ESTRUCTURA DE LA MATERIA 4

CURSO DE VERANO 2021

CLASE 10

RODOLFO SASSOT

CLASE 10: Teoría lagrangiana de campos

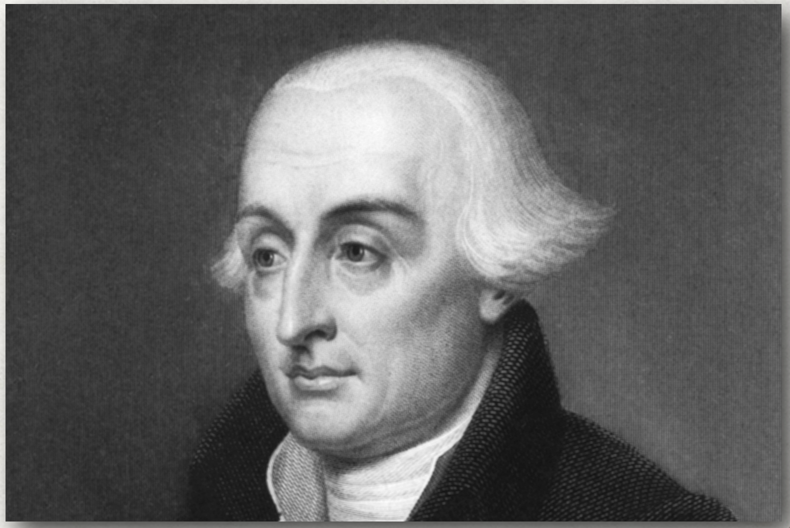
Temas: formalismo lagrangiano, lagrangiano de KG, Dirac y Maxwell.

herramienta para analizar las interacciones:

- extiende Ec. Dirac para ∞ grados de libertad
- pone en evidencia las simetrías (clave para las interacciones)
- su cuantización a) explica la relación entre spin-estadísitica
  b) imagen más satisfactoria de las antipartículas

porqué no simplemente Dirac? un sistema cuántico y relativista implica ∞ grados de libertad

$$\text{QFT} \xrightarrow[\text{bajas energías}]{} \text{Dirac} \xrightarrow[\text{no relativista}]{} \text{Schrödinger} \xrightarrow[\text{clásico}]{} \text{Newton}$$



Giuseppe Ludovico Lagrangia (1736-1813)
(alias Lagrange)

CLASE 10: Teoría lagrangiana de campos

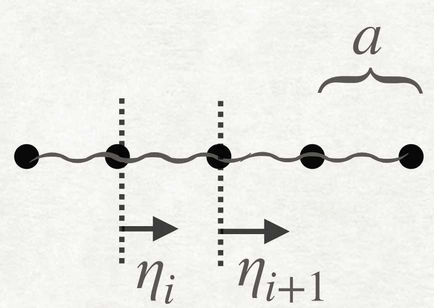
formalismo lagrangiano:

$$L(q,\dot q,t) = T - V$$

$$p \equiv \frac{\partial L}{\partial \dot q} \qquad H = p\dot q - L \qquad q, p$$ variables generalizadas $\{q,p\}_p = 1$

$$\frac{dg}{dt} = \{g, H\}_p$$

$$q(t),\ \dot q(t) \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} = 0 \qquad \delta \int dt\, L = 0$$



$$L = \sum_i^N \frac{1}{2} m_i \dot\eta_i^2 - \frac{1}{2} k\,(\eta_{i+1} - \eta_i)^2 = \sum_i^N a\,\frac{1}{2}\left[\frac{m_i}{a}\,\dot\eta_i^2 - ka\,(\frac{\eta_{i+1}-\eta_i}{a})^2\right]$$

$$\mu \equiv \frac{m}{a} \qquad Y \equiv ka$$

$$\frac{\eta_{i+1}-\eta_i}{a} \to \frac{\partial \eta}{\partial x}$$

$$\sum a \to \int dx$$

$$L \to \int dx\,\frac{1}{2}\left[\mu\,\dot\eta^2(x) - Y\left(\frac{\partial \eta}{\partial x}\right)^2\right] \equiv \int dx\,\mathscr{L}(\eta, \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial t})$$

$\eta, \dfrac{\partial L}{\partial \dot\eta}$ variables generalizadas

x, t parámetros

CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

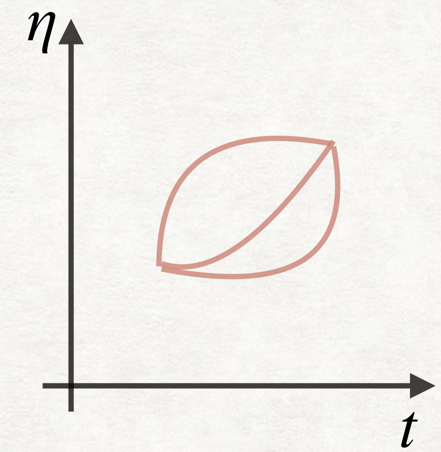
$$L = \int dx \, \frac{1}{2} \left[\mu \, \dot{\eta}^2(x) - Y \left(\frac{\partial \eta}{\partial x} \right)^2 \right] \equiv \int dx \, \mathscr{L}(\eta, \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial t})$$

$$\pi \equiv \frac{\partial \mathscr{L}}{\partial \dot{\eta}} = \mu \dot{\eta} \qquad p_i = \frac{\partial L}{\partial \dot{\eta}_i} \quad P = \sum_i p_i = \sum_i m \dot{\eta}_i = \sum_i a \, \frac{m}{a} \dot{\eta}_i \to \int dx \, \mu \dot{\eta} = \int dx \, \pi$$

$$\mathscr{H} \equiv \pi \, \dot{\eta} - \mathscr{L} \qquad H = \int dx \, \mathscr{H}$$

$$\{f, g\}_p \equiv \int dx \, \left[\frac{\partial f}{\partial \eta} \frac{\partial g}{\partial \pi} - \frac{\partial f}{\partial \pi} \frac{\partial g}{\partial \eta} \right]$$

CLASE 10: Teoría lagrangiana de campos



formalismo lagrangiano:

$$\delta \int dt\, L = \delta \int dt \int dx\, \mathscr{L} \qquad \delta\eta(x,t) \begin{cases} \delta\eta(x,t_1) = 0 \\ \delta\eta(x,t_2) = 0 \end{cases}$$

$$\delta \int dt\, L = \int dt \int dx \left[\frac{\partial \mathscr{L}}{\partial \eta}\delta\eta + \frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}}\delta\left(\frac{\partial \eta}{\partial x}\right) + \frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial t}}\delta\left(\frac{\partial \eta}{\partial t}\right) \right]$$

$$= \int dt \int dx \left[\frac{\partial \mathscr{L}}{\partial \eta}\delta\eta - \frac{\partial}{\partial x}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}} \right)\delta\eta - \frac{\partial}{\partial t}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial t}} \right)\delta\eta \right] = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}} \right) + \frac{\partial}{\partial t}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial t}} \right) - \frac{\partial \mathscr{L}}{\partial \eta} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}}\delta\eta \right) = \frac{\partial \delta\eta}{\partial x}\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}} + \frac{\partial}{\partial x}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}} \right)\delta\eta$$

$$0 = \underbrace{\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}}\delta\left(\frac{\partial \eta}{\partial x}\right)}_{\text{lo que tengo}} + \underbrace{\frac{\partial}{\partial x}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}} \right)\delta\eta}_{\text{como lo quiero}}$$

CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

$$\frac{\partial}{\partial x}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x}}\right) + \frac{\partial}{\partial t}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial t}}\right) - \frac{\partial \mathscr{L}}{\partial \eta} = 0$$

$$\mathscr{L} = \frac{1}{2}\mu\,\dot{\eta}^2(x) - \frac{Y}{2}\left(\frac{\partial \eta}{\partial x}\right)^2$$

$$\frac{\partial}{\partial x}\left(-Y\frac{\partial \eta}{\partial x}\right) + \frac{\partial}{\partial t}\left(\mu\dot{\eta}\right) = 0$$

$$-Y\frac{\partial^2 \eta}{\partial x^2} + \mu\frac{\partial^2 \eta}{\partial t^2} = 0$$

$$\sum_{k=1}^{3}\frac{\partial}{\partial x_k}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x_k}}\right) + \frac{\partial}{\partial t}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial t}}\right) - \frac{\partial \mathscr{L}}{\partial \eta} = 0$$

$$\frac{\partial}{\partial x^\mu}\left(\frac{\partial \mathscr{L}}{\partial \frac{\partial \eta}{\partial x^\mu}}\right) - \frac{\partial \mathscr{L}}{\partial \eta} = 0$$

$$\partial_\mu\left(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \eta)}\right) - \frac{\partial \mathscr{L}}{\partial \eta} = 0$$

CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

$$\partial_\mu \left(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathscr{L}}{\partial \phi} = 0 \qquad \eta = \phi(x,t) \qquad \mathscr{L} = \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi - \mu^2 \phi^2\right)$$

$$-\frac{\partial \mathscr{L}}{\partial \phi} = \mu^2 \phi \qquad \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} = \frac{\partial}{\partial(\partial_\mu \phi)}\left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi\right) = \frac{1}{2} g^{\mu\nu}\left(\partial_\nu \phi + \partial_\mu \phi \delta_{\mu\nu}\right)$$

$$= \frac{1}{2}\left(\partial^\mu \phi + \partial^\mu \phi\right) = \partial^\mu \phi$$

$$\partial_\mu \partial^\mu \phi + \mu^2 \phi = 0$$ Klein-Gordon?

$$\mathscr{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi = (\partial_\mu \phi_1 - i\partial_\mu \phi_2)(\partial^\mu \phi_1 + i\partial^\mu \phi_2) - \mu^2(\phi_1 - i\phi_2)(\phi_1 + i\phi_2)$$

$$\phi = \phi_1 + i\phi_2$$

$$\phi^* = \phi_1 - i\phi_2$$

$$= \partial_\mu \phi_1 \partial^\mu \phi_1 - \mu^2 \phi_1^2 + \partial_\mu \phi_2 \partial^\mu \phi_2 - \mu^2 \phi_2^2$$

$$= \mathscr{L}_1 + \mathscr{L}_2$$

$$\left.\begin{aligned} \partial_\mu \partial^\mu \phi^* + \mu^2 \phi^* &= 0 \\ \partial_\mu \partial^\mu \phi + \mu^2 \phi &= 0 \end{aligned}\right\} \longleftrightarrow \left\{\begin{aligned} \partial_\mu \partial^\mu \phi_1 + \mu^2 \phi_1 &= 0 \\ \partial_\mu \partial^\mu \phi_2 + \mu^2 \phi_2 &= 0 \end{aligned}\right. \qquad \partial_\mu \partial^\mu \phi + \mu^2 \phi = 0$$

CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

$$\partial_\mu \left(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \psi)} \right) - \frac{\partial \mathscr{L}}{\partial \psi} = 0 \qquad \eta = \psi(x,t) \qquad \mathscr{L} = \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$$\partial_\mu \left(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \overline{\psi})} \right) - \frac{\partial \mathscr{L}}{\partial \overline{\psi}} = 0 \qquad (i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\mathscr{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \qquad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \qquad A_\mu(\overrightarrow{x}, t)$$ campo cuadrivectorial

$$\Box^2 A_\mu - \partial^\mu(\partial_\nu A^\nu) = 0$$

Ec. Dirac $\psi \sim$ función de onda x, p

lagrangiano $\psi \sim$ variable dinámica (clásica!) $\psi, \pi \equiv \frac{\partial \mathscr{L}}{\partial \dot{\psi}}$