ESTRUCTURA DE LA MATERIA 4

CURSO DE VERANO 2021

CLASE 12

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CLASE 12: Simetría de gauge

Temas: Invariancia de gauge, derivada covariante, QED.

invariancia de fase *(invariancia ante transformaciones de fase)*

$$\mathscr{L} = \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$$\psi \longrightarrow \psi' = e^{iq\alpha}\,\psi \qquad \overline{\psi} \longrightarrow \overline{\psi}' = e^{-iq\alpha}\,\overline{\psi}$$

α parámetro infinitesimal

q constante

$$e^{iq\alpha} \in U(1)$$

$$\mathscr{L} \longrightarrow \mathscr{L}$$

$$\mathscr{L} \longrightarrow \mathscr{L} + \alpha\,\partial_\mu J^\mu \qquad \Longrightarrow \; J^\mu = 0$$

$$j^\mu \equiv \frac{\partial \mathscr{L}}{\partial(\partial_\mu \psi)}\Delta\psi - J^\mu \qquad \partial_\mu j^\mu = 0$$

$$\psi \longrightarrow \psi' = \psi + \alpha\,\Delta\psi \qquad \Longrightarrow \; \Delta\psi = iq\psi$$

$$= \psi + iq\alpha\,\psi + \mathscr{O}(\alpha^2)$$

$$j^\mu = i\overline{\psi}\gamma^\mu(iq\psi) = -\,q\overline{\psi}\gamma^\mu\psi$$

invariancia de fase “local”

$$\psi \longrightarrow \psi' = \psi\,e^{iq\alpha(x)}$$

$$\partial_\mu\psi \longrightarrow \partial_\mu(e^{iq\alpha(x)}\psi) = iq\,(\partial_\mu\alpha)\,e^{iq\alpha(x)}\psi + e^{iq\alpha(x)}\partial_\mu\psi$$

$$\mathscr{L} \longrightarrow \mathscr{L}'$$

$$= e^{iq\alpha(x)}\left(\partial_\mu\psi + \underline{iq(\partial_\mu\alpha)\,\psi}\right)$$

$$\partial_\mu \longrightarrow D_\mu \qquad D_\mu\psi \longrightarrow (D_\mu\psi)' = e^{iq\alpha(x)}\,D_\mu\psi$$

“derivada covariante”

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invariancia de fase "local"

$$D_\mu\psi \longrightarrow (D_\mu\psi)' = e^{iq\alpha(x)} D_\mu\psi$$ "derivada covariante"

$$D_\mu \equiv \partial_\mu - iq\,A_\mu$$ A_μ "algo" (pero cuadrivector)

$$(D_\mu\psi)' = (\partial_\mu\psi)' - iq\,(A_\mu\psi)'$$

$$= e^{iq\alpha(x)}\left(\partial_\mu\psi + \underline{iq(\partial_\mu\alpha)\,\psi\ - iq\,A'_\mu\,\psi}\right) = e^{iq\alpha(x)}\,D_\mu\psi\ = e^{iq\alpha(x)}\left(\partial_\mu\psi - \underline{iq\,A_\mu\psi}\right)$$

$$A'_\mu = A_\mu + \partial_\mu\alpha(x)$$

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi \qquad \begin{cases}\psi \longrightarrow \psi' = \psi\,e^{iq\alpha(x)}\\ A_\mu \longrightarrow A_\mu + \partial_\mu\alpha(x)\end{cases}$$

$$p_\mu \to p_\mu + q\,A_\mu$$

$$\partial_\mu \to \partial_\mu - iq\,A_\mu$$

$$= \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi + q\overline{\psi}\gamma^\mu\psi\,A_\mu$$

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi + ?\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$\sim A^\mu$ no es escalar

$\sim A_\mu A^\mu$ no es invariante gauge

$\sim \partial_\mu A^\mu$ no es invariante gauge

$\sim (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) = F_{\mu\nu}F^{\mu\nu}$ si!

$$\Box^2 A_\mu - \cancel{\partial_\mu\partial^\nu} A_\nu = 0 \qquad \underbrace{(\partial^\nu A_\nu) = 0}_{\text{gauge de Lorenz}}$$

"bosones" $\Box^2 A_\mu = 0$

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repaso de T1

$$\Box^2 A_\mu - \partial_\mu \partial^\nu A_\nu = 0 \qquad \Box^2 A_\mu - \partial_\mu \partial^\nu A_\nu + m^2 A_\mu = 0$$ (no es invariante de gauge!)

$$A_\mu = \epsilon^j_\mu e^{-ip_\nu x^\nu} \qquad p_\nu p^\nu = m^2 \qquad \epsilon^j_\mu$$ 4 cuadrivectores l.i.

$$\partial^\mu A_\mu = 0 \qquad \Box^2 A_\mu = 0 \qquad \Box^2 A_\mu + m^2 A_\mu = 0$$

$$\Rightarrow \quad p^\mu \epsilon^j_\mu = 0 \qquad \Rightarrow \quad \epsilon^j_\mu$$ 3 cuadrivectores l.i.

si $p^\mu = (E, 0, 0, p_z)$ $\quad p^\mu \epsilon^j_\mu = E\,\epsilon_0 - p_z\,\epsilon_3 = 0$ no hay restricciones sobre ϵ_1 y ϵ_2

$$\begin{cases} \epsilon^1_\mu = (0, 1, 0, 0\,) \\ \epsilon^2_\mu = (0, 0, 1, 0\,) \\ \epsilon^3_\mu = (p_z, 0, 0, E\,) \end{cases}$$ ~ las tres componentes de spin 1

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repaso de T1

si $m = 0$ $\qquad \partial^\mu A_\mu = 0$ $\qquad$ no restringe totalmente la invariancia de gauge

$A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu F$ $\qquad$ con $\Box^2 F = 0$ $\qquad$ pasa el filtro de Lorenz

por ejemplo: $F = iae^{-ip_\nu x^\nu}$ $\qquad \partial_\mu F = ap_\mu e^{-ip_\nu x^\nu}$ $\qquad \Box^2 F \sim p^2$

➜ le puedo sumar cualquier cosa $\propto p_\mu$ al vector de polarización ϵ_μ

$$\begin{cases} \epsilon^1_\mu = (0,1,0,0) \\ \epsilon^2_\mu = (0,0,1,0) \\ \epsilon^3_\mu = (p_z,0,0,E) \end{cases}$$

➜ el tercer vector de polarización ϵ^3_μ es lo mismo que 0: $\quad$ si $m = 0$ solo hay polarización transversal

si $m \neq 0$ hay un modo "longitudinal"

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generalizaciones de invariancia de fase "local"

$$e^{iq\alpha(x)} \in U(1) \qquad SU(2), SU(3)?$$

$$U(x) \qquad U^\dagger U = \mathbb{1} \qquad \begin{cases} \psi \longrightarrow \psi' = U\psi \\ D_\mu\psi \longrightarrow (D_\mu\psi)' = U(D_\mu\psi) \end{cases}$$

$$D_\mu = \partial_\mu - iqA_\mu \qquad A_\mu \longrightarrow ?$$

$$iqA_\mu\psi = \partial_\mu\psi - D_\mu\psi$$

$$iq(A_\mu\psi)' = (\partial_\mu\psi)' - (D_\mu\psi)'$$

$$iqA'_\mu\psi' = (\partial_\mu U)\psi + U\partial_\mu\psi - UD_\mu\psi$$

$$iqA'_\mu\psi' = (\partial_\mu U)\psi + U\partial_\mu\psi - U\partial_\mu\psi + iqUA_\mu\psi$$

$$iqA'_\mu\psi' = (\partial_\mu U)U^{-1}\psi' + iqUA_\mu U^{-1}\psi'$$

$$\boxed{A'_\mu = \frac{1}{iq}(\partial_\mu U)U^{-1} + UA_\mu U^{-1}}$$

Por ejemplo: $U(x) = e^{iq\alpha(x)}$

$$\partial_\mu U = iq(\partial_\mu\alpha)U$$

$$A'_\mu = \frac{1}{iq}iq(\partial_\mu\alpha)UU^{-1} + UA_\mu U^{-1}$$

$$A'_\mu = \partial_\mu\alpha + A_\mu$$

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generalizaciones de invariancia de fase "local"

$$[D_\mu, D_\nu]\psi \equiv D_\mu D_\nu \psi - D_\nu D_\mu \psi$$

$$D_\mu D_\nu \psi = (\partial_\mu - iq\,A_\mu)\,(\partial_\nu - iq\,A_\nu)\,\psi$$

$$= \partial_\mu \partial_\nu \psi - iqA_\mu \partial_\nu \psi - iq\partial_\mu A_\nu \psi - q^2 A_\mu A_\nu \psi$$

$$= \partial_\mu \partial_\nu \psi - iqA_\mu \partial_\nu \psi - iq(\partial_\mu A_\nu)\psi - iqA_\nu(\partial_\mu \psi) - q^2 A_\mu A_\nu \psi$$

$$D_\nu D_\mu \psi = \partial_\nu \partial_\mu \psi - iqA_\nu \partial_\mu \psi - iq(\partial_\nu A_\mu)\psi - iqA_\mu(\partial_\nu \psi) - q^2 A_\nu A_\mu \psi$$

$$[D_\mu, D_\nu]\psi = -iq[\partial_\mu A_\nu - \partial_\mu A_\nu]\psi = -iq\,F_{\mu\nu}\,\psi$$

$$(D_\mu \psi)' = U\,D_\mu\,\psi$$

$$(D_\mu \psi)' = D'_\mu \psi' = D'_\mu\,U\,\psi$$

$$D'_\mu\,U = U\,D_\mu \qquad D'_\mu = U\,D_\mu\,U^{-1}$$

$$D'_\mu D'_\nu = U\,D_\mu\,U^{-1}U\,D_\nu\,U^{-1} = U\,D_\mu D_\nu\,U^{-1}$$

$$\boxed{F'_{\mu\nu} = U\,F_{\mu\nu}\,U^{-1}}$$

si $U \in U(1)$ $\quad F'_{\mu\nu} = F_{\mu\nu}$

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generalizaciones de invariancia de fase "local"

a nivel del lagrangiano $\psi_i \longrightarrow \psi'_i = \sum_{j=1}^{N} U_{ij}\,\psi_j$ $\quad \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$ $\quad \Psi' = U\,\Psi \quad U_{N\times N}$

$$\mathscr{L} = \sum_{i=1}^{N} \overline{\psi}_i (i\gamma^\mu \partial_\mu - m_i)\psi_i \qquad \mathscr{L} = \overline{\Psi}(i\gamma^\mu\partial_\mu - M)\Psi \qquad M_{N\times N}$$

si $\det U = 1 \quad SU(N)$ $\qquad$ p.ej $\quad SU(3)$ $\quad \Psi = \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix}$

$$U(\alpha_1, \alpha_2, \ldots, \alpha_{N^2-1}) = e^{i(\alpha_1 T_1 + \alpha_2 T_2 + \ldots + \alpha_{N^2-1} T_{N^2-1})}$$
$$= e^{ig\,\alpha_a(x)\,T_a}$$

p.ej $\begin{cases} SU(2) & T_i = \frac{\sigma_i}{2} \\ SU(3) & T_i = \frac{\lambda_i}{2} \end{cases}$ $\qquad [T_a, T_b] = i f_{abc} T_c$

$U \in U(1)$

$$\psi \longrightarrow \psi' = U\psi = e^{iq\alpha(x)}\psi$$
$$\partial_\mu\psi \longrightarrow \partial_\mu\psi' = iq(\partial_\mu\alpha)U\psi + U\partial_\mu\psi$$
$$D_\mu \equiv \partial_\mu - iq\,A_\mu$$
$$A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu\alpha(x)$$

$U \in SU(N)$

$$\Psi \longrightarrow \Psi' = U\,\Psi = e^{ig\alpha_a(x)T_a}\Psi$$
$$\partial_\mu\Psi \longrightarrow \partial_\mu\Psi' = ig(\partial_\mu\alpha_a)T_a\,U\Psi + U\partial_\mu\Psi$$
$$D_\mu \equiv \partial_\mu - ig\,T_a\,W^a_\mu$$
$$T_a W^a_\mu \longrightarrow T_a W'^a_\mu = \frac{1}{ig}(\partial_\mu U)U^{-1} + U T_a W^a_\mu U^{-1}$$