ESTRUCTURA DE LA MATERIA 4

2DO CUATRIMESTRE 2025

CLASE 5

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CLASE 5: Funciones de onda SU(3) y Color.

funciones de onda de SU(3):

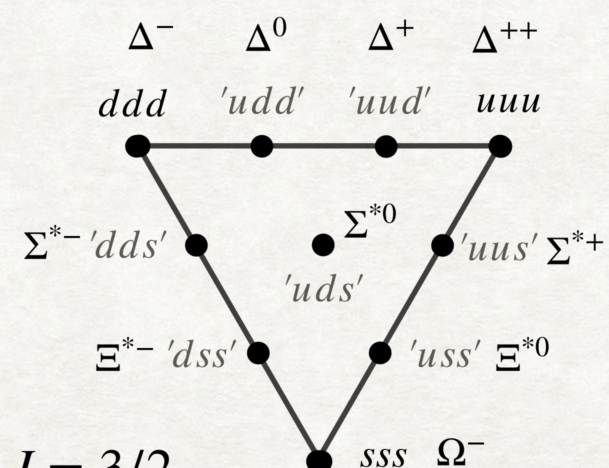
de la clase anterior: partículas spin 1/2 requieren simetría definida ante permutaciones

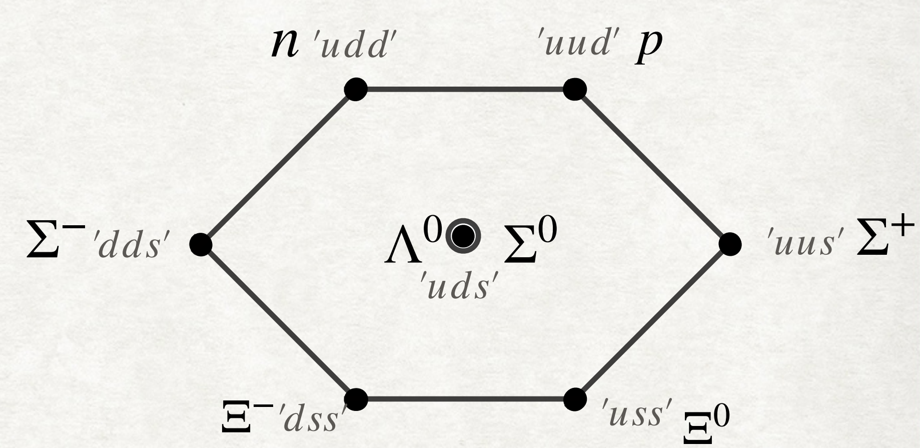
$SU(2) \qquad 2\otimes 2 = 3_S + 1_A \qquad\qquad 2\otimes 2\otimes 2 = 4_S + 2_{MA} + 2_{MS}$

	S	A
$u\ u$	uu	
$u\ d$ $d\ u$	$\frac{1}{\sqrt{2}}(ud+du)$	$\frac{1}{\sqrt{2}}(ud-du)$
$d\ d$	dd	
$s\ d$ $d\ s$	$\frac{1}{\sqrt{2}}(sd+ds)$	$\frac{1}{\sqrt{2}}(sd-ds)$
$s\ s$	ss	
$s\ u$ $u\ s$	$\frac{1}{\sqrt{2}}(us+su)$	$\frac{1}{\sqrt{2}}(us-su)$

	S	MA	MS	
$u\ u\ u$	uuu			$I_3=\frac{3}{2}$
$u\ u\ d$ $u\ d\ u$ $d\ u\ u$	$\frac{1}{\sqrt{3}}(uud+udu+duu)$	$\frac{1}{\sqrt{2}}(ud-du)u$	$\frac{1}{\sqrt{3}}[\frac{(ud+du)u}{\sqrt{2}}-\sqrt{2}uud]$	$I_3=\frac{1}{2}$
$d\ d\ u$ $d\ u\ d$ $u\ d\ d$	$\frac{1}{\sqrt{3}}(ddu+dud+udd)$	$\frac{1}{\sqrt{2}}(ud-du)d$	$\frac{-1}{\sqrt{3}}[\frac{(ud+du)d}{\sqrt{2}}-\sqrt{2}ddu]$	$I_3=-\frac{1}{2}$
$d\ d\ d$	ddd			$I_3=-\frac{3}{2}$

$SU(3) \qquad 3\otimes 3 = 6_S + \tilde{3}_A \qquad\qquad 3\otimes 3\otimes 3 = 10_S + 8_{MS} + 8_{MA} + 1_A$





$$|\chi^{3/2}_{SU(2)}> \qquad \begin{aligned} |3/2> &= |\uparrow\uparrow\uparrow> \\ |1/2> &= \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow) \\ |-1/2> &= \frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow+\downarrow\uparrow\downarrow+\uparrow\downarrow\downarrow) \\ |-3/2> &= |\downarrow\downarrow\downarrow> \end{aligned}$$

$J = 3/2$

$$|\psi> = |\phi^{3/2}_{SU(3)}>|\chi^{3/2}_{SU(2)}>$$

$$|\psi>_S = \frac{1}{\sqrt{2}}[|\phi_{MS}>|\chi_{MS}> + |\phi_{MA}>|\chi_{MA}>]$$

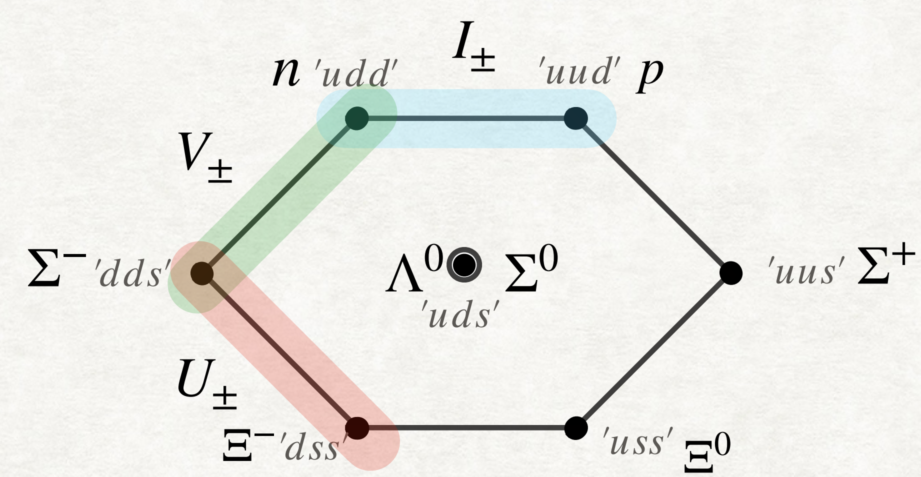
$$|\psi>_A = \frac{1}{\sqrt{2}}[|\phi_{MS}>|\chi_{MA}> - |\phi_{MA}>|\chi_{MS}>]$$

CLASE 5:

funciones de onda de los octetes SU(3):

$$\sigma_\pm \equiv \frac{1}{2}(\sigma_1 \pm i\sigma_2) \qquad \sigma_+|d> = |u> \qquad \sigma_-|u> = |d>$$

MA	MS	
$\frac{1}{\sqrt{2}}(ud-du)u$	$\frac{1}{\sqrt{3}}[\frac{(ud+du)u}{\sqrt{2}} - \sqrt{2}uud]$	$I_3 = \frac{1}{2}$
$\frac{1}{\sqrt{2}}(ud-du)d$	$\frac{-1}{\sqrt{3}}[\frac{(ud+du)d}{\sqrt{2}} - \sqrt{2}ddu]$	$I_3 = -\frac{1}{2}$



$$\sigma_\pm^{2\otimes2\otimes2} \equiv \sigma_\pm \mathbb{1}\mathbb{1} + \mathbb{1}\sigma_\pm\mathbb{1} + \mathbb{1}\mathbb{1}\sigma_\pm \qquad \sigma_-^{2\otimes2\otimes2}|\frac{1}{2},\frac{1}{2}>_{MS} = |\frac{1}{2},-\frac{1}{2}>_{MS}$$

$$\sigma_-^{2\otimes2\otimes2}|\frac{1}{2},\frac{1}{2}>_{MS} = \sigma_-^{2\otimes2\otimes2}\frac{1}{\sqrt{6}}[udu + duu - 2uud] = \frac{1}{\sqrt{6}}[ddu - 2dud + ddu - 2udd + udd + dud]$$

$$= \frac{1}{\sqrt{6}}[-dud - udd + 2ddu] = \frac{-1}{\sqrt{6}}[(du+ud)d - 2ddu]$$

$$SU(2) \to SU(3) \qquad I_\pm \equiv \frac{1}{2}(\lambda_1 \pm i\lambda_2) = \begin{pmatrix} \sigma_\pm^{11} & \sigma_\pm^{12} & 0 \\ \sigma_\pm^{21} & \sigma_\pm^{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad u \leftrightarrow d \qquad V_\pm \equiv \frac{1}{2}(\lambda_4 \pm i\lambda_5) = \begin{pmatrix} \sigma_\pm^{11} & 0 & \sigma_\pm^{12} \\ 0 & 0 & 0 \\ \sigma_\pm^{21} & 0 & \sigma_\pm^{22} \end{pmatrix} \quad u \leftrightarrow s$$

$$U_\pm \equiv \frac{1}{2}(\lambda_6 \pm i\lambda_7) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_\pm^{11} & \sigma_\pm^{12} \\ 0 & \sigma_\pm^{21} & \sigma_\pm^{22} \end{pmatrix} \quad d \leftrightarrow s$$

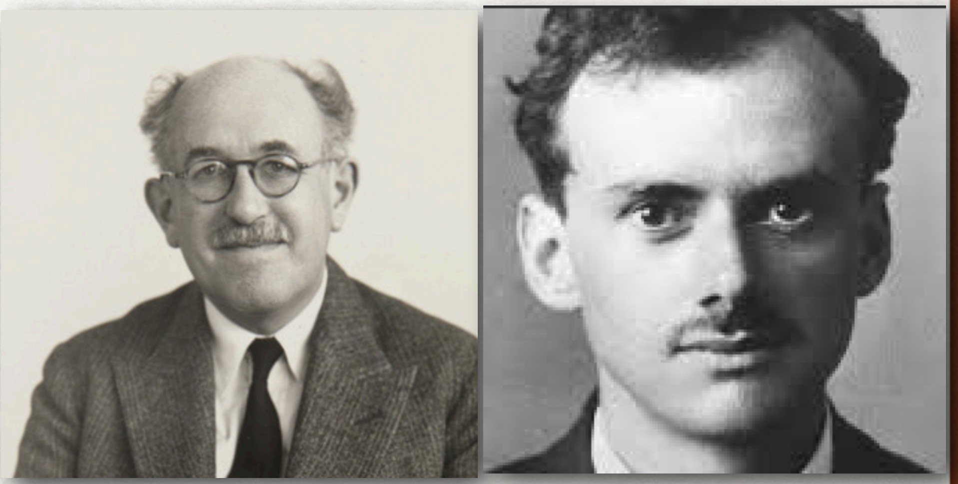
CLASE 5:

momentos magnéticos:

propiedad bien característica partículas ~*determina como de acopla a* $\vec{B}$

definición clásica μ ~ corriente en una espira, extiende en términos de $\vec{L}$

ec. de Dirac predice correctamente el μ para e^-, μ^- (e^+, μ^+) (*con nueve cifras significativas!*)



$$\mu = \frac{e}{2m} \qquad (\hbar = c = 1)$$

pero falla para $p, n, \ldots$ momento magnético "anómalo"
(Otto Stern 1933)

$$\mu_q = Q_q \frac{e}{2m_q} \qquad \mu_p = \sum_q < p\uparrow | \mu_q \sigma_3 | p\uparrow > \qquad \mu_n = \sum_q < n\uparrow | \mu_q \sigma_3 | n\uparrow >$$

$$m_u = ? \quad m_d = ? \qquad m_u \simeq m_d \qquad \left.\frac{\mu_p}{\mu_n}\right|_{\psi_S} = -\frac{3}{2} \qquad -1.45989806(34)$$

$$\left.\frac{\mu_n}{\mu_p}\right|_{\psi_A} = \frac{1}{2}$$

CLASE 5:

color:

$|\psi> = |\phi^{3/2}_{SU(3)}> |\chi^{3/2}_{SU(2)}> |\psi_c>$

decuplete incluye $I = 3/2$, y todas ellas tiene $J = 3/2$

$|\psi> = \frac{1}{\sqrt{2}}[|\phi_{MS}> |\chi_{MS}> + |\phi_{MA}> |\chi_{MA}>]|\psi_c>$

valor medido de μ_p/μ_n requiere $|\psi>_S$

quién antisimetriza la $|\psi>$ de los hadrones?

$3 \otimes 3 \otimes 3 = 10_S + 8_{MS} + 8_{MA} + 1_A$

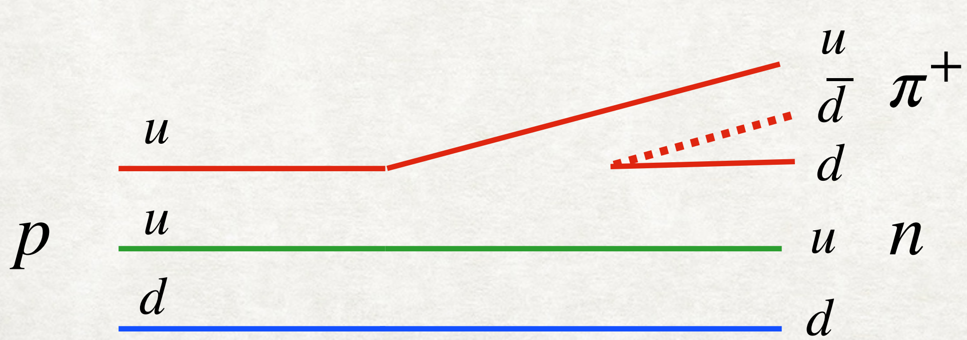
$3 \otimes \overline{3} = 8_S + 1_A$

$\begin{pmatrix} R \\ B \\ G \end{pmatrix}$

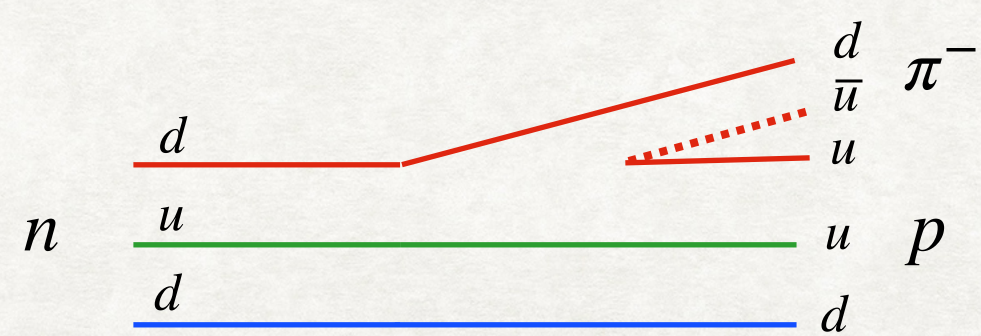
$|\psi_c> = \frac{1}{\sqrt{6}}[RGB - RBG + GBR - GRB + BRG - BGR]$

singlete de color $\equiv$ sin color

las interacciones fuertes “aborrecen” el color explícito



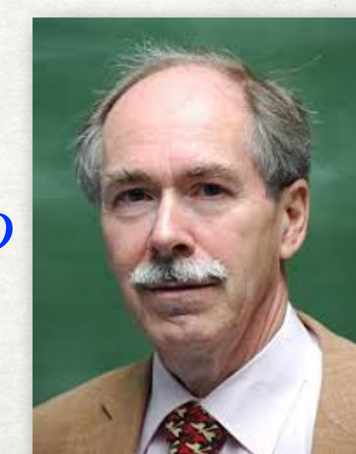
$p + p \rightarrow d + \pi^+$



$n + p \rightarrow d + \pi^-$

QCD

G. 't Hooft

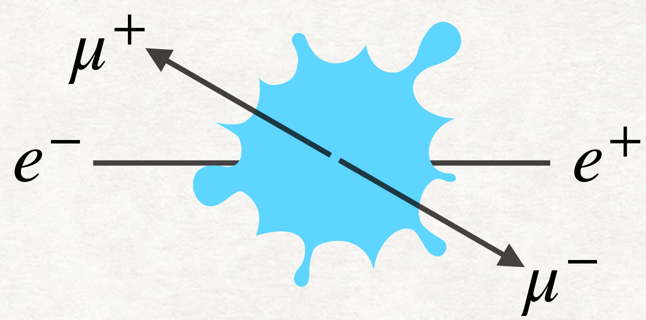


CLASE 5:

evidencias del color:

además el indicio estadístico, y anteriores a la formulación de QCD

$$e^- + e^+ \to \mu^- + \mu^+$$



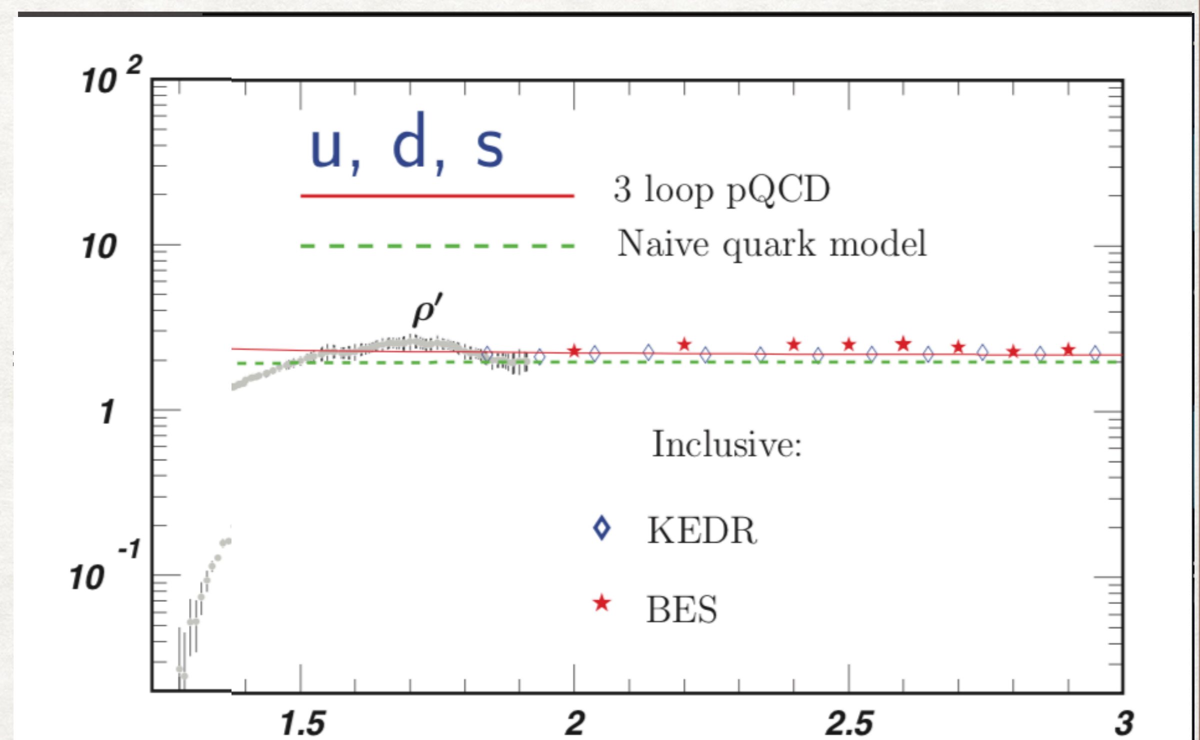
$$\sigma = \frac{\pi\alpha^2}{3E^2} \qquad (m_e = m_\mu = 0)$$

$$e^- + e^+ \to u + \bar{u} \ \to hadrones$$
$$\to d + \bar{d}$$
$$\to s + \bar{s}$$

$$\sigma = \frac{\pi\alpha^2}{3E^2} Q_q^2$$

$$\frac{\sigma(e^+ + e^- \to hadrones)}{\sigma(e^+ + e^- \to \mu^+ + \mu^-)} = \sum_q Q_q^2 = 4/9 + 1/9 + 1/9 = 2/3$$

$$= \sum_{colores} \sum_q Q_q^2 = 3(4/9 + 1/9 + 1/9)$$



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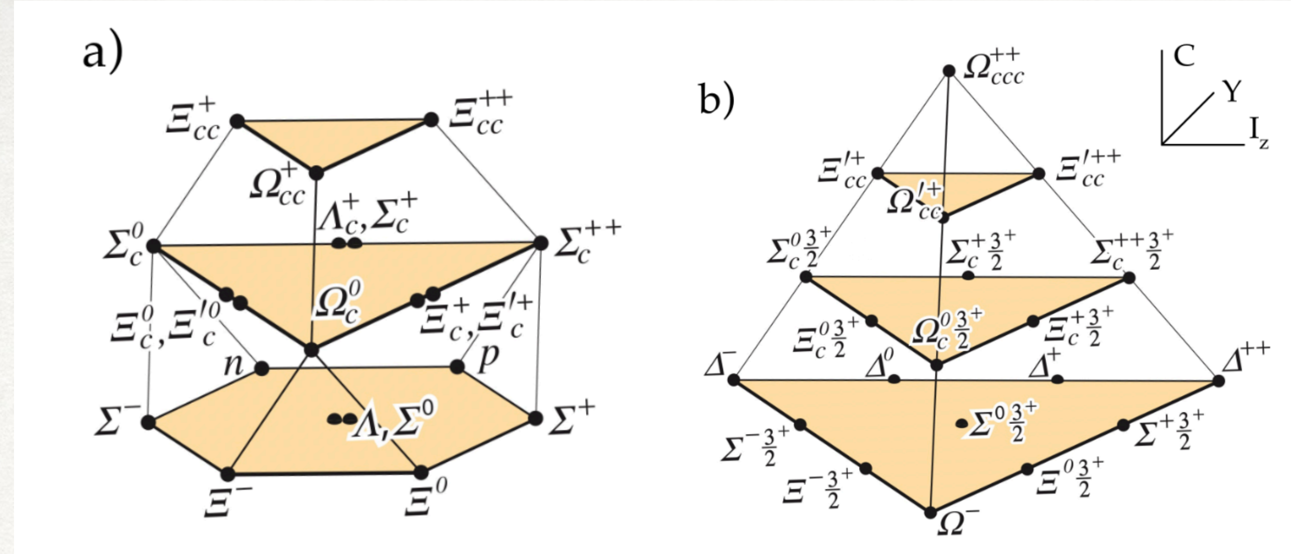


evidencias del color:

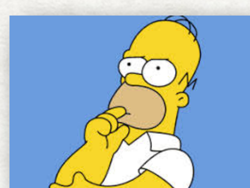
$$R = \sum_{colores} \sum_{q} Q_q^2 = 3(4/9 + 1/9 + 1/9) = 2$$

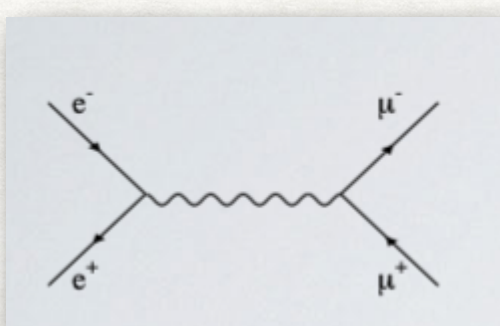
1974 $$R = \sum_{colores} \sum_{q} Q_q^2 = 3(4/9 + 1/9 + 1/9 + 4/9) = 3.33 \qquad m_c = 1.6\,GeV$$

1977 $$R = \sum_{colores} \sum_{q} Q_q^2 = 3(4/9 + 1/9 + 1/9 + 4/9 + 1/9) = 3.66 \qquad m_b = 4.5\,GeV$$

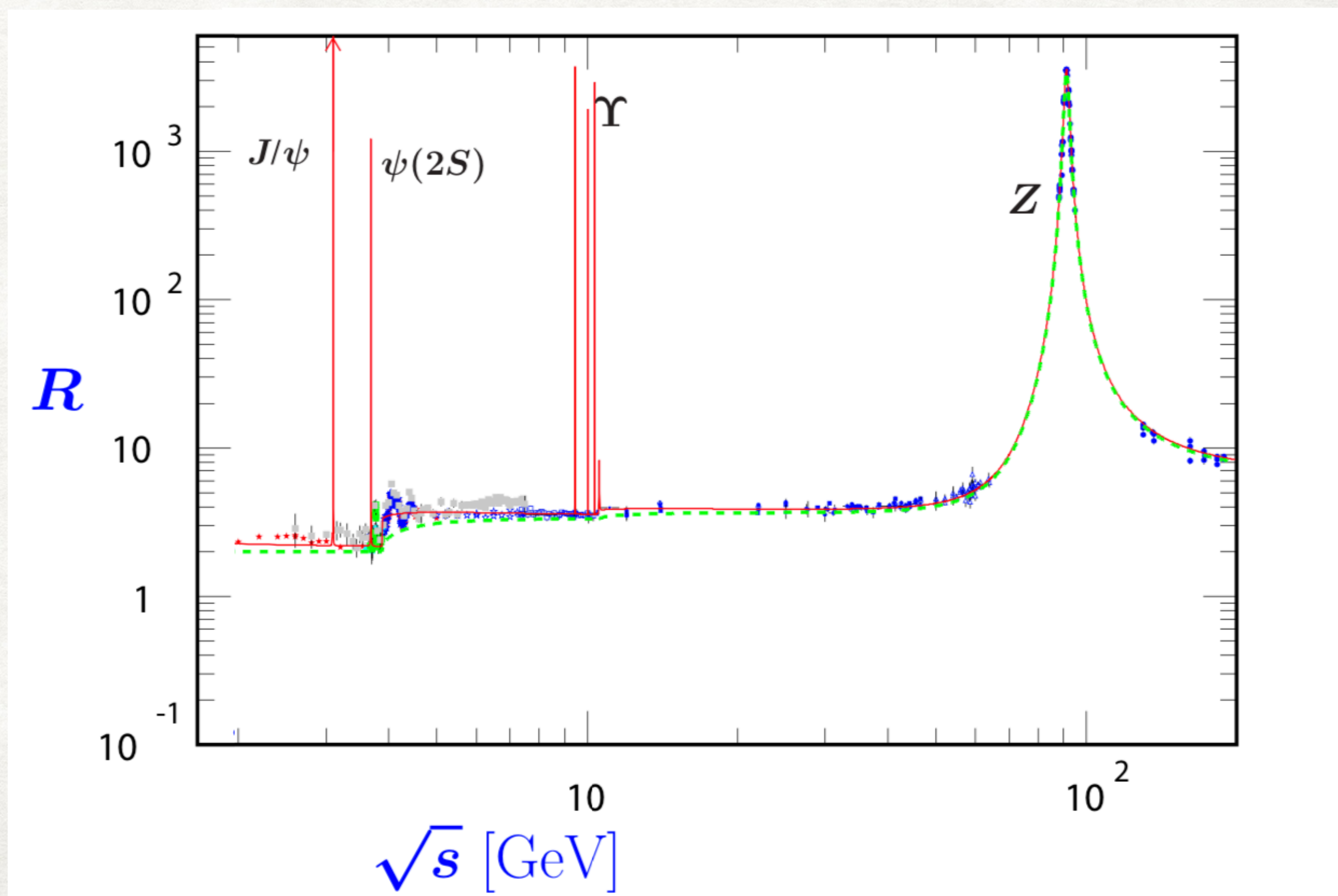
1995 $m_t = 178\,GeV$

$$\begin{array}{cccc} 2/3 & u & c & t \\ -1/3 & d & s & b \\ -1 & e & \mu & \tau \\ 0 & \nu_e & \nu_\mu & \nu_\tau \end{array}$$



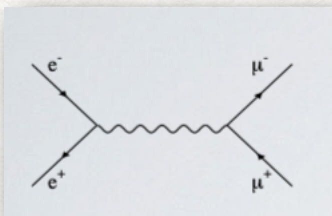


$$\sim \frac{1}{M^2 - s}$$



CLASE 5:

evidencias del color:



$$\sigma = \frac{\pi\alpha^2}{3E^2}$$