

ESTRUCTURA DE LA MATERIA 4

2DO CUATRIMESTRE 2025

CLASE 14

RODOLFO SASSOT

CLASE 14: Gauge no abeliano

de U(1) a SU(N)

$$\psi \longrightarrow \psi' = \psi e^{iq\alpha(x)}$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$D_\mu \equiv \partial_\mu - iq A_\mu$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\psi_i \longrightarrow \psi'_i = \sum_{j=1}^N U_{ij} \psi_j$$

$$\mathcal{L} = \sum_{i=1}^N \bar{\psi}_i(i\gamma^\mu \partial_\mu - m_i)\psi_i$$

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - M)\Psi \quad M_{N \times N} \quad \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix} \quad \Psi' = U \Psi \quad U_{N \times N}$$

$$U(\alpha_1, \alpha_2, \dots, \alpha_{N^2-1}) = e^{i(\alpha_1 T_1 + \alpha_2 T_2 + \dots + \alpha_{N^2-1} T_{N^2-1})}$$

$$U \in U(1)$$

$$\psi \longrightarrow \psi' = U \psi = e^{iq\alpha(x)} \psi$$

$$\partial_\mu \psi \longrightarrow \partial_\mu \psi' = iq(\partial_\mu \alpha) U \psi + U \partial_\mu \psi$$

$$D_\mu \equiv \partial_\mu - iq A_\mu$$

$$A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu \alpha(x)$$

$$U \in SU(N)$$

$$\Psi \longrightarrow \Psi' = U \Psi = e^{ig\alpha_a(x)T_a} \Psi$$

$$\partial_\mu \Psi \longrightarrow \partial_\mu \Psi' = ig(\partial_\mu \alpha_a) T_a U \Psi + U \partial_\mu \Psi$$

$$D_\mu \equiv \partial_\mu - ig T_a W_\mu^a$$

$$T_a W_\mu^a \longrightarrow T_a W_\mu'^a = \frac{1}{ig}(\partial_\mu U)U^{-1} + UT_a W_\mu^a U^{-1}$$

CLASE 14: Gauge no abelinao

de U(1) a SU(N)

$$A'_\mu = \frac{1}{iq}(\partial_\mu U)U^{-1} + UA_\mu U^{-1}$$

$$U = e^{ig \alpha^a(x) T^a}$$

$$U = 1 + ig \alpha^a T^a + \mathcal{O}(\alpha^2)$$

$$U^{-1} = 1 - ig \alpha^a T^a + \mathcal{O}(\alpha^2)$$

$$(W_\mu^a T^a)' = \frac{1}{ig} ig(\partial_\mu \alpha^a) T^a \cancel{U} U^{-1} + (1 + ig \alpha^b T^b) W_\mu^a T^a (1 - ig \alpha^c T^c)$$

$$= \partial_\mu \alpha^a T^a + W_\mu^a T^a + ig \alpha^b T^b T^a W_\mu^a - ig \alpha^c T^a T^c W_\mu^a + \mathcal{O}(\alpha^b \alpha^c)$$

$$= \partial_\mu \alpha^a T^a + W_\mu^a T^a + ig \alpha^b (T^b T^a - T^a T^b) W_\mu^a$$

$$[T^b, T^a] = if_{bac} T^c$$

$$= \partial_\mu \alpha^a T^a + W_\mu^a T^a - g \alpha^b f_{bac} T^c W_\mu^a$$

$$\begin{matrix} c \rightarrow a \\ a \rightarrow c \end{matrix}$$

$$= \partial_\mu \alpha^a T^a + W_\mu^a T^a - g \alpha^b f_{bca} T^a W_\mu^c$$



$$= \partial_\mu \alpha^a T^a + W_\mu^a T^a - g f_{abc} \alpha^b W_\mu^c T^a$$

$$W_\mu'^a = \partial_\mu \alpha^a + W_\mu^a - g f_{abc} \alpha^b W_\mu^c$$

$$\sim A'_\mu = \partial_\mu \alpha + A_\mu$$

CLASE 14: Gauge no abeliano

de U(1) a SU(N)

$$[D_\mu, D_\nu] \psi = -iq F_{\mu\nu} \psi \quad G_{\mu\nu} \equiv \frac{1}{-ig} [D_\mu, D_\nu]$$



C.N. Yang R. Mills

$$D_\mu D_\nu \Psi = \partial_\mu \partial_\nu \Psi - ig W_\mu^a T^a \partial_\nu \Psi - ig (\partial_\mu W_\nu^a) T^a \Psi - ig W_\nu^a T^a (\partial_\mu \Psi) - g^2 W_\mu^a T^a W_\nu^b T^b \Psi$$

$$D_\nu D_\mu \Psi = \partial_\nu \partial_\mu \Psi - ig W_\nu^a T^a \partial_\mu \Psi - ig (\partial_\nu W_\mu^a) T^a \Psi - ig W_\mu^a T^a (\partial_\nu \Psi) - g^2 W_\nu^a T^a W_\mu^b T^b \Psi$$

$$\begin{aligned} D_\mu D_\nu - D_\nu D_\mu = & -ig(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) T^a - g^2 W_\mu^a W_\nu^b [T^a, T^b] \\ & -ig^2 W_\mu^a W_\nu^b f_{abc} T^c \quad a \rightarrow b \quad b \rightarrow c \quad c \rightarrow a \\ & -ig^2 W_\mu^b W_\nu^c f_{bca} T^a \quad \text{circular arrow} \\ & -ig^2 W_\mu^c W_\nu^a f_{cab} T^b \end{aligned}$$

$$G_{\mu\nu} = \left[(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) + g W_\mu^b W_\nu^c f_{abc} \right] T^a = G_{\mu\nu}^a T^a$$

$$G'_{\mu\nu} = U G_{\mu\nu} U^{-1} \quad (G_{\mu\nu} G^{\mu\nu})' = U G_{\mu\nu} \cancel{U^{-1}} U G^{\mu\nu} U^{-1} \quad \text{Tr}[(G_{\mu\nu} G^{\mu\nu})'] = \text{Tr}[G_{\mu\nu} G^{\mu\nu}]$$

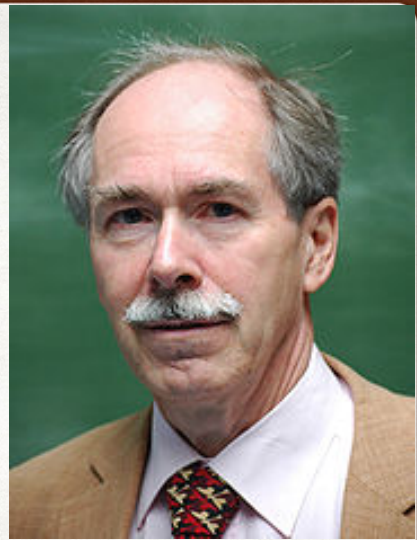
$$\text{Tr}[G_{\mu\nu} G^{\mu\nu}] = \frac{1}{2} G_{\mu\nu}^a G_a^{\mu\nu}$$

$$\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu \partial_\mu - M)\Psi + g(\bar{\Psi}\gamma^\mu T^a \Psi)W_\mu^a - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

CLASE 14: Gauge no abeliano

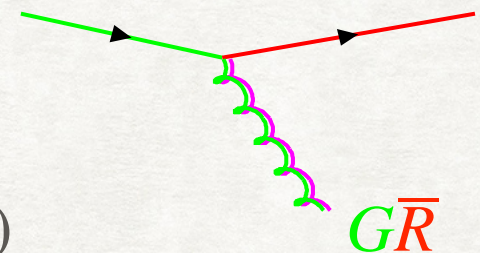
QCD

$$\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu \partial_\mu - M)\Psi + g(\bar{\Psi}\gamma^\mu T^a \Psi)W_\mu^a - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} \quad \Psi = \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix}$$



G. 't Hooft

$$\bar{\Psi}\gamma^\mu \tilde{T}^a \Psi = (\bar{\psi}_R, \bar{\psi}_B, \bar{\psi}_G) \gamma^\mu \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix} = \bar{\psi}_R \gamma^\mu \frac{1}{2} \psi_G$$



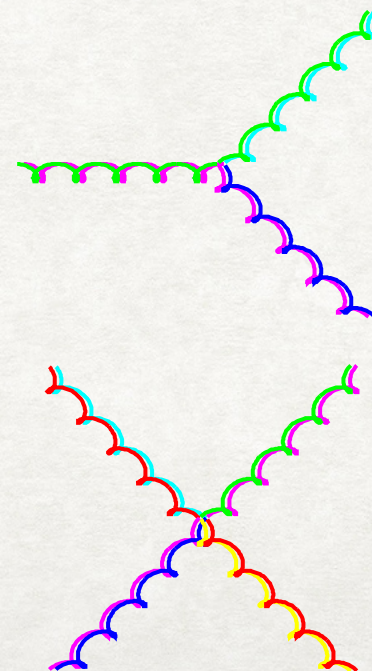
$$3 \otimes \bar{3} = 8 + 1 \quad 1 \sim R\bar{R} + B\bar{B} + G\bar{G} \quad \tilde{T} \sim \mathbb{1} \quad \notin SU(3)$$

$$G_{\mu\nu}^a G_a^{\mu\nu} = [(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) + gW_\mu^b W_\nu^c f_{abc}] [(\partial^\mu W_a^\nu - \partial^\nu W_a^\mu) + gW_b^\mu W_c^\nu f_{abc}]$$

$$= (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) (\partial^\mu W_a^\nu - \partial^\nu W_a^\mu)$$

$$+ gW_\mu^b W_\nu^c f_{abc} (\partial^\mu W_a^\nu - \partial^\nu W_a^\mu)$$

$$+ g^2 W_\mu^b W_\nu^c f_{abc} W_b^\mu W_c^\nu f_{abc}$$

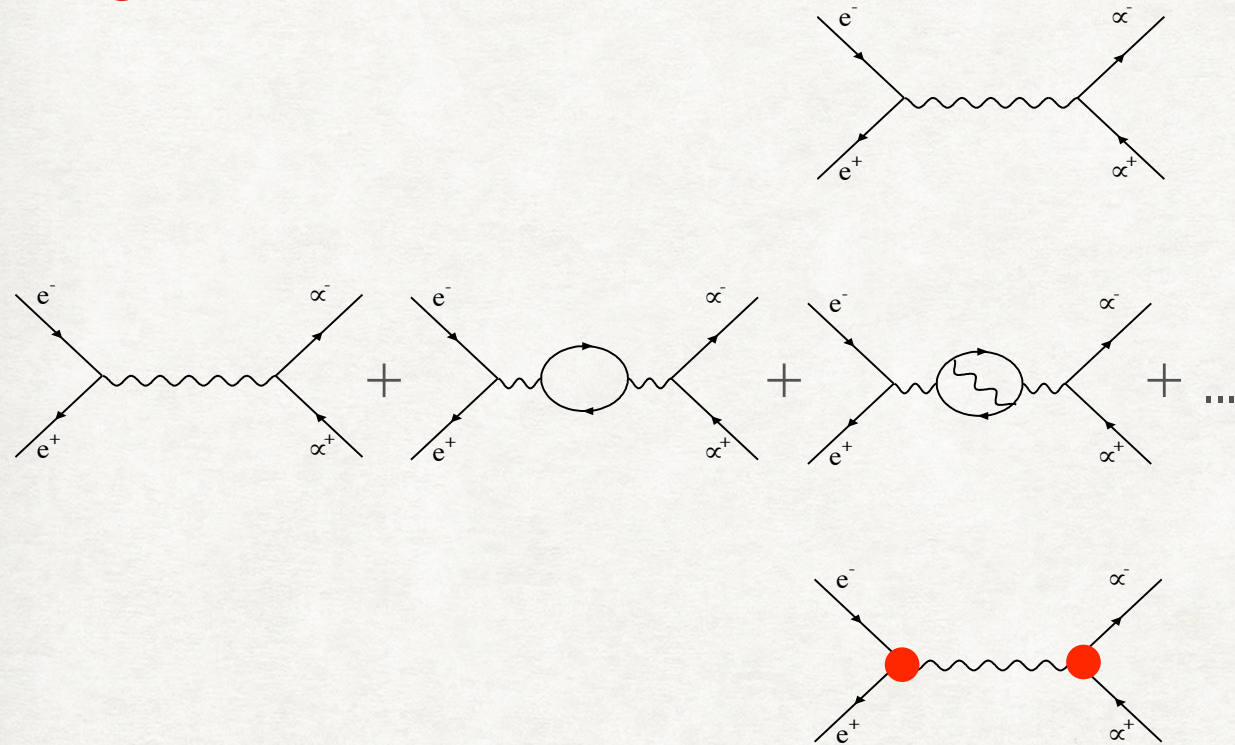


$\sim g$

$\sim g^2$

CLASE 14: Gauge no abeliano

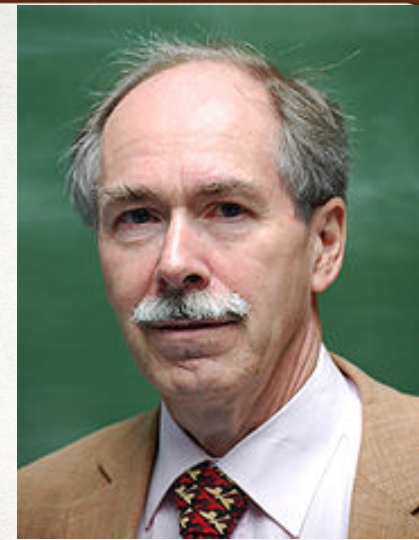
QCD



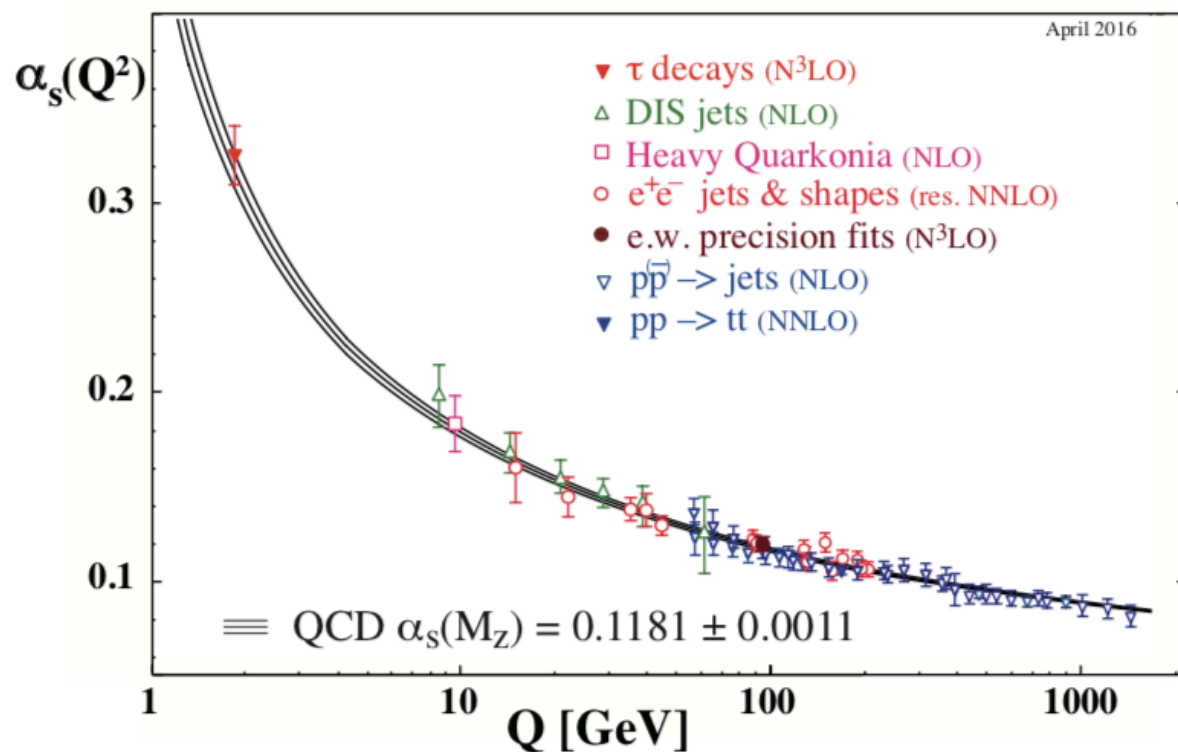
$$\sigma = \frac{4\pi}{3s} \alpha_0^2$$

$$\sigma = \frac{4\pi}{3s} \alpha_0^2 (1 + \alpha_0 \sigma^{(1)}(s) + \alpha_0^2 \sigma^{(2)}(s) + \dots)$$

$$\sigma = \frac{4\pi}{3s} \alpha_{\text{eff}}^2(s)$$



G. 't Hooft



$$\alpha_{\text{eff}}(s) = \frac{1}{\beta_0 \log\left(\frac{s}{\Lambda_{\text{QCD}}^2}\right)}$$

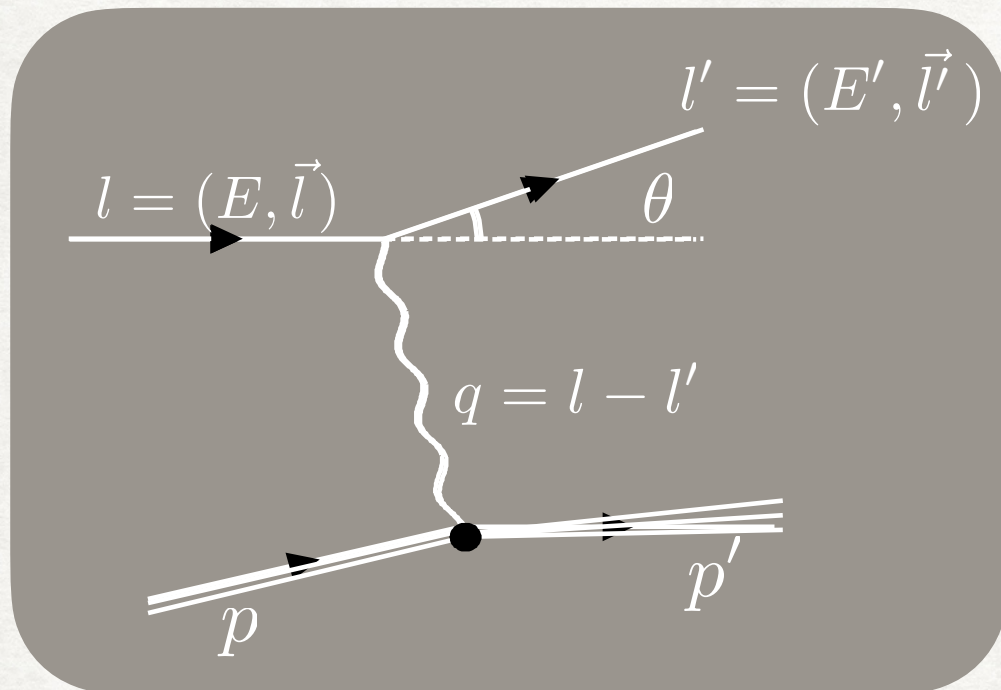
$$\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$$

D. Gross, F. Wilczek,
D. Politzer



CLASE 14: Gauge no abeliano

QCD: libertad asintótica



l^μ, p^μ fijos

l'^μ, p'^μ 8 variables

-4 conservación E-p

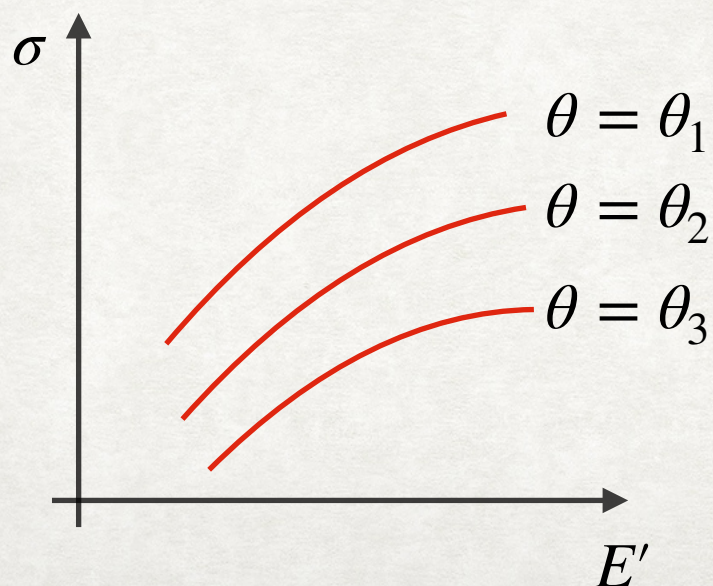
-1 $l^2 = m_e^2$

~~-1 $p^2 = m_p^2$~~

-1 en c.m.s. $l - p$, simetría rotaciones

~~1 variable independiente (θ o E')~~

2 variables independientes (θ y E')



$$Q^2 = 4EE' \sin(\theta/2)$$

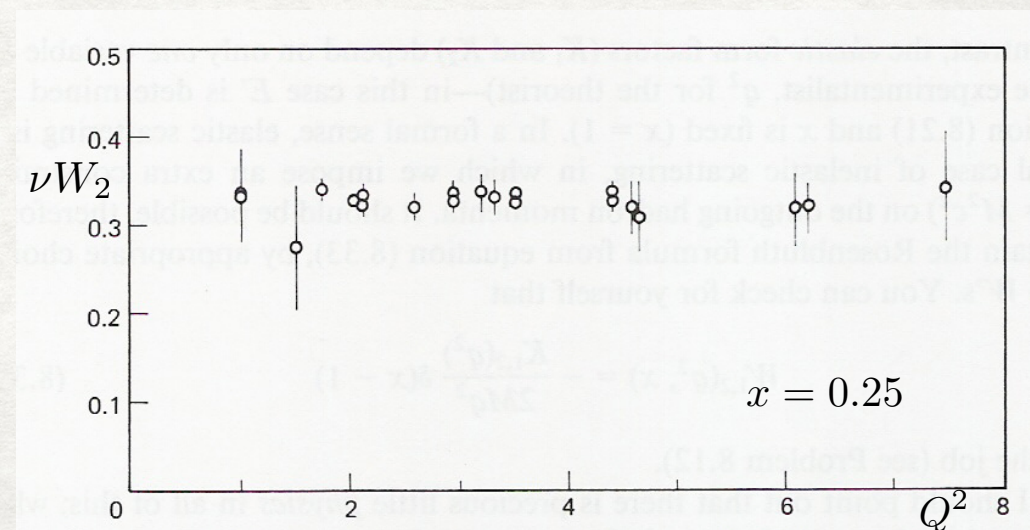
$$x = \frac{Q^2}{2M(E - E')}$$



R.P. Feynman



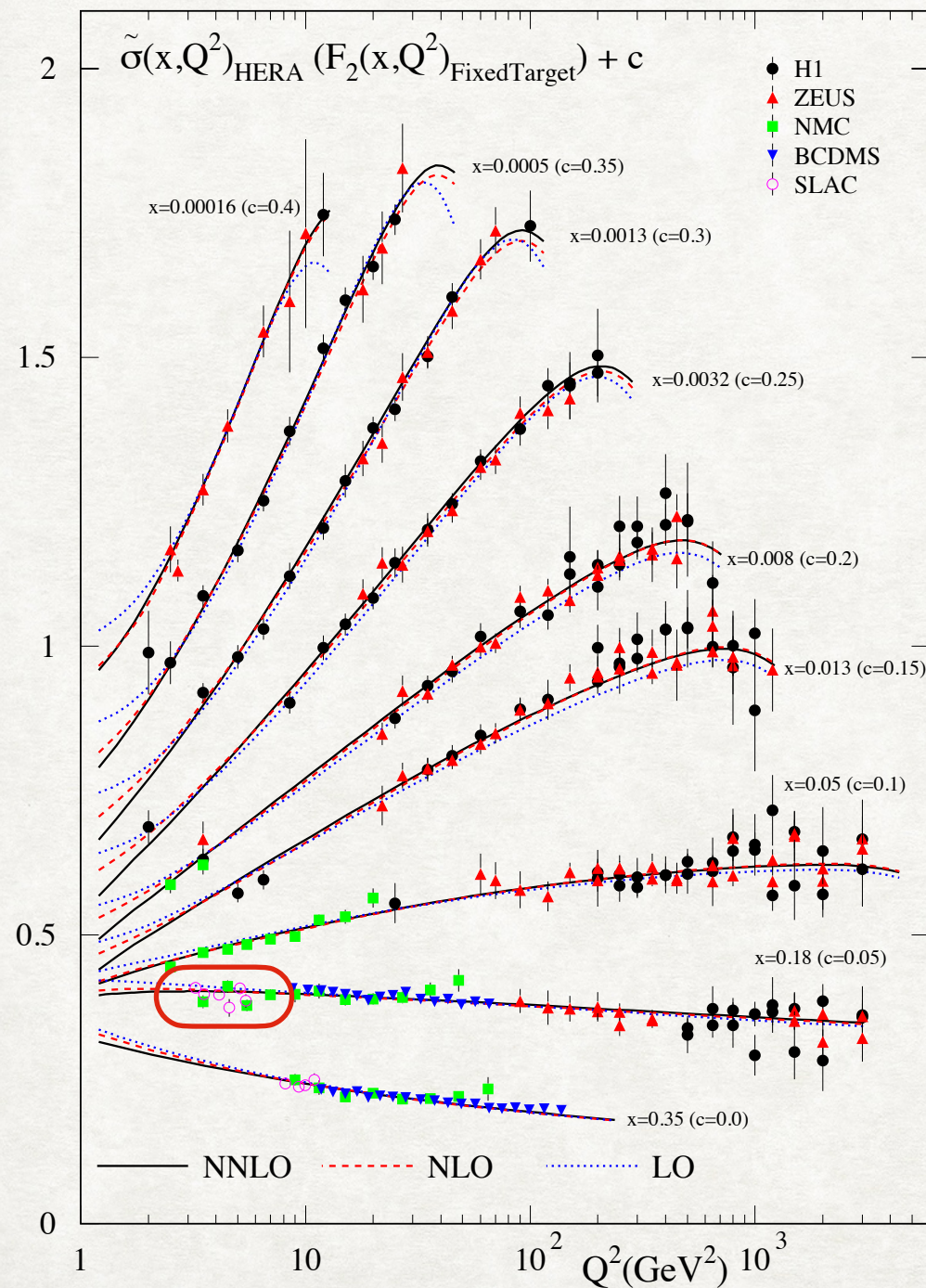
J. D. Bjorken



CLASE 14: Gauge no abeliano

QCD: libertad asintótica

MSTW 2008



R.P. Feynman

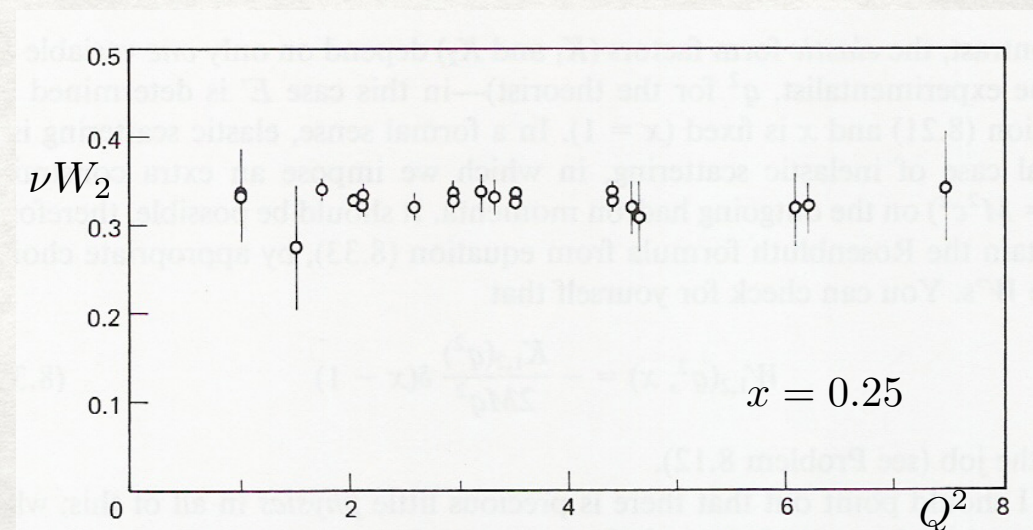
J. D. Bjorken

$$\sigma_{ep} \sim \sum_q \int dx e_q^2 \sigma_{eq} f_q(x)$$

$$f_q(x) \longrightarrow f_q(x, Q^2)$$

$$Q^2 = 4EE' \sin(\theta/2)$$

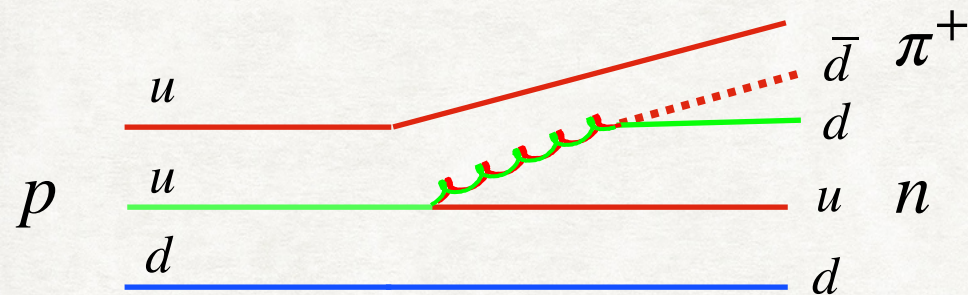
$$x = \frac{Q^2}{2M(E - E')}$$



CLASE 14: Gauge no abeliano

QCD: confinamiento

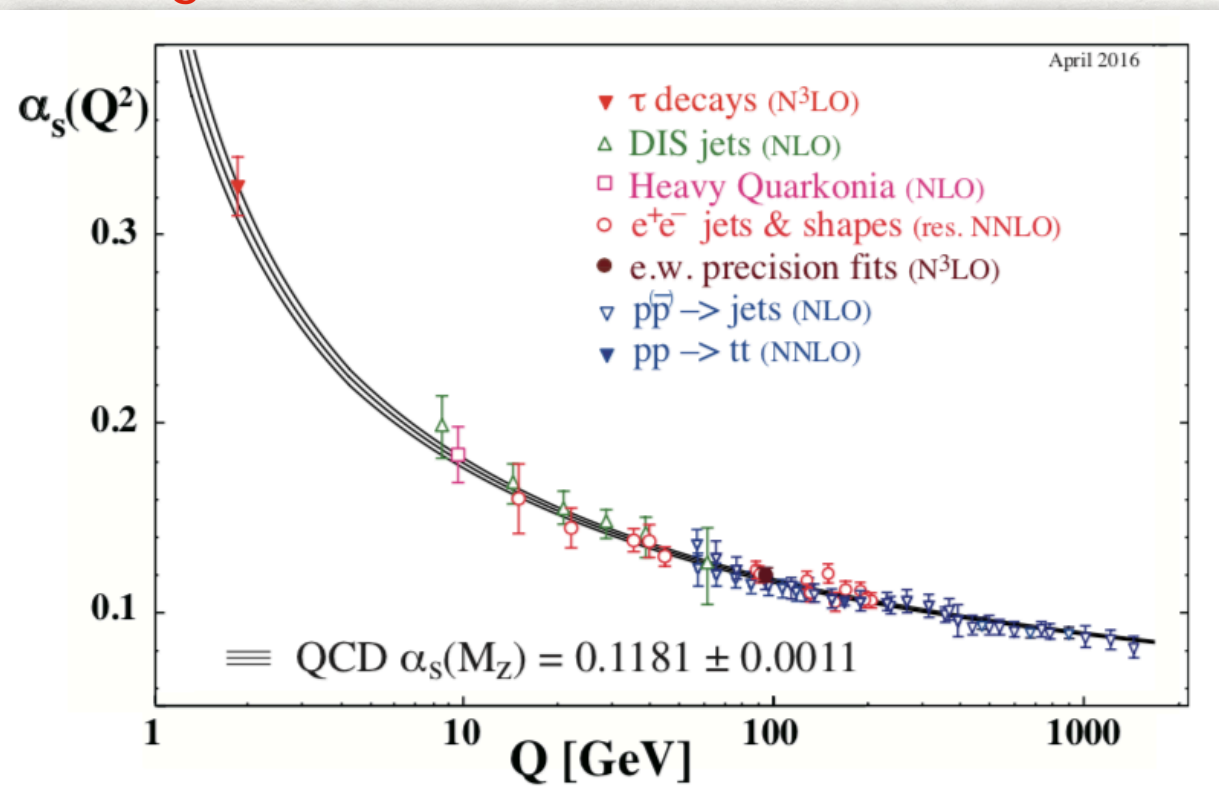
los estados "físicos" no pueden tener color explícito



los detalles del mecanismo de confinamiento no son perturbativos: factorización

distancias largas ← distancias cortas

lattice QCD / holografía / modelos efectivos



$$\alpha_{eff}(s) = \frac{1}{\beta_0 \log\left(\frac{s}{\Lambda_{QCD}^2}\right)}$$

$$\Lambda_{QCD} \sim 200 \text{ MeV}$$

$$200 \text{ MeV} \sim 1 \text{ fm}$$