Simple model for directed networks

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We study a model for directed networks based on the Watts-Stogatz model for small-world phenomena. We focus on some topological aspects of directed networks inspired in food web theory, namely, the fraction of basal and top nodes in the network and node level distributions. We argue that in directed networks basal nodes play an important role, collecting information or resources from the environment. We give analytical expressions for the fraction of basal and top nodes for the model, and study the node level distributions with numerical simulations.

I. INTRODUCTION

The subject of complex networks has become relevant in several branches of science. Sometimes, complex systems can be described as a large set of interacting units. Regardless of the nature of these units and the way they interact with each other, it is possible to define a network describing the interactions present in the system, thinking of the units as the nodes of the network and placing a link between two units whenever an interaction is present. The interest in complex networks has been triggered both by novel theoretical approaches [1–3] and by the availability of large datasets for networks of different origins. In the recent literature, a systematic study of the topological properties of neural [1,4], ecological [5–7], metabolic [8], transport [9], technological [1,10,11] and social [1,9,12,22,23] networks has been carried out, revealing a rich subject with a wide scope of applications.

Most of the models proposed to account for the observed features in real networks consist of undirected networks [13], i.e., networks in which the links connecting two nodes have no definite direction. However, in many cases of interest the interactions are not symmetric. A natural way to describe these networks is to consider directed links. One example of directed networks comes from ecology, where ecosystems can be represented by food webs [14]. In predator-prey food webs, each species is represented as a node of the network, and a link is placed between two species whenever one of them feeds on the other. Food webs describe in this way the who-eats-who interactions in the ecosystem. The natural way to describe the flow of resources from prey to predators is to consider a directed network.[1] Neural networks provide another example of directed networks. In fact, neurons connect to other cells with their axon, and receive connections from other axon cells through their dendrites. This asymmetry of the cell can be accounted for with a directed link. In energetic networks as the power grid, the energy flows in some

definite direction, from providers to consumers. Also in economy, directed networks play an important role. The network of goods necessary to produce other goods is an example. Hierarchical social networks, in which the relationships between people are not symmetric, give yet another example of directed networks.

The relevance of directed networks has already been recognized and emphasized [13,17]. A model for the World Wide Web with directed links has been proposed [16] and a spreading process occurring on a directed network has been studied [18]. The problem of percolation in a directed scale-free network [19] and the dynamics of a spin model in a directed small world network [20] have also been considered.

In this paper, we propose a model for directed networks based on the Watts-Stogatz model for small-world phenomena. It is a simple extension of the model, modified to account for the direction of the links. We focus our study on some static topological properties inspired in food web theory [15]. In the following section, we introduce some definitions concerning the topology of directed networks. In the following, we present the model and give analytical expressions for the fraction of basal and top nodes. Then we show numerical results for the fraction of basal nodes and the node level distribution. We finish the paper with a discussion of the results and some remarks.

II. SOME ASPECTS OF FOOD WEB TOPOLOGY

We therefore begin with a brief discussion of some aspects of network topology inspired in food web theory. A very coarse grained classification of the species in a food web is based on the fraction of basal species, the fraction of top species, and the fraction of intermediate species in the web. Basal species feed only on the environment, and have no prey. Top species have no predators. Intermediate species have both preys and predators. Here we define basal, top, and intermediate nodes in an analogous way. Basal nodes have only outgoing links, intermediate nodes have both outgoing and incoming links, and top nodes have only incoming links. The fractions of basal, intermediate, and top nodes in the network are noted as B, I, and T. Note that by definition, $B+T+I=1$.

In food web models, basal species feed only on the environment. The ecosystem is an open system, and resources

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[1]In some contexts, though, food webs can be represented by undirected networks because a fluctuation in the population of a species has consequences both on the populations of its preys and its predators.

enter through basal species. This is why basal nodes are expected to play an important role in general directed networks. It is through basal nodes that information, energy, or resources in general come into the system from the environment. Note that in general directed networks, the fraction of basal and top nodes could be zero.

It is possible to build a much more detailed description of the trophic structure of the food web in terms of the trophic level distribution. The trophic level of a given species is sometimes defined as the shortest path to the basal species of the network [15]. Note that this definition relies on the existence of basal species, and also on the existence of such a path. Here we define in an analogous way the level of a node, λ, as the minimum number of incoming links that has to be traversed to reach the basal nodes in the network. With this definition basal nodes are at level 0, nodes receiving links from basal nodes are at level 1, and so on. The node level distribution $\rho_\Lambda(\lambda)$ gives the frequency of nodes at node level λ in the network, providing a good description of its hierarchical structure.

Finally, and for future reference, we introduce some concepts that have proved very useful in the characterization of complex networks [1,2]. The clustering coefficient c_i of a given node i is defined as the fraction of all possible connections between the neighbors of site i that is actually present. The characteristic path length is the average number of connections that have to be traversed to join two nodes of the network. The degree k_i of a given node is simply the number of links attached to node i. The degree distribution of the network $\rho_K(k)$ gives the frequency of nodes with k links in the network. It has to be stressed that these concepts are defined for undirected networks. The degree distribution for a directed network splits in the distribution of incoming links and the distribution of outgoing links [9,16]. The average path length should be calculated considering the direction of the links [18]. The definition of a clustering coefficient for directed networks is less obvious.

III. THE MODEL

We consider a directed version of the Watts-Strogatz model (DWS) for small-world phenomena [1]. This model is an interpolation between regular and random networks, with a control parameter p that measures the amount of disorder in the network. Regular networks ($p=0$) have very high clustering coefficients, as relatively big fractions of the neighbors of a site are connected with each other. The characteristic path length in regular networks grows linearly with the number of nodes N. On the other hand, random networks display very short path lengths growing as $\ln(N)$ and very small clustering. The WS model generates networks which have both high clustering coefficient and short characteristic path lengths, in a wide range of the parameter values. These features have been shown to be common to many biological, technological, and social networks, and so the model has become paradigmatic to these phenomena.

Here we propose a simple way of building a directed network based on the WS rewiring procedure. We start with a regular network consisting of N nodes linked to their K near-

est neighbors and periodic boundary conditions. The nodes are arranged in a ring with $2K$ links per node. We then rewire the network following the procedure introduced in Ref. [1]. We select a node and the link that connects this node to its nearest neighbor in a counterclockwise sense. With probability p we rewire this link to a randomly chosen node in the network, and with probability $1-p$ we leave it as it is. Self-connections and repeated connections are not allowed. The direction of the link is then chosen at random. With probability b the link goes out from the current node, and with probability $1-b$ it goes into it. We move counterclockwise around the ring and repeat this procedure for each node, until one lap is completed. Then we repeat the process with the second nearest counterclockwise neighbors of each node, and so on up to the Kth nearest neighbors. At this point, each link in the network has been considered once, its direction has been chosen, and with probability p it has been rewired to another node.

Several realizations of this model are shown in Fig. 1. To make a clear picture, we choose $N=10$ and $K=1$. In the first row we show three networks with $p=0$. These are regular networks, with no rewired links. For $b=1$ all the links go counterclockwise, for $b=0.5$ some of the links go in one direction and some of them go in the other, and for $b=0$ all the links go in a clockwise direction. In the second row we show three realizations with $p=0.5$ and $b=1,0.5,0$ respectively. For this value of p some links have been rewired. It is clear that independently of K for $b=0$, no basal nodes exist in the network because each node has at least K incoming links. For $p=0$ and $b=0,1$, neither basal nor top nodes exist in the network. For other values of b and p, however, basal and top nodes can appear leading to a classification of the nodes in a hierarchical structure.

In order to avoid confusion in the following discussion, we introduce here a definition. Note that each node has at least K links. These are the links that for $p=0$ are connected to the K nearest counterclockwise neighbors of each node. We say that a given node owns the K links that connect it to its K nearest counterclockwise neighbors before the rewiring process.

It is possible to find an analytical expression for the fraction of basal nodes β in the network. We call it β to distinguish from the numerical results for B. In general, β is a function of the number of nodes N, the connectivity K, the disorder parameter p, and the bias b. As it was noted above, for $b=0$ all the links are set as incoming and no basal nodes can appear in the network, so $\beta(N,K,p,0)\equiv0$. Let us now consider the case $p=0$ in more detail. In this case, no links are rewired and the network remains regular. Each node has its own K links, and also keeps the K links connecting it to its K clockwise neighbors. The probability for a node to become a basal node is given by the product of the independent probabilities for each of its own links to be set as outgoing, and the other K links to be set as incoming to their respective owners. This gives the expression

$$\beta(N,K,0,b)=b^K(1-b)^K. \tag{1}$$

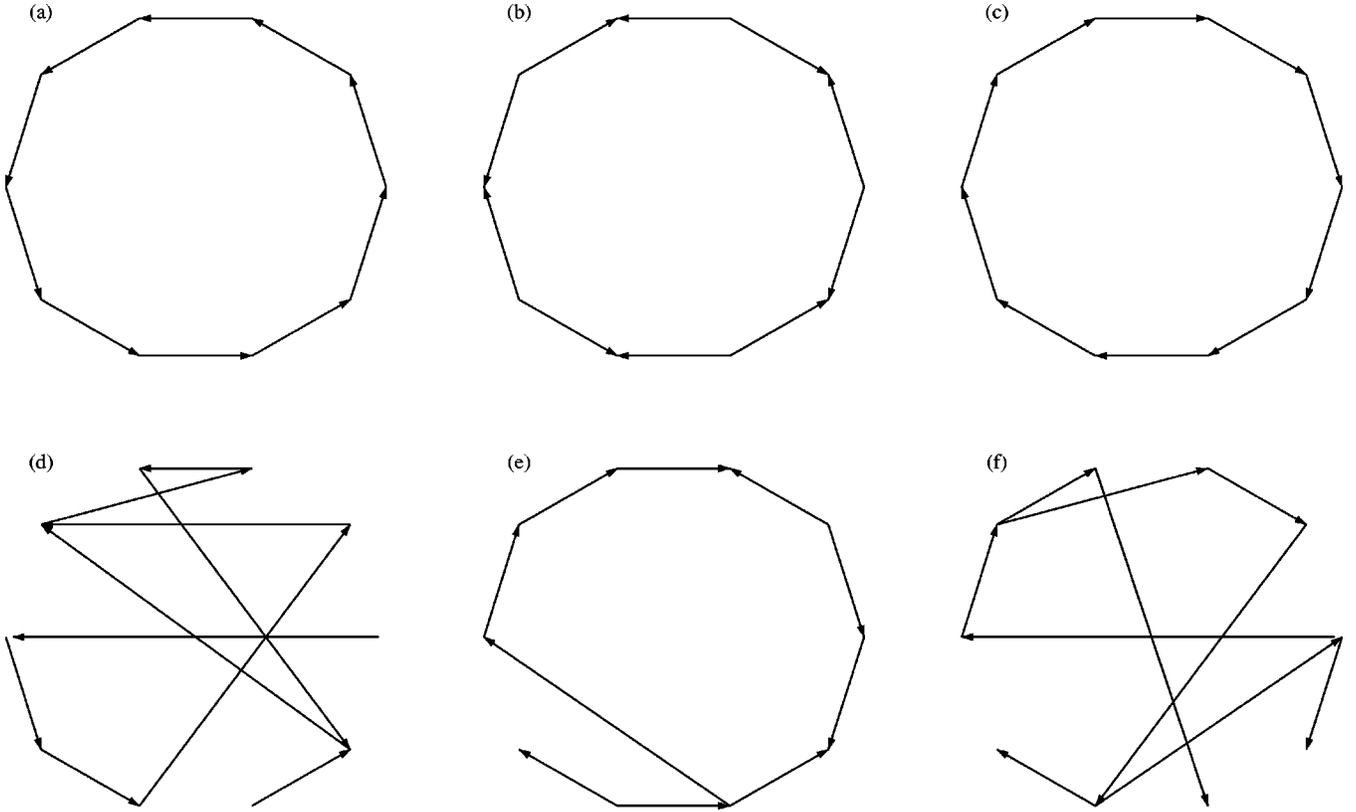The case $b=1$ is also straightforward. All the links are set as

FIG. 1. Some realizations of the DWS model with $N=10$ and $K=1$. In the first row, we have $p=0$, and (a) $b=1$, (b) $b=0.5$, (c) $b=0$. In the second row, $p=0.5$ and (d) $b=1$, (e) $b=0.5$, (f) $b=0$.

outgoing, so the probability for a node to become basal is given by the probability for losing its K clockwise links and not receiving any further rewired link from other nodes of the network. The probability for losing the K clockwise links is simply p^K. The probability of not receiving further links can be approximated by $(1-1/N)^{pNK}$, where pNK is the average number of rewired links and $(1-1/N)$ gives the probability for a link to miss a given node. This is an approximation valid for $K \ll N$, where the fact that repeated links are not allowed is neglected. Within this approximation we have

$$\beta(N,K,p,1)=p^K(1-1/N)^{pNK}, \qquad (2)$$

which can be further simplified, if $N \gg 1$, in the form

$$\beta(N,K,p,1)=p^K e^{-pK}. \qquad (3)$$

Note that the N dependence drops in this last expression. In the general case, it is still possible to write down an approximate expression for the fraction of basal nodes in the network,

$$\beta(K,p,b)=b^K[1-b(1-p)]^K e^{-pbK}. \qquad (4)$$

In this expression, the factor b^K stands, as in Eq. (1), for the probability to have the K own links of a given node set as outgoing. The second factor comes from the probability for the remaining (not rewired) of the K clockwise links to be set as incoming to their owners,

$$[1-b(1-p)]^K=\sum_{i=0}^{K}\binom{K}{i}p^i[(1-b)(1-p)]^{K-i}. \qquad (5)$$

The exponential factor stands for the probability to receive only rewired new links that are set as incoming to their owner nodes. Concerning the exponential factor, note that there are, on average, pKN rewired links. When $K \ll N$, the probability for each of these links to hit a given node is $1/N$, and the probability to miss it is $(1-1/N)$. The probability for a node to receive only rewired links that go out from it is

$$f(N,K,p,b)=\left[\frac{1-b}{N}+\left(1-\frac{1}{N}\right)\right]^{pKN}. \qquad (6)$$

For $N \gg 1$ this can be approximated by

$$f(N,K,p,b)=e^{-pbK}. \qquad (7)$$

Note that Eq. (4) reduces to Eqs. (1) and (3) in the corresponding cases. Note further that for $p=1$, we have the simple form

$$\beta(K,1,b)=b^K e^{-bK}. \qquad (8)$$

An expression for the fraction of top nodes τ in the network can also be written in a similar way, yielding

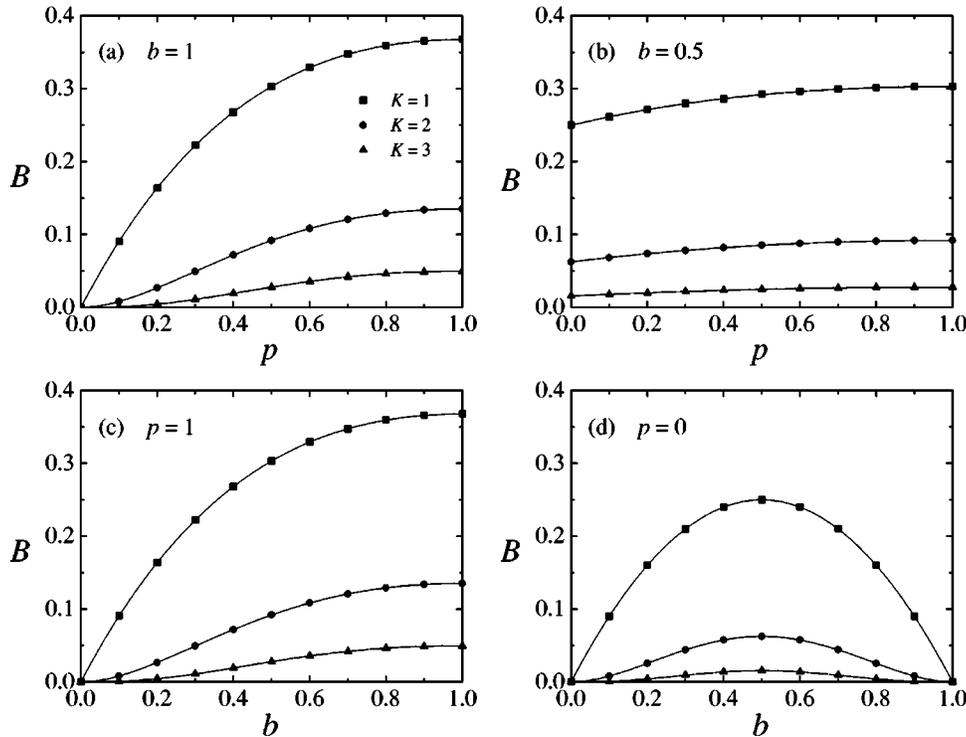$$\tau(K,p,b)=(1-b)^K[p+b(1-p)]^K e^{-p(1-b)K}. \qquad (9)$$

FIG. 2. In the top row, we plot numerical (dots) and analytical (lines) results for the fraction of basal nodes B vs disorder parameter p, for $K = 1,2,3$, and (a) $b = 1$, (b) $b = 0.5$. In the bottom, we plot B vs b for (c) $p = 1$ and (d) $p = 0$. The fraction of basal nodes is found to increase with disorder [(a) and (b)]. When $p = 1$ (c), the fraction of basal nodes grows with b, while for $p = 0$ (d) it has a maximum at $b = 0.5$.

Because of the symmetry of the network construction, we have

$$\tau(K, p, b) = \beta(K, p, 1 - b). \tag{10}$$

Note that knowing B and T, one can also compute the value of I.

The classification in basal, top, and intermediate nodes is very coarse. Nevertheless, it gives important information on the structure of the network. The knowledge of the fraction of basal and top nodes is important to understand the flow of resources in any directed network. The node level distribution gives a more precise picture of the position of the nodes in the network, and will be studied with the help of numerical simulations in the following section.

Some remarks have to be made on the DWS model. With the rewiring procedure described above, the network can become disconnected. We have discarded these events in the numerical simulations, since we are interested in connected networks of definite sizes. Besides, we have excluded from the model the possibility of self-connections and repeated connections, even with different directions. In a directed network this last interaction would result in a loop consisting of two nodes A and B, where A is connected to B and B is in turn connected to A. These features are known to be present in some real networks. Here we have neglected them in order to simplify the model, but they could be straightforwardly incorporated. Finally, we note that directed loops might appear in these networks. The simplest instance is a three-node loop, where A is connected to B, B is connected to C, C is connected to A, and other links go out from A, B, and C connecting the triangle to the network, but no links come from other nodes to A, B, or C. Thus, the triangle is connected to the network only by outgoing links. For nodes in

this kind of substructure the node level becomes ill defined, since there is no way to reach basal nodes from them following incoming links. In the numerical simulations with the parameters we have considered, we have seen that these structures are usually very rare except when $p = 0$ and $b \approx 0.5$.

IV. NUMERICAL RESULTS

In this section, we present results from numerical simulations of the model. Averages have been done over 10^4 realizations in $N = 10^3$ nodes networks. In all figures, dots correspond to numerical results and lines to analytical results, unless otherwise noted.

In Fig. 2, we plot the fraction of basal nodes B as a function of p and b for $K = 1,2,3$. Figure 2(a) shows B vs p for $b = 1$. For $p = 0$ no basal nodes exist, and the fraction of basal nodes grows monotonically with disorder in the network for $p \in [0,1]$. Note also that B decreases sensibly as the connectivity K is increased. The analytical results are in very good agreement with the numerical results for the values of K considered here. Figure 2(b) shows B vs p for $b = 0.5$. In this case, the direction of the links is chosen with equal probability in one way or the other. The fraction of basal nodes does not vary significantly with p in this case. Figure 2(c) shows the plot of the fraction of basal nodes as a function of the bias b for maximum disorder $p = 1$, and Fig. 2(d) shows B vs b for $p = 0$. In all cases, we find good agreement with the analytical result (4).

In Fig. 3, we plot B vs K for several values of p and b. Note that the scale is logarithmic in the vertical axis, to stress the exponential decay of B with K. In fact, Eq. (4) can be cast in the exponential form
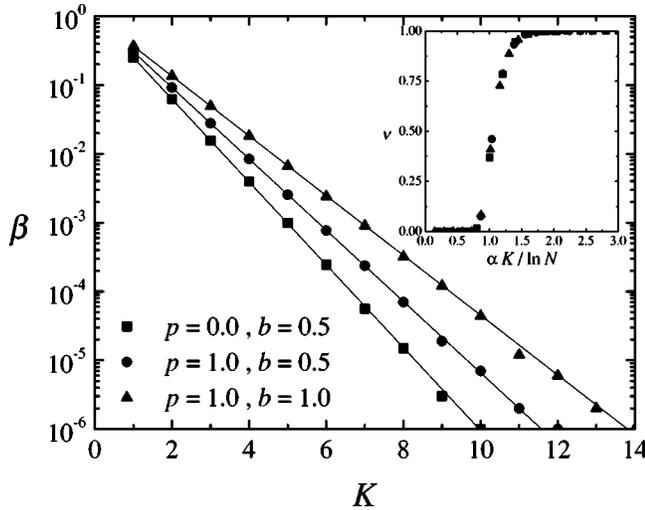
FIG. 3. Numerical (dots) and analytical (lines) results for the fraction of basal nodes B vs connectivity K. Note the logarithmic scale in the vertical axis. The fraction of basal nodes decays exponentially with connectivity. In the inset, we plot the fraction of realizations with no basal nodes ν vs normalized connectivity. When $\alpha K/\ln N < 1$, almost all realizations of the network have basal nodes, while for $\alpha K/\ln N > 1$ realizations with no basal nodes become more probable. For large enough connectivity, almost all realizations of the network have no basal nodes.

$$\beta(K,p,b) = e^{-\alpha(p,b)K}, \tag{11}$$

with

$$\alpha(p,b) = pb - \ln b - \ln[1 - b(1-p)]. \tag{12}$$

Numerical results are again in good agreement with Eq. (4) in the range considered. For large values of K we find $B < 1/N$. This means that in some realizations of the network there are no basal nodes, yielding an average value of B below $1/N$. From Eq. (11) we have that the condition $\beta = N^{-1}$ is met when $\alpha K = \ln N$. Thus for $\alpha K > \ln N$, a significant fraction of the realizations of the network has no basal nodes. This is shown in the inset of Fig. 3, where we plot the fraction of realizations with no basal nodes vs $\alpha K/\ln N$, for several values of p and b.

A detailed characterization of the hierarchical arrangement of the nodes in the network can be given in terms of the node level distribution $\rho_\Lambda(\lambda)$. In Fig. 4, we plot ρ_Λ for $K = 3$ and various values of p and b. Realizations with $B = 0$ have been discarded since the node level is ill defined in those cases, and averages have been done over realizations with $B \neq 0$. In Fig. 4(a), we plot $\rho_\Lambda(\lambda)$ for $K = 3$, $b = 0.5$, and $p = 0, 0.1, 1$. The distributions show exponential tails. For $p = 0$ an approximate fraction 0.18 of nodes belongs to directed loop structures as the one described at the end of Sec. III. As disorder increases, the fraction of basal nodes slightly increases [see Fig. 2(b)]. The average path length decreases with increasing disorder [1]. As a result of these two combined effects, the exponential decay is faster for higher levels of disorder. In Fig. 4(b), we plot $\rho_\Lambda(\lambda)$ for $b = 1$ and $p = 0, 0.2, 1$.
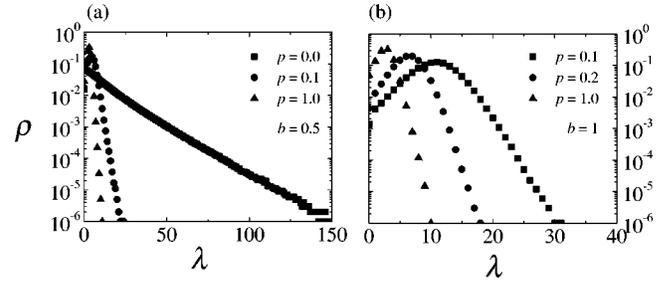

FIG. 4. Node level distributions for $K = 3$ networks, with (a) $b = 0.5$ and (b) $b = 1$. In the first case, note the tail of the distribution for $p = 0$. When the network is regular, a wide range of path lengths is found. Both for $b = 0.5$ and $b = 1$, the distribution becomes broader with increasing order, as a consequence of decreasing average path lengths.

Figure 5(a) shows $\rho_\Lambda(\lambda)$ for different connectivities $K = 1, 3, 5$ and fixed $p = 1$ and $b = 1$. We find, as expected, that the distribution is broader for smaller values of K. Next we plot, in Fig. 5(b), the level distribution for $p = 0$ and $b = 0.5$. Here we find that the distribution becomes broader as K increases from 1 to 5. This is unexpected at first sight, because the average path length is known to decrease as K grows. However, as K grows the fraction of basal nodes decays exponentially (Fig. 3). For $p = 0$ the network is regular, and as basal nodes become less abundant, the average distance to them grows. As K keeps growing, the point where $B < 1/N$ is reached. For $\alpha K/\ln N \gg 1$, the only realizations with basal nodes have most probably $B = 1/N$. The average maximum distance to the only basal node in the network can be estimated as $N/2K$, neglecting the direction of the links. The shape of $\rho_\Lambda(\lambda)$ is thus steplike, with $\rho \approx 2K/N$ for $\lambda < N/2K$ and $\rho = 0$ for $\lambda > N/2K$. When actually considering the direction of the links, the value of the average maximum level moves to higher values and the value of ρ is smaller. This can be seen in Fig. 5(b) for $K = 10$. Note that fluctuations in the direction of the links can result in realizations with a higher maximum level. This yields a monotonically
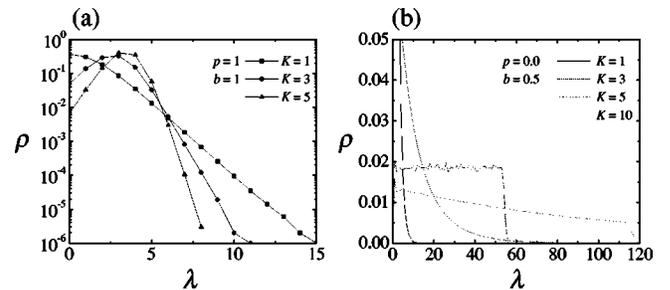

FIG. 5. (a) Node level distributions for $p = 1$, $b = 1$, and $K = 1, 3, 5$. With increasing connectivity, the distribution becomes sharper as average path lengths become shorter. (b) Node level distributions for $p = 0$, $b = 0.5$, and $K = 1, 3, 5, 10$. Here we plot numerical results as lines. This is the case of a regular network with no rewired links, and the direction set in one way or the other with equal probability. Here we find that for small values of K the distribution becomes broader as K increases, and then it becomes steplike for higher values of K.
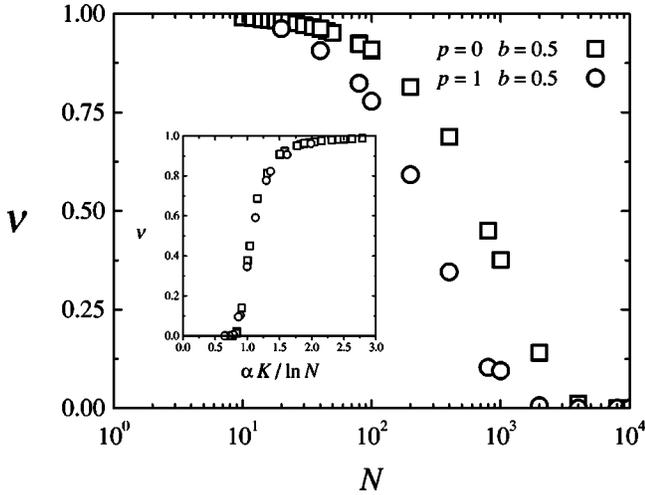
FIG. 6. Fraction of realizations with no basal nodes as a function of system size for $K=5$, $b=0.5$, and $p=0,1$. In the inset, we plot the same data plotted vs $\alpha K/\ln N$.

FIG. 7. Plot of the difference $\rho_\Lambda(0)-\beta$ vs $1/N$. For small system sizes, the difference grows as $1/N$. In the upper left corner, we plot the fraction of nodes at level λ as a function of system size, for $K=5$, $p=0$, $b=0.5$, and $\lambda=0$, 1, and 2. We find that $\rho_\Lambda(\lambda)$ goes to an asymptotic value as N grows. In the down right corner we show the difference $\rho_\Lambda(0)-\beta$ vs N, and find that for large N it falls faster than $1/N$, shown as a continuous line.

decreasing tail in the distribution of levels, not seen in the figure for reasons of scale. In many cases of interest, the size of the network does not exceed 10^3 nodes. This is the case for the neural network of *C. Elegans* and for the datasets available for some ecosystems. Nevertheless, it is interesting to study the behavior of the model for large N values. On the other hand, some systems show large connectivity values with $K/N\sim1$, so it is also interesting to understand the deviations from the analytical results in this case, where the approximations involved in the calculation of β are no longer valid.

In Fig. 6, we plot the fraction of realizations with no basal nodes as a function of the number of nodes N, for $K=5$, $p=0$ and 1, and $b=0.5$. For small values of N, the connectivity is relatively high and basal nodes rarely occur, so $\nu\approx1$. As N grows, the network becomes less connected and basal nodes become more frequent, so ν decays to zero. In the inset of Fig. 6, we show ν vs $\alpha K/\ln N$ for the same data. We find that the transition from networks with basal nodes to networks without basal nodes occurs around $\alpha K/\ln N=1$, as it was seen in the inset of Fig. 3. Note that now it is N that varies along the curve.

Now we would like to study how does the fraction of basal nodes varies with system size. In the numerical results presented in Figs. 2 and 3, the value of B is obtained normalizing over all realizations, including those with no basal nodes. The level distribution, on the other hand, is defined only when basal nodes exist, and so it is normalized over realizations with basal nodes. The fraction of nodes at level zero, $\rho_\Lambda(0)$, is then the fraction of basal nodes in realizations with $B>0$. In the upper inset of Fig. 7, we plot $\rho_\Lambda(\lambda)$ vs N for $\lambda=0$, 1, and 2. We see that $\rho_\Lambda(\lambda)$ goes to an asymptotic value as N grows. To compare the value of $\rho_\Lambda(0)$ with the analytical result β, we plot in Fig. 7 the difference $\rho_\Lambda(0)-\beta$ vs $1/N$. We find that this difference follows a straight line with slope 0.995 ± 0.005, also shown in the plot. This suggests that the first correction to the analytical calculation for small N values is of the order of $1/N$, so
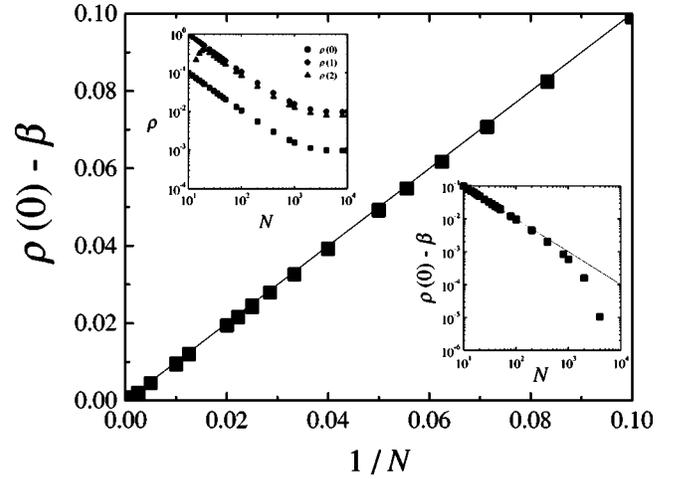
$$\rho_\Lambda(0)=\beta+\frac{1}{N}. \qquad (13)$$

In the lower inset of Fig. 7, we plot $\rho_\Lambda(0)-\beta$ vs N in a log-log plot in order to show the behavior for large N. We find that the difference goes to zero faster than N^{-1} for large N.

V. DISCUSSION

We presented a model for directed networks based on the Watts-Strogatz model for small-world phenomena. The direction of the links is set as outgoing from the owner node with probability b, and as incoming with probability $1-b$. We found analytical expressions for the fraction of basal and top nodes in the network, which are in good agreement with results from numerical simulations. We also studied node level distributions with the aid of numerical simulations. Finally, we considered the effects of size on these properties.

We stress the relevance of basal nodes in the structure of directed networks. In neural and some technological networks, information enters the system through the basal nodes and moves through the network following directed links, eventually reaching top nodes. In the case of food webs and economic networks, it is resources that enter the system through basal nodes. In any case, basal nodes determine the boundary of the network, the interphase between the environment and the system. In a similar way, top nodes are at the end of the chain. They represent the units where information ends up. For this reason, the fractions of basal and top nodes are expected to play an important role in dynamical processes occurring on directed networks. Furthermore, the node level distribution gives detailed information about the arrangement of the network relative to basal nodes. This distribution could give information on the organization and

efficiency of the network to distribute resources or information.

There is another quantity studied in food web theory which has not been considered here, namely, the degree of omnivory in the network. An omnivorous is a species feeding in more than one trophic level. It is possible to define the distribution $\rho_O(l)$ giving the frequency of species feeding on l different trophic levels. The mean of this distribution should give a measure of the omnivory degree in the food web. In a general directed network, this would give a measure of the interactions among different levels.

Finally, some remarks should be made on the limitations of the model we have studied. While many known networks present exponential or scale-free degree distributions, the WS model is known to yield Poissonian degree disributions. We have also studied the fraction of basal nodes and node level distributions in models of directed growing networks that have exponential degree distributions, but results will be published elsewhere [21].

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