SISTEMAS CONTINUOS

PROBLEMA 10

Los 10 primeros armonicos de Ψ(x,0) y $\frac{\partial}{\partial t}\Psi(x,0)$ (*hecho com Maple* 17)

Valores asumidos para los gráficos:

v (velocidad de propagación $\sqrt{\frac{T0}{\mu 0}}$) =100

v0 (velocidad inicial en t=0 para $\frac{L}{2} \leq$x$\leq$ L) = 1

L (longitus de la cuerda) = 10

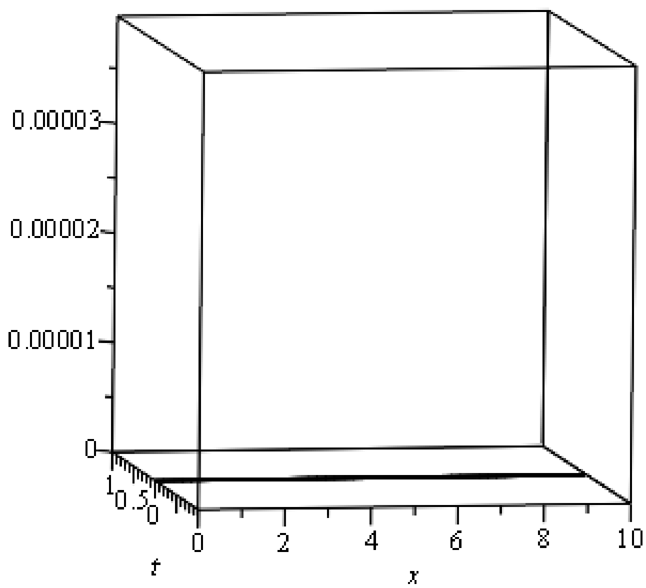
> **local** Ψ : $L := 10 : v0 := 1 : v := 100 :$

> $\Psi(x,t) := \sum_{i=1}^{10} (-1)^i \cdot \frac{8 \cdot v0 \cdot L}{v \cdot (2 \cdot i - 1)^2 \cdot \pi^2} \left(\cos\left((2 \cdot i - 1) \cdot \frac{\pi}{2} \right) - \frac{4}{(2 \cdot i - 1) \cdot \pi} \cdot \sin\left((2 \cdot i - 1) \cdot \frac{\pi}{4} \right) \right) \cdot \sin\left((2 \cdot i - 1) \cdot \frac{\pi}{2 \cdot L} \cdot x \right) \cdot \cos\left((2 \cdot i - 1) \cdot \frac{\pi}{2} \cdot \left(\frac{v}{L} \cdot t - 1 \right) \right);$

$$\Psi := (x,t) \rightarrow \sum_{i=1}^{10} \frac{1}{v\,(2\,i-1)^2\,\pi^2} \left(8\,(-1)^i\,v0\,L \left(\cos\left(\frac{1}{2}\,(2\,i-1)\,\pi \right) - \frac{4 \sin\left(\frac{1}{4}\,(2\,i-1)\,\pi \right)}{(2\,i-1)\,\pi} \right) \sin\left(\frac{1}{2}\,\frac{(2\,i-1)\,\pi\,x}{L} \right) \cos\left(\frac{1}{2}\,(2\,i-1)\,\pi \left(\frac{v\,t}{L} - 1 \right) \right) \right) \quad \textbf{(1)}$$

> $plot3d(\Psi(x,t), x = 0..L, t = 0..0);$

Ψ(x,0)
valores asumidos $v = 100$, $v0 = 1$, $L = 10$



> $d(x, t) := \frac{d}{dt}\Psi(x, t);$

$$d := (x, t) \rightarrow \frac{\partial}{\partial t}\Psi(x, t) \quad \textbf{(2)}$$

> $d(x, t);$

$$-\frac{8\sqrt{2}\sin\left(\frac{1}{20}\pi x\right)\sin\left(\frac{1}{2}\pi(10t-1)\right)}{\pi^2} + \frac{8}{9}\frac{\sqrt{2}\sin\left(\frac{3}{20}\pi x\right)\sin\left(\frac{3}{2}\pi(10t-1)\right)}{\pi^2} + \frac{8}{25}\frac{\sqrt{2}\sin\left(\frac{1}{4}\pi x\right)\sin\left(\frac{5}{2}\pi(10t-1)\right)}{\pi^2} - \frac{8}{49}\frac{\sqrt{2}\sin\left(\frac{7}{20}\pi x\right)\sin\left(\frac{7}{2}\pi(10t-1)\right)}{\pi^2} - \frac{8}{81}\frac{\sqrt{2}\sin\left(\frac{9}{20}\pi x\right)\sin\left(\frac{9}{2}\pi(10t-1)\right)}{\pi^2} \quad \textbf{(3)}$$

$$+\frac{8}{121}\,\frac{\sqrt{2}\,\sin\left(\frac{11}{20}\,\pi\,x\right)\,\sin\left(\frac{11}{2}\,\pi\,(10\,t-1)\right)}{\pi^2}$$

$$+\frac{8}{169}\,\frac{\sqrt{2}\,\sin\left(\frac{13}{20}\,\pi\,x\right)\,\sin\left(\frac{13}{2}\,\pi\,(10\,t-1)\right)}{\pi^2}$$

$$-\frac{8}{225}\,\frac{\sqrt{2}\,\sin\left(\frac{3}{4}\,\pi\,x\right)\,\sin\left(\frac{15}{2}\,\pi\,(10\,t-1)\right)}{\pi^2}$$

$$-\frac{8}{289}\,\frac{\sqrt{2}\,\sin\left(\frac{17}{20}\,\pi\,x\right)\,\sin\left(\frac{17}{2}\,\pi\,(10\,t-1)\right)}{\pi^2}$$

$$+\frac{8}{361}\,\frac{\sqrt{2}\,\sin\left(\frac{19}{20}\,\pi\,x\right)\,\sin\left(\frac{19}{2}\,\pi\,(10\,t-1)\right)}{\pi^2}$$

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> plot3d(d(x, t), x = 0 ..L, t = 0 ..0);
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$$\frac{\partial}{\partial t}\,\Psi(x, 0)$$

valores asumidos $v = 100$, $v0 = 1$, $L = 10$