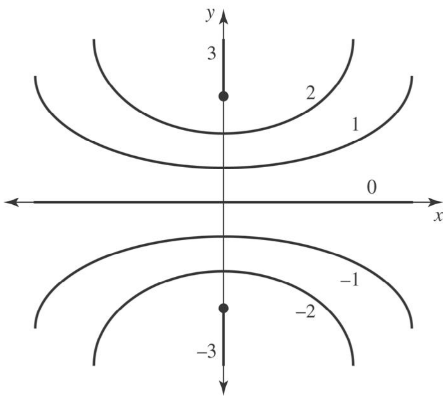
$$s_1' = 2.00 \text{ cm} + s_2 = 5.00 \text{ cm} \Rightarrow \frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1'} \Rightarrow \frac{1}{\infty} + \frac{1}{5.00 \text{ cm}} = \frac{1}{f_1'} \Rightarrow f_1' = 5.00 \text{ cm}$$

This is the focal length in water, but to get it in air, we use the formula from Problem 34.98: $f_1 = f_1'\left[\frac{n - n_{\text{liq}}}{n_{\text{liq}}(n-1)}\right] = (5.00 \text{ cm})\left[\frac{1.52 - 1.333}{1.333(1.52-1)}\right] = 1.34 \text{ cm}$.

35.1: Measuring with a ruler from both S_1 and S_2 to there different points in the antinodal line labeled m = 3, we find that the difference in path length is three times the wavelength of the wave, as measured from one crest to the next on the diagram.

35.2: a) At $S_1, r_2 - r_1 = 4\lambda$, and this path difference stays the same all along the y-axis, so $m = +4$. At $S_2, r_2 - r_1 = -4\lambda$, and the path difference below this point, along the negative y-axis, stays the same, so $m = -4$.

b)



c) The maximum and minimum m-values are determined by the largest integer less than or equal to $\frac{d}{\lambda}$.

d) If $d = 7\frac{1}{2}\lambda \Rightarrow -7 \le m \le +7$, so there will be a total of 15 antinodes between the sources. (Another antinode cannot be squeezed in until the separation becomes six times the wavelength.)

35.3: a) For constructive interference the path diference is $m\lambda, n = 0, \pm 1, \pm 2, \ldots$ The separation between sources is 5.00 m, so for points between the sources the largest possible path difference is 5.00 m. Thus only the path difference of zero is possible. This occurs midway between the two sources, 2.50 m from *A*.

b) For destructive interference the path difference is $(m + \frac{1}{2})\lambda, m = 0, \pm 1, \pm 2, \ldots$

A path difference of $\pm \lambda/2 = 3.00$ m is possible but a path difference as large as $3\lambda/2 = 9.00\,\text{m}$ is not possible. For a point a distance x from A and $5.00 - x$ from B the path difference is

$x - (5.00\ \text{m} - x)$.

$x - (5.00\ \text{m} - x) = +3.00$ m gives $x = 4.00$ m

$x - (5.00\ \text{m} - x) = -3.00$ m gives $x = 1.00$ m

35.4: a) The path difference is 120 m, so for destructive interference:

$$\frac{\lambda}{2} = 120 \text{ m} \Rightarrow \lambda = 240 \text{ m}.$$

b) The longest wavelength for constructive interference is $\lambda = 120$ m.

35.5: For constructive interference, we need $r_2 - r_1 = m\lambda \Rightarrow (9.00 \text{ m} - x) - x = m\lambda$

$$\Rightarrow x = 4.5 \text{ m} - \frac{m\lambda}{2} = 4.5\text{m} - \frac{mc}{2f} = 4.5\text{m} - \frac{m(3.00\times 10^8 \text{ m/s})}{2(120\times 10^6 \text{ Hz})} = 4.5 \text{ m} - m(1.25 \text{ m}).$$

$\Rightarrow x = 0.75$ m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m, 8.25 m. For $m = 3, 2, 1, 0, -1, -2, -3.$ (Don't confuse this m with the unit meters, also represented by an "m").

35.6: a) The brightest wavelengths are when constructive interference occurs:

$$d = m\lambda \Rightarrow \lambda = \frac{d}{m} \Rightarrow \lambda_3 = \frac{2040 \text{ nm}}{3} = 680 \text{ nm}, \ \lambda_4 = \frac{2040 \text{ nm}}{4} = 510 \text{ nm and}$$

$$\lambda_s = \frac{2040 \text{ nm}}{5} = 408 \text{ nm}.$$

b) The path-length difference is the same, so the wavelengths are the same as part (a).

35.7: Destructive interference occurs for:

$$\lambda = \frac{d}{m + 1/2} \Rightarrow \lambda_3 = \frac{2040 \text{ nm}}{3.5} = 583 \text{ nm and } \lambda_4 = \frac{2040 \text{ nm}}{4.5} = 453 \text{ nm}.$$

35.8: a) For the number of antinodes we have:

$$\sin\theta = \frac{m\lambda}{d} = \frac{mc}{df} = \frac{m(3.00\times 10^8 \text{ m/s})}{(12.0 \text{ m})(1.079\times 10^8 \text{ Hz})} = 0.2317\, m, \text{ so, setting } \theta = 90°,$$

the maximum integer value is four. The angles are $\pm 13.4°, \pm 27.6°, \pm 44.0°$, and $\pm 67.9°$ for $m = 0, \pm 1, \pm 2, \pm 3, \pm 4.$

b) The nodes are given by $\sin\,\theta = \dfrac{(m + 1/2)\lambda}{d} = 0.2317\,(m + 1/2).$ So the angles are $\pm 6.65°, \pm 20.3°, \pm 35.4°, 54.2°$ for $m = 0, \pm 1, \pm 2, \pm 3.$

35.9: $$\Delta y = \frac{R\lambda}{d} \Rightarrow \lambda = \frac{d\Delta y}{R} = \frac{(4.60\times 10^{-4} \text{ m})(2.82\times 10^{-3} \text{ m})}{2.20 \text{ m}} = 5.90\times 10^{-7} \text{ m}.$$

35.10: For bright fringes:

$$d = \frac{Rm\lambda}{y_m} = \frac{(1.20 \text{ m})(20)(5.02\times 10^{-7} \text{ m})}{0.0106 \text{ m}} = 1.14\times 10^{-3} \text{ m} = 1.14 \text{ mm}.$$

35.11: Recall $y_m = \dfrac{Rm\lambda}{d} \Rightarrow \Delta y_{23} = y_3 - y_2 = \dfrac{R\lambda(3-2)}{d} = \dfrac{(0.750\text{ m})\ (5.00\times 10^{-7}\text{ m})}{4.50\times 10^{-4}\text{ m}}$

$$\Rightarrow \Delta y_{23} = 8.33\times 10^{-4}\text{ m} = 0.833\text{ mm.}$$

35.12: The width of a bright fringe can be defined to be the distance between its two adjacent destructive minima. Assuming the small angle formula for destructive interference

$$y_m = R\frac{(m+\frac{1}{2})\lambda}{d},$$

the distance between any two successive minima is

$$y_{n+1} - y_n = R\frac{\lambda}{d} = (4.00\text{ m})\frac{(400\times 10^{-9}\text{m})}{(0.200\times 10^{-3}\text{ m})} = 8.00\text{ mm.}$$

Thus, the answer to both part (a) and part (b) is that the width is 8.00 mm.

35.13: Use the information given about the bright fringe to find the distance *d* between the two slits: $y_1 = \dfrac{R\lambda_1}{d}$ (Eq.35.6), so $d = \dfrac{R\lambda_1}{y_1} = \dfrac{(3.00\text{ m})(600\times 10^{-9}\text{ m})}{4.84\times 10^{-3}\text{ m}} = 3.72\times 10^{-4}\text{m.}$

(*R* is much greater than *d*, so Eq.35.6 is valid.)

The dark fringes are located by $d\sin\theta = (m+\frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots$ The first order dark fringe is located by $\sin\theta = \lambda_2/2d$, where λ_2 is the wavelength we are seeking.

$$y = R\tan\theta \approx R\sin\theta = \frac{\lambda_2 R}{2d}$$

We want λ_2 such that $y = y_1$. This gives $\dfrac{R\lambda_1}{d} = \dfrac{R\lambda_2}{2d}$ and $\lambda_2 = 2\lambda_1 = 1200\text{ nm.}$

35.14: Using Eq.35.6 for small angles,

$$y_m = R\frac{m\lambda}{d},$$

we see that the distance between corresponding bright fringes is

$$\Delta_y = \frac{Rm}{d}\Delta\lambda = \frac{(5.00\text{ m})(1)}{(0.300\times 10^{-3}\text{ m})}(660-470)\times(10^{-9}\text{ m}) = 3.17\text{ mm.}$$

35.15: We need to find the positions of the first and second dark lines:

$$\theta_1 = \arcsin\left(\frac{\lambda}{2d}\right) = \arcsin\left(\frac{5.50\times10^{-7}\ \text{m}}{2(1.80\times10^{-6}\ \text{m}}\right) = 8.79°$$

$$\Rightarrow y_1 = R\tan\theta_1 = (0.350\ \text{m})\ \tan(8.79°) = 0.0541\ \text{m}.$$

Also $\theta_2 = \arcsin\left(\frac{3\lambda}{2d}\right) = \arcsin\left(\frac{3(5.50\times10^{-7}\ \text{m})}{2(1.80\times10^{-6}\ \text{m})}\right) = 27.3°$

$$\Rightarrow y_2 R\tan\theta_2 = (0.350\ \text{m})\ \tan(27.30) = 0.1805\ \text{m}.$$

The fringe separation is then $\Delta y = y_2 - y_1 = 0.1805\ \text{m} - 0.0541\ \text{m} = 0.1264\ \text{m}.$

35.16: (a) Dark fringe implies destructive interference.

$$d\ \sin\theta = \frac{1}{2}\lambda$$

$$d = \frac{\lambda}{2\ \sin\theta} = \frac{624\times10^{-9}\text{m}}{2\ \sin 11.0°} = 1.64\ \times10^{-6}\text{m}$$

(b) Bright fringes: $d\ \sin\theta_{\text{max}} = m_{\text{max}}\lambda$

The largest that θ can be is $90°$, so $m_{\text{max}} = d/\lambda = \frac{1.64\times10^{-6}\ \text{m}}{624\times10^{-9}\ \text{m}} = 2.6$ Since m is an integer, its maximum value is 2. There are 5 bright fringes, the central spot and 2 on each side of it. Dark fringes: $d\ \sin\theta = \left(m + \frac{1}{2}\right)\lambda.$ This equation has solutions for $\theta = \pm 11.0°; \pm 34.9°;$ and $\pm 72.6°.$ Therefore, there are 6 dark fringes.

35.17: Bright fringes for wavelength λ are located by $d\ \sin\theta = m\lambda.$ First-order $(m = 1)$ is closest to the central bright line, so $\sin\theta = \lambda / d.$

$\lambda = 400$ nm gives $\sin\theta = (400\times10^{-9}\ \text{m})/(0.100\times10^{-3}\ \text{m})$ and $\theta = 0.229°$

$\lambda = 700$ nm gives $\sin\theta = (700\times10^{-9}\ \text{m})/(0.100\times10^{-3}\ \text{m})$ and $\theta = 0.401°$

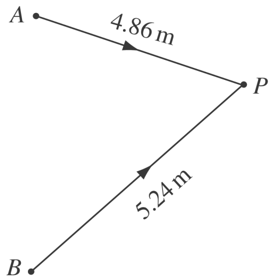
The angular width of the visible spectrum is thus $0.401° - 0.229° = 0.172°.$

35.18: $y = \frac{R\lambda}{d} \Rightarrow d = \frac{R\lambda}{y} = \frac{(1.80\ \text{m})\ (4.50\times10^{-7}\ \text{m})}{4.20\times10^{-3}\ \text{m}} - 1.93\times10^{-4}\text{m} = 0.193\ \text{m}.$

35.19: The phase difference ϕ is given by $\phi = (2\pi d/\lambda)\sin\theta$(Eq.35.13)

$$\phi = [2\pi(0.340\times10^{-3}\text{m})/(500\times10^{-9}\ \text{m})]\sin 23.0° = 1670\ \text{rad}$$

35.20:



$$\frac{\Delta\phi}{2\pi} = \frac{\text{Path difference}}{\lambda}$$

$$\Delta\phi = 2\pi\left(\frac{524\text{cm} - 486\text{ cm}}{2\text{ cm}}\right) = 119 \text{ radians}$$

35.21: a) Eq.(35.10): $I = I_0 \cos^2(\phi/2) = I_0(\cos 30.0°)^2 = 0.750 I_0$

b) $60.0° = (\pi/3)$ rad

Eq. (35.11) : $\phi = (2\pi/\lambda)(r_2 - r_1)$, so

$(r_2 - r_1) = (\phi/2\pi)\lambda = [(\pi/3)/2\pi]\lambda = \lambda/6 = 80$ nm

35.22: a) The source separation is 9.00 m, and the wavelength of the wave is $\lambda = \frac{c}{f} = \frac{3.00\times10^8 \text{ m/s}}{1.50\times10^7 \text{ Hz}} = 20.0$ m. So there is only one antinode between the sources $(m = 0)$, and it is a perpendicular bisector of the line connecting the sources.

b) $I = I_0 \cos^2\left(\frac{\phi}{2}\right) = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = I_0 \cos^2\left(\frac{\pi(9.00 \text{ m})}{(20.0 \text{ m})}\sin\theta\right)$

$= I_0 \cos^2 \left((1.41)\sin\theta\right)$

So, for $\theta = 30°, I = 0.580\ I_0; \theta = 45°, I = 0.295\ I_0;$

$\theta = 60°, I = 0.117\ I_0; \theta = 90°, I = 0.026\ I_0.$

35.23: a) The distance from the central maximum to the first minimum is half the distance to the first maximum, so:

$$y = \frac{R\lambda}{2d} = \frac{(0.700\ \text{m})(6.60\times10^{-7}\ \text{m})}{2(2.60\times10^{-4}\ \text{m})} = 8.88\times10^{-4}\ \text{m}.$$

b) The intensity is half that of the maximum intensity when you are halfway to the first minimum, which is 4.44×10^{-4} m. Remember, all angles are *small*.

35.24: a) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.20\times10^{8}\ \text{Hz}} = 2.50\ \text{m}$, and we have:

$$\phi = \frac{2\pi}{\lambda}(r_1 - r_2) = \frac{2\pi}{2.50\ \text{m}}(1.8\ \text{m}) = 4.52\ \text{rad}.$$

b) $I = I_0\cos^2\left(\dfrac{\phi}{2}\right) = I_0\cos^2\left(\dfrac{4.52\ \text{rad}}{2}\right) = 0.404I_0.$

35.25: a) To the first maximum: $y_1 = \dfrac{R\lambda}{d} = \dfrac{(0.900\ \text{m})\,(5.50\times10^{-7}\ \text{m})}{1.30\times10^{-4}\ \text{m}} = 3.81\times10^{-3}\ \text{m}.$ So the distance to the first minimum is one half this, 1.91 mm.

b) The first maximum and minimum are where the waves have phase differences of zero and pi, respectively. Halfway between these points, the phase difference between the waves is $\dfrac{\pi}{2}$. So:

$$I = I_0\cos^2\left(\frac{\phi}{2}\right) = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2} = 2.00\times10^{-6}\ \text{W/m}^2.$$

35.26: From Eq. (35.14), $I = I_0\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta\right)$. So the intensity goes to zero when the cosine's argument becomes an odd integer of $\dfrac{\pi}{2}$. That is: $\dfrac{\pi d}{\lambda}\sin\theta = (m + 1/2)\pi \Rightarrow$ $d\sin\theta = \lambda(m + 1/2)$, which is Eq. (35.5).

35.27: By placing the paper between the pieces of glass, the space forms a cavity whose height varies along the length. If *twice* the height at any given point is one wavelength (recall it has to make a return trip), constructive interference occurs. The distance between the maxima (i.e., the # of meters per fringe) will be

$$\Delta x = \frac{\lambda l}{2h} = \frac{\lambda}{2\tan\theta} \Rightarrow \theta = \arctan\left(\frac{\lambda}{2\Delta x}\right) = \arctan\left(\frac{5.46\times10^{-7}\ \text{m}}{2((1/1500)\ \text{m})}\right) = 4.095\times10^{-4}\ \text{rad} = 0.02$$

35.28: The distance between maxima is

$$\Delta x = \frac{\lambda l}{2h} = \frac{(6.56\times10^{-7}\ \text{m})\,(9.00\ \text{cm})}{2(8.00\times10^{-5}\ \text{m})} = 0.0369\ \text{cm}.$$

So the number of fringes per centimeter is $\dfrac{1}{\Delta x} = 27.1\ \text{fringes/cm}.$

35.29: Both parts of the light undergo half-cycle phase shifts when they reflect, so for destructive interference $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{6.50 \times 10^{-7}\ \text{m}}{4(1.42)} = 1.14 \times 10^{-7}\ \text{m} = 114\ \text{nm}.$

35.30: There is a half-cycle phase shift at both interfaces, so for destructive interference:

$$t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{480\ \text{nm}}{4(1.49)} = 80.5\ \text{nm}.$$

35.31: Destructive interference for $\lambda_1 = 800\,\text{nm}$ incident light. Let n be the refractive index of the oil. There is a $\lambda/2$ phase shift for the reflection at the air-oil interface but no phase shift for the reflection at the oil-water interface. Therefore, there is a net $\lambda/2$ phase difference due to the reflections, and the condition for destructive interference is $2t = m(\lambda/n)$. Smallest nonzero thickness means $m = 1$, so $2tn = \lambda_1$.
The condition for constructive interference with incident wavelength λ is $2t = (m + \frac{1}{2})(\lambda/n)$ and $2tn = (m + \frac{1}{2})\lambda$.

But $2tn = \lambda_1$, so $\lambda = \lambda_1/(m + \frac{1}{2})$, where $\lambda_1 = 800$ nm.

for $m = 0$, $\lambda = 1600$ nm

for $m = 1$, $\lambda = 533$ nm

for $m = 2$, $\lambda = 320$ nm, and so on

The visible wavelength for which there is constructive interference is 533 nm.

35.32: a) The number of wavelengths is given by the total extra distance traveled, divided by the wavelength, so the number is

$$\frac{x}{\lambda} = \frac{2tn}{\lambda_0} = \frac{2(8.76 \times 10^{-6}\ \text{m})\,(1.35)}{6.48 \times 10^{-7}\text{m}} = 36.5.$$

b) The phase difference for the two parts of the light is zero because the path difference is a half-integer multiple of the wavelength and the top surface reflection has a half-cycle phase shift, while the bottom surface does not.

35.33: Both rays, the one reflected from the pit and the one reflected from the flat region between the pits, undergo the same phase change due to reflection. The condition for destructive interference is $2t = (m + \frac{1}{2})(\lambda/n)$, where n is the refractive index of the plastic substrate. The minimum thickness is for $m = 0$, and equals $t = \lambda/(4n) = 790\ \text{nm}/[(4)(1.8)] = 110\ \text{nm} = 0.11\mu\text{m}.$

35.34: A half-cycle phase change occurs, so for destructive interference

$$t = \frac{\lambda}{2} = \frac{\lambda_0}{2n} = \frac{480\text{nm}}{2(1.33)} = 180\ \text{nm}.$$

35.35: a) To have a strong reflection, constructive interference is desired. One part of the light undergoes a half-cycle phase shift, so:

$$2d = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \Rightarrow \lambda = \frac{2dn}{\left(m + \frac{1}{2}\right)} = \frac{2(290\text{ nm})\,(1.33)}{\left(m + \frac{1}{2}\right)} = \frac{771\text{ nm}}{\left(m + \frac{1}{2}\right)}.$$ For an integer value of zero, the wavelength is not visible (infrared) but for $m = 1$, the wavelength is 514 nm, which is green.

b) When the wall thickness is 340 nm, the first visible constructive interference occurs again for $m = 1$ and yields $\lambda = \dfrac{904\text{ nm}}{\left(m + \frac{1}{2}\right)} = 603$ nm, which is orange.

35.36: a) Since there is a half-cycle phase shift at just one of the interfaces, the minimum thickness for constructive interference is:

$$t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{550\text{ nm}}{4(1.85)} = 74.3\text{ nm}.$$

b) The next smallest thickness for constructive interference is with another half wavelength thickness added: $t = \dfrac{3\lambda}{4} = \dfrac{3\lambda_0}{4n} = \dfrac{3(550\text{ nm})}{4(1.85)} = 223$ nm.

35.37: $x = \dfrac{m\lambda}{2} = \dfrac{1800(6.33\times10^{-7}\text{ m})}{2} = 5.70\times10^{-4}\text{ m} = 0.570\text{ mm}.$

35.38: a) For Jan, the total shift was $\Delta x_1 = \dfrac{m\lambda 1}{2} = \dfrac{818(6.06\times10^{-7}\text{ m})}{2} = 2.48\times10^{-4}\text{m}.$

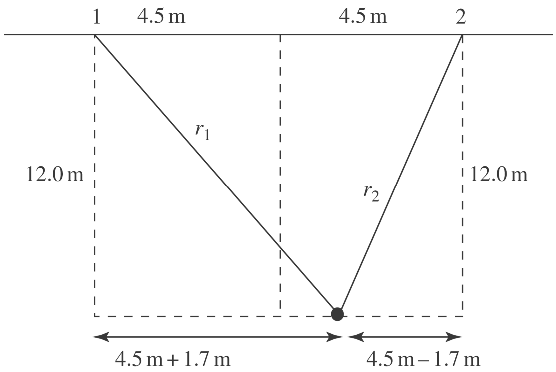
For Linda, the total shift was $\Delta x_2 = \dfrac{m\lambda^2}{2} = \dfrac{818(5.02\times10^{-7}\text{ m})}{2} = 2.05\times10^{-4}\text{ m}.$

b) The net displacement of the mirror is the difference of the above values:

$\Delta x = \Delta x_1 = \Delta x_2 = 0.248\text{ mm} - 0.205\text{ mm} = 0.043\text{ mm}.$

35.39: Immersion in water just changes the wavelength of the light from Exercise 35.11, so: $y = \dfrac{R\lambda}{dn} = \dfrac{y_{\text{vacuum}}}{n} = \dfrac{0.833\text{ mm}}{1.33} = 0.626$ mm, using the solution from Exercise 35.11.

35.40: Destructive interference occurs 1.7 m from the centerline.



$$r_1 = \sqrt{(12.0\ \text{m})^2 + (6.2\ \text{m})^2} = 13.51\ \text{m}$$
$$r_2 = \sqrt{(12.0\ \text{m})^2 + (2.8\ \text{m})^2} = 12.32\ \text{m}$$

For destructive interference, $r_1 - r_2 = \lambda/2 = 1.19\ \text{m}$ and $\lambda = 2.4\ \text{m}$. The wavelength we have calculated is the distance between the wave crests.

Note: The distance of the person from the gaps is not large compared to the separation of the gaps, so the path length is not accurately given by $d \sin\theta$.

35.41: a) Hearing minimum intensity sound means that the path lengths from the individual speakers to you differ by a half-cycle, and are hence out of phase by $180°$ at that position.

b) By moving the speakers toward you by 0.398 m, a maximum is heard, which means that you moved the speakers one-half wavelength from the min and the signals are back in phase. Therefore the wavelength of the signals is 0.796 m, and the frequency is

$$f = \frac{v}{\lambda} = \frac{340\ \text{m/s}}{0.796\ \text{m}} = 427\ \text{Hz}.$$

c) To reach the next maximum, one must move an additional distance of one wavelength, a distance of 0.796 m.

35.42: To find destructive interference, $d = r_2 - r_1 = \sqrt{(200\ m)^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda$

$$\Rightarrow (200\ \text{m})^2 + x^2 = x^2 + \left[\left(m + \frac{1}{2}\right)\lambda\right]^2 + 2x\left(m + \frac{1}{2}\right)\lambda$$

$$\Rightarrow x = \frac{20{,}000\ \text{m}^2}{\left(m + \frac{1}{2}\right)\lambda} - \frac{1}{2}\left(m + \frac{1}{2}\right)\lambda.$$

The wavelength is calculated by $\lambda = \frac{c}{f} = \frac{3.00\times10^8\ \text{m/s}}{5.80\times10^6\ \text{Hz}} = 51.7\ \text{m}.$

$\Rightarrow m = 0 : x = 761\ \text{m}$, and $m = 1 : x = 219$ m, and $m = 2 : x = 90.1$ m, and $m = 3;\ x = 20.0$ m.

35.43: At points on the same side of the centerline as point A, the path from B is longer than the path from A, and the path difference $d \sin\theta$ puts speaker A ahead of speaker B in phase. Constructive interference occurs when

$$d \sin\theta - \lambda/6 = \left(m + \frac{1}{2}\right)\lambda,\ m = 0, 1, 2, \ldots$$

$$\sin\theta = \left(m + \frac{2}{3}\right)(\lambda/d) = \left(m + \frac{2}{3}\right)(0.2381),\ m = 0, 1, 2, \ldots$$

$m = 0, 9.13°;\ m = 1, 23.4°;\ m = 2, 39.4°;\ m = 3, 60.8°;\ m = 4,$ no solution

At points on the other side of the centerline, the path from A is longer than the path from B, and the path difference $d \sin\theta$ puts speaker A behind speaker B in phase. Constructive interference occurs when

$$d \sin\theta + \lambda/6 = \left(m + \frac{1}{2}\right)\lambda,\ m = 0, 1, 2, \ldots$$

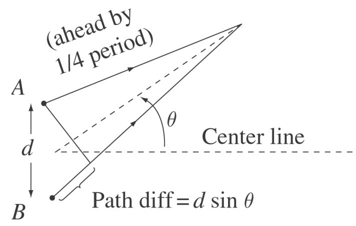
$$\sin\theta = \left(m + \frac{2}{3}\right)(\lambda/d) = \left(m + \frac{1}{3}\right)(0.2381),\ m = 0, 1, 2, \ldots$$

$m = 0,\ 4.55°;\ m = 1, 18.5°;\ m = 2,\ 33.7°;\ m = 3,\ 52.5°;\ m = 4,$ no solution

35.44: First find out what fraction the 0.159 ms time lag is of the period.

$$\Delta t = \frac{0.159\times10^{-3}\,s}{T} = (0.159\times10^{-3}\,\mathrm{s})\,f = (0.159\times10^{-3}\,\mathrm{s})\,(1570\ \mathrm{Hz})$$

$\Delta t = 0.250$, so the speakers are $1/4$ period out of phase. Let A be ahead of B in phase.



$$\lambda = v/f = \frac{330\ \mathrm{m/s}}{1570\ \mathrm{Hz}} = 0.210\ \mathrm{m}$$

On A's side of centerline: Since A is ahead by $1/4$ period, the path difference must retard B's phase enough so the waves are in phase.

$$d\sin\theta = \frac{3}{4}\lambda, \frac{7}{4}\lambda, ...$$

$$\sin\theta_1 = \frac{3}{4}\left(\frac{0.210\ \mathrm{m}}{0.422\ \mathrm{m}}\right) \rightarrow \theta_1 = 21.9°$$

$$\sin\theta_2 = \frac{7}{4}\left(\frac{0.210\ \mathrm{m}}{0.422\ \mathrm{m}}\right) \rightarrow \theta_2 = 60.6°$$

On B's side of centerline: The path difference must now retard A's sound by $\frac{1}{4}\lambda, \frac{5}{4}\lambda, \ldots$

$$-d\sin\theta = \frac{1}{4}\lambda, \frac{5}{4}\lambda, \ldots \text{ gives } -7.2°, -38.5°$$

35.45: a) If the two sources are out of phase by one half-cycle, we must add an extra half a wavelength to the path difference equations Eq. (35.1) and Eq. (35.2).

This exactly changes one for the other, for $m \rightarrow m + \frac{1}{2}$ and $m + \frac{1}{2} \rightarrow m,$ since m in any integer.

b) If one source leads the other by a phase angle ϕ, the fraction of a cycle difference is $\frac{\phi}{2\pi}$. Thus the path length difference for the two sources must be adjusted for both destructive and constructive interference, by this amount. So for constructive inference: $r_1 - r_2 = (m + \phi/2\pi)\lambda,$ and for destructive interference, $r_1 - r_2 = (m + 1/2 + \phi/2\pi)\lambda.$

35.46: a) The electric field is the sum of the two wave functions, and can be written: $E_p(t) = E_2(t) + E_1(t) = E\cos(\omega t) + E\cos(\omega t + \phi) \Rightarrow E_p(t) = 2E\cos(\phi/2)\cos(\omega t + \phi/2)$.

b) $E_p(t) = A\cos(\omega t + \phi/2)$, so comparing with part (a), we see that the amplitude of the wave (which is always positive) must be $A = 2E\,|\cos(\phi/2)|$.

c) To have an interference maximum, $\frac{\phi}{2} = 2\pi m$. So, for example, using $m = 1$, the relative phases are $E_2 : \varphi = 0;\ E_1 : \varphi = \phi = 4\pi; E_p : \varphi = \frac{\phi}{2} = 2\pi$, and all waves are in phase.

d) To have an interference minimum, $\frac{\phi}{2} = \pi\left(m + \frac{1}{2}\right)$. So, for example using $m = 0$, relative phases are $E_2 : \varphi = 0;\ E_1 : \varphi = \phi = \pi; E_p : \varphi = \phi/2 = \pi/2$, and the resulting wave is out of phase by a quarter of a cycle from both of the original waves.

e) The instantaneous magnitude of the Poynting vector is:

$$|\vec{S}| = \varepsilon_0 c E_p{}^2(t) = \varepsilon_0 c(4E^2\cos^2(\phi/2)\cos^2(\omega t + \phi/2)).$$

For a time average, $\cos^2(\omega t + \phi/2) = \frac{1}{2}$, so $|S_{av}| = 2\varepsilon_0 c E^2 \cos^2(\phi/2)$.

35.47: a)

$$\Delta r = m\lambda$$
$$r_1 = \sqrt{x^2 + (y-d)^2}.$$
$$r_2 = \sqrt{x^2 + (y+d)^2}.$$
$$\text{So } \Delta r = \sqrt{x^2 + (y+d)^2} - \sqrt{x^2 + (y-d)^2} = m\lambda.$$

b) The definition of hyperbola is the locus of points such that the difference between P to S_2 and P to S_1 is a constant. So, for a given m and λ we get a hyperbola. Or, in the case of all m for a given λ, a family of hyperbola.

c) $\sqrt{x^2 + (y+d)^2} - \sqrt{x^2 + (y-d)^2} = (m + \frac{1}{2})\lambda.$

35.48: a) $E_p^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(\pi - \phi)$

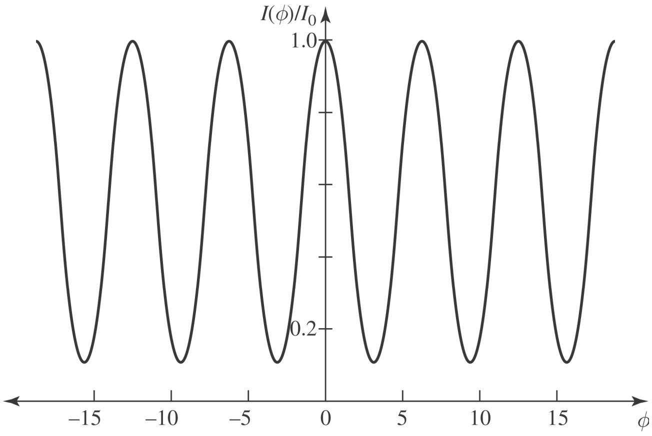
$$= E^2 + 4E^2 + 4E^2\cos\phi = 5E^2 + 4E^2\cos\phi.$$

$$I = \frac{1}{2}\varepsilon_0 c E_p{}^2 = \varepsilon_0 c\left[\left(\frac{5}{2}E^2\right) + \left(\frac{4}{2}E^2\right)\cos\phi\right].$$

$$\phi = 0 \Rightarrow I_0 = \frac{9}{2}\varepsilon_0 c E^2.$$

So $I = I_0\left[\frac{5}{9} + \frac{4}{9}\cos\phi\right].$

b) $I_{\min} = \frac{1}{9}I_0$ which occurs when $\phi = n\pi (n \text{ odd})$.



35.49: For this film on this glass, there is a net $\lambda/2$ phase change due to reflection and the condition for destructive interference is $2t = m(\lambda/n)$, where $n = 1.750$.
Smallest nonzero thickness is given by $t = \lambda/2n$.
At 20.0°C, $t_0 - (582.4\text{ nm})/[(2)\ (1.750)] = 166.4\text{ nm}$.
At 170°C, $t_0 = (588.5\text{ nm})/[(2)\ (1.750)] = 168.1\text{ nm}$.
$t = t_0(1 + \alpha\Delta T)$ so
$\alpha = (t - t_0)/(t_0\Delta T) = (1.7\text{ nm})/[(166.4\text{ nm})\ (150\text{C}^\circ)] = 6.8\times10^{-5}\ (\text{C}^\circ)^{-1}$

35.50: For constructive interference: $d\sin\theta = m\lambda_1 \Rightarrow d\sin\theta = 3(700\ \text{nm}) = 2100\ \text{nm}$.

For destructive interference: $d\sin\theta = \left(m+\dfrac{1}{2}\right)\lambda_2 \Rightarrow \lambda_2 = \dfrac{d\sin\theta}{m+\frac{1}{2}} = \dfrac{2100\ \text{nm}}{m+\frac{1}{2}}$.

So the possible wavelengths are $\lambda_2 = 600$ nm, for $m = 3$, and $\lambda_2 = 467$ nm, for $m = 4$.

Both d and θ drop out of the calculation since their combination is just the path difference, which is the same for both types of light.

35.51: First we need to find the angles at which the intensity drops by one-half from the value of the m th bright fringe.

$$I = I_0\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = \frac{I_0}{2} \Rightarrow \frac{\pi d}{\lambda}\sin\theta \approx \frac{\pi d\theta_m}{\lambda} = (m+1/2)\frac{\pi}{2}.$$

$$\Rightarrow m = 0 : \theta = \theta_m^- = \frac{\lambda}{4d};\ m = 1 : \theta = \theta_m^+ = \frac{3\lambda}{4d} \Rightarrow \Delta\theta_m = \frac{\lambda}{2d'}$$

so there is no dependence on the m - value of the fringe.

35.52: There is just one half-cycle phase change upon reflection, so for constructive interference: $2t = (m_1+\frac{1}{2})\lambda_1 = (m_2+\frac{1}{2})\lambda_2$. But the two different wavelengths differ by just one m - value, $m_2 = m_1 - 1$.

$$\Rightarrow \left(m_1+\frac{1}{2}\right)\lambda_1 = \left(m_1-\frac{1}{2}\right)\lambda_2 \Rightarrow m_1(\lambda_2-\lambda_1) = \frac{\lambda_1+\lambda_2}{2} \Rightarrow m_1 = \frac{\lambda_1+\lambda_2}{2(\lambda_2-\lambda_1)}$$

$$\Rightarrow m_1 = \frac{477.0\ \text{nm} + 540.6\ \text{nm}}{2(540.6\ \text{nm} - 477.0\ \text{nm})} = 8.$$

$$\Rightarrow 2t = \left(8+\frac{1}{2}\right)\frac{\lambda_1}{n} \Rightarrow t = \frac{17(477.0\ \text{nm})}{4(1.52)} = 1334\ \text{nm}.$$

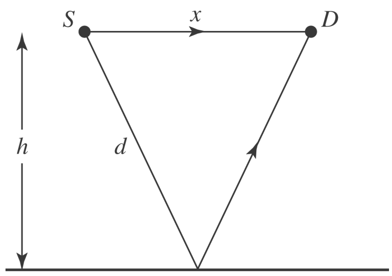
35.53: a) There is a half-cycle phase change at the glass, so for constructive interference:

$$2d - x = 2\sqrt{h^2 + \left(\frac{x}{2}\right)^2} - x = \left(m + \frac{1}{2}\right)\lambda$$

$$\Rightarrow \sqrt{x^2 + 4h^2} - x = \left(m + \frac{1}{2}\right)\lambda.$$

Similarly for destructive interference:

$$\sqrt{x^2 + 4h^2} - x = m\lambda.$$



b) The longest wavelength for constructive interference is when $m = 0$:

$$\lambda = \frac{\sqrt{x^2 + 4h^2} - x}{m + \frac{1}{2}} = \frac{\sqrt{(14\text{ cm})^2 + 4(24\text{ cm})^2} - 14\text{ cm}}{1/2} = 72\text{ cm}.$$

35.54: a) At the water (or cytoplasm) to guanine interface, is a half-cycle phase shift for the reflected light, but there is not one at the guanine to cytoplasm interface. Therefore there will always be one half-cycle phase difference between two neighboring reflected beams. For the guanine layers:

$$2t_g = (m + \frac{1}{2})\frac{\lambda}{n_g} \Rightarrow \lambda = \frac{2t_g n_g}{(m + \frac{1}{2})} = \frac{2(74\text{ nm})\,(1.80)}{(m + \frac{1}{2})} = \frac{266\text{ nm}}{(m + \frac{1}{2})} \Rightarrow \lambda = 533\text{ nm}(m = 0).$$

For the cytoplasm layers:

$$2t_c = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_c} \Rightarrow \lambda = \frac{2t_c n_c}{(m + \frac{1}{2})} = \frac{2(100\text{ nm})\,(1.333)}{(m + \frac{1}{2})} = \frac{267\text{ nm}}{(m + \frac{1}{2})} \Rightarrow \lambda = 533\text{ nm}(m = 0).$$

b) By having many layers the reflection is strengthened, because at each interface some more of the transmitted light gets reflected back, increasing the total percentage reflected.

c) At different angles, the path length in the layers change (always to a larger value than the normal incidence case). If the path length changes, then so do the wavelengths that will interfere constructively upon reflection.

35.55: a) Intensified reflected light means we have constructive interference. There is one half-cycle phase shift, so:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \Rightarrow \lambda = \frac{2tn}{(m+\frac{1}{2})} = \frac{2(485\text{ nm})\,(1.53)}{(m+\frac{1}{2})} = \frac{1484\text{ nm}}{(m+\frac{1}{2})}.$$

$$\Rightarrow \lambda = 593\text{ nm}(m=2), \text{ and } \lambda = 424\text{ nm } (m=3).$$

b) Intensified transmitted light means we have destructive interference at the upper surface. There is still a one half-cycle phase shift, so:

$$2t = \frac{m\lambda}{n} \Rightarrow \lambda = \frac{2tn}{m} = \frac{2(485\text{ nm})\,(1.53)}{m} = \frac{1484\text{ nm}}{m}.$$

$$\Rightarrow \lambda = 495\text{ nm } (m=3)$$

is the only wavelength of visible light that is intensified. We could also think of this as the result of internal reflections interfering with the outgoing ray *without* any extra phase shifts.

35.56: a) There is one half-cycle phase shift, so for constructive interference:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{n} \Rightarrow \lambda = \frac{2tn}{(m+\frac{1}{2})} = \frac{2(380\text{ nm})\,(1.45)}{(m+\frac{1}{2})} = \frac{1102\text{ nm}}{(m+\frac{1}{2})}.$$

Therefore, we have constructive interference at $\lambda = 441\text{ nm } (m=2)$, which corresponds to blue-violet.

b) Beneath the water, looking for maximum intensity means that the reflected part of the wave at the wavelength must be weak, or have interfered destructively. So:

$$2t = \frac{m\lambda_0}{n} \Rightarrow \lambda_0 = \frac{2tn}{m} = \frac{2(380\text{ nm})\,(1.45)}{m} = \frac{1102\text{ nm}}{m}.$$

Therefore, the strongest transmitted wavelength (as measured in air) is $\lambda = 551\text{ nm } (m=2)$, which corresponds to green.

35.57: For maximum intensity, with a half-cycle phase shift,

$$2t = \left(m + \frac{1}{2}\right)\lambda \text{ and } t = R - \sqrt{R^2 - r^2} \Rightarrow \frac{(2m+1)\lambda}{4} = R - \sqrt{R^2 - r^2}$$

$$\Rightarrow \sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4} \Rightarrow R^2 - r^2 = R^2 + \left[\frac{(2m+1)\lambda}{4}\right]^2 - \frac{(2m+1)\lambda R}{2}$$

$$\Rightarrow r = \sqrt{\frac{(2m+1)\lambda R}{2} - \left[\frac{(2m+1)\lambda}{4}\right]^2} \Rightarrow r \approx \sqrt{\frac{(2m+1)\lambda R}{2}}, \text{ for } R >> \lambda.$$

The second bright ring is when $m = 1$:

$$r \approx \sqrt{\frac{(2(1)+1)\,(5.80\times10^{-7}\text{ m})\,(0.952\text{ m})}{2}} = 9.10\times10^{-4}\text{ m} = 0.910\text{ mm}.$$

So the diameter of the third bright ring is 1.82 mm.

35.58: As found in Problem (35.51), the radius of the mth bright ring is in general:

$$r \approx \sqrt{\frac{(2m+1)\lambda R}{2}},$$

for $R >> \lambda$. Introducing a liquid between the lens and the plate just changes the wavelength from $\lambda \to \frac{\lambda}{n}$.

So:

$$r(n) \approx \sqrt{\frac{(2m+1)\lambda R}{2n}} = \frac{r}{\sqrt{n}} = \frac{0.850\text{ mm}}{\sqrt{1.33}} = 0.737\text{ mm}.$$

35.59: a) Adding glass over the top slit increases the effective path length from that slit to the screen. The interference pattern will therefore change, with the central maximum shifting downwards.

b) Normally the phase shift is $\phi = \frac{2\pi d}{\lambda}\sin\theta$, but now there is an added shift from the glass, so the total phase shift is now

$$\phi = \frac{2\pi d}{\lambda}\sin\theta + \left(\frac{2\pi Ln}{\lambda} - \frac{2\pi L}{\lambda}\right) = \frac{2\pi d}{\lambda}\sin\theta + \frac{2\pi L(n-1)}{\lambda} = \frac{2\pi}{\lambda}(d\sin\theta + L(n-1)).$$

So the intensity becomes $I = I_0\cos^2\frac{\phi}{2} = I_0\cos^2\left(\frac{\pi}{\lambda}(d\sin\theta + L(n-1))\right)$.

c) The maxima occur at $\frac{\pi}{\lambda}(d\sin\theta + L(n-1)) = m\pi \Rightarrow d\sin\theta = m\lambda - L(n-1)$

35.60: The passage of fringes indicates an effective change in path length, since the wavelength of the light is getting shorter as more gas enters the tube.

$$\Delta m = \frac{2L}{\lambda/n} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(n-1) \Rightarrow (n-1) = \frac{\Delta m\lambda}{2L}.$$

So here:

$$(n-1) = \frac{48(5.46\times10^{-7}\text{ m})}{2(0.0500\text{ m})} = 2.62\times10^{-4}.$$

35.61: There are two effects to be considered: first, the expansion of the rod, and second, the change in the rod's refractive index. The extra length of rod replaces a little of the air so that the change in the number of wavelengths due to this is given by:

$$\Delta N_1 = \frac{2n_{\text{glass}}\Delta L}{\lambda_0} - \frac{2n_{\text{air}}\Delta L}{\lambda_0} = \frac{2(n_{\text{glass}} - 1)L_0\alpha\Delta T}{\lambda_0}$$

$$\Rightarrow \Delta N_1 = \frac{2(1.48 - 1)\ (0.030\ \text{m})\ (5.00 \times 10^{-6}/\text{C°})\ (5.00\ \text{C°})}{5.89 \times 10^{-7}\ \text{m}} = 1.22.$$

The change in the number of wavelengths due to the change in refractive index of the rod is:

$$\Delta N_2 = \frac{2\Delta n_{\text{glass}} L_0}{\lambda_0} = \frac{2(2.50 \times 10^{-5}/\text{C°})\ (5.00\ \text{C°/min})\ (1.00\ \text{min})\ (0.0300\ \text{m})}{5.89 \times 10^{-7}\ \text{m}} = 12.73.$$

So the total change in the number of wavelengths as the rod expands is $\Delta N = 12.73 + 1.22 = 14.0$ fringes/minute.

35.62: a) Since we can approximate the angles of incidence on the prism as being small, Snell's Law tells us that an incident angle of θ on the flat side of the prism enters the prism at an angle of θ/n, where n is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n - A$ from the normal, and the outgoing, relative to the prism, is $n(\theta/n - A)$. So the beam leaving the prism is at an angle of $\theta' = n(\theta/n - A) + A$ from the optical axis. So $\theta - \theta' = (n-1)A.$

At the plane of the source S_0, we can calculate the height of one image above the source:

$$\frac{d}{2} = \tan(\theta - \theta')a \approx (\theta - \theta')a = (n-1)Aa \Rightarrow d = 2aA(n-1).$$

b) To find the spacing of fringes on a screen, we use:

$$\Delta y = \frac{R\lambda}{d} = \frac{R\lambda}{2aA(n-1)} = \frac{(2.00\ \text{m} + 0.200\ \text{m})\ (5.00 \times 10^{-7}\ \text{m})}{2(0.200\ \text{m})\ (3.50 \times 10^{-3}\ \text{rad})\ (1.50 - 1.00)} = 1.57 \times 10^{-3}\ \text{m}.$$

36.1: $y_1 = \dfrac{x\lambda}{a} \Rightarrow \lambda = \dfrac{y_1 a}{x} = \dfrac{(1.35\times10^{-3}\ \text{m})\ (7.50\times10^{-4}\ \text{m})}{2.00\ \text{m}} = 5.06\times10^{-7}\ \text{m}.$

36.2: $y_1 = \dfrac{x\lambda}{a} \Rightarrow a\ \dfrac{x\lambda}{y_1} = \dfrac{(0.600\ \text{m})\ (5.46\times10^{-7}\ \text{m})}{10.2\times10^{-3}\text{m}} = 3.21\times10^{-5}\text{m}.$

36.3: The angle to the first dark fringe is simply:

$$\theta = \arctan\left(\frac{\lambda}{a}\right) = \arctan\left(\frac{633\times10^{-9}\,\text{m}}{0.24\times10^{-3}\,\text{m}}\right) = 0.15°.$$

36.4: $D = 2y_1 = \dfrac{2x\lambda}{a} = \dfrac{2(3.50\ \text{m})\ (6.33\times10^{-7}\,\text{m})}{7.50\times10^{-4}\,\text{m}} = 5.91\times10^{-3}\text{m}.$

36.5: The angle to the first minimum is $\theta = \arcsin\left(\dfrac{\lambda}{a}\right) = \arcsin\left(\dfrac{9.00\ \text{cm}}{12.00\ \text{cm}}\right) = 48.6°.$

So the distance from the central maximum to the first minimum is just $y_1 = x\tan\theta =$ (40.0 cm) tan (48.6°) = ±45.4 cm.

36.6: a) According to Eq. 36.2

$$\sin(\theta) = \frac{m\lambda}{a} = \sin(90.0°) = 1 = \frac{m\lambda}{a} = \frac{\lambda}{a}$$

.

Thus, $a = \lambda = 580\ \text{nm} = 5.80\times10^{-4}\text{mm}.$

b) According to Eq. 36.7

$$\frac{I}{I_0} = \left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2 = \left\{\frac{\sin[\pi(\sin\frac{\pi}{4})]}{\pi(\sin\frac{\pi}{4})}\right\}^2 = 0.128.$$

36.7: The diffraction minima are located by $\sin\theta = m\lambda/a,\ m = \pm1, \pm2, \ldots$

$\lambda = v/f = (344\ \text{m/s})/(1250\ \text{Hz}) = 0.2752\ \text{m};\ a = 1.00\ \text{m}$

$m = \pm1,\ \theta = \pm16.0°;\ m = \pm2,\ \theta = \pm33.4°;\ m = \pm3,\ \theta = \pm55.6°;$ no solution for larger m

36.8: a) $E = E_{\max} \sin(kx - \omega t)$

$$k = \frac{2\pi}{\lambda} \to \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.20\times 10^{7}\ \text{m}^{-1}} = 5.24\times 10^{-7}\ \text{m}$$

$$f\lambda = c \to f = \frac{c}{\lambda} = \frac{3.0\times 10^{8}\ \text{m/s}}{5.24\times 10^{-7}\ \text{m}} = 5.73\times 10^{14}\ \text{Hz}$$

b) $a \sin\theta = \lambda$

$$a = \frac{\lambda}{\sin\theta} = \frac{5.24\times 10^{-7}\ \text{m}}{\sin 28.6°} = 1.09\times 10^{-6}\ \text{m}$$

c) $a \sin\theta = m\lambda\ (m = 1,2,3,\ldots)$

$$\sin\theta_2 = \pm 2\tfrac{\lambda}{D} = \pm 2\tfrac{5.24\times 10^{-7}\ \text{m}}{1.09\times 10^{-6}\ \text{m}}$$

$$\theta_2 = \pm 74°$$

36.9: $\sin\theta = \lambda/a$ locates the first minimum

$y = x\tan\theta$, $\tan\theta = y/x = (36.5\ \text{cm})/(40.0\ \text{cm})$ and $\theta = 42.38°$

$a = \lambda/\sin\theta = (620\times 10^{-9}\ \text{m})/(\sin 42.38°) = 0.920\ \mu\text{m}$

36.10: a) $y_1 = \dfrac{x\lambda}{a} \Rightarrow a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})\ (5.00\times 10^{-7}\ \text{m})}{3.00\times 10^{-3}\ \text{m}} = 4.17\times 10^{-4}\ \text{m}.$

b) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times 10^{-5}\ \text{m})}{3.00\times 10^{-3}\ \text{m}} = 4.17\times 10^{-2}\ \text{m} = 4.2\ \text{cm}.$

c) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times 10^{-10}\ \text{m})}{3.00\times 10^{-3}\ \text{m}} = 4.17\times 10^{-7}\ \text{m}.$

36.11: a) $y_1 = \dfrac{x\lambda}{a} = \dfrac{(3.00\ \text{m})(6.33\times 10^{-7}\ \text{m})}{3.50\times 10^{-4}\ \text{m}} = 5.43\times 10^{-3}\ \text{m}.$ So the width of the brightest fringe is twice this distance to the first minimum, 0.0109 m.

b) The next dark fringe is at $y_2 = \dfrac{2x\lambda}{a} = \dfrac{2(3.00\ \text{m})(6.33\times 10^{-7}\ \text{m})}{3.50\times 10^{-4}\ \text{m}} = 0.0109\ \text{m}$.

So the width of the first bright fringe on the side of the central maximum is the distance from y_2 to y_1, which is $5.43\times 10^{-3}\ \text{m}$.

36.12: $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \approx \dfrac{2\pi a}{\lambda}\cdot\dfrac{y}{x} = \dfrac{2\pi(4.50\times10^{-4}\,\text{m})}{(6.20\times10^{-7}\,\text{m})(3.00\,\text{m})}y = (1520\,\text{m}^{-1})y.$

a) $y = 1.00\times10^{-3}\,\text{m}: \dfrac{\beta}{2} = \dfrac{(1520\,\text{m}^{-1})\,(1.00\times10^{-3}\,\text{m})}{2} = 0.760.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(0.760)}{0.760}\right)^2 = 0.822I_0$$

b) $y = 3.00\times10^{-3}\,\text{m}: \dfrac{\beta}{2} = \dfrac{(1520\,\text{m}^{-1})(3.00\times10^{-3}\,\text{m})}{2} = 2.28.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(2.28)}{2.28}\right)^2 = 0.111I_0.$$

c) $y = 5.00\times10^{-3}\,\text{m}: \dfrac{\beta}{2} = \dfrac{(1520\,\text{m}^{-1})\,(5.00\times10^{-3}\,\text{m})}{2} = 3.80.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(3.80)}{3.80}\right)^2 = 0.0259I_0.$$

36.13: a) $y_1 = \dfrac{x\lambda}{a} = \dfrac{(3.00\,\text{m})(5.40\times10^{-7}\,\text{m})}{2.40\times10^{-4}\,\text{m}} = 6.75\times10^{-3}\,\text{m}.$

b) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \approx \dfrac{2\pi a}{\lambda}\cdot\dfrac{y_1/2}{x} = \dfrac{2\pi a}{\lambda}\cdot\dfrac{x\lambda}{2ax} = \pi.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = (6.00\times10^{-6}\ \text{W/m}^2)\left(\frac{\sin\,(\pi/2)}{\pi/2}\right)^2 = 2.43\times10^{-6}\ \text{W/m}^2.$$

36.14: a) $\theta = 0: \beta = \dfrac{2\pi a}{\lambda}\sin\,0^\circ = 0.$

b) At the second minimum from the center $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\cdot\dfrac{2\lambda}{a} = 4\pi.$

c) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi(1.50\times10^{-4}\,\text{m})}{6.00\times10^{-7}\,\text{m}}\sin 7.0^\circ = 191\ \text{rad}.$

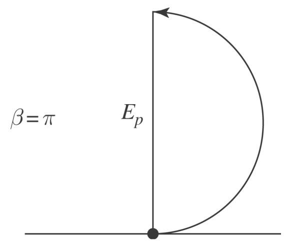
36.15: $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \Rightarrow \lambda = \dfrac{2\pi a}{\beta}\sin\theta = \dfrac{2\pi(3.20\times10^{-4}\,\text{m})}{\pi/2}\sin 0.24^\circ = 5.36\times10^{-6}\,\text{m}.$

36.16: The total intensity is given by drawing an arc of a circle that has length E_0 and finding the length of the cord which connects the starting and ending points of the curve. So graphically we can find the electric field at a point by examining the geometry as shown below for three cases.

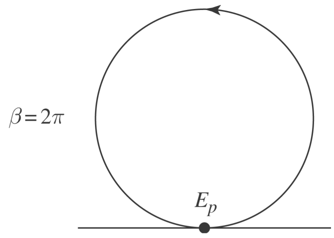
a) $\beta = \frac{2\pi a}{\lambda}\sin\theta = \frac{2\pi a}{\lambda}\cdot\frac{\lambda}{2a} = \pi$. From the diagram, $\pi\frac{E_p}{2} = E_0 \Rightarrow E_p = \frac{2}{\pi}E_0$.

So the intensity is just: $I = \left(\frac{2}{\pi}\right)^2 I_0 = \frac{4I_0}{\pi^2}$.

This agrees with Eq. (36.5).



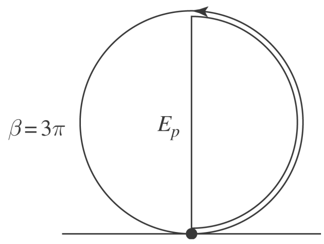
b) $\beta = \frac{2\pi a}{\lambda}\sin\theta = \frac{2\pi a}{\lambda}\cdot\frac{\lambda}{a} = 2\pi$. From the diagram, it is clear that the total amplitude is zero, as is the intensity. This also agrees with Eq. (36.5).



c) $\beta = \frac{2\pi a}{\lambda}\sin\theta = \frac{2\pi a}{\lambda}\cdot\frac{3\lambda}{2a} = 3\pi$. From the diagram, $3\pi\frac{E_p}{2} = E_0 \Rightarrow E_p = \frac{2}{3\pi}E_0$. So the intensity is just:

$$I = \left(\frac{2}{3\pi}\right)^2 I_0 = \frac{4}{9\pi^2}I_0.$$

This agrees with Eq. (36.5).



36.17: a) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \Rightarrow \lambda = \dfrac{2\pi a}{\beta}\sin\theta = \dfrac{2\pi(1.05\times10^{-4}\,\text{m})}{56.0\text{ rad}}\sin 3.25° =$

6.68×10^{-7} m.

b) $I = I_0\left(\dfrac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\dfrac{\sin(56.0/2)}{56.0/2}\right)^2 = (9.36\times10^{-5})\,I_0.$

36.18: a) Ignoring diffraction, the first five maxima will occur as given by:

$d\sin\theta = m\lambda \Rightarrow \theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{m\lambda}{4a}\right)$, for $m = 1, 2, 3, 4, 5.$

b) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\cdot\dfrac{m\lambda}{d} = \dfrac{m\pi}{2}$, and $\phi = \dfrac{2\pi d}{\lambda}\sin\theta = \dfrac{2\pi d}{\lambda}\cdot\dfrac{m\lambda}{d} = 2\pi m.$

So including diffraction, the intensity:

$$I = I_0\cos^2\frac{\phi}{2}\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\cos^2\frac{2\pi m}{2}\left(\frac{\sin(m\pi/4)}{m\pi/4}\right)^2 = I_0\left(\frac{\sin(m\pi/4)}{m\pi/4}\right)^2.$$

So for

$m = 1: I_1 = \left(\dfrac{\sin(\pi/4)}{\pi/4}\right)^2 I_0 = 0.811 I_0$; $m = 2: I_2 = \left(\dfrac{\sin(\pi/2)}{\pi/2}\right)^2 I_0 = 0.405 I_0$;

$m = 3: I_3 = \left(\dfrac{\sin(3\pi/4)}{3\pi/4}\right)^2 I_0 = 0.0901 I_0$; $m = 4: I_4 = \left(\dfrac{\sin(\pi)}{\pi}\right)^2 I_0 = 0$

$m = 5: I_5 = \left(\dfrac{\sin(5\pi/4)}{5\pi/4}\right)^2 I_0 = 0.0324 I_0.$

36.19: a) If $\dfrac{d}{a} = 3$, then there are five fringes: $m = 0, \pm1, \pm2.$

b) The $m = 6$ interference fringe coincides with the second diffraction minimum, so there are two fringes $(m = +4, m = +5)$ within the first diffraction maximum on one side of the central maximum.

36.20: By examining the diagram, we see that every fourth slit cancels each other.

36.21: a) If the slits are very narrow, then the first maximum is at

$$\frac{d}{\lambda}\sin\theta_1 = 1.$$

$$\Rightarrow \theta_1 = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{5.80\times10^{-7}\ \text{m}}{5.30\times10^{-4}\ \text{m}}\right) = \pm 0.0627°.$$

Also, the second maximum is at

$$\frac{d}{\lambda}\sin\theta_2 = 2$$

$$\Rightarrow \theta_2 = \arcsin\left(\frac{2\lambda}{d}\right) = \arcsin\left(\frac{2(5.80\times10^{-7}\ \text{m})}{5.30\times10^{-4}\text{m}}\right) = \pm\, 0.125°.$$

b) $I = I_0 \cos^2\frac{\phi}{2}\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$ but $\cos\frac{\phi}{2} = 1,$ since we are at the 2 slit maximum. So

$$I_1 = I_0\left(\frac{\sin(\pi a \sin\theta_1/\lambda)}{\pi a \sin\theta_1/\lambda}\right)^2 = I_0\left(\frac{\sin(\pi a/d)}{\pi a/d}\right)^2$$

$$\Rightarrow I_1 = I_0\left(\frac{\sin(\pi(3.20\times10^{-4}\ \text{m})/(5.30\times10^{-4}\ \text{m}))}{\pi(3.20\times10^{-4}\ \text{m})/(5.30\times10^{-4}\ \text{m})}\right)^2 = 0.249 I_0.$$

And

$$I_2 = I_0\left(\frac{\sin(\pi a \sin\theta_2/\lambda)}{\pi a \sin\theta_2/\lambda}\right)^2 = I_0\left(\frac{\sin(2\pi\, a/d)}{2\pi a/d}\right)^2$$

$$\Rightarrow I_2 = I_0\left(\frac{\sin\,(2\pi(3.20\times10^{-4}\ \text{m})/(5.30\times10^{-4}\ \text{m}))}{2\pi(3.20\times10^{-4}\ \text{m})\ /(5.30\times10^{-4}\ \text{m})}\right)^2 = 0.0256 I_0.$$

36.22: We will use $I = I_0 \cos^2\frac{\phi}{2}\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$, and must calculate the phases ϕ and β.

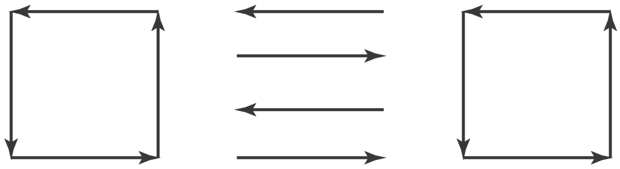
Using $\beta/2 = \frac{\pi a}{\lambda}\sin\theta$, and $\phi = \frac{2\pi d}{\lambda}\sin\theta$, we have:

a) $\theta = 1.25\times10^{-4}\ \text{rad}: \beta/2 = 0.177(10.1°)$, and $\phi = 1.01(57.9°) \Rightarrow I = 0.757 I_0$.

b) $\theta = 2.50\times10^{-4}\ \text{rad}: \beta/2 = 0.355(20.3°)$, and $\phi = 2.03(116.3°) \Rightarrow I = 0.268 I_0$.

c) $\theta = 3.00\times10^{-4}\ \text{rad}: \beta/2 = 0.426(24.1°)$, and $\phi = 2.43(139.2°) \Rightarrow I = (0.114) I_0$.

36.23: With four slits there must be four vectors in each phasor diagram, with the orientation of each successive one determined by the relative phase shifts. So:



(*i*) $\phi = \frac{\pi}{2}$ (*ii*) $\phi = \pi$ (*iii*) $\phi = \frac{3\pi}{2}$

We see that destructive interference occurs from adjacent slits in case (ii) and from alternate slits in cases (i) and (iii).

36.24: Diffraction dark fringes occur for $\sin\theta = \dfrac{m_d \lambda}{a}$, and interference maxima occur for $\sin\theta = \dfrac{m_i \lambda}{d}$. Setting them equal to each other yields a missing bright spot whenever the destructive interference matches the bright spots. That is:
$\dfrac{m_i \lambda}{d} = \dfrac{m_d \lambda}{a} \Rightarrow m_i = \dfrac{d}{a} m_d = 3m_d$. That is, the missing parts of the pattern occur for $m_i = 3, 6, 9\ldots = 3m$, for $m =$ integers.

36.25: a) Interference maxima: $d \sin \theta_i = m\lambda$ and Diffraction minima: $a \sin \theta_d = n\lambda$.

If the mth interference maximum corresponds to the nth diffraction minimum then $\theta_i = \theta_d$.

or
$$\frac{d}{a} = \frac{m}{n}$$
so
$$a = \frac{n}{m} d = \frac{1}{3}(0.840 \text{ mm}) = 0.280 \text{ mm}.$$

b) The diffraction minima will squelch the interference maxima for all $\frac{m}{n} = 3$ up to the highest seen order. For $\lambda = 630$ nm, the largest value of m will be when $\theta = 90°$.

$$m_{\max} = \frac{d}{\lambda} = \frac{8.40 \times 10^{-4} \text{ m}}{6.30 \times 10^{-7} \text{ m}} = 1333.$$

$$n_{\max} = \frac{a}{\lambda} = \frac{8.40 \times 10^{-4} \text{ m}}{3(6.30 \times 10^{-7} \text{ m})} = 444.$$

So after $m = 3$, $m = 6, 9, \ldots, 1332$ for $n = 1, 2, 3, \ldots, 444$ will also be missing.

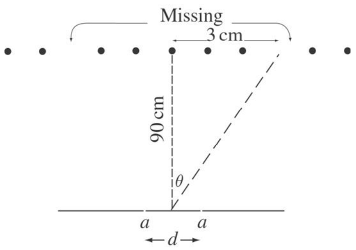
c) By changing λ we only change the highest order seen.

$$m_{\max} = \frac{d}{\lambda} = \frac{8.40 \times 10^{-4} \text{ m}}{4.20 \times 10^{-7} \text{ m}} = 2000.$$

$$n_{\max} = \frac{a}{\lambda} = \frac{2000}{3} = 666.$$

So $m = 3, 6, 9, \ldots, 1999$ for $n = 1, 2, 3, \ldots, 666$.

36.26: The third bright band is missing because the first order single slit minimum occurs at the same angle as the third order double slit maximum.



$$\tan\theta = \frac{3\text{ cm}}{90\text{ cm}}$$

$$\theta = 1.91°$$

Single-slit dark spot: $a \sin\theta = \lambda$

$$a = \frac{\lambda}{\sin\theta} = \frac{500\text{ nm}}{\sin 1.91°} = 1.50\times10^{4}\text{ nm(width)}$$

Double-slit bright fringe:

$$d\sin\theta = 3\lambda$$

$$d = \frac{3\lambda}{\sin\theta} = \frac{3(500\text{ nm})}{\sin 1.91°} = 4.50\times10^{4}\text{ nm(separation)}$$

36.27: a) Find $d : d\sin\theta = m\lambda$

$\theta = 78.4°$for $m = 3$ and $\lambda = 681$ nm, so $d = m\lambda/\sin\theta = 2.086\times10^{-4}$ cm

The number of slits per cm is $1/d = 4790$ slits/cm

b) 1st order: $m = 1$, so $\sin\theta = \lambda/d = (681\times10^{-9}\text{ m})/(2.086\times10^{-6}\text{ m})$ and $\theta = 19.1°$

2nd order: $m = 2$, so $\sin\theta = 2\lambda/d$ and $\theta = 40.8°$

c) For $m = 4, \sin\theta = 4/d$ is greater than 1.00, so there is no 4th-order bright band.

36.28: First-order: $d\sin\theta_1 = \lambda$

Fourth-order: $d\sin\theta_1 = 4\lambda$

$$\frac{d\sin\theta_4}{d\sin\theta_1} = \frac{4\lambda}{\lambda}$$

$$\sin\theta_4 = 4\sin\theta_1 = 4\sin 8.94°$$

$$\theta_4 = 38.4°$$

36.29: a) $d = \left(\frac{1}{900}\right)\text{cm} = 1.111\times 10^{-5}\ \text{m}$

For $\lambda = 700$ nm, $\lambda/d = 6.3\times 10^{-4}$. The first-order lines are located at $\sin\theta = \lambda/d$; $\sin\theta$ is small enough for $\sin\theta \approx \theta$ to be an excellent approximation.

b) $y = x\lambda/d$, where $x = 2.50$ m.

The distance on the screen between 1st order bright bands for two different wavelengths is $\Delta y = x(\Delta y)/d$, so $\Delta\lambda = d\,(\Delta y)/x$

$$= (1.111\times 10^{-5}\ \text{m})\ (3.00\times 10^{-3}\ \text{m})/(2.50\ \text{m}) = 13.3\ \text{nm}$$

36.30:

a)

$$R = \frac{\lambda}{\Delta\lambda} = Nm \Rightarrow N = \frac{\lambda}{m\Delta\lambda} = \frac{6.5645\times 10^{-7}\,\text{m}}{2(6.5645\times 10^{-7}\ \text{m} - 6.5627\times 10^{-7}\ \text{m})} = 1820\ \text{slits.}$$

b) $d = (500\ \text{slits/mm})^{-1} = (500{,}000\ \text{slits/m})^{-1} \cdot \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) \Rightarrow$

$$\theta_1 = \sin^{-1}((2)(6.5645\times 10^{-7}\ \text{m})\cdot 500{,}000) = 41.0297°$$

$$\theta_2 = \sin^{-1}((2)(6.5627\times 10^{-7}\ \text{m})\cdot 500{,}000) = 41.0160°$$

$$\Rightarrow \Delta\theta = 0.0137°.$$

36.31: $\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{\text{m}(6.328\times 10^{-7}\ \text{m})}{1.60\times 10^{-6}\ \text{m}}\right) = \arcsin\ ((0.396)m)$

$\Rightarrow m = 1: \theta_1 = 23.3°$; $m = 2: \theta = 52.3°$. All other m-values lead to angles greater than 90^o.

36.32: $5000\ \text{slits/cm} \Rightarrow d = \frac{1}{5.00\times 10^{5}\ \text{m}^{-1}} = 2.00\times 10^{-6}\ \text{m}.$

a) $d\sin\theta = m\lambda \Rightarrow \lambda = \frac{d\sin\theta}{m} = \frac{(2.00\times 10^{-6}\ \text{m})\sin 13.5°}{1} = 4.67\times 10^{-7}\ \text{m}.$

b) $m = 2: \theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{2(4.67\times 10^{-7}\ \text{m})}{2.00\times 10^{-6}\ \text{m}}\right) = 27.8^\text{o}.$

36.33: $350\ \text{slits/mm} \Rightarrow d = \frac{1}{3.50\times 10^{5}\ \text{m}^{-1}} = 2.86\times 10^{-6}\ \text{m. Then:}$

$$d\sin\theta = m\lambda \Rightarrow \theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(5.20\times 10^{-7}\ \text{m})}{2.86\times 10^{-6}\ \text{m}}\right) = \arcsin\ ((0.182)m)$$

$$\Rightarrow m = 1: \theta = 10.5°;\ \ m = 2: \theta = 21.3°;\ \ m = 3: \theta = 33.1°.$$

36.34: 350 slits/mm $\Rightarrow d = \dfrac{1}{3.50\times10^{5}\ \text{m}^{-1}} = 2.86\times10^{-6}\ \text{m.}$, and $d\sin\theta = m\lambda$.

$$\Rightarrow m=1:\theta_{400} = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{4.00\times10^{-7}\ \text{m}}{2.86\times10^{-6}\ \text{m}}\right) = 8.05^{\circ}.$$

$$\theta_{700} = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{7.00\times10^{-7}\ \text{m}}{2.86\times10^{-6}\ \text{m}}\right) = 14.18^{\circ}.$$

$$\Rightarrow \Delta\theta_1 = 14.18^{\circ} - 8.05^{\circ} = 6.13^{\circ}.$$

$$\Rightarrow m=3:\theta_{400} = \arcsin\left(\frac{3\lambda}{d}\right) = \arcsin\left(\frac{3(4.00\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = 24.8^{\circ}.$$

$$\theta_{700} = \arcsin\left(\frac{3\lambda}{d}\right) = \arcsin\left(\frac{3(7.00\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = 47.3^{\circ}.$$

$$\Rightarrow \Delta\theta_1 = 47.3^{\circ} - 24.8^{\circ} = 22.5^{\circ}.$$

36.35: 4000 slits/cm $\Rightarrow d = \dfrac{1}{4.00\times10^{5}\ \text{m}^{-1}} = 2.50\times10^{-6}\ \text{m.}$ So for the α-hydrogen line, we have:

$$\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(6.56\times10^{-7}\ \text{m})}{2.50\times10^{-6}\ \text{m}}\right) = \arcsin\left((0.262)m\right).$$

$$\Rightarrow m=1:\theta_1 = 15.2^{\circ};\ m=2:\theta = 31.6^{\circ}.$$

And for the β-hydrogen line, the angle is given by:

$$\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(4.86\times10^{-7}\ \text{m})}{2.50\times10^{-6}\ \text{m}}\right) = \arcsin\left((0.194)m\right).$$

$$\Rightarrow m=1:\theta_1 = 11.2^{\circ};\ m=2:\theta = 22.9^{\circ};\text{ so, a) } \Delta\theta_1 = 4.00^{\circ},\text{ b) } \Delta\theta_2 = 8.77^{\circ}.$$

36.36: $\dfrac{\Delta}{\Delta\lambda} = Nm \Rightarrow N = \dfrac{\lambda}{m\Delta\lambda} = \dfrac{587.8002\ \text{nm}}{(587.9782\ \text{nm} - 587.8002\ \text{nm})} = \dfrac{587.8002}{0.178}$

$$\Rightarrow N = 3302\ \text{slits.}$$

$$\frac{N}{1.20\ \text{cm}} = \frac{3302}{1.20\ \text{cm}} = 2752\,\frac{\text{slits}}{\text{cm}}.$$

36.37: For *x*-ray diffraction,

$$2d\sin\theta = m\lambda \Rightarrow d = \frac{m\lambda}{2\sin\theta} \Rightarrow d = \frac{2(8.50\times10^{-11}\ \text{m})}{2\sin 21.5^{\circ}} = 2.32\times10^{-10}\ \text{m.}$$

36.38: For the first order maximum in Bragg reflection:

$$2d\,\sin\theta = m\lambda \Rightarrow \lambda = \frac{2d\,\sin\theta}{m} = \frac{2(4.40\times10^{-10}\ \text{m})\sin 39.4^\circ}{1} = 5.59\times10^{-10}\ \text{m}.$$

36.39: The best resolution is 0.3 arcseconds, which is about $8.33\times10^{-5\circ}$.

a) $D = \dfrac{1.22\lambda}{\sin\theta_1} = \dfrac{1.22(5.5\times10^{-7}\ \text{m})}{\sin(8.33\times10^{-5\circ})} = 0.46\ \text{m} \approx 0.5\ \text{m}.$

b) The Keck telescopes are able to gather more light than the Hale telescope, and hence they can detect fainter objects. However, their larger size does not allow them to have greater resolution—atmospheric conditions limit the resolution.

36.40: $D = \dfrac{1.22\lambda}{\sin\theta_1} = \dfrac{1.22(5.5\times10^{-7}\ \text{m})}{\sin(1/60)^\circ} = 2.31\times10^{-3}\ \text{m} = 2.3\ \text{mm}.$

36.41: $\sin\theta_1 = 1.22\dfrac{\lambda}{D} \Rightarrow D = \dfrac{1.22\,\lambda}{\sin\theta_1} = 1.22\lambda\dfrac{h}{W} = 1.22(0.036\ \text{m})\dfrac{1.2\times10^6\ \text{m}}{2.8\times10^4\ \text{m}}$

$\Rightarrow D = 1.88\ \text{m}.$

36.42: $\sin\theta_1 = 1.22\dfrac{\lambda}{D} \Rightarrow \lambda = \dfrac{D\sin\theta_1}{1.22} \approx \dfrac{D\theta_1}{1.22} = \dfrac{(8.00\times10^6\ \text{m})(1.00\times10^{-8})}{1.22}$

$\Rightarrow \lambda = 0.0656\ \text{m} = 6.56\ \text{cm}.$

36.43: $\sin\theta_1 = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{6.20\times10^{-7}\ \text{m}}{7.4\times10^{-6}\ \text{m}} = 0.102.$ The screen is 4.5 m away, so the diameter of the Airy ring is given by trigonometry:

$$D = 2y = 2x\,\tan\theta \approx 2x\,\sin\theta = 2(4.5\ \text{m})(0.102) = 91.8\ \text{cm}.$$

36.44: The image is 25.0 cm from the lens, and from the diagram and Rayleigh's criteria, the diameter of the circles is twice the "height" as given by:

$$D = 2\,|\,y'\,| = \frac{2s'}{s}y = \frac{2\,fy}{s} = \frac{2(0.180\ \text{m})(8.00\times10^{-3}\ \text{m})}{25.0\ \text{m}} = 1.15\times10^{-4}\,\text{m} = 0.115\ \text{mm}.$$

36.45: $\sin\theta_1 = 1.22\dfrac{\lambda}{D} \Rightarrow D = \dfrac{1.22\lambda}{\sin\theta_1} \approx 1.22\lambda\dfrac{R}{W}$

$$= 1.22(5.0\times10^{-7}\,\text{m})\frac{5.93\times10^{11}\ \text{m}}{2.50\times10^5\,\text{m}} = 1.45\ \text{m}.$$

36.46: $\sin\theta_1 = 1.22\dfrac{\lambda}{D} \cong \dfrac{y}{s} \Rightarrow s = \dfrac{yD}{1.22\lambda} = \dfrac{(4.00\times10^{-3}\,\text{m})(0.0720\,\text{m})}{1.22(5.50\times10^{-7}\ \text{m})} = 429\ \text{m}.$

36.47: Let y be the separation between the two points being resolved and let s be their distance from the telescope. Then the limit of resolution corresponds to $1.22\dfrac{\lambda}{D} = \dfrac{y}{s}$

a) Let the two points being resolved be the opposite edges of the crater, so y is the diameter of the crater. For the moon, $s = 3.8\times10^{8}\ \text{m}$.

$y = 1.22\lambda\, s/D$

Hubble: $D = 2.4$ m and $\lambda = 400\,\text{nm}$ gives the maximum resolution, so $y = 77$ m

Arecibo: $D = 305$ m and $\lambda = 0.75$ m; $y = 1.1\times10^{6}\ \text{m}$

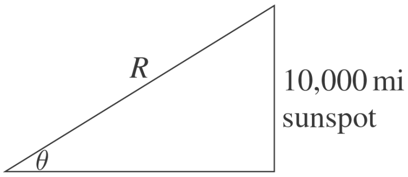
b) $s = \dfrac{yD}{1.22\lambda}$

Let $y \approx 0.30$ (the size of a license plate)

$s = (0.30\ \text{m})(2.4\ \text{m})/[(1.22)(400\times10^{-9}\ \text{m})] = 1500\ \text{km}$

36.48: Smallest resolving angle is for short-wavelength light(400 nm)

$$\theta \approx 1.22\frac{\lambda}{D} = (1.22)\frac{400\times10^{-9}\ \text{m}}{5.08\ \text{m}} = 9.61\times10^{-8}\ \text{rad}$$



$$\theta = \frac{10{,}000\ \text{mi}}{R}$$

$$R = \frac{10{,}000\ \text{mi}}{\theta} = \frac{16{,}000\ \text{km}}{9.6\times10^{-8}\,\text{rad}} = 1.7\times10^{11}\ \text{km}$$

This is less than a light year, so there are no stars this close.

36.49: Let y be the separation between the two points being resolved and let s be their distance from the telescope. The limit of resolution corresponds to $1.22\lambda/D = y/s$

$s = 4.28\ \text{ly} = 4.05\times10^{16}\ \text{m}$

Assume visible light, with $\lambda = 400\ \text{m}$

$y = 1.22\lambda s/D = 1.22(400\times10^{-9}\ \text{m})(4.05\times10^{16}\ \text{m}/(10.0\ \text{m}) = 2.0\times10^{9}\,\text{m}$

The diameter of Jupiter is $1.38\times10^{8}\ \text{m}$, so the resolution is insufficient, by about one order of magnitude.

36.50: a) For dark spots, $a \sin\theta = m\lambda,$ so $\sin\theta \propto 1/a.$ Heating the sheet causes the slit width to increase due to thermal expansion, so $\sin\theta$ and hence θ will decrease. Therefore the bright region gets *narrower.*

b) *At the lower temperature:*

$$a_1 = \frac{\lambda}{\sin\theta_1} \text{ where } \tan\theta_1 = \frac{5\text{ cm}}{800\text{ cm}} \rightarrow \theta_1 = 0.35809^\circ$$

$$a_1 = \frac{500\text{ nm}}{\sin 0.35809^\circ} = 80{,}002\text{ nm}$$

At the higher temperature:

$$\tan\theta_2 = \frac{5\text{ cm} - 0.001\text{ cm}}{800\text{ cm}} \rightarrow \theta_2 = 0.35802^\circ$$

$$a_2 = \frac{\lambda}{\sin\theta_1} = a_1 = \frac{500\text{ nm}}{\sin 0.35802^\circ} = 80{,}018\text{ nm}$$

Thermal expansion: $\Delta a = \alpha a_1 \Delta T$

$$\alpha = \frac{\Delta a}{a_1 \Delta T} = \frac{80,\,018\text{ nm} - 80,\,002\text{ nm}}{(80,\,002\text{ nm})(80°\text{C})}$$

$$= 2.5 \times 10^{-6}\,°C^{-1}$$

36.51: a) $I = I_0/2 \Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{\sin(\pi\ a\sin\theta/\lambda)}{\pi\ a\sin\theta/\lambda} \equiv \dfrac{\sin x}{x} = 0.7071.$ Solving for x through trial and error, and remembering to use radians throughout, one finds $x = 1.39$ rad and $\beta = 2x = 2.78$ rad. Also, $\Delta\theta = |\theta_+ - \theta_-| = 2\theta_+$, and

$$\beta = \frac{2\pi a}{\lambda}\sin\theta \Rightarrow \sin\theta_+ \frac{\lambda\beta}{2\pi a} = \frac{\lambda}{a}\left(\frac{2.78\text{ rad}}{2\pi\ \text{ rad}}\right) = 0.442\frac{\lambda}{a}.$$

i) $\dfrac{a}{\lambda} = 2 \Rightarrow \sin\theta_+ = 0.221 \Rightarrow \theta_+ = 0.223\text{ rad} \Rightarrow \Delta\theta = 0.446\text{ rad} = 25.3^\circ.$

ii) $\dfrac{a}{\lambda} = 5 \Rightarrow \sin\theta_+ = 0.0885 \Rightarrow \theta_+ = 0.0886\text{ rad} \Rightarrow \Delta\theta = 0.177\text{ rad} = 10.1^\circ.$

iii) $\dfrac{a}{\lambda} = 10 \Rightarrow \sin\theta_+ = 0.0442 \Rightarrow \theta_+ = 0.0443\text{ rad} \Rightarrow \Delta\theta = 0.885\text{ rad} = 5.07^\circ.$

b) For the first minimum, $\sin\theta_0 = \dfrac{\lambda}{a}.$

i) $\dfrac{a}{\lambda} = 2 \Rightarrow \theta_0 = \arcsin\left(\dfrac{1}{2}\right) = 0.524\text{ rad} \Rightarrow 2\theta_0 = 1.05\text{ rad} = 60.2^\circ.$

ii) $\dfrac{a}{\lambda} = 5 \Rightarrow \theta_0 = \arcsin\left(\dfrac{1}{5}\right) = 0.201\text{ rad} \Rightarrow 2\theta_0 = 0.402\text{ rad} = 23.0^\circ.$

iii) $\dfrac{a}{\lambda} = 10 \Rightarrow \theta_0 = \arcsin\left(\dfrac{1}{10}\right) = 0.100\text{ rad} \Rightarrow 2\theta_0 = 0.200\text{ rad} = 11.5^\circ.$

Both methods show the central width getting smaller as the slit width a is increased.

36.52: If the apparatus of Exercise 36.4 is placed in water, then all that changes is the wavelength $\lambda \to \lambda' = \dfrac{\lambda}{n}$. So : $D' = 2y_1' = \dfrac{2x\lambda'}{a} = \dfrac{2x\lambda}{an} = \dfrac{D}{n} = \dfrac{5.91\times10^{-3}\text{m}}{1.33} =$ $4.44\times10^{-3}\text{m} = 4.44$ mm.

36.53: $\sin\theta = \lambda/a$ locates the first dark band

$$\sin\theta_{\text{air}} = \frac{\lambda_{\text{air}}}{a};\ \sin\theta_{\text{liquid}} = \frac{\lambda_{\text{liquid}}}{a}$$

$$\lambda_{\text{liquid}} = \lambda_{\text{air}}\left(\frac{\sin\theta_{\text{liquid}}}{\sin\theta_{\text{air}}}\right) = 0.4836$$

$\lambda = \lambda_{\text{air}}/n$ (Eq.33.5), so $n = \lambda_{\text{air}}/\lambda_{\text{liquid}} = 1/0.4836 = 2.07$

36.54: For bright spots, $\frac{1}{N}\sin\theta = \lambda$

Red: $\frac{1}{N}\sin\theta_{\text{R}} = 700\text{ nm}$

Violet: $\frac{1}{N}\sin\theta_{\text{V}} = 400\text{ nm}$

$$\frac{\sin\theta_R}{\sin\theta_V} = \frac{7}{4}$$

$$\theta_{\text{R}} - \theta_{\text{V}} = 15° \to \theta_{\text{R}} = \theta_V + 15°$$

$$\frac{\sin(\theta_{\text{V}} + 15°)}{\sin\theta_{\text{V}}} = \frac{7}{4}$$

Expand: $\dfrac{\sin\theta_{\text{V}}\ \cos 15° + \cos\theta_{\text{V}}\ \sin 15°}{\sin\theta_{\text{V}}} = 7/4$

$$\cos 15° + \cot\theta_{\text{V}}\ \sin 15° = 7/4$$

$$\tan\theta_{\text{V}} = 0.330 \to \theta_{\text{V}} = 18.3°$$

$$\theta_{\text{R}} = \theta_{\text{V}} + 15° = 18.3° + 15° = 33.3°$$

Line density: $\frac{1}{N}\sin\theta_{\text{R}} = 700\text{nm}$

$$N = \frac{\sin\theta_{\text{R}}}{700\text{ nm}} = \frac{\sin 33.3^{\circ}}{700\times 10^{-9}\text{m}} = 7.84\times 10^{5}\ \text{lines/m}$$

$$= 7840\text{ lines/cm}$$

The spectrum begins at 18.3^{o} and ends at 33.3^{o}

36.55: a) $y_1 = \dfrac{x\lambda}{a} = \dfrac{(1.20\text{ m})(5.40\times 10^{-7}\text{ m})}{3.60\times 10^{-4}\text{ m}} = 1.80\times 10^{-3}\text{m}.$

b) $\dfrac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda} = \dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{\pi a\sin\theta}{\lambda} = 1.39$

$$\Rightarrow \sin\theta = \frac{(1.39)(5.40\times 10^{-7}\text{ m})}{\pi(3.60\times 10^{-4}\text{m})} = 6.64\times 10^{-4}$$

$$\Rightarrow y = x\tan\theta \approx x\sin\theta = (1.20\text{ m})(6.64\times 10^{-4}) = 7.97\times 10^{-4}\text{m} = 0.797\text{ mm}.$$

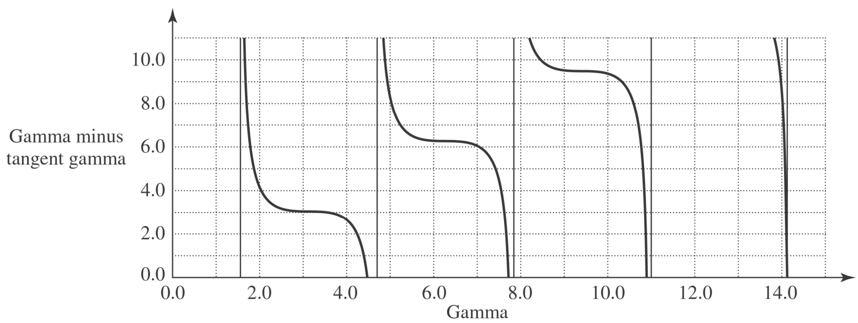
36.56: a) $I = I_0\left(\dfrac{\sin\gamma}{\gamma}\right)^2$. The maximum intensity occurs when the derivative of the intensity function with respect to γ is zero.

$$\frac{dI}{d\gamma} = I_0\frac{d}{d\gamma}\left(\frac{\sin\gamma}{\gamma}\right)^2 = 2\left(\frac{\sin\gamma}{\gamma}\right)\left(\frac{\cos\gamma}{\gamma} - \frac{\sin\gamma}{\gamma^2}\right) = 0$$

$$\Rightarrow \frac{\cos\gamma}{\gamma} - \frac{\sin\gamma}{\gamma^2} \Rightarrow \gamma\cos\gamma = \sin\gamma$$

$$\Rightarrow \gamma = \tan\gamma.$$

b)



The graph above is a plot of f(γ) = $\gamma - \tan\gamma$. So when it equals zero, one has an intensity maximum. Getting estimates from the graph, and then using trial and error to narrow in on the value, we find that the three smallest γ -values are γ = 4.49 rad 7.73 rad, and 10.9 rad.

36.57: The phase shift for adjacent slits is $\phi = \frac{2\pi d}{\lambda}\sin\theta \approx \frac{2\pi d\theta}{\lambda} \Rightarrow \theta = \frac{\phi\lambda}{2\pi d}$.

So, with the principal maxima at phase shift values of $\phi = 2\pi m,$ and $(\mathrm{N}-1)$ minima between the maxima, the phase shift between the minima adjacent to the maximum, and the maximum itself, must be $\pm\frac{2\pi}{N}$.

Therefore total phase shifts of these minima are $2\pi m \pm \frac{2\pi}{N}$.

Hence the angle at which they are found, and the angular width, will be:

$$\theta_{\pm} = \frac{\lambda}{2\pi d}\left(2\pi m \pm \frac{2\pi}{N}\right) = \frac{m\lambda}{d} \pm \frac{\lambda}{dN} \Rightarrow \Delta\theta_{\pm} = \frac{2\lambda}{dN}.$$

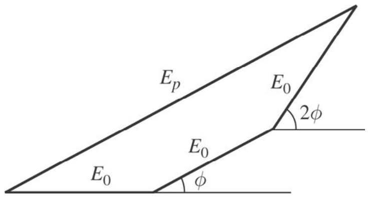
36.58: a) $E_p^2 = E_{p_x}^2 + E_{p_y}^2$. So, from the diagram at right, we have:

$$\frac{E_p^2}{E_0^2} = (1+\cos\phi+\cos 2\phi)^2 + (\sin\phi+\sin 2\phi)^2$$

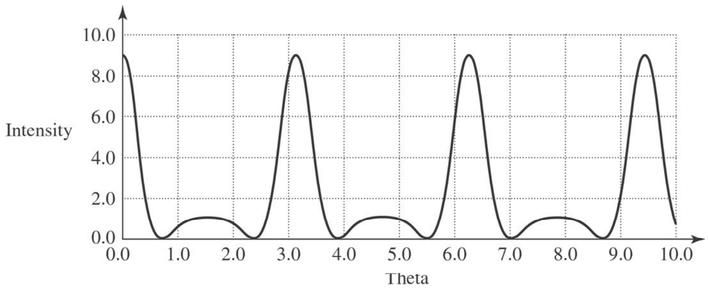
$$= (2\cos^2\phi+\cos\phi)^2 + (\sin\phi+2\sin\phi\cos\phi)^2$$

$$= (\cos^2\phi+\sin^2\phi)(1+2\cos\phi)^2$$

$$\Rightarrow \frac{E_p^2}{E_0^2} = (1+2\cos\phi)^2 \Rightarrow E_p = E_0(1+2\cos\phi).$$



b) $\phi = \dfrac{2\pi d}{\lambda}\sin\theta \Rightarrow I_p = I_0\left(1+2\cos\left(\dfrac{2\pi d\sin\theta}{\lambda}\right)\right)^2$. This is graphed below:



c) (i) At $\theta = 0, I_p = I_0(1+2\cos(0^0))^2 = 9I_0$.

(ii) The principal maximum is when $I_0 \dfrac{2\pi d\sin\theta}{\lambda} = 2\pi m \Rightarrow d\sin\theta = m\lambda$

(iii) & (iv) The minima occur at 2 cos $\left(\dfrac{2\pi d\sin\theta}{\lambda}\right) = -1 \Rightarrow \dfrac{2\pi d\sin\theta}{\lambda} = \dfrac{2\pi m}{3}$

$\Rightarrow d\sin\theta = \dfrac{m\lambda}{3}$, with m not divisible by 3. Thus there are two minima between every principal maximum.

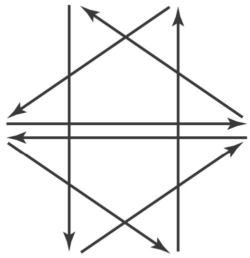
(v) The secondary maxima occur when $\cos\left(\dfrac{2\pi d\sin\theta}{\lambda}\right) = -1 \Rightarrow I_p = I_0 = \dfrac{I_{\max}}{9}$.

Also $\dfrac{2\pi d\sin\theta}{\lambda} = m\pi \Rightarrow d\sin\theta = \dfrac{m\lambda}{2}$.

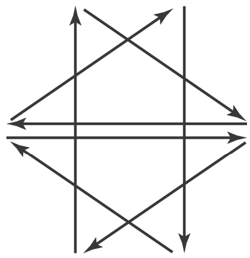
All of these findings agree with the N-slit statements in Section 35.5.

d) Below are phasor diagrams for specific phase shifts.

36.59: a) For eight slits, the phasor diagrams must have eight vectors:

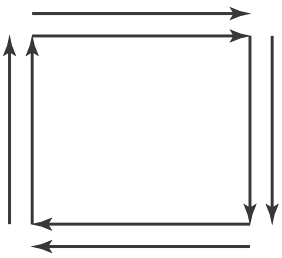


$\phi = \frac{3\pi}{4}$

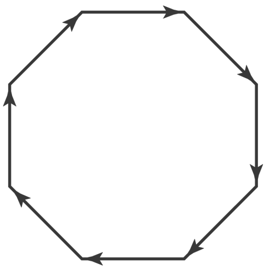


$\phi = \frac{5\pi}{4}$

b) For $\phi = \frac{3\pi}{4}$, $\phi = \frac{5\pi}{4}$, and $\phi = \frac{7\pi}{4}$, totally destructive interference occurs between slits four apart. For $\phi = \frac{3\pi}{2}$, totally destructive interference occurs with every second slit.



$\phi = \frac{3\pi}{2}$



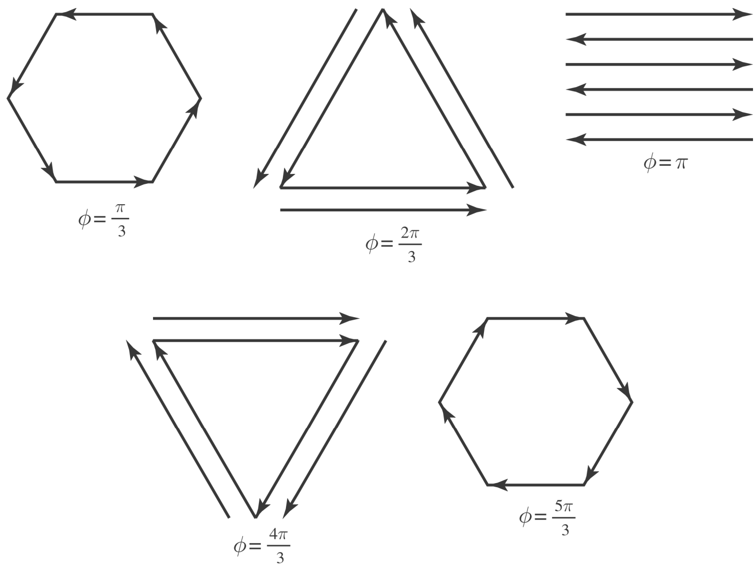
$\phi = \frac{7\pi}{4}$

36.60: For six slits, the phasor diagrams must have six vectors.

a) Zero phase difference between adjacent slits means that the total amplitude is $6E$, and the intensity is $36I$.

b) If the phase difference is 2π, then we have the same phasor diagram as above, and equal amplitude, $6E$, and intensity, $36I$.

c) There is an interference minimum whenever the phasor diagrams close on themselves, such as in the five cases below.

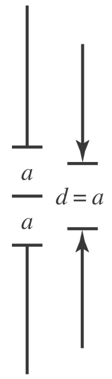


36.61: a) For the maxima to occur for N slits, the sum of all the phase differences between the slits must add to zero (the phasor diagram closes on itself). This requires that, adding up all the relative phase shifts, $N\phi = 2\pi m$, for some integer m. Therefore $\phi = \dfrac{2\pi m}{N}$, for m not an integer multiple of N, which would give a maximum.

b) The sum of N phase shifts $\phi = \dfrac{2\pi m}{N}$ brings you full circle back to the maximum, so only the $N-1$ previous phases yield minima between each pair of principal maxima.

36.62: As shown below, a pair of slits whose width and separation are equal is the same as having a single slit, of twice the width.

$$\phi = \frac{2\pi d}{\lambda}\sin\theta, \text{ so } \beta = \frac{2\pi a}{\lambda}\sin\theta = \phi.$$



So then the intensity is

$$I = I_0 \cos^2(\beta/2)\left(\frac{\sin^2(\beta/2)}{(\beta/2)^2}\right) = I_0 \frac{(2\sin(\beta/2)\cos(\beta/2))^2}{\beta^2}$$

$$\Rightarrow I = I_0 \frac{\sin^2 \beta}{\beta^2} = I_0 \frac{\sin^2(\beta'/2)}{(\beta'/2)^2},$$

where $\beta' = \frac{2\pi(2a)}{\lambda}\sin\theta$, which is Eq. (35.5) with double the slit width.

36.63: For 6500 slits/cm $\Rightarrow d = \frac{1}{6.50\times10^5 \text{ m}^{-1}} = 1.54\times10^{-6}$ m. When $\theta = 90^\circ$, $m = 3$ so one barely gets the 3rd order.

$$d \sin\theta = m\lambda \Rightarrow \lambda = \frac{d\sin\theta}{m} \Rightarrow \lambda_{3\max} = \frac{d}{3} = \frac{1.54\times10^{-6}\text{ m}}{3} = 5.13\times10^{-7}\text{ m}.$$

36.64: a) As the rays first reach the slits there is already a phase difference between adjacent slits of $\frac{2\pi d \sin\theta'}{\lambda}$.

This, added to the usual phase difference introduced after passing through the slits, yields the condition for an intensity maximum:

$$\frac{2\pi d \sin\theta}{\lambda} + \frac{2\pi d \sin\theta'}{\lambda} = 2\pi m \Rightarrow d(\sin\theta + \sin\theta') = m\lambda$$

b) $600 \text{ slits/mm} \Rightarrow d = \frac{1}{6.00\times10^{5} \text{ m}^{-1}} = 1.67\times10^{-6} \text{ m}.$

$\theta' = 0^\circ : m = 0 : \theta = \arcsin(0) = 0.$

$$m = 1 : \theta = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{6.50\times10^{-7} \text{ m}}{1.67\times10^{-6} \text{ m}}\right) = 22.9^\circ.$$

$$m = -1 : \theta = \arcsin\left(-\frac{\lambda}{d}\right) = \arcsin\left(-\frac{6.50\times10^{-7} \text{ m}}{1.67\times10^{-6} \text{ m}}\right) = -22.9^\circ.$$

$\theta' = 20.0^\circ : m = 0 : \theta = \arcsin(-\sin 20.0^\circ) = -20.0^\circ.$

$$m = 1 : \theta = \arcsin\left(\frac{6.50\times10^{-7} \text{ m}}{1.67\times10^{-6} \text{ m}} - \sin 20.0^\circ\right) = 2.71^\circ.$$

$$m = -1 : \theta = \arcsin\left(\frac{6.50\times10^{-7} \text{ m}}{1.67\times10^{-6} \text{ m}} - \sin 20.0^\circ\right) = -47.0^\circ.$$

36.65: For $650 \text{ slits/mm} \Rightarrow d = \frac{1}{6.50\times10^{5} \text{ m}^{-1}} = 1.53\times10^{-6} \text{ m}.$

We need $d \sin\theta = m\lambda \Rightarrow \sin\theta = \frac{m\lambda}{d} \le 1,$ if the whole spectrum is to be seen.

$$\lambda_1 = 4.00\times10^{-7}\text{m} : m = 1 : \frac{\lambda_1}{d} = 0.26;\ m = 2 : \frac{2\lambda_1}{d} = 0.52;\ m = 3 : \frac{3\lambda_1}{d} = 0.78.$$

$$\lambda_2 = 7.00\times10^{-7}\text{m} : m = 1 : \frac{\lambda_2}{d} = 0.46;\ m = 2 : \frac{2\lambda_2}{d} = 0.92;\ m = 3 : \frac{3\lambda_2}{d} = 1.37.$$

So the third order does not contain the violet end of the spectrum, and therefore only the first and second diffraction patterns contain all colors of the spectrum.

36.66: a) $2d\sin\theta = m\lambda \Rightarrow \theta = \arcsin\left(\frac{m\lambda}{2d}\right) = \arcsin\left(m\frac{0.125\text{ nm}}{2(0.282\text{ nm})}\right)$

$$= \arcsin(0.2216\,m).$$

So for $m = 1 : \theta = 12.8°$, $m = 2 : \theta = 26.3°$, $m = 3 : \theta = 41.7°$, and $m = 4 : \theta =$ $62.4°$. No larger m values yield answers.

b) If the separation $d = \frac{a}{\sqrt{2}}$, then $\theta = \arcsin\left(\frac{\sqrt{2}m\lambda}{2a}\right) = \arcsin(0.3134\,m)$.

So for $m = 1 : \theta = 18.3^\circ$, $m = 2 : \theta = 38.8^\circ$, and $m = 3 : \theta = 70.1^\circ$. No larger m values yield answers.

36.67: a) $d\sin\theta = m\lambda$. Place 1^{st} maximum at ∞ or $\theta = 90^\circ$.
$d = \lambda$. If $d < \lambda$, this puts the first maximum "beyond ∞." Thus, if $d < \lambda$, there is only a single principal maximum.

b) $\Phi_{\text{path}} = 2\pi\left(\frac{d\sin\theta}{\lambda}\right)$. This just scales 2π radians by the fraction the wavelength is of the path difference between adjacent sources.

If we add a relative phase δ between sources, we still must maintain a total phase difference of zero to keep our principal maximum.

$$\Phi_{\text{path}} \pm \delta = 0 \Rightarrow \frac{2\pi d\sin\theta}{\lambda} = \pm\delta \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{\delta\lambda}{2\pi d}\right)$$

c) $d = \frac{0.280\text{ m}}{14} = 0.0200\text{ m}$ (count the number of spaces between 15 points).

Let $\theta = 45^\circ$. Also recall $f\lambda = c$, so

$$\delta_{\max} = \pm\frac{2\pi(0.0200\text{ m})(8.800\times10^{9}\text{ Hz})\sin 45^\circ}{(3.00\times10^{8}\text{ m/s})} = \pm2.61\text{ radians.}$$

36.68: $\sin\theta = 1.22\frac{\lambda}{D} \Rightarrow \theta = \arcsin\left(1.22\frac{\lambda}{D}\right)$. So for

a) Mauna Kea: $\theta = \arcsin\left(1.22\frac{(5.00\times10^{-7}\text{ m})}{(8.3\text{ m})}\right) = (4.21\times10^{-6})^\circ$.

b) Arecibo: $\theta = \arcsin\left(1.22\frac{(0.210\text{ m})}{(305\text{ m})}\right) = 0.0481^\circ$.

36.69: To resolve two objects, according to Rayleigh's criterion, one must be located at the first minimum of the other. In this case, knowing the equation for the angle to the first minimum, and also the objects' separation and distance away, the sine of the angle subtended by them is calculated to be:

$$\sin\theta = \frac{\lambda}{a} = \frac{\Delta x}{R} \Rightarrow R = \frac{a\Delta x}{\lambda} = \frac{(3.50\times10^{-4}\ \text{m})(2.50\ \text{m})}{6.00\times10^{-7}\ \text{m}} = 1458\ \text{m} = 1.46\ \text{km}.$$

36.70: $\sin\theta = 1.22\dfrac{\lambda}{D} \approx \dfrac{\Delta x}{R} \Rightarrow \Delta x = \dfrac{1.22\,\lambda R}{D} = \dfrac{(1.22)cR}{Df}\ (\lambda f = c).$

$$\Delta x = \frac{(1.22)(3.00\times10^{5}\ \text{km/s})(7.2\times10^{8}\ \text{ly})}{(77.000\times10^{3}\ \text{km})(1.665\times10^{9}\ \text{Hz})} = 2.06\ \text{ly}.$$

$$\Rightarrow 9.41\times10^{12}\ \text{km/ly}\ \cdot 2.06\ \text{ly} = 1.94\times10^{13}\ \text{km}.$$

36.71: Diffraction limited seeing and Rayleigh's criterion tell us:

$$\sin\theta = 1.22\frac{\lambda}{D} = \frac{(1.22)(5.00\times10^{-7}\ \text{m})}{(4.00\times10^{-3}\ \text{m})} = 1.53\times10^{-4}.$$

But now the altitude of the astronaut can be calculated from the angle (above) and the object separation (75 m). We have:

$$\frac{\Delta x}{h} = \tan\theta \Rightarrow h = \frac{\Delta x}{\tan\theta} \approx \frac{\Delta x}{\sin\theta} = \frac{75.0\ \text{m}}{1.53\times10^{-4}} = 4.90\times10^{5}\ \text{m} = 490\ \text{km}.$$

36.72: a) $\sin\theta = 1.22\dfrac{\lambda}{D} \approx \dfrac{\Delta x}{R} \Rightarrow R = \dfrac{D\Delta x}{1.22\lambda} = \dfrac{(6.00\times10^{6}\ \text{m})(2.50\times10^{5}\ \text{m})}{(1.22)(1.0\times10^{-5}\ \text{m})} =$

$1.23\times10^{17}\ \text{m}$. But $9.41\times10^{15}\ \text{m/ly} \Rightarrow R = 13.1\ \text{ly}$.

b) $\Delta x = \dfrac{1.22\,\lambda R}{D} = \dfrac{(1.22)(1.0\times10^{-5}\ \text{m})(4.22\ \text{ly})(9.41\times10^{15}\ \text{m/ly})}{1.0\ \text{m}} =$

$4.84\times10^{11}\ \text{m} = 4.84\times10^{8}\ \text{km}$.

$\approx 10{,}000$ times the diameter of the earth! Not enough resolution to see an earth-like planet!

≈ 3 times the distance from the earth to the sun.

c) $\Delta x = \dfrac{(1.22)(1.0\times10^{-5}\ \text{m})(59\ \text{ly})(9.41\times10^{15}\ \text{m/ly})}{6.00\times10^{6}\ \text{m}} = 1.13\times10^{6}\ \text{m} = 1130\ \text{km}.$

$\dfrac{\Delta x}{D_{\text{planet}}} = \dfrac{1130\ \text{km}}{1.38\times10^{5}\ \text{km}} = 8.19\times10^{-3}$; Δx is small compared to the size of the planet.

36.73: a) From the segment dy', the fraction of the amplitude of E_0 that gets through is $E_0\left(\frac{dy'}{a}\right) \Rightarrow dE = E_0\left(\frac{dy'}{a}\right)\sin(kx-\omega t)$.

b) The path difference between each little piece is $y'\sin\theta \Rightarrow kx = k(D - y'\sin\theta) \Rightarrow dE = \frac{E_0 dy'}{a}\sin(k(D-y'\sin\theta)-\omega t)$. This can be rewritten as $dE = \frac{E_0 dy'}{a}(\sin(kD-\omega t)\cos(ky'\sin\theta)+\sin(ky'\sin\theta)\cos(kD-\omega t))$.

c) So the total amplitude is given by the integral over the slit of the above.

$$\Rightarrow E = \int_{-a/2}^{a/2} dE = \frac{E_0}{a}\int_{-a/2}^{a/2} dy'\ (\sin(kD-\omega t)\cos(ky'\sin\theta)+\sin(ky'\sin\theta)\times$$
$$\cos(kD-\omega t)).$$

But the second term integrates to zero, so we have:

$$E = \frac{E_0}{a}\sin(kD-\omega t)\int_{-a/2}^{a/2} dy'\ (\cos(ky'\sin\theta)) = E_0\sin(kD-\omega t)\left[\left(\frac{\sin(ky'\sin\theta}{ka\sin\theta/2}\right)\right]_{-a/2}^{a/2}$$

$$\Rightarrow E = E_0\sin(kD-\omega t)\left(\frac{\sin(ka(\sin\theta)/2}{ka(\sin\theta)/2}\right) = E_0\sin(kD-\omega t)\left(\frac{\sin(\pi a(\sin\theta)/\lambda)}{\pi a(\sin\theta)/\lambda}\right).$$

At $\theta = 0$, $\frac{\sin[\ldots]}{[\ldots]} = 1 \Rightarrow E = E_0\sin(kD-\omega t)$.

d) Since $I = E^2 \Rightarrow I = I_0\left(\frac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right)^2 = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$, where we have used $I_0 = E_0^2\sin^2(kx-\omega t)$.

36.74: a) Recall that the expression for the amplitude of a traveling wave is $\cos(kx-\omega t)$. Thus each source can be thought of as a traveling wave evaluated at $x=R$ with a maximum amplitude of E_0. However, each successive source will pick up an extra phase from its respective pathlength to point P. $\phi = 2\pi\left(\dfrac{d\sin\theta}{\lambda}\right)$ which is just 2π, the maximum phase, scaled by whatever fraction the path difference, $d\sin\theta$, is of the wavelength, λ. By adding up the contributions from each source (including the accumulating phase difference) this gives the expression provided.

b) $e^{i(kR-\omega t+n\phi)} = \cos(kR-\omega t+n\phi) + i\sin(kR-\omega t+n\phi)$.

The real part is just $\cos(kR-\omega t+n\phi)$.

So $\mathrm{Re}\left[\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)}\right] = \sum_{n=0}^{N-1} E_0\cos(kR-\omega t+n\phi)$.

(Note: *Re* means "the real part of . . ..")

but this is just $E_0\cos(kR-\omega t+\phi) + E_0\cos(kR-\omega t+\phi) + E_0\cos(kR-\omega t+2\phi) + \ldots + E_0\cos(kR-\omega t+(N-1)\phi)$.

c) $\sum_{n=0}^{N-1} E_0\, e^{i(kR-\omega t+n\phi)} = E_0\sum_{n=0}^{N-1} e^{-i\omega t}\, e^{+ikR} e^{in\phi} = E_0 e^{i(kR-\omega t)}\sum_{n=0}^{N-1} e^{in\phi}$.

but $\sum_{n=0}^{\infty} e^{in\phi} = \sum_{n=0}^{N-1}(e^{i\phi})^n$. But recall $\sum_{n=0}^{N-1} x^n = \dfrac{x^N-1}{x-1}$.

Let $x = e^{i\phi}$ so $\sum_{n=0}^{N-1}(e^{i\phi})^n = \dfrac{e^{iN\phi}-1}{e^{i\phi-1}}$ (nice trick!). But $\dfrac{e^{iN\phi}-1}{e^{i\phi}-1} = \dfrac{e^{\frac{iN\phi}{2}}(e^{\frac{iN\phi}{2}}-e^{-\frac{iN\phi}{2}})}{e^{i\frac{\phi}{2}}(e^{i\frac{\phi}{2}}-e^{-i\frac{\phi}{2}})} =$

$$e^{i(N-1)\frac{\phi}{2}}\frac{(e^{\frac{iN\phi}{2}}-e^{-\frac{iN\phi}{2}})}{(e^{i\frac{\phi}{2}}-e^{-i\frac{\phi}{2}})}.$$

Putting everything together:

$$E = \sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)} = E_0 e^{i(kR-\omega t+(N-1)\frac{\phi}{2})}\frac{(e^{iN\frac{\phi}{2}}-e^{-iN\frac{\phi}{2}})}{(e^{i\frac{\phi}{2}}-e^{-i\frac{\phi}{2}})}$$

$$= E_0[\cos(kR-\omega t+n\phi)+i\sin(kR-\omega t+n\phi)]\left[\frac{\cos\frac{N\phi}{2}+i\sin\frac{N\phi}{2}-\cos\frac{N\phi}{2}-i\sin\frac{N\phi}{2}}{\cos\frac{\phi}{2}+i\sin\frac{\phi}{2}-\cos\frac{\phi}{2}-i\sin\frac{\phi}{2}}\right]$$

Taking only the real part gives $\Rightarrow E_0\cos(kR-\omega t+n\phi)\dfrac{\sin(N\frac{\phi}{2})}{\sin\frac{\phi}{2}} = E$.

d) $I = |E|^2_{\text{ave}} = I_0\dfrac{\sin^2(N\frac{\phi}{2})}{\sin^2(\frac{\phi}{2})}$. (The $\cos^2$ term goes to $\frac{1}{2}$ in the time average and is included in the definition of I_0.) $I_0 \propto \dfrac{E_0^2}{2}$.

36.75: a) $I = I_0 \dfrac{\sin^2(N\frac{\phi}{2})}{\sin^2\frac{\phi}{2}} \lim_{\phi\to 0} I \to \dfrac{0}{0}$.

Use l'Hôpital's rule: $\lim_{\phi\to 0} \dfrac{\sin\,(N\frac{\phi}{2})}{\sin\,\frac{\phi}{2}} = \lim_{\phi\to 0}\left(\dfrac{N/2}{1/2}\right)\dfrac{\cos(N\frac{\phi}{2})}{\cos(\frac{\phi}{2})}$.

$$= N$$

So $\lim_{\phi\to 0} I = N^2 I_0$.

b) The location of the first minimum is when the numerator first goes to zero at $\dfrac{N}{2}\phi_{\text{min}} = \pi$ or $\phi_{\text{min}} = \dfrac{2\pi}{N}$. The width of the central maximum goes like $2\phi_{\text{min}}$, so it is proportional to $\dfrac{1}{N}$.

c) Whenever $\dfrac{N\phi}{2} = n\pi$ where n is an integer, the numerator goes to zero, giving a minimum in intensity. That is, I is a minimum wherever $\phi = \dfrac{2n\pi}{N}$. This is true assuming that the denominator doesn't go to zero as well, which occurs when $\dfrac{\phi}{2} = m\pi$, where m is an integer. When both go to zero, using the result from part(a), there is a maximum. That is, if $\dfrac{n}{N}$ is an integer, there will be a maximum.

d) From part c), if $\dfrac{n}{N}$ is an integer we get a maximum. Thus, there will be $N-1$ minima. (Places where $\dfrac{n}{N}$ is not an integer for fixed N and integer n.) For example, $n = 0$ will be a maximum, but $n = 1, 2 \ldots, N-1$ will be minima with another maximum at $n = N$.

e) Between maxima $\dfrac{\phi}{2}$ is a half-integer multiple of $\pi\left(\text{i.e. } \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \text{etc.}\right)$ and if N is odd then $\dfrac{\sin^2(N\frac{\phi}{2})}{\sin^2\frac{\phi}{2}} \to 1$, so $I \to I_0$.

37.1: If O' sees simultaneous flashes then O will see the $\mathrm{A}(\mathrm{A}')$ flash first since O would believe that the A' flash must have traveled longer to reach O', and hence started first.

37.2: a) $\gamma = \dfrac{1}{\sqrt{1-(0.9)^2}} = 2.29.$

$$t = \gamma\tau = (2.29)\,(2.20\times10^{-6}\,\text{s}) = 5.05\times10^{-6}\ \text{s}.$$

b) $d = vt = (0.900)\,(3.00\times10^{8}\ \text{m/s})\,(5.05\times10^{-6}\,\text{s}) = 1.36\times10^{3}\ \text{m} = 1.36\ \text{km}.$

37.3: $\sqrt{1-u^2/c^2} = (1-u^2/c^2)^{1/2} \approx 1-\dfrac{u^2}{2c^2}+\cdots$

$$\Rightarrow (\Delta t-\Delta t_0) = (1-\sqrt{1-u^2/c^2})(\Delta t) = \frac{u^2}{2c^2}\Delta t = \frac{(250\ \text{m/s})^2\ (4\ \text{hrs})\cdot(3600)}{2(3.00\times10^8\ \text{m/s})^2}$$

$$\Rightarrow (\Delta t-\Delta t_0) = 5.00\times10^{-9}\,\text{s}.$$

The clock on the plane shows the shorter elapsed time.

37.4: $\gamma = \dfrac{1}{\sqrt{1-(0.978)^2}} = 4.79.$

$$\gamma\Delta t = (4.79)\,(82.4\times10^{-6}\,\text{s}) = 3.95\times10^{-4}\,\text{s} = 0.395\ \text{ms}.$$

37.5: a) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}} \Rightarrow 1-\dfrac{u^2}{c^2} = \left(\dfrac{\Delta t_0}{\Delta t}\right)^2$

$$\Rightarrow u = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{2.6}{42}\right)^2}$$

$$\therefore u = 0.998c.$$

b) $\Delta x = u\Delta t = (0.998)\,(3.00\times10^8\ \text{m/s})\,(4.2\times10^{-7}\,\text{s}) = 126\ \text{m}.$

37.6: $\gamma = 1.667$ a) $\Delta t_0 = \dfrac{\Delta t}{\gamma} = \dfrac{1.20\times10^8\ \text{m}}{\gamma(0.800c)} = 0.300\ \text{s}.$

b) $(0.300\ \text{s})\,(0.800c) = 7.20\times10^7\ \text{m}.$

c) $\Delta t_0 = 0.300\ \text{s}/\gamma = 0.180\ \text{s}.$ (This is what the *racer* measures *your* clock to read at that instant.) At *your* origin you read the original $\dfrac{1.20\times10^8\text{m}}{(0.800)\,(3\times10^8\ \text{m/s})} = 0.5\ \text{s}.$

Clearly the observes (you and the racer) will not agree on the order of events!

37.7: $\Delta t_0 = \sqrt{1-u^2/c^2}\,\Delta t = \sqrt{1-\left(\dfrac{4.80\times10^6\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2}\ (1\ \text{yr})$

$$\Rightarrow (\Delta t - \Delta t_0) = (1.28\times10^{-4})\ \text{yr} = 1.12\ \text{hrs}.$$

The least time elapses on the rocket's clock because it had to be in two inertial frames whereas the earth was only in one.

37.8: a) The frame in which the source (the searchlight) is stationary is the spacecraft's frame, so 12.0 ms is the proper time. b) To three figures, $u = c$. Solving Eq. (37.7) for u/c in terms of γ,

$$\frac{u}{c} = \sqrt{1-(1/\gamma)^2} \approx 1-\frac{1}{2\gamma^2}.$$

Using $1/\gamma = \Delta t_0/\Delta t = 12.0\ \text{ms}/190\ \text{ms}$ gives $u/c = 0.998$.

37.9: $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = \dfrac{1}{\sqrt{1-(0.9860)^2}} = 6.00$

a) $l = \dfrac{l_0}{\gamma} = \dfrac{55\ \text{km}}{6.00} = 9.17\ \text{km}.$

b) In muon's frame:

$$d = u\Delta t = (0.9860c)(2.20\times10^{-6}\ \text{s}) = 0.651\ \text{km}.$$

$$\Rightarrow \% = \frac{d}{h} = \frac{0.651}{9.17} = 0.071 = 7.1\%.$$

c) In earth's frame:

$$\Delta t = \Delta t_0\gamma = (2.2\times10^{-6}\ \text{s})(6.00) = 1.32\times10^{-5}\,\text{s}$$

$$\Rightarrow d' = u\Delta t = (0.9860c)(1.32\times10^{-5}\ \text{s}) = 3.90\ \text{km}$$

$$\Rightarrow \% = \frac{d'}{h'} = \frac{3.90\ \text{km}}{55.0\ \text{km}} = 7.1\%.$$

37.10: a) $t = \dfrac{4.50\times10^4\ \text{m}}{0.99540c} = 1.51\times10^{-4}\ \text{s}.$

b) $\gamma = \dfrac{1}{\sqrt{1-(0.9954)^2}} = 10.44$

$$h' = \frac{h}{\gamma} = \frac{45\ \text{km}}{10.44} = 4.31\,\text{km}.$$

c) $\dfrac{h'}{0.99540c} = 1.44\times10^{-5}$ s, and $\dfrac{t}{\gamma} = 1.44\times10^{-5}$ s; so the results agree but the particle's lifetime is dilated in the frame of the earth.

37.11: a) $l_0 = 3600\ \text{m}$

$$\Rightarrow l = l_0\sqrt{1-\frac{u^2}{c^2}} = l_0(3600\ \text{m})\ \sqrt{1-\frac{(4.00\times10^7\ \text{m/s})^2}{(3.00\times10^8\ \text{m/s})^2}}$$

$$= (3600\ \text{m})(0.991) = 3568\ \text{m}.$$

b) $\Delta t_0 = \dfrac{l_0}{u} = \dfrac{3600\ \text{m}}{4.00\times10^7\ \text{m/s}} = 9.00\times10^{-5}\ \text{s}.$

c) $\Delta t = \dfrac{l}{u} = \dfrac{3568\ \text{m}}{4.00\times10^7\ \text{m/s}} = 8.92\times10^{-5}\ \text{s}.$

37.12: $\gamma = 1/0.3048, \text{so } u = c\sqrt{1-(1/\gamma)^2} = 0.952c = 2.86\times10^8\ \text{m/s}.$

37.13: $l = l_0\sqrt{1-\dfrac{u^2}{c^2}} \Rightarrow l_0 = \dfrac{l}{\sqrt{1-u^2/c^2}}$

$$\Rightarrow l_0 = \frac{74.0\ \text{m}}{\sqrt{1-\left(\dfrac{0.600c}{c}\right)^2}} = 92.5\,\text{m}.$$

37.14: Multiplying the last equation of (37.21) by *u* and adding to the first to eliminate *t* gives

$$x' + ut' = \gamma x\left(1-\frac{u^2}{c^2}\right) = \frac{1}{\gamma}x,$$

and multiplying the first by $\dfrac{u}{c^2}$ and adding to the last to eliminate *x* gives

$$t' + \frac{u}{c^2}x' = \gamma t\left(1-\frac{u^2}{c^2}\right) = \frac{1}{\gamma}t,$$

so $x = \gamma(x' + ut')$ and $t = \gamma(t' + ux'/c^2)$,

which is indeed the same as Eq. (37.21) with the primed coordinates replacing the unprimed, and a change of sign of *u*.

37.15: a) $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.400c + 0.600c}{1+(0.400)\,(0.600)} = 0.806c$

b) $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.900c + 0.600c}{1+(0.900)(0.600)} = 0.974c$

c) $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.990c + 0.600c}{1+(0.990)(0.600)} = 0.997c.$

37.16: $\gamma = 1.667 (\gamma = 5/3$ if $u = (4/5)c)$. a) In Mavis's frame the event "light on" has space-time coordinates $x' = 0$ and $t' = 5.00$ s, so from the result of Exercise 37.14 or Example 37.7, $x = \gamma(x' + ut')$ and $t = \gamma\left(t' + \dfrac{ux'}{c^2}\right) \Rightarrow x = \gamma ut' = 2.00\times 10^9\,\text{m}$, $t = \gamma t' =$ 8.33 s.

b) The 5.00-s interval in Mavis's frame is the proper time Δt_0 in Eq. (37.6), so $\Delta t = \gamma \Delta t_0 = 8.33$ s, as in part (a).

c) $(8.33\,\text{s})\,(0.800c) = 2.00\times 10^9$ m, which is the distance x found in part (a).

37.17: Eq. (37.18): $x' = \dfrac{x - ut}{\sqrt{1 - u^2/c^2}}$ Eq. (37.19): $x' = -ut' + x\sqrt{1 - u^2/c^2}$

Equate: $(x - ut)\,\gamma = -ut' + \dfrac{x}{\gamma}$

$$\Rightarrow t' = \left(-\frac{x\gamma}{u} + t\gamma + \frac{x}{u\gamma}\right) = t\gamma + \frac{x}{u}\left(\frac{1}{\gamma} - \gamma\right)$$

$$\frac{1}{\gamma} - \gamma = \sqrt{1 - (u/c)^2} - \frac{1}{\sqrt{1 - (u/c)^2}} = \frac{1 - (u/c)^2 - 1}{\sqrt{1 - (u/c)^2}} = \frac{-u^2/c^2}{\sqrt{1 - (u/c)^2}} = \gamma u^2/c^2$$

$$\Rightarrow t' = t\gamma + \frac{xu\gamma}{c^2} = \frac{t - ux/c^2}{\sqrt{1 - (u/c)^2}}.$$

37.18: Starting from Eq. (37.22),

$$v' = \frac{v - u}{1 - uv/c^2}$$

$$v'(1 - uv/c^2) = v - u$$

$$v' + u = v + v'uv/c^2$$

$$= v(1 + uv'/c^2)$$

from which Eq. (37.23) follows. This is the same as switching the primed and unprimed coordinates and changing the sign of u.

37.19: Let the unprimed frame be Tatooine and let the primed frame be the pursuit ship. We want the velocity v' of the cruiser knowing the velocity of the primed frame u and the velocity of the cruiser v in the unprimed frame (Tatooine).

$$v' = \frac{v - u}{1 - \frac{uv}{c^2}} = \frac{0.600c - 0.800c}{1 - (0.600)\,(0.800)} = -0.385c$$

$\Rightarrow$ the cruiser is moving toward the pursuit ship at $0.385c$.

37.20: In the frame of one of the particles, u and v are both $0.9520c$ but with opposite sign.

$$v' = \frac{-v-(u)}{1-(u)\,(-v)/c^2} = \frac{-0.9520c-0.9520c}{1-(0.9520)(-0.9520)} = -0.9988c.$$

Thus, one particle moves at a speed $0.9988c$ toward the other in the other particle's frame.

37.21: $v = \dfrac{v'+u}{1+\dfrac{uv'}{c^2}} = \dfrac{-0.950c+0.650c}{1+(-0.950)(0.650)} = -0.784c.$

37.22: a) In Eq.(39-24), $u = 0.400c,\, v' = 0.700c \Rightarrow v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.700c+0.400c}{1+(0.700)\,(0.400)}$ $= 0.859c.$

b) $\dfrac{\Delta x}{v} = \dfrac{8.00\times10^9\,\text{m}}{0.859c} = 31.0\ \text{s}.$

37.23: $v' = \dfrac{v-u}{1-uv/c^2} \Rightarrow v' - \dfrac{uvv'}{c^2} = v-u$

$$\Rightarrow u\left(1-\frac{vv'}{c^2}\right) = v-v' \Rightarrow u = \frac{v-v'}{(1-vv'/c^2)}$$

$$\Rightarrow u = \frac{0.360c-0.920c}{(1-(0.360)(0.920))} = -0.837c$$

$\Rightarrow$ moving opposite the rocket, i.e., away from Arrakis.

37.24: Solving Eq. (37.25) for u/c, (see solution to Exercise 37.25)

$$\frac{u}{c} = \frac{1-(f/f_0)^2}{1+(f/f_0)^2},$$

and so (a) if $f/f_0 = 0.98,\ (u/c) = 0.0202,$ the source and observer are moving away from each other. b) if $f/f_0 = 4,\ (u/c) = -0.882,$ they are moving toward each other.

37.25: a) $f = \sqrt{\dfrac{c+u}{c-u}}\, f_0 \Rightarrow (c-u)f^2 = (c+u)f_0^2$

$$\Rightarrow u = \frac{c(f^2-f_0^2)}{f_0^2+f^2} = \frac{c((f/f_0)^2-1)}{(f/f_0)^2+1} = \frac{c((\lambda_0/\lambda)^2-1)}{((\lambda_0/\lambda)^2+1)}$$

$$\therefore u = c\frac{((675/575)^2-1)}{((675/575)^2+1)} = 0.159c = 4.77\times10^7\ \text{m/s} = 4.77\times10^4\ \text{km/s} = 1.72\times10^8\ \text{km/h}.$$

b) $(1.72\times10^8\ \text{km/h} - 90\,\text{km/h})\ (\$1.00) = \$172$ million dollars!

37.26: Using $u = -0.600c = -(3/5)c$ in Eq. (37.25) gives

$$f = \sqrt{\frac{1-(3/5)}{1+(3/5)}} f_0 = \sqrt{\frac{2/5}{8/5}} f_0 = f_0/2.$$

37.27: a) $F = \frac{dp}{dt} = \frac{d}{dt}\left(\frac{mv}{\sqrt{1-v^2/c^2}}\right) = \frac{ma}{\sqrt{1-v^2/c^2}} - \frac{\frac{1}{2}mv \cdot \frac{1}{2}(2va/c^2)}{(1-v^2/c^2)^{3/2}}$

$$= ma\left(\frac{(1-v^2/c^2)+v^2/c^2}{(1-v^2/c^2)^{3/2}}\right) = ma/(1-v^2/c^2)^{3/2}$$

$$\Rightarrow a = \frac{F}{m}\left(1-\frac{v^2}{c^2}\right)^{3/2}.$$

b) If the force is perpendicular to velocity then denominator is constant $\Rightarrow F = \frac{dp}{dt} =$ $\frac{m\,dv/dt}{\sqrt{1-v^2/c^2}} \Rightarrow a = \frac{F}{m}\sqrt{1-v^2/c^2}.$

37.28: The force is found from Eq. (37.32) or Eq. (37.33). (a) Indistinguishable from $F = ma = 0.145\text{ N}$. b) $\gamma^3 ma = 1.75\text{ N}$. c) $\gamma^3 ma = 51.7\text{ N}$. d) $\gamma ma = 0.145\text{ N}$, $0.333\text{ N}, 1.03\text{ N}$.

37.29: a) $p = \frac{mv}{\sqrt{1-v^2/c^2}} = 2mv$

$$\Rightarrow 1 = 2\sqrt{1-v^2/c^2} \Rightarrow \frac{1}{4} = 1-\frac{v^2}{c^2}$$

$$\Rightarrow v^2 = \frac{3}{4}c^2 \Rightarrow v = \frac{\sqrt{3}}{2}c = 0.866c.$$

b) $F = \gamma^3 ma = 2ma \Rightarrow \gamma^3 = 2 \Rightarrow \gamma = (2)^{1/3}$ so $\frac{1}{1-\frac{v^2}{c^2}} = 2^{2/3} \Rightarrow \frac{v}{c}$

$$= \sqrt{1-2^{-2/3}} = 0.608.$$

37.30: a) $\gamma = 1.01$, so $(v/c) = 0.140$ and $v = 4.21\times10^8\text{ m/s}$. b) The relativistic expression is *always* larger in magnitude than the non-relativistic expression.

37.31: a) $K = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2$

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{3}{4}} = 0.866c.$$

b) $K = 5mc^2 \Rightarrow \dfrac{1}{\sqrt{1-v^2/c^2}} = 6 \Rightarrow \dfrac{1}{36} = 1 - \dfrac{v^2}{c^2} \Rightarrow v = \sqrt{\dfrac{35}{36}}c = 0.986c.$

37.32: $E = 2mc^2 = 2(1.67\times10^{-27}\,\text{kg})(3.00\times10^8\ \text{m/s})^2 = 3.01\times10^{-10}\,\text{J} = 1.88\times10^9\,\text{eV}.$

37.33: $K = (\gamma - 1)mc^2 \approx \dfrac{1}{2}mv^2 + \dfrac{3}{8}\dfrac{mv^4}{c^2} + \cdots$

$if\ K - K_0 = 1.02\dfrac{1}{2}mv^2 \Rightarrow \dfrac{3}{8}\dfrac{v^4}{c^2} = \dfrac{0.02}{2}v^2$

$\Rightarrow \dfrac{150}{4}v^2 = c^2 \Rightarrow v = \sqrt{\dfrac{4}{150}}c = 0.163c = 4.89\times10^7\ \text{m/s}.$

37.34: a) $W = \Delta K = (\gamma_\text{f} - 1)mc^2 = (4.07\times10^{-3})\ mc^2.$ b) $(\gamma_\text{f} - \gamma_\text{i})mc^2 = 4.79mc^2.$
c) The result of part (b) is far larger than that of part (a).

37.35: a) Your total energy E increases because your potential energy increases;
$\Delta E = mg\Delta y$

$\Delta E = (\Delta m)c^2$ so $\Delta m = \Delta E/c^2 = mg(\Delta y)/c^2$

$\Delta m/m = (g\Delta y)/c^2 = (9.80\ \text{m/s}^2)(30\ \text{m})/(2.998\times10^8\ \text{m/s})^2 = 3.3\times10^{-13}\ \%$

This increase is much, much too small to be noticed.

b) $\Delta E = \Delta U = \frac{1}{2}kx^2 = \frac{1}{2}(2.00\times10^2\ \text{N/m})(0.060\,\text{m})^2 = 36.0\ \text{J}$

$\Delta m = (\Delta E)/c^2 = 4.0\times10^{-16}\ \text{kg}$

Energy increases so mass increases. The mass increase is much, much too small to be noticed.

37.36: a) $E_0 = m_0c^2$

$$2E = mc^2 = 2m_0c^2$$

$$\therefore m = 2m_0 \rightarrow \frac{m_0}{\sqrt{1-v^2/c^2}} = 2m_0$$

$$\frac{1}{4} = 1 - \frac{v^2}{c^2} \rightarrow \frac{v^2}{c^2} = \frac{3}{4} \rightarrow v = c\sqrt{3/4}$$

$$v = 0.866\ c = 2.60 \times 10^8\ \text{m/s}$$

b) $10\ m_0c^2 = mc^2 = \dfrac{m_0}{\sqrt{1-v^2/c^2}}c^2$

$$1 - \frac{v^2}{c^2} = \frac{1}{100} \rightarrow \frac{v^2}{c^2} = \frac{99}{100}$$

$$v = c\sqrt{\frac{99}{100}} = 0.995\ c = 2.98 \times 10^8\ \text{m/s}$$

37.37: a) $E = mc^2 + K$, so $E = 4.00mc^2$ means $K = 3.00mc^2 = 4.50 \times 10^{-10}$ J

b) $E^2 = (mc^2)^2 + (pc)^2$; $\quad E = 4.00mc^2$, so $15.0(mc^2)^2 = (pc)^2$

$p = \sqrt{15}mc = 1.94 \times 10^{-18}\ \text{kg} \cdot \text{m/s}$

c) $E = mc^2 \big/ \sqrt{1-v^2/c^2}$

$E = 4.00mc^2$ gives $1 - v^2/c^2 = 1/16$ and $v = \sqrt{15/16c} = 0.968c$

37.38: The work that must be done is the kinetic energy of the proton.

a) $K = (\gamma - 1)m_0c^2 = \left(\dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)m_0c^2$

$$= (1.67 \times 10^{-27}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\frac{1}{\sqrt{1-\left(\frac{0.1\,c}{c}\right)^2}} - 1\right)$$

$$= (1.50 \times 10^{-10}\ \text{J})\left(\frac{1}{\sqrt{1-0.01}} - 1\right)$$

$$= 7.56 \times 10^{-13}\ \text{J}$$

b) $K = (1.50 \times 10^{-10}\text{J})\left(\frac{1}{\sqrt{1-(0.5)^2}} - 1\right)$

$$= 2.32 \times 10^{-11}\ \text{J}$$

c) $K = (1.50 \times 10^{-10}\ \text{J})\left(\frac{1}{\sqrt{1-(0.9)^2}} - 1\right)$

$$= 1.94 \times 10^{-10}\ \text{J}$$

37.39: $(m = 6.64\times10^{-27}\ \text{kg},\quad p = 2.10\times10^{-18}\ \text{kg}\cdot\text{m/s})$

a) $E = \sqrt{(mc^2)^2 + (pc)^2}$

$= 8.68\times10^{-10}\ \text{J}.$

b) $K = E - mc^2 = 8.68\times10^{-10} - (6.64\times10^{-27}\ \text{kg})c^2 = 2.70\times10^{-10}\ \text{J}.$

c) $\dfrac{K}{mc^2} = \dfrac{2.70\times10^{-10}\ \text{J}}{(6.64\times10^{-27}\ \text{kg})c^2} = 0.452.$

37.40: $E = (m^2c^4 + p^2c^2)^{1/2} = mc^2\left(1+\left(\dfrac{p}{mc}\right)^2\right)^{1/2}$

$\approx mc^2\left(1+\dfrac{1}{2}\dfrac{p^2}{m^2c^2}\right) = mc^2 + \dfrac{p^2}{2m} = mc^2 + \dfrac{1}{2}mv^2$

the sum of the rest mass energy and the classical kinetic energy.

37.41: a) $v = 8\times10^7\ \text{m/s} \Rightarrow \gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = 1.0376$

$m = m_p \qquad K_0 = \dfrac{1}{2}mv^2 = 5.34\times10^{-12}\ \text{J}$

$K = (\gamma - 1)mc^2 = 5.65\times10^{-12}\ \text{J} \ \therefore \dfrac{K}{K_0} = 1.06.$

b) $v = 2.85\times10^8\ \text{m/s} \ \therefore \gamma = 3.203$

$K_0 = \dfrac{1}{2}mv^2 = 6.78\times10^{-11}\ \text{J}$

$K = (\gamma - 1)mc^2 = 3.31\times10^{-10}\ \text{J} \qquad K/K_0 = 4.88.$

37.42: $(5.52\times10^{-27}\ \text{kg})(3.00\times10^8\ \text{m/s})^2 = 4.97\times10^{-10}\ \text{J} = 3105\ \text{MeV}.$

37.43: a) $K = q\Delta V = e\Delta V$

$K = mc^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right) = 4.025mc^2 = 3.295\times10^{-13}\ \text{J} = 2.06\ \text{MeV}$

$\Delta V = K/e = 2.06\times10^6\ \text{V}$

b) From part (a), $K = 3.30\times10^{-13}\ \text{J} = 2.06\ \text{MeV}$

37.44: a) According to Eq. 37.38 and conservation of mass-energy

$$2Mc^2 + mc^2 = \gamma 2Mc^2 \Rightarrow \gamma = 1 + \frac{m}{2M} = 1 + \frac{9.75}{2(16.7)} = 1.292.$$

Note that since $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, we have that

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.292)^2}} = 0.6331.$$

b) According to Eq. 37.36, the kinetic energy of each proton is

$$K = (\gamma - 1)Mc^2 = (1.292 - 1)(1.67 \times 10^{-27}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\frac{1.00\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}} \right) =$$

274 MeV.

c) The rest energy of η^0 is $mc^2 = (9.75 \times 10^{-28}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\frac{1.00\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}} \right) =$ 548 MeV.

d) The kinetic energy lost by the protons is the energy that produces the η^0,

$$548\ \text{MeV} = 2(274\ \text{MeV}).$$

37.45: a) $E = 0.420\ \text{MeV} = 4.20 \times 10^5\ \text{eV}.$

b) $$E = K + mc^2 = 4.20 \times 10^5\ \text{eV} + \frac{(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2}{1.6 \times 10^{-19}\ \text{J/eV}}$$

$$= 4.20 \times 10^5\ \text{eV} + 5.11 \times 10^5 \text{eV} = 9.32 \times 10^5\ \text{eV}.$$

c) $$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \Rightarrow v = c\sqrt{1 - \left(\frac{mc^2}{E} \right)^2}$$

$$= c\sqrt{1 - \left(\frac{5.11 \times 10^5\ \text{eV}}{9.32 \times 10^5\ \text{eV}} \right)^2} = 0.836c = 2.51 \times 10^8\ \text{m/s}.$$

d) Nonrel: $$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4.20 \times 10^5\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$= 3.84 \times 10^8\ \text{m/s}.$$

37.46: a) The fraction of the initial mass that becomes energy is

$1 - \frac{(4.0015\ \text{u})}{2(2.0136\ \text{u})} = 6.382 \times 10^{-3}$, and so the energy released per kilogram is

$(6.382 \times 10^{-3})(1.00\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 = 5.74 \times 10^{14}\ \text{J}.$

b) $$\frac{1.0 \times 10^{19}\ \text{J}}{5.74 \times 10^{14}\ \text{J/kg}} = 1.7 \times 10^4\ \text{kg}.$$

37.47: a) $E = mc^2$, $m = E/c^2 = (3.8\times10^{26}\ \text{J})/(2.998\times10^8\ \text{m/s})^2 = 4.2\times10^9\ \text{kg}$

1 kg is equivalent to 2.2 lbs, so $m = 4.6\times10^6$ tons

b) The current mass of the sun is 1.99×10^{30} kg, so it would take it $(1.99\times10^{30}\ \text{kg})/(4.2\times10^9\ \text{kg/s}) = 4.7\times10^{20}\,\text{s} = 1.5\times10^{13}$ y to use up all its mass.

37.48: a) Using the classical work-energy theorem we obtain

$$\Delta x = \frac{m(v^2 - v^2{}_0)}{2F} = \frac{(2.00\times10^{-12}\ \text{kg})[(0.920)(3.00\times10^8\ \text{m/s})]^2}{2(4.20\times10^4\ \text{N})} = 1.81\,\text{m}.$$

b) Using the relativistic work-energy theorem for a constant force (Eq. 37.35) we obtain

$$\Delta x = \frac{(\gamma - 1)mc^2}{F}.$$

For the given speed, $\gamma = \frac{1}{\sqrt{1-0.920^2}} = 2.55$, thus

$$\Delta x = \frac{(2.55-1)(2.00\times10^{-12}\ \text{kg})(3.00\times10^8\ \text{m/s})^2}{(4.20\times10^4\,\text{N})} = 6.65\ \text{m}.$$

c) According to Eq. 37.30,

$$a = \frac{F}{m}\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} = \frac{(4.20\times10^4\ \text{N})}{(2.00\times10^{-12}\ \text{kg})}\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} = (2.10\times10^{16}\ \text{m/s}^2)\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}},$$

which yields, i) $a = 2.07\times10^{16}\ \text{m/s}^2$ $(\beta = 0.100)$

ii) $a = 1.46\times10^{16}\ \text{m/s}^2$ $(\beta = 0.462)$

iii) $a = 0.126\times10^{16}\ \text{m/s}^2$ $(\beta = 0.920)$.

37.49: a) $d = c\Delta t \Rightarrow \Delta t = \dfrac{d}{c} = \dfrac{1200\ \text{m}}{3\times10^8\ \text{m/s}} = 4.00\times10^{-6}\,\text{s}$ (v is very close to c).

But $\dfrac{\Delta t_0}{\Delta t} = \sqrt{1-\dfrac{u^2}{c^2}} \Rightarrow \left(\dfrac{u}{c}\right)^2 = 1-\left(\dfrac{\Delta t_0}{\Delta t}\right)^2$

$$\Rightarrow (1-\Delta)^2 = 1-\left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$\Rightarrow \Delta = 1-\left[1-\left(\frac{\Delta t_0}{\Delta t}\right)^2\right]^{1/2} = 1-\left[1-\left(\frac{2.6\times10^{-8}}{4.00\times10^{-6}}\right)^2\right]^{1/2} = 2.11\times10^{-5}.$$

b) $E = \gamma mc^2 = \left(\dfrac{\Delta t}{\Delta t_0}\right)\text{mc}^2 = \left(\dfrac{4.00\times10^{-6}}{2.6\times10^{-8}}\right)139.6\ \text{MeV}$

$$\Rightarrow E = 2.15\times10^4\ \text{MeV}.$$

37.50: One dimension of the cube appears contracted by a factor of $\frac{1}{\gamma}$, so the volume in S' is $a^3/\gamma = a^3\sqrt{1-(u/c)^2}$.

37.51: Need $a = b \Rightarrow l_0 = a, l = b$

$$\therefore \frac{l}{l_0} = \frac{b}{a} = \frac{b}{1.40b} = \sqrt{1-u^2/c^2}$$

$$\Rightarrow u = c\sqrt{1-\left(\frac{b}{a}\right)^2} = c\sqrt{1-\left(\frac{1}{1.40}\right)^2} = 0.700c.$$

$$= 2.10\times10^8 \text{ m/s}.$$

37.52: The change in the astronaut's biological age is Δt_0 in Eq. (37.6), and Δt is the distance to the star as measured from earth, divided by the speed. Combining, the astronaut's biological age is

$$19 \text{ yr} + \frac{42.2 \text{ yr } c}{\gamma u} = 19 \text{ yr} + \frac{42.2 \text{ yr}}{\gamma(0.9910)} = 24.7 \text{ yr}.$$

37.53: a) $E = \gamma mc^2$ and $\gamma = 10 = \dfrac{1}{\sqrt{1-(v/c)^2}} \Rightarrow \dfrac{v}{c} = \sqrt{\dfrac{\gamma^2-1}{\gamma^2}} \Rightarrow \dfrac{v}{c} = c\sqrt{\dfrac{99}{100}}$

$$= 0.995.$$

b) $(pc)^2 = m^2v^2\gamma^2c^2,\ E^2 = m^2c^4\left(\left(\dfrac{v}{c}\right)^2\gamma^2+1\right)$

$$\Rightarrow \frac{E^2-(pc)^2}{E^2} = \frac{1}{1+\gamma^2\left(\frac{v}{c}\right)^2} = \frac{1}{1+(10/(0.995))^2} = 0.01 = 1\%.$$

37.54: a) Note that the initial velocity is parallel to the *x*-axis. Thus, according to Eqn. 37.30,

$$a_x = \frac{F_x}{m}\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} = \frac{(-3.00\times10^{-12}\ \text{N})}{(1.67\times10^{-27}\text{kg})}(1-0.900^2)^{\frac{3}{2}} = -1.49\times10^{14}\ \text{m/s}^2.$$

Now note that the initial velocity is perpendicular to the *y*-axis. Thus, according to Eqn. 37.33,

$$a_y = \frac{F_y}{m}\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}} = \frac{(5.00\times10^{-12}\ \text{N})}{(1.67\times10^{-27}\ \text{kg})}(1-0.900^2)^{\frac{1}{2}} = 1.31\times10^{15}\ \text{m/s}^2.$$

b) The angle between the force and acceleration is given by

$$\cos\theta = \frac{F_x a_x + F_y a_y}{Fa} = \frac{(-3.00\times10^{-12}\ \text{N})(-1.49\times10^{14}\ \text{m/s}^2)+(5.00\times10^{-12}\ \text{N})(1.31\times10^{15}\ \text{m/s}^2)}{\sqrt{(-3.00\times10^{-12}\ \text{N})^2+(5.00\times10^{-12}\ \text{N})^2}\sqrt{(-1.49\times10^{14}\ \text{m/s}^2)^2+(1.31\times10^{15}\ \text{m/s}^2)^2}} \Rightarrow$$

$\theta = 24.5°$.

37.55: a) $K = 20\times10^{12}\text{eV} = 3.204\times10^{-6}\ \text{J}$

$$K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right),\ \text{so}\ \frac{1}{\sqrt{1-v^2/c^2}} = 2.131\times10^4$$

$$1-\frac{v^2}{c^2} = \frac{1}{(2.131\times10^4)^2};\ 1-\frac{v^2}{c^2} = \frac{(c+v)(c-v)}{c^2} = \frac{2(c-v)}{c}\ \ \text{since}\ c+v\approx 2c$$

$$v = (1-\Delta)c\ \ \text{so}\ 1-v^2/c^2 = 2\Delta\ \ \text{and}\ \Delta = 1.1\times10^{-9}$$

b) $m_{\text{rel}} = \dfrac{m}{\sqrt{1-v^2/c^2}} = \dfrac{m}{\sqrt{2\Delta}} = (2.1\times10^4)m$

37.56: a) $(8.00\ \text{kg})(1.00\times10^{-4})(3.00\times10^8\ \text{m/s})^2 = 7.20\times10^{13}\ \text{J}.$

b) $(\Delta E/\Delta t) = (7.20\times10^{13}\ \text{J})/(4.00\times10^{-6}\ \text{s}) = 1.80\times10^{19}\ \text{W}.$

c) $M = \dfrac{\Delta E}{gh} = \dfrac{(7.20\times10^{13}\ \text{J})}{(9.80\ \text{m/s}^2)(1.00\times10^3\ \text{m})} = 7.35\times10^9\ \text{kg}.$

37.57: Heat in $Q = mL_f = (4.00\ \text{kg})(3.34\times10^5\ \text{J/kg}) = 1.34\times10^6\ \text{J}$

$$\Rightarrow \Delta m = \frac{Q}{c^2} = \frac{(1.34\times10^6\ \text{J})}{(3.00\times10^8\ \text{m/s})^2} = 1.49\times10^{-11}\ \text{kg}.$$

37.58: a) $v = \dfrac{p}{m} = \dfrac{(E/c)}{m} = \dfrac{E}{mc}$, where the atom and the photon have the same magnitude of momentum, E/c. b) $v = \dfrac{E}{mc} =\ll c$, so $E \ll mc^2$.

37.59: Speed in glass $v = \frac{c}{n} = \frac{c}{1.52} = 1.97 \times 10^{8} \text{ m/s}$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1.326$$

$$\Rightarrow K = (\gamma - 1)mc^2 = (0.326)(0.511 \text{ MeV}) = 0.167 \text{ MeV} = 1.67 \times 10^{5} \text{ eV}.$$

37.60: a) 80.0 m/s is non-relativistic, and $K = \frac{1}{2}mv^2 = 186 \text{ J}.$

b) $(\gamma - 1)mc^2 = 1.31 \times 10^{15} \text{ J}.$

c) In Eq. (37.23),

$v' = 2.20 \times 10^{8} \text{ m/s}$, $u = -1.80 \times 10^{8} \text{ m/s}$, and so $v = 7.14 \times 10^{7} \text{ m/s}$.

d) $\frac{20.0 \text{ m}}{\gamma} = 13.6 \text{ m}.$

e) $\frac{20.0 \text{ m}}{2.20 \times 10^{8} \text{ m/s}} = 9.09 \times 10^{-8} \text{ s}.$

f) $t' = \frac{t}{\gamma} = 6.18 \times 10^{-8} \text{ s}$, or $t' = \frac{13.6 \text{ m}}{2.20 \times 10^{8} \text{ m/s}} = 6.18 \times 10^{-8} \text{ s}.$

37.61: $x'^2 = c^2 t'^2$

$$\Rightarrow (x - ut)^2 \gamma^2 = c^2 \gamma^2 \left(t - ux/c^2\right)^2$$

$$\Rightarrow x - ut = c(t - ux/c^2)$$

$$\Rightarrow x\left(1 + \frac{u}{c}\right) = \frac{1}{c} x(u + c) = t(u + c) \Rightarrow x = ct$$

$$\Rightarrow x^2 = c^2 t^2.$$

37.62: a) From Eq. (37.37),

$$K - \frac{1}{2}mv^2 = \frac{3}{8}m\frac{v^4}{c^2} = \frac{3}{8}(90.0 \text{ kg})\frac{(3.00 \times 10^{4} \text{ m/s})^4}{(3.00 \times 10^{8} \text{ m/s})^2} = 304 \text{ J}.$$

b) $\frac{(3/8)\, mv^4/c^2}{(1/2)mv^2} = \frac{3}{4}\left(\frac{v}{c}\right)^2 = 7.50 \times 10^{-9}.$

37.63: $a = \dfrac{dv}{dt} = \dfrac{F}{m}(1 - v^2/c^2)^{3/2}$

$$\Rightarrow \int_0^v \frac{dv}{(1-(v^2/c^2))^{3/2}} = \frac{F}{m}\int_0^t dt = \frac{F}{m}t$$

$$\Rightarrow c\int_0^{v/c} \frac{dx}{(1-x^2)^{3/2}} = c\left.\frac{x}{\sqrt{1-x^2}}\right|_0^{v/c} = \frac{v}{\sqrt{1-(v/c)^2}} = \frac{F}{m}t$$

$$\Rightarrow v^2 = \left(\frac{Ft}{m}\right)^2\left(1 - \left(\frac{v}{c}\right)^2\right) = \left(\frac{Ft}{m}\right)^2 - v^2\left(\frac{Ft}{mc}\right)^2$$

$$\Rightarrow v = \frac{Ft/m}{\sqrt{1+(Ft/mc)^2}}. \qquad \text{So as } t \to \infty, v \to c.$$

37.64: Setting $x = 0$ in Eq. (37.21), the first equation becomes $x' = -\gamma ut$ and the last, upon multiplication by c, becomes $ct' = \gamma ct$. Squaring and subtracting gives

$$c^2t'^2 - x'^2 = \gamma^2(c^2t^2 - u^2t^2) = c^2t^2,$$

or $x' = c\sqrt{t'^2 - t^2} = 4.53\times10^8$ m.

37.65: a) Want $\Delta t' = t_2' = t_1'$

$$x_1' = (x_1 - ut_1)\gamma = x_2' = (x_2 - ut_2)\gamma \Rightarrow u = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

And $\Delta t' = \gamma\left(t_2 - t_1 - u\frac{(x_2 - x_1)}{c_2}\right)$. Since $u = \Delta x/\Delta t$,

$$\Delta t' = \gamma\left(\Delta t - \frac{u\Delta x}{c^2}\right) = \gamma\left(\Delta t - \frac{\Delta x^2}{c^2\Delta t}\right) \Rightarrow \Delta t' = \Delta t\sqrt{1 - \left(\tfrac{\Delta x}{\Delta t}\right)^2/c^2} = \sqrt{\Delta t^2 - \frac{\Delta x^2}{c^2}}$$

There's no physical solution for $\Delta t < \dfrac{\Delta x}{c} \Rightarrow \Delta x \geq c\Delta t.$

b) Simultaneously $\Rightarrow \Delta t' = 0 \;\therefore t_1 - \dfrac{ux_1}{c^2} = t_2 - \dfrac{ux_2}{c^2}$

$$\Rightarrow \Delta t = \frac{u}{c^2}\Delta x \Rightarrow u = \frac{c^2\Delta t}{\Delta t}.$$

Also

$$\Delta x' = x_2' - x_1' = \frac{1}{\sqrt{1 - (u/c)^2}}(\Delta x - u\Delta t) = \frac{1}{\sqrt{1 - c^2\Delta t^2/\Delta x^2}}\left(\Delta x - c^2\frac{\Delta t^2}{\Delta x}\right)$$

$$= \Delta x\sqrt{1 - \frac{c^2\Delta t^2}{\Delta x^2}} \Rightarrow \Delta x' = \sqrt{\Delta x^2 - c^2\Delta t^2}.$$

c) Part (b): $\Delta t = \dfrac{1}{c}\sqrt{(\Delta x)^2 - (\Delta x')^2} = \dfrac{1}{c}\sqrt{(5.00\text{ m})^2 - (2.50\text{ m})^2} = 1.44 \times 10^{-8}\text{s}.$

37.66: a) $(100\text{ s})(0.600)(3.00 \times 10^8\text{ m/s}) = 1.80 \times 10^{10}\text{ m}.$ b) In Sebulbas frame, the relative speed of the tachyons and the ship is $3.40c$, and so the time $t_2 = 100\text{ s} + \dfrac{1.80 \times 10^{10}\text{ m}}{3.4c} = 118\text{ s}.$ At t_2 Sebulba measures that Watto is a distance from him of $(118\text{ s})(0.600)(3.00 \times 10^8\text{ m/s}) = 2.12 \times 10^{10}\text{ m}.$ c) From Eq. (37.23), with $v' = -4.00c$ and $u = 0.600c$, $v = +2.43c$, with the plus sign indicating a direction in the same direction as Watto's motion (that is, away from Sebulba). d) As the result of part (c) suggests, Sebulba would see the tachyons moving toward Watto and hence t_3 is the time they would have left Sebulba in order to reach Watto at the distance found in part (b), or $118\text{ s} - \dfrac{2.12 \times 10^{10}\text{ m}}{2.43c} = 89\text{ s},$ and so Sebulba receives Watto's message before even sending it! Tachyons seem to violate causality.

37.67: Longer wavelength (redshift) implies recession. (The emitting atoms are moving away.) Using the result of Ex. 37.26: $u = c\dfrac{(\lambda_0/\lambda)^2 - 1}{(\lambda_0/\lambda) + 1}$

$$\Rightarrow u = c\left[\frac{(656.3/953.4)^2 - 1}{(656.3/953.4)^2 + 1}\right] = -0.3570c = -1.071\times 10^8\ \text{m/s}$$

37.68: The baseball had better be moving non-relativistically, so the Doppler shift formula (Eq. (37.25)) becomes $f \cong f_0(1-(u/c))$. In the baseball's frame, this is the frequency with which the radar waves strike the baseball, and the baseball reradiates at *f*. But in the coach's frame, the reflected waves are Doppler shifted again, so the detected frequency is $f(1-(u/c)) = f_0(1-(u/c))^2 \approx f_0(1-2(u/c))$, so $\Delta f = 2f_0(u/c)$ and the fractional frequency shift is $\dfrac{\Delta f}{f_0} = 2(u/c)$. In this case,

$$\begin{aligned} u &= \frac{\Delta f}{2f_0}c = \frac{(2.86\times 10^{-7})}{2}(3.00\times 10^8\ \text{m}) \\ &= 42.9\ \text{m/s} = 154\ \text{km/h} \\ &= 92.5\ \text{mi/h}. \end{aligned}$$

37.69: a) Since the two triangles are similar:

$$H = A\gamma = mc^2\gamma = E.$$

b) $O = \sqrt{H^2 - A^2} = \sqrt{E^2 - (mc^2)^2} = pc.$

c) $K = E - mc^2.$

The kinetic energy can be obtained by the difference between the hypoteneuse and adjacent edge lengths.

37.70: a) As in the hint, both the sender and the receiver measure the same distance. However, in our frame, the ship has moved between emission of successive wavefronts, and we can use the time $T = 1/f$ as the proper time, with the result that $f = \gamma f_0 > f_0$.

b) Toward: $f = f_0\sqrt{\dfrac{c+u}{c-u}} = 345\ \text{MHz}\left(\dfrac{1+0.758}{1-0.758}\right)^{1/2} = 930\ \text{MHz}$

$$f - f_0 = 930\ \text{MHz} - 345\ \text{MHz} = 585\ \text{MHz}.$$

Away:

$$f = f_0\sqrt{\frac{c-u}{c+u}} = 345\ \text{MHz}\left(\frac{1-0.758}{1+0.758}\right)^{1/2} = 128\ \text{MHz}$$

$f - f_0 = -217\ \text{MHz}.$

c) $\gamma f_0 = 1.53 f_0 = 528\ \text{MHz}$, $f - f_0 = 183\ \text{MHz}$. The shift is still bigger than f_0, but not as large as the approaching frequency.

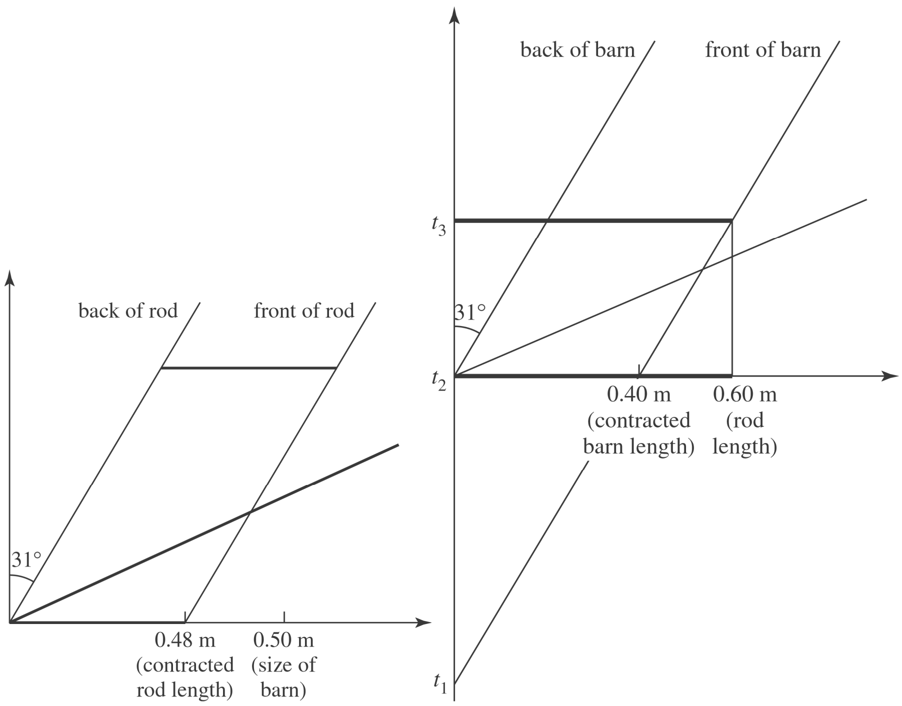
37.71: The crux of this problem is the question of simultaneity. To be "in the barn at one time" for the runner is different than for a stationary observer in the barn.

The diagram below, at left, shows the rod fitting into the barn at time $t = 0$, according to the stationary observer.

The diagram below, at right, is in the runner's frame of reference.

The front of the rod enters the barn at time t_1 and leaves the back of the barn at time t_2.

However, the back of the rod does not enter the front of the barn until the later time t_3.



37.72: In Eq. (37.23), $u = V,\ v' = (c/n),$ and so

$$v = \frac{(c/n)+V}{1+\dfrac{cV}{nc^2}} = \frac{(c/n)+V}{1+(V/nc)}.$$

For V non-relativistic, this is

$$\begin{aligned} v &\approx ((c/n)+V)(1-(V/nc)) \\ &= (c/n)+V-(V/n^2)-(V^2/nc) \\ &\approx \frac{c}{n}+\left(1-\frac{1}{n^2}\right)V \end{aligned}$$

so $k = \left(1-\dfrac{1}{n^2}\right)$. For water, $n = 1.333$ and $k = 0.437.$

37.73: a) $a' = \dfrac{dv}{dt'} \qquad dt' = \gamma(dt - udx/c^2)$

$$dv' = \frac{dv}{(1-uv/c^2)} + \frac{v-u}{(1-uv/c^2)^2}\,\frac{u}{c^2}dv$$

$$\frac{dv'}{dv} = \frac{1}{1-uv/c^2} + \frac{v-u}{(1-uv/c^2)^2}.\left(\frac{u}{c^2}\right)$$

$$\therefore dv' = dv\left(\frac{1}{1-uv/c^2} + \frac{(v-u)u/c^2}{(1-uv/c^2)^2}\right) = dv\left(\frac{1-u^2/c^2}{(1-uv/c^2)^2}\right)$$

$$\therefore a' = \frac{dv\dfrac{(1-u^2/c^2)}{(1-uv/c^2)^2}}{\gamma dt - u\gamma dx/c^2} = \frac{dv}{dt}.\frac{(1-u^2/c^2)}{(1-uv/c^2)^2}.\frac{1}{\gamma(1-uv/c^2)}$$

$$= a(1-u^2/c^2)^{3/2}\ (1-uv/c^2)^{-3}.$$

b) Changing frames from $S' \to S$ just involves changing

$$a \to a',\ v \to -v' \Rightarrow a = a'(1-u^2/c^2)^{3/2}\left(1+\frac{uv'}{c^2}\right)^{-3}.$$

37.74: a) The speed v' is measured relative to the rocket, and so for the rocket and its occupant, $v' = 0.$ The acceleration as seen in the rocket is given to be $a' = g,$ and so the acceleration as measured on the earth is

$$a = \frac{du}{dt} = g\left(1 - \frac{u^2}{c^2}\right)^{3/2}.$$

b) With $v_1 = 0$ when $t = 0$,

$$dt = \frac{1}{g}\frac{du}{(1-u^2/c^2)^{3/2}}$$

$$\int_0^{t_1} dt = \frac{1}{g}\int_0^{v_1}\frac{du}{(1-u^2/c^2)^{3/2}}$$

$$t_1 = \frac{v_1}{g\sqrt{1-v_1^2/c^2}}.$$

c) $dt' = \gamma dt = dt/\sqrt{1-u^2/c^2}$, so the relation in part (b) between *dt* and *du*, expressed in terms of dt' and *du*, is

$$dt' = \gamma dt = \frac{1}{\sqrt{1-u^2/c^2}}\frac{du}{g(1-u^2/c^2)^{3/2}} = \frac{1}{g}\frac{du}{(1-u^2/c^2)^2}.$$

Integrating as above (perhaps using the substitution $z = u/c$) gives

$$t_1' = \frac{c}{g}\text{ arctanh}\left(\frac{v_1}{c}\right).$$

For those who wish to avoid inverse hyperbolic functions, the above integral may be done by the method of partial fractions;

$$gdt' = \frac{du}{(1+u/c)(1-u/c)} = \frac{1}{2}\left[\frac{du}{1+u/c} + \frac{du}{1-uc}\right],$$

which integrates to

$$t_1' = \frac{c}{2g}\text{1n}\left(\frac{\text{c}+v_1}{c-v_1}\right).$$

d) Solving the expression from part (c) for v_1 in terms of t_1, $(v_1/c) = \tanh(gt_1'/c),$ so that $\sqrt{1-(v_1/c)^2} = 1/\cosh(gt_1'/c),$ using the appropriate indentities for hyperbolic functions. Using this in the expression found in part (b),

$$t_1 = \frac{c}{g}\frac{\tanh(gt_1'/c)}{1/\cosh(gt_1'/c)} = \frac{c}{g}\sinh(gt_1'/c),$$

which may be rearranged slightly as

$$\frac{gt_1}{c} = \sinh\left(\frac{gt_1'}{c}\right).$$

If hyperbolic functions are not used, v_1 in terms of t_1' is found to be

$$\frac{v_1}{c} = \frac{e^{gt_1'/c} - e^{-gt_1'/c_1}}{e^{gt_1'/c} + e^{-gt_1'/c'}}$$

37.75: a) $f_0 = 4.568110 \times 10^{14}$ Hz $f_+ = 4.568910 \times 10^{14}$ Hz $f_- = 4.567710 \times 10^{14}$ Hz

$$\left.\begin{aligned} f_+ &= \sqrt{\frac{c+(u+v)}{c-(u+v)}} \cdot f_0 \\ f_- &= \sqrt{\frac{c+(u-v)}{c-(u-v)}} \cdot f_0 \end{aligned}\right\} \Rightarrow \begin{aligned} f_+^2(c-(u+v)) &= f_0^2\ (c+(u+v)) \\ f_-^2(c-(u-v)) &= f_0^2(c+(u-v)) \end{aligned}$$

where u is the velocity of the center of mass and v is the orbital velocity.

$$\Rightarrow (u+v) = \frac{(f_+/f_0)^2 - 1}{(f_+/f_0)^2 + 1} c \text{ and } (u-v) = \frac{(f_-^2/f_0^2) - 1}{(f_-^2/f_0^2) + 1} c$$

$$\Rightarrow u+v = 5.25 \times 10^4 \text{ m/s} \quad u-v = -2.63 \times 10^4 \text{ m/s}$$

$$\Rightarrow u = +1.31 \times 10^4 \text{ m/s} \Rightarrow \text{moving toward at } 13.1 \text{ km/s}.$$

b) $v = 3.94 \times 10^4$ m/s $T = 11.0$ days.

$$\Rightarrow 2\pi R = vt \Rightarrow R = \frac{(3.94 \times 10^4 \text{ m/s})(11.0 \text{ days})(24 \text{ hrs/day})(3600 \text{ sec/hr})}{2\pi} =$$

$$5.96 \times 10^9 \text{ m} \approx 0.040 \text{ earth - sun distance}.$$

Also the gravitational force between them (a distance of $2R$) must equal the centripetal force from the center of mass:

$$\frac{(Gm^2)}{(2R)^2} = \frac{mv^2}{R} \Rightarrow m = \frac{4Rv^2}{G} =$$

$$\frac{4(5.96 \times 10^9 \text{ m})(3.94 \times 10^4 \text{ m/s})^2}{6.672 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 5.55 \times 10^{29} \text{ kg} = 0.279 \text{ m}_{\text{sun}}.$$

37.76: For any function $f = f(x, t)$ and $x = x(x', t'), t = t(x', t')$, let $F(x', t') = f(x(x', t'), t(x', t'))$ and use the standard (but mathematically improper) notation $F(x', t') = f(x', t')$. The chain rule is then

$$\frac{\partial f(x', t')}{\partial x} = \frac{\partial f(x, t)}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f(x', t')}{\partial t'}\frac{\partial t'}{\partial x},$$
$$\frac{\partial f(x', t')}{\partial t} = \frac{\partial f(x, t)}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial f(x', t')}{\partial t'}\frac{\partial t'}{\partial t}.$$

In this solution, the explicit dependence of the functions on the sets of dependent variables is suppressed, and the above relations are then

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial x}, \quad \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial t}.$$

a) $\frac{\partial x'}{\partial x} = 1, \frac{\partial x'}{\partial t} = -v, \frac{\partial t'}{\partial x} = 0$ and $\frac{\partial t'}{\partial t} = 1$. Then, $\frac{\partial E}{\partial x} = \frac{\partial E}{\partial x'}$, and $\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial x'^2}$. For the time derivative, $\frac{\partial f}{\partial t} = -v\frac{\partial E}{\partial x'} + \frac{\partial yE}{\partial t'}$. To find the second time derivative, the chain rule must be applied to both terms; that is,

$$\frac{\partial}{\partial t}\frac{\partial E}{\partial x'} = -v\frac{\partial^2 E}{\partial x'^2} + \frac{\partial^2 E}{\partial t'\partial x'},$$
$$\frac{\partial}{\partial t}\frac{\partial E}{\partial t'} = -v\frac{\partial^2 E}{\partial x'\partial t'} + \frac{\partial^2 E}{\partial t'^2}.$$

Using these in $\frac{\partial^2 E}{\partial t^2}$, collecting terms and equating the mixed partial derivatives gives

$$\frac{\partial^2 E}{\partial t^2} = v^2\frac{\partial^2 E}{\partial x'^2} - 2v\frac{\partial^2 E}{\partial x'\partial t'} + \frac{\partial^2 E}{\partial t'^2}$$

and using this and the above expression for $\frac{\partial^2 E}{\partial x'^2}$ gives the result.

b) For the Lorentz transformation, $\frac{\partial x'}{\partial x} = \gamma, \frac{\partial x'}{\partial t} = \gamma v, \frac{\partial t'}{\partial x} = \gamma v / c^2$ and $\frac{\partial t'}{\partial t} = \gamma.$ The first partials are then

$$\frac{\partial E}{\partial x} = \gamma\frac{\partial E}{\partial x'} - \gamma\frac{v}{c^2}\frac{\partial E}{\partial t'}, \frac{\partial E}{\partial t} = -\gamma v\frac{\partial E}{\partial x'} + \gamma\frac{\partial E}{\partial t'}$$

and the second partials are (again equating the mixed partials)

$$\frac{\partial^2 E}{\partial x^2} = \gamma^2\frac{\partial^2 E}{\partial x'^2} + \gamma^2\frac{v^2}{c^4}\frac{\partial^2 E}{\partial t'^2} - 2\gamma^2\frac{v}{c^2}\frac{\partial^2 E}{\partial x'\partial t'}$$
$$\frac{\partial^2 E}{\partial t^2} = \gamma^2 v^2\frac{\partial^2 E}{\partial x'^2} + \gamma^2\frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 v\frac{\partial^2 E}{\partial x'\partial t'}.$$

Substituting into the wave equation and combining terms (note that the mixed partials cancel),

37.77: a) In the center of momentum frame, the two protons approach each other with equal velocities (since the protons have the same mass). After the collision, the two protor are at rest—but now there are kaons as well. In this situation the kinetic energy of the protons must equal the total rest energy of the two kaons $\Rightarrow 2(\gamma_{\text{cm}} - 1)m_p c^2 = 2m_k c^2 \Rightarrow$

$\gamma_{\text{cm}} = 1 + \dfrac{m_k}{m_p} = 1.526.$ The velocity of a proton in the center of momentum frame is then

$v_{\text{cm}} = c\sqrt{\dfrac{\gamma_{\text{cm}}^2 - 1}{\gamma_{\text{cm}}^2}} = 0.7554c.$

To get the velocity of this proton in the lab frame, we must use the Lorentz velocity transformations. This is the same as "hopping" into the proton that will be our target and asking what the velocity of the projectile proton is. Taking the lab frame to be the unprimed frame moving to the left, $u = v_{\text{cm}}$ and $v' = v_{\text{cm}}$ (the velocity of the projectile proton in the center of momentum frame).

$$v_{\text{lab}} = \frac{v' + u}{1 + \frac{uv'}{c^2}} = \frac{2v_{\text{cm}}}{1 + \frac{v_{\text{cm}}^2}{c^2}} = 0.9619c$$

$$\Rightarrow \gamma_{\text{lab}} = \frac{1}{\sqrt{1 - \frac{v_{\text{lab}}^2}{c^2}}} = 3.658$$

$$\Rightarrow K_{\text{lab}} = (\gamma_{\text{lab}} - 1)m_p c^2 = 2494 \text{ MeV}.$$

b) $\dfrac{K_{\text{lab}}}{2m_k} = \dfrac{2494 \text{ MeV}}{2(493.7 \text{ MeV})} = 2.526.$

c) The center of momentum case considered in part (a) is the same as this situation. Thus, the kinetic energy required *is* just twice the rest mass energy of the kaons. $K_{\text{cm}} = 2(493.7 \text{ MeV}) = 987.4 \text{ MeV}.$ This offers a substantial advantage over the fixed target experiment in part (b). It takes less energy to create two kaons in the proton center of momentum frame.

38.1: $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{5.20\times10^{-7}\,\text{m}} = 5.77\times10^{14}\ \text{Hz}$

$p = \dfrac{h}{\lambda} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{5.20\times10^{-7}\ \text{m}} = 1.28\times10^{-27}\,\text{kg}\cdot\text{m/s}$

$E = pc = (1.28\times10^{-27}\ \text{kg}\cdot\text{m/s})\,(3.00\times10^{8}\ \text{m/s}) = 3.84\times10^{-19}\ \text{J} = 2.40\ \text{eV}.$

38.2: a) $Pt = (0.600\ \text{W})\,(20.0\times10^{-3}\text{s}) = 0.0120\ \text{J} = 7.49\times10^{16}\,\text{eV}.$

b) $hf = \dfrac{hc}{\lambda} = 3.05\times10^{-19}\,\text{J} = 1.90\ \text{eV}.$

c) $\dfrac{Pt}{hf} = 3.94\times10^{16}.$

38.3: a) $E = hf \Rightarrow f = \dfrac{E}{h} = \dfrac{(2.45\times10^{6}\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}{6.63\times10^{-34}\ \text{J}\cdot\text{s}} = 5.91\times10^{20}\ \text{Hz}.$

b) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{5.91\times10^{20}\ \text{Hz}} = 5.08\times10^{-13}\ \text{m}.$

c) λ is of the same magnitude as a nuclear radius.

38.4: $\dfrac{dN}{dt} = \dfrac{(dE/dt)}{(dE/dN)} = \dfrac{P}{hf} = \dfrac{P\lambda}{hc} = \dfrac{(12.0\,W)\,(2.48\times10^{-7}\,\text{m})}{hc}$

$= 1.50\times10^{19}\ \text{photons/sec}.$

38.5: $\dfrac{1}{2}mv_{\text{max}}^{2} = hf - \phi$

$= (6.63\times10^{-34}\ \text{J}\cdot\text{s})\left(\dfrac{c}{2.35\times10^{-7}\,\text{m}}\right) - (5.1\ \text{eV})\ (1.60\times10^{-19}\ \text{J/eV})$

$= 3.04\times10^{-20}\,\text{J}$

$\Rightarrow v_{\text{max}} = \sqrt{\dfrac{2(3.04\times10^{-20}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}} = 2.58\times10^{5}\ \text{m/s}.$

38.6: $E = hf - \phi = hf - \dfrac{hc}{\lambda_0} = h\left(1.45\times10^{15}\,\text{Hz} - \dfrac{c}{2.72\times10^{-7}\,\text{m}}\right)$

$= 2.30\times10^{-19}\ \text{J} = 1.44\ \text{eV}.$

38.7: a) $f = c/\lambda = 5.0\times10^{14}$ Hz

b) Each photon has energy $E = hf = 3.31\times10^{-19}$ J.

Source emits 75 J/s so $(75\text{ J/s})/(3.31\times10^{-19}\text{ photons/s}) = 2.3\times10^{20}$ photons/s

c) No, they are different. The frequency depends on the energy of each photon and the number of photons per second depends on the power output of the source.

38.8: For red light $\lambda = 700$ nm

$$\phi = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})\,(3.00\times10^{8}\text{ m/s})}{(700\times10^{-9}\text{ m})}$$

$$= 2.84\times10^{-19}\text{ J}\left(\frac{1\text{eV}}{1.6\times10^{-19}\text{ J}}\right)$$

$$= 1.77\text{ eV}$$

38.9: a) For a particle with mass, $K = p^2/2m$. $p_2 = 2p_1$ means $K_2 = 4K_1$.

b) For a photon, $E = pc$. $p_2 = 2p_1$ means $E_2 = 2E_1$.

38.10: $K_{\text{max}} = hf - \phi$

Use the information given for $\lambda = 400$ nm to find ϕ:

$$\phi = hf - K_{\text{max}} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})\,(2.998\times10^{8}\text{ m/s})}{400\times10^{-9}\text{ m}} - (1.10\text{ eV})\,(1.602\times10^{-19}\text{ J/eV})$$

$$= 3.204\times10^{-19}\text{ J}$$

Now calculate K_{max} for $\lambda = 300$ nm:

$$K_{\text{max}} = hf - \phi = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})\,(2.998\times10^{8}\text{ m/s})}{300\times10^{-9}\text{ m}} - 3.204\times10^{-19}\text{ J}$$

$$= 3.418\times10^{-19}\text{ J} = 2.13\text{ eV}$$

38.11: a) The work function $\phi = hf - \text{eV}_0 = \dfrac{hc}{\lambda} - \text{eV}_0$

$$\Rightarrow \phi = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})\,(3.00\times10^{8}\,\text{m/s})}{2.54\times10^{-7}\,\text{m}} - (1.60\times10^{-19}\,\text{C})\,(0.181\,\text{V})$$

$$= 7.53\times10^{-19}\,\text{J}.$$

The threshold frequency implies $\phi = hf_{th} \Rightarrow \dfrac{hc}{\lambda_{th}} \Rightarrow \lambda_{\text{th}} = \dfrac{hc}{\phi}$

$$\Rightarrow \lambda_{\text{th}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times10^{8}\ \text{m/s})}{7.53\times10^{-19}\ \text{J}} = 2.64\times10^{-7}\,\text{m}.$$

b) $\phi = 7.53\times10^{-19}\ \text{J} = 4.70\,\text{eV},$ as found in part (a), and this is the value from Table 38.1.

38.12: a) From Eq. (38.4),

$$V = \frac{1}{e}\left(\frac{hc}{\lambda} - \phi\right) = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})\,(3.00\times10^{8}\ \text{m/s})}{(2.50\times10^{-7}\ \text{m})} - 2.3\,\text{V} = 2.7\,\text{V}.$$

b) The stopping potential, multiplied by the electron charge, is the maximum kinetic energy, 2.7 eV.

c) $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2\,eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\ \text{C})\,(2.7\ \text{V})}{(9.11\times10^{-31}\ \text{kg})}} = 9.7\times10^{5}\ \text{m/s}.$

38.13: a) $E = pc = (8.24\times10^{-28}\ \text{kg}\cdot\text{m/s})\,(3.00\times10^{8}\ \text{m/s})$

$$= 2.47\times10^{-19}\ \text{J} = \frac{2.47\times10^{-19}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}} = 1.54\,\text{eV}$$

b) $p = \dfrac{h}{\lambda} \Rightarrow \lambda = \dfrac{h}{p} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(8.24\times10^{-28}\ \text{kg}\cdot\text{m/s})} = 8.05\times10^{-7}\ \text{m}.$

This is infrared radiation.

38.14: a) The threshold frequency is found by setting V = 0 in Eq. (40.4), $f_0 = \phi/h$.

b) $\phi = hf_0 = \dfrac{hc}{\lambda} = \dfrac{hc}{3.72\times10^{-7}\ \text{m}} = 5.35\times10^{-19} = 3.34\ \text{eV}.$

38.15: a) $E_\gamma = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})\,(3.00\times10^{8}\ \text{m/s})}{8.60\times10^{-7}\ \text{m}} = 2.31\times10^{-19}\,\text{J} = 1.44\ \text{eV}.$

So the internal energy of the atom increases by 1.44 eV to $E = -6.52\ \text{eV} + 1.44\ \text{eV} =$ -5.08 eV.

b) $E_\gamma = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times10^{8}\ \text{m/s})}{4.20\times10^{-7}\,\text{m}} = 4.74\times10^{-19}\,\text{J} = 2.96\ \text{eV}.$

So the final internal energy of the atom decreases to
$E = -2.68\ \text{eV} - 2.96\ \text{eV} = -5.64\ \text{eV}.$

38.16: a) $-E_1 = 20$ eV. b) The system starts in the $n = 4$ state. If we look at all paths to $n = 1$ we find the 4-3, 4-2, 4-1, 3-2, 3-1, and 2-1 transitions are possible (the last three are possible in combination with the others), with energies 3 eV, 8 eV, 18 eV, 5 eV, 15 eV, and 10 eV, respectively. c) There is no energy level 8 eV above the ground state energy, so the photon will not be absorbed. d) The work function must be more than 3 eV, but not larger than
5 eV.

38.17: a) $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)$ (Balmer series implies final state is $n = 2$)

$$H_\gamma \Rightarrow n = 5:\quad \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{25}\right) = \frac{21}{100}R$$

$$\Rightarrow \lambda = \frac{100}{21R} = \frac{100}{21(1.10\times10^{7})}\,\text{m} = 4.33\times10^{-7}\ \text{m} = 433\,\text{nm}$$

b) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{4.33\times10^{-7}\ \text{m}} = 6.93\times10^{14}\ \text{Hz}$

c) $E = hf = 2.87$ eV.

38.18: Lyman: largest is $n = 2, \lambda = \dfrac{(4/3)}{R} = \dfrac{(4/3)}{(1.097\times10^{7}\ \text{m}^{-1})} = 122\ \text{nm},$ in the ultraviolet.

Smallest is $n = \infty, \lambda = \dfrac{1}{R} = 91.2$ nm, also ultraviolet. Paschen: largest is

$n = 4, \lambda = \dfrac{(144/7)}{R} = 1875\ \text{nm},$ in the infrared. Smallest is $n = \infty, \lambda = \dfrac{9}{R} = 820\ \text{nm},$ also infrared.

38.19: $\Delta E_{\frac{3}{2}-g} = \dfrac{hc}{\lambda_1} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\,(3.000\times10^{8}\ \text{m/s})}{5.890\times10^{-7}\ \text{m}}$

$= 3.375\times10^{-19}\ \text{J} = 2.109\ \text{eV}.$

$\Delta E_{\frac{1}{2}-g} = \dfrac{hc}{\lambda_2} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\,(3.000\times10^{8}\ \text{m/s})}{5.896\times10^{-7}\ \text{m}}$

$= 3.371\times10^{-19}\ \text{J} = 2.107\ \text{eV}.$

$\Delta E_{\frac{3}{2}-\frac{1}{2}} = 2.00\times10^{-3}\ \text{eV}.$

38.20: a) Equating initial kinetic energy and final potential energy and solving for the separation radius r,

$$r = \frac{1}{4\pi\varepsilon_0}\frac{(92e)\,(2e)}{K}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(184)\,(1.60\times10^{-19}\ \text{C})}{(4.78\times10^{6}\ \text{J/C})} = 5.54\times10^{-14}\ \text{m}.$$

b) The above result may be substituted into Coulomb's law, or, the relation between the magnitude of the force and the magnitude of the potential energy in a Coulombic field is

$$F = \frac{K}{r} = \frac{(4.78\times10^{6}\ \text{eV})\,(1.6\times10^{-19}\ \text{J/ev})}{(5.54\times10^{-14}\ \text{m})} = 13.8\ \text{N}.$$

38.21: a) $U = \dfrac{q_1q_2}{4\pi\varepsilon_0 r} = \dfrac{(2e)\,(82e)}{4\pi\varepsilon_0 r} = \dfrac{164\,(1.60\times10^{-19}\ \text{C})^2}{4\pi\varepsilon_0(6.50\times10^{-14}\ \text{m})}$

$\Rightarrow U = 5.81\times10^{-13}\ \text{J} = 3.63\times10^{6}\ \text{eV} = 3.63\ \text{MeV}.$

b) $K_1 + U_1 = K_2 + U_2 \Rightarrow K_1 = U_2 = 5.81\times10^{-13}\ \text{J} = 3.63\ \text{MeV}.$

c) $K = \dfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(5.81\times10^{-13}\ \text{J})}{6.64\times10^{-27}\ \text{kg}}} = 1.32\times10^{7}\ \text{m/s}.$

38.22: $\dfrac{1}{\lambda} = R\left(1 - \dfrac{1}{(4)^2}\right)$, so $\lambda = 97.0\ \text{nm}$ and $f = \dfrac{c}{\lambda} = 3.09\times10^{15}\ \text{Hz}.$

38.23: a) Following the derivation for the hydrogen atom we see that for Be^{3+} all we need do is replace e^2 by $4e^2$. Then

$$E_n(\text{Be}^{3+}) = -\frac{1}{\varepsilon_0^2}\frac{m(4e^2)^2}{8n^2h^2} = 16\,E_n(\text{H}) \Rightarrow E_n(\text{Be}^{3+}) = 16\left(\frac{-13.60\text{ eV}}{n^2}\right).$$

So for the ground state, $E_1(\text{Be}^{3+}) = -218\,\text{eV}$.

b) The ionization energy is the energy difference between the $n \to \infty$ and $n = 1$ levels. So it is just 218 eV for Be^{3+}, which is 16 times that of hydrogen.

c) $\dfrac{1}{\lambda} = \dfrac{m(4e^2)^2}{8\varepsilon_0^2 h^3 c}\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right) = (1.74\times10^8\ \text{m}^{-1})\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right).$

So for $n = 2$ to $n = 1$, $\dfrac{1}{\lambda} = (1.74\times10^8\ \text{m}^{-1})\left(1 - \dfrac{1}{4}\right) = 1.31\times10^8\ \text{m}$

$$\Rightarrow \lambda = 7.63\times10^{-9}\,\text{m}.$$ This is 16 times shorter than that from the hydrogen atom.

d) $r_n = (\text{Be}^{3+}) = \dfrac{\varepsilon_0 n^2 h^2}{\pi m(4e^2)} = \dfrac{1}{4}r_n(\text{H}).$

38.24: a), b) For either atom, the magnitude of the angular momentum is $\dfrac{h}{2\pi} =$ $1.05\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}.$

38.25: $E_n = -(13.6\text{ eV})/n^2$, so this state has $n = \sqrt{13.6/1.51} = 3.$ In the Bohr model. $L = n\hbar$ so for this state $L = 3\hbar = 3.16\times10^{-34}\,\text{kg}\cdot\text{m}^2/\text{s}.$

38.26: a) We can find the photon's energy from Eq. 38.8

$$E = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = (6.63\times10^{-34}\ \text{J}\cdot\text{s})\ (3.00\times10^8\ \text{m/s})\ (1.097\times10^7\,\text{m}^{-1})\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$

$= 4.58\times10^{-19}$ J.

The corresponding wavelength is $\lambda = \frac{E}{hc} = 434$ nm.

b) In the Bohr model, the angular momentum of an electron with principal quantum number n is given by Eq. 38.10

$$L = n\frac{h}{2\pi}.$$

Thus, when an electron makes a transition from $n = 5$ to $n = 2$ orbital, there is the following loss in angular momentum (which we would assume is transferred to the photon):

$$\Delta L = (2-5)\frac{h}{2\pi} = -\frac{3(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi} = -3.17\times10^{-34}\ \text{J}\cdot\text{s}.$$

However, this prediction of the Bohr model is wrong (as shown in Chapter 41).

38.27: a) $v_n = \dfrac{1}{\varepsilon_0}\dfrac{e^2}{2nh} : n=1 \Rightarrow v_1 = \dfrac{(1.60\times10^{-19}\ \text{C})^2}{\varepsilon_0 2\,(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = 2.18\times10^6\ \text{m/s}$

$$h = 2 \Rightarrow v_2 = \frac{v_1}{2} = 1.09\times10^6\ \text{m/s}$$

$$h = 3 \Rightarrow v_3 = \frac{v_1}{3} = 7.27\times10^5\ \text{m/s}.$$

b) Orbital period $= \dfrac{2\pi r_n}{v_n} = \dfrac{2\varepsilon_0 n^2h^2/me^2}{1/\varepsilon_0 \cdot e^2/2nh} = \dfrac{4\varepsilon_0^2 n^3h^3}{me^4}$

$$n = 1 \Rightarrow T_1 = \frac{4\varepsilon_0^2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^3}{(9.11\times10^{-31}\ \text{kg})\,(1.60\times10^{-19}\ \text{C})^4} = 1.53\times10^{-16}\ \text{s}$$

$$n = 2:\ T_2 = T_1(2)^3 = 1.22\times10^{-15}\ \text{s}$$

$$n = 3:\ T_3 = T_1(3)^3 = 4.13\times10^{-15}\ \text{s}.$$

c) number of orbits $= \dfrac{1.0\times10^{-8}\,\text{s}}{1.22\times10^{-15}\ \text{s}} = 8.2\times10^6.$

38.28: a) Using the values from Appendix F, keeping eight significant figures, gives $R = 1.0973731\times10^7\ \text{m}^{-1}$. (Note: On some standard calculators, intermediate values in the calculation may have exponents that exceed 100 in magnitude. If this is the case, the numbers must be manipulated in a different order.)

b) Using the eight-figure value for R gives $E = \dfrac{hc}{\lambda} = hcR = 2.1798741\times10^{-18}$ $\text{J} = 13.605670\ \text{eV}$. c) Using the value for the proton mass as given in Appendix F give $R = 1.0967758\times10^7\,\text{m}^{-1}$, so $\Delta R = 5970\ \text{m}^{-1}$.

38.29: $\dfrac{n_{5s}}{n_{3p}} = e^{-(E_{5s}-E_{3p})/kT}$

But $$E_{5s} = 20.66 \text{ eV} = 3.306\times10^{-18}\text{ J}$$
$$E_{3p} = 18.70 \text{ eV} = 2.992\times10^{-18}\text{ J}$$
$$\Rightarrow \Delta E = 3.14\times10^{-19}\text{ J}$$

a) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.14\times10^{-19}\text{ J})/(1.38\times10^{-23}\text{ J/K}\cdot 300\text{ K})} = 1.15\times10^{-33}.$

b) $e^{-(3.14\times10^{-19}\text{ J})/(1.38\times10^{-23}\text{ J/K}\cdot 600\text{ K})} = 3.39\times10^{-17}.$

c) $e^{-(3.14\times10^{-19}\text{ J})/(1.38\times10^{-23}\text{ J/K}\cdot 1200\text{ K})} = 5.82\times10^{-9}.$

d) The $5s$ state is not highly populated compared to the $3p$ state, so very few atoms are able to make the required energy jump to produce the 632.8 nm light.

38.30: $\dfrac{n_{2p_{3/2}}}{n_{2p_{1/2}}} = e^{-(E_{2p3/2}-E_{2p1/2})/KT}.$

From the diagram $\Delta E_{3/2-g} = \dfrac{hc}{\lambda_1} = \dfrac{(6.626\times10^{-34}\text{ J})(3.000\times10^{8}\text{ m/s})}{5.890\times10^{-7}\text{ m}} = 3.375\times10^{-19}\text{ J}.$

$\Delta E_{1/2-g} = \dfrac{hc}{\lambda_2} = \dfrac{(6.626\times10^{-34}\text{ J})(3.000\times10^{8}\text{ m/s})}{5.896\times10^{-7}\text{ m}} = 3.371\times10^{-19}\text{ J}.$

so $\Delta E_{3/2-1/2} = 3.375\times10^{-19}\text{ J} - 3.371\times10^{-19}\text{ J} = 4.00\times10^{-22}\text{ J}.$

$\dfrac{n_{2p_{3/2}}}{n_{2p_{1/2}}} = e^{-(4.00\times10^{-22}\text{ J})/(1.38\times10^{-23}\ J/\text{K}\cdot 500\text{ K}).} = 0.944.$ So more atoms are in the $2p_{1/2}$ state.

38.31: $E_\gamma = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.06\times10^{-5}\text{ m}} = 1.88\times10^{-20}\text{ J}.$

Total energy in 1 second from the laser $E = Pt = 7.50\times10^{-3}$ J. So the number of photons emitted per second is $\dfrac{E}{E_\gamma} = \dfrac{7.50\times10^{-3}\text{ J}}{1.88\times10^{-20}\text{ J}} = 4.00\times10^{17}.$

38.32: $20.66\text{ eV} - 18.70\text{ eV} = 1.96\text{ eV} = 3.14\times10^{-19}$ J, and $\lambda = \dfrac{c}{f} = \dfrac{hc}{E} = 632$ nm, in good agreement.

38.33: $eV_{AC} = hf_{\max} = \dfrac{hc}{\lambda_{\min}}$

$$\Rightarrow \lambda_{\min} = \frac{hc}{eV_{AC}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(4000\ \text{V})} = 3.11\times10^{-10}\ \text{m}$$

This is the same answer as would be obtained if electrons of this energy were used. Electron beams are much more easily produced and accelerated than proton beams.

38.34: a) $\dfrac{hc}{e\lambda} = 8.29\times10^{3}\ \text{V} = 8.29\ \text{kV}.$ b) The shortest wavelength would correspond to the maximum electron energy, eV, and so $\lambda = \dfrac{hc}{eV} = 0.0414\ \text{nm}.$

38.35: An electron's energy after being accelerated by a voltage V is just $E = eV$. The most energetic photon able to be produced by the electron is just: $\lambda = \dfrac{hc}{E} = \dfrac{hc}{e\text{V}}$

$$\Rightarrow \lambda = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(1.5\times10^{4}\ \text{V})} = 8.29\times10^{-11}\ \text{m} = 0.0829\ \text{nm}.$$

38.36: a) From Eq. (38.23), $\cos\phi = 1 - \dfrac{\Delta\lambda}{(h/mc)}$, and so a)

$\Delta\lambda = 0.0542\ \text{nm} - 0.0500\ \text{nm},$

$$\cos\phi = 1 - \frac{0.0042\ \text{nm}}{0.002426\ \text{nm}} = -0.731, \text{ and } \phi = 137^\circ.$$

b) $\Delta\lambda = 0.0521\ \text{nm} - 0.0500\ \text{nm}.\ \cos\phi = 1 - \dfrac{0.0021\ \text{nm}}{0.002426\ \text{nm}} = 0.134.\ \phi = 82.3^\circ.$

c) $\Delta\lambda = 0,$ the photon is undeflected, $\cos\phi = 1$ and $\phi = 0.$

38.37: $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi)$

$$\Rightarrow \lambda'_{\max} = \lambda + \frac{h}{mc}(1-(-1))(\phi = 180^\circ \text{ is } \textit{chosen} \text{ to maximize } \lambda'.)$$

$$= \lambda + \frac{2h}{mc}$$

$$\Rightarrow \lambda'_{\max} = 6.65\times10^{-11}\ \text{m} + 2(2.426\times10^{-12}\ \text{m}) = 7.14\times10^{-11}\ \text{m}.$$

38.38: a) $\lambda = \dfrac{hc}{eV} = 0.0691\ \text{nm}$. b) $\lambda' - \lambda = (h/mc)(1-\cos\phi) = (2.426\times10^{-12}\ \text{m})\times$ $(1-\cos 45.0°) = 7.11\times10^{-13}\ \text{m}$, so $\lambda' = 0.0698\ \text{nm}$.

c) $E = \dfrac{hc}{\lambda'} = 17.8\ \text{keV}$.

38.39: $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi)$

$$\Delta\lambda = \lambda' - \lambda,\ \phi = 180^\circ;\ \Delta\lambda = \frac{2h}{mc}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{2h}{mc\lambda} = 9.70\times10^{-6}$$

38.40: The change in wavelength of the scattered photon is given by Eq. 38.23

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{mc\lambda}(1-\cos\phi) \Rightarrow \lambda = \frac{h}{mc\left(\frac{\Delta\lambda}{\lambda}\right)}(1-\cos\phi).$$

Thus,

$$\lambda = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(3.00\times10^{8}\ \text{m/s})(0.100)}(1+1) = 2.65\times10^{-14}\ \text{m}.$$

38.41: The derivation of Eq. (38.23) is explicitly shown in Equations (38.24) through (38.27) with the final substitution of

$p' = h/\lambda'$ and $p = h/\lambda$ yielding $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi)$.

38.42: $T = \dfrac{2.90\times10^{-3}\ \text{m}\cdot K}{\lambda_m} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{400\times10^{-9}\ \text{m}} = 7.25\times10^{3}\ \text{K}.$

38.43: $\lambda_\text{m} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{2.728\ \text{K}} = 1.06\times10^{-3}\ \text{m} = 1.06\ \text{mm}.$ This is in the microwave part of the electromagnetic spectrum.

38.44: From Eq. (38.30),

a) $\lambda_\text{m} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{3.00\ \text{K}} = 0.966\ \text{mm}$, and $f = \dfrac{c}{\lambda_m} = 3.10\times10^{11}\ \text{Hz}$.

Note that a more precise value of the Wien displacement law constant has been used. b) A factor of 100 increase in the temperature lowers λ_m by a factor of 100 to 9.66 μm and raises the frequency by the same factor, to 3.10×10^{13} Hz. c) Similarly, $\lambda_\text{m} = 966\ \text{nm}$ and $f = 3.10\times10^{14}$ Hz.

38.45: a) $H = Ae\sigma T^4$; $A = \pi r^2 l$

$$T = \left(\frac{H}{Ae\sigma}\right)^{1/4} = \left(\frac{100\text{ W}}{\pi(0.20\times10^{-3}\text{ m})^2(0.30\text{ m})(0.26)(5.671\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right)^{1/4}$$

$$T = 2.06\times10^4\text{ K}$$

b) $\lambda_{\text{m}}T = 2.90\times10^{-3}\text{ m}\cdot\text{K}$; $\lambda_{\text{m}} = 141\text{ nm}$

Much of the emitted radiation is in the ultraviolet.

38.46: (a) Wien's law: $\lambda_m = \frac{k}{T}$

$$\lambda_m = \frac{2.90\times10^{-3}\text{ K}\cdot\text{m}}{30{,}000\text{ K}} = 9.7\times10^{-8}\text{m} = 97\text{ nm}$$

This peak is in the ultraviolet region, which is *not* visible. The star is blue because the largest part of the visible light radiated is in the blue/violet part of the visible spectrum

(b) $P = \sigma AT^4$ (Stefan-Boltzmann law)

$$(100,000)(3.86\times10^{26}\text{ W}) = \left(5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\text{K}^4}\right)(4\pi R^2)(30{,}000\text{ K})$$

$$R = 8.2\times10^9\text{ m}$$

$$R_{\text{star}}/R_{\text{sun}} = \frac{8.2\times10^9\text{ m}}{6.96\times10^8\text{ m}} = 12$$

(c) The visual luminosity is proportional to the power radiated at visible wavelengths. Much of the power is radiated nonvisible wavelengths, which does not contribute to the visible luminosity.

38.47: Eq. (38.32): $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$ but $e^x = 1 + x + \dfrac{x^2}{2} + \ldots \approx 1 + x$ for

$x << 1 \Rightarrow I(\lambda) \approx \dfrac{2\pi hc^2}{\lambda^5(hc/\lambda kT)} = \dfrac{2\pi ckT}{\lambda^4}$ = Eq. (38.31), which is Rayleigh's distribution.

38.48: a) As in Example 38.10, using four-place values for the physical constants, $\frac{hc}{\lambda kT} = 95.80$, from which

$$\frac{I(\lambda)\Delta\lambda}{\sigma T^4} = 6.44\times 10^{-38}.$$

b) With $T = 2000$ K and the same values for λ and $\Delta\lambda$,

$$\frac{hc}{\lambda kT} = 14.37$$

and so

$$\frac{I(\lambda)\Delta\lambda}{\sigma T^4} = 7.54\times 10^{-6}.$$

c) With $T = 6000$ K, $\frac{hc}{\lambda kT} = 4.790$ and $\frac{I(\lambda)\Delta\lambda}{\sigma T^4} = 1.36\times 10^{-3}$.

d) For these temperatures, the intensity varies strongly with temperature, although for even higher temperatures the intensity in this wavelength interval would decrease. From the Wien displacement law, the temperature that has the peak of the corresponding distribution in this wavelength interval is 5800 K (see Example 38.10), close to that used in part (c).

38.49: a) To find the maximum in the Planck distribution:

$$\frac{dI}{d\lambda} = \frac{d}{d\lambda}\left(\frac{2\pi hc^2}{\lambda^5(e^{\alpha/\lambda}-1)}\right) = 0 = -5\frac{(2\pi hc^2)}{\lambda^5(e^{\alpha/\lambda}-1)} - \frac{2\pi hc^2(-\alpha/\lambda^2)}{\lambda^5(e^{\alpha/\lambda}-1)^2}$$

$$\Rightarrow -5(e^{\alpha/\lambda}-1)\lambda = \alpha$$

$$\Rightarrow -5e^{\alpha/\lambda} + 5 = \alpha/\lambda$$

$$\Rightarrow \text{Solve } 5 - x = 5e^x \text{ where } x = \frac{\alpha}{\lambda} = \frac{hc}{\lambda kT}.$$

Its root is 4.965, so $\frac{\alpha}{\lambda} = 4.965 \Rightarrow \lambda = \frac{hc}{(4.965)kT}$.

b) $\lambda_m T = \frac{hc}{(4.965)k} = \frac{(6.63\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^8\text{ m/s})}{(4.965)(1.38\times 10^{-23}\text{ J/K})} = 2.90\times 10^{-3}\text{ m}\cdot\text{K}.$

38.50: Combining Equations (38.28) and (38.30),

$$\lambda_m = \frac{2.90\times 10^{-3}\text{ m}\cdot\text{K}}{(I/\sigma)^{1/4}}$$

$$= \frac{(2.90\times 10^{-3}\text{ m}\cdot\text{K})}{(6.94\times 10^6\text{ W/m}^2/5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)^{1/4}}$$

$$= 8.72\times 10^{-7}\text{ m} = 872\text{ nm}.$$

38.51: a) Energy to dissociate an AgBr molecule is just $E = \frac{E(\text{mole})}{1\text{ mole}} = \frac{1.00\times10^{5}\text{ J}}{6.02\times10^{23}} =$

$$\frac{1.66\times10^{-19}\text{ J}}{(1.60\times10^{-19}\text{ J/eV})} = 1.04\text{ eV}.$$

b) $\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.66\times10^{-19}\text{ J}} = 1.20\times10^{-6}\text{ m}.$

c) $f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{1.20\times10^{-6}\text{ m}} = 2.51\times10^{14}\text{ Hz}.$

d) $E = hf = (6.63\times10^{-34}\text{ J}\cdot\text{s})(1.00\times10^{8}\text{ Hz}) = \frac{6.63\times10^{-26}\text{J}}{1.60\times10^{-19}\text{ J/eV}} = 4.14\times10^{-7}\text{ eV}.$

e) Even though a 50-kW radio station emits huge numbers of photons, each individual photon has insufficient energy to dissociate the AgBr molecule. However, the individual photons in faint visible light do have enough energy.

38.52: a) Assume a non-relativistic velocity and conserve momentum $\Rightarrow mv = \frac{h}{\lambda} \Rightarrow$

$$v = \frac{h}{m\lambda}.$$

b) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^2 = \frac{h^2}{2m\lambda^2}.$

c) $\frac{K}{E} = \frac{h^2}{2m\lambda^2}\cdot\frac{\lambda}{hc} = \frac{h}{2mc\lambda}.$ Recoil becomes an important concern for small m and small λ since this ratio becomes large in those limits.

d)

$$E = 10.2\text{ eV} \Rightarrow \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(10.2\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 1.22\times10^{-7}\text{ m} = 122\text{ nm}.$$

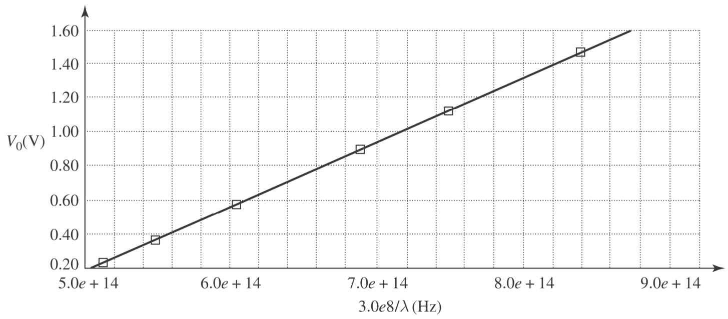
$$K = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.67\times10^{-27}\text{ kg})(1.22\times10^{-7}\text{ m})^2} = 8.84\times10^{-27}\text{ J} = 5.53\times10^{-8}\text{ eV}.$$

$$\frac{K}{E} = \frac{5.53\times10^{-8}\text{ eV}}{10.2\text{ eV}} = 5.42\times10^{-9}.$$ This is quite small so recoil can be neglected.

38.53: Given a source of spontaneous emission photons we can imagine we have a uniform source of photons over a long period of time (any one direction as likely as any other for emission). If a certain number of photons pass out though an area A, a distance D from the source, then at a distance 2D, those photons are spread out over an area $(2)^2 = 4$ times the original area A (because of how surface areas of spheres increase). Thus the number of photons per unit area DECREASES as the inverse square of the distance from the source.

38.54: a) $\lambda_0 = \frac{hc}{E}$, and the wavelengths are: cesium: 590 nm, copper: 264 nm, potassium: 539 nm, zinc: 288 nm. b) The wavelenghts of copper and zinc are in the ultraviolet, and visible light is not energetic enough to overcome the threshold energy of these metals.

38.55: a) Plot: Below is the graph of frequency versus stopping potential.



Threshold frequency is when the stopping potential is zero.

$$\Rightarrow f_{th} = \frac{1.89}{4.11\times10^{-15}} \text{ Hz} = 4.60\times10^{14} \text{ Hz}$$

b) Threshold wavelength is $\lambda_{\text{th}} = \frac{c}{f_{th}} = \frac{(3.00\times10^{8} \text{ m/s})}{4.60\times10^{14} \text{ Hz}} = 6.52\times10^{-7}$ m.

c) The work function is just $hf_{th} = (6.63\times10^{-34} \text{ J}\cdot\text{s})(4.60\times10^{14}\text{Hz}) = 3.05\times10^{-19}$ J $= 1.91\,\text{eV}$.

d) The slope of the graph is $m = \frac{V_0}{f} = \frac{h}{e} \Rightarrow h = me$

$$= (4.11\times10^{-15} \text{ V/Hz})(1.60\times10^{-19} \text{ C})$$
$$= 6.58\times10^{-34} \text{ J}\cdot\text{s}$$

38.56: a) See Problem 38.4: $\frac{dN}{dt} = \frac{(dE/dt)}{(dE/dN)} = \frac{P}{hf} = \frac{(200 \text{ W})(0.10)}{h(5.00\times10^{14} \text{ Hz})} =$ 6.03×10^{19} photons/sec.

b) Demand $\frac{(dN/dt)}{4\pi r^2} = 1.00\times10^{11}$ photons/sec/cm^2. So $r =$

$$\left(\frac{6.03\times10^{19} \text{ photons/sec}}{4\pi(1.00\times10^{11} \text{ photons/sec}\cdot\text{cm}^2)}\right)^{1/2} = 6930 \text{ cm} = 69.3 \text{ m}.$$

38.57: a) Recall $eV_0 = \dfrac{hc}{\lambda} - \phi$

$$\Rightarrow e(V_{02} - V_{01}) = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1} \Rightarrow \Delta V_0 = \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

b)

$$\Delta V_0 = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{(1.60 \times 10^{-11}\ \text{C})}\left(\frac{1}{2.65 \times 10^{-7}\ \text{m}} - \frac{1}{2.95 \times 10^{-7}\ \text{m}}\right) = 0.477\ \text{V}.$$

So the change in the stopping potential is an increase of 0.739 V.

38.58: From Eq. (38.13), the speed in the ground state is $v_1 = Z(2.19 \times 10^{6}\ \text{m/s})$. Setting $v_1 = \dfrac{c}{10}$ gives Z = 13.7, or 14 as an integer. b) The ionization energy is $E = Z^2$ (13.6 eV), and the rest mass energy of an electron is 0.511 MeV, and setting $E = \dfrac{mc^2}{100}$ gives Z = 19.4, or 19 as an integer.

38.59: $\lambda' = \lambda + \dfrac{h}{mc}(1 - \cos\phi)$

$\phi = 180°$ so $\lambda' = \lambda + \dfrac{2h}{mc} = 0.09485$ m

$p' = h/\lambda' = 6.99 \times 10^{-24}\ \text{kg}\cdot\text{m/s}$

b) $E = E' + E_e;\quad hc/\lambda = hc/\lambda' + E_e$

$$E_e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (hc)\frac{\lambda' - \lambda}{\lambda\lambda'} = 1.129 \times 10^{-16}\ \text{J} = 705\ \text{eV}$$

38.60: a) The change in wavelength of the scattered photon is given by Eq. 38.23

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \Rightarrow \lambda = \lambda' - \frac{h}{mc}(1 - \cos\phi) =$$

$$(0.0830 \times 10^{-9}\ \text{m}) - \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})}{(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})}(1 + 1) = 0.0781\,\text{nm}.$$

b) Since the collision is one-dimensional, the magnitude of the electron's momentum must be equal to the magnitude of the change in the photon's momentum. Thus,

$$p_e = h\left(\frac{1}{\lambda} - \frac{-1}{\lambda'}\right) = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})\left(\frac{1}{0.0781} + \frac{1}{0.0830}\right)(10^{9}\,\text{m}^{-1})$$

$$= 1.65 \times 10^{-23}\,\text{kg}\cdot\text{m/s} \approx 2 \times 10^{-23}\,\text{kg}\cdot\text{m/s}.$$

c) Since the electron is non relativistic $(\beta = 0.06)$,

$$K_e = \frac{p_e^2}{2m} = 1.49 \times 10^{-16}\ \text{J} \approx 10^{-16}\ \text{J}.$$

38.61: a) $m_r = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{207 m_e m_p}{207 m_e + m_p} = 1.69 \times 10^{-28}\ \text{kg}.$

b) The new energy levels are given by Eq. (38.18) with m_e replaced by m_r.

$$E_n = -\frac{1}{\varepsilon_0^2}\frac{m_r e^4}{8n^2h^2} = \left(\frac{m_r}{m_e}\right)\left(\frac{-13.60\ \text{eV}}{n^2}\right)$$

$$= \left(\frac{1.69\times 10^{-28}\ \text{kg}}{9.11\times 10^{-31}\ \text{kg}}\right)\left(\frac{-13.60\ \text{eV}}{n^2}\right)$$

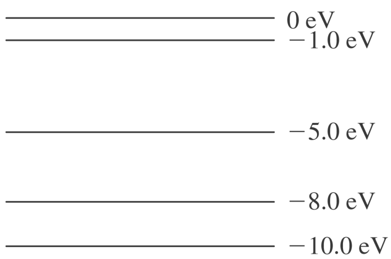
$$= \frac{-2.53\times 10^{3}\ \text{eV}}{n^2} \Rightarrow E_1 = -2.53\ \text{keV}.$$

c) $$\frac{1}{\lambda} = \frac{1}{hc}(E_1 - E_2) = \frac{(-2.53\times 10^3\text{e V}/4 - (-2.53\times 10^3\text{eV}))\,(1.60\times 10^{-19}\ \text{J/eV}))}{(6.63\times 10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times 10^8\ \text{m/s})}$$

$$= 1.53\times 10^{9}\ \text{m}^{-1}$$

$$\Rightarrow \lambda = 6.55\times 10^{-10}\ \text{m}.$$

38.62: a) The levels are
$E_4 = -1.0\ \text{eV}, E_3 = -5.0\ \text{eV}, E_2 = -8.0\ \text{eV}, \text{and } E_1 = -10.0\ \text{eV}.$



b) We can go from 4-3(4 eV), 4-2(7 eV), and 4-1(9 eV) directly, but also 3-2(3 eV), 3-1(5 eV), and 2-1(2 eV) after starting from 4.

38.63: a) The maximum energy available to be deposited in the atom is

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times 10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times 10^8\ \text{m/s})}{8.55\times 10^{-8}\ \text{m}} = 2.33\times 10^{-18}\ \text{J} = 14.5\ \text{eV}$$

but the inoization energy of hydrogen is 13.6 eV, so the maximum kinetic energy is 14.5 eV − 13.6 eV = 0.900 eV.

b) If some of the electrons were in the $n = 2$ state $\left(E_2 = \dfrac{-13.6\ \text{eV}}{4} = 3.4\ \text{eV}\right)$, then we would expect a maximum kinetic energy of $14.5\ \text{eV} - 3.4\ \text{eV} = 11.1\ \text{eV}$, which is exactly 10.2 eV above that measured in part (a), explaining the anomoly.

38.64: a) In terms of the satellite's mass M, orbital radius R and orbital period T,

$$n = \frac{2\pi}{h} L = \frac{2\pi}{h} MR^2 \left(\frac{2\pi}{T}\right) = \frac{4\pi^2 MR^2}{hT}.$$

Using the given numerical values, $n = 1.08\times 10^{46}$. b) The angular momentum of the satellite in terms of its orbital speed V, mass, and radius is $L = MVR$, so $V^2 = (L/MR)^2$, and its centripetal acceleration is $\frac{V^2}{R} = \frac{L^2}{M^2R^3}$.

Newton's law of gravitation can then be expressed as

$$\frac{GM_{\text{earth}}}{R^2} = \frac{ML^2}{M^2R^3}, \text{ or } R = \frac{L^2}{GM_{\text{earth}}M}.$$

If $L = nh/2\pi$,

$$R = n^2\left(\frac{h^2}{4\pi^2 GM_{\text{earth}}M}\right) = kn^2.$$

c) $\Delta R = 2kn\Delta n$, and for the next orbit, $\Delta n = 1$, and $\Delta R = n(h^2/4\pi^2\, GM_{\text{earth}}M)$. Insertion of numerical values from Appendix F and using n from part (a) gives $\Delta R = 1.5\times 10^{-39}$ m, which is (d) not observable. (e) The quantum and classical orbit do correspond, either would be correct, but only the classical calculation is useful.

38.65: a) Quantization of angular momentum implies $L = mvr = n\frac{h}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$.

But

$$F = +Dr = \frac{mv^2}{r} \Rightarrow r^2 = \frac{mv^2}{D} = \frac{n^2h^2}{4\pi^2mr^2D} \Rightarrow r_n = \left(\frac{n^2h^2}{4\pi^2 mD}\right)^{1/4}.$$

b) The energy $E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}Dr^2$, since $F = -Dr$ is completely analogous to

$$F = -kx \Rightarrow U = \frac{1}{2}kx^2. \text{ So } E = \frac{1}{2}m\cdot\left(\frac{D}{m}r^2\right) + \frac{1}{2}Dr^2 = Dr^2$$

$$\Rightarrow En = D\cdot\frac{nh}{2\pi\sqrt{mD}} = \sqrt{\frac{D}{m}}\cdot\frac{nh}{2\pi}.$$

c) Photon energies $E_\gamma = E_i - E_f = (n_i - n_f)\sqrt{\frac{D}{m}}\frac{h}{2\pi} \Rightarrow E_\gamma = n\sqrt{\frac{D}{m}}\frac{h}{2\pi}$ where $n =$ integers > 0.

d) This could describe a charged mass attached to a spring, being spun in a circle.

38.66: a) $m_{\mathrm{r,d}} = m_{\mathrm{e}} \dfrac{m_{\mathrm{d}}}{m_{\mathrm{e}} + m_{\mathrm{d}}} = m_{\mathrm{e}}(1 + (m_{\mathrm{e}}/m_{\mathrm{d}}))^{-1}$, and insertion of the numerical values gives $m_{\mathrm{r,d}} = 0.999728\, m_{\mathrm{e}}$. b) Let $\lambda_0 = \dfrac{(4/3)}{R}, \lambda' = \dfrac{(4/3)}{R'}$, where R' is the Rydberg constant evaluated with $m = m_{\mathrm{r,p}} = 0.999456$, so

$$R' = \frac{m_{\mathrm{r,p}}}{m_{\mathrm{e}}} R, \text{ and } \lambda'' = \frac{(4/3)}{R''}, R'' = \frac{m_{\mathrm{r,d}}}{m_{\mathrm{e}}} R \cdot$$

Then

$$\Delta\lambda = \frac{4}{3}\frac{1}{R}\left(\frac{R}{R'} - \frac{R}{R''}\right) = \lambda_0 \left(\frac{m_{\mathrm{e}}}{m_{\mathrm{r,p}}} - \frac{m_{\mathrm{e}}}{m_{\mathrm{r,d}}}\right) = 0.033\,\mathrm{nm}.$$

38.67: a) The H_α line is emitted by an electron in the $n = 3$ energy level, $E_3 = \dfrac{-13.60\ \mathrm{eV}}{(3)^2} = -1.51\,\mathrm{eV}$. The ground state energy is $E_1 = -13.60\ \mathrm{V}$, so one must add at least $13.60\,\mathrm{eV} - 1.51\ \mathrm{eV} = 12.09\ \mathrm{eV}$ if the H_α line is to be emitted.

b) The possible emitted photons are $3 \to 2, 3 \to 1$, and $2 \to 1$, with the wavelengths given by $\dfrac{1}{\lambda} = \dfrac{1}{hc} = (E_i - E_f) = \dfrac{+13.60\ \mathrm{eV}}{hc}\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$. This yields the wavelengths of 658 nm, 103 nm, and 122 nm for the respective photons above.

38.68: $$\frac{n_2}{n_1} = e^{-(E_{\text{ex}} - E_{\text{g}})/\text{kT}} \Rightarrow T = \frac{-(E_{\text{ex}} - E_{\text{g}})}{\text{k}\ln\left(\frac{n_2}{n_1}\right)}.$$

$$E_{\text{ex}} = E_2 = \frac{-13.6\ \text{eV}}{4} = -3.4\ \text{eV}.$$

$$E_{\text{g}} = -13.6\ \text{eV}.$$

$$E_{\text{ex}} - E_{\text{g}} = 10.2\ \text{eV} = 1.63 \times 10^{-18}\ \text{J}.$$

a) $\frac{n_2}{n_1} = 10^{-12}.$

$$T = \frac{-(1.63 \times 10^{-18}\ \text{J})}{(1.38 \times 10^{-23}\ \text{J/K})\ln(10^{-12})} = 4275\ \text{K}.$$

b) $\frac{n_2}{n_1} = 10^{-8}.$

$$T = \frac{-(1.63 \times 10^{-18}\ \text{J})}{(1.38 \times 10^{-23}\ \text{J/K})\ln(10^{-8})} = 6412\ \text{K}.$$

c) $\frac{n_2}{n_1} = 10^{-4}.$

$$T = \frac{-(1.63 \times 10^{-18}\ \text{J})}{(1.38 \times 10^{-23}\ \text{J/K})\ln(10^{-4})} = 12824\ \text{K}.$$

d) For absorption to take place in the Balmer series, hydrogen must *start* in the $n = 2$ state. From part (a), colder stars have fewer atoms in this state leading to weaker absorption lines.

38.69: The transition energy equals the sum of the recoiling atom's kinetic energy and the photon's energy.

$$E_{tr} = E_k + E_{\gamma'}$$

$$\Rightarrow E_{\gamma'} = \frac{hc}{\lambda'} = E_{tr} - E_k \Rightarrow \lambda' = \frac{hc}{E_{tr} - E_k}.$$

If the recoil is neglected $\lambda = \dfrac{hc}{E_{tr}}$

$$\Rightarrow \Delta\lambda = hc\left(\frac{1}{E_{tr} - E_k} - \frac{1}{E_{tr}}\right) = \frac{hc}{E_{tr}}\left(\frac{1}{1 - E_k/E_{tr}} - 1\right)$$

$$\approx \frac{hc}{E_{tr}}\left(\left(1 + \frac{E_k}{E_{tr}} - \cdots\right) - 1\right)$$

$$\Rightarrow \Delta\lambda = hc\left(\frac{E_k}{E_{tr}^2}\right) = hc\frac{E_k}{(hc/\lambda)^2} = \left(\frac{E_k}{hc}\right)\lambda^2$$

Conservation of momentum, assuming atom initially at rest, yields:

$$P_\gamma = \frac{h}{\lambda} = P_k \Rightarrow E_k = \frac{P_k^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$\Rightarrow \Delta\lambda = \left(\frac{h^2}{2mhc\lambda^2}\right)\lambda^2 = \frac{h}{2mc}$$

b) For the hydrogen atom: $\Delta\lambda = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2(1.67\times10^{-27}\ \text{kg})(3.00\times10^{8}\ \text{m/s})} = 6.6\times10^{-16}\ \text{m}$ and it doesn't depend on n.

38.70: a) $\phi = 180°$ so $(1-\cos\phi) = 2 \Rightarrow \Delta\lambda = \dfrac{2h}{mc} = 0.0049$ nm, so $\lambda' = 0.1849$ nm.

b) $\Delta E = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda'}\right) = 2.93\times10^{-17}\ \text{J} = 183\,\text{eV}.$

This will be the kinetic energy of the electron. c) The kinetic energy is far less than the rest mass energy, so a non-relativistic calculation is adequate;

$$v = \sqrt{2K/m} = 8.02\times10^{6}\ \text{m/s}.$$

38.71: a) Largest wavelength shift:

$$\Delta\lambda = \frac{h}{mc}(1-(-1)) = \frac{2(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})} = 4.85\times10^{-12}\text{ m}.$$

b) We want $\Delta\lambda = \lambda' - \lambda = 2\lambda - \lambda = \lambda = \frac{h}{mc}(1-\cos\phi).$

Smallest energy implies largest λ, so

$$\cos\phi = -1 \Rightarrow \lambda = \frac{2h}{mc} \Rightarrow E = \frac{hc}{\lambda} = \frac{mc^2}{2} = \frac{5.11\times10^{5}\text{ eV}}{2} = 2.56\times10^{5}\text{ eV}.$$

38.72: a) Power delivered = pIV.

b) $dE = mcdT \Rightarrow$ Power delivered $= \frac{dE}{dt} = mc\frac{dT}{dt}.$

So $\frac{dT}{dt} = \frac{1}{mc}\frac{dE}{dt} = \frac{pIV}{mc}.$

c) (a) Power$=pIV = (0.010)(0.0600\text{ A})(18.0\times10^{3}\text{ V}) = 10.8\text{ W};$

(b) $\frac{dT}{dt} = \frac{(0.01)(60.0\times10^{-3}\text{ A})(18.0\times10^{3}\text{ V})}{(0.250\text{ kg})(130\text{ J/kg/K})} = 0.332\text{ K/s}.$

d) A high melting point and heat capacity—tungsten and copper, for example.

38.73: a) The photon's energy change $\Delta E = h(f_2 - f_1) = hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) =$

$$(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})\left(\frac{1}{1.132\times10^{-10}\text{ m}} - \frac{1}{1.100\times10^{-10}\text{ m}}\right) = 5.111\times10^{-17}\text{ J} = -3$$

which is a loss for the photon, but which is a gain for the electron. So, the kinetic energy of the electron is $5.111\times10^{-17}\text{ J} = 319.5\text{ eV}.$

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(5.11\times10^{-17})\text{ J}}{9.11\times10^{-31}\text{ kg}}} = 1.06\times10^{7}\text{ m/s}.$$

b) If all the energy of the electron is lost in the emission of a photon, then

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{5.111\times10^{-17}\text{ J}}$$

$$= 3.892\times10^{-9}\text{ m} = 3.89\text{ nm}.$$

38.74: a) $\Delta\lambda_1 = (h/mc)(1-\cos\theta_1)$, $\Delta\lambda_2 = (h/mc)(1-\cos\theta_2)$, and so the overall wavelength shift is $\Delta\lambda = (h/mc)(2-\cos\theta_1 - \cos\theta_2)$.

(b) For a single scattering through angle θ, $\Delta\lambda_s = (h/mc)(1-\cos\theta)$.

For two successive scatterings through an angle of $\theta/2$, for each scattering,

$\Delta\lambda_t = 2(h/mc)(1-\cos\theta/2)$.

$1-\cos\theta = 2(1-\cos^2(\theta/2))$ and $\Delta\lambda_s = (h/mc)2(1-\cos^2(\theta/2))$

$\cos(\theta/2) \le 1$ so $1-\cos^2(\theta/2) \ge (1-\cos(\theta/2))$ and $\Delta\lambda_s \ge \Delta\lambda_t$

Equality holds only when $\theta = 180°$.

c) $(h/mc)2(1-\cos 30.0°) = 0.268(h/mc)$. d) $(h/mc)(1-\cos 60°) = 0.500(h/mc)$, which is indeed greater than the shift found in part (c).

38.75: a) The wavelength of the gamma rays is

$$\lambda_i = \frac{hc}{E_i} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.00\times10^{6}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 1.24\times10^{-12}\ \text{m}$$

and the wavelength of the final visible photons is 5.00×10^{-7} m. So, the increase in wavelength per interaction (assuming about 10^{26} interactions) is

$$\frac{5.00\times10^{-7}\ \text{m} - 1.24\times10^{-12}\ \text{m}}{10^{26}} = 5.00\times10^{-33}\ \text{m}.$$

b) $\Delta\lambda = \dfrac{h}{mc}(1-\cos\phi) \approx \dfrac{h}{mc}\left(1-1+\dfrac{\phi^2}{2}\right) = \dfrac{h}{2mc}\phi^2$ (for small ϕ).

$$\text{So}\ \phi \approx \sqrt{\frac{2mc\Delta\lambda}{h}} = \sqrt{\frac{2(9.11\times10^{-31}\ \text{Kg})(3.00\times10^{8}\ \text{m/s})(5.00\times10^{-33}\ \text{m})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}}$$

$$= 6.42\times10^{-11}\ \text{radians} = (3.68\times10^{-9})°.$$

c) 10^6 $years = 3.15\times10^{13}$ sec $\Rightarrow \dfrac{10^{26}\ \text{collisions}}{3.15\times10^{13}\ \text{sec}} = 3.17\times10^{12}$ collisions/sec $\Rightarrow$

3.15×10^{-13} sec/collision.

How far does light travel in this time? $(3.00\times10^{8}\ \text{m/s})\ (3.15\times10^{-13}\ \text{sec/collision}) =$ $9.45\times10^{-5}\,\text{m} = 0.0945\,\text{mm} \approx 0.1\text{mm}$.

38.76: a) The final energy of the photon is $E' = \dfrac{hc}{\lambda'}$, and $E = E' + K$, where K is the kinetic energy of the electron after the collision. Then,

$$\lambda = \frac{hc}{E' + K} = \frac{hc}{(hc/\lambda') + K} = \frac{hc}{(hc/\lambda') + (\gamma - 1)mc^2}$$

$$= \frac{\lambda'}{1 + \dfrac{\lambda' mc}{h}\left[\dfrac{1}{(1 - v^2/c^2)^{1/2}} - 1\right]}.$$

$(K = mc^2(\gamma - 1)$ since the relativistic expression must be used for three-figure accuracy).

b) $\phi = \arccos(1 - \Delta\lambda/(h/mc))$.

c) $\gamma - 1 = \dfrac{1}{\left(1 - \left(\frac{1.80}{3.00}\right)^2\right)^{1/2}} - 1 = 1.25 - 1 = 0.250,\ \dfrac{h}{mc} = 2.43 \times 10^{-12}\ \text{m}$

$$\Rightarrow \lambda = \frac{5.10 \times 10^{-3}\ \text{mm}}{1 + \dfrac{(5.10 \times 10^{-12}\ \text{m})(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})(0.250)}{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})}}$$

$$= 3.34 \times 10^{-3}\ \text{nm}.$$

$$\phi = \arccos\left(1 - \frac{(5.10 \times 10^{-12}\ \text{m} - 3.34 \times 10^{-12}\ \text{m})}{2.43 \times 10^{-12}\ \text{m}}\right) = 74.0°.$$

38.77: a) $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$ but $\lambda = \dfrac{c}{f}$

$$\Rightarrow I(f) = \frac{2\pi hc^2}{(c/f)^5 (e^{hf/kT} - 1)} = \frac{2\pi hf^5}{c^3 (e^{hf/kT} - 1)}$$

b) $\displaystyle\int_0^\infty I(\lambda)\, d\lambda = \int_\infty^0 I(f)\, df \left(\frac{-c}{f^2}\right)$

$$= \int_0^\infty \frac{2\pi hf^3\, df}{c^2 (e^{hf/kT} - 1)} = \frac{2\pi (kT)^4}{c^2 h^3} \int_0^\infty \frac{x^3}{e^x - 1}\, dx$$

$$= \frac{2\pi (kT)^4}{c^2 h^3} \frac{1}{240} (2\pi)^4 = \frac{(2\pi)^5 (kT)^4}{240 h^3 c^2} = \frac{2\pi^5 k^4 T^4}{15 c^2 h^3}$$

c) The expression $\dfrac{2\pi^5 k^4 T^4}{15 h^3 c^2} = \sigma$ as shown in Eq. (38.36). Plugging in the values for the constants we get $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$.

38.78: $I = \sigma T^4, P = IA,$ and $\Delta E = Pt$; combining,

$$t = \frac{\Delta E}{A\sigma T^4} = \frac{(100\ \text{J})}{(4.00\times 10^{-6}\ \text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(473\ \text{K})^4}$$

$$= 8.81\times 10^3\,\text{s} = 2.45\ \text{hrs}.$$

38.79: a) The period was found in Exercise 38.27b: $T = \dfrac{4\varepsilon_0^2 n^3 h^3}{me^4}$ and frequency is just

$$f = \frac{1}{T} = \frac{me^4}{4\varepsilon_0^2 n^3 h^3}.$$

b) Eq. (38.6) tells us that $f = \dfrac{1}{h}(E_2 - E_1)$. So $f = \dfrac{me^4}{8\varepsilon_0^2 h^3}\left(\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right)$ (from Eq. (38.18)).

If $n_2 = n$ and $n_1 = n+1$, then $\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2} = \dfrac{1}{n^2} - \dfrac{1}{(n+1)^2}$

$$= \frac{1}{n^2}\left(1 - \frac{1}{(1+1/n)^2}\right) \approx \frac{1}{n^2}\left(1 - \left(1 - \frac{2}{n} + \cdots\right)\right)$$

$$= \frac{2}{n^3}\ \text{for large}\ n \Rightarrow f \approx \frac{me^4}{4\varepsilon_0^2 n^3 h^3}.$$

38.80: Each photon has momentum $p = \dfrac{h}{\lambda}$, and if the rate at which the photons strike the surface is (dN/dt), the force on the surface is $(h/\lambda)(dN/dt)$, and the pressure is $(h/\lambda)(dN/dt)/A$. The intensity is

$$I = (dN/dt)(E)/A = (dN/dt)(hc/\lambda)/A,$$

and comparison of the two expressions gives the pressure as (I/c).

38.81: Momentum:

$$\vec{\boldsymbol{p}} + \vec{\boldsymbol{P}} = \vec{\boldsymbol{p}}' + \vec{\boldsymbol{P}}' \Rightarrow \ p - P = -p' - P'$$

$$\Rightarrow \ p' = P - (p + P')$$

energy: $pc + E = p'c + E'$

$$= p'c + \sqrt{(P'c)^2 + (mc^2)^2}$$

$$\Rightarrow (pc - p'c + E)^2 = (P'c)^2 + (mc^2)^2$$

$$= (Pc)^2 + ((p + p')c)^2 - 2P(p + p')c^2 + (mc^2)^2$$

$$(pc - p'c)^2 + E^2 = E^2 + (pc + p'c)^2 - 2(Pc^2)(p + p') + 2Ec(p - p') - 4pp'c^2 + 2Ec(p$$

$$+ 2(Pc^2)(p + p') = 0$$

$$\Rightarrow p'(Pc^2 - 2pc^2 - Ec) = p(-Ec - Pc^2)$$

$$\Rightarrow p' = p\frac{Ec + Pc^2}{2pc^2 + Ec - Pc^2} = p\frac{E + Pc}{2pc + (E - Pc)}$$

$$\Rightarrow \lambda' = \lambda\left(\frac{2hc/\lambda + (E - Pc)}{E + Pc}\right) = \lambda\left(\frac{E - Pc}{E + Pc}\right) + \frac{2hc}{E + Pc}$$

$$\Rightarrow \lambda' = \frac{(\lambda(E - Pc) + 2hc)}{E + Pc}$$

If

$$E >> mc^2, Pc = \sqrt{E^2 - (mc^2)^2} = E\sqrt{1 - \left(\frac{mc^2}{E}\right)^2}$$

$$\approx E\left(1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 + \cdots\right)$$

$$\Rightarrow E - Pc \approx \frac{1}{2}\frac{(mc^2)^2}{E}$$

$$\Rightarrow \lambda_1 \approx \frac{\lambda(mc^2)^2}{2E(2E)} + \frac{hc}{E} = \frac{hc}{E}\left(1 + \frac{m^2c^4\lambda}{4hcE}\right)$$

b) If $\lambda = 10.6 \times 10^{-6}$ m, $E = 1.00 \times 10^{10}$ eV $= 1.60 \times 10^{-9}$ J

$$\Rightarrow \lambda' \approx \frac{hc}{1.60 \times 10^{-9}\ \text{J}}\left(1 + \frac{(9.11 \times 10^{-31}\ \text{kg})^2 c^4 (10.6 \times 10^{-6}\ \text{m})}{4hc\,(1.6 \times 10^{-9}\ \text{J})}\right)$$

$$= (1.24 \times 10^{-16}\ \text{m})(1 + 56.0) = 7.08 \times 10^{-15}\ \text{m}.$$

c) These photons are gamma rays. We have taken infrared radiation and converted it into gamma rays! Perhaps useful in nuclear medicine, nuclear spectroscopy, or high energy physics: wherever controlled gamma ray sources might be useful.

39.1: a) $\lambda = \dfrac{h}{p} \Rightarrow p = \dfrac{h}{\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(2.80\times10^{-10}\ \text{m})} = 2.37\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

b) $K = \dfrac{p^2}{2m} = \dfrac{(2.37\times10^{-24}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 3.08\times10^{-18}\ \text{J} = 19.3\ \text{eV}.$

39.2: $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(6.64\times10^{-27}\ \text{kg})\,(4.20\times10^{6}\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}} = 7.02\times10^{-15}\ \text{m}.$$

39. 3: a) $\lambda_e = \dfrac{h}{m_e v} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})\,(4.70\times10^{6}\ \text{m/s})} = 1.55\times10^{-10}\ \text{m}.$

b) $\lambda_p = \dfrac{m_e}{m_p}\lambda_e = \left(\dfrac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) 1.55\times10^{-10}\ \text{m} = 8.46\times10^{-14}\ \text{m}.$

39.4: a) $E = \dfrac{hc}{\lambda} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})\,(3.00\times10^{8}\ \text{m/s})}{(0.20\times10^{-9}\ \text{m})} = 6.2\ \text{keV}.$

b) $K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m} = \dfrac{((6.626\times10^{-34}\ \text{J}\cdot\text{s})/(0.20\times10^{-9}\ \text{m}))^2}{2(9.11\times10^{-31}\ \text{kg})}$

$= 6.0\times10^{-18}\ \text{J} = 37\ \text{eV}.$

Note that the kinetic energy found this way is much smaller than the rest energy, so the nonrelativistic approximation is appropriate.

c) $K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m} = \dfrac{((6.626\times10^{-34}\ \text{J}\cdot\text{s})/(0.20\times10^{-9}\ \text{m}))^2}{2(6.64\times10^{-27}\ \text{kg})} = 8.3\times10^{-22}\ \text{J} =$
5.2 meV. Again, the nonrelativistic approximation is appropriate.

39.5: a) In the Bohr model $mv\,r_n = \dfrac{nh}{2\pi}$. The de Broglie wavelength is

$\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{2\pi\, r_n}{n}$ for $n = 1$: $r_1 = a_0 = 5.29\times10^{-11}\ \text{m} \Rightarrow \lambda_1 = 2\pi(5.29\times10^{-11}\ \text{m}) = 3.32\times10^{-10}\ \text{m}.$

This equals the orbit circumference.

b) $n = 4$: $r_4 = (4)^2 a_0 = 16a_0 \Rightarrow \lambda_4 = \dfrac{2\pi(16a_0)}{4} = 4\lambda,$

$$\Rightarrow \lambda_4 = 1.33\times10^{-9}\ \text{m}.$$

The de Broglie wavelength is a quarter of the circumference of the orbit, $2\pi r_4$.

39.6: a) For a nonrelativistic particle, $K = \dfrac{p^2}{2m}$, so

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Km}}.$$

b) $(6.63\times10^{-34}\ \text{J}\cdot\text{s})\Big/\sqrt{2(800\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})\,(9.11\times10^{-31}\ \text{Kg})} = 4.34\times10^{-11}\ \text{m}.$

39.7: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(0.005\ \text{kg})\ (340\ \text{m/s})} = 3.90 \times 10^{-34}\ \text{m}.$

We should not expect the bullet to exhibit wavelike properties.

39.8: Combining Equations 37.38 and 37.39 gives

$$p = mc\sqrt{\gamma^2 - 1}.$$

a) $\lambda = \dfrac{h}{p} = (h/mc)\big/\sqrt{\gamma^2 - 1} = 4.43 \times 10^{-12}\ \text{m}.$ (The incorrect nonrelativistic calculation gives 5.05×10^{-12} m.)

b) $(h/mc)\big/\sqrt{\gamma^2 - 1} = 7.07 \times 10^{-13}\ \text{m}.$

39.9: a) photon

$E = \dfrac{hc}{\lambda}$ so $\lambda = \dfrac{hc}{E} = \dfrac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})\ (2.998 \times 10^{8}\ \text{m/s})}{(20.0\ \text{eV})\ (1.602 \times 10^{-19}\ \text{J/eV})} = 62.0\ \text{nm}$

electron

$E = p^2/(2m)$ so $p = \sqrt{2mE} = \sqrt{2(9.109 \times 10^{-31}\ \text{kg})\ (20.0\ \text{eV})\ (1.602 \times 10^{-19}\ \text{J/eV})} =$ $2.416 \times 10^{-24}\ \text{kg}\cdot\text{m/s}$

$\lambda = h/p = 0.274\ \text{nm}$

b) photon $\quad E = hc/\lambda = 7.946 \times 10^{-19}\ \text{J} = 4.96\ \text{eV}$

electron $\quad \lambda = h/p$ so $p = h/\lambda = 2.650 \times 10^{-27}\ \text{kg}\cdot\text{m/s}$

$E = p^2/(2m) = 3.856 \times 10^{-24}\ \text{J} = 2.41 \times 10^{-5}\ \text{eV}$

c) You should use a probe of wavelength approximately 250 nm. An electron with $\lambda = 250$ nm has much less energy than a photon with $\lambda = 250$ nm, so is less likely to damage the molecule.

39.10: $\lambda = \dfrac{h}{mv} \to v = \dfrac{h}{m\lambda}$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^2 = \frac{h^2}{2m\lambda^2}$$

They will *not* have the same kinetic energy since they have different masses.

$$\frac{K_p}{K_e} = \frac{\left(\dfrac{h^2}{2m_p\lambda^2}\right)}{\left(\dfrac{h^2}{2m_e\lambda^2}\right)} = \frac{m_e}{m_p}$$

$$= \frac{9.11 \times 10^{-31}\ \text{kg}}{1.67 \times 10^{-27}\ \text{kg}} = 5.46 \times 10^{-4}$$

39.11: a) $\lambda = 0.10\ \text{nm}$

$p = mv = h/\lambda$ so $v = h/(m\lambda) = 7.3\times10^{6}\ \text{m/s}$

b) $E = \frac{1}{2}mv^2 = 150\ \text{eV}$

c) $E = hc/\lambda = 12\ \text{keV}$

d) The electron is a better probe because for the same λ it has less energy and is less damaging to the structure being probed.

39.12: (a) $\lambda = h/mv \rightarrow v = h/m\lambda$

Energy conservation: $e\Delta V = \frac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2e} = \frac{m(\frac{h}{m\lambda})^2}{2e} = \frac{h^2}{2em\lambda^2}$$

$$= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.60\times10^{-19}\ \text{C})\,(9.11\times10^{-31}\ \text{kg})\,(0.15\times10^{-9}\ \text{m})^2}$$

$$= 66.9\ \text{V}$$

(b) $$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\,(3.0\times10^{8}\ \text{m/s})}{0.15\times10^{-9}\ \text{m}} = 1.33\times10^{-15}\ \text{J}$$

$$e\Delta V = K = E_{\text{photon}}$$

$$\Delta V = \frac{E_{\text{photon}}}{e} = \frac{1.33\times10^{-15}\ \text{J}}{1.6\times10^{-19}\ \text{C}} = 8310\ \text{V}$$

39.13: For $m = 1$,

$$\lambda = d\sin\theta = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow E = \frac{h^2}{2md^2\sin^2\theta} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.675\times10^{-27}\ \text{kg})\,(9.10\times10^{-11}\ \text{m})^2\sin^2(28.6°)}$$

$$\Rightarrow E = 6.91\times10^{-20}\ \text{J} = 0.432\ \text{eV}.$$

39.14: Intensity maxima occur when $d\sin\theta = m\lambda.$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2ME}}\ \text{ so }\ d\sin\theta = \frac{mh}{\sqrt{2ME}}.$$

(Careful! Here, *m* is the order of the maxima, whereas *M* is the mass of the incoming particle.)

a) $$d = \frac{mh}{\sqrt{2ME}\sin\theta} = \frac{(2)\,(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})\,(188\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}\ \sin(60.6°)}$$

$$= 2.06\times10^{-10}\ \text{m} = 0.206\ \text{nm}.$$

b) $m = 1$ also gives a maximum.

$$\theta = \arcsin\left(\frac{(1)\,(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})\,(188\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}\ (2.06\times10^{-10}\ \text{m})}\right)$$

$= 25.8°$. This is the only other one.

If we let $m \geq 3,$ then there are no more maxima.

c) $E = \dfrac{m^2h^2}{2Md^2\sin^2\theta} = \dfrac{(1)^2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})\,(2.60\times10^{-10}\ \text{m})^2\ \sin^2(60.6°)}$

$= 7.49\times10^{-18}\ \text{J} = 46.8\ \text{eV}.$

Using this energy, if we let $m = 2$, then $\sin\theta > 1$. Thus, there is no $m = 2$ maximum in this case.

39.15: Surface scattering implies $d\ \sin\theta = m\lambda.$

If $m = 1$: $\quad\theta = \arcsin[\lambda/d]$

But $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(6.64\times10^{-27}\ \text{kg})\,(840\ \text{eV})\,(1.60\times10^{-19}\,\text{J/eV})}}$

$\Rightarrow \lambda = 4.96\times10^{-13}\ \text{m}.$

So $\theta = \arcsin\left[\dfrac{4.96\times10^{-13}\ \text{m}}{8.34\times10^{-11}\ \text{m}}\right] = 0.341°.$

39.16: The condition for a maximum is $d\ \sin\theta = m\lambda.\quad \lambda = \dfrac{h}{p} = \dfrac{h}{Mv}$, so $\theta = \arcsin\left(\dfrac{mh}{dMv}\right)$.

(Careful! Here, m is the order of the maximum, whereas M is the incoming particle mass.)

a) $m = 1 \Rightarrow \theta_1 = \arcsin\left(\dfrac{h}{dMv}\right)$

$= \arcsin\left(\dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.60\times10^{-6}\ \text{m})\,(9.11\times10^{-31}\ \text{kg})\,(1.26\times10^{4}\ \text{m/s})}\right) = 2.07°.$

$m = 2 \Rightarrow \theta_2 = \arcsin\left(\dfrac{(2)\,(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.60\times10^{-6}\ \text{m})\,(9.11\times10^{-31}\ \text{kg})\,(1.26\times10^{4}\ \text{m/s})}\right)$

$= 4.14°.$

b) For small angles (in radians!) $y \cong D\theta$, so

$y_1 \approx (50.0\ \text{cm})\,(2.07°)\left(\dfrac{\pi\ \text{radians}}{180°}\right) = 1.81\,\text{cm}.$

$y_2 \approx (50.0\ \text{cm})\,(4.14°)\left(\dfrac{\pi\ \text{radians}}{180°}\right) = 3.61\,\text{cm}$

$\Rightarrow y_2 - y_1 = 3.61\,\text{cm} - 1.81\,\text{cm} = 1.81\,\text{cm}.$

39.17: a) $\Delta p\Delta x = \dfrac{h}{2\pi} \Rightarrow m\Delta v_x\Delta x = \dfrac{h}{2\pi} \Rightarrow \Delta v_x = \dfrac{h}{2\pi m\Delta x} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(1200\ \text{kg})\,(1.00\times10^{-6}\ \text{m})}$

$= 8.79\times10^{-32}\ \text{m/s}.$

b) Knowing the position of a macroscopic object (like a car) to within 1.00 μm is, for all practical purposes, indistinguishable from knowing "exactly" where the object is. Even with this tiny position uncertainty of 1.00 μm, the velocity uncertainty is insanely small by our standards.

39.18: a) $\Delta p_y \Delta y = \dfrac{h}{2\pi}$ for minimum uncertainty $\Rightarrow m\Delta v_y \Delta y = \dfrac{h}{2\pi} \Rightarrow \Delta v_y = \dfrac{h}{2\pi m \Delta y}$.

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(1.67\times10^{-27}\ \text{kg})\ (2.0\times10^{-12}\ \text{m})} = 3.2\times10^{4}\ \text{m/s}$$

b) For minimum uncertainty,

$$\Delta z = \frac{h}{2\pi\Delta p_z} = \frac{h}{2\pi m\Delta v_z} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(9.11\times10^{-31}\ \text{kg})\ (0.250\ \text{m/s})} = 4.63\times10^{-4}\ \text{m}.$$

39.19: Heisenberg's Uncertainty Principles tells us that:

$\Delta x\Delta p_x \geq \dfrac{h}{2\pi}$. We can treat the standard deviation as a direct measure of uncertainty.

Here $\Delta x\Delta p_x = (1.2\times10^{-10}\ \text{m})\ (3.0\times10^{-25}\ \text{kg}\cdot\text{m/s}) = 3.6\times10^{-35}\ \text{J}\cdot\text{s}$ but $\dfrac{h}{2\pi} = 1.05\times10^{-34}\ \text{J}\cdot\text{s}$

Therefore $\Delta x\Delta p_x < \dfrac{h}{2\pi}$ so the claim is *not valid*.

39.20: a) $(\Delta x)\ (m\Delta v_x) \geq h/2\pi$, and setting $\Delta v_x = (0.010)v_x$ and the product of the uncertainties equal to $h/2\pi$ (for the minimum uncertainty) gives $v_x = h/(2\pi m(0.010)\Delta x) = 57.9\ \text{m/s}$.

b) Repeating with the proton mass gives 31.6 mm/s.

39.21: $\Delta p\Delta x = \dfrac{h}{2\pi} \Rightarrow \Delta p = \dfrac{h}{2\pi\Delta x} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi\left(\dfrac{0.215\times10^{-9}\ \text{m}}{2}\right)} = 9.82\times10^{-25}\ \text{kg}\cdot\text{m/s}.$

b) $K = \dfrac{p^2}{2m} = \dfrac{(9.82\times10^{-25}\ \text{m})^2}{2(9.75\times10^{-26}\ \text{kg})} = 4.95\times10^{-24}\ \text{J} = 3.09\times10^{-5}\ \text{eV}.$

c) $K_{\text{total}} = NK = \dfrac{1.00\ \text{kg}}{9.75\times10^{-26}\ \text{kg/Ni}}(4.95\times10^{-24}\ \text{J}) = 50.8\ \text{J}.$

d) $mgh = K_{\text{total}} \Rightarrow h = \dfrac{K_{\text{total}}}{mg} = \dfrac{50.8\ \text{J}}{(1.00\ \text{kg})\ (9.81\ \text{m/s}^2)} = 5.18\ \text{m}.$

e) One is claiming to know both an exact momentum for each atom (giving rise to an exact kinetic energy of the system) and an exact position of each atom (giving rise to an exact potential energy of the system), in violation of Heisenberg's uncertainty principle.

39.22: $\Delta E\Delta t = \dfrac{h}{2\pi} \quad \Delta E = \Delta mc^2 \quad \Delta m = 2.06\times10^{9}\ \text{eV}/c^2 = 3.30\times10^{-10}\ \text{J}/c^2.$

$$\Delta t = \frac{h}{2\pi\Delta mc^2} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(3.30\times10^{-10}\ \text{J})} = 3.20\times10^{-25}\ \text{s}.$$

39.23: $\Delta E \Delta t = \dfrac{h}{2\pi} \Rightarrow \Delta E = \dfrac{h}{2\pi \Delta t} = \dfrac{(6.63\times 10^{-34}\ \text{J}\cdot\text{s})}{2\pi(7.6\times 10^{-21}\ \text{s})} = 1.39\times 10^{-14}\ \text{J} = 8.69\times$ $10^{4}\ \text{eV} = 0.0869\ \text{MeV}.$

$$\frac{\Delta E}{E} = \frac{0.0869\ \text{MeV}/c^2}{3097\ \text{MeV}/c^2} = 2.81\times 10^{-5}.$$

39.24: $\Delta E > \dfrac{h}{2\pi \Delta t} = \dfrac{(6.63\times 10^{-34}\ \text{J}\cdot\text{s})}{2\pi(5.2\times 10^{-3}\ \text{s})} = 2.03\times 10^{-32}\ \text{J} = 1.27\times 10^{-13}\ \text{eV}.$

39.25: To find a particle's lifetime we need to know the uncertainty in its energy.

$$\Delta E = (\Delta m)c^2 = (0.145)\ (4.5)\ (1.67\times 10^{-27}\ \text{kg})\ (3.00\times 10^{8}\ \text{m/s})^2$$
$$= 9.81\times 10^{-11}\ \text{J}$$
$$\Rightarrow \Delta t \approx \frac{h}{2\pi \Delta E} = \frac{6.63\times 10^{-34}\ \text{J}\cdot\text{s}}{2\pi(9.81\times 10^{-11}\ \text{J})} = 1.08\times 10^{-24}\ \text{s}.$$

39.26: a) $eV = K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m}$, so $V = \dfrac{(h/\lambda)^2}{2me} = 419\ \text{V}.$

b) The voltage is reduced by the ratio of the particle masses,

$$(419\ \text{V})\frac{9.11\times 10^{-31}\ \text{kg}}{1.67\times 10^{-27}\ \text{kg}} = 0.229\ \text{V}.$$

39.27: a) We recall $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$. But the energy of an electron accelerated through a potential is just $E = e\Delta V$

$$\Rightarrow \lambda_e = \frac{h}{\sqrt{2m\Delta V}} = \frac{(6.63\times 10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times 10^{-31}\ \text{kg})\ (1.60\times 10^{-19}\ \text{C})\ (800\ \text{V})}}$$
$$\Rightarrow \lambda_e = 4.34\times 10^{-11}\ \text{m}.$$

b) For a proton, all that changes is the mass, so

$$\lambda_p = \sqrt{\frac{m_e}{m_p}}\lambda_e = \sqrt{\frac{9.11\times 10^{-31}\ \text{kg}}{1.67\times 10^{-27}\ \text{kg}}}.\left(4.34\times 10^{-11}\ \text{m}\right)$$
$$\Rightarrow \lambda_e = 1.01\times 10^{-12}\ \text{m}.$$

39.28: $\Psi^* = \psi^* \sin \omega t$, so

$$|\Psi|^2 = |\Psi^*\Psi| = \psi^*\psi \sin^2 \omega t = |\psi|^2 \sin^2 \omega t.$$

$|\Psi|^2$ is not time-independent, so Ψ is not the wavefunction for a stationary state.

39.29: a) $\psi(x) = A\sin kx$. The probability density is $|\psi|^2 = A^2 \sin^2 kx$, and this is greatest when $\sin^2 kx = 1 \Rightarrow kx = \dfrac{n\pi}{2}, n = 1, 3, 5\ldots$

$$\Rightarrow x = \frac{n\pi}{2k} = \frac{n\pi}{2(2\pi/\lambda)} = \frac{n\lambda}{4}, n = 1, 3, 5\ldots$$

b) The probability is zero when $|\psi|^2 = 0$, which requires

$$\sin^2 kx = 0 \Rightarrow kx = n\pi \Rightarrow x = \frac{n\pi}{k} = \frac{n\lambda}{2},\ n = 0, 1, 2 \ldots$$

39.30: a) The uncertainty in the particle position is proportional to the width of $\psi(x)$, and is inversely proportional to $\sqrt{\alpha}$. This can be seen by either plotting the function for different values of α, finding the expectation value $\langle x^2 \rangle = \int \psi^2 x^2 dx$ for the normalized wave function or by finding the full width at half-maximum. The particle's uncertainty in position decreases with increasing α. The dependence of the expectation value $\langle x^2 \rangle$ on α may be found by considering

$$\begin{aligned}\langle x^2 \rangle &= \frac{\int_{-\infty}^{\infty} x^2 e^{-2\alpha x^2} dx}{\int_{-\infty}^{\infty} e^{-2\alpha x^2} dx} \\ &= -\frac{1}{2}\frac{\partial}{\partial \alpha}\ln\left[\int_{-\infty}^{\infty} e^{-2\alpha x^2} dx\right] \\ &= -\frac{1}{2}\frac{\partial}{\partial \alpha}\ln\left[\frac{1}{\sqrt{2\alpha}}\int_{-\infty}^{\infty} e^{-u^2} du\right] = \frac{1}{4\alpha},\end{aligned}$$

where the substitution $u = \sqrt{\alpha x}$ has been made. (b) Since the uncertainty in position decreases, the uncertainty in momentum must increase.

39.31: $f(x, y) = \left(\dfrac{x - iy}{x + iy}\right)$ and $f^*(x, y) = \left(\dfrac{x + iy}{x - iy}\right)$

$$\Rightarrow |f|^2 = f\ f^* = \left(\frac{x - iy}{x + iy}\right) \cdot \left(\frac{x + iy}{x - iy}\right) = 1.$$

39.32: The same.

$$\begin{aligned}|\psi(x, y, z)|^2 &= \psi^*(x, y, z)\psi(x, y, z) \\ |\psi(x, y, z)e^{i\phi}|^2 &= (\psi^*(x, y, z)e^{-i\phi})(\psi(x, y, z)e^{+i\phi}) \\ &= \psi^*(x, y, z)\psi(x, y, z).\end{aligned}$$

The complex conjugate means convert all i's to $-i$'s and vice-versa. $e^{i\phi} \cdot e^{-i\phi} = 1.$

39.33: Following the hint:

If we Taylor expand $\sin(ax)$ about a point x_0, we get $f(x_0) + f'(x_0)(x - x_0) + \cdots =$ $\sin(ax_0) + a\cos(ax_0)(x - x_0) + \cdots$. If $x - x_0$ is small we can even ignore the first order term and $\sin(ax) \approx \sin(ax_0)$.

For us $x - x_0 \approx 0.01L$ which is small compared to L so $\psi(x, y, z) \cong \left(\dfrac{2}{L}\right)^{3/2} \sin\left(\dfrac{\pi x_0}{L}\right)$ $\times \sin\left(\dfrac{\pi y_0}{L}\right)\sin\left(\dfrac{\pi z_0}{L}\right)$.

a) $x_0 = y_0 = z_0 = \dfrac{L}{4}$

$$P = \int |\psi|^2 \, dV = \left(\frac{2}{L}\right)^3 \sin^6\left(\frac{\pi}{4}\right) V = \left(\frac{2}{L}\right)^3 \left(\frac{\sqrt{2}}{2}\right)^6 (0.01L)^3 = 1.00 \times 10^{-6}.$$

b) $x_0 = y_0 = z_0 = \dfrac{1}{2}$

$$P = \int |\psi|^2 \, dV = \left(\frac{2}{L}\right)^3 \sin^6\left(\frac{\pi}{2}\right) V = \left(\frac{2}{L}\right)^3 (0.01L)^3 = 8.00 \times 10^{-6}.$$

39.34: Eq. (39.18): $\dfrac{-\hbar^2}{2m} \dfrac{d^2\psi}{dx^2} + U\psi = E\psi$

Let $\psi = A\psi_1 + B\psi_2$

$$\Rightarrow \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} (A\psi_1 + B\psi_2) + U(A\psi_1 + B\psi_2) = E(A\psi_1 + B\psi_2)$$

$$\Rightarrow A\left(-\frac{\hbar^2}{2m} \frac{d^2\psi_1}{dx^2} + U\psi_1 - E\psi_1\right) + B\left(-\frac{\hbar^2}{2m} \frac{d^2\psi_2}{dx^2} + U\psi_2 - E\psi_2\right) = 0.$$

But each of ψ_1 and ψ_2 satisfy Schröedinger's equation separately so the equation still holds true, for any A or B.

39.35: $-\dfrac{\hbar^2}{2m} \dfrac{d^2\psi}{dx^2} + U\psi = BE_1\psi_1 + CE_2\psi_2.$

If ψ were a solution with energy E, then

$$BE_1\psi_1 + CE_2\psi_2 = BE\psi_1 + CE\psi_2 \text{ or}$$
$$B(E_1 - E)\psi_1 = C(E - E_2)\psi_2.$$

This would mean that ψ_1 is a constant multiple of ψ_2, and ψ_1 and ψ_2 would be wave functions with the same energy. However, $E_1 \neq E_2$, so this is not possible, and ψ cannot be a solution to Eq. (39.18).

39.36: a) $\lambda = \dfrac{h}{\sqrt{2mK}} = \dfrac{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})}{\sqrt{2(9.11 \times 10^{-31}\,\text{kg})(40\text{ eV})(1.60 \times 10^{-19}\text{ J/eV})}}$

$= 1.94 \times 10^{-10}\,\text{m}.$

b) $\dfrac{R}{v} = \dfrac{R}{\sqrt{2E/m}} = \dfrac{(2.5\text{m})(9.11 \times 10^{-31}\,\text{kg})^{1/2}}{\sqrt{2(40\text{eV})(1.6 \times 10^{-19}\text{ J/eV})}} = 6.67 \times 10^{-7}\,\text{s}.$

c) The width w is $w = 2R\dfrac{\lambda}{a}$' and $w = \Delta v_y t = \Delta p_y t/m,$ where t is the time found in part (b) and a is the slit width. Combining the expression for w, $\Delta p_y = \dfrac{2m\lambda R}{at} = 2.65 \times 10^{-28}\,\text{kg}\cdot\text{m/s}.$

d) $\Delta y = \dfrac{h}{2\pi\Delta p_y} = 0.40\ \mu\text{m},$ which is the same order of magnitude.

39.37: a) $E = hc/\lambda = 12\text{eV}$

b) Find E for an electron with $\lambda = 0.10\times10^{-6}\,\text{m}$.

$\lambda = h/p$ so $p = h/\lambda = 6.626\times10^{-27}\,\text{kg}\cdot\text{m/s}$

$E = p^2/(2m) = 1.5\times10^{-4}\,\text{eV}$

$E = q\Delta V$ so $\Delta V = 1.5\times10^{-4}\,\text{V}$

$v = p/m = (6.626\times10^{-27}\,\text{kg}\cdot\text{m/s})/(9.109\times10^{-31}\,\text{kg}) = 7.3\times10^3\,\text{m/s}$

c) Same λ so same p.

$E = p^2/(2m)$ but now $m = 1.673\times10^{-27}\,\text{kg}$ so $E = 8.2\times10^{-8}\,\text{eV}$and $\Delta V = 8.2\times10^{-8}\,\text{V}$.

$v = p/m = (6.626\times10^{-27}\,\text{kg}\cdot\text{m/s})/(1.673\times10^{-27}\,\text{kg}) = 4.0\,\text{m/s}$

39.38: (a) Single slit diffraction: $a\sin\theta = m\lambda$

$$\lambda = a\sin\theta = (150\times10^{-9}\,\text{m})\sin20° = 5.13\times10^{-8}\,\text{m}$$

$$\lambda = h/mv \rightarrow v = h/m\lambda$$

$$v = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{(9.11\times10^{-31}\text{kg})(5.13\times10^{-8}\,\text{m})} = 1.42\times10^4\,\text{m/s}$$

(b) $a\sin\theta_2 = 2\lambda$

$$\sin\theta_2 = \pm2\frac{\lambda}{a} = \pm2\left(\frac{5.13\times10^{-8}\,\text{m}}{150\times10^{-9}\,\text{m}}\right) = \pm0.684$$

$$\theta_2 = \pm43.2°$$

39.39: a) The first dark band is located by $\sin\theta = \lambda/a$

$$a = \frac{\lambda}{\sin\theta} = \frac{150\text{nm}}{\sin25.0°} = 355\text{nm}$$

b) Find λ for the electrons

$$E = \frac{hc}{\lambda_{\text{photon}}} = 1.324\times10^{-18}\,\text{J}$$

$E = p^2/(2m)$ so $p = \sqrt{2mE} = 1.553\times10^{-24}\,\text{kg}\cdot\text{m/s}$

$\lambda = h/p = 4.266\times10^{-10}\,\text{m}$

No electrons at locations of minima in the diffraction pattern. The angular position of these minima are given by:

$\sin\theta = m\lambda/a = m(4.266\times10^{-10}\,\text{m})/(355\times10^{-9}\,\text{m}) = m(0.00120),\ m = \pm1, \pm2, \pm3, ...$

$m = \pm1, \theta = 0.0689°;\ m = \pm2, \theta = 0.138°;\ m = \pm3, \theta = 0.207°; ...$

39.40: According to Eq. 35.4

$$\lambda = \frac{d\sin\theta}{m} = \frac{(40.0\times10^{-6}\,\text{m})\sin(0.0300)}{2} = 600\text{ nm}.$$

The velocity of an electron with this wavelength is given by Eq. 39.1

$$v = \frac{p}{m} = \frac{h}{m\lambda} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})}{(9.11\times10^{-31}\,\text{kg})(600\times10^{-9}\,\text{m})} = 1.21\times10^3\,\text{m/s}.$$

Since this velocity is much smaller than c we can calculate the energy of the electron classically

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\,\text{kg})(1.21\times10^3\,\text{m/s})^2 = 6.70\times10^{-25}\,\text{J} = 4.19\ \mu\text{eV}.$$

39.41: The de Broglie wavelength of the blood cell is

$$\lambda = \frac{h}{mv} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.00\times10^{-14}\ \text{kg})(4.00\times10^{-3}\ \text{m/s})} = 1.66\times10^{-17}\ \text{m}.$$

We need not be concerned about wave behavior.

39.42: a) $\lambda = \dfrac{h}{p} = \dfrac{h\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}{mv}$

$$\Rightarrow \lambda^2 m^2 v^2 = h^2\left(1-\frac{v^2}{c^2}\right) = h^2 - \frac{h^2 v^2}{c^2}$$

$$\Rightarrow \lambda^2 m^2 v^2 + h^2\frac{v^2}{c^2} = h^2$$

$$\Rightarrow v^2 = \frac{h^2}{\left(\lambda^2 m^2 + \dfrac{h^2}{c^2}\right)} = \frac{c^2}{\left(\dfrac{\lambda^2 m^2 c^2}{h^2}+1\right)}$$

$$\Rightarrow v = \frac{c}{\left(1+\left(\dfrac{mc\lambda}{h}\right)^2\right)^{1/2}}.$$

b) $v = \dfrac{c}{\left(1+\left(\dfrac{\lambda}{(h/mc)}\right)^2\right)^{1/2}} \approx c\left(1-\dfrac{1}{2}\left(\dfrac{mc\lambda}{h}\right)^2\right) = (1-\Delta)c.$

$$\Delta = \frac{m^2c^2\lambda^2}{2h^2}.$$

c) $\lambda = 1.00\times10^{-15}\,m << \dfrac{h}{mc}.$

So $\Delta = \dfrac{(9.11\times10^{-31}\ \text{kg})^2(3.00\times10^8\ \text{m/s})^2(1.00\times10^{-15}\ \text{m})^2}{2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2} = 8.50\times10^{-8}$

$\Rightarrow v = (1-\Delta)c = (1-8.50\times10^{-8})c.$

39.43: a) Recall $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{\sqrt{2mq\Delta V}}$. So for an electron:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{C})(125\ \text{V})}} \Rightarrow \lambda = 1.10\times10^{-10}\ \text{m}.$$

b) For an alpha particle:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(6.64\times10^{-27}\ \text{kg})2(1.60\times10^{-19}\ \text{C})(125\ \text{V})}} = 9.10\times10^{-13}\ \text{m}.$$

39.44: a) $E^2 = p^2c^2 + m^2c^4$ and $E = K + mc^2 \Rightarrow (K + mc^2)^2 = p^2c^2 + m^2c^4$

$$\Rightarrow p = \frac{[(K+mc^2)^2 - m^2c^4]^{1/2}}{c} = \frac{[K^2 + 2Kmc^2 + m^2c^4 - m^2c^4]^{1/2}}{c}$$

$$= \frac{[K(K+2mc^2)]^{1/2}}{c}$$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{hc}{[K(K+2mc^2)]^{1/2}}.$$

b) i) $K << mc^2$ $\lambda \approx \dfrac{hc}{(2Kmc^2)^{1/2}} = \dfrac{h}{(2Km)^{1/2}}.$

ii) $K >> mc^2$ $\lambda \approx \dfrac{hc}{(K^2)^{1/2}} = \dfrac{hc}{K}.$

c) $K = 7.00 \times 10^9\,\text{eV} = 1.12 \times 10^{-9}\,\text{J}.$

$m = 1.67 \times 10^{-27}\,\text{kg}.$

$$\lambda = \frac{hc}{[K(K+2mc^2)]^{1/2}}$$

$$= \frac{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^8\,\text{m/s})}{[(1.12 \times 10^{-9}\,\text{J})(1.12 \times 10^{-9}\,\text{J} + 2(1.67 \times 10^{-27}\,\text{kg})(3.00 \times 10^8\,\text{m/s})^2)]^{1/2}}$$

$= 1.57 \times 10^{-16}\,\text{m}.$

d) $K = 25.0 \times 10^6\,\text{eV} = 4.00 \times 10^{-12}\,\text{J}.$

$m = 9.11 \times 10^{-31}\,\text{kg}.$

$$\lambda = \frac{(6.63 \times 10^{-34}\,\text{J})(3.00 \times 10^8\,\text{m/s})}{[4.00 \times 10^{-12}\,\text{J}(4.00 \times 10^{-12}\,\text{J} + 2(9.11 \times 10^{-31}\,\text{kg})(3.00 \times 10^8\,\text{m/s})]^{1/2}}$$

$= 4.87 \times 10^{-14}\,\text{m}.$

39.45: a) Since $K > mc^2$ we must use the relativistic expression for energy.

$$E^2 = p^2c^2 + m^2c^4 \text{ but } E = K + mc^2 \Rightarrow (K + mc^2)^2 = p^2c^2 + m^2c^4$$

$$\Rightarrow p = \frac{[(K+mc^2)^2 - m^2c^4)]^{1/2}}{c} \Rightarrow \lambda = \frac{h}{p} = \frac{hc}{[(K+mc^2)^2 - m^2c^4]^{1/2}}.$$

If $K = 3mc^2$ then $\lambda = \dfrac{hc}{[(4mc^2)^2 - m^2c^4]^{1/2}} = \dfrac{h}{\sqrt{15}mc}.$

b) i) $m = 9.11 \times 10^{-31}\,\text{kg}$ $K = 3mc^2 = 3(9.11 \times 10^{-31}\,\text{kg})(3.00 \times 10^8\,\text{m/s})^2$

$= 2.46 \times 10^{-13}\,\text{J}$

$= 1.54\,\text{MeV}.$

$$\lambda = \frac{h}{\sqrt{15}mc} = \frac{(6.63 \times 10^{-13}\,\text{J}\cdot\text{s})}{\sqrt{15}(9.11 \times 10^{-31}\,\text{kg})(3.00 \times 10^8\,\text{m/s})}$$

$= 6.2 \times 10^{-13}\,\text{m}.$

ii) $m = 1.67\times10^{-27}\,\text{kg} \qquad K = 3mc^2 = 3(1.67\times10^{-27}\,\text{kg})(3.00\times10^{8}\,\text{m/s})^2$

$$= 4.51\times10^{-10}\,\text{J}$$
$$= 2.82\times10^{3}\,\text{MeV}.$$

$$\lambda = \frac{h}{\sqrt{15}mc} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})}{\sqrt{15}(1.67\times10^{-27}\,\text{kg})(3.00\times10^{8}\,\text{m/s})}$$
$$= 3.42\times10^{-16}\,\text{m}.$$

39.46: $\Delta p \sim \dfrac{h}{2\pi a_0} = \dfrac{(6.626\times10^{-34}\,\text{J}\cdot\text{s})}{2\pi(0.5292\times10^{-10}\,\text{m})} = 2.0\times10^{-24}\,\text{kg}\cdot\text{m/s}$, which is comparable to $mv_1 = 2.0\times10^{-24}\,\text{kg}\cdot\text{m/s}.$

39.47: $\Delta x = 0.40\lambda = 0.40\dfrac{h}{p}$. But $\Delta x\Delta p_x \geq \dfrac{h}{2\pi} \Rightarrow \Delta p_x(\text{min}) = \dfrac{h}{2\pi\Delta x} = \dfrac{p}{2\pi(0.4)} = 0.40p.$

39.48: a) $\dfrac{(6.626\times10^{-34}\,\text{J}\cdot\text{s})}{2\pi(5.0\times10^{-15}\,\text{m})} = 2.1\times10^{-20}\,\text{kg}\cdot\text{m/s}.$

b) $K = \sqrt{(pc)^2 + (mc^2)^2} - mc^2 = 1.3\times10^{-13}\,\text{J} = 0.82\,\text{MeV}.$

c) The result of part (b), about $1\,\text{MeV} = 1\times10^{6}\,\text{eV}$, is many orders of magnitude larger than the potential energy of an electron in a hydrogen atom.

39.49: a) $\Delta p(\text{min}) = \dfrac{h}{2\pi\Delta x} = \dfrac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{2\pi(5.0\times10^{-15}\,\text{m})} = 2.1\times10^{-20}\,\text{kg}\cdot\text{m/s}$

b) $E = \sqrt{(pc)^2 + (mc^2)^2}$

$$= \sqrt{[(2.1\times10^{-20}\,\text{kg}\cdot\text{m/s})(3.0\times10^{8}\,\text{m/s})]^2 + [(9.11\times10^{-31}\,\text{kg})(3.0\times10^{8}\,\text{m/s})^2]^2}$$
$$= 6.3\times10^{-12}\,\text{J} = 39.5\text{MeV}.$$
$$K = E - mc^2 = 38.8\,\text{MeV}$$

c) The coulomb potential energy is $U = \dfrac{q_1q_2}{4\pi\varepsilon_0 V} \Rightarrow U = -\dfrac{(1.60\times10^{-19}\,\text{C})^2}{4\pi\varepsilon_0(5.0\times10^{-15}\,\text{m})} =$

$-4.60\times10^{-14}\,\text{J} = -0.29\,\text{MeV}$

Hence there is not enough energy to “hold” the electron in the nucleus.

39.50: a) Take the direction of the electron beam to be the *x*-direction and the direction of motion perpendicular to the beam to be the *y*-direction. Then, the uncertainty Δr in the position of the point where the electrons strike the screen is

$$\Delta r = \Delta v_y t = \frac{\Delta p_y}{m}\frac{x}{v_x}$$
$$= \frac{h}{2\pi m\Delta y}\frac{x}{\sqrt{2K/m}}$$
$$= 9.56\times10^{-10}\ \text{m},$$

which is (b) far too small to affect the clarity of the picture.

39.51: $\Delta E = \dfrac{h}{2\pi\Delta t} = \dfrac{(6.63\times10^{-34}\,\mathrm{J\cdot s})}{2\pi(8.4\times10^{-17}\,\mathrm{s})} = 1.26\times10^{-18}\,\mathrm{J}$

But $\Delta E = (\Delta m)c^2 \Rightarrow \Delta m = \dfrac{\Delta E}{c^2} = \dfrac{1.26\times10^{-18}\,\mathrm{J}}{(3.0\times10^{8}\,\mathrm{m/s})^2} = 1.4\times10^{-35}\,\mathrm{kg}$

$\Rightarrow \dfrac{\Delta m}{m} = \dfrac{1.4\times10^{-35}\,\mathrm{kg}}{264(9.11\times10^{-31}\,\mathrm{kg})} = 5.8\times10^{-8}$

39.52: a) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{\sqrt{2m(\frac{3}{2}kT)}} = \dfrac{h}{\sqrt{3mkT}}.$

b) We would roughly expect the length scale of the problem to go like $V^{1/3}$ (e.g., for a cube $V = l^3$ so $l = v^{1/3}$ and for a sphere $V = \frac{4}{3}\pi R^3$, so $R = (\frac{3}{4\pi}V^{1/3} \propto V^{1/3})$. Let n be the number of molecules along one length (again, think of a cube) so that $n^3 = N$, the total number of particles in the volume. So $n = N^{1/3}$. Thus, the typical spacing between particles is $\dfrac{l}{n} = \left(\dfrac{V}{N}\right)^{1/3}$. The exact relationship will change depending on the geometry, but the scaling is correct up to a multiplicative constant.

c) $\lambda = \left(\dfrac{V}{N}\right)^{1/3} \Rightarrow \dfrac{h}{\sqrt{3mkT}} = \left(\dfrac{V}{N}\right)^{1/3} \Rightarrow V = \dfrac{Nh^3}{(3mkT)^{3/2}}.$

d) $$V_{\mathrm{wave}} = \frac{\left(6.02\times10^{23}\ \mathrm{particles/mol}\right)\left(1.00\ \mathrm{mol}\right)\left(6.63\times10^{-34}\ \mathrm{J\cdot s}\right)^3}{\left[(3)\left(2.66\times10^{-26}\ \mathrm{kg}\right)\left(1.38\times10^{-23}\ \mathrm{J/K}\right)\left(293\,\mathrm{K}\right)\right]^{3/2}} = 3.03\times10^{-8}\ \mathrm{m}^3$$

$= 30.3\,\mathrm{mm}^3.$

$$PV = NkT \Rightarrow V_{\mathrm{STP}} = \frac{NkT}{P}$$

$$= \frac{\left(6.02\times10^{23}\ \mathrm{particles/mol}\right)\left(1.00\ \mathrm{mol}\right)\left(1.38\times10^{-23}\ \mathrm{J/K}\right)\left(293\,\mathrm{K}\right)}{\left(1.01\times10^{5}\mathrm{Pa}\right)}$$

$\Rightarrow V_{\mathrm{STP}} = 0.0241\,\mathrm{m}^3 = 2.41\times10^{7}\ \mathrm{mm}^3.$

$\Rightarrow V_{\mathrm{STP}}$ is far larger than V_{wave} so the wave nature is not important.

e) $N = \dfrac{1.00\ \mathrm{kg}}{1.79\times10^{-25}\ \mathrm{kg/atom}} = 5.59\times10^{24}$ Ag atoms($=$ conduction e).

$$V_{\mathrm{wave}} = \frac{(5.59\times10^{24}\ \mathrm{electrons})(6.63\times10^{-34}\ \mathrm{J\cdot s})^3}{[(3)(9.11\times10^{-31}\ \mathrm{kg})(1.38\times10^{-23}\ \mathrm{J/K})(293\,\mathrm{K})]^{3/2}} = 1.40\ \mathrm{m}^3.$$

$$V_{\mathrm{Real}} = \frac{1.00\ \mathrm{kg}}{1.05\times10^{4}\ \mathrm{kg/m}^3} = 9.52\times10^{-5}\ \mathrm{m}^3.$$

The real volume is much smaller than the wave limit volume. So, the wave nature of the electrons must be accounted for.

39.53: a) $\lambda = \dfrac{h}{mv} \Rightarrow v = \dfrac{h}{m\lambda} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(60\ \text{kg})(1.0\ \text{m})} = 1.1\times10^{-35}\ \text{m/s}.$

b) $t = \dfrac{d}{v} = \dfrac{0.80\ \text{m}}{1.1\times10^{-35}\ \text{m/s}} = 7.2\times10^{34}\ \text{s} = 2.3\times10^{27}\ \text{years}.$

Therefore, we will not notice diffraction effects while passing through doorways.

39.54: a) $E = 2.58\ \text{eV} = 4.13\times10^{-19}\ \text{J}$, with a wavelength of $\lambda = \dfrac{hc}{E} = 4.82\times10^{-7}\ \text{m} =$ 482 nm.

b) $\Delta E = \dfrac{h}{2\pi\Delta t} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(1.64\times10^{-7}\ \text{s})} = 6.43\times10^{-28}\ \text{J} = 4.02\times10^{-9}\ \text{eV}.$

c) $\lambda E = hc$, so $(\Delta\lambda)E + \lambda\Delta E = 0$, and $|\Delta E/E| = |\Delta\lambda/\lambda|$, so $\Delta\lambda = \lambda|\Delta E/E| =$

$$(4.82\times10^{-7}\ \text{m})\left(\frac{6.43\times10^{-28}\ \text{J}}{4.13\times10^{-19}\ \text{J}}\right) = 7.50\times10^{-16}\ \text{m} = 7.50\times10^{-7}\ \text{nm}.$$

39.55: $\Delta E = \dfrac{h'}{2\pi\Delta t} = \dfrac{6.63\times10^{-22}\ \text{J}\cdot\text{s}}{2\pi(2.24\times10^{-3}\ \text{s})} = 4.71\times10^{-20}\ \text{J} = 0.294\ \text{eV}.$

Note that this uncertainty is *much* larger than the real uncertainty as compared to the 4.50 eV.

39.56: $\sin\theta' = \dfrac{\lambda'}{\lambda}\sin\theta$, and $\lambda' = (h/p') = (h/\sqrt{2mE'})$, and so

$$\theta' = \arcsin\left(\frac{h}{\lambda\sqrt{2mE'}}\sin\theta\right)$$

$$= \arcsin\left(\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\sin 35.8^\circ}{(3.00\times10^{-11}\ \text{m})\sqrt{2(9.11\times10^{-31}\ \text{kg})(4.50\times10^{+3})(1.60\times10^{-19}\ \text{J/eV})}}\right)$$

$= 20.9^\circ.$

39.57: a) The maxima occur when $2d\sin\theta = m\lambda$ as described in Section 38.7.

b) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-37}\ \text{kg})(71.0\ \text{eV})\left(1.60\times10^{-19}\ \frac{\text{J}}{\text{eV}}\right)}} = 1.46\times$ $10^{-10}\,\text{m} = 0.146\ \text{nm}.$ $\theta = \sin^{-1}\left(\dfrac{m\lambda}{2d}\right)$ (Note: This m is the order of the maximum, not the mass.)

$$\Rightarrow \sin^{-1}\left(\frac{(1)(1.46\times10^{-10}\ \text{m})}{2(9.10\times10^{-11}\ \text{m})}\right) = 53.3^\circ.$$

c) The work function of the metal acts like an attractive potential increasing the kinetic energy of incoming electrons by $e\phi$. An increase in kinetic energy is an increase in momentum that leads to a smaller wavelength. A smaller wavelength gives a smaller angle θ (see part (b)).

39.58: a) Using the given approximation, $E = \frac{1}{2}\left((h/x)^2/m + kx^2\right)$, $(dE/dx) = kx - (h^2/mx^3)$, and the minimum energy occurs when $kx = (h^2/mx^3)$, or $x^2 = \frac{h}{\sqrt{mk}}$. The minimum energy is then $h\sqrt{k/m}$.

b) They are the same.

39.59: a) $U = A|x|$ but $F = -\frac{dU}{dx}$. For $x > 0$, $|x| = x \Rightarrow F = -A$. For $x < 0$, $|x| = -x \Rightarrow F = A$. So $F(x) = -\frac{A|x|}{x}$ for $x \neq 0$.

b) From Problem 39.58, $E = K + U = \frac{p^2}{2m} + A|x|$, and $px \approx h \Rightarrow E = \frac{h^2}{2mx^2} + A|x|$.

For $x > 0$; $E = \frac{h^2}{2mx^2} + Ax$. The minimum energy occurs when $\frac{dE}{dx} = 0 \Rightarrow \frac{dE}{dx} = 0 = -\frac{h^2}{mx^3} + A \Rightarrow x' = \left(\frac{h^2}{mA}\right)^{1/3}$. So $E_{\min} = \frac{h^2}{2m(h^2/mA)^{2/3}} + A\left(\frac{h^2}{mA}\right)^{1/3} = \frac{3}{2}\left(\frac{h^2A^2}{m}\right)^{1/3}$

39.60: For this wave function, $\Psi^* = \psi_1^* e^{i\omega_1 t} + \psi_2^* e^{i\omega_2 t}$, so

$$\begin{aligned}\Psi^2 &= \Psi^*\Psi \\ &= (\psi_1^* e^{i\omega_1 t} + \psi_2^* e^{i\omega_2 t})(\psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t}) \\ &= \psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_1^*\psi_2 e^{i(\omega_1 - \omega_2)t} + \psi_2^*\psi_1 e^{i(\omega_2 - \omega_1)t}.\end{aligned}$$

The frequencies ω_1 and ω_2 are given as not being the same, so $|\Psi|^2$ is not time-independent, and Ψ is not the wave function for a stationary state.

39.61: The time-dependent equation, with the separated form for $\Psi(x, t)$ as given becomes

$$i\hbar\psi(-i\omega) = \left(-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi\right).$$

Since ψ is a solution of the time-independent solution with energy E, the term in parenthesis is $E\psi$, and so $\omega\hbar = E$, and $\omega - (E/\hbar)$.

39.62: a)

$$\omega = 2\pi\ f = \frac{2\pi\ E}{h} = \frac{E}{\hbar}.$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p = \frac{p}{\hbar}.$$

$$\hbar\omega = E = KE = \frac{p^2}{2m} = \frac{(\hbar k)^2}{2m}$$

$$\Rightarrow \omega = \frac{\hbar k^2}{2m}.$$

b) From Problem 39.61 the time-dependent Schrödinger's equation is $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\psi(x,t)}{\partial x^2} +$ $U(x)\psi(x,t) = i\hbar\dfrac{\partial\psi(x,t)}{\partial t}$. $U(x) = 0$ for a free particle, so

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = -\frac{2mi}{\hbar}\frac{\partial\psi(x,t)}{\partial t}.$$

Try $\psi(x,t) = \cos(kx - \omega t)$

$$\frac{\partial\psi}{\partial t}(x,t) = A\omega\sin(kx - \omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = -Ak\sin(kx - \omega t) \text{ and } \frac{\partial^2\psi}{\partial x^2} = Ak^2\cos(kx - \omega t).$$

Putting this into the Schrödinger's equation, $Ak^2\cos(kx - \omega t) = -\left(\dfrac{2mi}{\hbar}\right)A\omega\sin(kx - \omega t)$.

This is not generally true for all x and t so is not a solution.

c) Try $\psi(x,t) = A\sin(kx - \omega t)$

$$\frac{\partial\psi(x,t)}{\partial t} = -A\omega\cos(kx - \omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = Ak\cos(kx - \omega t)$$

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = -Ak^2\sin(kx - \omega t).$$

Again, $-Ak^2\sin(kx - \omega t) = -\left(\dfrac{2mi}{\hbar}\right)A\omega\cos(kx - \omega t)$ is not generally true for *all* x and t so is not a good solution.

d) Try $\psi(x,t) = A\cos(kx - \omega t) + B\sin(kx - \omega t)$

$$\frac{\partial\psi(x,t)}{\partial t} = +A\omega\sin(kx - \omega t) - B\omega\cos(kx - \omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = -Ak\sin(kx - \omega t) + Bk\cos(kx - \omega t)$$

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = -Ak^2\cos((kx - \omega t) - Bk^2\sin(kx - \omega t).$$

Putting this into the Schrödinger's equation,

$$-Ak^2\cos(kx - \omega t) - Bk^2\sin(kx - \omega t) = -\frac{2mi}{\hbar}(+A\omega\sin(kx - \omega t) - B\omega\cos(kx - \omega t)).$$

Recall that $\omega = \dfrac{\hbar k^2}{2m}$. Collect sin and cos terms. $(A + iB)k^2\cos(kx - \omega t) + (iA - B)k^2\sin(kx - \omega t) = 0$. This is only true if $B = iA$.

39.63: a) The ball is in a cube of volume $125\,\text{cm}^3$ to start with, and hence has an uncertainty of 5 cm in any direction. $\Delta x = 0.05\text{ m}$. (The x-direction in the horizontal, side-to-side direction.)

Now $\Delta p_x = \dfrac{h}{2\pi\Delta x} = \dfrac{0.0663\text{ J.}}{2\pi(0.05\text{m})} = 0.21\text{ kg}\cdot\text{m/s}.$

b) The time of flight is $t = \dfrac{12\text{m}}{6.0\text{ m/s}} = 2.0\text{ s}.$

So the uncertainty in the x-direction at the catcher is

$$\Delta x = (\Delta v)t = \left(\frac{\Delta p}{m}\right)t = \left(\frac{0.21\text{ kg}\cdot\text{m/s}}{0.25\text{ kg}}\right)(2.0\text{ s}) \Rightarrow \Delta x = 1.7\text{ m}.$$

39.64: a) $\left|\psi^2\right| = A^2 x^2 e^{-2(\alpha x^2+\beta y^2+\gamma z^2)}$. To save some algebra, let $u = x^2$, so that $|\psi|^2 =$ $ue^{-2\alpha u} f(y,z)$, and $\dfrac{\partial}{\partial u}|\psi|^2 = (1-2\alpha u)|\psi|^2$; the maximum occurs at $u_0 = \dfrac{1}{2\alpha}$,

$x_0 = \pm\dfrac{1}{\sqrt{2\alpha}}.$

b) ψ vanishes at $x = 0$, so the probability of finding the particle in the $x = 0$ plane is zero. The wave function vanishes for $x = \pm\infty$.

39.65: a) $\psi(x,y,z) = Ae^{-\alpha(x^2+y^2+z^2)}$ but the distance to the origin is just $r = \sqrt{x^2+y^2+z^2}$.

So, doing a change of variable.

$$\psi(r) = Ae^{-\alpha r^2}.$$

However, the probability is the integral of this function over all space. It is best to think of the volume integral in spherical coordinates where $dV = dx\,dy\,dz \Rightarrow r^2 \sin\theta dr d\theta d\omega$ Now

$p = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\,\theta d\theta \int |\psi(r)|^2 r^2 dr = 4\pi|A^2| \int_e^{-2\alpha r^2} r^2 dr.$ Thus we interpret the probability of finding a particle in a spherical shell between r and $r+dr$ as $4\pi|A|^2 r^2 e^{-2\alpha r^2} dr.$

b) $\dfrac{d}{dr}(4\pi|A|^2 r^2 e^{-2\alpha r^2}) = 0 \Rightarrow 4\pi|A|^2(2re^{-2\alpha r^2} - 4\alpha r^3 e^{-2\alpha r^2}) = 0$

$\Rightarrow r = \dfrac{1}{\sqrt{2\alpha}}$. This is not the same as the maximum value of just $|\psi(r)|^2$ (which has a maximum only at $r = 0$). The "extra" r^2 from looking at the function in spherical coordinates (that is, having the variable be "distance from the origin" rather than the cartesian coordinates) makes all the difference.

39.66: a) $B(k) = e^{-\alpha^2 k^2}$ $\quad B(0) = B_{\text{max}} = 1$

$B(k_h) = \dfrac{1}{2} = e^{-\alpha^2 k_h^2} \Rightarrow \ln(1/2) = -\alpha^2 k_h^2$

$\Rightarrow k_h = \dfrac{1}{\alpha}\sqrt{\ln(2)} = \omega_k.$

Using tables: (b) $\psi(x) = \int_0^{\infty} e^{-\alpha^2 k^2} \cos kx dk = \dfrac{\sqrt{\pi}}{2\alpha}(e^{-x^2/4\alpha^2})$.

$\psi(x)$ is a maximum when $x = 0$.

c) $\psi(x_h) = \dfrac{\sqrt{\pi}}{4\alpha}$ when $e^{-x_h^2/4\alpha^2} = \dfrac{1}{2} \Rightarrow \dfrac{-x_h^2}{4\alpha^2} = \ln(1/2)$

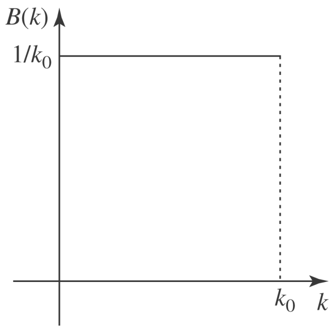
$\Rightarrow x_h = 2\alpha\sqrt{\ln 2} = \omega_x$

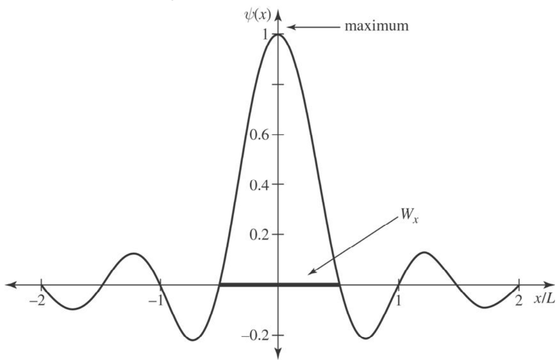
d) $\omega_p \omega_x = \left(\dfrac{h\omega_k}{2\pi}\right)\omega_x = \dfrac{h}{2\pi}\left(\dfrac{1}{\alpha}\sqrt{\ln 2}\right)\left(2\alpha\sqrt{\ln 2}\right) = \dfrac{h}{2\pi}(2\ln 2) = \dfrac{h\ln 2}{\pi}$.

39.67: a) $\psi(x) = \int_0^{\infty} B(k)\cos kx dk = \int_0^{k0} (\dfrac{1}{k_0}) \cos kx dk = \left.\dfrac{\sin kx}{k_0 x}\right|_0^{k_0} = \dfrac{\sin k_0 x}{k_0 x}$

b) $\psi(x)$ has a maximum value at the origin $x = 0$. $\psi(x_0) = 0$ when $k_0 x_0 = \pi$ so $x_0 = \dfrac{\pi}{k_0}$. Thus the width of this function $w_x = 2x_0 = \dfrac{2\pi}{k_0}$.

If $k_0 = \dfrac{2\pi}{L}, w_x = L.$





c) If $k_0 = \dfrac{\pi}{L} w_x = 2L.$

d) $w_p w_x = \left(\dfrac{hw_k}{2\pi}\right)\left(\dfrac{2\pi}{k_0}\right) = \dfrac{hw_k}{k_0} = \dfrac{hk_0}{k_0} = h.$

The uncertainty principle states that $w_p w_x \geq \dfrac{h}{2\pi}$. For us, no matter what k_0 is, $w_p w_x = h$, which is greater than $\dfrac{h}{2\pi}$.

39.68: a) For a standing wave, $n\lambda = 2L$, and

$$E_n = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{n^2h^2}{8mL^2}.$$

b) With $L = a_0 = 0.5292\times10^{-10}\text{ m}$, $E_1 = 2.15\times10^{-17}\text{ J} = 134\text{ eV}$.

39.69: Time of flight of the marble, from free-fall kinematic equation is just $t = \sqrt{\dfrac{2y}{g}} = \sqrt{\dfrac{2(25.0\text{ m})}{9.81\text{ m/s}^2}} = 2.26\text{ s}$

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\frac{\Delta p_x}{m}\right)t = \frac{ht}{2\pi \Delta x_i m} + \Delta x_i$$

to minimize Δx_f with respect to Δx_i

$$\frac{d(\Delta x_f)}{d(\Delta x_i)} = 0 = \frac{-ht}{2\pi m(\Delta x_i)^2} + 1 \Rightarrow \Delta x_i(\text{min}) = \sqrt{\left(\frac{ht}{2\pi m}\right)}$$

$$\Rightarrow \Delta x_f(\text{min}) = \sqrt{\frac{ht}{2\pi m}} + \sqrt{\frac{ht}{2\pi m}} = \sqrt{\frac{2ht}{\pi m}}$$

$$\Delta x_f(\text{min}) = \sqrt{\frac{2(6.63\times10^{-34}\text{ J}\cdot\text{s})(2.26\text{ s})}{\pi(0.0200\text{ kg})}} = 2.18\times10^{-16}\text{ m} = 2.18\times10^{-7}\text{ nm}.$$

40.1: a) $E_n = \dfrac{n^2h^2}{8mL^2} \Rightarrow E_1 = \dfrac{h^2}{8mL^2} = \dfrac{(6.63\times10^{-34}\text{J}\cdot\text{s})^2}{8(0.20\text{ kg})(1.5\text{ m})^2}$

$\Rightarrow E_1 = 1.22\times10^{-67}$ J.

b) $E = \dfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2(1.2\times10^{-67}\text{ J})}{0.20\text{ kg}}} = 1.1\times10^{-33}\text{m/s}$

$\Rightarrow t = \dfrac{d}{v} = \dfrac{1.5\text{ m}}{1.1\times10^{-33}\text{ m/s}} = 1.4\times10^{33}$ s.

c) $E_2 - E_1 = \dfrac{h^2}{8mL^2}(4-1) = \dfrac{3h^2}{8mL^2} = 3(1.22\times10^{-67}\text{ J}) = 3.7\times10^{-67}$ J.

d) No. The spacing between energy levels is so small that the energy appears continuous and the balls particle-like (as opposed to wave-like).

40.2: $L = \dfrac{h}{\sqrt{8mE_1}}$

$$= \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})}{\sqrt{8(1.673\times10^{-27}\text{ kg})(5.0\times10^{6}\text{eV})(1.602\times10^{-19}\text{ J/eV})}} = 6.4\times10^{-15}\text{ m.}$$

40.3: $E_2 - E_1 = \dfrac{h^2}{8mL^2}(4-1) = \dfrac{3h^2}{8mL^2} \Rightarrow L = h\sqrt{\dfrac{3}{8m(E_2-E_1)}}$

$$= (6.63\times10^{-34}\text{ J}\cdot\text{s})\sqrt{\frac{3}{8(9.11\times10^{-31}\text{ kg})(3.0\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}$$

$\Rightarrow L = 6.1\times10^{-10}\text{ m} = 0.61\,\text{nm}.$

40.4: a) The energy of the given photon is

$$E = hf = h\frac{c}{\lambda} = (6.63\times10^{-34}\text{ J}\cdot\text{s})\frac{(3.00\times10^{3}\text{ m/s})}{(122\times10^{-9}\text{ m})} = 1.63\times10^{-18}\text{ J.}$$

The energy levels of a particle in a box are given by Eq. 40.9

$$\Delta E = \frac{h^2}{8mL^2}(n^2 - m^2) \Rightarrow L = \sqrt{\frac{h^2(n^2-m^2)}{8m\Delta E}} = \sqrt{\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2(2^2-1^2)}{8(9.11\times10^{-31}\text{ kg})(1.63\times10^{-20}\text{J})}}$$

$= 3.33\times10^{-10}$ m.

b) The ground state energy for an electron in a box of the calculated dimensions is $E = \frac{h^2}{8mL^2} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{8(9.11\times10^{-31}\text{ kg})(3.33\times10^{-10}\text{m})^2} = 5.43\times10^{-19}\text{ J} = 3.40\text{ eV}$ (one-third of the original photon energy), which does not correspond to the -13.6 eV ground state energy of the hydrogen atom. Note that the energy levels for a particle in a box are proportional to n^2, whereas the energy levels for the hydrogen atom are proportional to $-\frac{1}{n^2}$.

40.5: $E = \frac{1}{2}mv^2 = \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(0.010\,\text{m/s})^2 = 2.5\times10^{-7}\ \text{J}$

b) $E_1 = \dfrac{h^2}{8mL^2}$ so $L = \dfrac{h}{\sqrt{8mE_1}}$

$E_1 = 2.5\times10^{-7}$ J gives $L = 6.6\times10^{-30}$ m

40.6: a) The wave function for $n = 1$ vanishes only at $x = 0$ and $x = L$ in the range $0 \le x \le L$. b) In the range for x, the sine term is a maximum only at the middle of the box, $x = L/2$. c) The answers to parts (a) and (b) are consistent with the figure.

40.7: The first excited state or $(n = 2)$ wave function is $\psi_2(x) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{2\pi x}{L}\right)$.

So $|\psi_2(x)|^2 = \dfrac{2}{L}\sin^2\left(\dfrac{2\pi x}{L}\right)$.

a) If the probability amplitude is zero, then $\sin^2\left(\dfrac{2\pi x}{L}\right) = 0$

$\Rightarrow \dfrac{2\pi x}{L} = m\pi \Rightarrow x = \dfrac{Lm}{2}, m = 0, 1, 2\ \ldots$

So probability is zero for $x = 0, \dfrac{L}{2}, L.$

b) The probability is largest if $\sin\left(\dfrac{2\pi x}{L}\right) = \pm1.$

$\Rightarrow \dfrac{2\pi x}{L} = (2m+1)\dfrac{\pi}{2} \Rightarrow x = (2m+1)\dfrac{L}{4}.$

So probability is largest for $x = \dfrac{L}{4}$ and $\dfrac{3L}{4}$.

c) These answers are consistent with the zeros and maxima of Fig. 40.5.

40.8: a) The third excited state is $n = 4$, so

$$\Delta E = (4^2 - 1)\frac{h^2}{8mL^2}$$
$$= \frac{15(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})(0.125\times10^{-9}\ \text{m})^2} = 5.78\times10^{-17}\ \text{J} = 361\,\text{eV}.$$

b) $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{5.78\times10^{-17}\ \text{J}}$

$\lambda = 3.44$ nm.

40.9: Recall $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$.

a) $E_1 = \dfrac{h^2}{8mL^2} \Rightarrow \lambda_1 = \dfrac{h}{\sqrt{2mh^2/8mL^2}} = 2L = 2(3.0\times10^{-10}\,\text{m}) = 6.0\times10^{-10}\,\text{m}$

$p_1 = \dfrac{h}{\lambda_1} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{6.0\times10^{-10}\ \text{m}} = 1.1\times10^{-24}\ \text{kg}\cdot\text{m/s}$

b) $E_2 = \dfrac{4h^2}{8mL^2} \Rightarrow \lambda_2 = L = 3.0\times10^{-10}\ \text{m}$

$p_2 = \dfrac{h}{\lambda_2} = 2p_1 = 2.2\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

c) $E_3 = \dfrac{9h^2}{8mL^2} \Rightarrow \lambda_3 = \dfrac{2}{3}L = 2.0\times10^{-10}\ \text{m}$

$p_3 = 3p_1 = 3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

40.10: $\dfrac{d^2\psi}{dx^2} = -k^2\psi,$ and for ψ to be a solution of Eq. (40.3), $k^2 = E\dfrac{8\pi^2 m}{h^2} = E\dfrac{2m}{\hbar^2}.$

b) The wave function must vanish at the rigid walls; the given function will vanish at $x = 0$ for any $k,$ but to vanish at $x = L,\ kL = n\pi$ for integer $n.$

40.11: a) Eq.(40.3): $-\dfrac{h^2}{8\pi^2 m}.\dfrac{d^2\psi}{dx^2} = E\psi.$

$$\frac{d^2}{dx^2}\psi = \frac{d^2}{dx^2}(A\cos kx) = \frac{d}{dx}(-Ak\sin kx) = -Ak^2\cos kx$$

$$\Rightarrow \frac{Ak^2h^2}{8\pi^2 m}\cos kx = EA\cos kx$$

$$\Rightarrow E = \frac{k^2h^2}{8\pi^2 m} \Rightarrow k = \sqrt{\frac{8\pi^2 mE}{h^2}} = \frac{\sqrt{2mE}}{\hbar}.$$

b) This is not an acceptable wave function for a box with rigid walls since we need $\psi(0) = \psi(L) = 0,$ but this $\psi(x)$ has maxima there. It doesn't satisfy the boundary condition.

40.12: $$-\frac{\hbar^2}{2m}\frac{d^2\psi'}{dx^2} + U\psi' = -\frac{\hbar^2}{2m}C\frac{d^2\psi}{dx^2} + UC\psi$$

$$= C\left[-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi\right]$$

$$= CE\psi = EC\psi = E\psi',$$

and so ψ' is a solution to Eq. (40.1) with the same energy.

40.13: a) Eq.(40.1): $\dfrac{-\hbar^2}{2m}\dfrac{d^2\psi}{dx^2}+U\psi=E\psi$

Left-hand side: $\dfrac{-\hbar^2}{2m}\dfrac{d^2}{dx^2}(A\sin kx)+U_0 A\sin kx$

$$=\frac{\hbar^2k^2}{2m}A\sin kx+U_0A\sin kx$$

$$=\left(\frac{\hbar^2k^2}{2m}+U_0\right)\psi.$$

But $\dfrac{\hbar^2k^2}{2m}+U_0>U_0>E$ for constant k. But $\dfrac{\hbar^2k^2}{2m}+U_0$ should equal $E\Rightarrow$ no solution.

b) If $E>U_0$, then $\dfrac{\hbar^2k^2}{2m}+U_0=E$ is consistent and so $\psi=A\sin kx$ is a solution of Eq.(40.1) for this case.

40.14: According to Eq.40.17, the wavelength of the electron inside of the square well is given by

$$k=\frac{\sqrt{2mE}}{\hbar}\Rightarrow\lambda_{\text{in}}=\frac{h}{\sqrt{2m(3U_0)}}.$$

By an analysis similar to that used to derive Eq.40.17, we can show that outside the box

$$\lambda_{\text{out}}=\frac{h}{\sqrt{2m(E-U_0)}}=\frac{h}{\sqrt{2m(2U_0)}}.$$

Thus, the ratio of the wavelengths is

$$\frac{\lambda_{\text{out}}}{\lambda_{\text{in}}}=\frac{\sqrt{2m(3U_0)}}{\sqrt{2m(2U_0)}}=\sqrt{\frac{3}{2}}.$$

40.15: $E_1=0.625E_\infty=0.625\dfrac{\pi^2\hbar^2}{2mL^2};\quad E_1=2.00\text{ eV}=3.20\times10^{-19}\text{ J}$

$$L=\pi\hbar\left(\frac{0.625}{2(9.109\times10^{-31}\text{ kg})(3.20\times10^{-19}\text{ J})}\right)^{1/2}=3.43\times10^{-10}\text{ m}$$

40.16: Since $U_0=6E_\infty$ we can use the result $E_1=0.625\,E_\infty$ from Section 40.3, so $U_0-E_1=5.375\,E_\infty$ and the maximum wavelength of the photon would be

$$\lambda=\frac{hc}{U_0-E_1}=\frac{hc}{(5.375)(h^2/8mL^2)}=\frac{8mL^2c}{(5.375)h}$$

$$=\frac{8(9.11\times10^{-31}\text{ kg})(1.50\times10^{-9}\text{m})^2(3.00\times10^8\text{m/s})}{(5.375)(6.63\times10^{-34}\text{ J}\cdot\text{s})}=1.38\times10^{-6}\text{ m}.$$

40.17: Since $U_0 = 6E_\infty$, we can use the results from Section 40.3, $E_1 = 0.625E_\infty, E_3 = 5.0$
and

$$E_\infty = \frac{\pi^2\hbar^2}{2mL^2} = \frac{\pi^2(1.054\times10^{-34}\ \text{J}\cdot\text{s.})^2}{2(1.67\times10^{-27}\ \text{kg})(4.0\times10^{-15}\ \text{m})^2}$$

$$\Rightarrow E_\infty = 2.05\times10^{-12}\ \text{J}.$$

The transition energy is $E_3 - E_1 = (5.09 - 0.625)(2.05\times10^{-12}\,\text{J}) = 9.15\times10^{-12}$ J.

The wavelength of the photon absorbed is then

$$\lambda = \frac{hc}{E_3 - E_1} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{9.15\times10^{-12}\ \text{J}} = 2.17\times10^{-14}\text{m}.$$

40.18: Since $U_0 = 6E_\infty$, we can use the results from Section 40.3.

$$E_\infty = \frac{h^2}{8mL^2}.$$

$E_2 = 2.43E_\infty$ and $E_1 = 0.625E_\infty$.

$$E_\gamma = E_2 - E_1 \Rightarrow \frac{hc}{\lambda} = (2.43 - 0.625)E_\infty = \frac{1.805h^2}{8mL^2}.$$

So $L = \sqrt{\dfrac{(1.805)h\lambda}{8mc}} = \sqrt{\dfrac{(1.805)(6.63\times10^{-34}\ \text{J}\cdot\text{s})(4.55\times10^{-7}\ \text{m})}{8(9.11\times10^{-31}\ \text{kg})(3.00\times10^8\ \text{m/s})}} = 4.99\times10^{-10}$ m

40.19: Eq.(40.16)$: \psi = A\sin\dfrac{\sqrt{2m\text{E}}}{\hbar}x + B\cos\dfrac{\sqrt{2mE}}{\hbar}x$

$$\frac{d^2\psi}{dx^2} = -A\left(\frac{2mE}{\hbar^2}\right)\sin\frac{\sqrt{2mE}}{\hbar}x - B\left(\frac{2mE}{\hbar^2}\right)\cos\frac{\sqrt{2mE}}{\hbar}x$$

$$= \frac{-2mE}{\hbar^2}(\psi) = \text{Eq.(40.15)}.$$

40.20: $\dfrac{d\psi}{dx} = \kappa(Ce^{\kappa x} - De^{-\kappa x}),$

$$\frac{d^2\psi}{dx^2} = \kappa^2(Ce^{\kappa x} + De^{-\kappa x}) = \kappa^2\psi$$

for all constants C and D. Hence ψ is a solution to Eq. (40.1) for

$$-\frac{\hbar^2}{2m}\kappa^2 + U_0 = E, \text{ or } \kappa = [2m(U_0 - E)]^{1/2}/\hbar,$$

and κ is real for $E < U_0$.

40.21: $T = 16\dfrac{E}{U_0}\left(1 - \dfrac{E}{U_0}\right)e^{\frac{-2\sqrt{2m(U_0-E)}}{\hbar}}L.$

$$\frac{E}{U_0} = \frac{6.0 \text{ eV}}{11.0 \text{ eV}} \text{ and } E - U_0 = 5 \text{ eV} = 8.0\times10^{-19} \text{ J.}$$

a) $L = 0.80\times10^{-9}$ m

$$T = 16\left(\frac{6.0 \text{ eV}}{11.0 \text{ eV}}\right)\left(1 - \frac{6.0 \text{ ev}}{11.0 \text{ eV}}\right)e^{\frac{-2\sqrt{2(9.11\times10^{-31}\text{ kg})(8.0\times10^{-19}\text{ J})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}}(0.80\times10^{-9}\text{ m})}$$

$= 4.4\times10^{-8}.$

b) $L = 0.40\times10^{-9}$ m

$T = 4.2\times10^{-4}.$

40.22: (Also see Problem 40.25). The transmission coefficient is

$T = 16\dfrac{E}{U_0}\left(1 - \dfrac{E}{U_0}\right)e^{\frac{-2\sqrt{2m(U_0-E)}L}{\hbar}}$ with $E = 5.0$ eV, $L = 0.60\times10^{-9}$ m, and

$\text{m} = 9.11\times10^{-31}$ kg

a) $U_0 = 7.0 \text{ eV} \Rightarrow T = 5.5\times10^{-4}.$

b) $U_0 = 9.0 \text{ eV} \Rightarrow T = 1.8\times10^{-5}$

c) $U_0 = 13.0 \text{ eV} \Rightarrow T = 1.1\times10^{-7}.$

40.23: $\lambda = h/p = h/\sqrt{2mK}$, so $\lambda\sqrt{K}$ is constant

$\lambda_1\sqrt{K_1} = \lambda_2\sqrt{K_2}$; λ_1 and K_1 are for $x > L$ where $K_1 = 2U_0$ and λ_2 and K_2 are for $0 < x < L$ where $K_2 = E - U_0 = U_0$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{U_0}{2U_0}} = \frac{1}{\sqrt{2}}$$

40.24: Using Eq. 40.21

$$G = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) = 16\frac{12.0 \text{ eV}}{15.0 \text{ eV}}\left(1 - \frac{12.0 \text{ eV}}{15.0 \text{ eV}}\right) = 2.56$$

$$\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar} = \frac{\sqrt{2(9.11\times10^{-31}\text{ kg})(15.0 - 12.0 \text{ eV})(1.60\times10^{-19}\text{J/eV})}}{(6.63\times10^{-34}\text{ J}\cdot\text{s})/2\pi} =$$

$8.9\times10^{9}\,\text{m}^{-1}$

$$T = Ge^{-2\kappa L} \Rightarrow L = \frac{1}{2\kappa}\ln(G/T) = \frac{1}{2(8.9\times10^{8}\text{ m}^{-1})}\ln\left(\frac{2.56}{0.025}\right) = 0.26 \text{ nm.}$$

40.25: a) Probability of tunneling is $T = Ge^{-2\kappa L}$

where $G = 16\dfrac{E}{U_0}\left(1-\dfrac{E}{U_0}\right) = 16\left(\dfrac{32}{41}\right)\left(1-\dfrac{32}{41}\right) = 2.74$

and $\kappa = \dfrac{\sqrt{2m(U_0-E)}}{\hbar} = \dfrac{\sqrt{2(9.11\times10^{-31}\text{ kg})(41\text{eV}-32\text{ eV})(1.60\times10^{-9}\text{ J/eV}).}}{1.054\times10^{-34}\text{ J}\cdot\text{s}}$

$= 1.54\times10^{10}\,\text{m}^{-1}.$

So $T_e = 2.74\, e^{-2(1.54\times10^{10}\text{ m}^{-1})(2.5\times10^{-10}\text{ m})} = 2.74e^{-7.7} = 1.2\times10^{-3}.$

b) For a proton, $\kappa' = \sqrt{\dfrac{m_p}{m_e}}\kappa = \sqrt{\dfrac{1.67\times10^{-27}}{9.11\times10^{-31}}}.\kappa$

$\Rightarrow \kappa' = 6.59\times10^{11}\text{ m}^{-1}$

$\Rightarrow T = 2.74e^{-2(6.59\times10^{11}\text{ m}^{-1})(2.5\times10^{-10}\text{ m})} = 2.74e^{-330}$

$\Rightarrow T \approx 10^{-143}$. The is *small* and for all practical purposes equal to zero.

40.26: $T = Ge^{-2\kappa L}$ with $G = 16\dfrac{E}{U_0}\left(1-\dfrac{E}{U_0}\right)$ and $\kappa = \dfrac{\sqrt{2m(U_0-E)}}{\hbar}$,

Giving $T = 16\dfrac{E}{U_0}\left(1-\dfrac{E}{U_0}\right)e^{\frac{-2\sqrt{2m(U_0-E)}}{\hbar}L}$.

a) If $U_0 = 30.0\times10^6$ eV, $L = 2.0\times10^{-15}$ m, $m = 6.64\times10^{-27}$ kg and $U_0 - E =$ 1.0×10^6 eV($E = 29.0\times10^6$ eV), $T = 0.090.$

b) If $U_0 - E = 10.0\times10^6$ eV $\quad (E = 20.0\times10^6$ eV) $T = 0.014.$

40.27: The ground state energy of a simple harmonic oscillator is, with $n = 0$,

$E_0 = \dfrac{1}{2}\hbar\omega = \dfrac{1}{2}\hbar\sqrt{\dfrac{k'}{m}} = \dfrac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{2}\sqrt{\dfrac{110\text{ N/m}}{0.250\text{ kg}}}$

$\Rightarrow E_0 = 1.11\times10^{-33}\text{J} = 6.91\times10^{-15}$ eV

Also $E_{n+1} - E_n = \hbar\omega = 2E_0 = 2.22\times10^{-33}$ J

$= 1.38\times10^{-14}$ eV.

Such tiny energies are unimportant for the motion of the block so quantum effects are not important.

40.28: Let $\sqrt{mk'}/2\hbar = \delta$, and so $\dfrac{d\psi}{dx} = -2x\delta\psi$ and $\dfrac{d^2\psi}{dx^2} = (4x^2\delta^2 - 2\delta)\psi$, and ψ is a solution of Eq.(40.21) if $E = \dfrac{\hbar^2}{m}\delta = \dfrac{1}{2}\hbar\sqrt{k'/m} = \dfrac{1}{2}\hbar\omega.$

40.29: The photon's energy is $E_\gamma = \dfrac{hc}{\lambda}$.

The transition energy is $\Delta E = E_1 - E_0 = \hbar\omega\left(\dfrac{3}{2} - \dfrac{1}{2}\right) = \hbar\omega = \hbar\sqrt{\dfrac{k'}{m}}$

$$\Rightarrow \frac{2\pi\hbar c}{\lambda} = \hbar\sqrt{\frac{k'}{m}} \Rightarrow k' = \frac{4\pi^2 c^2 m}{\lambda^2} = \frac{4\pi^2(3.00\times10^8\ \text{m/s})^2(9.4\times10^{-26}\ \text{kg})}{(5.25\times10^{-4}\ \text{m})^2}$$

$\Rightarrow k' = 1.2\ \text{N/m}$.

40.30: According to Eq.40.26, the energy released during the transition between two adjacent levels is twice the ground state energy

$$E_3 - E_2 = \hbar\omega = 2E_0 = 11.2\ \text{eV}.$$

For a photon of energy E

$$E = hf \Rightarrow \lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J.s})(3.00\times10^8\ \text{m/s})}{(11.2\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 111\ \text{nm}.$$

40.31: a) $E = (n + \frac{1}{2})\,\hbar\omega$

For $\Delta n = 1$, $\Delta E = \hbar\omega = \hbar\sqrt{k/m} = 1.544\times10^{-21}$ J

$\Delta E = hc/\lambda$ so $\lambda = (hc)/\Delta E = 129\ \mu$m

b) $E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}(1.544\times10^{-21}\ \text{J}) = 4.82$ meV

40.32: a) $\dfrac{|\psi(A)|^2}{|\psi(0)|^2} = \exp\left(-\dfrac{\sqrt{mk'}}{\hbar}A^2\right) = \exp\left(-\sqrt{mk'}\,\dfrac{\omega}{k'}\right) = e^{-1} = 0.368.$

This is more or less what is shown in Fig. (40.19).

b) $$\frac{|\psi(2A)^2|}{|\psi(0)|^2} = \exp\left(-\frac{\sqrt{mk'}}{\hbar}(2A)^2\right) = \exp\left(-\sqrt{mk'}\,4\frac{\omega}{k'}\right) = e^{-4} = 1.83\times10^{-2}.$$

This figure cannot be read this precisely, but the qualitative decrease in amplitude with distance is clear.

40.33: For an excited level of the harmonic oscillator

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega = \frac{1}{2}k'A^2$$

$$\Rightarrow A = \sqrt{\frac{(2n+1)\hbar\omega}{k'}}.$$ This is the uncertainty in the position.

Also $E_n = \left(n + \frac{1}{2}\right)\hbar\omega = \frac{1}{2}mv^2{}_{\max}$

$$\Rightarrow v_{\max} = \sqrt{\frac{(2n+1)\hbar\omega}{m}}$$

$$\Rightarrow \Delta x \Delta p = A \cdot (mv_{\max}) = \sqrt{\frac{(2n+1)\hbar\omega}{k'}} \cdot \sqrt{(2n+1)\hbar\omega m}$$

$$= (2n+1)\hbar\omega\sqrt{\frac{m}{k'}} = (2n+1)\hbar.$$

So $\Delta x \Delta p = (2n+1)\hbar,$ which agrees for the ground state $(n = 0)$ with $\Delta x \Delta p = \hbar.$ The uncertainty is seen to increase with n.

40.34: a) $$\frac{2}{L}\int_0^{L/4} \sin^2 \frac{\pi x}{L} dx = \frac{2}{L}\int_0^{L/4} \frac{1}{2}\left(1 - \cos\frac{2\pi x}{L}\right) dx$$

$$= \frac{1}{L}\left(x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)_0^{L/4}$$

$$= \frac{1}{4} - \frac{1}{2\pi},$$

about 0.0908.

b) Repeating with limits of $L/4$ and $L/2$ gives

$$\frac{1}{L}\left(x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)_{L/4}^{L/2} = \frac{1}{4} + \frac{1}{2\pi},$$

about 0.0409. c) The particle is much likely to be nearer the middle of the box than the edge. d) The results sum to exactly $1/2$, which means that the particle is as likely to be between $x = 0$ and $L/2$ as it is to be between $x = L/2$ and $x = L.$ e) These results are represented in Fig. (40.5b).

40.35: a) $P=\int_{L/4}^{3L/4} |\psi_1|^2\ dx=\int_{L/4}^{3L/4} \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx$

Let $z=\frac{\pi x}{L}\Rightarrow dz=dx\frac{\pi}{L}$

$$\Rightarrow P=\frac{2}{\pi}\int_{\pi/4}^{3\pi/4} \sin^2 z\ dz=\frac{1}{\pi}\left[z-\frac{1}{2}\sin 2z\right]_{\pi/4}^{3\pi/4}$$

$$=\frac{1}{2}+\frac{1}{\pi}=0.818.$$

b) $P=\int_{L/4}^{3L/4} |\psi_2|^2\ dx=\int_{L/4}^{3L/4} \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx$

$$\Rightarrow P=\frac{1}{\pi}\int_{\pi/2}^{3\pi/2} \sin^2 z\ dz=\frac{1}{2\pi}\left[z-\frac{1}{2}\sin 2z\right]_{\pi/2}^{3\pi/2}$$

$$=\frac{1}{2}$$

c) This is consistent with Fig. 40.5(b) since more of ψ_1 is between $x=\frac{L}{4}$ and $\frac{3L}{4}$ than ψ and the proportions appear correct.

40.36: Using the normalized wave function $\psi_1=\sqrt{2/L}\ \sin(\pi x/L)$, the probabilities $|\psi|^2\ dx$ are a) $(2/L)\sin^2(\pi/4)dx=dx/L$, b) $(2/L)\sin^2(\pi/2)\ dx=2\ dx/L$ and c) $(2/L)\sin^2(3\pi/4)=dx/L$.

40.37: $\psi_2(x)=\sqrt{\frac{2}{L}}\cdot\sin\left(\frac{2\pi x}{L}\right)$

$$\Rightarrow|\psi_2|^2\ dx=\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx$$

a) $x=\frac{L}{4}:|\psi_2|^2\ dx=\frac{2}{L}\sin^2\left(\frac{\pi}{2}\right)dx=\frac{2dx}{L}$

b) $x=\frac{L}{2}:|\psi_2|^2\ dx=\frac{2}{L}\sin^2(\pi)\ dx=0$

c) $x=\frac{3L}{4}:|\psi_2|^2\ dx=\frac{2}{L}\sin^2\left(\frac{3\pi}{2}\right)dx=\frac{2dx}{L}$

40.38: a) $R_n=\frac{(n+1)^2-n^2}{n^2}=\frac{2n+1}{n^2}=\frac{2}{n}+\frac{1}{n^2}.$

This is never larger than it is for $n=1$, and $R=3$. b) R approaches zero; in the classical limit, there is no quantization, and the spacing of successive levels is vanishingly small compared to the energy levels.

40.39: a) The transition energy $\Delta E = E_2 - E_1 = \dfrac{h^2}{8mL^2}(4-1)$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{8mcL^2}{3h} = \frac{8(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})(4.18\times10^{-9}\ \text{m})^2}{3(6.63\times10^{-34}\ \text{J}\cdot\text{s})}$$

$$\Rightarrow \lambda = 1.92\times10^{-5}\ \text{m}.$$

b) $\lambda = \dfrac{8(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})(4.18\times10^{-9}\ \text{m})^2}{(3^2-2^2)(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = 1.15\times10^{-5}\ \text{m}.$

40.40: a) $\psi = 0$, so $\dfrac{d\psi}{dx} = 0.$ b) $\dfrac{d\psi}{dx} = \dfrac{\pi}{L}\sqrt{\dfrac{2}{L}}\cos(\pi x/L)$, and as $x \to L$, $\dfrac{d\psi}{dx} \to -\sqrt{2\pi^2/L^3}.$ Clearly not. In Eq. (40.1), any point where $U(x)$ is singular necessitates a discontinuity in $\dfrac{d\psi}{dx}$.

40.41: a) $\psi_1 = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{\pi x}{L}\right)$

$$\Rightarrow \frac{d\psi_1}{dx} = \sqrt{\frac{2}{L}}\cdot\frac{\pi}{L}\cos\left(\frac{\pi x}{L}\right) = \sqrt{\frac{2\pi^2}{L^3}}\left(1-\frac{1}{2}\left(\frac{\pi x}{L}\right)^2+\cdots\right)$$

$$\Rightarrow \frac{d\psi_1}{dx} \approx \sqrt{\frac{2\pi^2}{L^3}}\text{for } x\approx 0$$

b) $\psi_2 = \sqrt{\dfrac{2}{L}}\sin\dfrac{2\pi x}{L} \Rightarrow \dfrac{d\psi_2}{dx} = \sqrt{\dfrac{2}{L}}\cdot\dfrac{2\pi}{L}\cdot\cos\dfrac{2\pi x}{L}$

$$\Rightarrow \frac{d\psi_2}{dx} \approx \sqrt{\frac{8\pi^2}{L^3}}\text{for } x\approx 0.$$

c) For $x \approx L$, $\cos\left(\dfrac{\pi x}{L}\right) \approx \cos\pi = -1$

$$\Rightarrow \frac{d\psi_1}{dx} \approx -\sqrt{\frac{2\pi^2}{L^3}}\text{for } x\approx L$$

d) $\dfrac{d\psi_2}{dx} = \sqrt{\dfrac{8\pi^2}{L^3}}\cos\left(\dfrac{2\pi x}{L}\right) \approx \sqrt{\dfrac{8\pi^2}{L^3}}$for $x \approx L$, since $\cos 2\pi = 1$

e) The slope of the wave function is greatest for $\psi_2 (n=2)$ close to the walls of the box, as shown in Fig. 40.5.

40.42: $\Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_{\text{final}} - \vec{\boldsymbol{p}}_{\text{initial.}}$

$$\left|\vec{p}\right| = \hbar k = \frac{\hbar n\pi}{L} = \frac{hn}{2L}.$$

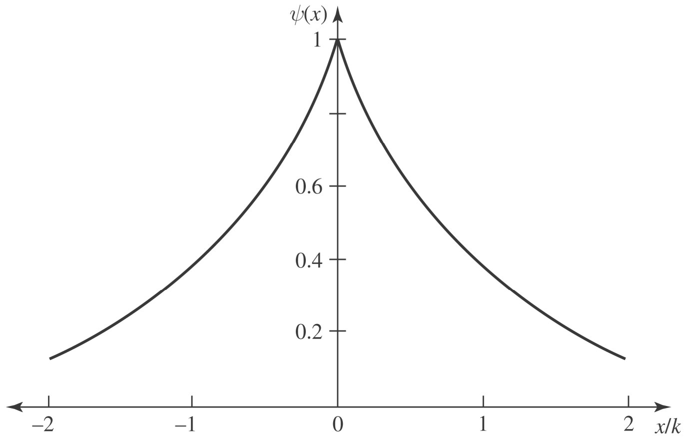
At $x = 0$ the initial momentum at the wall is $\vec{\boldsymbol{p}}_{\text{initial}} = -\frac{hn}{2L}\hat{\boldsymbol{i}}$ and the final momentum, after turning around, is $\vec{\boldsymbol{p}}_{\text{final}} = +\frac{hn}{2L}\hat{\boldsymbol{i}}$. So $\Delta\vec{\boldsymbol{p}} = +\frac{hn}{2L}\hat{\boldsymbol{i}} - \left(-\frac{hn}{2L}\hat{\boldsymbol{i}}\right) = +\frac{hn}{L}\hat{\boldsymbol{i}}$**.**

At $x = L$ the initial momentum is $\vec{\boldsymbol{p}}_{\text{initial}} = +\frac{hn}{2L}\hat{\boldsymbol{i}}$ and the final momentum, after turning around, is $\vec{\boldsymbol{p}}_{\text{final}} = -\frac{hn}{2L}\hat{\boldsymbol{i}}$. So $\Delta\vec{\boldsymbol{p}} = -\frac{hn}{2L}\hat{\boldsymbol{i}} - \frac{hn}{2L}\hat{\boldsymbol{i}} = -\frac{hn}{L}\hat{\boldsymbol{i}}$.

40.43: a) For a free particle, $U(x) = 0$ so Schrödinger's equation becomes $\dfrac{d^2\psi(x)}{dx^2} =$

$-\dfrac{2m}{h^2} E\psi(x).$

See graph below.



b) For $x < 0$ $\psi(x) = e^{+kx}$.

$$\frac{d\psi(x)}{dx} = ke^{+kx}.$$

$$\frac{d^2\psi(x)}{dx} = k^2 e^{+kx}.$$

So $k^2 = -\dfrac{2m}{\hbar^2} E \Rightarrow E = -\dfrac{\hbar^2 k^2}{2m}.$

c) For $x > 0$ $\psi(x) = e^{-kx}$.

$$\frac{d\psi(x)}{dx} = -ke^{-kx}.$$

$$\frac{d^2\psi(x)}{dx} = k^2 e^{-kx}$$

So again $k^2 = -\dfrac{2m}{\hbar^2} E \Rightarrow E = \dfrac{-\hbar^2 k^2}{2m}.$

Parts (c) and (d) show $\psi(x)$ satisfies Schrödinger's equation, provided $E = \dfrac{-\hbar^2 k^2}{2m}.$

d) Note $\dfrac{d\psi(x)}{dx}$ is discontinuous at $x = 0$. (That is, negative for $x > 0$ and positive for $x <$

40.44: b) $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x)$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2}=-\frac{2m}{\hbar^2}(E-U(x))\psi(x)$$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2}=-A\psi(x) \text{ where } A=-\frac{2m}{h^2}(E-U(x)).$$

Note that if $E>U(x)$ we are inside the well and $A>0$. From the above equation, this implies $\dfrac{d^2\psi(x)}{dx^2}$ has the opposite sign of $\psi(x)$.

If $E<U(x)$, we are outside the well and $A<0$. From the above equation, this implies $\dfrac{d^2\psi(x)}{dx^2}$ has the same sign as $\psi(x)$.

40.45: a) We set the solutions for inside and outside the well equal to each other at the well boundaries, $x=0$ and L.

$$x=0: A\sin(0)+B=C\Rightarrow B=C, \text{ since we must have } D=0 \text{ for } x<0.$$

$$x=L: A\sin\frac{\sqrt{2mE}L}{\hbar}+B\cos\frac{\sqrt{2mE}L}{\hbar}=+De^{-\kappa L} \text{ since } C=0 \ for \ x>L$$

$$\Rightarrow \frac{A\sin kL+B\cos kL=De^{-\kappa L}}{\text{where } k=\dfrac{\sqrt{2mE}}{\hbar}}.$$

b) Requiring continuous derivatives at the boundaries yields

$$x=0: \frac{d\psi}{dx}=kA\cos(k\cdot 0)-kB\sin(k\cdot 0)=kA=\kappa Ce^{k\cdot 0}\Rightarrow kA=\kappa C$$

$$x=L: kA\cos kL-kB\sin kL=-\kappa De^{-\kappa L}.$$

40.46: $T=Ge^{-2\kappa L}$ with $G=16\dfrac{E}{U_0}\left(1-\dfrac{E}{U_0}\right)$ and $\kappa=\dfrac{\sqrt{2m(U_0-E)}}{\hbar}\Rightarrow L=-\dfrac{1}{2\kappa}$ $\ln\left(\dfrac{T}{G}\right)$. If $E=5.5$ eV, $U_0=10.0$ eV, $m=9.11\times10^{-31}$ kg, and $T=0.0010$.

$$\text{Then } \kappa=\frac{\sqrt{2(9.11\times10^{-31}\text{ kg})(4.5\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}{(1.054\times10^{-34}\text{ J}\cdot\text{s})}=1.09\times10^{10}\text{ m}^{-1}$$

$$\text{and } G=16\frac{5.5\text{ eV}}{10.0\text{ eV}}\left(1-\frac{5.5\text{ eV}}{10.0\text{ eV}}\right)=3.96$$

$$\text{so } L=-\frac{1}{2(1.09\times10^{10}\text{ m}^{-1})}\ln\left(\frac{0.0010}{3.96}\right)=3.8\times10^{-10}\text{ m}=0.38\text{ nm}.$$

40.47: a) $T = \left[1 + \dfrac{(U_0 \sin \mathrm{h}\kappa L)^2}{4E(U_0 - E)}\right]^{-1}$

If $\kappa L >> 1$, then $\sinh \kappa L \approx \dfrac{1}{2} e^{\kappa L} \Rightarrow T \approx \left[1 + \dfrac{U_0^2 e^{2\kappa L}}{16E(U_0 - E)}\right]^{-1} = \left[\dfrac{U_0^2 e^{2\kappa L}}{16E(U_0 - E)}\right]^{-1} \Rightarrow$

$T \approx Ge^{-2\kappa L}$ where $G = 16 \dfrac{E}{U_0}\left(1 - \dfrac{E}{U_0}\right)$.

b) $\kappa L >> 1$ implies either κ is large or L is large (or both are large). If L is large, the barrier is wide. If κ is large, $U_0 - E$ is big, which implies E is small compared to U_0.

c) As $E \to U_0, \kappa \to 0 \Rightarrow \sinh \kappa L \to \kappa L$

$$\Rightarrow T \approx \left[1 + \frac{U_0^2 \kappa^2 L^2}{4E(U_0 - E)}\right]^{-1} \approx \left[\frac{1 + 2U_0^2 L^2 m}{4E\hbar^2}\right]^{-1}$$

since $\kappa^2 = \dfrac{2m(U_0 - E)}{\hbar^2}$.

But $U_0 \approx E \Rightarrow \dfrac{U_0^2}{E} = E \Rightarrow T \approx \left[1 + \left(\dfrac{2mE}{\hbar^2}\right)\left(\dfrac{L}{2}\right)^2\right]^{-1} = \left[1 + \left(\dfrac{kL}{2}\right)^2\right]^{-1}$ since

$k^2 = \dfrac{2mE}{\hbar^2}$.

40.48: For a wave function ψ with wave number k, $\dfrac{d^2\psi}{dx^2} = -k^2\psi$, and using this in Eq. (40.1) with $U = U_0 > E$ gives $k^2 = 2m(E - U_0)/\hbar^2$, or $k = i\sqrt{2m(U_0 - E)}/\hbar = i\kappa$.

40.49: The angular frequency $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.500 \text{ s}} = 12.6 \text{ rad/s}$. Ground state harmonic oscillator energy is given by

$$E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}(1.054 \times 10^{-34} \text{ J} \cdot \text{s})(12.6 \text{ rad/s}) = 6.62 \times 10^{-34} \text{ J}$$

$$= \frac{6.62 \times 10^{-34} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 4.14 \times 10^{-15} \text{ eV}$$

$\Delta E = E_{n+1} - E_n = \hbar\omega\left(\left(n + \dfrac{3}{2}\right) - \left(n + \dfrac{1}{2}\right)\right) = \hbar\omega \Rightarrow \Delta E = 2E_0 = 1.32 \times 10^{-33} \text{ J} =$

8.28×10^{-15} eV. These values are too small to be detected.

40.50: a) $E = \frac{1}{2}mv^2 = (n + (1/2))\hbar\omega = (n + (1/2))hf$, and solving for n,

$$n = \frac{\frac{1}{2}mv^2}{hf} - \frac{1}{2} = \frac{(1/2)(0.020\ \text{kg})(0.360\ \text{m/s})^2}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(1.50\ \text{Hz})} - \frac{1}{2} = 1.3\times10^{30}.$$

b) The difference between energies is
$\hbar\omega = hf = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(1.50\ \text{Hz}) = 9.95\times10^{-34}\ \text{J}$. This energy is too small to be detected with current technology

40.51: a) Eq. (40.21) : $\dfrac{-\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + \dfrac{1}{2}k'x^2\psi = E\psi$

Now, $\dfrac{d}{dx^2}(Cxe^{-m\omega x^2/2\hbar})$

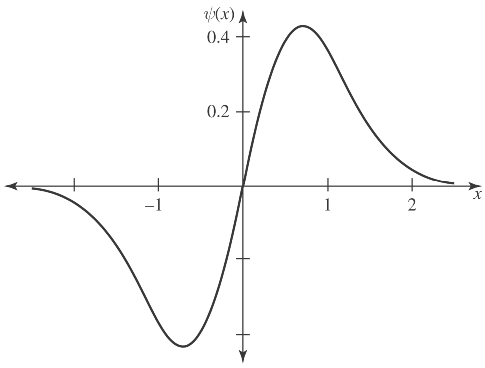
$$= \frac{d}{dx}\left[Ce^{-m\omega x^2/2\hbar} - \frac{Cm\omega 2x^2 e^{-m\omega x^2/2\hbar}}{2\hbar}\right]$$

$$= (Ce^{-m\omega x^2/2\hbar})\left[\frac{-2xm\omega}{2\hbar} - \frac{4m\omega x}{2\hbar} + \frac{4m^2\omega^2 x^3}{4\hbar^2}\right]$$

$$= (Cxe^{-m\omega x^2/2\hbar})\left[\frac{-3m\omega}{\hbar} + \left(\frac{m\omega}{\hbar}\right)^2 x^2\right]$$

$$\Rightarrow \text{Eq. (42-25)} : \psi\left[\frac{+3\omega\hbar}{2} - \frac{m\omega^2}{2}x^2 + \frac{1}{2}k'x^2\right] = \psi E \text{ but } \omega^2 = \frac{k'}{m} \Rightarrow E = \frac{3\hbar\omega}{2}.$$

b) Similar to the second graph in Fig. 40.18.



40.52: With $u(x) = x^2$, $a = m\omega/\hbar$, $|\psi|^2$ is of the form $C^*Cu(x)e^{-au(x)}$, which has a maximu at $au = 1$, or $\frac{m\omega}{\hbar}x^2 = 1$, $x = \pm\sqrt{\hbar}/m\omega$. b) ψ and hence $|\psi|^2$ vanish at $x = 0$, and as $x \to \pm\infty$ $|\psi|^2 \to 0$. the result of part (a) is $\pm A/\sqrt{3}$, found from $n = 1$ in Eq. (40.26), and this is cons with Fig. (40.19). The figure also shows a minimum at $x = 0$ and a rapidly decreasing $|\psi|^2$

40.53: a) Eq. (42-32): $\frac{-\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\psi = E\psi . \psi_{n_x}, \psi_{n_y}$ and ψ_{n_z} are all solutions to the 1-D Schrodinger equation, so $\frac{-\hbar}{2m}\frac{d^2\psi_{n_x}}{dx^2}+\frac{1}{2}k'x^2\psi_{n_x} = E_{n_x}\psi_{n_x}$, and similarly for ψ_{n_y} and ψ_{n_z}. Now if $\psi = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z)$ then $\frac{\partial^2\psi}{\partial x^2} = \left(\frac{d^2\psi_{n_x}}{dx}\right)\psi_{n_y}\psi_{n_z}, \frac{\partial^2\psi}{\partial y^2}$ $\left(\frac{d^2\psi_{n_y}}{dy^2}\right)\psi_{n_x}\psi_{n_z}$ and $\frac{\partial^2\psi}{dz^2} = \left(\frac{d^2\psi_{n_x}}{dz^2}\right)\psi_{n_x}\psi_{n_y}$.

Therefore:

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\frac{1}{2}k'(x^2+y^2+z^2)\psi$$

$$=\left[\frac{-\hbar^2}{2m}\left(\frac{d^2\psi_{n_x}}{dx^2}\right)+\frac{1}{2}k'x^2\psi_{n_x}\right]\psi_{n_y}\psi_{n_z}$$

$$+\left[\frac{-\hbar^2}{2m}\left(\frac{d^2\psi_{n_y}}{dy^2}\right)+\frac{1}{2}k'y^2\psi_{n_y}\right]\psi_{n_x}\psi_{n_z}$$

$$+\left[\frac{-\hbar^2}{2m}\left(\frac{d^2\psi_{n_z}}{dz^2}\right)+\frac{1}{2}k'z^2\psi_{n_z}\right]\psi_{n_x}\psi_{n_y}$$

$$=[E_{n_x}+E_{n_y}+E_{n_z}]\psi_{n_x}\psi_{n_y}\psi_{n_z}$$

$$=[E_{n_x}+E_{n_y}+E_{n_z}]\psi = E_{n_xn_yn_z}\psi$$

$$\Rightarrow E_{n_xn_yn_z} = \left[\left(n_x+\frac{1}{2}\right)+\left(n_y+\frac{1}{2}\right)+\left(n_z+\frac{1}{2}\right)\right]\hbar\omega$$

$$\Rightarrow E_{n_xn_yn_z} = \left[n_x+n_y+n_z+\frac{3}{2}\right]\hbar\omega.$$

b) Ground state energy $E_{000} = \frac{3}{2}\hbar\omega.$

First excited state energy $E_{100} = E_{001} = E_{010} = \frac{5}{2}\hbar\omega.$

c) As seen in b) there is just one set of quantum numbers (0, 0, 0) for the ground state and three possibilities (1, 0, 0), (0, 1, 0) and (0, 0, 1) for the first excited state.

40.54: Let $\omega_1 = \sqrt{k_1'/m}$, $\omega_2 = \sqrt{k_2'/m}$, $\psi_{n_x}(x)$ be a solution of Eq. (40.21) with $E_{n_x} = \left(n_x + \frac{1}{2}\right)\hbar\omega_1$, $\psi_{n_x}(y)$ be a similar solution, and $\psi_{n_z}(z)$ be a solution of Eq.(42 - 25) but with z as the independent variable instead of x, and energy $E_{n_z} = \left(n_z + \frac{1}{2}\right)\omega_{2.}$ (a) As in Problem 40.53, look for a solution of the form $\psi(x, y, z) = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z)$.

Then,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \left(E_{n_x} - \frac{1}{2}k_1'x^2\right)\psi$$

with similar relations for $\dfrac{\partial^2\psi}{\partial y^2}$ and $\dfrac{\partial^2\psi}{\partial z^2}$. Adding,

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = \left(E_{n_x} + E_{n_y} + E_{n_z} - \frac{1}{2}k_1'x^2 - \frac{1}{2}k_1'y^2 - \frac{1}{2}k_2'z^2\right)\psi$$

$$= (E_{n_x} + E_{n_y} + E_{n_z} - U)\psi$$

$$= (E - U)\psi$$

where the energy E is

$$E = E_{n_x} + E_{n_y} + E_{n_z} = \hbar\left[(n_x + n_y + 1)\omega_1^2 + \left(n_z + \frac{1}{2}\right)\omega_2^2\right],$$

n_x, n_y and n_z all nonnegative integers. b) The ground level corresponds to $n_x = n_y = n_z = 0$, and $E = \hbar(\omega_1^2 + \omega_2^2/2)$. The first excited level corresponds to $n_x = n_y = 0$ and $n_z = 1$, as $\omega_2^2 < \omega_1^2$, and $E = \hbar(\omega_1^2 + (3/2)\omega_2^2)$. There is only one set of quantum numbers for both the ground state and the first excited state.

40.55: a) $\psi(x) = A\sin kx$ and $\psi(-L/2) = 0 = \psi(+L/2)$

$$\Rightarrow 0 = A\sin\left(\frac{+kL}{2}\right) \Rightarrow \frac{+kL}{2} = n\pi \Rightarrow k = \frac{2n\pi}{L} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{L}{n} \Rightarrow p_n = \frac{h}{\lambda n} = \frac{nh}{L} \Rightarrow E_n = \frac{p_n^2}{2m} = \frac{n^2h^2}{2mL^2} = \frac{(2n)^2h^2}{8mL^2}$$, where

$n = 1, 2...$

b) $\psi(x) = A\cos kx$ and $\psi(-L/2) = 0 = \psi(+L/2)$

$$\Rightarrow 0 = A\cos\left(\frac{kL}{2}\right) \Rightarrow \frac{kL}{2} = (2n+1)\frac{\pi}{2} \Rightarrow k = \frac{(2n+1)\pi}{L} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{2L}{(2n+1)} \Rightarrow p_n = \frac{(2n+1)h}{2L}$$

$$\Rightarrow E_n = \frac{(2n+1)^2h^2}{8mL^2} \quad n = 0, 1, 2...$$

c) The combination of all the energies in parts (a) and (b) is the same energy levels as given in Eq. (40.9), where $E_n = \dfrac{n^2h^2}{8mL^2}$.

d) Part (a)'s wave functions are odd, and part (b)'s are even.

40.56: a) As with the particle in a box, $\psi(x) = A\sin kx$, where A is a constant and $k^2 = 2mE/\hbar^2$. Unlike the particle in a box, however, k and hence E do not have simple forms. b) For $x > L$, the wave function must have the form of Eq. (40.18). For the wave function to remain finite as $x \to \infty$, $C = 0$. The constant $\kappa^2 = 2m(U_0 - E)/\hbar$, as in Eq. (14.17) and Eq. (40.18). c) At $x = L$, $A\sin kL = De^{-\kappa L}$ and $kA\cos kL = -\kappa De^{-\kappa L}$. Dividing the second of these by the first gives

$$k\cot kL = -\kappa,$$

a transcendental equation that must be solved numerically for different values of the length L and the ratio E/U_0.

40.57: a) $E = K + U(x) = \dfrac{p^2}{2m} + U(x) \Rightarrow p = \sqrt{2m(E - U(x)).}$

$$\lambda = \frac{h}{p} \Rightarrow \lambda(x) = \frac{h}{\sqrt{2m(E - U(x))}}.$$

b) As $U(x)$ gets larger (i.e., $U(x)$ approaches E from below—recall $k \geq 0$), $E - U(x)$ gets smaller, so $\lambda(x)$ gets larger.

c) When $E = U(x)$, $E - U(x) = 0$, so $\lambda(x) \to \infty$.

d) $\displaystyle\int_a^b \frac{dx}{\lambda(x)} = \int_a^b \frac{dx}{h/\sqrt{2m(E - U(x))}} = \frac{1}{h}\int_a^b \sqrt{2m(E - U(x))}\ dx = \frac{n}{2}$

$$\Rightarrow \int_a^b \sqrt{2m(E - U(x))}\ dx = \frac{hn}{2}.$$

e) $U(x) = 0$ for $0 < x < L$ with classical turning points at $x = 0$ and $x = L$. So, $\displaystyle\int_a^b \sqrt{2m(E - U(x))}\ dx = \int_0^L \sqrt{2mE}dx = \sqrt{2mE}\int_0^L dx = \sqrt{2mE}L$. So, from part ($d$),

$$\sqrt{2mE}L = \frac{hn}{2} \Rightarrow E = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2n^2}{8mL^2}.$$

f) Since $U(x) = 0$ in the region between the turning points at $x = 0$ and $x = L$, the results is the *same* as part (e). The height U_0 never enters the calculation. WKB is best used with *smoothly* varying potentials $U(x)$.

40.58: a) At the turning points $E=\frac{1}{2}k'x_{\text{TP}}^2 \Rightarrow x_{\text{TP}}=\pm\sqrt{\frac{2E}{k'}}$.

b) $\int_{-\sqrt{\frac{2E}{K'}}}^{+\sqrt{\frac{2E}{K'}}}\sqrt{2m\left(E-\frac{1}{2}k'x^2\right)}\,dx=\frac{nh}{2}$.

To evaluate the integral, we want to get it into a form that matches the standard integral given.

$$\sqrt{2m\left(E-\frac{1}{2}k'x^2\right)}=\sqrt{2mE-mk'x^2}=\sqrt{mk'}\sqrt{\frac{2mE}{mk'}-x^2}=\sqrt{mk'}\sqrt{\frac{2E}{k'}-x^2}.$$

Letting $A^2=\frac{2E}{k'}$, $a=-\sqrt{\frac{2E}{k'}}$, and $b=+\sqrt{\frac{2E}{k'}}$

$$\Rightarrow \sqrt{mk'}\int_a^b\sqrt{A^2-x^2}\,dx=2\frac{\sqrt{mk'}}{2}\left[x\sqrt{A^2-x^2}+A^2\arcsin\left(\frac{x}{|A|}\right)\right]\Bigg|_0^b$$

$$=\sqrt{mk'}\left[\sqrt{\frac{2E}{k'}}\sqrt{\frac{2E}{k'}-\frac{2E}{k'}}+\frac{2E}{k'}\arcsin\left(\frac{\sqrt{2E/k'}}{\sqrt{2E/k'}}\right)\right]$$

$$=\sqrt{mk'}\,\frac{2E}{k'}\arcsin(1)$$

$$=2E\sqrt{\frac{m}{k'}}\left(\frac{\pi}{2}\right).$$

Using WKB, this is equal to $\frac{hn}{2}$, so $E\sqrt{\frac{m}{k'}}\pi=\frac{hn}{2}$. Recall $\omega=\sqrt{\frac{k'}{m}}$, so $E=\frac{h}{2\pi}\omega n=\hbar\omega n.$

c) We are missing the zero-point-energy offset of $\frac{\hbar\omega}{2}\left(\text{recall } E=\hbar\omega\left(n+\frac{1}{2}\right)\right)$.

However, our approximation isn't bad at all!

40.59: a) At the turning points $E = A|x_{\text{TP}}| \Rightarrow x_{\text{TP}} = \pm \frac{E}{A}$.

b) $\int_{-\frac{E}{A}}^{+\frac{E}{A}} \sqrt{2m(E - A|x|}dx = 2\int_0^{\frac{E}{A}} \sqrt{2m(E - Ax)}\ dx$. Let $y = 2m(E - Ax) \Rightarrow dy =$

$-2mA\ dx$ when $x = \frac{E}{A}$, $y = 0$, and when $x = 0$, $y = 2mE$. So

$$2\int_0^{\frac{E}{A}} \sqrt{2m(E - Ax)}dx = -\frac{1}{mA}\int_{2mE}^{0} y^{1/2}dy$$

$$= -\frac{2}{3mA} y^{3/2}\Big|_{2mE}^{0} = \frac{2}{3mA}(2mE)^{3/2}.$$ Using WKB, this is equal to $\frac{hn}{2}$.

So, $\frac{2}{3mA}(2mE)^{3/2} = \frac{hn}{2} \Rightarrow E = \frac{1}{2m}\left(\frac{3mAh}{4}\right)^{2/3} n^{2/3}$.

c) The difference in energy decreases between successive levels. For example:
$1^{2/3} - 0^{2/3} = 1,\ 2^{2/3} - 1^{2/3} = 0.59,\ 3^{3/2} - 2^{3/2} = 0.49, \ldots$

- A sharp ∞ step gave ever-increasing level differences $(\sim n^2)$.
- A parabola $(\sim x^2)$ gave evenly spaced levels $(\sim n)$.
- Now, a linear potential $(\sim x)$ gives ever – decreasing level differences $(\sim n^{2/3})$.

Roughly: If the curvature of the potential (~ second derivative) is bigger than that of a parabola, then the level differences will increase. If the curvature is less than a parabola, the differences will decrease.

41.1: $L = \sqrt{l(l+1)}\hbar \Rightarrow l(l+1) = \left(\frac{L}{\hbar}\right)^2 = \left(\frac{4.716\times10^{-34}\ \text{J}\cdot\text{s}}{1.054\times10^{-34}\ \text{J}\cdot\text{s}}\right)^2$

$\Rightarrow l(l+1) = 20.0 \Rightarrow l = 4.$

41.2: a) $m_{l_{\max}} = 2$, so $L_{z_{\max}} = 2\hbar$. b) $\sqrt{l(l+1)}\hbar = \sqrt{6}\hbar = 2.45\hbar$. c) The angle is arccc $\left(\frac{L_z}{L}\right) = \arccos\left(\frac{m_l}{\sqrt{6}}\right)$, and the angles are, for $m_l = -2$ to $m_l = 2$, 144.7°, 114.1°, 90.0°, 65.9°, 35.3°. The angle corresponding to $m_l = l$ will always be larger for larger l.

41.3: $L = \sqrt{l(l+1)}\hbar$. The maximum orbital quantum number $l = n-1$. So if :

$n = 2 \quad l = 1 \quad \Rightarrow L = \sqrt{l(l+1)}\ \hbar = \sqrt{2}\hbar = 1.41\hbar$

$n = 20 \quad l = 19 \quad \Rightarrow L = \sqrt{19(20)}\ \hbar = 19.49\hbar$

$n = 200 \quad l = 199 \Rightarrow L = \sqrt{199(200)}\ \hbar = 199.5\hbar$

The maximum angular momentum value gets closer to the Bohr model value the larger the value of n.

41.4: The (l, m_l) combinations are (0, 0), (1, 0), (1, ±1), (2, 0), (2, ±1), (2, ±2), (3, 0), (3, ±1), (3, ±2), (3, ±3), (4, 0), (4, ±1), (4, ±2), (4, ±3), and (4, ±4), a total of 25.

b) Each state has the same energy (n is the same), $-\frac{13.60\ \text{eV}}{25} = -0.544\ \text{eV}.$

41.5: $U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r} = \frac{-1}{4\pi\varepsilon_0}\frac{(1.60\times10^{-19}\text{C})^2}{1.0\times10^{-10}\ \text{m}} = -2.3\times10^{-18}\ \text{J}$

$= \frac{-2.3\times10^{-18}\,\text{J}}{1.60\times10^{-19}\ \text{J/eV}} = -14.4\ \text{eV}.$

41.6: a) As in Example 41.3, the probability is

$$P = \int_0^{a/2} |\psi_{1s}|^2\ 4\pi r^2 dr = \frac{4}{a^3}\left[\left(-\frac{ar^2}{2} - \frac{a^2 r}{2} - \frac{a^3}{4}\right)e^{-2r/a}\right]_0^{a/2}$$

$$= 1 - \frac{5e^{-1}}{2} = 0.0803.$$

b) The difference in the probabilities is

$$(1-5e^{-2}) - (1-(5/2)e^{-1}) = (5/2)(e^{-1} - 2e^{-2}) = 0.243.$$

41.7: a) $|\psi|^2 = \psi^*\psi = |R(r)|^2|\Theta(\theta)|^2\ (Ae^{-im_l\phi})(Ae^{+im_l\phi})$

$= A^2\,|R(r)|^2|\Theta(\theta)|^2,$ which is independent of ϕ

b) $\int_0^{2\pi}|\Phi(\theta)|^2\,d\phi = A^2\int_0^{2\pi}d\phi = 2\pi A^2 = 1 \Rightarrow A = \dfrac{1}{\sqrt{2\pi}}.$

41.8: $E_n = -\dfrac{1}{(4\pi\varepsilon_0)^2}\dfrac{m_r e^4}{2n^2\hbar^2}\ \Delta E_{12} = E_2 - E_1 = \dfrac{E_1}{2^2} - E_1 = -(0.75)E_1.$

a) If $m_r = m = 9.11\times10^{-31}$ kg

$$\frac{m_r e^4}{(4\pi\varepsilon_0)^2\hbar^2} = \frac{9.109\times10^{-31}\text{kg})(1.602\times10^{-19}\text{C})^4}{2(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}\left(8.988\times10^9\ \text{Nm}^2/\text{C}\right)^2$$

$= 2.177\times10^{-18}\ \text{J} = 13.59\ \text{eV}.$

For $2\to1$ transition, the coefficient is (0.75)(13.59 eV)=10.19 eV.

b) If $m_r = \dfrac{m}{2}$, using the result from part (a),

$$\frac{m_r e^4}{(4\pi\varepsilon_0)^2\hbar^2} = (13.59\ \text{eV})\left(\frac{m/2}{m}\right) = \left(\frac{13.59\ \text{eV}}{2}\right) = 6.795\ \text{eV}.$$

Similarly, the $2\to1$ transition, $\Rightarrow\left(\dfrac{10.19\ \text{eV}}{2}\right) = 5.095\ \text{eV}.$

c) If $m_r = 185.8m,$ using the result from part (a),

$$\frac{m_r e^4}{(4\pi\varepsilon_0)^2\hbar^2} = (13.59\ \text{eV})\left(\frac{185.8m}{m}\right) = 2525\ \text{eV},$$

and the $2\to1$ transition gives $\Rightarrow$ (10.19 eV)(185.8)=1893 eV.

41.9: a) $m_r = m : a_1 = \dfrac{\varepsilon_0 h^2}{\pi m e^2} = \dfrac{\varepsilon_0(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{\pi(9.109\times10^{-31}\ \text{kg})(1.602\times10^{-19}\text{C})^2}$

$\Rightarrow a_1 = 5.293\times10^{-11}\,\text{m}$

b) $m_r = \dfrac{m}{2} \Rightarrow a_2 = 2a_1 = 1.059\times10^{-10}$ m.

c) $m_r = 185.8\ \text{m} \Rightarrow a_3 = \dfrac{1}{185.8}a_1 = 2.849\times10^{-13}\,\text{m}.$

41.10: $e^{im_l\phi} = \cos(m_l\phi) + i\sin(m_l\phi),$ and to be periodic with period $2\pi, m_l 2\pi$ must be an integer multiple of $2\pi,$ so m_l must be an integer.

41.11: $P(a)=\int_0^a |\psi_{1s}| dV = \int_0^a \frac{1}{\pi a^3} e^{-2r/a}(4\pi r^2 dr)$

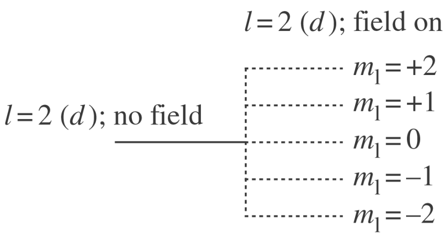
$$\Rightarrow P(a) = \frac{4}{a^3}\int_o^a r^2 e^{-2r/a} dr = \frac{4}{a^3}\left[\left(\frac{-ar^2}{2}-\frac{a^2 r}{2}-\frac{a^2}{4}\right)e^{-2r/a}\right]_0^a$$

$$= \frac{4}{a^3}\left[\left(\frac{-a^3}{2}-\frac{a^3}{2}-\frac{a^3}{4}\right)e^{-2}+\frac{a^3}{4}e^0\right]$$

$$\Rightarrow P(a) = 1-5e^{-2}.$$

41.12: a) $\Delta E = \mu_B B = (5.79\times10^{-5}\ \mathrm{eV/T})(0.400\ \mathrm{T}) = 2.32\times10^{-5}\ \mathrm{eV}$, b) $m_l = -2$, the lowest possible value of m_l.

c)



41.13: a) g - state $\Rightarrow l = 4 \Rightarrow$ # of states is $(2l+1) = 9$ ($m_l = 0, \pm1, \pm2, \pm3, \pm4$).

b) $\Delta U = \mu_B B = (5.79\times10^{-5}\ \mathrm{eV/T})(0.600\ \mathrm{T}) = 3.47\times10^{-5}\ \mathrm{eV} = 5.56\times10^{-24}\ \mathrm{J}$.

c) $\Delta U_{-4+4} = 8\mu_B B = 8(5.79\times10^{-5}\ \mathrm{eV/T})(0.600\ \mathrm{T}) = 2.78\times10^{-4}\ \mathrm{eV}$
$= 4.45\times10^{-23}\ \mathrm{J}$.

41.14: a) According to Fig. 41.8 there are three different transitions that are consistent with the selection rules. The initial m_l values are 0, ±1; and the final m_l value is 0.

b) The transition from $m_l = 0$ to $m_l = 0$ produces the same wavelength (122 nm) that was seen without the magnetic field.

c) The larger wavelength (smaller energy) is produced from the $m_l = -1$ to $m_l = 0$ transition.

d) The shorter wavelength (greater energy) is produced from the $m_l = +1$ to $m_l = 0$ transition.

41.15: a) $3p \Rightarrow n = 3, l = 1, \Delta U = \mu_B B \Rightarrow B = \frac{U}{\mu_B}$

$$B = \frac{(2.71\times10^{-5}\ \mathrm{eV})}{(5.79\times10^{-5}\ \mathrm{eV/T})} = 0.468\ \mathrm{T}.$$

b) Three $m_l = 0, \pm1$.

41.16: a) $U = +(2.00232)\left(\frac{e}{2m}\right)\left(\frac{-\hbar}{2}\right)B$

$$= -\frac{(2.00232)}{2}\mu_{\text{B}}B$$

$$= -\frac{(2.00232)}{2}(5.788\times10^{-5}\ \text{eV/T})(0.480\ \text{T})$$

$$= -2.78\times10^{-5}\ \text{eV}.$$

b) Since $n = 1$, $l = 0$ so there is no orbital magnetic dipole interaction. But if $n \neq 0$ there could be since $l < n$ allows for $l \neq 0$.

41.17: $U = -\mu_z B = (2.00232)\frac{e}{2m}S_z B$

$$\Rightarrow U = (2.00232)\frac{e}{2m}(m_s\hbar)B$$

$$\Rightarrow U = (2.00232)\mu_{\text{B}}\ m_s B \text{ where } \mu_{\text{B}} = \frac{e\hbar}{2m}.$$

So the energy difference

$$\Delta U = (2.00232)\mu_{\text{B}}B\left(\frac{1}{2}-\left(-\frac{1}{2}\right)\right)$$

$$\Rightarrow \Delta U = (2.00232)(5.788\times10^{-5}\ \text{eV/T})(1.45\ \text{T})$$

$$= 1.68\times10^{-4}\ \text{eV}.$$

And the lower energy level is $m_s = +\frac{1}{2}$ (since B points in the $-\hat{z}$ direction).

41.18: The allowed (l, j) combinations are $\left(0,\frac{1}{2}\right), \left(1,\frac{1}{2}\right), \left(1,\frac{3}{2}\right), \left(2,\frac{3}{2}\right)$ and $\left(2,\frac{5}{2}\right)$.

41.19: j quantum numbers are either $l+\frac{1}{2}$ or $l-\frac{1}{2}$, So if $j = 9/2$ and $7/2$, then $l = 4$. The letter used to describe $l = 4$ is "g"

41.20: a) $\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(300\times10^{8}\ \text{m/s})}{(5.9\times10^{-6}\ \text{eV})} = 21\,\text{cm},\ f = \frac{c}{\lambda} =$

$\frac{(3.00\times10^{8}\ \text{m/s})}{0.21\text{m}} = 1.4\times10^{9}$ Hz, a short radio wave.

b) As in Example 41.6, the effective field is $B \cong \Delta E/2\mu_{\text{B}} = 5.1\times10^{-2}$ T, for smaller than that found in the example.

41.21: a) Classically $L = I\omega$, and $I = \frac{2}{5}mR^2$ for a uniform sphere.

$$\Rightarrow L = \frac{2}{5}m\omega R^2 = \sqrt{\frac{3}{4}}\hbar$$

$$\Rightarrow \omega = \frac{5\hbar}{2mR^2}\sqrt{\frac{3}{4}} = \frac{5(1.054\times10^{-34}\,\text{J}\cdot\text{s})}{2(9.11\times10^{-31}\ \text{kg})\,(1.0\times10^{-17}\,\text{m})^2}\sqrt{\frac{3}{4}}$$

$$\Rightarrow \omega = 2.5\times10^{30}\ \text{rad/s}.$$

b) $v = r\omega = (1.0\times10^{-17}\ \text{m})\,(2.5\times10^{30}\ \text{rad/s}) = 2.5\times10^{13}\ \text{m/s}$. Since this is faster than the speed of light this model is invalid.

41.22: For the outer electrons, there are more inner electrons to screen the nucleus.

41.23: Using Eq. (41.27) for the ionization energy: $E_n = \frac{-Z_{\text{eff}}^2}{n^2}(13.6\,\text{eV})$. The $5s$ electron sees $Z_{\text{eff}} = 2.771$ and $n = 5$

$$\Rightarrow E_5 = \frac{-(2.771)^2}{5^2}(13.6\ \text{eV}) = 4.18\ \text{eV}.$$

41.24: However the number of electrons is obtained, the results must be consistent with Table (43-3); adding two more electrons to the zinc configuration gives $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6 4s^2 3d^{10} 4p^2$.

41.25: The ten lowest energy levels for electrons are in the n = 1 and n = 2 shells.

$$n = 1,\ l = 0,\ m_l = 0, \qquad m_s = \pm\frac{1}{2}: 2\ \text{states}.$$

$$n = 2,\ l = 0,\ m_l = 0, \qquad m_s = \pm\frac{1}{2}: 2\ \text{states}.$$

$$n = 2,\ l = 1,\ m_l = 0, \pm 1, \qquad m_s = \pm\frac{1}{2}: 6\ \text{states}.$$

41.26: For the $4s$ state, $E = -4.339$ eV and $Z_{\text{eff}} = 4\sqrt{(-4.339)/(-13.6)} = 2.26$. Similarly, $Z_{\text{eff}} = 1.79$ for the 4p state and 1.05 for the 4d state. The electrons in the states with higher l tend to be further away from the filled subshells and the screening is more complete.

41.27: a) Nitrogen is the seventh element (Z = 7). N^{2+} has two electrons removed, so there are 5 remaining electrons $\Rightarrow$ electron configuration is $1s^2 2s^2 2p$.

b) $E = \dfrac{-Z_{\text{eff}}^2}{n^2}(13.6\text{ eV}) = \dfrac{-(7-4)^2}{2^2}(13.6\text{ eV}) = -30.6\text{ eV}$

c) Phosphorous is the fifteenth element (Z = 15). P^{2+} has 13 electrons, so the electron configuration is $1s^2 2s^2 2p^6 3s^2 3p$.

d) The least tightly held electron: $E = -\dfrac{(15-12)^2}{3^2}(13.6\text{ eV}) = -13.6\text{ eV}.$

41.28: a) $E_2 = -\dfrac{13.6\text{ e}V}{4}Z_{\text{eff}}^2$, so $Z_{\text{eff}} = 1.26$. b) Similarly, $Z_{\text{eff}} = 2.26$. c) Z_{eff} becomes larger going down the columns in the periodic table.

41.29: a) Again using $E_n = \dfrac{-Z_{\text{eff}}^2}{n^2}(13.6\text{ eV})$, the outermost electron of the $\text{Be}^+ L$ shell (n = 2) sees the inner two electrons shield two protons so $Z_{\text{eff}} = 2$.

$$\Rightarrow E_2 = \frac{-2^2}{2^2}(13.6\text{ eV}) = -13.6\text{ eV}.$$

b) For Ca^+, outer shell has n = 4, so $E_4 = \dfrac{-2^2}{4^2}(13.6\text{ eV}) = -3.4\text{ eV}.$

41.30: $E_{kx} \cong (Z-1)^2(10.2\text{ eV})$

$$Z \approx 1 + \sqrt{\frac{7.46\times10^3\text{ eV}}{10.2\text{ eV}}} = 28.0,$$

which corresponds to the element Nickel (Ni).

41.31: a) $Z = 20: f = (2.48\times10^{15}\text{ Hz})(20-1)^2 = 8.95\times10^{17}\text{ Hz}$

$$E = hf = (4.14\times10^{-15}\text{ eV}\cdot\text{s})\,(8.95\times10^{17}\text{ Hz}) = 3.71\text{ keV}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^8\text{ m/s}}{8.95\times10^{17}\text{ Hz}} = 3.35\times10^{-10}\text{ m}.$$

b) Z = 27:

$$f = 1.68\times10^{18}\text{ Hz}$$
$$E = 6.96\text{ keV}$$
$$\lambda = 1.79\times10^{-10}\text{ m}.$$

c) $Z = 48: f = 5.48\times10^{18}\text{Hz}, E = 22.7\text{ keV}, \lambda = 5.47\times10^{-11}\text{ m}.$

41.32: See Example 41.3; $r^2|\psi|^2 = Cr^2e^{-2r/a}$, $\dfrac{dr^2|\psi|^2}{dr} = Ce^{-2r/a}(2r-(2r^2/a))$, and for a maximum, $r = a$, the distance of the electron from the nucleus in the Bohr model.

41.33: a) $E_{1s} = -\dfrac{1}{(4\pi\varepsilon_0)^2}\dfrac{me^4}{2\hbar^2}$ and $U(r) = \dfrac{-1}{4\pi\varepsilon_0}\dfrac{e^2}{r}$. *If* $E_{1s} = U(r)$, then $\dfrac{1}{(4\pi\varepsilon_0)^2}\dfrac{me^4}{2\hbar^2} =$

$$\frac{+1}{(4\pi\varepsilon_0)}\frac{e^2}{r} \Rightarrow r = \frac{4\pi\varepsilon_0 2\hbar^2}{me^2} = 2a$$

b) $P(r > 2a) = \displaystyle\int_{2a}^{\infty}|\psi_{1s}|^2 dV = 4\pi\int_{2a}^{\infty}|\psi_{1s}|^2 r^2 dr$ and ψ is $= \dfrac{1}{\sqrt{\pi a^3}}e^{-r/a}$.

$$\Rightarrow P(r > 2a) = \frac{4}{a^3}\int_{2a}^{\infty} r^2 e^{-2r/a} dr$$

$$\Rightarrow P(r > 2a) = -\frac{4}{a^3}\left[e^{-2r/a}\left(\frac{ar^2}{2} + \frac{a^2r}{2} + \frac{a^3}{4}\right)\right]_{2a}^{\infty}$$

$$\Rightarrow P(r > 2a) = \frac{4}{a^3}e^{-4}\left(2a^3 + a^3 + \frac{a^3}{4}\right).$$

$$= 13e^{-4} = 0.238.$$

41.34: a) For large values of n, the inner electrons will completely shield the nucleus, so $Z_{\text{eff}} = 1$ and the ionization energy would be $\dfrac{13.60\ \text{eV}}{n^2}$.

b) $\dfrac{13.60\ \text{eV}}{350^2} = 1.11\times10^{-4}$ eV, $r_{350} = (350)^2 a_0 = (350)^2(0.529\times10^{-10}\text{m}) = 6.48\times10^{-6}$ m .

c) Similarly for n = 650, $\dfrac{13.60\ \text{eV}}{(650)^2} = 3.22\times10^{-5}$ eV, $r_{650} = (650)^2(0.529\times$ 10^{-10} m) $= 2.24\times10^{-5}$ m.

41.35: a) If normalized, then

$$\int_0^{\infty}|\psi_{2s}|^2 dV = 4\pi\int_0^{\infty}|\ \psi_{2s}|^2 r^2 dr = 1$$

$$\Rightarrow I = \int_0^{\infty}\frac{4\pi}{32\pi a^3}r^2\left(2-\frac{r}{a}\right)^2 e^{-r/a}dr$$

$$= \frac{1}{8a^3}\int_0^{\infty}\left(4r^2 - \frac{4r^3}{a}+\frac{r^4}{a^2}\right)e^{-r/a}dr.$$

But recall $\int_0^{\infty}x^n e^{-ax}dx = \dfrac{n!}{\alpha^{n+1}}.$

$$\text{So}\ \ I = \frac{1}{8a^3}\left[4(2)a^3 - \frac{4}{a}(6)a^4 + \frac{1}{a^2}(24)a^5\right]$$

$$\Rightarrow I = \frac{1}{8a^3}\left[8a^3 - 24a^3 + 24a^3\right] = 1 \text{ and } \psi_{2s} \text{ is normalized.}$$

b) We carry out the same calculation as part (a) except now the upper limit on the integral is 4a, not infinity.

$$\text{So}\ \ I = \frac{1}{8a^3}\int_0^{4a}\left(4r^2 - 4\frac{r^3}{a}+\frac{r^4}{a^2}\right)e^{-r/a}dr.$$

Now the necessary integral formulas are:

$$\int r^2 e^{-r/a}dr = -e^{-r/a}(r^2 a + 2ra^2 + 2a^3)$$

$$\int r^3 e^{-r/a}dr = -e^{-r/a}(r^3 a + 3r^2 a^2 + 6ra^3 + 6a^3)$$

$$\int r^4 e^{-r/a}dr = -e^{-r/a}(r^4 a + 4r^3 a^2 + 12r^2 a^3 + 24ra^4 + 24a^5)$$

All the integrals are evaluated at the limits $r = 0$ and $4a$. After carefully plugging in the limits and collecting like terms we have:

$$I = \frac{1}{8a^3}\cdot a^3[(8-24+24)+e^{-4}(-104+568-824)]$$

$$\Rightarrow I = \frac{1}{8}(8-360e^{-4}) = 0.176 = \text{Prob}(r < 4a).$$

41.36: a) Since the given $\psi(r)$ is real, $r^2 \,|\, \psi \,|^2 = r^2\psi^2$. The probability density will be an extreme when

$$\frac{d}{dr}(r^2\psi^2) = 2\left(r\psi^2 + r^2\psi\frac{\psi}{dr}\right) = 2r\psi\left(\psi + r\frac{d\psi}{dr}\right) = 0.$$

This occurs at $r = 0$, a minimum, and when $\psi = 0$, also a minimum. A maximum must correspond to $\psi + r\frac{d\psi}{dr} = 0.$ Within a multiplicative constant,

$$\psi(r) = (2 - r/a)e^{-r/2a}, \frac{d\psi}{dr} = -\frac{1}{a}(2 - r/2a)e^{-r/2a},$$

and the condition for a maximum is

$$(2 - r/a) = (r/a)\,(2 - r/2a), \text{ or } r^2 - 6ra + 4a^2 = 0.$$

The solutions to the quadratic are $r = a(3 \pm \sqrt{5})$. The ratio of the probability densities at these radii is 3.68, with the larger density at $r = a(3 + \sqrt{5})$. *b*) $\psi = 0$ at $r = 2a$. Parts (a) and (b) are consistent with Fig.(41.4); note the two relative maxima, one on each side of the minimum of zero at $r = 2a$.

41.37: a) $\theta_L = \arccos\left(\frac{L_z}{L}\right)$. This is smaller for L_z and L as large as possible. Thus $l = n - 1$ and $m_l = l = n - 1$

$$\Rightarrow L_z = m_l\hbar = (n-1)\hbar \text{ and } L = \sqrt{l(l+1)}\hbar = \sqrt{(n-1)n}\hbar$$

$$\Rightarrow \theta_L = \arccos\left(\frac{n-1}{\sqrt{n(n-1)}}\right).$$

b) The largest angle implies $l = n-1, m_l = -l = -(n-1)$

$$\Rightarrow \theta_L = \arccos\left[\frac{-(n-1)}{\sqrt{n(n-1)}}\right]$$

$$= \arccos\left[-\sqrt{(1-1/n)}\right].$$

41.38: a) $L_x^2 + L_y^2 = L^2 - L_z^2 = l(l+1)\hbar^2 - m_l^2\hbar^2$ so $\sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m_l^2}\,\hbar.$

b) This is the magnitude of the component of angular momentum perpendicular to the z-axis. c) The maximum value is $\sqrt{l(l+1)}\hbar = L,$ when $m_l = 0.$ That is, if the electron is known to have no z-component of angular momentum, the angular momentum must be perpendicular to the z-axis. The minimum is $\sqrt{l}\hbar$ when $m_l = \pm l.$

41.39: $P(r)=\left(\frac{1}{24a^5}\right)r^4e^{-r/2a}$

$$\frac{dP}{dr}=\left(\frac{1}{24a^5}\right)(4r^3-\frac{r^4}{a})e^{-r/2a}$$

$$\frac{dP}{dr}=0 \text{ when } 4r^3-\frac{r^4}{a}=0; \qquad r=4a,$$

In the Bohr model, $r_n=n^2a$ so $r_2=4a$, which agrees.

41.40: The time required to transit the horizontal 50 cm region is

$$t=\frac{\Delta x}{v_x}=\frac{0.500\ \text{m}}{525\ \text{m/s}}=0.952\ \text{ms}.$$

The force required to deflect each spin component by 0.50 mm is

$$F_z=ma_z=\pm m\frac{2\Delta z}{t^2}=\pm\left(\frac{0.1079\ \text{kg/mol}}{6.022\times10^{23}\ \text{atoms/mol}}\right)\frac{2(0.50\times10^{-3}\ \text{m})}{(0.952\times10^{-3}\ \text{s})^2}=\pm1.98\times10^{-22}\,\text{N}.$$

According to Eq. 41.22, the value of μ_z is

$$|\mu_z|=9.28\times10^{-24}\,\text{A}\cdot\text{m}^2.$$

Thus, the required magnetic-field gradient is

$$\left|\frac{dB_z}{dz}\right|=\left|\frac{F_z}{\mu_z}\right|=\frac{1.98\times10^{-22}\,\text{N}}{9.28\times10^{-24}\ \text{J/T}}=21.3\ \text{T/m}.$$

41.41: Decay from a $3d$ to $2p$ state in hydrogen means that $n=3\rightarrow n=2$ and $m_l=\pm2,\pm1,0\rightarrow m_l=\pm1,0.$ However selection rules limit the possibilities for decay. The emitted photon carries off one unit of angular momentum so l must change by 1 and hence m_l must change by 0 or $\pm1.$ The shift in the transition energy from the zero field value is just

$$U=(m_{l_3}-m_{l_2})\mu_\text{B}B=\frac{e\hbar B}{2m}(m_{l_3}-m_{l_2})$$

where m_{l_3} is the $3d$ m_l value and m_{l_2} is the $2p$ m_l value. Thus there are only three different energy shifts. They and the transitions that have them, labeled by $m,$ are:

$$\frac{e\hbar B}{2m}: 2\rightarrow1, \quad 1\rightarrow0, \quad 0\rightarrow-1$$

$$0: 1\rightarrow1, \quad 0\rightarrow0, \quad -1\rightarrow-1$$

$$-\frac{e\hbar B}{2m}: 0\rightarrow1, \quad -1\rightarrow0, \quad -2\rightarrow-1$$

41.42: a) The energy shift from zero field is $\Delta U_0 = m_l \mu_B B$. For $m_l = 2$, $\Delta U_0 =$ $(2)\,(5.79\times 10^{-5}\ \text{eV/T})\,(1.40\ \text{T}) = 1.62\times 10^{-4}\ \text{eV}$. For $m_l = 1$, $\Delta U_0 = (1)\times$ $(5.79\times 10^{-5}\ \text{eV/T})\,(1.40\ \text{T}) = 8.11\times 10^{-5}\text{eV}$.

b) $|\Delta\lambda| = \lambda_0 \frac{|\Delta E|}{E_0}$, where $E_0 = (13.6\ \text{eV}((1/4)-(1/9)),\ \lambda_0 = \left(\frac{36}{5}\right)\frac{1}{R} =$ 6.563×10^{-7} m and $\Delta E = 1.62\times 10^{-4}\ \text{eV} - 8.11\times 10^{-5}\ \text{eV} = 8.09\times 10^{-5}$ eV from part (a).

Then, $|\Delta\lambda| = 2.81\times 10^{-11}\ \text{m} = 0.0281\,\text{nm}$. The wavelength corresponds to a larger energy change, and so the wavelength is smaller.

41.43: From Section 38.6: $\dfrac{n_1}{n_0} = e^{-(E_1-E_0)/kT}$. We need to know the difference in energy between the $m_s = +\dfrac{1}{2}$ and $m_s = -\dfrac{1}{2}$ states. $U = -\mu_z B = 2.00232\mu_B m_s B.$

So $U_{\frac{1}{2}} - U_{-\frac{1}{2}} = 2.00232\mu_B B$

$$\Rightarrow \frac{n_{1/2}}{n_{-1/2}} = e^{-(2.00232)\mu_B B/kT}$$

$$= e^{-(2.00232)\,(9.274\times 10^{-24}\ \text{J/T})\text{B}/(1.381\times 10^{-23}\ \text{J/K})\,(300\ \text{K})}$$

$$= e^{-(4.482\times 10^{-3}\text{T}^{-1})B}$$

a) $B = 5.00\times 10^{-5}\,\text{T} \Rightarrow \dfrac{n_{1/2}}{n_{-1/2}} = 0.9999998.$

b) $B = 0.500\ \text{T} \Rightarrow \dfrac{n_{1/2}}{n_{-1/2}} = 0.9978$

c) $B = 5.00\ \text{T} \Rightarrow \dfrac{n_{1/2}}{n_{-1/2}} = 0.978$

41.44 Using Eq. 41.4

$$L = mvr = \sqrt{l(l+1)}\hbar,$$

and the Bohr radius from Eq. 38.15, we obtain the following value for v

$$v = \frac{\sqrt{l(l+1)}\hbar}{m(n^2 a_0)} = \frac{\sqrt{2}(6.63\times 10^{-34}\ \text{J}\cdot\text{s})}{2\pi(9.11\times 10^{-31}\ \text{kg})\,(4)\,(5.29\times 10^{-11}\ \text{m})} = 7.74\times 10^5\ \text{m/s}.$$

The magnetic field generated by the "moving" proton at the electrons position can be calculated from Eq. 28.1

$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2} = (10^{-7A}\text{T}\cdot\text{m/A})\frac{(1.60\times 10^{-19}\text{C})\,(7.74\times 10^5\ \text{m/s})\sin(90°)}{(4)^2(5.29\times 10^{-11}\ \text{m})^2} = 0.277\ \text{T}.$$

41.45: m_s can take on 4 different values: $m_s = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$. Each nlm_l state can have 4 electrons, each with one of the four different m_s values.

a) For a filled $n = 1$ shell, the electron configuration would be $1s^4$; four electrons and $Z = 4$. For a filled $n = 2$ shell, the electron configuration would be $1s^4 2s^4 2p^{12}$; twenty electrons and $Z = 20$.

b) Sodium has $Z = 11$; 11 electrons. The ground-state electron configuration would be $1s^4 2s^4 2p^3$.

41.46: a) Z^2 (–13.6 eV) = $(7)^2$ (–13.6 eV) = –666 eV. b) The negative of the result of part (a), 666 eV. c) The radius of the ground state orbit is inversely proportional to the nuclear charge, and $\frac{a}{Z} = (0.529 \times 10^{-10}\ \text{m})/7 = 7.56 \times 10^{-12}$ m.

d) $\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_0\left(\frac{1}{1^2} - \frac{1}{2^2}\right)}$, where E_0 is the energy found in part (b), and $\lambda = 2.49$ nm.

41.47: a) The photon energy equals the atom's transition energy. The hydrogen atom decays from $n = 2$ to $n = 1$, so:

$$\Delta E = -13.60\ \text{eV}\left(\frac{1}{(2)^2} - \frac{1}{(1)^2}\right) = (10.2\ \text{eV})\ (1.60 \times 10^{-19}\ \text{J/eV})$$

$$= 1.63 \times 10^{-18}\ \text{J}$$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{(6.33 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^8\ \text{m/s})}{1.63 \times 10^{-18}\ \text{J}} = 1.22 \times 10^{-7}\ \text{m}.$$

b) The change in an energy level due to an external magnetic field is just $U = m_l \mu_B B$. The ground state has $m_l = 0$, and it is not shifted. The $n = 2$ state has $m_l = -1$, so it is shifted by

$$U = (-1)(9.274 \times 10^{-24}\ \text{J/T})(2.20\ \text{T})$$

$$= -2.04 \times 10^{-23}\ \text{J}$$

and since

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta E}{E}$$

$$\Rightarrow \Delta\lambda = \lambda\frac{\Delta E}{E} = (1.22 \times 10^{-7}\ \text{m})\left(\frac{2.04 \times 10^{-23}\ \text{J}}{1.63 \times 10^{-18}\ \text{J}}\right) = 1.53 \times 10^{-12}\ \text{m}.$$

Since the $n = 2$ level is lowered in energy (brought closer to the $n = 1$ level) the change in energy is less, and the photon wavelength increases due to the magnetic field.

41.48: The effective field is that which gives rise to the observed difference in the energy level transition,

$$B = \frac{\Delta E}{\mu_B} = \frac{hc}{\mu_B}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right) = \frac{2\pi mc}{e}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right).$$

Substitution of numerical values gives $B = 3.64 \times 10^{-3}$ T, much smaller than that for sodium.

41.49: a) The minimum wavelength means the largest transition energy. If we assume that the electron makes a transition from a high shell, then using the screening approximation outlined in Section 41.5, the transition energy is approximately the ionization energy of hydrogen. Then $\Delta E = E_1 = (Z-1)^2(13.6 \text{ eV})$. For vanadium, $Z = 23$.

$$\Rightarrow \Delta E = 6.58 \times 10^3 \text{ eV} = 1.05 \times 10^{-15} \text{ J}$$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{1.05 \times 10^{-15} \text{ J}} = 1.89 \times 10^{-10} \text{ m}.$$

For the longest wavelength, we need the smallest transition energy, so this is the $n = 2 \rightarrow n = 1$ transition (K_α). So we use Moseley's Law:

$$f = (2.48 \times 10^{15} \text{ Hz})(23-1)^2 = 1.20 \times 10^{18} \text{ Hz}$$

$$\Rightarrow \lambda = \frac{c}{f} = 2.50 \times 10^{-10} \text{ m}.$$

b) The rhenium, $Z = 45$, the minimum wavelength is

$$\lambda = \frac{hc}{(Z-1)^2(13.6 \text{ eV})} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(44)^2(13.6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}$$

$$\Rightarrow \lambda = 4.72 \times 10^{11} \text{ m}.$$

The maximum wavelength is $\lambda = \frac{c}{f} = \frac{(3.00 \times 10^8 \text{ m/s})}{(2.48 \times 10^{15} \text{ Hz})(45-1)^2}$

$$\Rightarrow \lambda = 6.25 \times 10^{-11} \text{ m}.$$

41.50: a) $\Delta E = (2.00232)\mu_B B \Delta S_Z \approx \frac{e\hbar}{m} B = \frac{hc}{\lambda} \Rightarrow B = \frac{2\pi mc}{\lambda e}$.

b) $B = \frac{2\pi\left(9.11 \times 10^{-31} \text{ kg}\right)(3.00 \times 10^8 \text{ m/s})}{(0.0350 \text{ m})(1.60 \times 10^{-19} \text{ C})} = 0.307 \text{ T}.$

41.51: a) To calculate the total number of states for the n^{th} principle quantum number shell we must multiply all the possibilities. The spin states multiply everything by 2. The maximum l value is (n –1), and each l value has $(2l+1)m_l$ values.

So the total number of states is

$$N = 2\sum_{l=0}^{n-1}(2l+1) = 2\sum_{l=0}^{n-1}1 + 4\sum_{l=0}^{n-1} l$$

$$= 2n + \frac{4(n-1)(n)}{2} = 2n + 2n^2 - 2n$$

$$N = 2n^2.$$

b) The n = 5 shell (O-shell) has 50 states.

41.52: a) Apply Coulomb's law to the orbiting electron and set it equal to the centripetal force. There is an attractive force with charge $+2e$ a distance r away and a repulsive force a distance $2r$ away. So,

$$\frac{(+2e)(-e)}{4\pi\varepsilon_0 r^2}+\frac{(-e)(-e)}{4\pi\varepsilon_0 (2r)^2}=\frac{-mv^2}{r}.$$

But, from the quantization of angular momentum in the first Bohr orbit, $L = mvr = \hbar \Rightarrow v$.

So $\dfrac{-2e^2}{4\pi\varepsilon_0 r^2}+\dfrac{e^2}{4\pi\varepsilon_0 (4r)^2}=\dfrac{-mv^2}{r}=\dfrac{-m\left(\frac{\hbar}{mr}\right)^2}{r}=-\dfrac{\hbar^2}{mr^3}$

$$\Rightarrow \frac{-7}{4}\frac{e^2}{r^2}=-\frac{4\pi\varepsilon_0\hbar^2}{mr^3}\Rightarrow r=\frac{4}{7}\left(\frac{4\pi\varepsilon_0\hbar^2}{me^2}\right)$$

$$=\frac{4}{7}a_0=\frac{4}{7}(0.529\times 10^{-10}\text{ m})=3.02\times 10^{-11}\text{ m}.$$

And $v=\dfrac{-\hbar}{mr}=\dfrac{7}{4}\dfrac{\hbar}{ma_0}=\dfrac{7}{4}\dfrac{(1.054\times 10^{-34}\text{ J}\cdot\text{s})}{(9.11\times 10^{-31}\text{ kg})(0.529\times 10^{-10}\text{ m})}=3.83\times 10^6\text{ m/s}.$

b) $KE=2\left(\dfrac{1}{2}mv^2\right)=9.11\times 10^{-31}\text{ kg }(3.83\times 10^6\text{ m/s})^2=1.34\times 10^{-17}\text{ J}=83.5\text{ eV}.$

c) $PE=2\left(\dfrac{-2e^2}{4\pi\varepsilon_0 r}\right)+\dfrac{e^2}{4\pi\varepsilon_0 (2r)}=\dfrac{-4e^2}{4\pi\varepsilon_0 r}+\dfrac{e^2}{4\pi E_0 (2r)}$

$$=\frac{-7}{2}\left(\frac{e^2}{4\pi\varepsilon_0 r}\right)=-2.67\times 10^{-17}\text{ J}=-166.9\text{ eV}.$$

d) $E_\infty=-[-166.9\text{ eV}+83.5\text{ eV}]=83.4\text{ eV},$ which is only off by about 5% from the real value of 79.0 eV.

41.53: The potential $U(x) = \frac{1}{2}k'x^2$ is that of a simple harmonic oscillator. Treated quantum mechanically (see Section 40.4) each *energy* state has energy $E_n = \hbar\omega\,(n + \frac{1}{2})$. Since electrons obey the exclusion principle, this allows us to put *two* electrons (one for each $m_s = \pm\frac{1}{2}$) for every value of *n*—each quantum state is then defined by the ordered pair of quantum numbers $(n,\, m_s)$.

By placing two electrons in each energy level the lowest energy is then

$$2\left(\sum_{n=0}^{N-1} E_n\right) = 2\left(\sum_{n=0}^{N-1} \hbar w\left(n + \frac{1}{2}\right)\right) = 2\hbar w\left[\sum_{n=0}^{N-1} n + \sum_{n=0}^{N-1} \frac{1}{2}\right]$$

$$= 2\hbar w\left[\frac{(N-1)(N)}{2} + \frac{N}{2}\right] = \hbar w\,[N^2 - N + N]$$

$$= \hbar w N^2 = \hbar N^2\sqrt{\frac{k'}{m}}.$$

Here we used the hint from Problem 41.51 to do the first sum, realizing that the first value of *n* is zero and the last value of *n* is $N - 1$, giving us a total of *N* energy levels filled.

41.54: a) The radius is inversely proportional to *Z*, so the classical turning radius is $2a/Z$.

b) The normalized wave function is

$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a^3/Z^3}}\, e^{-Zr/a}$$

and the probability of the electron being found outside the classical turning point is

$$P = \int_{2a/Z}^{\infty} \left|\psi_{1s}\right|^2 4\pi r^2 dr = \frac{4}{a^3/Z^3}\int_{2a/Z}^{\infty} e^{-2Zr/a} r^2 dr.$$

Making the change of variable $u = Zr/a$, $dr = (a/Z)du$ changes the integral to

$$P = 4\int_2^{\infty} e^{-2u} u^2 du,$$

which is independent of *Z*. The probability is that found in Problem 41.33, 0.238, independent of *Z*.

42.1: a) $K = \frac{3}{2}kT \Rightarrow T = \frac{2K}{3k} = \frac{2(7.9\times10^{-4}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{3(1.38\times10^{-23}\text{ J/K})}$

$\Rightarrow T = 6.1\text{ K}$

b) $T = \frac{2(4.48\text{ eV})\,(1.60\times10^{-19}\text{ J/eV})}{3(1.38\times10^{-23}\text{ J/K})} = 34{,}600\text{ K}.$

c) The thermal energy associated with room temperature (300 K) is much greater than the bond energy of He_2 (calculated in part (a)), so the typical collision at room temperature will be more than enough to break up He_2. However, the thermal energy at 300 K is much less than the bond energy of H_2, so we would expect it to remain intact at room temperature.

42.2: a) $U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -5.0\text{ eV}.$

b) $-5.0\text{ eV} + (4.3\text{ eV} - 3.5\text{ eV}) = -4.2\text{ eV}.$

42.3: Let 1 refer to C and 2 to O.

$$m_1 = 1.993\times10^{-26}\text{ kg},\ m_2 = 2.656\times10^{-26}\text{ kg},\ r_0 = 0.1128\text{ nm}$$

$$r_1 = \left(\frac{m_2}{m_1+m_2}\right)r_0 = 0.0644\text{ nm (carbon)}$$

$$r_2 = \left(\frac{m_1}{m_1+m_2}\right)r_0 = 0.0484\text{ nm (carbon)}$$

b) $I = m_1r_1^2 + m_2r_2^2 = 1.45\times10^{-46}\text{ kg}\cdot\text{m}^2$; yes, this agrees with Example 42.2.

42.4: The energy of the emitted photon is $1.01\times10^{-5}\text{ eV}$, and so its frequency and wavelength are

$$f = \frac{E}{h} = \frac{(1.01\times10^{-5}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})} = 2.44\text{ GHz}$$

$$\lambda = \frac{c}{f} = \frac{(3.00\times10^{8}\text{ m/s})}{(2.44\times10^{9}\text{ Hz})} = 0.123\text{ m}.$$

This frequency corresponds to that given for a microwave oven.

42.5: a) From Example 42.2,

$$E_1 = 0.479 \text{ meV} = 7.674 \times 10^{-23} \text{ J and } I = 1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2$$

$$K = \tfrac{1}{2} I\omega^2 \text{ and } K = E \text{ gives } \omega = \sqrt{2E_1/I} = 1.03 \times 10^{12} \text{ rad/s}$$

b) $v_1 = r_1\omega_1 = (0.0644 \times 10^{-9} \text{ m})(1.03 \times 10^{12} \text{ rad/s}) = 66.3 \text{ m/s (carbon)}$

$v_2 = r_2\omega_2 = (0.0484 \times 10^{-9} \text{ m})(1.03 \times 10^{12} \text{ rad/s}) = 49.8 \text{ m/s (oxygen)}$

c) $T = 2\pi/\omega = 6.10 \times 10^{-12} \text{ s}$

42.6: a) $E_0 = \dfrac{1}{2}\hbar\omega = \dfrac{1}{2}(0.2690 \text{ eV}) = 2.083 \times 10^{-20} \text{ J}$

$$E_0 = \frac{1}{2} m_{\text{r}} v_{\text{max}}^2 \text{ gives } v_{\text{max}} = \sqrt{\frac{2E_0}{m_{\text{r}}}} = \sqrt{\frac{2(2.083 \times 10^{-2} \text{ J})}{1.139 \times 10^{-26} \text{ kg}}} = 1.91 \times 10^3 \text{ m/s}$$

b) According the Eq. 42.7 the spacing between adjacent vibrational energy levels is twice the ground state energy:

$$E_n = (n + \frac{1}{2})\,\hbar\omega,$$
$$\Delta E = E_{n+1} - E_n = \hbar\omega = hf.$$

Thus, using the ΔE specified in Example 42.3, it follows that its vibrational period is

$$T = \frac{1}{f} = \frac{h}{\Delta E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(0.2690 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 1.54 \times 10^{-14} \text{ s}.$$

c) The vibrational period is shorter than the rotational period.

42.7: a) $I = m_r r^2 = \left(\dfrac{m_{\text{Li}} m_{\text{H}}}{m_{\text{Li}} + m_{\text{H}}}\right) r^2$

$$= \frac{(1.17\times10^{-26}\,\text{kg})(1.67\times10^{-27}\,\text{kg})(1.59\times10^{-10}\,\text{m})^2}{(1.17\times10^{-26}\,\text{kg}+1.67\times10^{-27}\,\text{kg})}$$

$$= 3.69\times10^{-47}\,\text{kg}\cdot\text{m}^2$$

$$\Delta E = E_4 - E_3 = \frac{\hbar^2}{2I}(4(4+1)-(3)\,(3+1)) = \frac{4\hbar^2}{I}$$

$$\Rightarrow \Delta E = \frac{4(1.054\times10^{-34}\,\text{J}\cdot\text{s})^2}{3.69\times10^{-47}\,\text{kg}\cdot\text{m}^2}$$

$$\Rightarrow \Delta E = 1.20\times10^{-21}\ \text{J} = 7.53\times10^{-3}\,\text{eV}.$$

b) $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\text{m/s})}{1.20\times10^{-21}\,\text{J}} \Rightarrow \lambda = 1.66\times10^{-4}\,\text{m}.$

42.8: Each atom has a mass m and is at a distance $L/2$ from the center, so the moment of inertia is $2(m)(L/2)^2 = mL^2/2 = 2.21\times10^{-44}\,\text{kg}\cdot\text{m}^2$.

42.9: a) $E_l = \dfrac{l(l+1)\hbar^2}{2I},\ E_{l-1} = \dfrac{l(l-1)\hbar^2}{2I} \Rightarrow \Delta E = \dfrac{\hbar^2}{2I}(l^2+l-l^2+l) = \dfrac{l\hbar^2}{I}$

b) $f = \dfrac{\Delta E}{h} = \dfrac{\Delta E}{2\pi\hbar} = \dfrac{l\hbar}{2\pi I}.$

42.10: a) $\Delta E = \dfrac{hc}{\lambda} = \hbar\sqrt{k'/m_r},$ and solving for k',

$$k' = \left(\frac{2\pi c}{\lambda}\right)^2 m_r = 205\ \text{N/m}.$$

42.11: Energy levels are $E = E_n + E_l = \left(n + \frac{1}{2}\right)\hbar\omega + l(l+1)\frac{\hbar^2}{2I}$

$= \left(n + \frac{1}{2}\right)(0.269\text{ eV}) + l(l+1)\,(2.395\times10^{-4}\text{ eV})$ where the values are from Examples 42.2 and 42.3.

a) $n = 0, l = 1 \rightarrow n = 1, l = 2$:

$$\Rightarrow \Delta E = E_f - E_i = (1)(0.2690\text{ eV}) + (4)(2.395\times10^{-4}\text{eV}) = 0.2700\text{ eV}$$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{(0.2700\text{ eV})(1.602\times10^{-19}\text{ J/eV})}$$

$$= 4.592\times10^{-6}\text{m.}$$

b) $n = 0, l = 2 \rightarrow n = 1, l = 1$:

$$\Rightarrow \Delta E = E_f - E_i = (1)(0.2690\text{ eV}) + (-4)(2.395\times10^{-4}\text{eV}) = 0.2680\text{ eV}$$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{(0.2680\text{ eV})(1.602\times10^{-19}\text{ J/eV})}$$

$$\Rightarrow \lambda = 4.627\times10^{-6}\text{m}$$

c) $n = 0, l = 3 \rightarrow n = 1, l = 2$:

$$\Delta E = E_f - E_i = (1)(0.2690\text{ eV}) + (-6)(2.395\times10^{-4}\text{eV}) = 0.2676\text{ eV}$$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{(0.2676\text{ eV})(1.602\times10^{-19}\text{ J/eV})} = 4.634\times10^{-6}\text{m.}$$

42.12: $\hbar\omega = \hbar\sqrt{k'/m_\text{r}} = \hbar\sqrt{2k'/m} = 3.14\times10^{-20}\text{ J} = 0.196\text{ eV}$, where $m_\text{r} = m/2$ has been used.

42.13: a) $f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k'}{m_r}}$

$$\Rightarrow k'=m_r(2\pi f)^2=\frac{m_1m_2(2\pi f)^2}{m_1+m_2}$$

$$\Rightarrow k'=\frac{(1.67\times10^{-27}\text{ kg})\ (3.15\times10^{-26}\text{ kg})[2\pi(1.24\times10^{14}\text{ Hz})]^2}{(1.67\times10^{-27}\text{ kg}+3.15\times10^{-26}\text{kg})}$$

$$\Rightarrow k'=963\text{ N/m}$$

b) $\Delta E=\left(n+\frac{3}{2}\right)\hbar\omega-\left(n+\frac{1}{2}\right)\hbar\omega=\hbar\omega=hf$

$$\Rightarrow \Delta E=(6.63\times10^{-34}\text{ J}\cdot\text{s})(1.24\times10^{14}\text{ Hz})=8.22\times10^{-20}\text{ J}$$
$$=0.513\text{ eV}$$

c) $\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\text{ m/s}}{1.24\times10^{14}\text{ Hz}}=2.42\times10^{-6}\text{ m (infrared)}$

42.14: a) As a photon,

$$\lambda=\frac{hc}{E}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(6.20\times10^{3}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}=0.200\text{ nm.}$$

b) As a matter wave,

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mE}}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\text{ kg})(37.6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}=0.200\text{ nm and}$$

c) as a matter wave,

$$\lambda=\frac{h}{\sqrt{2mE}}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{\sqrt{2(1.67\times10^{-27}\text{ kg})(0.0205\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}=0.200\text{ nm}.$$

42.15: The volume enclosing a single sodium and chlorine atom $=2(2.82\times10^{-10}\text{ m})^3=$ $4.49\times10^{-29}\text{ m}^3$. So the density

$$\rho=\frac{m_{\text{Na}}+m_{\text{Cl}}}{V}=\frac{3.82\times10^{-26}\text{ kg}+5.89\times10^{-26}\text{ kg}}{4.49\times10^{-29}\text{ m}^3}$$

$$\Rightarrow \rho=2.16\times10^{3}\text{ kg/m}^3.$$

42.16: For an average spacing *a*, the density is $\rho = m/a^3$, where *m* is the average of the ionic masses, and so

$$a^3 = \frac{m}{\rho} = \frac{\left(6.49\times10^{-26}\ \text{kg} + 1.33\times10^{-25}\ \text{kg}\right)/2}{(2.75\times10^3\ \text{kg/m}^3)} = 3.60\times10^{-29}\ \text{m}^3,$$

and $a = 3.30\times10^{-10}\ \text{m} = 0.330\ \text{nm}$. b) The larger (higher atomic number) atoms have the larger spacing.

42.17: $$\Delta E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times10^8\ \text{m/s})}{9.31\times10^{-13}\ \text{m}} = 2.14\times10^{-13}\ \text{J}$$

$$= 1.34\times10^6\ \text{eV}$$

So the number of electrons that can be excited to the conduction band is $n = \dfrac{1.34\times10^6\ \text{eV}}{1.12\ \text{eV}}$

$= 1.20\times10^6$ electrons.

42.18: a) $\dfrac{hc}{\Delta E} = 2.27\times10^{-7}\ \text{m} = 227\ \text{nm}$, in the ultraviolet.

b) Visible light lacks enough energy to excite the electrons into the conduction band, so visible light passes through the diamond unabsorbed.

c) Impurities can lower the gap energy making it easier for the material to absorb shorter wavelength visible light. This allows longer wavelength visible light to pass through, giving the diamond color.

42.19: a) To be detected the photon must have enough energy to bridge the gap width $\Delta E = 1.12\ \text{eV}$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{\left(1.12\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 1.11\times10^{-6}\ \text{m}, \text{ in the infrared.}$$

b) Visible photons have more than enough energy to excite electrons from the valence to conduction band. Thus visible light is absorbed, making silicon opaque.

42.20: $v_{\text{rms}} = \sqrt{3kT/m} = 1.17\times10^5\ \text{m/s}$, as found in Example 42.9. The equipartition theorem does not hold for the electrons at the Fermi energy. Although these electrons are very energetic, they cannot lose energy, unlike electrons in a free electron gas.

42.21: a) $\dfrac{-\hbar^2}{2m}\left(\dfrac{\partial^2\psi}{\partial x^2}+\dfrac{\partial^2\psi}{\partial y^2}+\dfrac{\partial^2\psi}{\partial z^2}\right)=E\psi$

where $\psi = A\sin\dfrac{n_x\pi x}{L}\sin\dfrac{n_y\pi y}{L}\sin\dfrac{n_z\pi z}{L}$

$\dfrac{\partial^2\psi}{\partial x^2}=-\left(\dfrac{n_x\pi}{L}\right)^2\psi,$ and similarly for $\dfrac{\partial^2\psi}{\partial y^2}$ and $\dfrac{\partial^2\psi}{\partial z^2}$.

$$\Rightarrow -\frac{\hbar^2}{2m}\left(-\left(\frac{n_x\pi}{L}\right)^2-\left(\frac{n_y\pi}{L}\right)^2-\left(\frac{n_z\pi}{L}\right)^2\right)\psi = E\psi$$

$$\Rightarrow E=\frac{\left(n_x^2+n_y^2+n_z^2\right)\pi^2\hbar^2}{2mL^2}$$

b) Ground state $\Rightarrow n_x \Rightarrow n_y \Rightarrow n_z = 1 \Rightarrow E=\dfrac{3\pi^2\hbar^2}{2mL^2}$,

The only degeneracy is from the two spin states. The first excited state $\Rightarrow (2,1,1)$ or $(1,2,1$

or $(1,1,2)\Rightarrow E=\dfrac{3\pi^2\hbar^2}{mL^2}$ and the degeneracy is $(2)\times(3)=6$.

The second excited state $\Rightarrow (2,2,1)$ or $(2,1,2)$ or $(1,2,2)\Rightarrow E=\dfrac{9\pi^2\hbar^2}{2mL^2}$ and the degeneracy is $(2)\times(3)=6$.

42.22: $1=\int|\psi|^2 dV$

$$=A^2\left(\int_0^L \sin^2\left(\frac{n_x\pi x}{L}\right)dx\right)\left(\int_0^L \sin^2\left(\frac{n_y\pi y}{L}\right)dy\right)\left(\int_0^L \sin^2\left(\frac{n_z\pi z}{L}\right)dz\right)$$

$$=A^2\left(\frac{L}{2}\right)^3,$$

so $A=(2/L)^{3/2}$ (assuming A to be real positive).

42.23: Density of states:

$$g(E) = \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2}$$

$$\Rightarrow g(E) = \frac{(2(9.11\times10^{-31}\text{ kg}))^{3/2}(1.0\times10^{-6}\text{ m}^3)(5.0\text{ eV})^{1/2}(1.60\times10^{-19}\text{ J/eV})^{1/2}}{2\pi^2(1.054\times10^{-34}\text{ J}\cdot\text{s})^3}$$

$$= \left(9.5\times10^{40}\text{ states/J}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)$$

$$= 1.5\times10^{22}\text{ states/eV}.$$

42.24: Equation (42.13) may be solved for $n_{\text{rs}} = (2mE)^{1/2}\,(L/\hbar\pi)$, and substituting this into Eq. (42.12), using $L^3 = V$, gives Eq. (42.14).

42.25: Eq.(42.13): $E = \dfrac{n_{\text{rs}}^2\pi^2\hbar^2}{2mL^2}$

$$\Rightarrow n_{\text{rs}} = \frac{L}{\pi\hbar}\sqrt{2mE}$$

$$= \frac{0.010\text{ m}}{\pi(1.054\times10^{-34}\text{ J}\cdot\text{s})}\sqrt{2(9.11\times10^{-31}\text{ kg})\,(7.0\text{ eV})\,(1.60\times10^{-19}\text{ J/eV})}$$

$$\Rightarrow n_{\text{rs}} = 4.3\times10^7.$$

42.26: a) From Eq. (42.22), $E_{\text{av}} = \dfrac{3}{5}E_{\text{F}} = 1.94\text{ eV}.$

b) $\sqrt{2E/m} = \sqrt{\dfrac{2(1.94)\left(1.60\times10^{-19}\text{ J/eV}\right)}{9.11\times10^{-31}\text{ kg}}} = 8.25\times10^5\text{ m/s}.$

c) $\dfrac{E_{\text{F}}}{k} = \dfrac{(3.23\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}{\left(1.38\times10^{-23}\text{ J/K}\right)} = 3.74\times10^4\text{ K}.$

42.27: a) $C_V = \left(\dfrac{\pi^2 kT}{2E_{\text{F}}}\right)R = \left(\dfrac{\pi^2\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})}{2(5.48\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}\right)R$

$$\Rightarrow C_V = 0.0233\,R = 0.194\text{ J/mol K}.$$

b) $\dfrac{0.194\text{ J/mol K}}{25.3\text{ J/mol K}} = 7.68\times10^{-3}.$

c) Mostly ions (see Section 18.4).

42.28: a) See Example 42.10: The probabilities are 1.78×10^{-7}, 2.37×10^{-6}, and 1.51×10^{-5}. b) The Fermi distribution, Eq. (42.17), has the property that $f(E_F - E) = 1 - f(E)$ (see Problem (42.46)), and so the probability that a state at the top of the valence band is occupied is the same as the probability that a state of the bottom of the conduction band is filled (this result depends on having the Fermi energy in the middle of the gap).

42.29: $$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$\Rightarrow E_F = E - kT\ \ln\left(\frac{1}{f(E)} - 1\right)$$

$$E_F = E - \left(1.38\times10^{-23}\ \text{J/K}\right)(300\ \text{K})\ln\left(\frac{1}{4.4\times10^{-4}} - 1\right)$$

$$E_F = E - \left(3.20\times10^{-20}\ \text{J}\right) = E - 0.20\ \text{eV}.$$

So the Fermi level is 0.20 eV below the conduction band.

42.30: a) Solving Eq. (42.23) for the voltage as a function of current,

$$V = \frac{kT}{e}\ln\left(\frac{I}{I_S} + 1\right) = \frac{kT}{e}\ln\left(\frac{40.0\ \text{mA}}{3.60\ \text{mA}} + 1\right) = 0.0645\ \text{V}.$$

b) From part (a), the quantity $e^{eV/kT} = 12.11$, so far a reverse-bias voltage of the same magnitude,

$$I = I_S\left(e^{-eV/kT} - 1\right) = I_S\left(\frac{1}{12.11} - 1\right) = -3.30\ \text{mA}.$$

42.31: $I = I_S\left(e^{eV/kT} - 1\right) \Rightarrow I_S = \dfrac{1}{e^{eV/kT - 1}}$

a) $\dfrac{eV}{kT} = \dfrac{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.50\times10^{-2}\ \text{V}\right)}{\left(1.38\times10^{-23}\ \text{J/K}\right)\left(300\ \text{K}\right)} = 0.580$

$$\Rightarrow I_S = \frac{9.25\times10^{-3}\ \text{A}}{e^{0.580} - 1} = 0.0118\ \text{A}.$$

Now for $V = 0.0100\ \text{V}$, $eV/kT = 0.387$

$$\Rightarrow I = \left(0.0118\ \text{A}\right)\left(e^{+0.387} - 1\right) = 5.56\times10^{-3}\ \text{A} = 5.56\ \text{mA}$$

b) Now with $V = -15.0\ \text{mV}$, $\dfrac{eV}{kT} = -0.580$

$$\Rightarrow I = \left(0.0118\ \text{A}\right)\left(e^{-0.580} - 1\right) = -5.18\times10^{-3}\ \text{A}\ .$$

If $V = -10.0\ \text{mV} \Rightarrow \dfrac{eV}{kT} = -0.387$

$$\Rightarrow I = \left(0.0118\ \text{A}\right)\left(e^{-0.387} - 1\right) = -3.77\times10^{-3}\ \text{A}\ .$$

42.32: See Problem (42.7): $I = \dfrac{2\hbar^2}{\Delta E} = \dfrac{h\lambda}{2\pi^2 c} = 7.14\times10^{-48}\ \text{kg}\cdot\text{m}^2$.

42.33: a) $p = qd = \left(1.60\times10^{-19}\ \text{C}\right)\left(2.4\times10^{-10}\ \text{m}\right) = 3.8\times10^{-29}\ \text{C}\cdot\text{m}$

b) $q = \dfrac{p}{d} = \dfrac{3.0\times10^{-29}\ \text{C}\cdot\text{m}}{2.4\times10^{-10}\ \text{m}} = 1.3\times10^{-19}\ \text{C}$

c) $q/e = 0.78$

d) $q = \dfrac{p}{d} = \dfrac{1.5\times10^{-30}\ \text{C}\cdot\text{m}}{1.6\times10^{-10}\ \text{m}} = 9.4\times10^{-21}\ \text{C}$

$\Rightarrow q/e = 0.059$

This is much less than for sodium chloride (part (c)). Therefore the bond for hydrogen iodide is more covalent in nature than ionic.

42.34: The electrical potential energy is $U = -5.13\,\text{eV}$, and

$$r = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{U} = 2.8\times10^{-10}\ \text{m}.$$

42.35: a) For maximum separation of Na^+ and Cl^- for stability:

$$U = \frac{-e^2}{4\pi\varepsilon_0 r} = -(5.1\,\text{eV} - 3.6\,\text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right) = -2.40\times10^{-19}\ \text{J}$$

$$\Rightarrow r = \frac{(1.60\times10^{-19}\ \text{C})^2}{4\pi\varepsilon_0(2.40\times10^{-19}\ \text{J})} = 9.6\times10^{-10}\ \text{m}.$$

b) For K^+ and Br^-:

$$U = -\frac{e^2}{4\pi\varepsilon_0 r} = -(4.3\,\text{eV} - 3.5\,\text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right) = -1.28\times10^{-19}\ \text{J}$$

$$\Rightarrow r = \frac{(1.60\times10^{-19}\ \text{C})^2}{4\pi\varepsilon_0(1.28\times10^{-19}\ \text{J})} = 1.8\times10^{-9}\ \text{m}.$$

42.36: The energies corresponding to the observed wavelengths are 3.29×10^{-21} J,

2.87×10^{-21}J, 2.47×10^{-21} J, 2.06×10^{-21} J and 1.65×10^{-21} J. The average spacing of thes energies is 0.410×10^{-21} J and using the result of Problem (44-4), these are seen to corresp to transition from levels 8, 7, 6, 5 and 4 to the respective next lower levels.

Then, $\dfrac{\hbar^2}{I} = 0.410\times10^{-21}$ J, from which $I = 2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2$.

42.37: a) Pr. (44.36) yields $I = 2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2$, and so $r = \sqrt{\dfrac{I}{m_\text{r}}} = \sqrt{\dfrac{I(m_\text{H}+m_\text{Cl})}{m_\text{H}m_\text{Cl}}}$

$$\Rightarrow r = \sqrt{\frac{(2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2)\,(1.67\times10^{-27}\,\text{kg} + 5.81\times10^{-26}\ \text{kg})}{(1.67\times10^{-27}\ \text{kg} + 5.81\times10^{-26}\ \text{kg})}}$$

$$\Rightarrow r = 1.29\times10^{-10}\ \text{m}.$$

b) From $l \to l-1 : \Delta E = \dfrac{\hbar^2}{2I}\left[l(l+1)-(l-1)l\right] = \dfrac{l\hbar^2}{I}$. But $\Delta E = \dfrac{hc}{\lambda} \Rightarrow l = \dfrac{2\pi cI}{\hbar\lambda} =$

$\dfrac{4.84\times10^{-4}\ \text{m}}{\lambda}$. So the l-values that lead to the wavelength of Pr. (44-32) are:

$$\lambda = 6.04\times10^{-5}\ \text{m}: \quad l = \frac{4.84\times10^{-4}\ \text{m}}{6.04\times10^{-5}\ \text{m}} = 8.$$

Similarly for:

$\lambda = 6.90\times10^{-5}\ \text{m}: \ l = 7; \ \lambda = 8.04\times10^{-5}\,\text{m}: \ l = 6$

$\lambda = 9.64\times10^{-5}\ \text{m}: \ l = 5; \ \lambda = 1.204\times10^{-4}\,\text{m}: \ l = 4.$

c) The longest wavelength means the least transition energy $(l = 1 \to l = 0)$

$$\Rightarrow \Delta E = \frac{(1)\,(1.054\times10^{-34}\,\text{J}\cdot\text{s})^2}{2.71\times10^{-47}\,\text{kg}\cdot\text{m}^2} = 4.10\times10^{-22}\ \text{J}$$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = 4.85\times10^{-4}\,\text{m}.$$

d) If the hydrogen atom is replaced by deuterium, then the reduced mass changes to $m'_\text{r} = 3.16\times10^{-27}\,\text{kg}$. Now,

$$\Delta E = \frac{l\hbar^2}{I'} = \frac{hc}{\lambda'} \Rightarrow \lambda' = \frac{2\pi cI'}{l\hbar} = \frac{2\pi c m'_r r^2}{l\hbar}$$

$$\Rightarrow \lambda' = \left(\frac{m'_\text{r}}{m_\text{r}}\right)\lambda = \left(\frac{3.16\times10^{-27}\,\text{kg}}{1.62\times10^{-27}\ \text{kg}}\right)\lambda = (1.95)\lambda$$

So for $l = 8 \to l = 7$: $\quad \lambda = (60.4\ \mu\text{m})\,(1.95) = 118\ \mu\text{m}.$

$l = 7 \to l = 6$: $\quad \lambda = (69.0\ \mu\text{m})\,(1.95) = 134\ \mu\text{m}.$

$l = 6 \to l = 5$: $\quad \lambda = (80.4\ \mu\text{m})\,(1.95) = 156\ \mu\text{m}.$

$l = 5 \to l = 4$: $\quad \lambda = (96.4\ \mu\text{m})\,(1.95) = 188\ \mu\text{m}.$

$l = 4 \to l = 3$: $\quad \lambda = (120.4\ \mu\text{m})\,(1.95) = 234\ \mu\text{m}.$

42.38: From the result of Problem (42.9), the moment inertia of the molecule is

$$I = \frac{\hbar^2 l}{\Delta E} = \frac{hl\lambda}{4\pi^2 c} = 6.43\times 10^{-46}\,\text{kg}\cdot\text{m}^2$$

and from Eq. (42.6) the separation is

$$r_0 = \sqrt{\frac{I}{m_\text{r}}} = 0.193\,\text{nm}.$$

42.39: a) $E_{ex} = \dfrac{L^2}{2I} = \dfrac{\hbar^2 l(l+1)}{2I}.$

$E_g = 0\ (l = 0),$

and there is an additional multiplicative factor of $2l + 1$ because for each l state there are really $2l+1\ m_l$ states with the same energy.

$$\text{So}\ \frac{n_l}{n_0} = (2l+1)e^{-\left[\frac{\hbar^2 l(l+1)}{2I}\right]\Big/kT}.$$

b) $T = 300\ \text{K}, I = 1.449\times 10^{-46}\ \text{kg}\cdot\text{m}^2.$

(i) $E_{l=1} = \dfrac{\hbar^2(1)\,(1+1)}{2(1.449\times 10^{-46}\,\text{kg}\cdot\text{m}^2)} = 7.67\times 10^{-23}\ \text{J}.$

$$\frac{E_{l=1}}{kT} = \frac{7.67\times 10^{-23}\,\text{J}}{(1.38\times 10^{-23}\ \text{J/K})\ (300\ \text{K})} = 0.0185.$$

$(2l+1) = 3$

so $\dfrac{n_{l=1}}{n_0} = (3)e^{-0.0185} = 2.95.$

(ii) $\dfrac{E_{l=2}}{kT} = \dfrac{\hbar^2(2)\ (2+1)}{2(1.449\times 10^{-46}\ \text{kg}\cdot\text{m}^2)\ (1.38\times 10^{-23}\ \text{J/K})\ (300\ \text{K})} = 0.0556.$

$(2l+1) = 5.$

$$\frac{n_{l=1}}{n_0} = (5)(e^{-0.0556}) = 4.73.$$

(iii) $\dfrac{E_{l=10}}{kT} = \dfrac{\hbar^2(10)\ (10+1)}{2(1.449\times 10^{-46}\ \text{kg}\cdot\text{m}^2)\ (1.38\times 10^{-23}\ \text{J/K})\ (300\ \text{K})} = 1.02.$

$(2l+1) = 21$ so $\dfrac{n_{l=10}}{n_0} = (21)\ (e^{-1.02}) = 7.57.$

(iv) $\dfrac{E_{l=20}}{kT} = \dfrac{\hbar^2(20)\ (20+1)}{2(1.449\times 10^{-46}\ \text{kg}\cdot\text{m}^2)\ (1.38\times 10^{-23}\ \text{J/K})\ (300\ \text{K})} = 3.89.$

$(2l+1) = 41.$

$$\frac{n_{l=20}}{n_0} = (41)e^{-3.89} = 0.838.$$

42.40: a) $I_{co} = 1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2.$

$$E_{l=1} = \frac{\hbar^2 l(l+1)}{2I} = \frac{1.054\times10^{-34}\ \text{J}\cdot\text{s})^2(1)\,(1+1)}{2(1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2)} = 7.67\times10^{-23}\ \text{J}.$$

$$\text{E}_{l=0} = 0.$$

$$\Delta E = 7.67\times10^{-23}\ \text{J} = 4.79\times10^{-4}\ \text{eV}.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times10^{8}\ \text{m/s})}{(7.67\times10^{-23}\ \text{J})} = 2.59\times10^{-3}\ \text{m} = 2.59\ \text{mm}.$$

b) Let's compare the value of kT when T=20 K to that of ΔE for the $l = 1 \rightarrow l = 0$ rotational transition:

$$kT = (1.38\times10^{-23}\ \text{J/K})\,(20\ \text{K}) = 2.76\times10^{-22}\ \text{J}.$$

$$\Delta E = 7.67\times10^{-23}\,\text{J (from part (a)). So}\,\frac{kT}{\Delta E} = 3.60.$$

Therefore, although T is quite small, there is still plenty of energy to excite CO molecules into the first rotational level. This allows astronomers to detect the 2.59 mm wavelength radiation from such molecular clouds.

42.41: a) $I = m_r r^2 = \dfrac{m_{Na} m_{Cl}}{m_{Na} + m_{Cl}} r^2$

$$= \frac{(3.8176\times10^{-26}\,\text{kg})\,(5.8068\times10^{-26}\,\text{kg})\,(2.361\times10^{-10}\,\text{m})^2}{(3.8176\times10^{-26}\,\text{kg} + 5.8068\times10^{-26}\,\text{kg})}$$

$$\Rightarrow I = 1.284\times10^{-45}\ \text{kg}\cdot\text{m}^2$$

For $l = 2 \rightarrow l = 1$:

$$\Rightarrow \Delta E = E_2 - E_1 = (6-2)\frac{\hbar^2}{2I} = \frac{2(1.054\times10^{-34}\ \text{J}\cdot\text{s})^2}{1.284\times10^{-45}\,\text{kg}\cdot\text{m}^2} = 1.730\times10^{-23}\ \text{J}$$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = 0.01148\,\text{m} = 1.148\,\text{cm}.$$

For $l = 1 \rightarrow l = 0: \Delta E = E_1 - E_0 = (2-0)\dfrac{\hbar^2}{2I} = \dfrac{1}{2}(1.730\times10^{-23}\ \text{J})$

$$= 8.650\times10^{-24}\ \text{J}$$

$$\lambda = \frac{hc}{\Delta E} = 2.297\ \text{cm}.$$

b) Carrying out exactly the same calculation for $Na^{37}Cl$, where $m_r(37) = 2.354\times 10^{-26}$ kg and $I(37) = 1.312\times10^{-45}\,\text{kg}\cdot\text{m}^2$ we find for

$l = 2 \rightarrow l = 1$: $\Delta E = 1.693\times10^{-23}$ J and $\lambda = 1.173$ cm. For

$l = 1 \rightarrow l = 0$: $\Delta E = 8.465\times10^{-24}$ J and $\lambda = 2.347$ cm.

So the differences in wavelength are:

$l = 2 \rightarrow l = 1$: $\Delta\lambda = 1.173\,\text{cm} - 1.148\,\text{cm} = 0.025\,\text{cm}.$

$l = 1 \rightarrow l = 0$: $\Delta\lambda = 2.347\ \text{cm} - 2.297\ \text{cm} = 0.050\ \text{cm}.$

42.42: The vibration frequency is, from Eq. (42.8), $f = \dfrac{\Delta E}{h} = 1.12\times10^{14}$ Hz. The force constant is

$$k' = (2\pi f)^2 m_r = 777\,\text{N/m}.$$

42.43: $E_n = \left(n + \dfrac{1}{2}\right)\hbar\sqrt{\dfrac{k'}{m_r}} \Rightarrow E_0 = \dfrac{1}{2}\hbar\sqrt{\dfrac{2k'}{m_H}}$

$$\Rightarrow E_0 = \frac{1}{2}(1.054\times10^{-34}\ \text{J}\cdot\text{s})\sqrt{\frac{2(576\ \text{N/m})}{1.67\times10^{-27}\ \text{kg}}} = 4.38\times10^{-20}\ \text{J} = 0.274\ \text{eV}.$$

This is much less than the H_2 bond energy.

42.44: a) The frequency is proportional to the reciprocal of the square root of the reduced mass, and in terms of the atomic masses, the frequency of the isotope with the deuterium atom is

$$f = f_0\left(\frac{m_F m_H/(m_H + m_F)}{m_F m_D/(m_D + m_F)}\right)^{1/2} = f_0\left(\frac{1+(m_F/m_D)}{1+(m_F/m_H)}\right)^{1/2}.$$

Using f_0 from Exercise (42.13) and the given masses, $f = 8.99\times10^{13}$ Hz.

42.45: a) $I = m_r r^2 = \dfrac{m_H m_I r^2}{m_H + m_I} = (1.657\times10^{-27}\ \text{kg})\ (0.160\times10^{-9}\ \text{m})^2$

$$= 4.24\times10^{-47}\ \text{kg}\cdot\text{m}^2$$

b) Vibration-rotation energy levels are:

$$E_l = l(l+1)\frac{\hbar^2}{2I} + \left(h + \frac{1}{2}\right)\ \hbar\sqrt{\frac{k'}{m_r}}$$

$$= l(l+1)\frac{\hbar^2}{2I} + \left(n + \frac{1}{2}\right)\ hf\ \left(\text{since}: \omega = 2\pi f = \sqrt{\frac{k'}{m_r}}\right)$$

i) $n = 1, l = 1 \rightarrow n = 0, l = 0$:

$$\Delta E = (2-0)\frac{\hbar^2}{2I} + \left(\frac{3}{2} - \frac{1}{2}\right)\ hf = \frac{\hbar^2}{I} + hf.$$

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{hc}{\frac{\hbar^2}{I} + hf} = \frac{c}{\frac{\hbar}{2\pi I} + f} = \frac{3.00\times10^8\ \text{m/s}}{3.96\times10^{11}\ \text{Hz} + 6.93\times10^{13}\ \mu}.$$

$$\Rightarrow \lambda = 4.30\times10^{-6}\ \text{m}.$$

ii) $n = 1, l = 2 \rightarrow n = 0, l = 1$:

$$\Delta E = (6-2)\frac{\hbar^2}{2I} + hf$$

$$\Rightarrow \lambda = \frac{c}{2\left(\frac{\hbar^2}{2\pi I}\right) + f} = \frac{3.00\times10^8\ \text{m/s}}{2(3.96\times10^{11}\text{Hz}) + 6.93\times10^{13}\text{Hz}} = 4.28\times10^{-6}\text{m}.$$

iii) $n = 2, l = 2 \rightarrow n = 1, l = 3$

$$\Delta E = (6-12)\frac{\hbar^2}{2I} + hf$$

$$\Rightarrow \lambda = \frac{c}{-3\left(\frac{\hbar^2}{2\pi I}\right) + f} = \frac{3.00\times10^8\ \text{m/s}}{-3(3.96\times10^{11}\ \text{Hz}) + 6.93\times10^{13}\ \text{Hz}} = 4.40\times10^{-6}\ \text{m}.$$

42.46: The sum of the probabilities is

$$f(E_F+\Delta E)+f(E_F-\Delta E)=\frac{1}{e^{-\Delta E/kT}+1}+\frac{1}{e^{\Delta E/kT}+1}$$
$$=\frac{1}{e^{-\Delta E/kT}+1}+\frac{e^{-\Delta E/kT}}{1+e^{-\Delta E/kT}}$$
$$=1.$$

42.47: Since potassium is a metal we approximate $E_F=E_{F0}$.

$$\Rightarrow E_F=\frac{3^{2/3}\pi^{4/3}\hbar^2 n^{2/3}}{2m}$$

But the electron concentration $n=\dfrac{\rho}{m}$

$$\Rightarrow n=\frac{851\ \text{kg/m}^3}{6.49\times10^{-26}\ \text{kg}}=1.31\times10^{28}\ \text{electron/m}^3$$

$$\Rightarrow E_F=\frac{3^{2/3}\pi^{4/3}(1.054\times10^{-34}\ \text{J}\cdot\text{s})^2(1.31\times10^{28}/\text{m}^3)^{2/3}}{2(9.11\times10^{-31}\ \text{kg})}=3.24\times10^{-19}\,\text{J}=2.03\,\text{eV}.$$

42.48: a) First we calculate the number-density of neutrons from the given mass-density:

$$n=(7.0\times10^{17}\ \text{kg/m}^3)/1.67\times10^{-27}\ \text{kg/neutron})=4.2\times10^{44}\ \text{m}^{-3}.$$

Now use Eq. 44.21

$$E_{F0}=\frac{3^{\frac{2}{3}}\pi^{\frac{4}{3}}\hbar^2 n^{\frac{1}{3}}}{2m}=\frac{3^{\frac{2}{3}}\pi^{\frac{4}{3}}(6.63\times10^{-34}\ \text{J}\cdot\text{s}/2\pi)^2\,(4.2\times10^{44}\text{m}^{-3})^{\frac{2}{3}}}{2(1.67\times10^{-27}\ \text{kg})}=1.8\times10^{-11}\ \text{J}.$$

b) Set $kT=E_{F0}$ (see Exercise 42.26) to obtain

$$T=\frac{E_{F0}}{k}=\frac{(1.8\times10^{-11}\ \text{J})}{(1.38\times10^{-28}\ \text{J/K})}=1.3\times10^{12}\,\text{K}.$$

42.49: a) Each unit cell has one atom at its center and 8 atoms at its corners that are each shared by 8 other unit cells. So there are $1+8/8=2$ atoms per unit cell.

$$\frac{n}{V}=\frac{2}{(0.35\times10^{-9}\ \text{m})^3}=4.66\times10^{28}\ \text{atoms/m}^3$$

b) $$E_{F0}=\frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\frac{N}{V}\right)^{2/3}$$

In this equation N/V is the number of free electrons per m^3. But the problem says to assume one free electron per atom, so this is the same as n/V calculated in part (a).

$m=9.109\times10^{-31}$ kg (the electron mass), so $E_{F0}=7.563\times10^{-19}\ \text{J}=4.7\ \text{eV}$

42.50: a) $\dfrac{d}{dr}U_{\text{tot}} = \dfrac{\alpha e^2}{4\pi\varepsilon_0}\dfrac{1}{r^2} - 8A\dfrac{1}{r^9}$.

Setting this equal to zero when $r = r_0$ gives

$$r_0^7 = \frac{8A4\pi\varepsilon_0}{\alpha e^2}$$

and so

$$U_{\text{tot}} = \frac{\alpha e^2}{4\pi\varepsilon_0}\left(-\frac{1}{r} + \frac{r_0^7}{8r^8}\right).$$

At $r = r_0$,

$$U_{\text{tot}} = -\frac{7\alpha e^2}{32\pi\varepsilon_0 r_0} = -1.26\times10^{-18}\ \text{J} = -7.85\ \text{eV}.$$

b) To remove a Na^+Cl^- ion pair from the crystal requires 7.85 eV. When neutral Na and Cl atoms are formed from the Na^+ and Cl^- atoms there is a net release of energy $-5.14\ \text{eV} + 3.61\ \text{eV} = -1.53\ \text{eV}$, so the net energy required to remove a neutral Na, Cl pair from the crystal is $7.85\ \text{eV} - 1.53\ \text{eV} = 6.32\ \text{eV}$.

42.51: a) $E_{av} = \dfrac{3}{5}E_{\text{F0}} = \dfrac{3}{5}\dfrac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\dfrac{N}{V}\right)^{2/3}$. $E_{\text{tot}} = NE_{av}$.

$$\frac{dE_{\text{tot}}}{dV} = N\frac{2}{3}\cdot\frac{3}{5}\cdot\frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\frac{N}{V}\right)^{-1/3}\left(\frac{-N}{V^2}\right)$$

$$= -\frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3}.$$

$$P = -\frac{dE_{\text{tot}}}{dV} = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3}.$$

b) $\dfrac{N}{V} = 8.45\times10^{28}\ m^{-3}$

$$\Rightarrow p = \frac{3^{2/3}\pi^{4/3}(1.054\times10^{-34}\ \text{J}\cdot\text{s})^2}{5(9.11\times10^{-31}\ \text{kg})}(8.45\times10^{28}\ \text{m}^{-3})^{5/3}$$

$$= 3.80\times10^{10}\ \text{Pa} = 3.76\times10^5\ \text{atm (!)}$$

c) There is a large attractive force on the electrons by the copper ions.

42.52: a) From Problem (42.51):

$$p = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3}.$$

$$B = -V\frac{dp}{dV} = -V\left[\frac{5}{3}\cdot\frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\cdot\left(\frac{N}{V}\right)^{2/3}\left(\frac{-N}{V^2}\right)\right]$$

$$= \frac{5}{3}p.$$

b) $\dfrac{N}{V} = 8.45\times10^{28}\ \text{m}^{-3}.$

$$B = \frac{5}{3}\cdot\frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}(8.45\times10^{28}\,\text{m}^{-3})^{5/3} = 6.33\times10^{10}\ \text{Pa}.$$

c) $\dfrac{6.33\times10^{10}\ \text{Pa}}{1.4\times10^{11}\ \text{Pa}} = 0.45.$ The copper ions themselves make up the remaining fraction.

42.53: a) $E_{\text{F0}} = \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\dfrac{N}{V}\right)^{2/3}.$

Let $E_{\text{F0}} = \dfrac{1}{100}mc^2 \Rightarrow \left(\dfrac{N}{V}\right) = \left[\dfrac{2m^2c^2}{(100)3^{2/3}\pi^{4/3}\hbar^2}\right]^{3/2} = \dfrac{2^{3/2}m^3c^3}{100^{3/2}3\pi^2\hbar^3} = \dfrac{2^{3/2}m^3c^3}{3000\pi^2\hbar^3} =$ $1.67\times10^{33}\ \text{m}^{-3}.$

b) $\dfrac{8.45\times10^{28}\ \text{m}^{-3}}{1.67\times10^{33}\ \text{m}^{-3}} = 5.06\times10^{-5}.$

Since the real concentration of electrons in copper is less than one part in 10^{-4} of the concentration where relativistic effects are important, it is safe to ignore relativistic effects for most applications.

c) The number of electrons is $N_e = \dfrac{6(2\times10^{30}\ \text{kg})}{1.99\times10^{-26}\ \text{kg}} = 6.03\times10^{56}.$ The concentration is $\dfrac{N_e}{V} = \dfrac{6.03\times10^{56}}{\frac{4}{3}\pi(6.00\times10^{6}\ \text{m})^3} = 6.66\times10^{35}\,\text{m}^{-3}.$

d) Comparing this to the result from part (a) $\dfrac{6.66\times10^{35}\ \text{m}^{-3}}{1.67\times10^{33}\ \text{m}^{-3}} \cong 400$ so relativistic effects will be very important.

42.54: a) Following the hint,

$$k'dr = -d\left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)_{r=r_0} = \frac{1}{2\pi\varepsilon_0}\frac{e^2}{r_0^3}dr$$

and

$$\hbar\omega = \hbar\sqrt{2k'/m} = \hbar\sqrt{\frac{1}{\pi\varepsilon_0}\frac{e^2}{mr_0^3}} = 1.23\times10^{-19}\text{ J} = 0.77\text{ eV},$$

where $(m/2)$ has been used for the reduced mass. b) The reduced mass is doubled, and the energy is reduced by a factor of $\sqrt{2}$ to 0.54 eV.

42.55: a) $U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{-1}{d}+\frac{1}{r}-\frac{1}{r+d}-\frac{1}{r-d}+\frac{1}{r}-\frac{1}{d}\right)$

$$= \frac{q^2}{4\pi\varepsilon_0}\left(\frac{2}{r}-\frac{2}{d}-\frac{1}{r+d}-\frac{1}{r-d}\right)$$

but $\frac{1}{r+d}+\frac{1}{r-d} = \frac{1}{r}\left(\frac{1}{1+\frac{d}{r}}+\frac{1}{1-\frac{d}{r}}\right)$

$$\approx \frac{1}{r}\left(1-\frac{d}{r}+\frac{d^2}{r^2}+\cdots+1+\frac{d}{r}+\frac{d^2}{r^2}\right) \approx \frac{2}{r}+\frac{2d^2}{r^3}$$

$$\Rightarrow U = \frac{-2q^2}{4\pi\varepsilon_0}\left(\frac{1}{d}+\frac{d^2}{r^3}\right) = \frac{-2p^2}{4\pi\varepsilon_0 r^3}-\frac{2p^2}{4\pi\varepsilon_0 d^3}-$$

b) $U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$

$$= \frac{q^2}{4\pi\varepsilon_0}\left(\frac{-1}{d}-\frac{1}{r}+\frac{1}{r+d}+\frac{1}{r-d}-\frac{1}{r}-\frac{1}{d}\right)$$

$$= \frac{q^2}{4\pi\varepsilon_0}\left(\frac{-2}{d}-\frac{2}{r}+\frac{2}{r}+\frac{2d^2}{r^3}\right) = \frac{-2q^2}{4\pi\varepsilon_0}\left(\frac{1}{d}-\frac{d^2}{r^3}\right)$$

$$\Rightarrow U = \frac{-2p^2}{4\pi\varepsilon_0 d^3}+\frac{2p^2}{4\pi\varepsilon_0 r^3}.$$

If we ignore the potential energy involved in forming each individual molecule, which just involves a different choice for the zero of potential energy, then the answers are:

a) $U = \frac{-2p^2}{4\pi\varepsilon_0 r^3}$. b) $U = \frac{+2p^2}{4\pi\varepsilon_0 r^3}$.

43.1: a) ${}^{28}_{14}\text{Si}$ has 14 protons and 14 neutrons.

b) ${}^{85}_{37}\text{Rb}$ has 37 protons and 48 neutrons.

c) ${}^{205}_{81}\text{Tl}$ has 81 protons and 124 neutrons.

43.2: a) Using $R = (1.2 \text{ fm})A^{1/3}$, the radii are roughly 3.6 fm, 5.3 fm, and 7.1 fm.

b) Using $4\pi R^2$ for each of the radii in part (a), the areas are 163 fm^2, 353 fm^2 and 633 fm^2.

c) $\frac{4}{3}\pi R^3$ gives 195 fm^3, 624 fm^3 and 1499 fm^3.

d) The density is the same, since the volume and the mass are both proportional to A: $2.3\times10^{17} \text{ kg/m}^3$ (see Example 43.1).

e) Dividing the result of part (d) by the mass of a nucleon, the number density is $0.14/\text{fm}^3 = 1.40\times10^{44}/\text{m}^3$.

43.3: $\Delta E = \mu_z B - (-\mu_z B) = 2\mu_z B$

But $\Delta E = hf$, so $B = \dfrac{hf}{2\mu_z}$

$$\Rightarrow B = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(2.27\times10^{7} \text{ Hz})}{2(2.7928)(5.051\times10^{-27} \text{ J/T})} = 0.533 \text{ T}$$

43.4: a) As in Example 43.2,

$$\Delta E = 2(1.9130)(3.15245\times10^{-8} \text{ eV/T})(2.30 \text{ T}) = 2.77\times10^{-7} \text{ eV}.$$

Since $\vec{\boldsymbol{N}}$ and $\vec{\boldsymbol{S}}$ are in opposite directions for a neutron, the antiparallel configuration is lower energy. This result is smaller than but comparable to that found in the example for protons.

b) $f = \dfrac{\Delta E}{h} = 66.9 \text{ MHz}$, $\lambda = \dfrac{c}{f} = 4.48 \text{ m}$.

43.5: a) $U = \vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}} = -\mu_z B$. $\vec{\boldsymbol{N}}$ and $\vec{\boldsymbol{S}}$ point in the same direction for a proton. So if the spin magnetic moment of the proton is parallel to the magnetic field, $U < 0$, and if they are antiparallel, $U > 0$. So the parallel case has lower energy.

The frequency of an emitted photon has a transition of the protons between the two states given by:

$$f = \frac{\Delta E}{h} = \frac{E_+ - E_-}{h} = \frac{2\mu_z B}{h}$$

$$= \frac{2(2.7928)(5.051\times10^{-27}\ \text{J/T})(1.65\ \text{T})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = 7.02\times10^{7}\ \text{Hz}.$$

$$\Rightarrow \lambda = \frac{c}{f} = \frac{3.00\times10^{+8}\ \text{m/s}}{7.02\times10^{7}\ \text{Hz}} = 4.27\ \text{m. This is a radio wave.}$$

b) For electrons, the negative charge means that the argument from part (a) leads to the $m_s = -\frac{1}{2}$ state (antiparallel) having the lowest energy, since $\vec{\boldsymbol{N}}$ and $\vec{\boldsymbol{S}}$ point in opposite directions. So an emitted photon in a transition from one state to the other has a frequency

$$f = \frac{\Delta E}{h} = \frac{E_{-1/2} - E_{+1/2}}{h} = \frac{-2\mu_z B}{h}$$

But from Eq. (41.22),

$$\mu_z = -(2.00232)\frac{e}{2m_e}S_z = \frac{-(2.00232)e\hbar}{4m_e}$$

$$\Rightarrow f = \frac{(2.00232)eB}{4\pi m_e} = \frac{(2.00232)(1.60\times10^{-19}\ \text{C})(1.65\ \text{T})}{4\pi(9.11\times10^{-31}\ \text{kg})}$$

$$\Rightarrow f = 4.62\times10^{10}\ \text{Hz}$$

$$\text{so } \lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{4.62\times10^{10}\ \text{Hz}} = 6.49\times10^{-3}\ \text{m}.$$

This is a microwave.

43.6: a) $(13.6\ \text{eV})/(0.511\times10^{6}\ \text{eV}) = 2.66\times10^{-5} = 0.0027\%$.

b) $(8.795\ \text{MeV})/(938.3\ \text{MeV}) = 9.37\times10^{-3} = 0.937\%$.

43.7: The binding energy of a deuteron is 2.224×10^{6} eV. The photon with this energy has wavelength equal to

$$\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{(2.224\times10^{6}\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})} = 5.576\times10^{-13}\ \text{m}.$$

43.8: a) $7(m_n + m_H) - m_N = 0.112\ \text{u}$, which is 105 MeV, or 7.48 MeV per nucleon.

b) Similarly, $2(m_H + m_n) - m_{He} = 0.03038\ \text{u} = 28.3\ \text{MeV}$, or 7.07 MeV per nucleon, slightly lower (compare to Fig. (43.2)).

43.9: a) For ${}^{11}_{5}\text{B}$ the mass defect is:

$$\Delta m = 5m_p + 6m_n + 5m_e - M({}^{11}_{5}B)$$

$$= 5(1.007277\ \text{u}) + 6(1.008665\ \text{u}) + 5(0.000549\ \text{u}) - 11.009305\ \text{u}$$
$$= 0.081815\ \text{u}$$
$$\Rightarrow \text{The binding energy } E_B = (0.081815\ \text{u})(931.5\ \text{MeV/u})$$
$$= 76.21\ \text{MeV}$$

b) From Eq. (43.11): $E_B = C_1 A - C_2 A^{2/3} - C_3 \dfrac{Z(Z-1)}{A^{1/3}} - C_4 \dfrac{(A-2Z)^2}{A}$ and there is no fifth term since Z is odd and A is even.

$$\Rightarrow E_B = (15.75\ \text{MeV})11 - (17.80\ \text{MeV})(11)^{2/3} - (0.7100\ \text{MeV})\frac{5(4)}{11^{1/3}}$$
$$- (23.69\ \text{MeV})\frac{(11-10)^2}{(11)}$$
$$\Rightarrow E_B = 76.68\ \text{MeV}.$$

So the percentage difference is $\dfrac{76.68\ \text{MeV} - 76.21\ \text{MeV}}{76.21\ \text{MeV}} \times 100 = 0.62\%$

Eq. (43.11) has a greater percentage accuracy for ${}^{62}\text{Ni}$.

43.10: a) $34m_{\mathrm{n}}+29m_{\mathrm{H}}-m_{\mathrm{Cu}}=34(1.008665)\ \mathrm{u}+29(1.007825)\ \mathrm{u}-62.929601\,\mathrm{u}=$

$0.592\ \mathrm{u}$, which is $551\ \mathrm{MeV}$, or $8.75\ \mathrm{MeV}$ per nucleon (using $931.5\,\frac{\mathrm{MeV}}{\mathrm{u}}$ and 63 nucleons).

b) In Eq. (43.11), Z = 29 and N = 34, so the fifth term is zero. The predicted binding energy is

$$E_{\mathrm{B}}=(15.75\ \mathrm{MeV})(63)-(17.80\ \mathrm{MeV})(63)^{2/3}-(0.7100\ \mathrm{MeV})\frac{(29)(28)}{(63)^{1/3}}$$

$$-(23.69\ \mathrm{MeV})\frac{(5)^2}{(63)}$$

$$=556\ \mathrm{MeV}.$$

(The fifth term is zero since the number of neutrons is even while the number of protons is odd, making the pairing term zero.)

This result differs from the binding energy found from the mass deficit by 0.86%, a very good agreement comparable to that found in Example 43.4.

43.11: Z is a magic number of the elements helium (Z = 2), oxygen (Z = 8), calcium (Z = 20), nickel (Z =28), tin (Z = 50) and lead (Z = 82). The elements are especially stable, with large energy jumps to the next allowed energy level. The binding energy for these elements is also large. The protons' net magnetic moments are zero.

43.12: a) $146m_{\mathrm{n}}+92m_{\mathrm{H}}-m_{\mathrm{U}}=1.93\ \mathrm{u}$, which is

b) 1.80×10^{3} MeV, or c) 7.56 MeV per nucleon (using $931.5\,\frac{\mathrm{MeV}}{\mathrm{u}}$ and 238 nucleons).

43.13: a) α decay: Z decreases by 2, A decreases by 4

$$\Rightarrow {}^{239}_{94}\mathrm{Pb}\rightarrow{}^{235}_{92}\mathrm{U}+\alpha$$

b) β^- decay: Z decreases by 1, A remains the same:

$$\Rightarrow {}^{24}_{11}\mathrm{Na}\rightarrow{}^{24}_{12}\mathrm{Mg}+\beta^-$$

c) β^+ decay: Z decreases by 1, A remains the same:

$$\Rightarrow {}^{15}_{8}\mathrm{O}\rightarrow{}^{15}_{7}\mathrm{N}+\beta^+$$

43.14: a)The energy released is the energy equivalent of $m_{\mathrm{n}} - m_{\mathrm{p}} - m_{\mathrm{e}} = 8.40 \times 10^{-4}$ u, or 783 keV. b) $m_{\mathrm{n}} > m_{\mathrm{p}}$, and the decay is not possible.

43.15:

$$\Delta m = 2M\,({}^{4}_{2}\mathrm{He}) - M\,({}^{8}_{4}\mathrm{Be})$$
$$= 2(4.002603\,\mathrm{u}) - 8.005305\,\mathrm{u}$$
$$\Rightarrow \Delta m = -9.9 \times 10^{-5}\,\mathrm{u}$$

43.16: a) A proton changes to a neutron, so the emitted particle is a positron (β^{+}).

b) The number of nucleons in the nucleus decreases by 4 and the number of protons by 2, so the emitted particle is an alpha-particle. c) A neutron changes to a proton, so the emitted particle is an electron (β^{-}).

43.17: If β^{-} decay ${}^{14}\mathrm{C}$ is possible, then we are considering the decay ${}^{14}_{6}\mathrm{C} \to {}^{14}_{7}\mathrm{N} + \beta^{-}$.

$$\begin{aligned}\Delta m &= M\,({}^{14}_{6}\mathrm{C}) - M\,({}^{14}_{7}\mathrm{N}) - m_{\mathrm{e}} \\ &= (14.003242\,\mathrm{u} - 6(0.000549\,\mathrm{u})) - (14.003074\,\mathrm{u} - 7(0.000549\,\mathrm{u})) - 0.0005491 \\ &= +1.68 \times 10^{-4}\ \mathrm{u} : \mathrm{So}\ E = (1.68 \times 10^{-4}\ \mathrm{u})(931.5\,\mathrm{MeV/u}) = 0.156\,\mathrm{MeV} \\ &= 156\,\mathrm{keV}\end{aligned}$$

43.18: a) As in the example, $(0.000898\ \mathrm{u})(931.5\ \mathrm{MeV/u}) = 0.836\,\mathrm{MeV}$.

b) $0.836\,\mathrm{MeV} - 0.122\,\mathrm{MeV} - 0.014\,\mathrm{MeV} = 0.700\,\mathrm{MeV}$.

43.19: a) If tritium is to be unstable with respect to β^{-}decay, then the mass of the products of the decay must be less than the parent nucleus.

$$M\,({}^{3}_{1}\mathrm{H}^{+}) = 3.016049\,\mathrm{u} - 0.00054858\,\mathrm{u} = 3.015500\,\mathrm{u}$$
$$M\,({}^{3}_{2}\mathrm{He}^{2+}) = 3.016029\,\mathrm{u} - 2(0.00054858\ \mathrm{u}) = 3.014932\,\mathrm{u}$$
$$\Rightarrow \Delta m = M\,({}^{3}_{1}\mathrm{H}^{+}) - M\,({}^{3}_{2}\mathrm{He}^{-}) - m_{\mathrm{e}} = 2.0 \times 10^{-5}\ \mathrm{u},$$

so the decay is possible.

b) The energy of the products is just

$$E = (2.0 \times 10^{-5}\ \mathrm{u})(931.5\,\mathrm{MeV/u}) = 0.019\,\mathrm{MeV} = 19\,\mathrm{keV}.$$

43.20: Note that Eq. 43.17 can be written as follows

$$N = N_0 2^{-t/T_{1/2}}.$$

The amount of elapsed time since the source was created is roughly 2.5 years. Thus, we expect the current activity to be

$$N = (5000\text{ Ci})2^{-\frac{2.6\text{years}}{5.271\text{years}}} = 3600\text{ Ci}.$$

The source is barely usable. Alternatively, we could calculate $\lambda = \frac{\ln(2)}{T_{1/2}} = 0.132(\text{years})^{-1}$ and use the Eq. 43.17 directly to obtain the same answer.

43.21: For $^{14}\text{C}, T_{1/2} = 5730\ y$

$$A = A_0 e^{-\lambda t};\ \lambda = \ln 2 / T_{1/2} \text{ so } A = A_0 e^{-(\ln 2)t/T_{1/2}};\ A_0 = 180.0\text{ decays/min}$$

a) $t = 1000\text{ y},\ A = 159\text{ decays/min}$

b) $t = 50{,}000\text{ y},\ A = 0.43\text{ decays/min}$

43.22: (a) $^{90}_{39}\text{Sr} \rightarrow \beta^- + ^{90}_{39}\text{X}$

X has 39 protons and 90 protons plus neutrons, so it must be ^{90}Y.

(b) Use base 2 because we know the half life.

$$\begin{aligned} A &= A_0 2^{-t/T_{1/2}} \\ 0.01A_0 &= A_0 2^{-t/T_{1/2}} \\ t &= -\frac{T_{1/2}\log 0.01}{\log 2} \\ &= -\frac{(28\text{ yr})\log 0.01}{\log 2} = 190\text{ yr} \end{aligned}$$

43.23: a) $^{3}_{1}\text{H} \rightarrow ^{0}_{-1}\text{e} + ^{3}_{2}\text{He}$

b) $N = N_0 e^{-\lambda t}, N = 0.100 N_0$ and $\lambda = (\ln 2)/T_{1/2}$

$$0.100 = e^{-t(\ln 2)/T_{1/2}}; \qquad -t(\ln 2)/T_{1/2} = \ln(0.100); \qquad t = \frac{-\ln(0.100)T_{1/2}}{\ln 2} = 40.9\text{ y}$$

43.24: a) $\dfrac{dN}{dt} = 500\mu\text{Ci} = (500\times10^{-6})(3.70\times10^{10}\,\text{s}^{-1})$

$$dN/dt = 1.85\times10^{7}\ \text{dec/s}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} \rightarrow \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{12\,d\,(86{,}400\text{s/d})} = 6.69\times10^{-7}\ \text{s}$$

$$\frac{dN}{dt} = \lambda N \rightarrow N = \frac{dN/dt}{\lambda} = \frac{1.85\times10^{7}\ \text{dec / s}}{6.69\times10^{-7}\text{s}^{-1}} = 2.77\times10^{13}\text{nuclei}$$

The mass of this many ^{131}Ba nuclei is

$$m = 2.77\times10^{13}\text{nuclei}\times(131\times1.66\times10^{-27}\ \text{kg/nucleus})$$

$$= 6.0\times10^{-12}\text{kg} = 6.0\times10^{-9}\ \text{g} = 6.0\ \text{ng}$$

(b) $A = A_0 e^{-\lambda t}$

$$1\mu\text{Ci} = (500\mu\text{Ci})\ e^{-\lambda t}$$

$$\ln(1/500) = -\lambda t$$

$$t = -\frac{\ln(1/500)}{\lambda} = -\frac{\ln(1/500)}{6.69\times10^{-7}\text{s}^{-1}}$$

$$= 9.29\times10^{6}\text{s}\left(\frac{1\text{d}}{86{,}400\,\text{s}}\right) = 108\,\text{days}$$

43.25: $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$

$$-\frac{(\ln 2)t}{T_{1/2}} = \ln(A/A_0)$$

$$T_{1/2} = -\frac{(\ln 2)t}{\ln(A/A_0)} = -\frac{(\ln 2)(4.00\ \text{days})}{\ln(3091/8318)} = 2.80\ \text{days}$$

43.26: $$\frac{dN}{dt} = \lambda N$$

$$\lambda = \frac{\ln 2}{T_{½}} = \frac{\ln 2}{1620\text{ yr}\left(\dfrac{3.15\times10^{7}\text{ s}}{1\text{yr}}\right)}$$

$$\lambda = 1.36\times10^{-11}\text{ s}^{-1}$$

$$N = 1\text{ g}\left(\frac{6.022\times10^{23}\text{atoms}}{226\text{ g}}\right) = 2.665\times10^{25}\text{ atoms}$$

$$\frac{dN}{dt} = \lambda N = (2.665\times10^{25})(1.36\times10^{-11}\text{s}^{-1}) = 3.62\times10^{10}\,\frac{\text{dec}}{\text{s}}$$

$$= 3.62\times10^{10}\,\text{Bq}$$

Convert to Ci:

$$3.62\times10^{10}\,\text{Bq}\left(\frac{1\text{ Ci}}{3.70\times10^{10}\text{ Bq}}\right) = 0.98\text{ Ci}$$

43.27: Find the total number of carbon atoms in the sample.
$n = m/M$;

$$N_{\text{tot}} = nN_{\text{A}} = m\,N_{\text{A}}/M = (12.0\times10^{-3}\text{ kg})\ (6.022\times10^{23}\text{ atoms/mol})/(12.011\times10^{-3}\text{ kg/m}$$

$$N_{\text{tot}} = 6.016\times10^{23}\text{ atoms, so } (1.3\times10^{-12})\ (6.016\times10^{23}) = 7.82\times10^{11}\text{ carbon-14 atoms}$$

$$\Delta N/\Delta t = -180\text{ decays/min} = -3.00\text{ decays/s}$$

$$\Delta N/\Delta t = -\lambda N;\qquad \lambda = \frac{-(\Delta N/\Delta t)}{N} = 3.836\times10^{-12}\ (\text{s})^{-1}$$

$$T_{1/2} = (\ln 2)/\lambda = 1.807\times10^{11}\text{ s} = 5730\text{ y}$$

43.28: a) Solving Eq. (43.19) for λ,

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(5.27\text{ y})\ (365\text{ days/year})(24\text{ hrs/day})\ (3600\text{ sec/hr})} = 4.17\times10^{-9}\text{ s}^{-1}.$$

b) $$N = \frac{m}{A\text{u}} = \frac{3.60\times10^{-5}\text{ g}}{(60)\ (1.66\times10^{-24}\text{g})} = 3.61\times10^{17}.$$

c) $\dfrac{dN}{dt} = \lambda N = 1.51\times10^{9}$ Bq, which is d) 0.0408 Ci. The same calculation for radium, with larger A and longer half-life (lower λ) gives

$$\lambda_{\text{RA}}N_{\text{Ra}} = \lambda_{\text{Co}}N_{\text{Co}}\left(\frac{T_{1/2\text{Co}}A_{\text{Co}}}{T_{1/2\text{Ra}}A_{\text{Ra}}}\right) = 0.0408\text{ Ci}\left(\frac{(5.27\text{ yrs})\ (60)}{(1.600\text{ yrs})\ (226)}\right) = 3.57\times10^{-5}\text{ Ci}.$$

43.29: a) $\left|\frac{dN(0)}{dt}\right| = 7.56\times10^{11}\ \text{Bq} = 7.56\times10^{11}\ \text{decays/s}$

and

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(30.8\ \text{min})(60\ \text{s/min})} = 3.75\times10^{-4}\text{s}^{-1}.$$

so

$$N(0) = \frac{1}{\lambda}\left|\frac{dN(0)}{dt}\right| = \frac{7.56\times10^{11}\ \text{decays/s}}{3.75\times10^{-4}\text{s}^{-1}} = 2.02\times10^{15}\text{nuclei.}$$

b) The number of nuclei left after one half-life is $\frac{N(0)}{2} = 1.01\times10^{15}$ nuclei, and the activity is half: $\left|\frac{dN}{dt}\right| = 3.78\times10^{11}\ \text{decays/s}$.

c) After three half lives (92.4 minutes) there is an eighth of the original amount $N = 2.53\times10^{14}$ nuclei, and an eighth of the activity:

$$\left(\frac{dN}{dt}\right) = 9.45\times10^{10}\ \text{decays/s}.$$

43.30: The activity of the sample is $\frac{3070\ \text{decays/min}}{(60\ \text{sec/min})\ (0.500\ \text{kg})} = 102\ \text{Bq/kg}$, while the activity of atmospheric carbon is $255\ \text{Bq/kg}$ (see Example 43.9). The age of the sample is then

$$t = -\frac{\ln\ (102/255)}{\lambda} = -\frac{\ln\ (102/255)}{1.21\times10^{-4}/\text{y}} = 7573\ \text{y}.$$

43.31: a) $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(1.28\times10^{9}\ \text{y})(3.156\times10^{7}\ \text{s/y})}$

$\Rightarrow \lambda = 1.72\times10^{-17}\ \text{s}^{-1}.$

In $m = 1.63\times10^{-6}\text{g}$ of ^{40}K there are

$$N = \frac{1.63\times10^{-9}\ \text{kg}}{40(1.66\times10^{-27}\ \text{kg})} = 2.45\times10^{16}\ \text{nuclei.}$$

So $\left|\frac{dN}{dt}\right| = \lambda N = (1.72\times10^{-17}\ \text{s}^{-1})(2.45\times10^{16}\ \text{nuclei}) = 0.421\ \text{decays/s}.$

b) $\left|\frac{dN}{dt}\right| = \frac{0.421\ \text{Bq}}{3.70\times10^{10}\ \text{Bq/Ci}} = 1.14\times10^{-11}\text{Ci.}$

43.32: $$\frac{360\times10^{6}\ \text{decays}}{86{,}400\ \text{s}} = 4.17\times10^{3}\text{Bq} = 1.13\times10^{-7}\text{Ci} = 0.113\ \mu\text{Ci.}$$

43.33: a) $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{(4.47\times10^{9}\ \text{y})(3.156\times10^{7}\ \text{s/y})} = 4.91\times10^{-18}\text{s}^{-1}.$

b) $1.20\times10^{-5}\text{Ci} = (1.20\times10^{-5}\text{Ci})\left(3.70\times10^{10}\ \dfrac{\text{decays}}{\text{s}\cdot\text{Ci}}\right) = 4.44\times10^{5}$ decays/s.

$$\text{But} \left|\frac{dN}{dt}\right| = \lambda N \Rightarrow N = \frac{1}{\lambda}\left|\frac{dN}{dt}\right| = \frac{4.44\times10^{5}\ \text{decays/s}}{4.91\times10^{-18}\ \text{s}^{-1}}.$$

$$\Rightarrow N = 9.04\times10^{22}\ \text{nuclei} \Rightarrow m = (238\ \text{u})\ N$$

$$\Rightarrow m = (238)(1.66\times10^{-27}\ \text{kg})(9.04\times10^{22}) = 0.0357\ \text{kg}.$$

c) Each decay emits one alpha particle. In 60.0 g of uranium there are

$$N = \frac{0.0600\ \text{kg}}{238(1.66\times10^{-27}\ \text{kg})} = 1.52\times10^{23}\text{nuclei}$$

$$\Rightarrow \left|\frac{dN}{dt}\right| = \lambda N = (4.91\times10^{-18}\text{s}^{-1})(1.52\times10^{23}\text{nuclei}) = 7.46\times10^{5}$$

alpha particles emitted each second.

43.34: (a) rem = rad × RBE

200 = x(10)

x = 20 rad

(b) 1 rad deposits $0.010\ \text{J/kg}$, so 20 rad deposit $0.20\ \text{J/kg}$. This radiation affects 25 g (0.025 kg) of tissue, so the total energy is

$$(0.025\ \text{kg})(0.20\ \text{J/kg}) = 5.0\times10^{-3}\ \text{J} = 5.0\ \text{mJ}$$

(c) Since RBE = 1 for β-rays, so rem = rad. Therefore

20 rad = 20 rem

43.35: 1 rad = 10^{-2} Gy, so 1 Gy = 100 rad and the dose was 500 rad

rem=(rad) (RBE) = (500 rad) (4.0) = 2000 rem

$1\ \text{Gy} = 1\ \text{J/kg}$, so 5.0 J/kg

43.36: a) $5.4\ \text{Sv}\ (100\ \text{rem/Sv}) = 540\ \text{rem}$. b) The RBE of 1 gives an absorbed dose of 540 rad. c) The absorbed dose is 5.4 Gy, so the total energy absorbed is $(5.4\ \text{Gy})\ (65\ \text{kg}) =$ $351\ \text{J}$. The energy required to raise the temperature of 65 kg 0.010° C is $(65\ \text{kg})$ (4190 $\text{J/kg}\cdot\text{K}$) $(0.01^{\circ}\text{C}) = 3\ \text{kJ}$.

43.37: a) We need to know how many decays per second occur.

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(12.3\text{ y})\,(3.156\times10^{7}\text{ s/y})} = 1.79\times10^{-9}\text{s}^{-1}.$$

The number of tritium atoms is $N(0) = \frac{1}{\lambda}\left|\frac{dN}{dt}\right| = \frac{(0.35\text{ Ci})\,(3.70\times10^{10}\text{ Bq/Ci})}{1.79\times10^{-9}\text{s}^{-1}}$

$\Rightarrow N(0) = 7.2540\times10^{18}$ nuclei. The number of remaining nuclei after one week is just $N(1\text{ week}) = N(0)e^{-\lambda t} = (7.25\times10^{18})e^{-(1.79\times10^{-9}\text{s}^{-1})(7)(24)(3600\text{s})} \Rightarrow N\ (1\text{ week}) = 7.2462\times10^{18}$ nuclei $\Rightarrow \Delta N = N(0) - N(1\text{ week}) = 7.8\times10^{15}$ decays. So the energy absorbed is $E_{\text{total}} = \Delta N\cdot E_{\gamma} = (7.8\times10^{15})\,(5000\text{ eV})\,(1.60\times10^{-19}\text{ J/eV}) = 6.24\text{ J}.$ So the absorbed dose is $\frac{(6.24\text{ J})}{(50\text{ kg})} = 0.125\text{ J/kg} = 12.5\text{ rad}.$ Since RBE = 1, then the equivalent dose is 12.5 rem.

b) In the decay, antinentrinos are also emitted. These are not absorbed by the body, and so some of the energy of the decay is lost (about 12 keV).

43.38: a) From Table (43.3), the absorbed dose is 0.0900 rad. b) The energy absorbed is $(9.00\times10^{-4}\text{ J/kg})\,(0.150\text{ kg}) = 1.35\times10^{-4}\text{ J};$ each proton has energy $1.282\times10^{-13}\text{ J},$ so the number absorbed is $1.05\times10^{9}\cdot$ c) The RBE for alpha particles is twice that for protons, so only half as many, $5.27\times10^{8},$ would be absorbed.

43.39: a) $E_{\text{total}} = NE_{\gamma} = \frac{Nhc}{\lambda} = \frac{(6.50\times10^{10})\,(6.63\times10^{-34}\text{ J}\cdot\text{s})\,(3.00\times10^{8}\text{ m/s})}{2.00\times10^{11}\text{m}}$

$$\Rightarrow E_{\text{total}} = 6.46\times10^{-4}\text{ J}.$$

b) The absorbed dose is the energy divided by tissue mass:

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$$\text{dose} = \frac{6.46\times10^{-4}\text{ J}}{0.600\text{ kg}} = (1.08\times10^{-3}\text{ J/kg})\left(\frac{100\text{ rad}}{\text{J/kg}}\right) = 0.108\text{ rad}.$$

The rem dose for x rays (RBE = 1) is just 0.108 rem.

43.40: $(0.72\times10^{-6}\text{Ci})\,(3.7\times10^{10}\text{ Bq/Ci})\,(3.156\times10^{7}\text{s}) = 8.41\times10^{11}\ \alpha$ particles. The absorbed dose is

$$\frac{(8.41\times10^{11})\,(4.0\times10^{6}\text{eV})\,(1.602\times10^{-19}\text{ J/eV})}{(0.50\text{ kg})} = 1.08\text{ Gy} = 108\text{ rad}.$$

The equivalent dose is (20) (108 rad) = 2160 rem.

43.41: a) ${}^2_1\text{H}+{}^9_4\text{Be} \to {}^A_Z\text{X} + {}^4_2\text{He}$. So $2+9 = A+4 \Rightarrow A = 7$ and $1+4 = Z+2 \Rightarrow$ $Z = 3$, so X is ${}^7_3$ Li.

b) $\Delta m = M({}^2_1\text{H}) + M({}^9_4\text{Be}) - M({}^7_3\text{Li}) - M({}^4_2\text{He})$

$= 2.014102\text{ u} + 9.012182\text{ u} - 7.016003\text{ u} - 4.002603\text{ u}$

$= 7.678 \times 10^{-3}\text{ u}.$

So $E = (\Delta m)c^2 = (7.678 \times 10^{-3}\text{ u})\,(931.5\text{ MeV/u}) = 7.152\text{ MeV}.$

c) The threshold energy is taken to be the potential energy of the two reactants when they just "touch." So we need to know their radii:

$r_{2_\text{H}} = (1.2 \times 10^{-15}\text{ m})\,(2)^{1/3} = 1.5 \times 10^{-15}\text{ m}$

$r_{9_\text{Be}} = (1.2 \times 10^{-15}\text{ m})\,(9)^{1/3} = 2.5 \times 10^{-15}\text{ m}$

So the centers' separation is $r = 4.0 \times 10^{-15}\text{ m}$

Thus $U = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_\text{H} q_\text{Be}}{r} = \dfrac{4(1.60 \times 10^{-19}\text{ C})^2}{4\pi\varepsilon_0 (4.0 \times 10^{-15}\text{ m})} = 2.3 \times 10^{-13}\text{ J}$

$\Rightarrow U = 1.4 \times 10^6\text{ eV} = 1.4\text{ MeV}.$

43.42: $m_{{}^3_2\text{He}} + m_{{}^2_1\text{H}} - m_{{}^4_2\text{He}} - m_{{}^1_1\text{H}} = 1.97 \times 10^{-2}\text{ u}$, so the energy released is 18.4 MeV.

43.43: a) As in Ex. (43.41a), $2 + 14 = A + 10 \Rightarrow A = 6$, and $1 + 7 = Z \to 5 \Rightarrow Z = 3$, so $\text{X} = {}^6_3$ Li.

b) As in Ex. (43.41b), using $M({}^2_1\text{H}) = 2.014102\text{ u}$, $M({}^{14}_7\text{N}) = 14.003074\text{ u}$, $M({}^6_3\text{Li}) =$, 6.01521 u, *and* $M({}^{10}_5\text{B}) = 10.012937\text{ u}$, $\Delta m = -0.010882\text{ u}$, so energy is absorbed in the reaction. $\Rightarrow Q = (-0.010882\text{ u})\,(931.5\text{ MeV/u}) = -10.14\text{ MeV}.$

c) From Eq. (43.24): $K_\text{cm} = \dfrac{M}{M+m} K$

so

$$K = \left(\frac{M+m}{M}\right) K_\text{cm} = \frac{14.0\text{ u} + 2.01\text{ u}}{14.0\text{ u}} (10.14\text{ MeV}) = 11.6\text{ MeV}$$

43.44: $(200 \times 10^6\text{ eV})\,(1.602 \times 10^{-19}\text{ J/eV})\,(6.023 \times 10^{23}\text{ molecules/mol}) = 1.93 \times 10^{13}\text{ J/mol}$, which is far higher than typical heats of combustion.

43.45: The mass defect is $\Delta m = M({}^{235}_{92}\text{U}) + m_\text{n} - M({}^{236}_{92}\text{U}^*)$

$\Rightarrow \Delta m = 235.043923\text{ u} + 1.008665\text{ u} - 236.045562\text{ u}$

$= 0.007025\text{ u}$

So the internal excitation of the nucleus is: $Q = (\Delta m)c^2 = (0.007025\text{ u})\,(931.5\text{ MeV/u})$

$= 6.544\text{ MeV}$

43 .46: a) $Z = 3 + 2 - 0 = 5$ and $A = 4 + 7 - 1 = 10.$ b) The nuclide is a boron nucleus, and $m_{\text{He}} + m_{\text{Li}} - m_{\text{n}} - m_{\text{B}} = -3.00 \times 10^{-3}\,\text{u},$ and so 2.79 MeV of energy is absorbed.

43.47: The energy liberated will be

$$\begin{aligned} M({}^3_2\text{He}) + M({}^4_2\text{He}) - M({}^7_4\text{Be}) &= (3.016029\text{ v} + 4.002603\text{ v} - 7.016929\text{ v}) \\ &\quad \times (931.5\text{ MeV/v}) \\ &= 1.586\text{ MeV}. \end{aligned}$$

43.48: a) ${}^{28}_{14}\text{Si} + \gamma \Rightarrow {}^{24}_{12}\text{Mg} + {}^{A}_{Z}\text{X}$. $A + 24 = 28$ so $A = 4$. $Z + 12 = 14$ so $Z = 2$. X is an α particle.

b) $$\begin{aligned} KE_\gamma = -\Delta mc^2 &= (23.985042\text{ v} + 4.002603\text{ v} - 27.976927\text{ v})\ (931.5\text{ MeV/v}) \\ &= 9.984\text{ MeV} \end{aligned}$$

43.49: Nuclei: ${}^{A}_{Z}\text{X}^{z+} \to {}^{A-4}_{Z-2}\text{Y}^{(Z-2)} + {}^4_2\text{He}^{2+}$

Add the mass of Z electrons to each side and we find: $\Delta m = M({}^{A}_{Z}\text{X}) - M({}^{A-4}_{Z-2}\text{Y}) - M({}^4_2\text{He}),$ where now we have the mass of the neutral atoms. So as long as the mass of the original neutral atom is greater than the sum of the neutral products masses, the decay can happen.

43.50: Denote the reaction as

$${}^{A}_{Z}\text{X} \to {}_{Z+1}{}^{A}\text{Y} + e^-.$$

The mass defect is related to the change in the neutral atomic masses by

$$[m_{\text{X}} - Zm_{\text{e}}] - [m_{\text{Y}} - (Z+1)m_{\text{e}}] - m_{\text{e}} = (m_{\text{X}} - m_{\text{Y}}),$$

where m_{X} and m_{Y} are the masses as tabulated in, for instance, Table (43.2).

43.51: ${}^{A}_{Z}\text{X}^{z+} \to {}_{Z-1}{}^{A}\text{Y}^{(Z-1)+} + \beta^+$

Adding $(Z-1)$ electron to both sides yields

${}^{A}_{Z}\text{X}^{+} \to {}_{Z-1}{}^{A}\text{Y} + \beta^+$

So in terms of masses:

$$\begin{aligned} \Delta m &= M\left({}^{A}_{Z}\text{X}^{+}\right) - M\left({}_{Z-1}{}^{A}\text{Y}\right) - m_{\text{e}} \\ &= \left(M\left({}^{A}_{Z}\text{X}\right) - m_{\text{e}}\right) - M\left({}_{Z-1}{}^{A}\text{Y}\right) - m_{\text{e}} \\ &= M\left({}^{A}_{Z}\text{X}\right) - M\left({}_{Z-1}{}^{A}\text{Y}\right) - 2m_{\text{e}}. \end{aligned}$$

So the decay will occur as long as the original neutral mass is greater than the sum of the neutral product mass and two electron masses.

43.52: Denote the reaction as ${}^{A}_{Z}\text{X} + e^- \to {}_{Z-1}{}^{A}\text{Y}.$

The mass defect is related to the change in the neutral atomic masses by

$$[m_{\text{X}} - Zm_{\text{e}}] + m_{\text{e}} - [m_{\text{Y}} - (Z-1)m_{\text{e}}] = (m_{\text{X}} - m_{\text{Y}}),$$

where m_{X} and m_{Y} are the masses as tabulated in, for instance, Table (43.2).

43.53: a) Only the heavier one $({}^{25}_{13}\text{Al})$ can decay into the lighter one $({}^{25}_{12}\text{Mg})$.

b) $({}^{25}_{13}\text{Al}) \to ({}^{25}_{12}\text{Mg}) + {}^{A}_{Z}\text{X} \Rightarrow A = 0,\ Z = +1 \Rightarrow X$ is a positron

$\Rightarrow \beta^{+}$ decay

or $({}^{25}_{13}\text{Al}) + {}^{A}_{Z}\text{X}' \to {}^{25}_{12}\text{Mg} \Rightarrow A = 0,\ Z = -1 \Rightarrow X'$ is an electron

$\Rightarrow$ electron capture

c) Using the nuclear masses, we calculate the mass defect for β^{+} decay:

$$\Delta m = (M({}^{25}_{13}\text{Al}) - 13m_{\text{e}}) - (M({}^{25}_{12}\text{Mg}) - 12m_{\text{e}}) - m_{\text{e}}$$
$$= 24.990429\ \text{u} - 24.985837\ \text{u} - 2(0.00054858\ \text{u})$$
$$= 3.495 \times 10^{-3}\ \text{u}$$
$$\Rightarrow Q = (\Delta m)c^{2} = (3.495 \times 10^{-3}\ \text{u})\ (931.5\ \text{MeV/u}) = 3.255\ \text{MeV}.$$

For electron capture:

$$\Delta m = M({}^{25}_{13}\text{Al}) - M({}^{25}_{12}\text{Mg}) = 24.990429\ \text{u} - 24.985837\ \text{u}$$
$$= 4.592 \times 10^{-3}\ \text{u}$$
$$\Rightarrow Q = (\Delta m)c^{2} = (4.592 \times 10^{-3}\ \text{u})\ (931.5\ \text{MeV/u}) = 4.277\ \text{MeV}.$$

43.54: a) $m_{{}^{210}_{84}\text{Po}} - m_{{}^{206}_{82}\text{Pb}} - m_{{}^{4}_{2}\text{He}} = 5.81 \times 10^{-3}\ \text{u}$, or $Q = 5.41\ \text{MeV}$. The energy of the alpha particle is $(206/210)$ times this, or $5.30\ \text{MeV}$ (see Example 43.5)

b) $m_{{}^{210}_{84}\text{Po}} - m_{{}^{209}_{83}\text{Bi}} - m_{{}^{1}_{1}\text{H}} = -5.35 \times 10^{-3}\ \text{u} < 0$, so the decay is not possible.

c) $m_{{}^{210}_{84}\text{Po}} - m_{{}^{209}_{84}\text{Po}} - m_{\text{n}} = -8.22 \times 10^{-3}\ \text{u} < 0$, so the decay is not possible.

d) $m_{{}^{210}_{85}\text{At}} > m_{{}^{210}_{84}\text{Po}}$, so the decay is not possible (see Problem (43.50)).

e) $m_{{}^{210}_{83}\text{Bi}} + 2m_{\text{e}} > m_{{}^{210}_{84}\text{Po}}$, so the decay is not possible (see Problem (43.51)).

43.55: Using Eq: (43.12): ${}^{A}_{Z}M = ZM_{\text{H}} + Nm_{\text{n}} - E_{\text{B}}/c^{2} \Rightarrow M({}^{24}_{11}\text{Na}) = 11M_{\text{H}} + 13m_{\text{n}} - E_{\text{B}}/c^{2}$. But $E_{\text{B}} = 15.75\ \text{MeV}(24) - (17.80\ \text{MeV})(24)^{2/3} - (0.7100\ \text{MeV}\dfrac{(11)(10)}{(24)^{1/3}} - (23.69\ \text{MeV})\dfrac{(24-2(11))^{2}}{24} - (39\ \text{MeV})(24)^{-4/3} = 198.31\ \text{MeV}$.

$$\Rightarrow M({}^{24}_{11}\text{Na}) = 11(1.007825\ \text{u}) + 13(1.008665\ \text{u}) - \frac{(198.31\ \text{MeV})}{931.5\ \text{MeV/u}} = 23.9858\ \text{u}$$

$$\%\ \text{error} = \frac{23.990963 - 23.9858}{23.990963} \times 100 = 0.022\%.$$

If the binding energy term is neglected, $M({}^{24}_{11}\text{Na}) = 24.1987\ \text{u}$ and so the percentage error would be $\dfrac{24.1987 - 23.990963}{23.990963} \times 100 = 0.87\%.$

43.56: The α-particle will have $\dfrac{226}{230}$ of the mass energy (see Example 45.5)

$$\frac{226}{230}(m_{\text{Th}} - m_{\text{Ra}} - m_{\alpha}) = 5.032\times10^{-3}\ \text{u or } 4.69\ \text{MeV}.$$

43.57: ${}^{198}_{79}\text{Au} \to {}^{198}_{80}\text{Hg} + \beta^{-}$ $\Delta m = M({}^{198}_{79}\text{Au}) - M({}^{198}_{80}\text{Hg}) = 197.968225\ \text{u} - 197.966752\ \text{u} =$ 1.473×10^{-3} u. And the total energy available was $Q = (\Delta m)c^2 \Rightarrow Q = (1.473\times$ 10^{-3} u)(931.5 MeV/u $= 1.372$ MeV.

The emitted photon has energy 0.412 MeV, so the emitted electron must have kinetic energy equal to 1.372 MeV – 0.412 MeV = 0.960 MeV.

43.58: (See Problem (43.51)) $m_{{}^{11}_{6}\text{C}} - m_{{}^{11}_{5}\text{B}} - 2m_{\text{e}} = 1.03\times10^{-3}$ u. Decay is energetically possible.

43.59: ${}^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + \beta^{+}$ As in Problem 43.51, β^{+} decay has a mass defect in terms of neutral atoms of

$$\begin{aligned}\Delta m &= M({}^{13}_{7}\text{N}) - M({}^{13}_{6}\text{C}) - 2m_{\text{e}}\\ &= 13.005739\ \text{u} - 13.003355 - 2(0.00054858\ \text{u})\\ &= 1.287\times10^{-3}\ \text{u}\end{aligned}$$

Therefore the decay is possible because the initial mass is greater than the final mass.

43.60: a) A least-squares fit to log of the activity *vs.* time gives a slope of $\lambda = 0.5995\ \text{hr}^{-1}$, for a half-life of $\dfrac{\ln 2}{\lambda} = 1.16$ hr. b) The initial activity is $N_0\lambda$, so

$$N_0\ \frac{(2.00\times10^{4}\ \text{Bq})}{(0.5995\ \text{hr}^{-1})(1\ \text{hr}/3600\ \text{s})} = 1.20\times10^{8}.$$

c) $N_0 e^{-\lambda t} = 1.81\times10^{6}$.

43.61: The activity $A(t) \equiv \dfrac{dN(t)}{dt}$ but $\dfrac{dN(t)}{dt} = -\lambda N(t)$ so $\dfrac{dN(0)}{dt} = -\lambda N(0) \equiv$ $-\lambda N_0 = A_0$. Taking the derivative of $N(t) = N_0 e^{-\lambda t} \Rightarrow \dfrac{dN(t)}{dt} = -\lambda N_0 e^{-\lambda t} = \dfrac{dN(0)}{dt} e^{-\lambda t}$ or $A(t) = A_0 e^{-\lambda t}$.

43.62: From Eq.43.17 $N(t) = N_0 e^{-\lambda t}$ but $N_0 e^{-\lambda t} = N_0 e^{-(\ln 2)\left(\frac{t}{T_{1/2}}\right)}$

$$= N_0\left[e^{-(\ln 2)}\right]^{\left(\frac{t}{T_{1/2}}\right)} = N_0\left[e^{\ln\left(\frac{1}{2}\right)}\right]^{\left(\frac{t}{T_{1/2}}\right)}.\ \text{So } N(t) = N_0\left(\frac{1}{2}\right)^{n}\ \text{where } n = \frac{t}{T_{1/2}}.$$

Recall $a\ln x = \ln(x^a), e^{ax} = (e^{ax})^a$, and $e^{\ln x} = x$.

43.63: $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{(4.75\times10^{10}\ \text{y})(3.156\times10^{7}\ \text{s/y})} = 4.62\times10^{-19}\ \text{s}^{-1}$

$N_{87} = N_{0_{87}}e^{-\lambda t} \Rightarrow N_{0_{87}} = N_{87}e^{+\lambda t}$

$\Rightarrow N_{0_{87}} = N_{0_{87}}e^{(4.62\times10^{-19}\ \text{s}^{-1})(4.6\times10^{9}\ \text{y})(3.156\times10^{7}\ \text{s/y})} \Rightarrow N_{087} = 1.0694\, N_{87}$

But we also know that $\dfrac{N_{87}}{N_{85}+N_{87}} = 0.2783 \Rightarrow N_{87} = \dfrac{0.2783\, N_{85}}{(1-0.2783)} = 0.3856\, N_{85} =$

$0.3856\ N_{0_{85}}$. So $\dfrac{N_{0_{87}}}{N_{0_{85}}+N_{0_{87}}} = \dfrac{1.0694(0.3856)}{(1+1.0694(0.3856))} = 0.2920.$

So the original percentage of ^{87}Rb is 29%. ($N_{0_{85}} = N_{85}$ since it doesn't decay.)

43.64: a) $(6.25\times10^{12})(4.77\times10^{6}\ \text{MeV})(1.602\times10^{-19}\ \text{J/eV})/(70.0\ \text{kg}) = 0.0682\ \text{Gy} =$ 6.82 rad. b) (20)(6.82 rad)=136 rem (c) $N\lambda = \dfrac{m}{Au}\dfrac{\ln(2)}{T_{1/2}} = 1.17\times10^{9}\ \text{Bq} =$

31.6 mCi. d) $\dfrac{6.25\times10^{12}}{1.17\times10^{9}\ \text{Bq}} = 5.34\times10^{3}$ s, about an hour and a half. Note that this time is so small in comparison with the half-life that the decrease in activity of the source may be neglected.

43.65: a) $\left|\dfrac{dN}{dt}\right| = 2.6\times10^{-4}\ \text{Ci}(3.70\times10^{10}\ \text{decays/s}\cdot\text{Ci}) = 9.6\times10^{6}\ \text{decays/s}$ so in one second there is an energy delivered of

$$E = \frac{1}{2}\left(\frac{dN}{dt}\right)\cdot t\cdot E_{\gamma} = \frac{1}{2}(9.6\times10^{6}\ \text{s}^{-1})(1.00\ \text{s})(1.25\times10^{6}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})$$

$$= 9.62\times10^{-7}\ \text{J/s}.$$

b) Absorbed dose $= \dfrac{E}{m} = \dfrac{9.6\times10^{-7}\ \text{J/s}}{0.500\ \text{kg}}$

$$= 1.9\times10^{-6}\ \text{J/kg}\cdot\text{s}\left(100\frac{\text{rad}}{\text{J/kg}\cdot\text{s}}\right) = 1.9\times10^{-4}\ \text{rad}.$$

c) Equivalent dose $= 0.7(1.9\times10^{-4})\text{rad} = 1.3\times10^{-4}\ \text{rem}.$

d) $\dfrac{200\ \text{rem}}{1.3\times10^{-4}\ \text{rem/s}} = 1.5\times10^{6}\ \text{s} = 17\ \text{days}.$

43.66: a) After 4.0 min = 240 s, the ratio of the number of nuclei is

$$\frac{2^{-240/122.2}}{2^{-240/26.9}} = 2^{(240)\left(\frac{1}{26.9}-\frac{1}{122.2}\right)} = 124.$$

b) After 15.0 min = 900 s, the ratio is 7.15×10^{7}.

43.67: $$\frac{N}{N_0} = 0.21 = e^{-\lambda t}$$

$$\Rightarrow t = -\frac{\ln(0.21)}{\lambda} = -\ln(0.21)\cdot\frac{5730\ y}{0.693} = 13000\ \text{y}$$

43.68: The activity of the sample will have decreased by a factor of

$$\frac{(4.2\times10^{-6}\ \text{Ci})(3.70\times10^{10}\ \text{Bq/Ci})}{(8.5\ \text{counts/min})(1\ \text{min}/60\text{s})} = 1.097\times10^{6} = 2^{20.06};$$

this corresponds to 20.06 half-lifes, and the elapsed time is 40.1 h. Note the retention of extra figures in the exponent to avoid roundoff error. To the given two figures the time is 40 h.

43.69: For deuterium:

a) $$U = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{(1.60\times10^{-19}\ \text{C})^2}{4\pi\varepsilon_0\left[2(1.2\times10^{-15}\ \text{m})(2)^{1/3}\right]} = 7.61\times10^{-14}\ \text{J}$$
$$= 0.48\,\text{MeV}$$

b) $$\begin{aligned}\Delta m &= 2M({}^2_1\text{H}) - M({}^3_2\text{He}) - m_\text{n}\\ &= 2(2.014102\,\text{u}) - 3.016029\,\text{u} - 1.008665\,\text{u}\\ &= 3.51\times10^{-3}\ \text{u}\\ \Rightarrow E &= (\Delta m)c^2 = (3.51\times10^{-3}\ \text{u})(931.5\,\text{MeV/u}) = 3.270\,\text{MeV}\\ &= 5.231\times10^{-13}\ \text{J}\end{aligned}$$

c) A mole of deuterium has 6.022×10^{23} molecules, so the energy per mole is $(6.022\times10^{23})(5.231\times10^{-13}\ \text{J}) = 3.150\times10^{11}\ \text{J}.$ This is over a million times more than the heat of combustion.

43.70: a) $m_{{}^{16}_{8}\text{O}} - m_{{}^{15}_{7}\text{N}} - m_{{}^{1}_{1}\text{H}} = -1.30\times10^{-2}\ \text{u},$ so the proton separation energy is $12.1\,\text{MeV}.$ b) $m_{{}^{16}_{8}\text{O}} - m_{{}^{15}_{8}\text{O}} - m_\text{n} = -1.68\times10^{-2}\ \text{u},$ so the neutron separation energy is 15.7 MeV. c) It takes less energy to remove a proton.

43.71: Mass of ${}^{40}\text{K}$ atoms in 1.00 kg is $(2.1\times10^{-3})(1.2\times10^{-4})\text{kg} = 2.52\times10^{-7}\ \text{kg}.$

Number of atoms $N = \dfrac{2.52\times10^{-7}\ \text{kg}}{40\ \text{u}(1.661\times10^{-27}\ \text{kg/u})} = 3.793\times10^{18}.$

$$\frac{dN}{dt} = \lambda N = \frac{(0.693)(3.793\times10^{18})}{1.28\times10^{9}\ y} = 2.054\times10^{9}\ \text{decays/y}.$$

So in 50 years the energy absorbed is: $E = (0.50\ \text{MeV/decay})(50\ \text{y})(2.054\times10^{9}\ \text{decay/y}$ $5.14\times10^{10}\ \text{MeV} = 8.22\times10^{-3}\ \text{J}.$ So the absorbed dose is $(8.22\times10^{-3}\ \text{J})(100\ \text{J/rad}) =$ 0.82 rad and since the RBE = 1.0, the equivalent dose is 0.82 rem.

43.72: In terms of the number N of cesium atoms that decay in one week and the mass $m = 1.0$ kg, the equivalent dose is

$$3.5\ \text{Sv} = \frac{N}{m}((\text{RBE})_{\gamma}\text{E}_{\gamma} + (\text{RBE})_{\text{e}}\text{E}_{\text{e}})$$

$$= \frac{N}{m}((1)(0.66\ \text{MeV}) + (1.5)(0.51\ \text{MeV}))$$

$$= \frac{N}{m}(2.283\times10^{-13}\ \text{J}),\ \text{so}$$

$$N = \frac{(1.0\ \text{kg})(3.5\ \text{Sv})}{(2.283\times10^{-13}\ \text{J})} = 1.535\times10^{13}.$$

The number N_0 of atoms present is related to N by $N_0 = Ne^{\lambda t}$, so

$\lambda = \dfrac{0.693}{(30.07\ \text{yr})(3.156\times10^{7}\ \text{sec/yr})} = 7.30\times10^{-10}\ \text{sec}^{-1} \Rightarrow N_0 = Ne^{\lambda t} = (1.535\times$

$10^{13})e^{(7.30\times10^{-10}\ \text{sec}^{-1})(7\ \text{days})(8.64\times10^{4}\ \text{sec/days})} = 1.536\times10^{13}.$

43.73: a) $v_{cm} = v\cdot\dfrac{m}{m+M}$

$$v'_m = v - v\frac{m}{m+M} = \left(\frac{M}{m+M}\right)v \qquad v'_m = \frac{vm}{m+M}$$

$$K' = \frac{1}{2}mv'^2_m + \frac{1}{2}Mv'^2_M = \frac{1}{2}\frac{mM^2}{(m+M)^2}v^2 + \frac{1}{2}\frac{Mm^2}{(m+M)^2}v^2$$

$$= \frac{1}{2}\frac{M}{(m+M)}\left(\frac{mM}{m+M} + \frac{m^2}{m+M}\right)v^2$$

$$= \frac{M}{m+M}\left(\frac{1}{2}mv^2\right) \Rightarrow K' = \frac{M}{m+M}K \equiv K_{cm}$$

b) For an endoenergetic reaction $K_{\text{cm}} = -Q(Q<0)$ at threshold. Putting this into part (a) gives $-Q = \dfrac{M}{M+m}K_{th} \Rightarrow K_{th} = \dfrac{-(M+m)}{M}Q$

43.74: $K = \dfrac{M_{\alpha}}{M_{\alpha}+m}K_{\infty}$, where K_{∞} is the energy that the α-particle would have if the nucleus were infinitely massive. Then, $M = M_{\text{Os}} - M_{\alpha} - K_{\infty} = M_{\text{Os}} - M_{\alpha} -$

$\dfrac{186}{182}\left(2.76\ \text{MeV}/c^2\right) = 181.94821\ \text{u}$

43.75: $\Delta m = M\left({}^{235}_{92}\text{U}\right) - M\left({}^{140}_{54}\text{Xe}\right) - M\left({}^{94}_{38}\text{Sr}\right) - m_{\text{n}}$

$$= 235.043923\ \text{u} - 139.921636\ \text{u} - 93.915360\ \text{u} - 1.008665\ \text{u}$$

$$= 0.1983\ \text{u}$$

$$\Rightarrow E = (\Delta m)c^2 = (0.1983\ \text{u})(931.5\ \text{MeV/u}) = 185\ \text{MeV}.$$

43.76: a) A least-squares fit of the log of the activity *vs.* time for the times later than 4.0 h gives a fit with correlation $-\left(1-2\times10^{-6}\right)$ and decay constant of $0.361\ \text{hr}^{-1}$, corresponding to a half-life of 1.92 hr. Extrapolating this back to time 0 gives a contribution to the rate of about 2500/s for this longer-lived species. A least-squares fit of the log of the activity *vs.* time for times earlier than 2.0 hr gives a fit with correlation = 0.994, indicating the presence of only two species.

b) By trial and error, the data is fit by a decay rate modeled by

$$R = (5000\ \text{Bq})e^{-t(1.733/\text{hr})} + (2500\ \text{Bq})e^{-t(0.361/\text{hr})}.$$

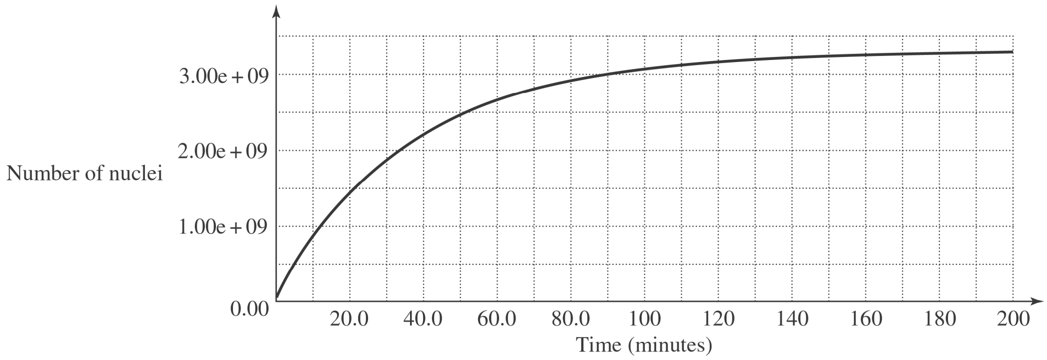
This would correspond to half-lives of 0.400 hr and 1.92 hr.

c) In this model, there are 1.04×10^{7} of the shorter-lived species and 2.49×10^{7} of the longer-lived species.

d) After 5.0 hr, there would be 1.80×10^{3} of the shorter-lived species and 4.10×10^{6} of the longer-lived species.

43.77: (a) There are two processes occurring: the creation of $^{128}\mathrm{I}$ by the neutron irradiation, and they decay of the newly produced $^{128}\mathrm{I}$. So $\dfrac{dN}{dt} = K - \lambda N$ where K is the rate of production by the neutron irradiation. Then $\int_0^N \dfrac{dN'}{K - \lambda N'} = \int_0^t dt.$

$$\Rightarrow \left[\ln\left(K - \lambda N'\right)\right]_0^N = -\lambda t$$
$$\Rightarrow \ln\left(K - \lambda N\right) = \ln K - \lambda t$$
$$\Rightarrow N(t) = \frac{K\left(1 - e^{-\lambda t}\right)}{\lambda}.$$



b) The activity of the sample is $\lambda N(t) = K\left(1 - e^{-\lambda t}\right) = \left(1.5 \times 10^6 \text{ decays/s}\right) \times \left(1 - e^{-\left(\frac{0.693}{25\,\text{min}}\right)t}\right)$. So the activity is $\left(1.5 \times 10^6 \text{ decays/s}\right)\left(1 - e^{-0.02772\,t}\right)$, with t in minutes. So the activity $\left(\dfrac{-dN'}{dt}\right)$ at various times is:

$$\frac{-dN'}{dt}(t = 1\text{ min}) = 4.1 \times 10^4 \text{ Bq}; \frac{-dN'(t = 10\text{ min})}{dt} = 3.6 \times 10^5 \text{ Bq};$$
$$\frac{-dN'}{dt}(t = 25\text{ min}) = 7.5 \times 10^5 \text{ Bq}; \frac{-dN'(t = 50\text{ min})}{dt} = 1.1 \times 10^6 \text{ Bq};$$
$$\frac{-dN'}{dt}(t = 75\text{ min}) = 1.3 \times 10^6 \text{ Bq}; \frac{-dN'(t = 180\text{ min})}{dt} = 1.5 \times 10^6 \text{ Bq};$$

c) $N_{\max} = \dfrac{K}{\lambda} = \dfrac{(1.5 \times 10^6)\,(60)}{(0.02772)} = 3.2 \times 10^9$ atoms.

d) The maximum activity is at saturation, when the rate being produced equals that decaying and so it equals 1.5×10^6 decays/s.

43.78: The activity of the original iron, after 1000 hours of operation, would be $(9.4\times10^{-6}\,\text{Ci})\,(3.7\times10^{10}\ \text{Bq/Ci})2^{-(1000\ \text{hr})/(45\ \text{d}\times 24\ \text{hr/d})} = 1.8306\times10^{5}\ \text{Bq}$.

The activity of the oil is 84 Bq, or 4.5886×10^{-4} of the total iron activity, and this must be the fraction of the mass worn, or mass of $4.59\times10^{-2}\ \text{g}$. The rate at which the piston rings lost their mass is then $4.59\times10^{-5}\ \text{g/hr}$.

44.1: a) $K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right) = 0.1547mc^2$

$m = 9.109\times10^{-31}$ kg, so $K = 1.27\times10^{-14}$ J

b) The total energy of each electron or positron is $E = K + mc^2 = 1.1547mc^2 =$ 9.46×10^{-14} J. The total energy of the electron and positron is converted into the total energy of the two photons. The initial momentum of the system in the lab frame is zero (since the equal-mass particles have equal speeds in opposite directions), so the final momentum must also be zero. The photons must have equal wavelengths and must be traveling in opposite directions. Equal λ means equal energy, so each photon has energy 9.46×10^{-14} J.

c) $E = hc/\lambda$ so $\lambda = hc/E = hc/(9.46\times10^{-14}\text{ J}) = 2.10$ pm

The wavelength calculated in Example 44.1 is 2.43 pm. When the particles also have kinetic energy, the energy of each photon is greater, so its wavelength is less.

44.2: The total energy of the positron is

$$E = K + mc^2 = 5.00\text{ MeV} + 0.511\text{ MeV} = 5.51\text{ MeV}.$$

We can calculate the speed of the positron from Eq. 37.38

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{v}{c} = \sqrt{1-\left(\frac{mc^2}{E}\right)^2} = \sqrt{1-\left(\frac{0.511\text{ MeV}}{5.51\text{ MeV}}\right)^2} = 0.996.$$

44.3: Each photon gets half of the energy of the pion

$$E_\gamma = \frac{1}{2}m_\pi c^2 = \frac{1}{2}(270\ m_e)c^2 = \frac{1}{2}(270)(0.511\text{ MeV}) = 69\text{ MeV}$$

$$\Rightarrow f = \frac{E}{h} = \frac{(6.9\times10^7\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})} = 1.7\times10^{22}\text{ Hz}$$

$$\Rightarrow \lambda = \frac{c}{f} = \frac{3.00\times10^8\text{ m/s}}{1.7\times10^{22}\text{ Hz}} = 1.8\times10^{-14}\text{m gamma ray.}$$

44.4: a) $\lambda = \frac{hc}{E} = \frac{hc}{m_\mu c^2} = \frac{h}{m_\mu c} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})}{(207)(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})}$

$$= 1.17\times10^{-14}\text{ m} = 0.0117\text{ pm.}$$

In this case, the muons are created at rest (no kinetic energy). b) Shorter wavelengths would mean higher photon energy, and the muons would be created with non-zero kinetic energy.

44.5: a) $\Delta m = m_{\pi+} - m_{\mu+} = 270\ m_e - 207\ m_e = 63\ m_e$

$\Rightarrow E = 63(0.511\text{ MeV}) = 32$ MeV.

b) A positive muon has less mass than a positive pion, so if the decay from muon to pion was to happen, you could always find a frame where energy was not conserved. This cannot occur.

44.6: a) The energy will be the proton rest energy, 938.3 MeV, corresponding to a frequency of 2.27×10^{23} Hz and a wavelength of 1.32×10^{-15} m. b) The energy of each photon will be $938.3\ \text{MeV}+830\ \text{MeV}=1768\ \text{MeV},$ with frequency 42.8×10^{22} Hz and wavelength 7.02×10^{-16} m.

44.7: $E=(\Delta m)c^2=(400\ \text{kg}+400\ \text{kg})(3.00\times10^{8}\ \text{m/s})^2=7.20\times10^{19}\ \text{J}.$

44.8: ${}^4_2\text{He}+{}^9_4\text{Be}\rightarrow{}^{12}_6\text{C}+{}^1_0\text{n}$

We take the masses for these reactants from Table 43.2, and use Eq. 43.23

$$Q=(4.002603\ \text{u}+9.012182\ \text{u}-12.000000\ \text{u}-1.008665\ \text{u})(931.5\ \text{MeV/u})$$
$$=5.701\ \text{MeV. This is an exoergic reaction.}$$

44.9: ${}^1_0\text{n}+{}^{10}_5\text{B}\rightarrow{}^7_3\text{Li}+{}^4_2\text{He}$

$$m({}^1_0\text{n}+{}^{10}_5\text{B})=1.008665\ \text{u}+10.012937\ \text{u}=11.021602\ \text{u}$$
$$m({}^7_3\text{Li}+{}^4_2\text{He})=7.016004\ \text{u}+4.002603\ \text{u}=11.018607\ \text{u}$$
$$\Delta m=0.002995\ \text{u};\ (0.002995\ \text{u})(931.5\ \text{MeV/u})=2.79\ \text{MeV}$$

The mass decreases so energy is released and the reaction is exoergic.

44.10: a) The energy is so high that the total energy of each particle is half of the available energy, 50 GeV. b) Equation (44.11) is applicable, and $E_\text{a}=226\ \text{MeV}.$

44.11: a) $\omega=\dfrac{|q|B}{m}\Rightarrow B=\dfrac{m\omega}{|q|}=\dfrac{2\pi mf}{|q|}$

$$\Rightarrow B=\frac{2\pi(2.01\ \text{u})(1.66\times10^{-27}\ \text{kg/u})(9.00\times10^{6}\ \text{Hz})}{1.60\times10^{-19}\ \text{C}}$$
$$\Rightarrow B=1.18\ \text{T}$$

b) $K=\dfrac{q^2B^2R^2}{2m}=\dfrac{(1.60\times10^{-19}\ \text{C})^2(1.18\ \text{T})^2(0.32\ \text{m})^2}{2(2.01\ \text{u})(1.66\times10^{-27}\ \text{kg/u})}=5.47\times10^{-13}\ \text{J}$

$$=3.42\times10^{6}\ \text{eV}=3.42\ \text{MeV}$$

and $v=\sqrt{\dfrac{2K}{m}}=\sqrt{\dfrac{2(5.47\times10^{-13}\ \text{J})}{(2.01\ \text{u})(1.66\times10^{-27}\ \text{kg/u})}}=1.81\times10^{7}\ \text{m/s}.$

44.12: a) $2f=\dfrac{\omega}{\pi}=\dfrac{eB}{m\pi}=3.97\times10^{7}/\text{s}.$ b) $\omega R=\dfrac{eBR}{m}=3.12\times10^{7}\ \text{m/s}.$ c) For three-figure precision, the relativistic form of the kinetic energy must be used,

$eV=(\gamma-1)mc^2,$ so $eV=(\gamma-1)mc^2,$ so $V=\dfrac{(\gamma-1)mc^2}{e}=5.11\times10^{6}\ \text{V}.$

44.13: a) $E_a^2 = 2mc^2(E_m + mc^2)$

$$\Rightarrow E_m = \frac{E_a^2}{2mc^2} - mc^2$$

The mass of the alpha particle is that of a $^4_2$He atomic mass, minus two electron masses. But to 3 significant figures this is just

$M(^4_2\text{He}) = 4.00\ \text{u} = (4.00\ \text{u})(0.9315\ \text{GeV/u}) = 3.73\ \text{GeV}.$

So $E_m = \dfrac{(16.0\ \text{GeV})^2}{2(3.73\ \text{GeV})} - 3.73\ \text{GeV} = 30.6\ \text{GeV}.$

b) For colliding beams of equal mass, each has half the available energy, so each has 8.0 GeV.

44.14: a) $\gamma = \dfrac{1000 \times 10^3\ \text{MeV}}{938.3\ \text{MeV}} = 1065.8$, so $v = 0.999999559c$.

b) Nonrelativistic:

$$\omega = \frac{eB}{m} = 3.83 \times 10^8\ \text{rad/s}.$$

Relativistic:

$$\omega = \frac{eB}{m}\frac{1}{\gamma} = 3.59 \times 10^5\ \text{rad/s}.$$

44.15: a) With $E_m >> mc^2$, $E_m = \dfrac{E_a^2}{2mc^2}$ Eq. (44.11).

So $E_m = \dfrac{[2(38.7\ \text{GeV})]^2}{2(0.938\ \text{GeV})} = 3190\ \text{GeV}.$

b) For colliding beams the available energy E_a is that of both beams. So two proton beams colliding would each need energy of 38.7 GeV to give a total of 77.4 GeV.

44.16: The available energy E_a must be $(m_{\eta^0} + 2m_p)c^2$, so Eq. (44.10) becomes

$(m_{\eta^0} + 2m_p)^2 c^4 = 2m_p c^2 (E_t + 2m_p c^2)$, or

$$E_t = \frac{(m_{\eta^0} + 2m_p)^2 c^2}{2m_p} - 2m_p c^2$$

$$= \frac{(547.3\ \text{MeV} + 2(938.3\ \text{MeV}))^2}{2(938.3\ \text{MeV})} - 2(938.3\ \text{MeV}) = 1254\ \text{MeV}.$$

44.17: Section 44.3 says $m(\text{Z}^0) = 91.2\ \text{GeV}/c^2$.

$E = 91.2 \times 10^9\ \text{eV} = 1.461 \times 10^{-8}\ \text{J};\ m = E/c^2 = 1.63 \times 10^{-25}\ \text{kg}$

$m(\text{Z}^0)/m(\text{p}) = 97.2$

44.18: a) We shall assume that the kinetic energy of the Λ^0 is negligible. In that case we can set the value of the photon's energy equal to *Q*.

$$Q = (1193 - 1116)\text{ MeV} = 77\text{ MeV} = E_{\text{photon}}\ .$$

b) The momentum of this photon is

$$p = \frac{E_{\text{photon}}}{c} = \frac{(77\times10^{6}\text{ eV})(1.60\times10^{-18}\text{ J/eV})}{(3.00\times10^{8}\text{ m/s})} = 4.1\times10^{-20}\text{ kg}\cdot\text{m/s}$$

To justify our original assumption, we can calculate the kinetic energy of a Λ^0 that has this value of momentum

$$K_{\Lambda^0} = \frac{p^2}{2m} = \frac{E^2}{2mc^2} = \frac{(77\text{ MeV})^2}{2(1116\,\text{MeV})} = 2.7\text{ MeV} << Q = 77\text{ MeV}.$$

Thus, we can ignore the momentum of the Λ^0 without introducing a large error.

44.19: $\Delta m = M(\Sigma^+) - m_p - m_{\pi^0}$. Using Table (44.3):

$$\Rightarrow E = (\Delta m)c^2 = 1189\text{ MeV} - 938.3\text{ MeV} - 135.0\text{ MeV}$$
$$= 116\text{ MeV}.$$

44.20: From Table (44.2), $(m_\mu - m_{\text{e}} - 2m_\nu)c^2 = 105.2\text{ MeV}.$

44.21: Conservation of lepton number.

a) $\mu^- \to e^- + \nu_e + \overline{\nu}_\mu \Rightarrow L_u : +1 \neq -1,\ L_e : 0 \neq +1 + 1$
so lepton numbers are not conserved.

b) $\tau^- \to e^- + \overline{\nu}_e + \nu_\tau \Rightarrow L_e : 0 = +1 - 1$
$L_\tau : +1 = +1$
so lepton numbers are conserved.

c) $\pi^+ \to e^+ + \gamma$. Lepton numbers are not conserved since just one lepton is produced from zero original leptons.

d) $\text{n} \to p + e^- + \overline{\gamma}_e \Rightarrow L_e : 0 = +1 - 1,$ so the lepton numbers are conserved.

44.22: a) Conserved: Both the neutron and proton have baryon number 1, and the electron and neutrino have baryon number 0. b) Not conserved: The initial baryon number is 1 +1 = 2 and the final baryon number is 1. c) Not conserved: The proton has baryon number 1, and the pions have baryon number 0. d) Conserved: The initial and final baryon numbers are 1+1 = 1+1+0.

44.23: Conservation of strangeness:

a) $K^+ \to \mu^+ + \nu_\mu$. Strangeness is not conserved since there is just one strange particle, in the initial states.

b) $\mathrm{n} + K^+ \to p + \pi^0$. Again there is just one strange particle so strangeness cannot be conserved.

c) $K^+ + K^- \to \pi^0 + \pi^0 \Rightarrow S: +1 - 1 = 0,$ so strangeness is conserved.

d) $p + K^- \to \Lambda^0 + \pi^0 \Rightarrow S: 0 - 1 = -1 + 0,$ so strangeness is conserved.

44.24: a) Using the values of the constants from Appendix F,

$$\frac{e^2}{4\pi\varepsilon_0 \hbar c} = 7.29660475 \times 10^{-3} = \frac{1}{137.050044},$$

or $1/137$ to three figures.

b) From Section 38.5

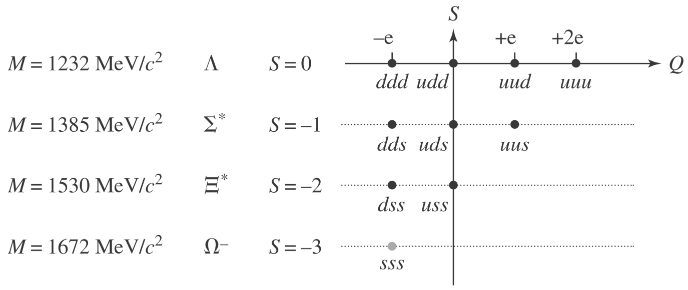
$$v_1 = \frac{e^2}{2\varepsilon_0 h}$$

but notice this is just $\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right) c$ as claimed $\left(\text{rewriting } \hbar \text{ as } \frac{h}{2\pi}\right)$.

44.25: $\left[\frac{f^2}{\hbar c}\right] = \frac{(\mathrm{J \cdot m})}{(\mathrm{J \cdot s})(\mathrm{m \cdot s^{-1}})} = 1$

and thus $\frac{f^2}{\hbar c}$ is dimensionless. (Recall f^2 has units of energy times distance.)

44.26: a)



The Ω^- particle has $Q = -1$ (as its label suggests) and $S = -3$. Its appears as a "hole"in an otherwise regular lattice in the $S - Q$ plane. The mass difference between each S row is around 145 MeV (or so). This puts the Ω^- mass at about the right spot. As it turns out, all the other particles on this lattice had been discovered already and it was this "hole" and mass regularity that led to an accurate prediction of the properties of the Ω!

b) See diagram. Use quark charges $u = +\dfrac{2}{3}$, $d = \dfrac{-1}{3}$, and $s = \dfrac{-1}{3}$ as a guide.

44.27: a) uds :

$$Q/e = \frac{2}{3} + \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right) = 0;$$

$$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1;$$

$$S = 0 + 0 + (-1) = -1$$

$$C = 0 + 0 + 0 = 0.$$

b) $c\bar{u}$:

$$\frac{Q}{e} = \frac{2}{3} + \frac{-2}{3} = 0;$$

$$B = \frac{1}{3} + \left(\frac{-1}{3}\right) = 0;$$

$$S = 0 + 0 = 0;$$

$$C = 1 + 0 = 1.$$

c) ddd:

$$\frac{Q}{e} = 3\left(\frac{-1}{3}\right) = -1;\ B = 3\left(\frac{1}{3}\right) = 1;$$

$$S = 3(0) = 0;\ C = 3(0) = 0.$$

d) $d\bar{c}$:

$$\frac{Q}{e} = \frac{-1}{3} + \left(\frac{-2}{3}\right) = -1;\ B = \frac{1}{3} + \left(\frac{-1}{3}\right) = 0;$$

$$S = 0 + 0 = 0;\ C = 0 + (-1) = -1.$$

44.28: a) $S = 1$ indicates the presence of one $\bar{s}$ antiquark and no *s* quark. To have baryon number 0 there can be only one other quark, and to have net charge + *e* that quark must be a *u*, and the quark content is $u\bar{s}$. b) The particle has an $\bar{s}$ antiquark, and for a baryon number of –1 the particle must consist of three antiquarks. For a net charge of –*e*, the quark content must be $\bar{d}\bar{d}\bar{s}$. c) $S = -2$ means that there are two *s* quarks, and for baryon number 1 there must be one more quark. For a charge of 0 the third quark must be a *u* quark and the quark content is *uss*.

44.29: a) The antiparticle must consist of the antiquarks so:

$\bar{\text{n}} = \bar{u}\bar{d}\bar{d}$.

b) So $\text{n} = udd$ is not its own antiparticle

c) $\psi = c\bar{c}$ so $\bar{\psi} = \bar{c}c = \psi$ so the ψ is its own antiparticle.

44.30: $(m_\gamma - 2m_\tau)c^2 = (9460\text{ MeV} - 2(1777\text{ MeV})) = 5906\text{ MeV}$ (see Sections 44.3 and 44.4 for masses).

44.31: In β^+ decay, ${}^1_1\text{p} \to {}^{\;\,0}_{+1}\text{e} + {}^1_0\text{ n} + \nu_e$

${}^1_1\text{p} = \boldsymbol{uud}$, ${}^1_0\text{n} = \boldsymbol{udd}$, so in β^+ decay a ***u*** quark changes to a ***d*** quark.

44.32: a) Using the definition of z from Example 44.9 we have that

$$1 + z = 1 + \frac{(\lambda_0 - \lambda_s)}{\lambda_0} = \frac{\lambda_0}{\lambda_s}.$$

Now we use Eq. 44.13 to obtain

$$1 + z = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} = \sqrt{\frac{1+\beta}{1-\beta}}.$$

b) Solving the above equation for β we obtain

$$\beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \frac{1.5^2 - 1}{1.5^2 + 1} = 0.3846.$$

Thus, $v = 0.3846\ c = 1.15 \times 10^8\ \text{m/s}$.

c) We can use Eq. 44.15 to find the distance to the given galaxy,

$$r = \frac{v}{H_0} = \frac{(1.15 \times 10^8\ \text{m/s})}{(7.1 \times 10^4\ (\text{m/s})/\text{Mpc})} = 1.6 \times 10^3\ \text{Mpc}$$

44.33: a) $v = H_0 r = (20\ (\text{km/s})/\text{Mly})(5210\ \text{Mly}) = 1.04 \times 10^5\ \text{km/s}$.

b) $\dfrac{\lambda_0}{\lambda_s} = \sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{3.0 \times 10^5\ \text{km/s} + 1.04 \times 10^5\ \text{km/s}}{3.0 \times 10^5\ \text{km/s} - 1.04 \times 10^5\ \text{km/s}}} = 1.44.$

44.34: From Eq. (44.15), $r = \dfrac{c}{H_0} = \dfrac{3.00 \times 10^8\ \text{m/s}}{20(\text{km/s})/\text{Mly}} = 1.5 \times 10^4\ \text{Mly}.$ b) This distance represents looking back in time so far that the light has not been able to reach us.

44.35: a) $v = \left[\frac{(\lambda_0/\lambda_s)^2 - 1}{(\lambda_0/\lambda_s)^2 + 1}\right] c = \left[\frac{(658.5/590.0)^2 - 1}{(658.5/590.0)^2 - 1}\right](2.998 \times 10^8 \text{ m/s})$

$= 3.280 \times 10^7 \text{ m/s}.$

b) $r = \frac{v}{H_0} = \frac{3.280 \times 10^7 \text{ m/s}}{2.0 \times 10^4 \text{ m/Mly}} = 1640 \text{ Mly}.$

44.36: Squaring both sides of Eq. (44.13) and multiplying by $c - v$ gives $\lambda_0^2(c - v) = \lambda_s^2(c + v),$ and solving this for v gives Eq. (44.14).

44.37: a) $\Delta m = M(^1_1\text{H}) + M(^2_1\text{H}) - M(^3_2\text{He})$ where atomic masses are used to balance electron masses.

$\Rightarrow \Delta m = 1.007825 \text{ u} + 2.014102 \text{ u} - 3.16029 \text{ u} = 5.898 \times 10^{-3} \text{ u}$

$\Rightarrow E = (\Delta m)c^2 = (5.898 \times 10^{-3} \text{ u})(931.5 \text{ MeV/u}) = 5.494 \text{ MeV}.$

b) $\Delta m = m_\text{n} + M(^3_2\text{He}) - M(^4_2\text{He})$

$= 1.0086649 \text{ u} + 3.016029 \text{ u} - 4.002603 \text{ u}$

$= 0.022091 \text{ u}$

$\Rightarrow E = (\Delta m)c^2 = (0.022091 \text{ u})(931.5 \text{ MeV/u}) = 20.58 \text{ MeV}.$

44.38: $3m(^4\text{He}) - m(^{12}\text{C}) = 7.80 \times 10^{-3}$ u, or 7.27 MeV.

44.39: $\Delta m = m_\text{e} + m_\text{p} - m_\text{n} - m_{\nu_\text{e}}$ so assuming $m_{\nu_\text{e}} \approx 0,$

$\Delta m = 0.0005486 \text{ u} + 1.007276 \text{ u} - 1.008665 \text{ u} = -8.40 \times 10^{-4} \text{ u}$

$\Rightarrow E = (\Delta m)c^2 = (-8.40 \times 10^{-4} \text{ u})(931.5 \text{ MeV/u}) = -0.783 \text{ MeV}$ and is endoergic.

44.40: $m_{^{12}_6\text{C}} + m_{^4_2\text{He}} - m_{^{16}_8\text{O}} = 7.69 \times 10^{-3}$ u, or 7.16 MeV, an exoergic reaction.

44.41: For blackbody radiation $\lambda_m T = 2.90 \times 10^{-3} \text{m} \cdot K,$ so $\lambda_{m_1} T_1 = \lambda_{m_2} T_2 \Rightarrow \lambda_{m_1} =$

$(1.062 \times 10^{-3} \text{m})\frac{2.728 \text{ K}}{3000 \text{ K}} = 9.66 \times 10^{-7} \text{m}.$

44.42: a) The dimensions of $\hbar$ are energy times time, the dimensions of G are energy tim time per mass squared, and so the dimensions of $\sqrt{\hbar G / c^3}$ are

$$\left[\frac{(\text{E} \cdot \text{T})(\text{E} \cdot \text{L/M}^2)}{(\text{L/T})^3}\right]^{1/2} = \left[\frac{\text{E}}{\text{M}}\right]\left[\frac{\text{T}^2}{\text{L}}\right] = \left[\frac{\text{L}}{\text{T}}\right]^2 \left[\frac{\text{T}^2}{\text{L}}\right] = \text{L}.$$

b) $\left(\frac{\hbar G}{c^3}\right)^{1/2} = \left(\frac{(6.626 \times 10^{-34} \text{J} \cdot \text{s})(6.673 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2)}{2\pi(3.00 \times 10^8 \text{ m/s})^3}\right)^{1/2} = 1.616 \times 10^{-35} \text{m}.$

44.43: a) $\text{E}_\text{a} = 2(7 \text{ TeV}) = 14 \text{ TeV}$

b) Fixed target; equal mass particles,

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(1.4 \times 10^7 \text{ MeV})^2}{2(938.3 \text{ MeV})} - 938.3\text{MeV}$$

$$= 1.04 \times 10^{11} \text{ MeV} = 1.04 \times 10^5 \text{TeV}.$$

44.44: $K + m_{\text{p}}c^2 = \dfrac{hc}{\lambda}, K = \dfrac{hc}{\lambda} - m_{\text{p}}c^2 = 652$ MeV.

44.45: The available energy must be the sum of the final rest masses: (at least)

$$E_{\text{a}} = 2m_{\text{e}}c^2 + m_{\pi^0}c^2$$
$$= 2(0.511\,\text{MeV}) + 135.0\ \text{MeV}$$
$$= 136.0\ \text{MeV}.$$

For alike target and beam particles: $E_{m_e} = \dfrac{E_{\text{a}}^2}{2m_{\text{e}}c^2} - m_{\text{e}}c^2 = \dfrac{(136.0\ \text{MeV})^2}{2(0.511\,\text{MeV})} - 0.511$ MeV $= 1.81\times10^4$ MeV. So $K = (1.81\times10^4\ \text{MeV}) - m_{\text{e}}c^2 = 1.81\times10^4$ MeV.

44.46: In Eq.(44.9),

$E_{\text{a}} = (m_{\Sigma^0} + m_{K^0})c^2$, and with $M = m_{\text{p}}, m = m_{\pi^-}$ and $E_m = (m_{\pi^-})c^2 + K$,

$$K = \frac{E_{\text{a}}^2 - (m_\pi - c^2)^2 - (m_{\text{p}}c^2)^2}{2m_{\text{p}}c^2} - (m_{\pi^-})c^2$$
$$= \frac{(1193\,\text{MeV} + 497.7\ \text{MeV})^2 - (139.6\,\text{MeV})^2 - (938.3\,\text{MeV})^2}{2(938.3\,\text{MeV})} - 139.6\text{MeV}$$
$$= 904\,\text{MeV}.$$

44.47: The available energy must be at least the sum of the final rest masses.

$E_{\text{a}} = (m_{\Lambda^0})c^2 + (m_{K^+})c^2 + (m_{K^-})c^2 = 1116\ \text{MeV} + 2(493.7\ \text{MeV}) = 2103\ \text{MeV}. E_{\text{a}}^2 = 2(m_{\text{p}})$

$((m_{\text{p}})c^2)^2 + ((m_{K^-})c^2)^2$.

$$\text{So } E_{K^-} = \frac{E_{\text{a}}^2 - ((m_{\text{p}})c^2)^2 - ((m_{K^-})c^2)^2}{2(m_{\text{p}})c^2} = \frac{(2103)^2 - (938.3)^2 - (493.7)^2}{2(938.3)}\text{MeV}$$
$$\Rightarrow E_{K^-} = 1759\ \text{MeV} = (m_{K^-})c^2 + K.$$

So the threshold energy K = 1759 MeV– 493.7 MeV=1265 MeV.

44.48: a) The decay products must be neutral, so the only possible combinations are $\pi^0\pi^0\pi^0$ *or* $\pi^0\pi^+\pi^-$

b) $m_{\eta_0} - 3m_{\pi^0} = 142.3\text{Me V}/c^2$, so the kinetic energy of the π^0 mesons is 142.3 MeV. For the other reaction, $K = (m_{\eta_0} - m_{\pi^0} - m_{\pi^+} - m_{\pi^-})c^2 = 133.1\,\text{MeV}$.

44.49: a) If the π^- decays, it must end in an electron and neutrinos. The rest energy of π (139.6 MeV) is shared between the electron rest energy (0.511 MeV) and kinetic energy (assuming the neutrino masses are negligible). So the energy released is 139.6 MeV – 0.511 MeV = 139.1 MeV.

b) Conservation of momentum leads to the neutrinos carrying away most of the energy.

44.50: $\dfrac{\hbar}{\Delta E}=\dfrac{(1.054\times10^{-34}\,\text{J}\cdot\text{s})}{(4.4\times10^{6}\,\text{eV})(1.6\times10^{-19}\,\text{J/eV})}=1.5\times10^{-22}\,\text{s}.$

44.51: a) $E=(\Delta m)c^2=(m_{\text{p}})c^2-(m_{K+})c^2-(m_{K^-})c^2$

$$=1019.4\,\text{MeV}-2(493.7\,\text{MeV})$$

$$=32.0\,\text{MeV}.$$

Each kaon gets half the energy so the kinetic energy of the K^+ is 16.0 MeV.

b) Since the π^0 mass is greater than the energy left over in part (a), it could not have been produced in addition to the kaons.

c) Conservation of strangeness will not allow $\phi\to K^+ +\pi^- or\ \phi\to K^+ +\mu^-$.

44.52: a) The baryon number is 0, the charge is $+e$, the strangeness is 1, all lepton numbers are zero, and the particle is K^+. b) The baryon number is 0, the charge is $-e$, the strangeness is 0, all lepton numbers are zero, and the particle is π^-. c) The baryon numbers is –1, the charge is 0, the strangeness is zero, all lepton numbers are 0, and the particle is an antineutron. d) The baryon number is 0, the charge is $+e$, the strangeness is 0, the muonic lepton number is –1, all other lepton numbers are 0, and the particle is μ^+.

44.53: $\Delta t=7.6\times10^{-21}\text{s}\Rightarrow\Delta E=\dfrac{\hbar}{\Delta t}=\dfrac{1.054\times10^{-34}\,\text{J}\cdot\text{s}}{7.6\times10^{-21}\,\text{s}}=1.39\times10^{-14}\ \text{J}=87\ \text{keV}$

$\dfrac{\Delta E}{m_\psi c^2}=\dfrac{0.087\,\text{MeV}}{3097\,\text{MeV}}=2.8\times10^{-5}.$

44.54: a) The number of protons in a kilogram is

$$(1.00\ \text{kg})\left(\frac{6.023\times10^{23}\ \text{molecules/mol}}{18.0\times10^{-3}\ \text{kg/mol}}\right)(2\ \text{protons/molecule})=6.7\times10^{25}.$$

Note that only the protons in the hydrogen atoms are considered as possible sources of proton decay. The energy per decay is $m_{\text{p}}c^2=938.3\,\text{MeV}=1.503\times10^{-10}\,\text{J}$, and so the energy deposited in a year, per kilogram, is $(6.7\times10^{25})\left(\dfrac{\ln(2)}{1.0\times10^{18}\,\text{y}}\right)(1\ \text{y})\ (1.50\times10^{-10}\ \text{J})=$ $7.0\times10^{-3}\text{Gy}=0.70\ \text{rad}.$

b) For an RBE of unity, the equivalent dose is (1) (0.70 rad) = 0.70 rem.

44.55: a) $E = (\Delta m)c^2 = (m_{\Xi^-})c^2 - (m_{\Lambda^0})c^2 - (m_{\pi^-})c^2$

$= 1321\,\text{MeV} - 1116\,\text{MeV} - 139.6\,\text{MeV} \Rightarrow E = 65\,\text{MeV}.$

b) Using (nonrelativistic) conservation of momentum and energy: $P_{\Lambda^0} = 0 = P_f =$

$m_{\Lambda^0} v_{\Lambda^0} - m_{\pi^-} - v_{\pi^-} \Rightarrow v_{\pi^-} = \left(\dfrac{m_{\Lambda^0}}{m_{\pi^-}}\right) v_{\Lambda^0}.$

Also $K_{\Lambda^0} + K_{\pi^-} = E$ from part (a).

$$\text{So } K_{\Lambda^0} + \frac{1}{2} m_{\pi^-} v_{\pi^-}^2 = K_{\Lambda^0} + \frac{1}{2}\left(\frac{m_{\Lambda^0}}{m_{\pi^-}}\right) m_{\Lambda^0} v_{\Lambda^0}^2 = K_{\Lambda^0}\left(1 + \frac{m_{\Lambda^0}}{m_{\pi^-}}\right)$$

$$\Rightarrow K_{\Lambda^0} = \frac{E}{1 + \dfrac{m_{\Lambda^0}}{m_{\pi^-}}} = \frac{65\,\text{MeV}}{1 + \dfrac{1116\,\text{MeV}}{139.6\,\text{MeV}}} = 7.2\,\text{MeV}$$

$$\Rightarrow K_{\pi^-} = 65 - 7.2\,\text{MeV} = 57.8\,\text{MeV}.$$

So the fractions of energy carried off by the particles are $\dfrac{7.2}{65} = 0.11$ for the Λ^0 and 0.89 for the π^-.

44.56: a) For this model, $\dfrac{dR}{dt} = HR$, so $\dfrac{dR/dt}{R} = \dfrac{HR}{R} = H$, presumed to be the same for all points on the surface. b) For constant θ, $\dfrac{dr}{dt} = \dfrac{dR}{dt}\theta = HR\theta = Hr$. c) See part (a), $H_0 = \dfrac{dR/dt}{R}$. d) The equation $\dfrac{dR}{dt} = H_0 R$ is a differential equation, the solution to which, for constant H_0 is $R(t) = R_0 e^{H_0 t}$, where R_0 is the value of R at $t = 0$. This equation may be solved by separation of variables, as $\dfrac{dR/dt}{R} = \dfrac{d}{dt} \ln(R) = H_0$ and integrating both sides with respect to time. e) A constant H_0 would mean a constant critical density, which is inconsistent with uniform expansion.

44.57: From Pr.(44.56): $r = R\theta \Rightarrow R = \dfrac{r}{\theta}$.

So $\dfrac{dR}{dt} = \dfrac{1}{\theta}\dfrac{dr}{dt} - \dfrac{r}{\theta^2}\dfrac{d\theta}{dt} = \dfrac{1}{\theta}\dfrac{dr}{dt}$ since $\dfrac{d\theta}{dt} = 0$.

So $\dfrac{1}{R}\dfrac{dR}{dt} = \dfrac{1}{R\theta}\dfrac{dr}{dt} = \dfrac{1}{r}\dfrac{dr}{dt} \Rightarrow v = \dfrac{dr}{dt} = \left(\dfrac{1}{R}\dfrac{dR}{dt}\right)r = H_0 r$.

Now $\dfrac{dv}{d\theta} = 0 = \dfrac{d}{d\theta}\left(\dfrac{r}{R}\dfrac{dR}{dt}\right) = \dfrac{d}{d\theta}\left(\theta\dfrac{dR}{dt}\right)$

$\Rightarrow \theta\dfrac{dR}{dt} = K$ where K is a constant.

$\Rightarrow \dfrac{dR}{dt} = \dfrac{K}{\theta} \Rightarrow R = \left(\dfrac{K}{\theta}\right)t$ since $\dfrac{d\theta}{dt} = 0$.

$\Rightarrow H_0 = \dfrac{1}{R}\dfrac{dR}{dt} = \dfrac{\theta}{Kt}\dfrac{K}{\theta} = \dfrac{1}{t}$.

So the current value of the Hubble constant is $\dfrac{1}{T}$ where T is the present age of the universe.

44.58: a) For mass *m*, in Eq. (37.23) $u = -v_{\text{cm}}, v' = v_0$, and so $v_m = \dfrac{v_0 - v_{\text{cm}}}{1 - v_0 v_{\text{cm}} / c^2}$.

For mass M, $u = -v_{\text{cm}}, v' = 0$, so $v_M = -v_{\text{cm}}$. b) The condition for no net momentum in the center of mass frame is $m\gamma_m v_m + M\gamma_M v_M = 0$, where γ_m *and* γ_M correspond to the velocities found in part (a). The algebra reduces to $\beta_m \gamma_m = (\beta_0 - \beta')\gamma_0 \gamma_M$, where

$\beta_0 = \dfrac{v_0}{c}, = \beta' = \dfrac{v_{\text{cm}}}{c}$, and the condition for no net momentum becomes

$m(\beta_0 - \beta')\gamma_0 \gamma_M =$

$M\beta'\gamma_M$, *or* $\beta' = \dfrac{\beta_0}{1 + \dfrac{M}{m\gamma_0}} = \beta_0 \dfrac{m}{m + M\sqrt{1 - \beta_0^2}}$, and $v_{\text{cm}} = \dfrac{mv_0}{m + M\sqrt{1 - (v_0 / c)^2}}$.

c) Substitution of the above expression into the expressions for the velocities found in part (a) gives the relatively simple forms

$$v_m = v_0 \gamma_0 \frac{M}{m + M_{\gamma_0}}, v_M = -v_0 \gamma_0 \frac{m}{m\gamma_0 + M}.$$

After some more algebra,

$\gamma_m = \dfrac{m + M_{\gamma_0}}{\sqrt{m^2 + M^2 + 2mM_{\gamma_0}}}, \gamma_M = \dfrac{M + m_{\gamma_0}}{\sqrt{m^2 + M^2 + 2mM_{\gamma_0}}}$, from which

$m\gamma_m + M\gamma_M = \sqrt{m^2 + M^2 + 2mM\gamma_0}$.

This last expression, multiplied by c^2, is the available energy E_{a} in the center of mass frame, so that

$$\begin{aligned} E_{\text{a}}^2 &= (m^2 + M^2 + 2mM\gamma_0)c^4 \\ &= (mc^2)^2 + (Mc^2)^2 + (2Mc^2)(m\gamma_0 c^2) \\ &= (mc^2)^2 + (Mc^2)^2 + 2Mc^2 E_m, \end{aligned}$$

which is Eq. (44.9).

44.59: $\Lambda^0 \to \mathrm{n} + \pi^0$

a) $E = (\Delta m)c^2 = (m_{\Lambda^0})c^2 - (m_{\mathrm{n}})c^2 (m_{\pi^0})c^2$

$= 1116\ \mathrm{MeV} - 939.6\ \mathrm{MeV} - 135.0\ \mathrm{MeV}$

$= 41.4\ \mathrm{MeV}.$

b) Using conservation of momentum and kinetic energy; we know that the momentum of the neutron and pion must have the same magnitude, $p_n = p_\pi$

$$K_n = E_n - m_n c^2 = \sqrt{(m_n c^2)^2 + (p_n c)^2} - m_n c^2$$

$$= \sqrt{(m_n c^2)^2 + (p_n c)^2} - m_n c^2$$

$$= \sqrt{(m_n c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi} - m_n c^2$$

$$= K_\pi + K_n = K_\pi + \sqrt{(m_n c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi} - m_n c^2 = E.$$

$$(m_n c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi = E^2 + (m_n c^2)^2 + K_\pi^2 + 2Em_\pi c^2 - 2EK_\pi - 2m_n c^2 K_\pi.$$

Collecting terms we find : $K_\pi (2m_\pi c^2 + 2E + 2m_n c^2) = E^2 + 2Em_n c^2$

$$\Rightarrow K_\pi = \frac{(41.4\ \mathrm{MeV})^2 + 2(41.4\ \mathrm{MeV})(939.6\ \mathrm{MeV})}{2(135.0\ \mathrm{MeV}) + 2(41.4\ \mathrm{MeV}) + 2(939.6\ \mathrm{MeV})}.$$

$$\Rightarrow K_\pi = 35.62\ \mathrm{MeV}.$$

So the fractional energy carried by the pion is $\frac{35.62}{41.4} = 0.86$, and that of the neutron is 0.14