ESTRUCTURA DE LA MATERIA 4

CURSO DE VERANO 2021

CLASE 13

RODOLFO SASSOT

CLASE 13: Gauge no abeliano

de U(1) a SU(N)

$$\psi \longrightarrow \psi' = \psi\, e^{iq\alpha(x)} \qquad \mathscr{L} = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi \qquad D_\mu \equiv \partial_\mu - iq\,A_\mu$$

$$\mathscr{L} = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\psi_i \longrightarrow \psi_i' = \sum_{j=1}^{N} U_{ij}\,\psi_j \qquad \mathscr{L} = \sum_{i=1}^{N} \overline{\psi}_i(i\gamma^\mu \partial_\mu - m_i)\psi_i$$

$$\mathscr{L} = \overline{\Psi}(i\gamma^\mu\partial_\mu - M)\Psi \quad M_{N\times N} \qquad \Psi = \begin{pmatrix}\psi_1\\ \psi_2\\ \vdots \\ \psi_N\end{pmatrix} \qquad \Psi' = U\,\Psi \quad U_{N\times N}$$

$$U(\alpha_1, \alpha_2, \ldots, \alpha_{N^2-1}) = e^{i(\alpha_1 T_1 + \alpha_2 T_2 + \ldots + \alpha_{N^2-1}T_{N^2-1})}$$

$U \in U(1)$

$$\psi \longrightarrow \psi' = U\,\psi = e^{iq\alpha(x)}\,\psi$$

$$\partial_\mu\psi \longrightarrow \partial_\mu\psi' = iq(\partial_\mu\alpha)U\,\psi + U\partial_\mu\psi$$

$$D_\mu \equiv \partial_\mu - iq\,A_\mu$$

$$A_\mu \longrightarrow A_\mu' = A_\mu + \partial_\mu\alpha(x)$$

$U \in SU(N)$

$$\Psi \longrightarrow \Psi' = U\,\Psi = e^{ig\alpha_a(x)T_a}\,\Psi$$

$$\partial_\mu\Psi \longrightarrow \partial_\mu\Psi' = ig(\partial_\mu\alpha_a)T_a\,U\Psi + U\partial_\mu\Psi$$

$$D_\mu \equiv \partial_\mu - ig\,T_a\,W_\mu^a$$

$$T_a W_\mu^a \longrightarrow T_a W_\mu^{\prime a} = \frac{1}{ig}(\partial_\mu U)U^{-1} + UT_aW_\mu^a U^{-1}$$

CLASE 13: Gauge no abelinao

de U(1) a SU(N)

$$A'_\mu = \frac{1}{iq}(\partial_\mu U)U^{-1} + UA_\mu U^{-1} \qquad U = e^{ig\,\alpha^a(x)\,T^a} \qquad U = 1 + ig\,\alpha^a\,T^a + \mathcal{O}(\alpha^2)$$

$$U^{-1} = 1 - ig\,\alpha^a\,T^a + \mathcal{O}(\alpha^2)$$

$$(W^a_\mu T^a)' = \frac{1}{ig} ig(\partial_\mu \alpha^a) T^a \cancel{U} U^{-1} + (1 + ig\,\alpha^b\,T^b)\,W^a_\mu T^a\,(1 - ig\,\alpha^c\,T^c)$$

$$= \partial_\mu \alpha^a T^a + W^a_\mu T^a + ig\,\alpha^b\,T^b T^a\;W^a_\mu - ig\,\alpha^c\,T^a T^c\;W^a_\mu + \mathcal{O}(\alpha^b \alpha^c)$$

$$= \partial_\mu \alpha^a T^a + W^a_\mu T^a + ig\,\alpha^b\,(T^b T^a - T^b T^a)\,W^a_\mu \qquad c \to b$$

$$[T^b, T^a] = if_{bac} T^c$$

$$= \partial_\mu \alpha^a T^a + W^a_\mu T^a - g\,\alpha^b f_{bac} T^c\,W^a_\mu \qquad \begin{matrix} c \to a \\ a \to c \end{matrix}$$

$$= \partial_\mu \alpha^a T^a + W^a_\mu T^a - g\,\alpha^b f_{bca} T^a\,W^c_\mu$$

$$= \partial_\mu \alpha^a T^a + W^a_\mu T^a - g\,f_{abc} \alpha^b W^c_\mu\,T^a$$

$$W'^a_\mu = \partial_\mu \alpha^a + W^a_\mu - g\,f_{abc} \alpha^b W^c_\mu \qquad \sim A'_\mu = \partial_\mu \alpha + A_\mu$$

CLASE 13: Gauge no abeliano

de U(1) a SU(N)

C.N. Yang R. Mills

$$[D_\mu, D_\nu]\psi \; = - iq\, F_{\mu\nu}\, \psi \qquad G_{\mu\nu} \equiv \frac{1}{-ig}\,[D_\mu, D_\nu]$$

$$D_\mu D_\nu \Psi = \partial_\mu \partial_\nu \Psi - igW^a_\mu T^a \partial_\nu \Psi - ig(\partial_\mu W^a_\nu) T^a \Psi - igW^a_\nu T^a(\partial_\mu \Psi) - g^2 W^a_\mu T^a W^b_\nu T^b \Psi$$

$$D_\nu D_\mu \Psi = \partial_\nu \partial_\mu \Psi - igW^a_\nu T^a \partial_\mu \Psi - ig(\partial_\nu W^a_\mu) T^a \Psi - igW^a_\mu T^a(\partial_\nu \Psi) - g^2 W^a_\nu T^a W^b_\mu T^b \Psi$$

$$D_\mu D_\nu - D_\nu D_\mu = -ig(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu) T^a - g^2 W^a_\mu W^b_\nu [T^a, T^b]$$

$$-ig^2 W^a_\mu W^b_\nu f_{abc} T^c \qquad a \to b \quad b \to c \quad c \to a$$

$$-ig^2 W^b_\mu W^c_\nu f_{bca} T^a$$

$$-ig^2 W^b_\mu W^c_\nu f_{abc} T^a$$

$$G_{\mu\nu} = \left[(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu) + g W^b_\mu W^c_\nu f_{abc} \right] T^a = G^a_{\mu\nu} T^a$$

$$G'_{\mu\nu} = U\, G_{\mu\nu}\, U^{-1} \qquad (G_{\mu\nu} G^{\mu\nu})' = U\, G_{\mu\nu}\, \cancel{U^{-1} U}\, G^{\mu\nu}\, U^{-1}$$

$$Tr[(G_{\mu\nu} G^{\mu\nu})'] = Tr[G_{\mu\nu} G^{\mu\nu}]$$

$$Tr[G_{\mu\nu} G^{\mu\nu}] = \frac{1}{2} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$\mathscr{L}_{YM} = \overline{\Psi}(i\gamma^\mu \partial_\mu - M)\Psi + g(\overline{\Psi}\gamma^\mu T^a \Psi) W^a_\mu - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

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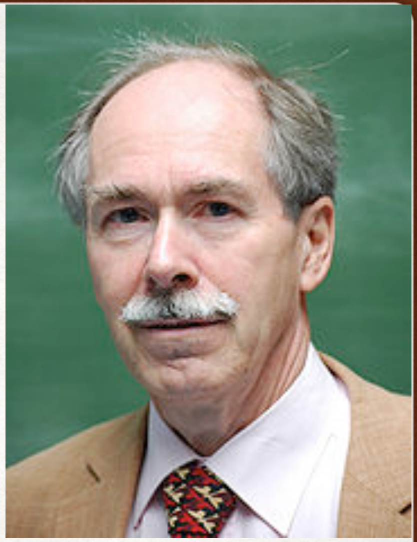
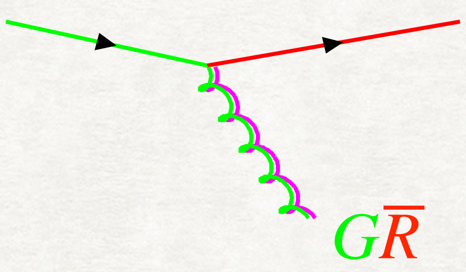


G. 't Hooft

QCD

$$\mathscr{L}_{YM} = \overline{\Psi}(i\gamma^\mu\partial_\mu - M)\Psi + g(\overline{\Psi}\gamma^\mu T^a\Psi)W^a_\mu - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \qquad \Psi = \begin{pmatrix}\psi_R\\ \psi_B\\ \psi_G\end{pmatrix}$$

$$\overline{\Psi}\gamma^\mu\tilde{T}^a\Psi = (\overline{\psi_R}, \overline{\psi_B}, \overline{\psi_G})\ \gamma^\mu\frac{1}{2}\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}\psi_R\\ \psi_B\\ \psi_G\end{pmatrix} = \overline{\psi_R}\,\gamma^\mu\frac{1}{2}\psi_G$$

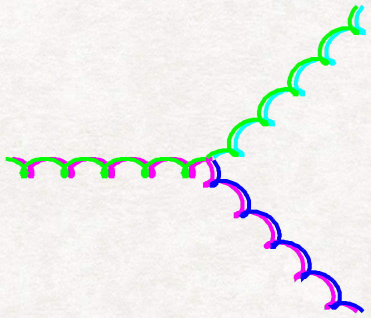


$$3\otimes\overline{3} = 8+1 \qquad 1 \sim R\overline{R} + B\overline{B} + G\overline{G} \qquad \tilde{T} \sim 1\!\!1 \qquad \notin SU(3)$$

$$G^a_{\mu\nu}G^{\mu\nu}_a = [(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu) + gW^b_\mu W^c_\nu f_{abc}][(\partial^\mu W^\nu_a - \partial^\nu W^\mu_a) + gW^\mu_b W^\nu_c f_{abc}]$$

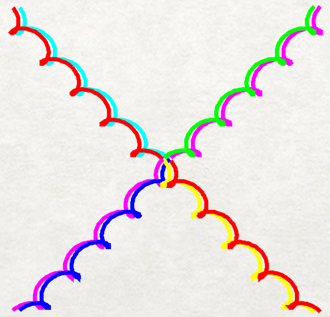
$$= (\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(\partial^\mu W^\nu_a - \partial^\nu W^\mu_a)$$

$$+gW^b_\mu W^c_\nu f_{abc}(\partial^\mu W^\nu_a - \partial^\nu W^\mu_a)$$



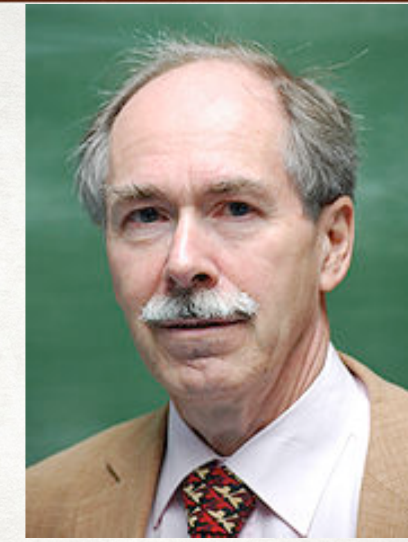
$\sim g$

$$+g^2 W^b_\mu W^c_\nu f_{abc} W^\mu_b W^\nu_c f_{abc}$$



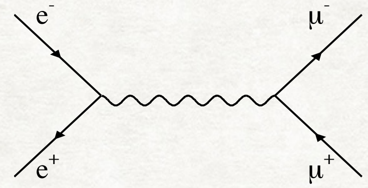
$\sim g^2$

CLASE 13: Gauge no abeliano

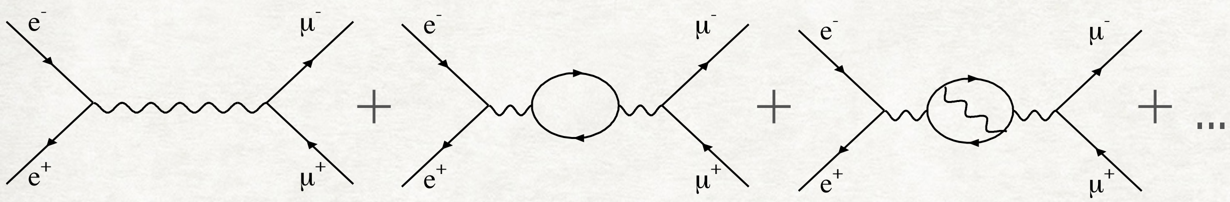


G. 't Hooft

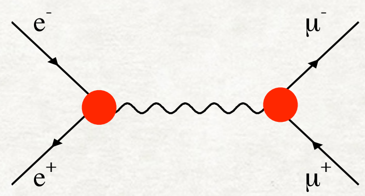
QCD



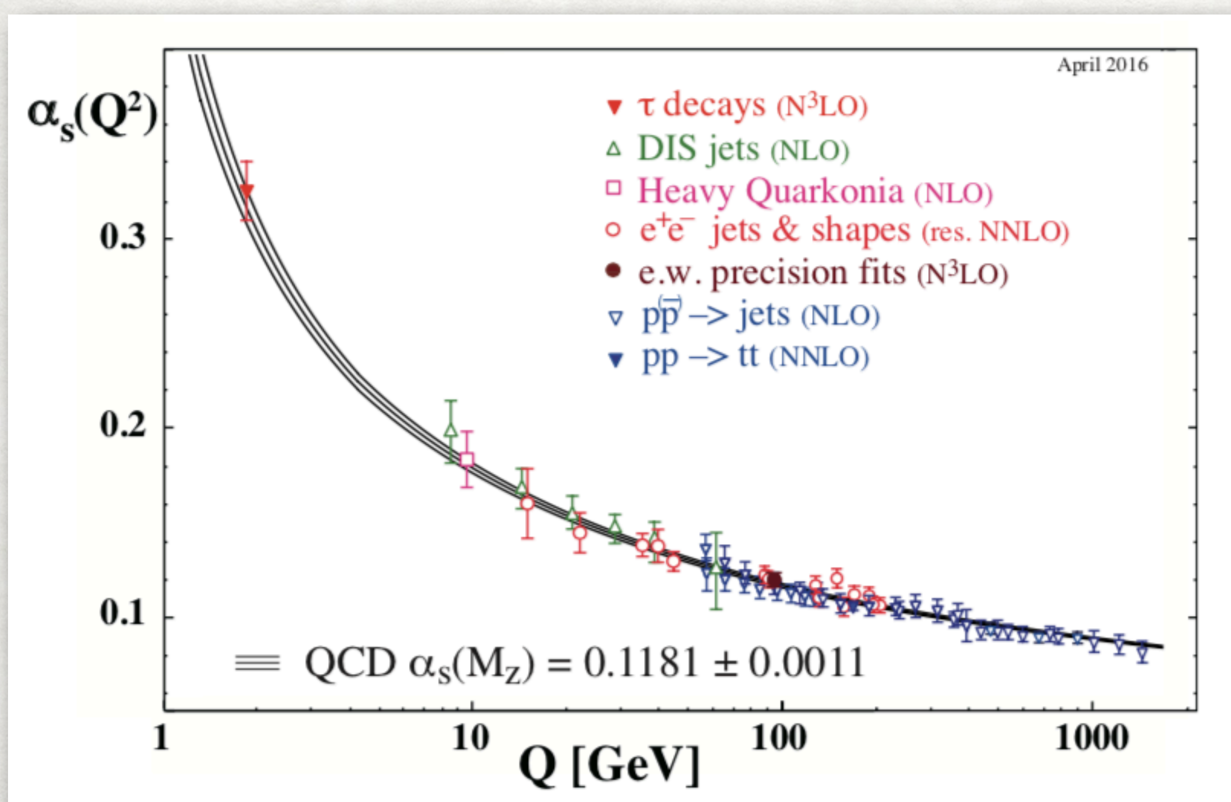
$$\sigma = \frac{4\pi}{3s}\alpha_0^2$$



$$\sigma = \frac{4\pi}{3s}\alpha_0^2(1 + \alpha_0\sigma^{(1)}(s) + \alpha_0^2\sigma^{(2)}(s) + \ldots)$$



$$\sigma = \frac{4\pi}{3s}\alpha_{\rm eff}^2(s)$$



$$\alpha_{eff}(s) = \frac{1}{\beta_0 \log(\frac{s}{\Lambda^2_{QCD}})}$$

$$\Lambda_{QCD} \sim 200\, MeV$$

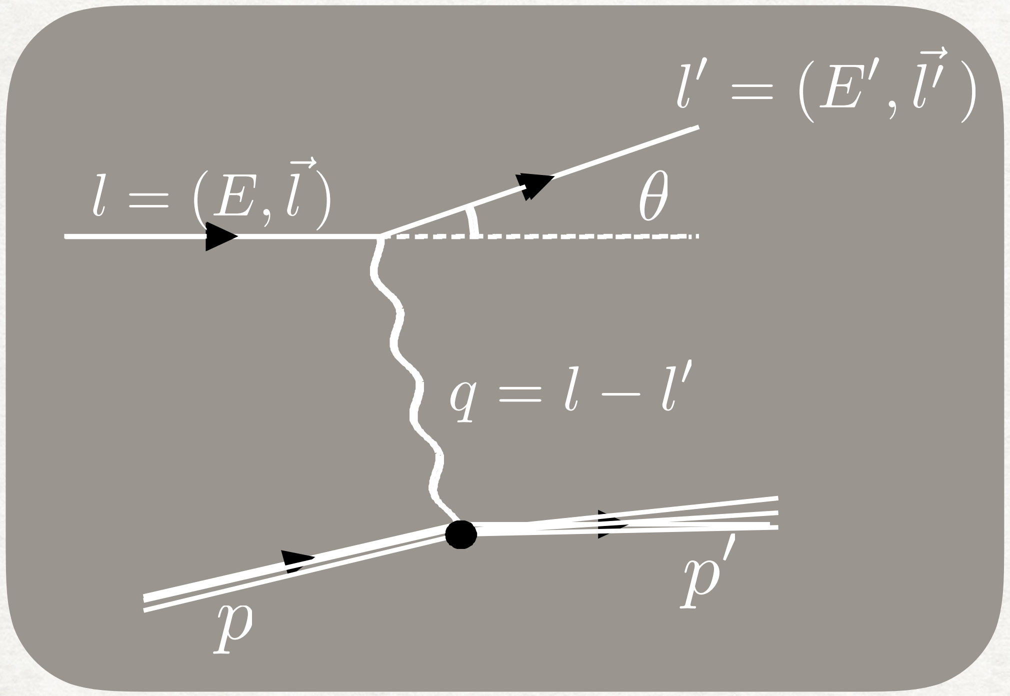
D. Gross, F. Wilczek,
D. Politzer

CLASE 13: Gauge no abeliano

QCD: libertad asintótica



R.P. Feynman J. D. Bjorken



$l^\mu \ p^\mu$ fijos

$l'^\mu \ p'^\mu$ 8 variables

-4 conservación E-p

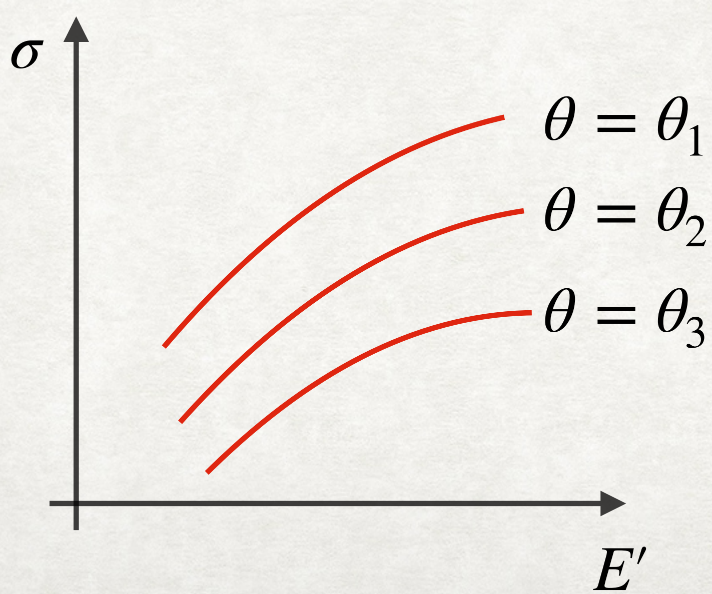
-1 $l^2 = m_e^2$

~~-1 $p^2 = m_p^2$~~

-1 en c.m.s. $l - p$, simetría rotaciones

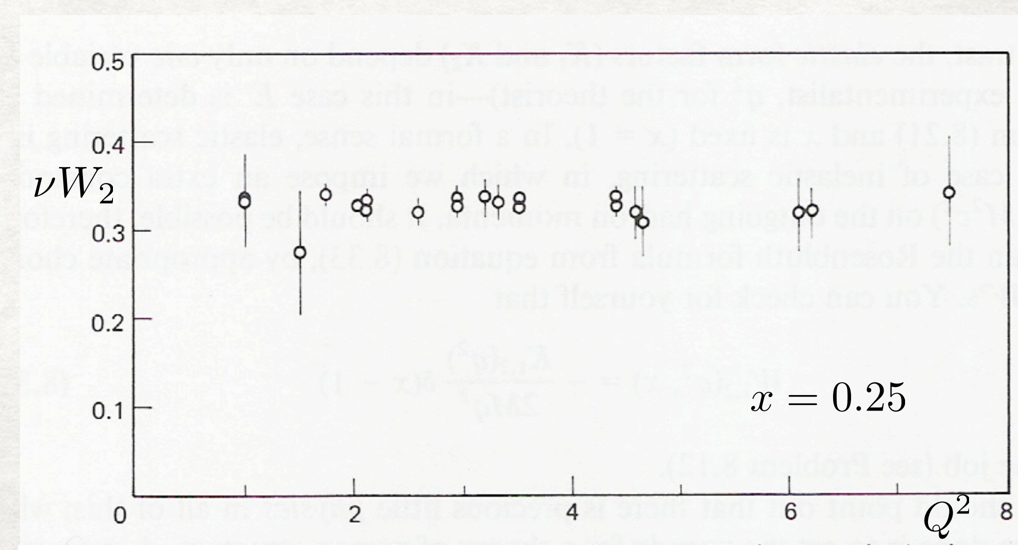
~~1 variable independiente (θ o E')~~

2 variables independientes (θ y E')



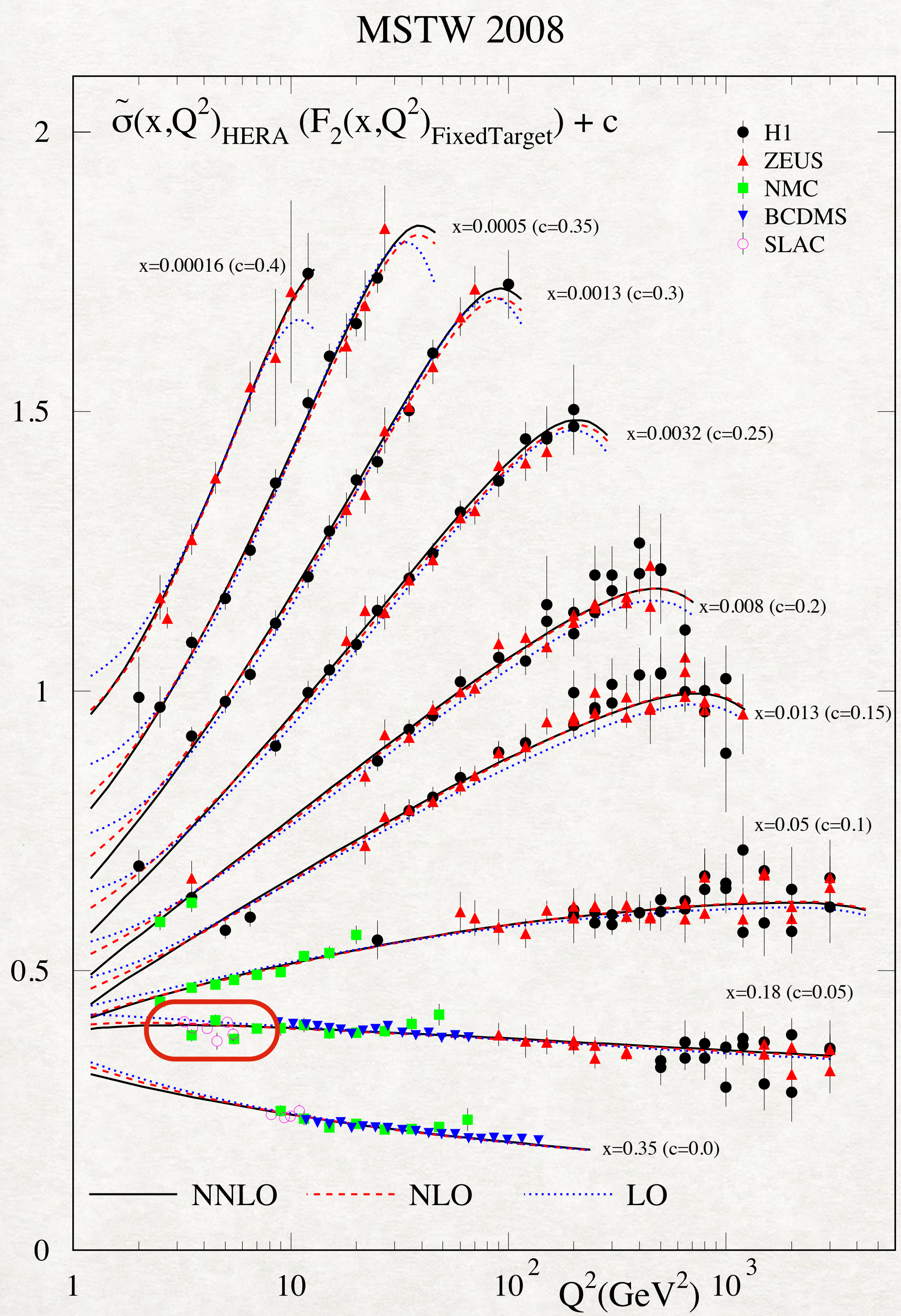
$$Q^2 = 4EE' \sin(\theta/2)$$

$$x = \frac{Q^2}{2M(E - E')}$$



CLASE 13: Gauge no abeliano

QCD: libertad asintótica



R.P. Feynman

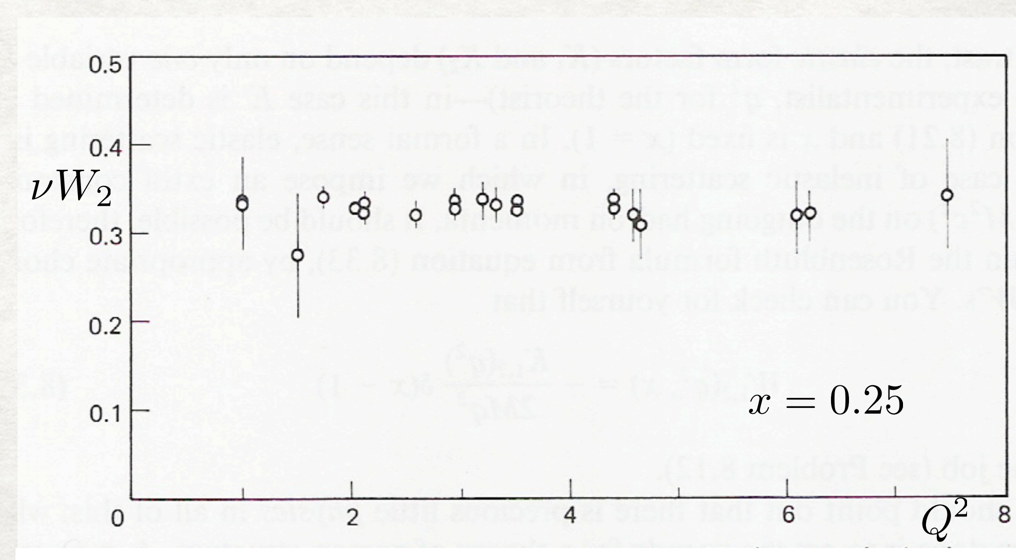
J. D. Bjorken

$$\sigma_{ep} \sim \sum_q \int dx\, e_q^2\, \sigma_{eq}\, f_q(x)$$

$$f_q(x) \longrightarrow f_q(x, Q^2)$$

$$Q^2 = 4EE' \sin(\theta/2)$$

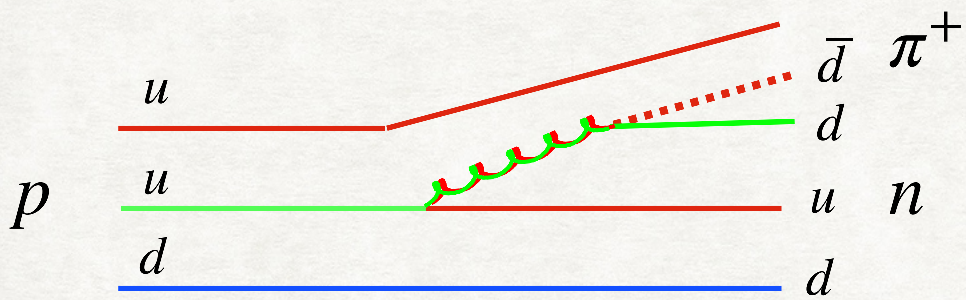
$$x = \frac{Q^2}{2M(E-E')}$$



CLASE 13: Gauge no abeliano

QCD: confinamiento

los estados "físicos" no pueden tener color explícito

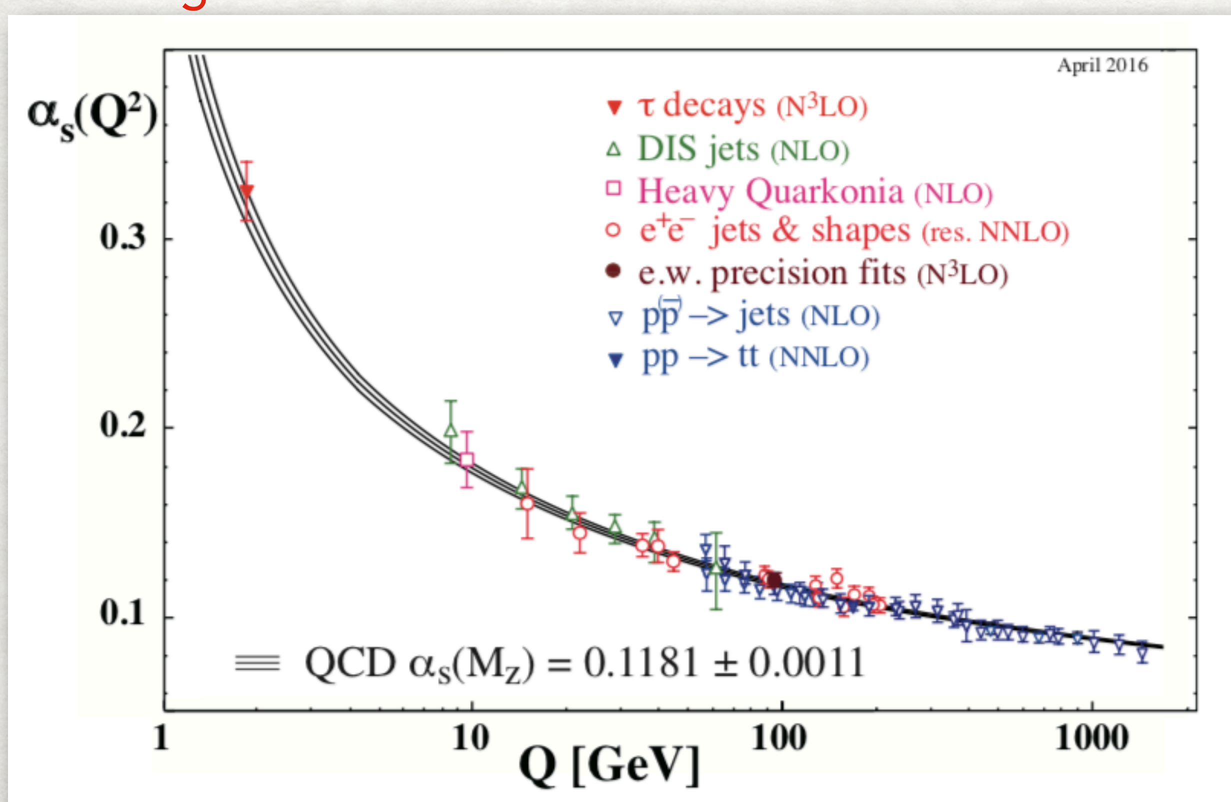


los detalles del mecanismo de confinamiento no son perturbativos:

factorización

lattice QCD /holografía/modelos efectivos

distancias largas ← → distancias cortas



$$\alpha_{eff}(s) = \frac{1}{\beta_0 \log(\frac{s}{\Lambda^2_{QCD}})}$$

$$\Lambda_{QCD} \sim 200\, MeV$$

$$200\, MeV \sim 1\, fm$$