

ESTRUCTURA DE LA MATERIA 4

2DO CUATRIMESTRE 2025

CLASE 10

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CLASE 10: Teoría de perturbaciones.

Temas: Perturbación covariante, propagadores, electrodinámica

formalismo no relativista:

$$E_n \phi_n = H_0 \phi_n$$

$$\int d^3x \phi_m^* \phi_n = \delta_{mn}$$

$$i \frac{\partial \psi}{\partial t} = (H_0 + V(\vec{x}, t)) \psi$$

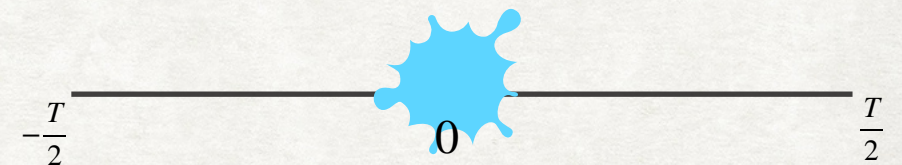
$$\psi(\vec{x}, t) = \sum_n a_n(t) \phi_n(\vec{x}) e^{-iE_n t} \quad a_n(t)$$

~~$$i \sum_n \frac{da_n}{dt} \phi_n(\vec{x}) e^{-iE_n t} + \sum_n E_n a_n(t) \phi_n(\vec{x}) e^{-iE_n t} = \sum_n E_n a_n(t) \phi_n(\vec{x}) e^{-iE_n t} + \sum_n a_n(t) V(\vec{x}, t) \phi_n(\vec{x}) e^{-iE_n t}$$~~

mult. m.a.m
por ϕ_f^*

$$\frac{da_f}{dt} = -i \sum_n a_n(t) \int d^3x \phi_f^* V(\vec{x}, t) \phi_n(\vec{x}) e^{i(E_f - E_n)t} \simeq -i \int d^3x \phi_f^* V(\vec{x}, t) \phi_i(\vec{x}) e^{i(E_f - E_i)t}$$

supongamos que a $t_i = -\frac{T}{2} \sim \phi_i$ $a_i(-\frac{T}{2}) = 1$ $a_n(-\frac{T}{2}) = 0 \quad \forall n \neq i$

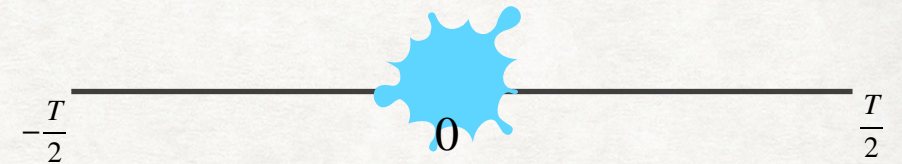


$$a_f(t) \simeq -i \int_{-\frac{T}{2}}^t dt' \int d^3x \phi_f^*(\vec{x}) V(\vec{x}, t') \phi_i(\vec{x}) e^{i(E_f - E_i)t'}$$

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formalismo no relativista:

$$a_f(t) \simeq -i \int_{-\frac{T}{2}}^t dt' \int d^3x \phi_f^*(\vec{x}) V(\vec{x}, t') \phi_i(\vec{x}) e^{i(E_f - E_i)t'}$$



$$T_{fi} \equiv a_f\left(\frac{T}{2}\right) = -i \int_{-\frac{T}{2}}^{\frac{T}{2}} dt' \int d^3x \left(\phi_f e^{-iE_f t'}\right)^* V(\vec{x}, t') \left(\phi_i e^{-iE_i t'}\right) = -i \int d^4x \Phi_f^*(x^\mu) V \Phi_i(x^\mu)$$

$$\text{si } V \sim \text{cte} \quad T_{fi} = -i V_{fi} \int dt e^{i(E_f - E_i)t} \quad V_{fi} \equiv \int d^3x \phi_f^*(\vec{x}) V \phi_i(\vec{x})$$

$$\text{si } T \rightarrow \infty \quad T_{fi} = -2\pi i V_{fi} \delta(E_f - E_i) \quad E_f = E_i \text{ sólo si } T \rightarrow \infty!!$$

probabilidad de transición
por unidad de tiempo

$$W \equiv \lim_{T \rightarrow \infty} \frac{|T_{fi}|^2}{T}$$

$$W = \lim_{T \rightarrow \infty} 2\pi \frac{|V_{fi}|^2}{T} \delta(E_f - E_i) \int_{-\frac{T}{2}}^{\frac{T}{2}} dt e^{i(E_f - E_i)t} = \lim_{T \rightarrow \infty} 2\pi \frac{|V_{fi}|^2}{\cancel{T}} \delta(E_f - E_i) \int_{-\frac{T}{2}}^{\frac{T}{2}} \cancel{dt}$$

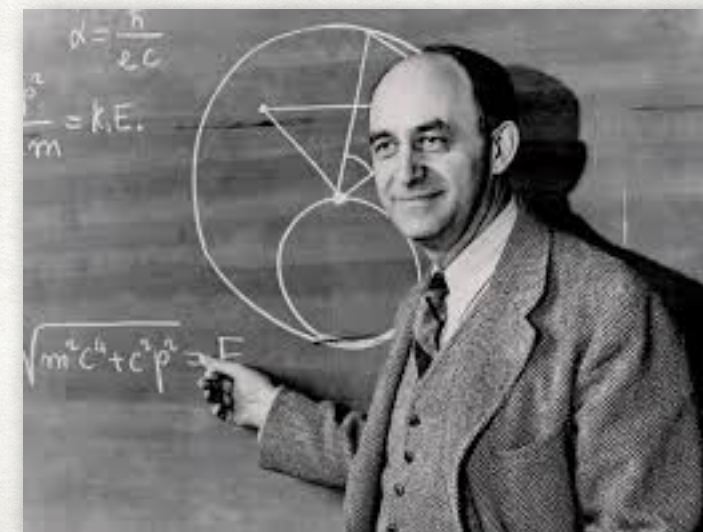
$$= 2\pi |V_{fi}|^2 \delta(E_f - E_i)$$

para un estado inicial fijo i
y un continuo de estados
finales $\rho(E_f)$

$$W_{if} = 2\pi \int dE_f \rho(E_f) |V_{fi}|^2 \delta(E_f - E_i) = 2\pi \rho(E_i) |V_{fi}|^2$$

$\rho(E_f)$ número de estados
entre E_f y $E_f + dE_f$

"regla de oro de Fermi"



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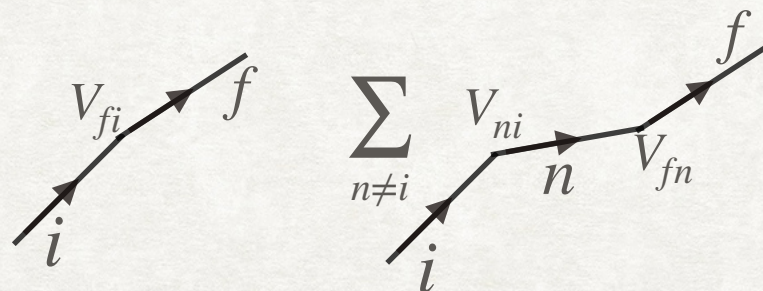
formalismo no relativista:

$$a_f^{(1)}(t) \simeq -i V_{fi} \int_{-\frac{T}{2}}^t dt' e^{i(E_f - E_i)t'} \quad \frac{da_f}{dt} = -i \sum_n a_n(t) V_{fn} e^{i(E_f - E_n)t} \quad a_n(t) \simeq 0 \quad \forall n \neq i$$

$$\frac{da_f^{(2)}}{dt} \simeq \underbrace{-i V_{fi} e^{i(E_f - E_i)t}}_{n=i} + \underbrace{(-i) \sum_{n \neq i} \overbrace{(-i) V_{ni} \int_{-\frac{T}{2}}^t dt' e^{i(E_n - E_i)t'} V_{fn} e^{i(E_f - E_n)t}}^{a_n^{(1)}}}_{n \neq i}$$

$$T_{fi} \equiv a_f\left(\frac{T}{2}\right) = -2\pi i V_{fi} \delta(E_f - E_i) - \sum_{n \neq i} V_{fn} V_{ni} \int_{-\frac{T}{2}}^{\frac{T}{2}} dt e^{i(E_f - E_n)t} \int_{-\frac{T}{2}}^t dt' e^{i(E_n - E_i)t'}$$

$$T_{fi} = -2\pi i V_{fi} \delta(E_f - E_i) - 2\pi i \sum_{n \neq i} \frac{V_{fn} V_{ni}}{E_i - E_n} \delta(E_f - E_i) \quad \int_{-\frac{T}{2}}^t dt' e^{i(E_n - E_i)t'} \xrightarrow{T \rightarrow \infty} i \frac{e^{i(E_n - E_i)t}}{E_i - E_n}$$



$E_f = E_i$ sólo si $T \rightarrow \infty$!!

$\frac{1}{E_i - E_n}$ "propagador"

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formalismo relativista:



$$H_0 = \alpha \cdot p + \beta m \quad (\gamma_\mu p^\mu - m) \psi = 0$$

$$H = \alpha \cdot p + \beta m + V \quad (\gamma_\mu p^\mu - m - \gamma^0 V) \psi' = 0$$

$$p^\mu \rightarrow p^\mu + e A^\mu \quad (\gamma_\mu p^\mu - m + e \gamma_\mu A^\mu) \psi' = 0$$

$$-\gamma^0 V = e \gamma_\mu A^\mu$$

$$V = -e \gamma^0 \gamma_\mu A^\mu$$

$$T_{fi} = -i \int d^4x \Phi_f^*(x^\mu) V \Phi_i(x^\mu) \longrightarrow -i \int d^4x \psi_f^\dagger(x^\mu) V \psi_i(x^\mu) = -i \int d^4x \psi_f^\dagger(x^\mu) (-e \gamma^0 \gamma_\mu A^\mu) \psi_i(x^\mu)$$

$$\begin{aligned} J_\mu^{fi} &\equiv -e \bar{\psi}_f \gamma_\mu \psi_i = -e \bar{u}_f \gamma_\mu u_i e^{i(p^f - p^i)_\mu x^\mu} \\ &= -e (p_\mu^f - p_\mu^i) e^{i(p^f - p^i)_\mu x^\mu} \end{aligned}$$

$$\begin{aligned} &= -i \int d^4x (-e) \bar{\psi}_f(x^\mu) \gamma_\mu \psi_i(x^\mu) A^\mu \\ &= -i \int d^4x J_\mu^{fi} A^\mu \end{aligned}$$

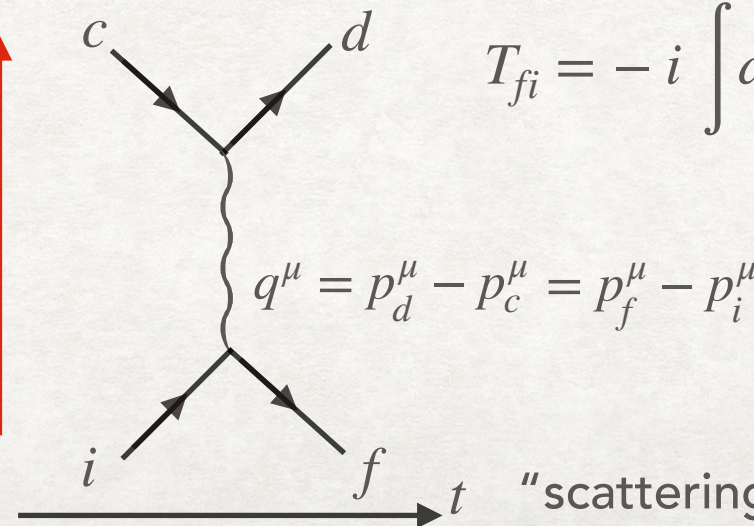
$$\square A^\mu = J^\mu \quad J^\mu = -e (p_d^\mu - p_c^\mu) e^{i(p^d - p^c)_\mu x^\mu}$$

$$\square e^{iq \cdot x} = -q^2 e^{iq \cdot x} \quad (q^2 = q_\mu q^\mu)$$

$$A^\mu = -\frac{1}{q^2} J^\mu \quad q^\mu = p_d^\mu - p_c^\mu$$

$\bar{u}_f \rightarrow \bar{v}_f$
 $u_c \rightarrow v_c$
 "aniquilación"
 $i + \bar{f} \rightarrow \bar{c} + d$

$$T_{fi} = -i \int d^4x J_\mu^{fi} \left(-\frac{1}{q^2} \right) J_{dc}^\mu$$

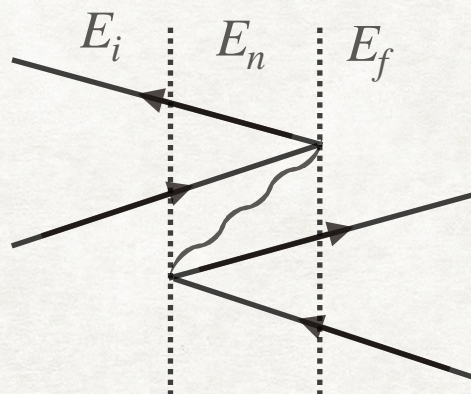
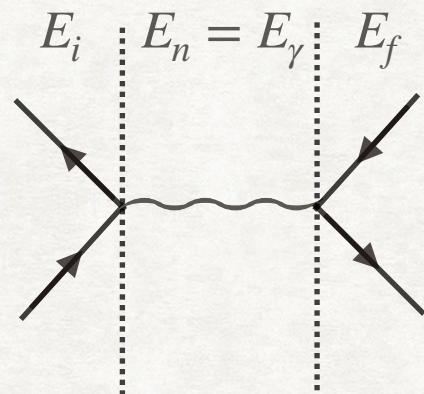
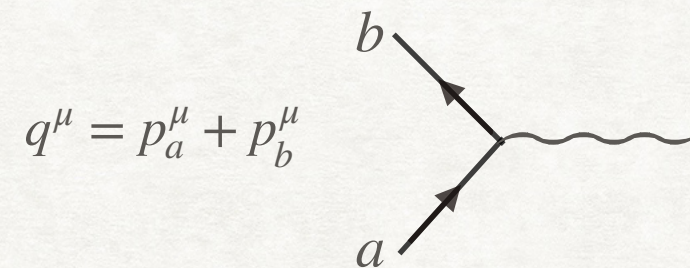
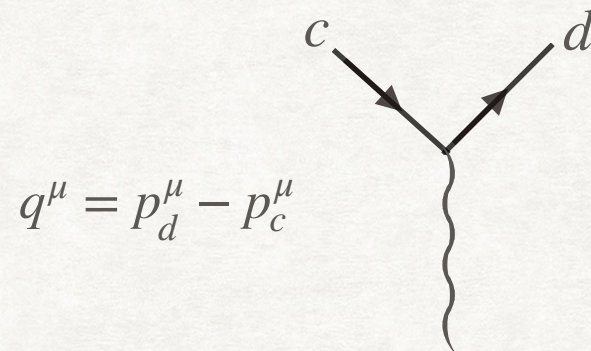


"scattering" $i + c \rightarrow f + d$

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propagadores:

$$\frac{1}{E_i - E_f} \longrightarrow -\frac{1}{q^2}$$



$$E_n = E_i + E_f + E_\gamma$$

$$= 2E_i + E_\gamma$$

$$T \sim V_{fn} \frac{1}{E_i - E_\gamma} V_{ni} + V_{fn} \frac{1}{E_i - 2E_i - E_\gamma} V_{ni} = V_{fn} \frac{-2E_i}{E_i^2 - E_\gamma^2} V_{ni}$$

en términos de p_a y p_b

$$p^2 = E^2 - \vec{p}^2$$

$$E_i^2 = (p_a + p_b)^2 + (\vec{p}_a + \vec{p}_b)^2$$

$$E_\gamma^2 = m_\gamma^2 + \vec{p}_\gamma^2$$

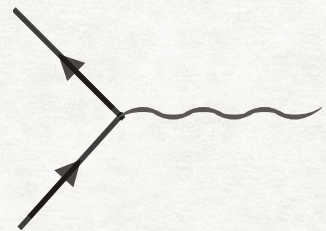
$$\frac{1}{E_i^2 - E_\gamma^2} = \frac{1}{(p_a + p_b)^2 - m_\gamma^2} = \frac{1}{q^2 - m_\gamma^2}$$

$-\frac{1}{q^2}$ propagador de un mediador de $m=0$
(incluyendo todos los ordenamientos)

q^2 sería el cuadri-impulso al cuadrado del fotón si se conservara la energía en cada vértice

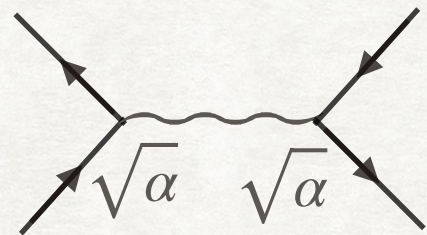
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electrodinámica (QED):



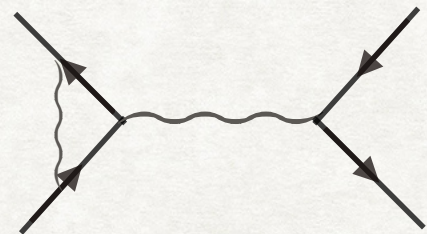
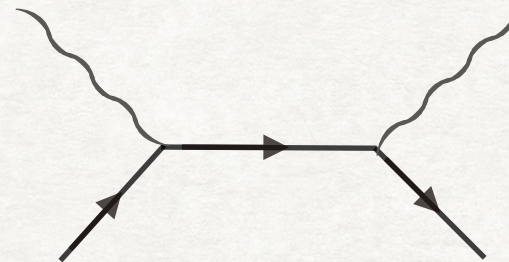
$$\mathcal{A} \propto \sqrt{\alpha_{QED}}$$

$$\alpha_{QED} = \frac{e^2}{\hbar c} \simeq \frac{1}{137}$$

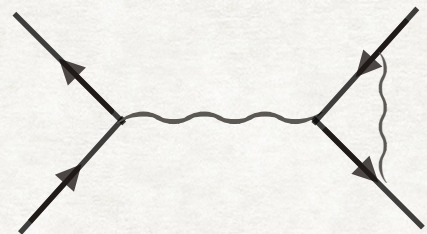


$$\mathcal{A} \propto \alpha$$

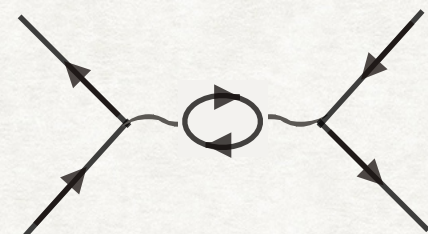
$$\sigma \propto \alpha^2$$



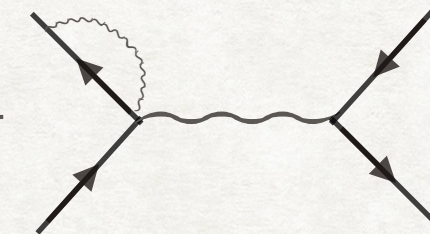
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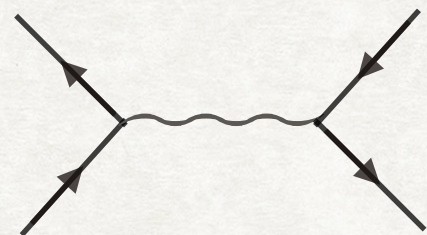
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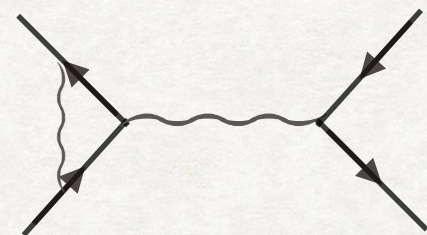
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$$\sigma \propto \alpha^4$$

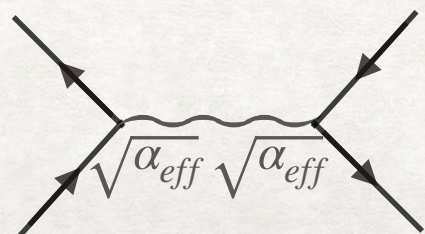
$$\sigma \sim \left| \sum \mathcal{A}_i \right|^2$$



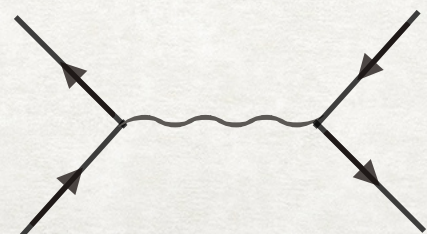
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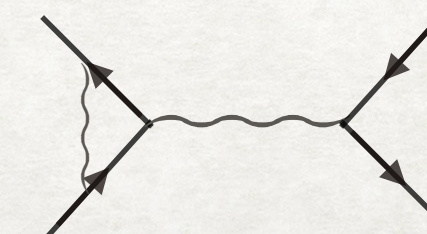
$$\sigma \propto \alpha^3$$



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$$\alpha_{eff}(E)$$

$$\frac{1}{137} \rightarrow \frac{1}{128}$$



