ESTRUCTURA DE LA MATERIA 4

CURSO DE VERANO 2021

CLASE 6

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CLASE 6: Fenomenología de Dirac

Temas: Límite no relativista, Fenomenología.

recapitulación:

$$\begin{cases} E \to i\hbar \frac{\partial}{\partial t} \\ \overrightarrow{p} \to -i\hbar \overrightarrow{\nabla} \end{cases} \qquad H = c\,\overrightarrow{\alpha}\cdot\overrightarrow{p} + \beta mc^2 \qquad E^2 = c^2p^2 + m^2c^4$$

$$\alpha_i^2 = 1 \quad \beta^2 = 1 \quad \{\alpha_i, \alpha_j\} = 2\delta_{ij} \quad \{\alpha_i, \beta\} = 0$$

$$H\psi = E\psi \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

2 sol. l.i. $E > 0$

2 sol. l.i. $E < 0$

$$\rho \equiv \psi^\dagger \psi \geq 0$$

$$\psi^{(1)}(\overrightarrow{x}, t) = N \begin{pmatrix} 1 \\ 0 \\ \left(\frac{c\overrightarrow{\sigma}\,\overrightarrow{p}}{E + mc^2}\right)\begin{matrix}1\\0\end{matrix} \end{pmatrix} e^{-\frac{i}{\hbar}p_\mu x^\mu}$$

$$\psi^{(2)}(\overrightarrow{x}, t) = N \begin{pmatrix} 0 \\ 1 \\ \left(\frac{c\overrightarrow{\sigma}\,\overrightarrow{p}}{E + mc^2}\right)\begin{matrix}0\\1\end{matrix} \end{pmatrix} e^{-\frac{i}{\hbar}p_\mu x^\mu}$$

$$\Sigma_3 \equiv \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \qquad [H, \Sigma_i] \neq 0 \qquad \overrightarrow{J} \equiv \overrightarrow{L} + \frac{\hbar}{2}\overrightarrow{\Sigma} \qquad [H, J_i] = 0$$

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$$\left(c\,\vec{\alpha}\cdot\vec{p} + \beta\,mc^2\right)\psi = E\,\psi$$

$$\vec{p} \longrightarrow \vec{p} - \frac{q}{c}\vec{A} \qquad E \longrightarrow E - q\phi \qquad \psi = \begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix}$$

$$\longrightarrow \left[c\,\vec{\alpha}\cdot\underbrace{\left(\vec{p} - \frac{q}{c}\vec{A}\right)}_{\equiv\,\vec{\pi}} + \beta\,mc^2\right]\begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix} = (\epsilon_{NR} + mc^2 - q\phi)\begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix}$$

$$\left[c\begin{pmatrix}0 & \vec{\sigma}\cdot\vec{\pi} \\ \vec{\sigma}\cdot\vec{\pi} & 0\end{pmatrix} + \begin{pmatrix}mc^2 & 0 \\ 0 & -mc^2\end{pmatrix} + q\phi\right]\begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix} = (\epsilon_{NR} + mc^2)\begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix}$$

$$c\,\vec{\sigma}\cdot\vec{\pi}\psi_B + \cancel{mc^2\psi_A} + q\phi\psi_A = (\epsilon_{NR} \cancel{+ mc^2})\psi_A \qquad (\epsilon_{NR} - q\phi)\psi_A = c\,\vec{\sigma}\cdot\vec{\pi}\psi_B$$

$$c\,\vec{\sigma}\cdot\vec{\pi}\psi_A - mc^2\psi_B + q\phi\psi_B = (\epsilon_{NR} + mc^2)\psi_B \qquad (\epsilon_{NR} + 2mc^2 - q\phi)\psi_B = c\,\vec{\sigma}\cdot\vec{\pi}\psi_A$$

$$\psi_A = \frac{c\,\vec{\sigma}\cdot\vec{\pi}}{(\epsilon_{NR} - q\phi)}\psi_B \qquad \psi_A = \frac{c\,\vec{\sigma}\cdot\vec{\pi}}{(\epsilon_{NR} - q\phi)}\frac{c\,\vec{\sigma}\cdot\vec{\pi}}{(\epsilon_{NR} + 2mc^2 - q\phi)}\psi_A$$

$$\psi_B = \frac{c\,\vec{\sigma}\cdot\vec{\pi}}{(\epsilon_{NR} + 2mc^2 - q\phi)}\psi_A \qquad \psi_A = \frac{c^2\left(\pi^2 + i\,\vec{\sigma}\cdot(\vec{\pi}\times\vec{\pi})\right)}{(\epsilon_{NR} - q\phi)(\epsilon_{NR} + 2mc^2 - q\phi)}\psi_A \qquad \vec{\pi}\times\vec{\pi} \neq 0$$

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$$\vec{\pi} \times \vec{\pi} = (\vec{p} - \frac{q}{c}\vec{A}) \times (\vec{p} - \frac{q}{c}\vec{A})$$

$$= \cancel{\vec{p} \times \vec{p}} - \frac{q}{c}\vec{p} \times \vec{A} - \frac{q}{c}\vec{A} \times \vec{p} + \frac{q^2}{c^2}\cancel{\vec{A} \times \vec{A}}$$

$$= \frac{q}{c} i\hbar\, (\vec{\nabla} \times \vec{A} + \vec{A} \times \vec{\nabla})$$

$$(\vec{\pi} \times \vec{\pi})\psi = \frac{q}{c} i\hbar\, (\vec{\nabla} \times \vec{A}\psi + \vec{A} \times \vec{\nabla}\psi) = \frac{q}{c} i\hbar \left[(\vec{\nabla} \times \vec{A})\psi + \cancel{\vec{\nabla}\psi \times \vec{A}} + + \cancel{\vec{A} \times \vec{\nabla}\psi}\right]$$

$$= \frac{q}{c} i\hbar\, (\vec{\nabla} \times \vec{A})\psi$$

$$\psi_A = \frac{c^2\left(\pi^2 + i\vec{\sigma} \cdot (\vec{\pi} \times \vec{\pi})\right)}{(\epsilon_{NR} - q\phi)(\epsilon_{NR} + 2mc^2 - q\phi)}\psi_A = \frac{c^2\left(\pi^2 + i\vec{\sigma} \cdot (i\hbar\frac{q}{c}(\vec{\nabla} \times \vec{A})\right)}{(\epsilon_{NR} - q\phi)(\epsilon_{NR} + 2mc^2 - q\phi)}\psi_A$$

$$\psi_A = \frac{\cancel{c^2}\left((\vec{p} - \frac{q}{c}\vec{A})^2 - \frac{q}{c}\hbar\vec{\sigma} \cdot \vec{B}\right)}{(\epsilon_{NR} - q\phi)(\cancel{\epsilon_{NR}} + 2m\cancel{c^2} - \cancel{q\phi})}\psi_A \qquad \psi_A(\epsilon_{NR} - q\phi) = \frac{1}{2m}\left((\vec{p} - \frac{q}{c})^2 - \frac{q}{c}\hbar\vec{\sigma} \cdot \vec{B}\right)\psi_A$$

$$\left[\frac{1}{2m}\left((\vec{p} - \frac{q}{c}\vec{A})^2 - \frac{q}{c}\hbar\vec{\sigma} \cdot \vec{B}\right) + q\phi\right]\psi_A = \epsilon_{NR}\psi_A$$

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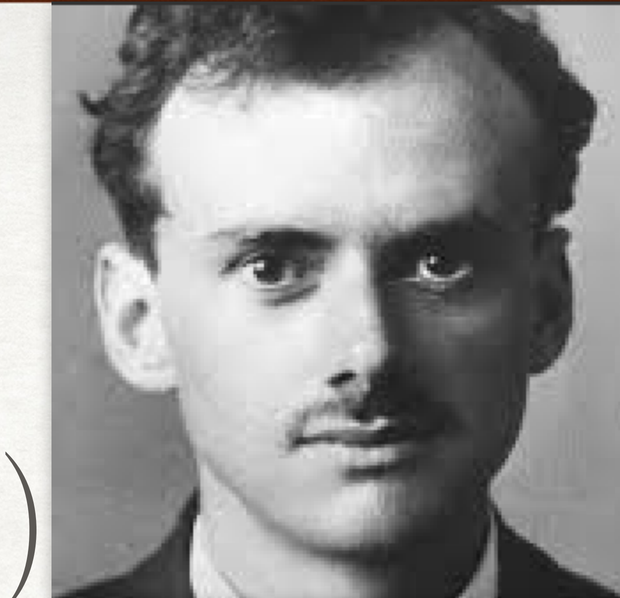


límite no relativista

$$\left[\frac{1}{2m}\left((\vec{p}-\frac{q}{c})^2-\frac{q}{c}\hbar\vec{\sigma}\cdot\vec{B}\right)+q\phi\right]\psi_A=\epsilon_{NR}\psi_A \quad \psi_A=\psi_{schroedinger}\begin{pmatrix}1\\0\end{pmatrix}$$

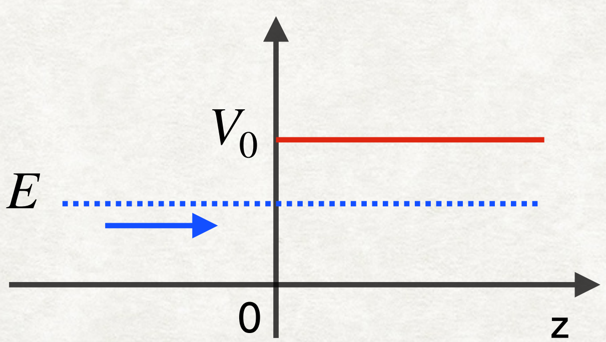
$$\frac{q}{2mc}\hbar\vec{\sigma}\cdot\vec{B}=\frac{gq}{2mc}\vec{S}\cdot\vec{B} \qquad g=2$$

$g = 2.00321930436182(52)$ (NIST 2018)

↑ teoría (3 loops)

paradoja de Klein: *cuando las componentes débiles no son tan débiles…*

soluciones estacionarias (E=fijo) a ambos lados



$$\psi_{inc}=\begin{pmatrix}1\\0\\\frac{k}{E+m}\\0\end{pmatrix}e^{-ikz}$$

$\vec{k}=k\hat{z} \quad S_z=1/2 \quad \hbar=c=1$

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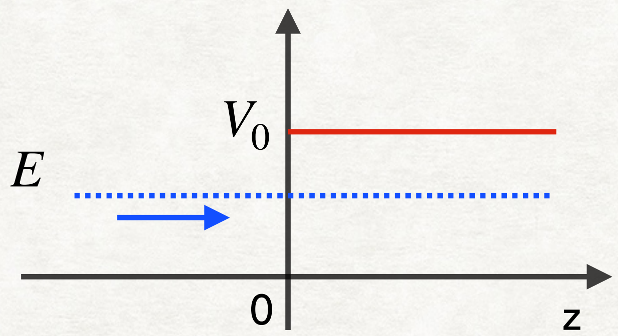

$$\psi_{inc} = \begin{pmatrix} 1 \\ 0 \\ \frac{k}{E+m} \\ 0 \end{pmatrix} e^{-ikz} \qquad \vec{k} = k\hat{z} \qquad S_z = 1/2 \qquad \hbar = c = 1$$

$$\psi_{refl} = a \begin{pmatrix} 1 \\ 0 \\ \frac{-k}{E+m} \\ 0 \end{pmatrix} e^{ikz} + b \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{k}{E+m} \end{pmatrix} e^{ikz}$$

$$E = \sqrt{c^2q^2 + m^2c^4} + V_0$$

$$E - V_0 = \sqrt{c^2q^2 + m^2c^4}$$

$$cq = \sqrt{(E-V_0)^2 - m^2c^4}$$

$$q = \sqrt{(E-V_0)^2 - m^2}$$

$$\psi_{trans} = c \begin{pmatrix} 1 \\ 0 \\ \frac{q}{E-V_0+m} \\ 0 \end{pmatrix} e^{iqz} + d \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-q}{E-V_0+m} \end{pmatrix} e^{iqz}$$

continuidad de ψ en z=0 $\quad \psi_{inc}(z=0) + \psi_{refl}(z=0) = \psi_{trans}(z=0)$

$$1 + a = c \qquad k(1-a)/(E+m) = cq/(E-V_0+m) \quad \rightarrow (1-a) = cr$$

$$b = d \qquad bk/(E+m) = -dq/(E-V_0+m) \quad \Rightarrow b = d = 0$$

$$r \equiv \frac{q}{k}\frac{E+m}{E-V_0+m}$$

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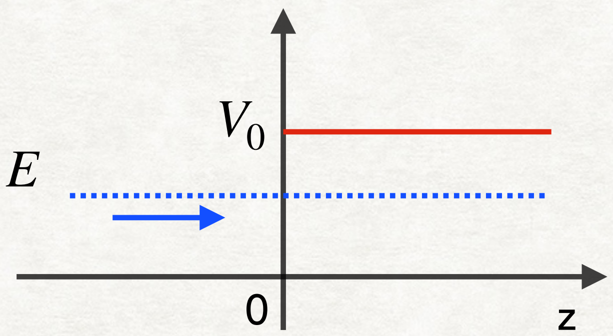

$$1 + a = c \qquad r \equiv \frac{q}{k}\frac{E+m}{E-V_0+m}$$

$$(1-a) = cr \qquad q = \sqrt{(E-V_0)^2 - m^2}$$

corrientes: $J^\mu \equiv \overline{\psi}\gamma^\mu\psi \qquad J^3 \equiv \overline{\psi}\gamma^3\psi = \psi^\dagger\gamma^0\gamma^3\psi = \psi^\dagger\beta\beta\alpha_3\psi = \psi^\dagger\alpha_3\psi$

$$\alpha_i = \left(\begin{array}{c|c} 0 & \sigma_i \\ \hline \sigma_i & 0 \end{array}\right)$$

$$J^3_{inc} = \left(1,0,\frac{k}{E+m},0\right)\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ \frac{k}{E+m} \\ 0 \end{pmatrix} = \frac{2k}{E+m}$$

$$J^3_{refl} = \frac{2a^2k}{E+m}$$

$$J^3_{inc} = J^3_{refl} + J^3_{trans}$$

$$J^3_{trans} = \frac{2c^2q}{E-V_0+m}$$

$$T \equiv \frac{J^3_{trans}}{J^3_{inc}} = \frac{4r}{(1+r)^2} \qquad R \equiv \frac{J^3_{refl}}{J^3_{inc}} = \frac{(1-r)^2}{(1+r)^2}$$

a) $V_0 < E + m \quad r > 0 \quad T > 0$ hay transmisión

$R < 1$ se reflejan menos que las incidentes

b) $V_0 > E + m \quad r < 0 \quad T < 0$ transmitidas en dirección opuesta?

$R > 1$ más partículas reflejadas que las incidentes?

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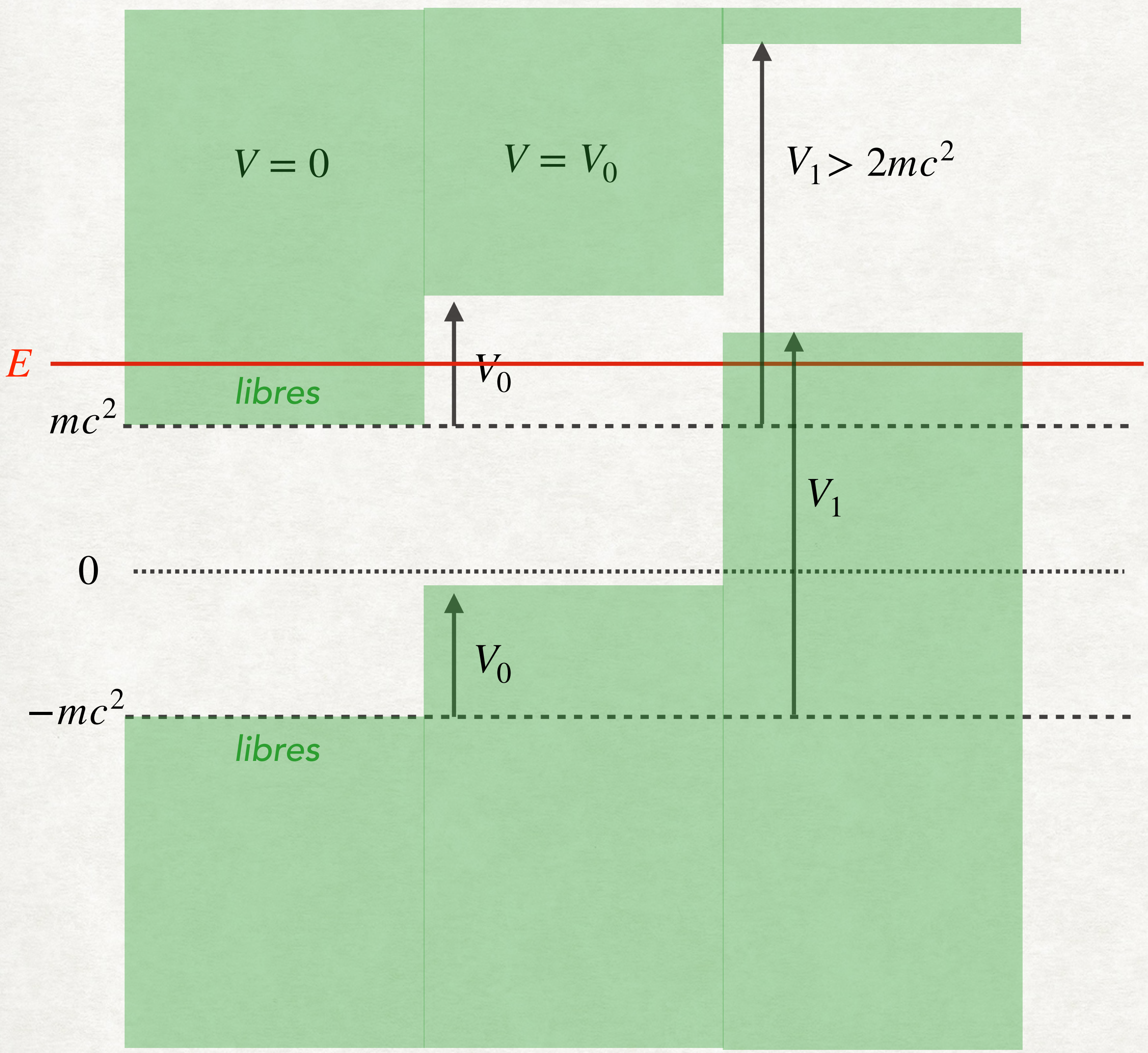

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trayectoria de una partícula de Dirac: *zitterbewegung*

velocidad de una partícula de Dirac:

$$\dot{x}_k \equiv \frac{i}{\hbar}[H, x_k] = \frac{ic}{\hbar}[\alpha_j p_j, x_k] = c\alpha_k$$

nunca va estar en autotestado de velocidad (salvo que $m = 0$)

$$H = c\,\overrightarrow{\alpha}\cdot\overrightarrow{p} + \beta\, mc^2 \qquad [x, p] = i\hbar$$

además, $[\alpha_i, \alpha_j] \neq 0$ ....

$$[H, p_k] = 0 \qquad [H, \alpha_k] \neq 0$$

para una partícula libre, la velocidad no es constante de movimiento (salvo que $m = 0$)

aceleración de una partícula de Dirac:

$$\dot{\alpha}_k \equiv \frac{i}{\hbar}[H, \alpha_k] = \frac{i}{\hbar}(-2\alpha_k H + 2cp_k) \neq 0$$

partícula libre…

$$\alpha_k(t) = \frac{cp_k}{H} + (\alpha_k(0) - \frac{cp_k}{H})e^{-2iHt/\hbar} \qquad (\nu \sim 1.5\;\, 10^{-21}s^{-1})$$

$$x_k(t) = x_k(0) + \frac{c^2 p_k}{H}t + \frac{ic\hbar}{2H}(\alpha_k(0) - \frac{cp_k}{H})e^{-2iHt/\hbar} \qquad (\Delta x \sim 3.9\;\, 10^{-11}cm)$$