

ESTRUCTURA DE LA MATERIA 4

2DO CUATRIMESTRE 2025

CLASE 11

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CLASE 11: Teoría lagrangiana de campos

Temas: formalismo lagrangiano, lagrangiano de KG, Dirac y Maxwell.

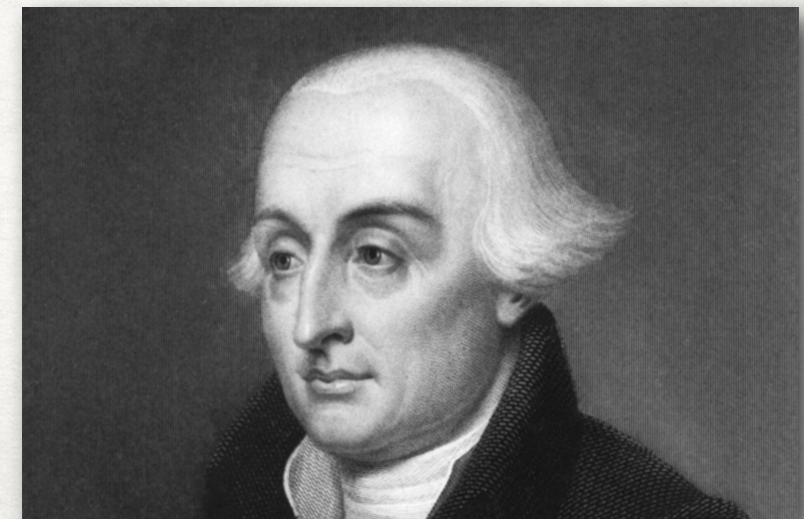
herramienta para analizar las interacciones: extiende Ec. Dirac para ∞ grados de libertad
pone en evidencia las simetrías (clave para las interacciones)
su cuantización a) explica la relación entre spin-estadística
b) imagen más satisfactoria de las antipartículas

porqué no simplemente Dirac? un sistema cuántico y relativista implica ∞ grados de libertad

QFT $\xrightarrow{\text{bajas energías}}$ Dirac $\xrightarrow{\text{no relativista}}$ Schrödinger $\xrightarrow{\text{clásico}}$ Newton



Giuseppe Ludovico Lagrangia (1736-1813)
(alias Lagrange)



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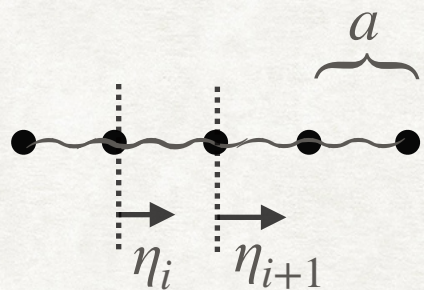
formalismo lagrangiano:

$$L(q, \dot{q}, t) = T - V$$

$$p \equiv \frac{\partial L}{\partial \dot{q}} \quad H = p\dot{q} - L \quad q, p \quad \text{variables generalizadas} \quad \{q, p\}_p = 1$$

$$\frac{dg}{dt} = \{g, H\}_p$$

$$q(t), \dot{q}(t) \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0 \quad \delta \int dt L = 0$$



$$L = \sum_i^N \frac{1}{2} m_i \dot{\eta}_i^2 - \frac{1}{2} k (\eta_{i+1} - \eta_i)^2 = \sum_i^N a \frac{1}{2} \left[\frac{m_i}{a} \dot{\eta}_i^2 - k a \left(\frac{\eta_{i+1} - \eta_i}{a} \right)^2 \right]$$

$$\mu \equiv \frac{m}{a} \quad Y \equiv ka$$

$$\frac{\eta_{i+1} - \eta_i}{a} \rightarrow \frac{\partial \eta}{\partial x}$$

$$\sum a \rightarrow \int dx$$

$$L \rightarrow \int dx \frac{1}{2} \left[\mu \dot{\eta}^2(x) - Y \left(\frac{\partial \eta}{\partial x} \right)^2 \right] \equiv \int dx \mathcal{L}(\eta, \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial t})$$

$$\eta, \frac{\partial L}{\partial \dot{\eta}} \quad \text{variables generalizadas}$$

$$x, t \quad \text{parámetros}$$

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formalismo lagrangiano:

$$L = \int dx \frac{1}{2} \left[\mu \dot{\eta}^2(x) - Y \left(\frac{\partial \eta}{\partial x} \right)^2 \right] \equiv \int dx \mathcal{L}(\eta, \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial t})$$

$$\pi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\eta}} = \mu \dot{\eta} \quad p_i = \frac{\partial L}{\partial \dot{\eta}_i} \quad P = \sum_i p_i = \sum_i m \dot{\eta}_i = \sum_i a \frac{m}{a} \dot{\eta}_i \rightarrow \int dx \mu \dot{\eta} = \int dx \pi$$

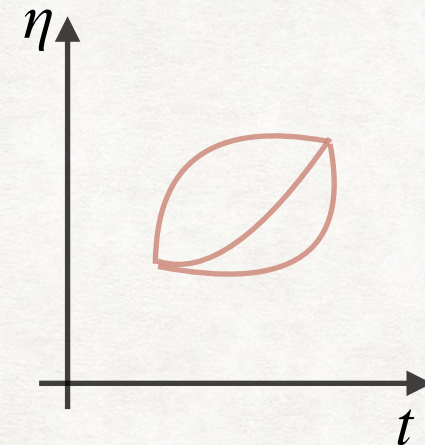
$$\mathcal{H} \equiv \pi \dot{\eta} - \mathcal{L} \quad H = \int dx \mathcal{H}$$

$$\{f, g\}_p \equiv \int dx \left[\frac{\partial f}{\partial \eta} \frac{\partial g}{\partial \pi} - \frac{\partial f}{\partial \pi} \frac{\partial g}{\partial \eta} \right]$$

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formalismo lagrangiano:

$$\delta \int dt L = \delta \int dt \int dx \mathcal{L} \quad \delta \eta(x, t) \quad \begin{cases} \delta \eta(x, t_1) = 0 \\ \delta \eta(x, t_2) = 0 \end{cases}$$



$$\delta \int dt L = \int dt \int dx \left[\frac{\partial \mathcal{L}}{\partial \eta} \delta \eta + \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \delta \left(\frac{\partial \eta}{\partial x} \right) + \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \delta \left(\frac{\partial \eta}{\partial t} \right) \right]$$

$$= \int dt \int dx \left[\frac{\partial \mathcal{L}}{\partial \eta} \delta \eta - \frac{\partial}{\partial x} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) \delta \eta - \frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \right) \delta \eta \right] = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) + \frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \delta \eta \right) = \frac{\partial \delta \eta}{\partial x} \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} + \frac{\partial}{\partial x} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) \delta \eta$$

$$0 = \underbrace{\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \delta \left(\frac{\partial \eta}{\partial x} \right)}_{\text{lo que tengo}} + \underbrace{\frac{\partial}{\partial x} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) \delta \eta}_{\text{como lo quiero}}$$

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formalismo lagrangiano:

$$\frac{\partial}{\partial x} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) + \frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0$$

$$\mathcal{L} = \frac{1}{2} \mu \dot{\eta}^2(x) - \frac{Y}{2} \left(\frac{\partial \eta}{\partial x} \right)^2$$

$$\frac{\partial}{\partial x} \left(-Y \frac{\partial \eta}{\partial x} \right) + \frac{\partial}{\partial t} (\mu \dot{\eta}) = 0 \quad -Y \frac{\partial^2 \eta}{\partial x^2} + \mu \frac{\partial^2 \eta}{\partial t^2} = 0$$

$$\sum_{k=1}^3 \frac{\partial}{\partial x_k} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x_k}} \right) + \frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0$$

$$\frac{\partial}{\partial x^\mu} \left(\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x^\mu}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \eta)} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0$$

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formalismo lagrangiano:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \quad \eta = \phi(x, t) \quad \mathcal{L} = \frac{1}{2} \left(\partial_\mu \phi \partial^\mu \phi - \mu^2 \phi^2 \right)$$

$$-\frac{\partial \mathcal{L}}{\partial \phi} = \mu^2 \phi \quad \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{\partial}{\partial(\partial_\mu \phi)} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) = \frac{1}{2} g^{\mu\nu} \left(\partial_\nu \phi + \partial_\mu \phi \delta_{\mu\nu} \right) \\ = \frac{1}{2} (\partial^\mu \phi + \partial^\mu \phi) = \partial^\mu \phi$$

$$\partial_\mu \partial^\mu \phi + \mu^2 \phi = 0 \quad \text{Klein-Gordon?}$$

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi = (\partial_\mu \phi_1 - i \partial_\mu \phi_2)(\partial^\mu \phi_1 + i \partial^\mu \phi_2) - \mu^2 (\phi_1 - i \phi_2)(\phi_1 + i \phi_2)$$

$$\begin{aligned} \phi &= \phi_1 + i \phi_2 \\ \phi^* &= \phi_1 - i \phi_2 \end{aligned} \quad \begin{aligned} &= \partial_\mu \phi_1 \partial^\mu \phi_1 - \mu^2 \phi_1^2 + \partial_\mu \phi_2 \partial^\mu \phi_2 - \mu^2 \phi_2^2 \\ &= \mathcal{L}_1 + \mathcal{L}_2 \end{aligned}$$

$$\left. \begin{aligned} \partial_\mu \partial^\mu \phi^* + \mu^2 \phi^* &= 0 \\ \partial_\mu \partial^\mu \phi + \mu^2 \phi &= 0 \end{aligned} \right\} \iff \begin{cases} \partial_\mu \partial^\mu \phi_1 + \mu^2 \phi_1 = 0 \\ \partial_\mu \partial^\mu \phi_2 + \mu^2 \phi_2 = 0 \end{cases} \quad \partial_\mu \partial^\mu \phi + \mu^2 \phi = 0$$

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formalismo lagrangiano:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0 \quad \eta = \psi(x, t) \quad \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} \right) - \frac{\partial \mathcal{L}}{\partial \bar{\psi}} = 0 \quad (i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \quad A_\mu(\vec{x}, t) \quad \text{campo cuadvectorial}$$

$$\square A_\mu - \partial^\mu(\partial_\nu A^\nu) = 0$$

Ec. Dirac

$\psi \sim$ función de onda

x, p

lagrangiano

$\psi \sim$ variable dinámica (clásica!)

$$\psi, \pi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\psi}}$$