

ESTRUCTURA DE LA MATERIA 4

2DO CUATRIMESTRE 2025

CLASE 12

RODOLFO SASSOT

CLASE 12: Cuantización y simetrías

Temas: cuantización del campo, relación con la estadística, simetrías.

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi$$

$$\partial_\mu \partial^\mu \phi + \mu^2 \phi = 0$$

programa de cuantización:

1) identificar las variables dinámicas

oscilador armónico

$$\vec{x}, \vec{p}$$

formalismo de campos

$$\phi(\vec{x}, t), \pi(\vec{x}, t)$$

2) promover las variables dinámicas a operadores en espacio de Hilbert

3) imponer restricciones a la conmutación inspirándose en los corchetes

$$[x_i, p_j] = i\delta_{ij}$$

$$[x_i, x_j] = [p_i, p_j] = 0$$

$$[\phi(x), \pi(y)] = i\delta(x - y)$$

$$[\phi(x), \phi(y)] = [\pi(x), \pi(y)] = 0$$

4) "resolver" p.ej. formalismo $a, a^\dagger$

$$\begin{aligned} x &= \frac{1}{\sqrt{2\omega}}(a + a^\dagger) & [x, p] = i \leftrightarrow [a, a^\dagger] &= 1 \\ p &= -i\sqrt{\frac{\omega}{2}}(a - a^\dagger) & H &= \frac{1}{2}p^2 + \frac{1}{2}\omega^2 x^2 \leftrightarrow \omega(a^\dagger a + \frac{1}{2}) \end{aligned}$$

$$\phi \sim (a + a^\dagger)$$

$$\pi \sim (a - a^\dagger)$$

CLASE 12: Cuantización y simetrías

cuantización del campo escalar (a vuelo de pájaro):

$$\phi(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot \vec{x}} \phi(\vec{p}, t) \quad \phi(\vec{p}, t) \text{ componente de Fourier}$$

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0 \longrightarrow \left[\frac{\partial^2}{\partial t^2} + \left(|\vec{p}|^2 + m^2 \right) \right] \phi(\vec{p}, t) = 0$$

$$\ddot{\phi}(\vec{p}, t) + \omega_p^2 \phi(\vec{p}, t) = 0 \quad \text{oscilador armónico! } \omega_p = \sqrt{\vec{p}^2 + m^2}$$

$$\begin{aligned} x &= \frac{1}{\sqrt{2\omega}}(a + a^\dagger) & H &= \omega(a^\dagger a + \frac{1}{2}) & |0\rangle &\therefore a|0\rangle = 0 & H|0\rangle &= \frac{\omega}{2}|0\rangle \\ p &= -i\sqrt{\frac{\omega}{2}}(a - a^\dagger) & & & |n\rangle &\equiv (a^\dagger)^n |0\rangle & H|n\rangle &= (n + \frac{1}{2})\omega |n\rangle \end{aligned}$$

$$\phi(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} (a_p e^{i\vec{p} \cdot \vec{x}} + a_p^\dagger e^{-i\vec{p} \cdot \vec{x}}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} (a_p + a_{-p}^\dagger) e^{i\vec{p} \cdot \vec{x}}$$

$$\pi(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} (-i)\sqrt{\frac{\omega}{2}} (a_p - a_{-p}^\dagger) e^{i\vec{p} \cdot \vec{x}} \quad [a_p, a_{p'}^\dagger] = (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \leftrightarrow [\phi(\vec{x}), \pi(\vec{y})] = i\delta^3(\vec{x} - \vec{y})$$

$$H = \int d^3x \mathcal{H} = \int d^3x (\pi \dot{\phi} - \mathcal{L}) = \int \frac{d^3p}{(2\pi)^3} \omega_p \left(a_p^\dagger a_p + \frac{1}{2} [a_p, a_p^\dagger] \right)$$

CLASE 12: Cuantización y simetrías

cuantización del campo escalar (a vuelo de pájaro):

$$H = \int d^3x \mathcal{H} = \int \frac{d^3p}{(2\pi)^3} \omega_p \left(a_p^\dagger a_p + \frac{1}{2} [a_p, a_p^\dagger] \right) \quad |0\rangle \text{ vacío}$$

$$\text{impulso } \vec{P} = \int \frac{d^3p}{(2\pi)^3} \vec{p} a_p^\dagger a_p$$

$$a_p^\dagger |0\rangle \text{ estado de energía } \omega_p = \sqrt{\vec{p}^2 + m^2} \text{ e impulso } \vec{p}$$

$$a_p^\dagger a_q^\dagger |0\rangle = a_q^\dagger a_p^\dagger |0\rangle \quad (a_p^\dagger)^n |0\rangle \quad \text{bosones}$$

cuantización del campo de Dirac (a vuelo de pájaro²):

$$\psi(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=1,2} \left(a_p^s u^s(p) + b_{-p}^{s\dagger} v^s(-p) \right) e^{i\vec{p} \cdot \vec{x}}$$

$$\psi^\dagger(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=1,2} \left(a_p^{s\dagger} u^{s\dagger}(p) + b_{-p}^s v^{s\dagger}(-p) \right) e^{i\vec{p} \cdot \vec{x}}$$

$$[a_p^s, a_{p'}^{s'\dagger}] = (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \delta_{ss'}$$

$$[b_p^s, b_{p'}^{s'\dagger}] = (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \delta_{ss'}$$

$$H = \int d^3x \mathcal{H} = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_p a_p^{s\dagger} a_p^s - E_p b_p^{s\dagger} b_p^s \right)$$

CLASE 12: Cuantización y simetrías

cuantización del campo de Dirac (a vuelo de pájaro):

$$\begin{aligned} \{a_p^s, a_{p'}^{s'\dagger}\} &= (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \delta_{ss'} \\ \{b_p^s, b_{p'}^{s'\dagger}\} &= (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \delta_{ss'} \end{aligned} \quad H = \int d^3x \mathcal{H} = \int \frac{d^3p}{(2\pi)^3} \sum_s \left(E_p a_p^{s\dagger} a_p^s - E_p b_p^{s\dagger} b_p^s \right)$$

$$\tilde{b}_p^s \equiv b_p^{s\dagger} \quad \tilde{b}_p^{s\dagger} \equiv b_p^s \quad \{b, b^\dagger\} = bb^\dagger + b^\dagger b = \{b^\dagger, b\} = \{\tilde{b}, \tilde{b}^\dagger\}$$

$$-E_p b_p^\dagger b_p = -E_p (1 - b_p b_p^\dagger) = -E_p (1 - \tilde{b}_p^\dagger \tilde{b}_p) = E_p \tilde{b}_p^\dagger \tilde{b}_p - E_p \quad E > 0$$

$$\{b, b^\dagger\} = 1 \quad \{b, b\} = \{b^\dagger, b^\dagger\} = 0$$

$$|0\rangle \quad \therefore b|0\rangle = 0 \quad b^\dagger|0\rangle = |1\rangle$$

$$b|1\rangle = bb^\dagger|0\rangle = (1 - b^\dagger b)|0\rangle = |0\rangle$$

$$b^\dagger|1\rangle = b^\dagger b^\dagger|0\rangle = 0 \quad \{|0\rangle, |1\rangle\} \quad \text{vacío/partícula}$$

$$(b^\dagger)^2|0\rangle = 0 \quad \text{fermiones}$$



CLASE 12: Cuantización y simetrías



simetrías: transformaciones de los campos

$$\phi(x) \longrightarrow \phi'(x) = \phi(x) + \alpha \Delta\phi(x)$$

α parámetro infinitesimal (cuánto)

$\Delta\phi(x)$ deformación del campo (cómo)

simetría $\iff$ deja invariantes las ec. de movimiento

la acción es invariante

o cambia en un término de superficie

$$\mathcal{L} \longrightarrow \mathcal{L} + \alpha \partial_\mu J^\mu$$

$$j^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - J^\mu$$

$$\partial_\mu j^\mu = 0$$

$$\frac{\partial}{\partial t} \int d^3x j^0 = \int d^3x \vec{\nabla} \cdot \vec{j} = 0$$

$$Q \equiv \int d^3x j^0 \quad \frac{\partial Q}{\partial t} = 0$$

$$\alpha \Delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \phi} \alpha \Delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \alpha \Delta(\partial_\mu \phi)$$

$$= \alpha \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi \right) + \alpha \frac{\partial \mathcal{L}}{\partial \phi} \Delta\phi - \alpha \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \Delta\phi$$

$$= \alpha \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi \right) + \alpha \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right)}_0 \Delta\phi$$

CLASE 12: Cuantización y simetrías



simetrías: transformaciones de los campos

ejemplos:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \quad \phi(x) \longrightarrow \phi'(x) = \phi(x) + \alpha \quad \implies \Delta\phi(x) = 1$$

$$\phi(x) \longrightarrow \phi'(x) = \phi(x) + \alpha \Delta\phi(x)$$

$$\mathcal{L} \longrightarrow \mathcal{L} \quad \implies J^\mu = 0$$

$$\mathcal{L} \longrightarrow \mathcal{L} + \alpha \partial_\mu J^\mu$$

$$j^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - J^\mu = \partial^\mu \phi \quad \partial_\mu \partial^\mu \phi = 0 \quad Q = \int d^3x \partial^0 \phi$$

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \quad \phi(x) \longrightarrow \phi'(x) = \phi(x) e^{i\alpha q} \simeq \phi(x) (1 + i\alpha q)$$

$$\Delta\phi(x) = iq\phi(x) \quad \Delta\phi^*(x) = -iq\phi^*(x)$$

$$\mathcal{L} \longrightarrow \mathcal{L} \quad J^\mu = 0$$

$$j^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - J^\mu = iq \partial^\mu \phi^* \phi - iq \phi^* \partial^\mu \phi = j_{KG}^\mu \quad \partial_\mu j_{KG}^\mu = 0$$

CLASE 12: Cuantización y simetrías

simetrías: transformaciones espacio-temporales



$$x^\mu \longrightarrow x^\mu + a^\mu$$

$$\phi(x^\mu) \longrightarrow \phi'(x^\mu) = \phi(x^\mu + a^\mu) = \phi(x) + a^\mu \partial_\mu \phi(x)$$

$$\mathcal{L} \longrightarrow \mathcal{L} + a^\mu \partial_\mu \mathcal{L} = \mathcal{L} + a^\nu \partial_\mu (\delta_\nu^\mu \mathcal{L}) = \mathcal{L} + a^\nu \partial_\mu (J^\mu)_\nu \quad \sim \text{cuatro corrientes } J^\mu$$

una para cada dirección del espacio ν

~ cuatro corrientes conservadas j^μ

$$j^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta \phi - J^\mu$$

$$T_\nu^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \mathcal{L} \delta_\nu^\mu$$

tensor de energía impulso

carga conservada ante traslaciones temporales

$$\int d^3x T^{00} \equiv H$$

hamiltoniano

carga conservada ante traslaciones espaciales

$$\int d^3x T^{0i} \equiv P^i$$

impulso