ESTRUCTURA DE LA MATERIA 4

CURSO DE VERANO 2021

CLASE 11

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CLASE 11: Cuantización y simetrías

Temas: cuantización del campo, relación con la estadística, simetrías.

$$\mathscr{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi \qquad \qquad \partial_\mu \partial^\mu \phi + \mu^2 \phi = 0$$

programa de cuantización:

1) identificar las variables dinámicas

oscilador armónico	formalismo de campos
$\overrightarrow{x}, \overrightarrow{p}$	$\phi(\overrightarrow{x}, t), \pi(\overrightarrow{x}, t)$

2) promover las variables dinámicas a operadores en espacio de Hilbert

3) imponer restricciones a la conmutación inspirándose en los corchetes

$[x_i, p_j] = i\delta_{ij}$	$[\phi(x), \pi(y)] = i\delta(x-y)$
$[x_i, x_j] = [p_i, p_j] = 0$	$[\phi(x), \phi(y)] = [\pi(x), \pi(y)] = 0$

4) "resolver" p.ej. formalismo $a, a^\dagger$

$$x = \frac{1}{\sqrt{2\omega}}(a + a^\dagger) \qquad [x, p] = i \leftrightarrow [a, a^\dagger] = 1$$

$$p = -i\sqrt{\frac{\omega}{2}}(a - a^\dagger) \qquad H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 x^2 \leftrightarrow \omega(a^\dagger a + \frac{1}{2})$$

$$\phi \sim (a + a^\dagger)$$

$$\pi \sim (a - a^\dagger)$$

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cuantización del campo escalar (a vuelo de pájaro):

$$\phi(\overrightarrow{x},t) = \int \frac{d^3p}{(2\pi)^3}\, e^{i\overrightarrow{p}\cdot\overrightarrow{x}}\, \phi(\overrightarrow{p},t)$$

$\phi(\overrightarrow{p},t)$ componente de Fourier

$$\partial_\mu\partial^\mu\phi + m^2\phi = 0 \longrightarrow \left[\frac{\partial^2}{\partial t^2} + \left(|\overrightarrow{p}|^2 + m^2\right)\right]\phi(\overrightarrow{p},t) = 0$$

$$\ddot{\phi}(\overrightarrow{p},t) + \omega_p^2\,\phi(\overrightarrow{p},t) = 0$$

oscilador armónico! $\omega_p = \sqrt{\overrightarrow{p}^2 + m^2}$

$$x = \frac{1}{\sqrt{2\omega}}(a + a^\dagger)$$

$$p = -i\sqrt{\frac{\omega}{2}}(a - a^\dagger)$$

$$H = \omega(a^\dagger a + \frac{1}{2})$$

$$|0> \quad \therefore \quad a|0> = 0$$

$$H|0> = \frac{\omega}{2}|0>$$

$$|n> \equiv (a^\dagger)^n|0>$$

$$H|n> = (n + \frac{1}{2})\,\omega\,|n>$$

$$\phi(\overrightarrow{x},t) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega}}(a_p\,e^{i\overrightarrow{p}\cdot\overrightarrow{x}} + a_p^\dagger\,e^{-i\overrightarrow{p}\cdot\overrightarrow{x}}) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega}}(a_p + a_{-p}^\dagger)\,e^{i\overrightarrow{p}\cdot\overrightarrow{x}}$$

$$\pi(\overrightarrow{x},t) = \int \frac{d^3p}{(2\pi)^3}(-i)\sqrt{\frac{\omega}{2}}(a_p - a_{-p}^\dagger)\,e^{i\overrightarrow{p}\cdot\overrightarrow{x}}$$

$$[a_p, a_{p'}^\dagger] = (2\pi)^3\delta^3(\overrightarrow{p}-\overrightarrow{p}') \leftrightarrow [\phi(\overrightarrow{x}), \pi(\overrightarrow{y})] = i\delta^3(\overrightarrow{x}-\overrightarrow{y})$$

$$H = \int d^3x\mathscr{H} = = \int d^3x(\pi\dot{\phi} - \mathscr{L}) = \int \frac{d^3p}{(2\pi)^3}\,\omega_p\left(a_p^\dagger a_p + \frac{1}{2}[a_p, a_p^\dagger]\right)$$

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cuantización del campo escalar (a vuelo de pájaro):

$$H = \int d^3x \mathscr{H} = \int \frac{d^3p}{(2\pi)^3}\, \omega_p \left(a_p^\dagger a_p + \frac{1}{2}[a_p, a_p^\dagger] \right)$$

impulso $\overrightarrow{P} = \int \frac{d^3p}{(2\pi)^3} \overrightarrow{p}\, a_p^\dagger a_p$

$|0>$ vacío

$a_p^\dagger |0>$ estado de energía $\omega_p = \sqrt{\overrightarrow{p}^2 + m^2}$
e impulso $\overrightarrow{p}$

$a_p^\dagger a_q^\dagger |0> = a_q^\dagger a_p^\dagger |0> \quad (a_p^\dagger)^n |0>$ bosones

cuantización del campo de Dirac (a vuelo de pájaro^2):

$$\psi(\overrightarrow{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=1,2} \left(a_p^s\, u^s(p) + b^{s\dagger}_{-p}\, v^s(-p) \right) e^{i\overrightarrow{p}\cdot\overrightarrow{x}}$$

$$\psi^\dagger(\overrightarrow{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=1,2} \left(a^{s\dagger}_p\, u^{s\dagger}(p) + b^s_{-p}\, v^{s\dagger}(-p) \right) e^{i\overrightarrow{p}\cdot\overrightarrow{x}}$$

$$[a_p^s, a_{p'}^{s'\dagger}] = (2\pi)^3 \delta^3(\overrightarrow{p} - \overrightarrow{p}\,')\delta_{ss'}$$
$$[b_p^s, b_{p'}^{s'\dagger}] = (2\pi)^3 \delta^3(\overrightarrow{p} - \overrightarrow{p}\,')\delta_{ss'}$$

$$H = \int d^3x \mathscr{H} = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_p a_p^{s\dagger} a_p^s - E_p b_p^{s\dagger} b_p^s \right)$$

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cuantización del campo de Dirac (a vuelo de pájaro):

$$\{a_p^s, a_{p'}^{s'\dagger}\} = (2\pi)^3\delta^3(\vec{p}-\vec{p}\,')\delta_{ss'}$$

$$\{b_p^s, b_{p'}^{s'\dagger}\} = (2\pi)^3\delta^3(\vec{p}-\vec{p}\,')\delta_{ss'}$$

$$H = \int d^3x\mathscr{H} = \int \frac{d^3p}{(2\pi)^3}\sum_s \left(E_p a_p^{s\dagger}a_p^s - E_p b_p^{s\dagger}b_p^s\right)$$

$$\tilde{b}_p^s \equiv b_p^{s\dagger} \qquad \tilde{b}_p^{s\dagger} \equiv b_p^s \qquad \{b, b^\dagger\} = bb^\dagger + b^\dagger b = \{b^\dagger, b\} = \{\tilde{b}, \tilde{b}^\dagger\}$$

$$-E_p\, b_p^\dagger b_p = -E_p(1 - b_p b_p^\dagger) = -E_p(1 - \tilde{b}_p^\dagger \tilde{b}_p) = E_p\,\tilde{b}_p^\dagger\tilde{b}_p - E_p \qquad E > 0$$

$$\{b, b^\dagger\} = 1 \qquad \{b, b\} = \{b^\dagger, b^\dagger\} = 0$$

$|0> \;\therefore\; b|0> = 0$ $\qquad b^\dagger|0> = |1>$

$b|1> = bb^\dagger|0> = (1 - b^\dagger b)|0> = |0>$

$b^\dagger|1> = b^\dagger b^\dagger|0> = 0$ $\qquad \{|0>, |1>\}$ vacío/partícula

$(b^\dagger)^2|0> = 0$ fermiones

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simetrías: transformaciones de los campos

$$\phi(x) \longrightarrow \phi'(x) = \phi(x) + \alpha\, \Delta\phi(x)$$

α parámetro infinitesimal (cuánto)

$\Delta\phi(x)$ deformación del campo (cómo)

simetría $\Longleftrightarrow$ deja invariantes las ec. de movimiento

la acción es invariante

o cambia en un término de superficie

$$\mathscr{L} \longrightarrow \mathscr{L} + \alpha\, \partial_\mu J^\mu$$

$$j^\mu \equiv \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \Delta\phi - J^\mu$$

$$\partial_\mu j^\mu = 0$$

$$\alpha \Delta \mathscr{L} = \frac{\partial \mathscr{L}}{\partial \phi} \alpha\, \Delta\phi + \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \alpha\, \Delta(\partial_\mu \phi)$$

$$= \alpha\, \partial_\mu \left(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \Delta\phi \right) + \alpha \frac{\partial \mathscr{L}}{\partial \phi} \Delta\phi - \alpha\, \partial_\mu \left(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \right) \Delta\phi$$

$$= \alpha\, \partial_\mu \left(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \Delta\phi \right) + \alpha \underbrace{\left(\frac{\partial \mathscr{L}}{\partial_\mu \phi} - \partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \right)}_{0} \Delta\phi$$

$$\frac{\partial}{\partial t} \int d^3x\, j^0 = \int d^3x\, \vec{\nabla} \cdot \vec{j} = 0$$

$$Q \equiv \int d^3x\, j^0 \qquad \frac{\partial Q}{\partial t} = 0$$

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simetrías: transformaciones de los campos

ejemplos:

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \qquad \phi(x) \longrightarrow \phi'(x) = \phi(x) + \alpha$$

$$\phi(x) \longrightarrow \phi'(x) = \phi(x) + \alpha\,\Delta\phi(x) \qquad \Longrightarrow \qquad \Delta\phi(x) = 1$$

$$\mathscr{L} \longrightarrow \mathscr{L}$$

$$\mathscr{L} \longrightarrow \mathscr{L} + \alpha\,\partial_\mu J^\mu \qquad \Longrightarrow \qquad J^\mu = 0$$

$$j^\mu \equiv \frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi)}\Delta\phi - J^\mu = \partial^\mu\phi \qquad \partial_\mu\partial^\mu\phi = 0 \qquad Q = \int d^3x\,\partial^0\phi$$

$$\mathscr{L} = \partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi \qquad \phi(x) \longrightarrow \phi'(x) = \phi(x)e^{i\alpha q} \simeq \phi(x)(1 + i\alpha q)$$

$$\Delta\phi(x) = iq\phi(x) \quad \Delta\phi^*(x) = -iq\phi^*(x)$$

$$\mathscr{L} \longrightarrow \mathscr{L} \qquad J^\mu = 0$$

$$j^\mu \equiv \frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi)}\Delta\phi - J^\mu = iq\partial^\mu\phi^*\,\phi - iq\phi^*\,\partial^\mu\phi = j^\mu_{KG} \qquad \partial_\mu j^\mu_{KG} = 0$$

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simetrías: transformaciones espacio-temporales

$$x^\mu \longrightarrow x^\mu + a^\mu$$

$$\phi(x^\mu) \longrightarrow \phi'(x^\mu) = \phi(x^\mu + a^\mu) = \phi(x) + a^\mu \partial_\mu \phi(x)$$

$$\mathscr{L} \longrightarrow \mathscr{L} + a^\mu \partial_\mu \mathscr{L} \quad = \mathscr{L} + a^\nu \partial_\mu (\delta^\mu_\nu \mathscr{L}) = \mathscr{L} + a^\nu \partial_\mu (J^\mu)_\nu$$

~ cuatro corrientes J^μ

una para cada dirección del espacio ν

~ cuatro corrientes conservadas j^μ

$$j^\mu \equiv \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \Delta\phi - J^\mu$$

$$T^\mu_\nu \equiv \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \mathscr{L} \delta^\mu_\nu$$

tensor de energía impulso

carga conservada ante traslaciones temporales $\int d^3x\, T^{00} \equiv H$ hamiltoniano

carga conservada ante traslaciones espaciales $\int d^3x\, T^{0i} \equiv P^i$ impulso