ESTRUCTURA DE LA MATERIA 4

2DO CUATRIMESTRE 2025

CLASE 4

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CLASE 4: Modelo de Quarks

Repaso:

$SU(2)$: matrices unitarias de 2x2 y det=1 $\quad U(\theta_i) = e^{-i\,\theta_i J_i} \quad i = 1,2,3$

$$\begin{array}{cc} |p> & |n> \\ |\uparrow> & |\downarrow> \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{array} \equiv 2$$

$J_i = \dfrac{1}{2}\sigma_i$ (σ_i matrices de Pauli) $\quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad [\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\,\sigma_k$

$$2 \otimes 2 = 3_S + 1_A$$

$$2 \otimes 2 \otimes 2 = 4_S + 2_{MS} + 2_{MA}$$

$$J_1^{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad J_2^{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad J_3^{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$[J_i, J_j] = i\epsilon_{ijk}\,J_k$$

$SU(3)$: matrices unitarias de 3x3 y det=1 $\quad U(\alpha_a) = e^{-i\alpha_a J_a} \quad a = 1,\ldots,8$

$$\begin{array}{ccc} |u> & |d> & |s> \\ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{array} \equiv 3$$

$J_a = \dfrac{1}{2}\lambda_a$ (λ_a matrices de Gell-Mann)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\frac{1}{\sqrt{3}}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$[J_a, J_b] = if_{abc}\,J_c$$

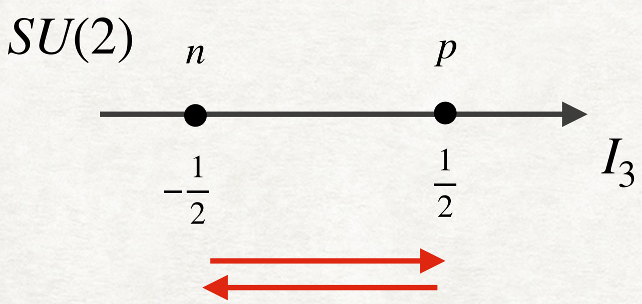
$$f_{123} = 1 \qquad f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

$$f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{378} = \frac{1}{2}$$

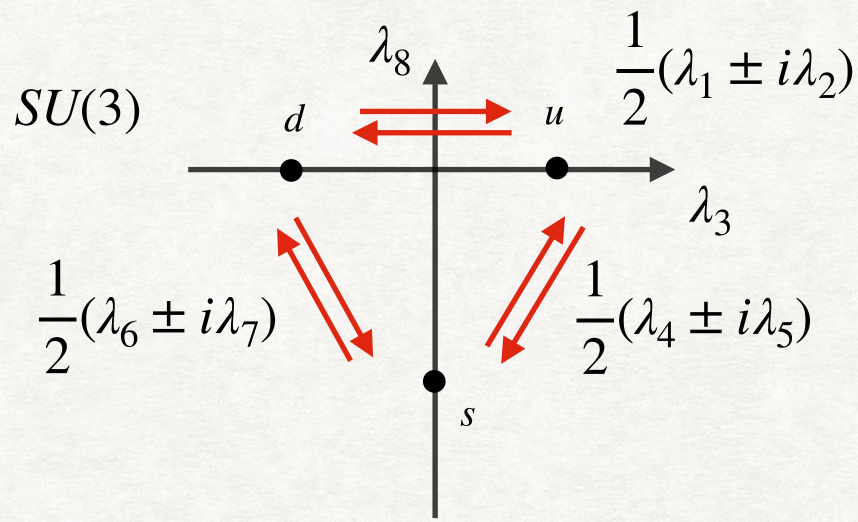
CLASE 4: Modelo de Quarks

Recapitulación:

grupo de interés $SU(3)$: tres subgrupos $SU(2)$ que se muerden la cola



$$\sigma_\pm \equiv \frac{1}{2}(\sigma_1 \pm i\sigma_2)$$



$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \frac{1}{\sqrt{3}}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$2 \otimes 2 = 3_S + 1_A \qquad |uu> \quad \frac{1}{\sqrt{2}}|ud + du> \quad |dd>$$

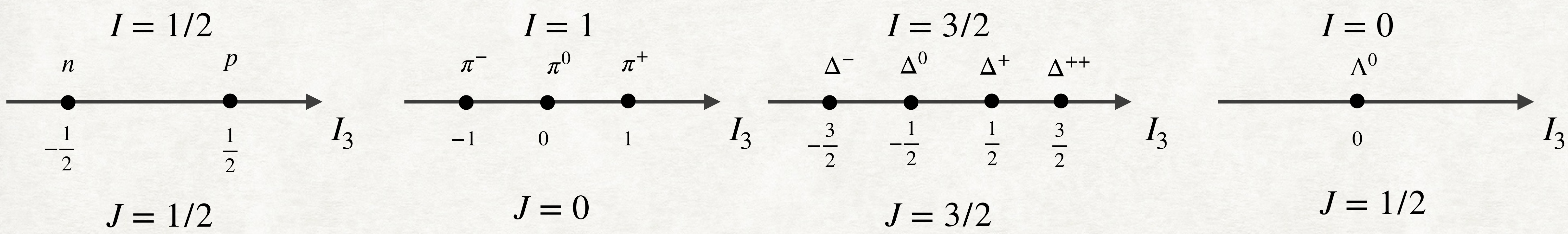
$$3 \otimes 3 = 6_S + 3_A \qquad \frac{1}{\sqrt{2}}|sd + ds> \quad |ss> \quad \frac{1}{\sqrt{2}}|us + su>$$

$$2 \otimes 2 \otimes 2 = 4_S + 2_{MS} + 2_{MA}$$

$$3 \otimes 3 \otimes 3 = 10_S + 8_{MS} + 8_{MA} + 1_A$$

CLASE 4: Modelo de Quarks

isospín, carga eléctrica y extrañeza:

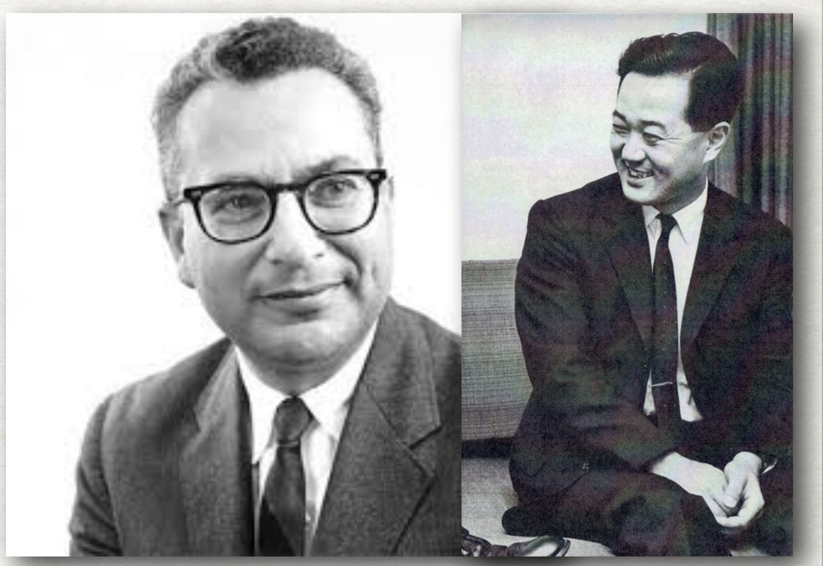


$$Q = e\left(I_3 + \frac{1}{2}\right) \qquad n, p, \Delta$$

$$Q = e\left(I_3 + \frac{B}{2}\right) \qquad \begin{array}{l} B = 1 \text{ bariones} \\ B = 0 \text{ mesones} \end{array}$$

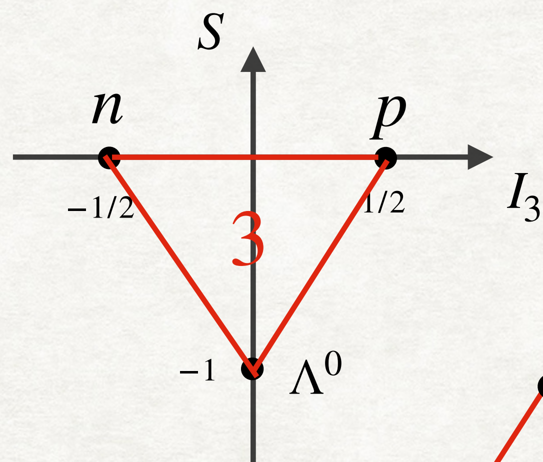
$$Q = e\left(I_3 + \frac{B+S}{2}\right) \qquad \begin{array}{ll} S = 1 & K^+ \\ S = 0 & K^0,\ \Lambda^0 \end{array}$$

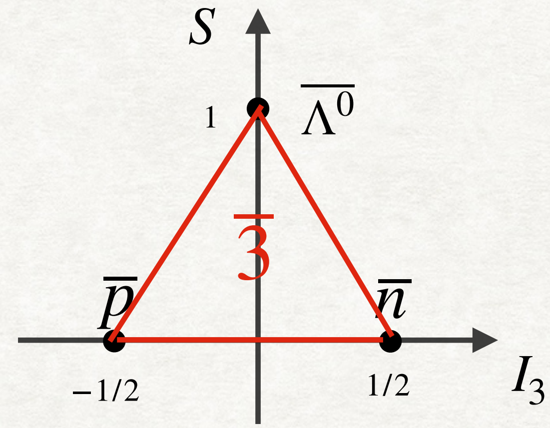
Gell-Mann Nishijima



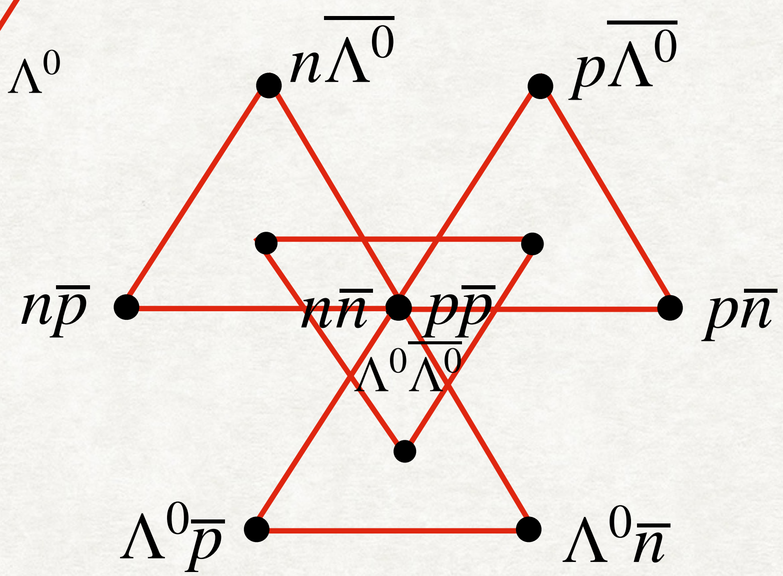
modelo SU(3) de Sakata (1956):

$$\begin{pmatrix} p \\ n \\ \Lambda^0 \end{pmatrix}$$





composición: $3 \otimes \bar{3} = 8 + 1$



$B = 0$ mesones

$S = 0$ $\quad p\bar{n} = |1,1>_I \quad \pi^+$

$p\bar{p} + n\bar{n} = |1,0>_I \quad \pi^0$

$p\bar{n} = |1,-1>_I \quad \pi^-$

$3 \otimes 3 \otimes \bar{3}$ $\quad B = 1$ bariones

$pn\overline{\Lambda^0}$ $\quad S = 1$ $\quad$ no existen!

modelo de quarks (1964): $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$

$$3 \otimes 3 \otimes 3 = 10_S + 8_{MS} + 8_{MA} + 1_A$$

$$3 \otimes \bar{3} = 8_S + 1_A$$

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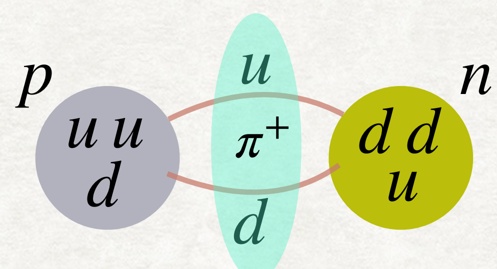
modelo de quarks (1964):

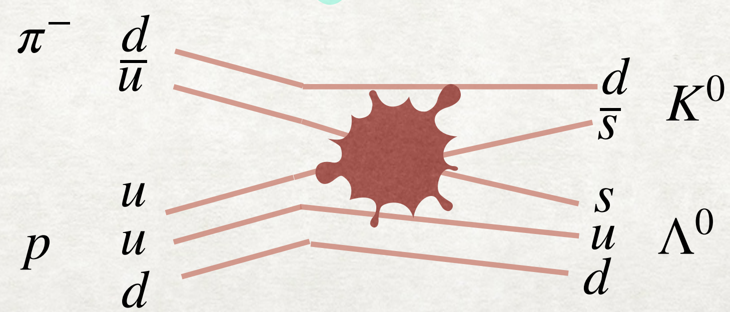
los quarks: tienen carga eléctrica fraccionaria
no existen de manera aislada
sólo se combinan de a tres o como $q\bar{q}$

	I_3	S	Q	B	J
u	$1/2$	0	$2/3$	$1/3$	$1/2$
d	$-1/2$	0	$-1/3$	$1/3$	$1/2$
s	0	-1	$-1/3$	$1/3$	$1/2$

uuu	$3/2$	0	2	1	Δ^{++}	$I = 3/2$	
uud	$1/2$	0	1	1	Δ^{+}, p		$SU(2)_{u-d}$
udd	$-1/2$	0	0	1	Δ^{0}, n		
ddd	$-3/2$	0	-1	1	Δ^{-}	$I = 3/2$	
dds	-1	-1	-1	1	Σ^{*-}, Σ^{-}		$SU(2)_{d-s}$
dss	$-1/2$	-2	-1	1	Ξ^{*-}, Ξ^{-}		
sss	0	-3	-1	1	Ω^{-}	$I = 3/2$	
ssu	$1/2$	-2	0	1	Ξ^{*0}, Ξ^{0}		$SU(2)_{u-s}$
suu	1	-1	1	1	Σ^{*+}, Σ^{+}		
uds	0	-1	0	1	Λ^{0}, Σ^{0}		

	I_3	S	Q	B	
$u\bar{d}$	1	0	1	0	π^+
$d\bar{u}$	-1	0	-1	0	π^-
$u\bar{u}$, $d\bar{d}$	0	0	0	0	π^0





CLASE 4: Modelo de Quarks.

funciones de onda de SU(3):

partículas spin 1/2 requieren simetría definida ante permutaciones

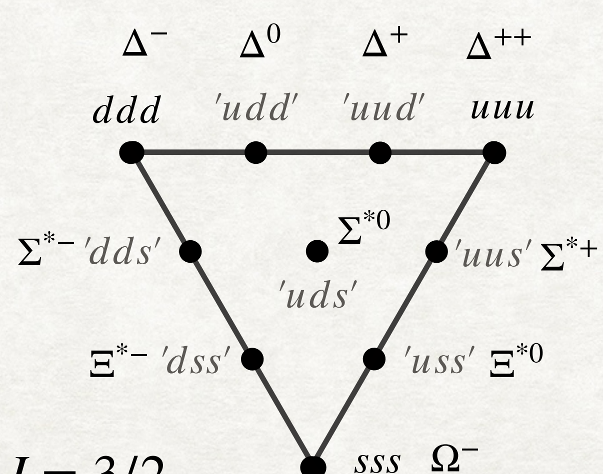
$SU(2)$ $\quad 2\otimes 2 = 3_S + 1_A$

	S	A
$u\ u$	uu	
$u\ d$ / $d\ u$	$\frac{1}{\sqrt{2}}(ud+du)$	$\frac{1}{\sqrt{2}}(ud-du)$
$d\ d$	dd	
$s\ d$ / $d\ s$	$\frac{1}{\sqrt{2}}(sd+ds)$	$\frac{1}{\sqrt{2}}(sd-ds)$
$s\ s$	ss	
$s\ u$ / $u\ s$	$\frac{1}{\sqrt{2}}(us+su)$	$\frac{1}{\sqrt{2}}(us-su)$

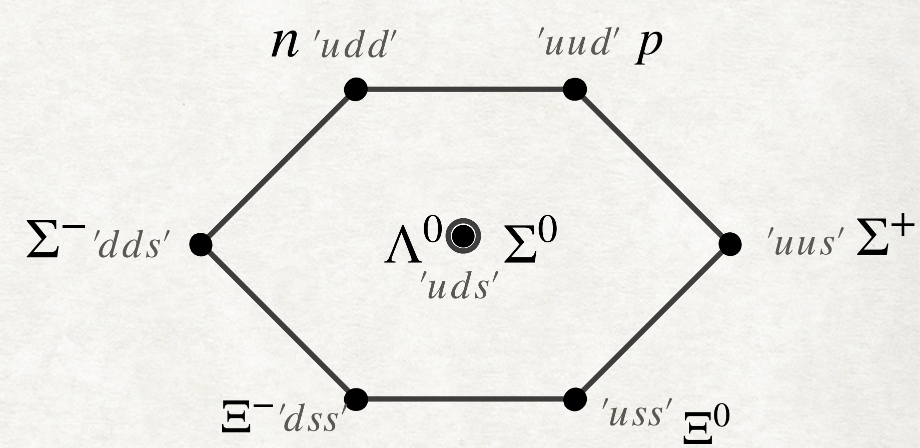
$2\otimes 2\otimes 2 \;= 4_S + 2_{MA} + 2_{MS}$

	S	MA	MS	
$u\ u\ u$	uuu			$I_3 = \frac{3}{2}$
$u\ u\ d$ / $u\ d\ u$ / $d\ u\ u$	$\frac{1}{\sqrt{3}}(uud+udu+duu)$	$\frac{1}{\sqrt{2}}(ud-du)u$	$\frac{1}{\sqrt{3}}[\frac{(ud+du)u}{\sqrt{2}} - \sqrt{2}uud]$	$I_3 = \frac{1}{2}$
$d\ d\ u$ / $d\ u\ d$ / $u\ d\ d$	$\frac{1}{\sqrt{3}}(ddu+dud+udd)$	$\frac{1}{\sqrt{2}}(ud-du)d$	$\frac{-1}{\sqrt{3}}[\frac{(ud+du)d}{\sqrt{2}} - \sqrt{2}ddu]$	$I_3 = -\frac{1}{2}$
$d\ d\ d$	ddd			$I_3 = -\frac{3}{2}$

$SU(3)$ $\quad 3\otimes 3 = 6_S + \tilde{3}_A$ $\qquad 3\otimes 3\otimes 3 = 10_S + 8_{MS} + 8_{MA} + 1_A$



$J = 3/2$



$$|\chi^{3/2}_{SU(2)}> \quad \begin{aligned} &|3/2> = |\uparrow\uparrow\uparrow> \\ &|1/2> = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \\ &|-1/2> = \frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow) \\ &|-3/2> = |\downarrow\downarrow\downarrow> \end{aligned}$$

$$|\psi> = |\phi^{3/2}_{SU(3)}> |\chi^{3/2}_{SU(2)}>$$

$$|\psi>_S = \frac{1}{\sqrt{2}}[|\phi_{MS}>|\chi_{MS}> + |\phi_{MA}>|\chi_{MA}>]$$

$$|\psi>_A = \frac{1}{\sqrt{2}}[|\phi_{MS}>|\chi_{MA}> - |\phi_{MA}>|\chi_{MS}>]$$