

ESTRUCTURA DE LA MATERIA 4

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CLASE 13

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CLASE 13: Simetría de gauge

Temas: Invariancia de gauge, derivada covariante, QED.

invariancia de fase (invariancia ante transformaciones de fase)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \quad \begin{array}{l} \psi \longrightarrow \psi' = e^{iq\alpha} \psi \\ \bar{\psi} \longrightarrow \bar{\psi}' = e^{-iq\alpha} \bar{\psi} \end{array} \quad \begin{array}{l} \alpha \text{ parámetro infinitesimal} \\ q \text{ constante} \end{array} \quad e^{iq\alpha} \in U(1)$$

$$\mathcal{L} \longrightarrow \mathcal{L}$$

$$\mathcal{L} \longrightarrow \mathcal{L} + \alpha \partial_\mu J^\mu \quad \implies J^\mu = 0$$

$$j^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \Delta\psi - J^\mu \quad \partial_\mu j^\mu = 0$$

$$\begin{aligned} \psi \longrightarrow \psi' &= \psi + \alpha \Delta\psi \\ &= \psi + iq\alpha \psi + \mathcal{O}(\alpha^2) \end{aligned} \quad \implies \Delta\psi = iq\psi$$

$$j^\mu = i\bar{\psi}\gamma^\mu(iq\psi) = -q\bar{\psi}\gamma^\mu\psi$$

invariancia de fase "local"

$$\psi \longrightarrow \psi' = \psi e^{iq\alpha(x)} \quad \partial_\mu \psi \longrightarrow \partial_\mu(e^{iq\alpha(x)}\psi) = iq(\partial_\mu \alpha) e^{iq\alpha(x)}\psi + e^{iq\alpha(x)}\partial_\mu \psi$$

$$\mathcal{L} \longrightarrow \mathcal{L}' = e^{iq\alpha(x)} \left(\partial_\mu \psi + \underline{iq(\partial_\mu \alpha) \psi} \right)$$

$$\partial_\mu \longrightarrow D_\mu \quad D_\mu \psi \longrightarrow (D_\mu \psi)' = e^{iq\alpha(x)} D_\mu \psi \quad \text{"derivada covariante"}$$

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invariancia de fase "local"

$$D_\mu \psi \longrightarrow (D_\mu \psi)' = e^{iq\alpha(x)} D_\mu \psi$$

"derivada covariante"

$$D_\mu \equiv \partial_\mu - iq A_\mu$$

A_μ "algo" (pero cuadrivector)

$$(D_\mu \psi)' = (\partial_\mu \psi)' - iq (A_\mu \psi)'$$

$$= e^{iq\alpha(x)} \left(\partial_\mu \psi + \underline{iq(\partial_\mu \alpha) \psi} - iq A'_\mu \psi \right) = e^{iq\alpha(x)} D_\mu \psi = e^{iq\alpha(x)} \left(\partial_\mu \psi - \underline{iq A_\mu \psi} \right)$$

$$A'_\mu = A_\mu + \partial_\mu \alpha(x)$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \quad \begin{cases} \psi \longrightarrow \psi' = \psi e^{iq\alpha(x)} \\ A_\mu \longrightarrow A_\mu + \partial_\mu \alpha(x) \end{cases}$$

$$p_\mu \rightarrow p_\mu + q A_\mu$$

$$\partial_\mu \rightarrow \partial_\mu - iq A_\mu$$

$$= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + q \bar{\psi} \gamma^\mu \psi A_\mu$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$\sim A^\mu$ no es escalar

$\sim A_\mu A^\mu$ no es invariante gauge

$\sim \partial_\mu A^\mu$ no es invariante gauge

$\sim (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) = F_{\mu\nu} F^{\mu\nu}$ si!

$$\square A_\mu - \cancel{\partial_\mu \partial^\nu A_\nu} = 0 \quad (\partial^\nu A_\nu) = 0$$

"bosones" $\square A_\mu = 0$ $\underbrace{\hspace{10em}}$
gauge de Lorenz

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repaso de T1

$$\square A_\mu - \partial_\mu \partial^\nu A_\nu = 0$$

$$\square A_\mu - \partial_\mu \partial^\nu A_\nu + m^2 A_\mu = 0 \quad (\text{no es invariante de gauge!})$$

$$A_\mu = \epsilon_\mu^j e^{-ip_\nu x^\nu} \quad p_\nu p^\nu = m^2 \quad \epsilon_\mu^j \quad 4 \text{ cuadvectores l.i.}$$

$$\partial^\mu A_\mu = 0 \quad \square A_\mu = 0 \quad \square A_\mu + m^2 A_\mu = 0$$

$$\Rightarrow p^\mu \epsilon_\mu^j = 0 \quad \Rightarrow \epsilon_\mu^j \quad 3 \text{ cuadvectores l.i.}$$

$$\text{si } p^\mu = (E, 0, 0, p_z) \quad p^\mu \epsilon_\mu^j = E \epsilon_0 - p_z \epsilon_3 = 0 \quad \text{no hay restricciones sobre } \epsilon_1 \text{ y } \epsilon_2$$

$$\begin{cases} \epsilon_\mu^1 = (0, 1, 0, 0) \\ \epsilon_\mu^2 = (0, 0, 1, 0) \\ \epsilon_\mu^3 = (p_z, 0, 0, E) \end{cases} \quad \sim \text{ las tres componentes de spin 1}$$

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repaso de T1

si $m = 0$ $\partial^\mu A_\mu = 0$ no restringe totalmente la invariancia de gauge

$A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu F$ con $\square F = 0$ pasa el filtro de Lorenz

por ejemplo: $F = iae^{-ip_\nu x^\nu}$ $\partial_\mu F = ap_\mu e^{-ip_\nu x^\nu}$ $\square F \sim p^2$

→ le puedo sumar cualquier cosa $\propto p_\mu$ al vector de polarización ϵ_μ

$$\begin{cases} \epsilon_\mu^1 = (0, 1, 0, 0) \\ \epsilon_\mu^2 = (0, 0, 1, 0) \\ \epsilon_\mu^3 = (p_z, 0, 0, E) \end{cases}$$

→ el tercer vector de polarización ϵ_μ^3 es lo mismo que 0: si $m = 0$ solo hay polarización transversal
si $m \neq 0$ hay un modo "longitudinal"

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generalizaciones de invariancia de fase "local"

$$e^{iq\alpha(x)} \in U(1) \quad SU(2), SU(3)?$$

$$U(x) \quad U^\dagger U = \mathbb{1} \quad \begin{cases} \psi \longrightarrow \psi' = U \psi \\ D_\mu \psi \longrightarrow (D_\mu \psi)' = U (D_\mu \psi) \end{cases}$$

$$D_\mu = \partial_\mu - iq A_\mu \quad A_\mu \longrightarrow ?$$

$$iq A_\mu \psi = \partial_\mu \psi - D_\mu \psi$$

$$iq (A_\mu \psi)' = (\partial_\mu \psi)' - (D_\mu \psi)'$$

$$iq A'_\mu \psi' = (\partial_\mu U) \psi + U \partial_\mu \psi - U D_\mu \psi$$

$$iq A'_\mu \psi' = (\partial_\mu U) \psi + \cancel{U \partial_\mu \psi} - \cancel{U \partial_\mu \psi} + iq U A_\mu \psi$$

$$iq A'_\mu \psi' = (\partial_\mu U) U^{-1} \psi' + iq U A_\mu U^{-1} \psi'$$

$$A'_\mu = \frac{1}{iq} (\partial_\mu U) U^{-1} + U A_\mu U^{-1}$$

Por ejemplo: $U(x) = e^{iq\alpha(x)}$

$$\partial_\mu U = iq (\partial_\mu \alpha) U$$

$$A'_\mu = \frac{1}{iq} \cancel{iq} (\partial_\mu \alpha) \cancel{U} \cancel{U^{-1}} + \cancel{U} A_\mu \cancel{U^{-1}}$$

$$A'_\mu = \partial_\mu \alpha + A_\mu$$

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generalizaciones de invariancia de fase "local"

$$[D_\mu, D_\nu] \psi \equiv D_\mu D_\nu \psi - D_\nu D_\mu \psi$$

$$D_\mu D_\nu \psi = (\partial_\mu - iq A_\mu) (\partial_\nu - iq A_\nu) \psi$$

$$= \partial_\mu \partial_\nu \psi - iq A_\mu \partial_\nu \psi - \underline{iq \partial_\mu A_\nu \psi} - q^2 A_\mu A_\nu \psi$$

$$= \cancel{\partial_\mu \partial_\nu \psi} - \cancel{iq A_\mu \partial_\nu \psi} - \underline{iq (\partial_\mu A_\nu) \psi} - \cancel{iq A_\nu (\partial_\mu \psi)} - \cancel{q^2 A_\mu A_\nu \psi}$$

$$D_\nu D_\mu \psi = \cancel{\partial_\nu \partial_\mu \psi} - \cancel{iq A_\nu \partial_\mu \psi} - iq (\partial_\nu A_\mu) \psi - \cancel{iq A_\mu (\partial_\nu \psi)} - \cancel{q^2 A_\nu A_\mu \psi}$$

$$[D_\mu, D_\nu] \psi = -iq [\partial_\mu A_\nu - \partial_\nu A_\mu] \psi = -iq F_{\mu\nu} \psi$$

$$(D_\mu \psi)' = U D_\mu \psi$$

$$D'_\mu U = U D_\mu$$

$$D'_\mu = U D_\mu U^{-1}$$

$$(D_\mu \psi)' = D'_\mu \psi' = D'_\mu U \psi$$

$$D'_\mu D'_\nu = U D_\mu \cancel{U^{-1}} U D_\nu U^{-1} = U D_\mu D_\nu U^{-1}$$

$$F'_{\mu\nu} = U F_{\mu\nu} U^{-1}$$

$$\text{si } U \in U(1) \quad F'_{\mu\nu} = F_{\mu\nu}$$

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generalizaciones de invariancia de fase "local"

a nivel del lagrangiano $\psi_i \longrightarrow \psi'_i = \sum_{j=1}^N U_{ij} \psi_j$ $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$ $\Psi' = U \Psi$ $U_{N \times N}$

$$\mathcal{L} = \sum_{i=1}^N \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i \quad \mathcal{L} = \bar{\Psi} (i\gamma^\mu \partial_\mu - M) \Psi \quad M_{N \times N}$$

si $\det U = 1$ $SU(N)$ p.ej $SU(3)$ $\Psi = \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix}$

$$U(\alpha_1, \alpha_2, \dots, \alpha_{N^2-1}) = e^{i(\alpha_1 T_1 + \alpha_2 T_2 + \dots + \alpha_{N^2-1} T_{N^2-1})}$$

$$= e^{ig \alpha_a(x) T_a}$$

p.ej $\begin{cases} SU(2) & T_i = \frac{\sigma_i}{2} \\ SU(3) & T_i = \frac{\lambda_i}{2} \end{cases} \quad [T_a, T_b] = if_{abc} T_c$

$$U \in U(1)$$

$$\psi \longrightarrow \psi' = U \psi = e^{iq\alpha(x)} \psi$$

$$\partial_\mu \psi \longrightarrow \partial_\mu \psi' = iq(\partial_\mu \alpha) U \psi + U \partial_\mu \psi$$

$$D_\mu \equiv \partial_\mu - iq A_\mu$$

$$A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu \alpha(x)$$

$$U \in SU(N)$$

$$\Psi \longrightarrow \Psi' = U \Psi = e^{ig\alpha_a(x) T_a} \Psi$$

$$\partial_\mu \Psi \longrightarrow \partial_\mu \Psi' = ig(\partial_\mu \alpha_a) T_a U \Psi + U \partial_\mu \Psi$$

$$D_\mu \equiv \partial_\mu - ig T_a W_\mu^a$$

$$T_a W_\mu^a \longrightarrow T_a W_\mu'^a = \frac{1}{ig} (\partial_\mu U) U^{-1} + U T_a W_\mu^a U^{-1}$$