ESTRUCTURA DE LA MATERIA 4

CURSO DE VERANO 2021

CLASE 9

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CLASE 9: Teoría de perturbaciones.

Temas: Perturbación covariante, propagadores, electrodinámica

formalismo no relativista:

$$E_n\,\phi_n = H_0\,\phi_n \qquad\qquad \int d^3x\,\phi_m^*\phi_n = \delta_{mn}$$

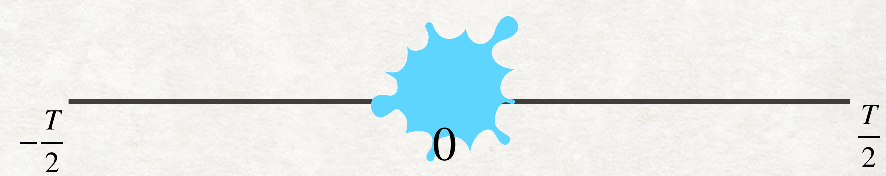
$$i\frac{\partial\psi}{\partial t} = \big(H_0 + V(\vec{x},t)\big)\,\psi \qquad\qquad \psi(\vec{x},t) = \sum_n a_n(t)\,\phi_n(\vec{x})\,e^{-iE_nt} \qquad\qquad a_n(t)$$

$$i\sum_n \frac{da_n}{dt}\,\phi_n(\vec{x})\,e^{-iE_nt} + E_n\sum_n a_n(t)\,\phi_n(\vec{x})\,e^{-iE_nt} = E_n\sum_n a_n(t)\,\phi_n(\vec{x})\,e^{-iE_nt} + \sum_n a_n(t)\,V(\vec{x},t)\,\phi_n(\vec{x})\,e^{-iE_nt}$$

mult. m.a.m por ϕ_f^*

$$\frac{da_f}{dt} = -i\sum_n a_n(t)\int d^3x\,\phi_f^*\,V(\vec{x},t)\,\phi_n(\vec{x})\,e^{i(E_f-E_n)t} \simeq -i\int d^3x\,\phi_f^*\,V(\vec{x},t)\,\phi_i(\vec{x})\,e^{i(E_f-E_i)t}$$

supongamos que a $t_i = -\frac{T}{2} \sim \phi_i$ $\qquad a_i(-\frac{T}{2}) = 1 \qquad a_n(-\frac{T}{2}) = 0 \quad \forall n \neq i$



$$a_f(t) \simeq -i\int_{-\frac{T}{2}}^{t} dt'\int d^3x\;\phi_f^*(\vec{x})\,V(\vec{x},t')\,\phi_i(\vec{x})\,e^{i(E_f-E_i)t'}$$

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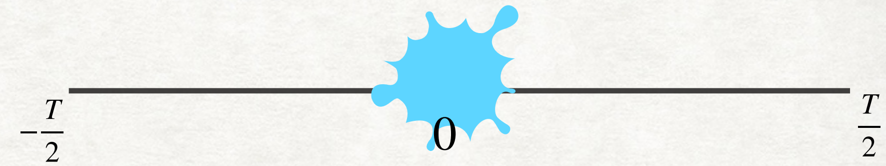
formalismo no relativista:

$$a_f(t) \simeq -i \int_{-\frac{T}{2}}^{t} dt' \int d^3x \; \phi_f^*(\vec{x}) \, V(\vec{x}, t') \, \phi_i(\vec{x}) \, e^{i(E_f - E_i)t'}$$



$$T_{fi} \equiv a_f(\frac{T}{2}) = -i \int_{-\frac{T}{2}}^{\frac{T}{2}} dt' \int d^3x \; \left(\phi_f e^{-iE_f}\right)^* \; V(\vec{x}, t') \; \left(\phi_i \, e^{-iE_i t'}\right) \; = -i \int d^4x \, \Phi_f^*(x^\mu) \, V \, \Phi_i(x^\mu)$$

si $V \sim cte$ $\qquad T_{fi} = -i \, V_{fi} \int dt \, e^{i(E_f - E_i)t}$ $\qquad V_{fi} \equiv \int d^3x \phi_f^*(\vec{x}) \, V \, \phi_i(\vec{x})$

si $T \to \infty$ $\qquad T_{fi} = -2\pi i \, V_{fi} \, \delta(E_f - E_i)$ $\qquad E_f = E_i$ sólo si $T \to \infty$!!

probabilidad de transición por unidad de tiempo $\qquad W \equiv \lim_{T \to \infty} \frac{|T_{fi}|^2}{T}$

$$W = \lim_{T \to \infty} 2\pi \frac{|V_{fi}|^2}{T} \delta(E_f - E_i) \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \, e^{i(E_f - E_i)t} = \lim_{T \to \infty} 2\pi \frac{|V_{fi}|^2}{T} \delta(E_f - E_i) \int_{-\frac{T}{2}}^{\frac{T}{2}} dt$$

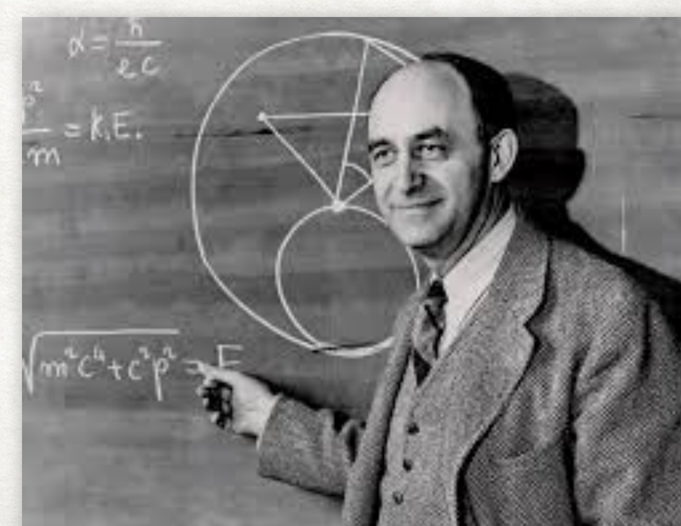
$$= 2\pi \, |V_{fi}|^2 \delta(E_f - E_i)$$

para un estado inicial fijo i y un continuo de estados finales $\rho(E_f)$

$$W_{if} = 2\pi \int dE_f \, \rho(E_f) \, |V_{fi}|^2 \, \delta(E_f - E_i) = 2\pi \; \rho(E_i) \, |V_{fi}|^2$$

$\rho(E_f)$ número de estados entre E_f y $E_f + dE_f$

"regla de oro de Fermi"



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formalismo no relativista:

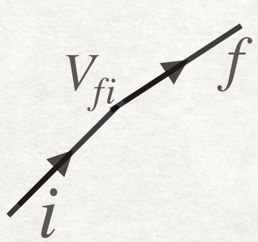
$$a_f^{(1)}(t) \simeq -\, i\, V_{fi} \int_{-\frac{T}{2}}^{t} dt'\, e^{i(E_f - E_i)t'} \qquad \frac{da_f}{dt} = -i \sum_n a_n(t)\, V_{fn}\, e^{i(E_f - E_n)t} \qquad a_n(t) \simeq 0 \quad \forall n \neq i$$

$$\frac{da_f^{(2)}}{dt} \simeq \underbrace{-i\, V_{fi}\, e^{i(E_f - E_i)t}}_{n=i} \quad \underbrace{+(-i) \sum_{n\neq i} \overbrace{(-i) V_{ni} \int_{-\frac{T}{2}}^{t} dt'\, e^{i(E_n - E_i)t'}}^{a_n^{(1)}}\, V_{fn}\, e^{i(E_f - E_n)t}}_{n \neq i}$$

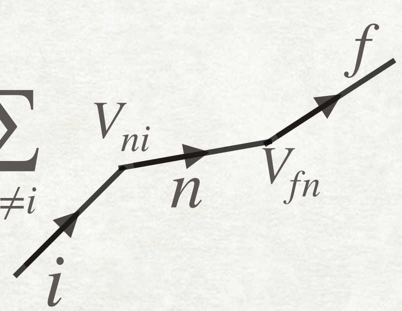
$$T_{fi} \equiv a_f(\frac{T}{2}) = -\, 2\pi i\, V_{fi}\, \delta(E_f - E_i) - i \sum_{n\neq i} V_{fn} V_{ni} \int_{-\frac{T}{2}}^{\frac{T}{2}} dt\, e^{i(E_f - E_n)t} \int_{-\frac{T}{2}}^{t} dt'\, e^{i(E_n - E_i)t'}$$

$$\int_{-\frac{T}{2}}^{t} dt'\, e^{i(E_n - E_i)t'} \xrightarrow[T \to \infty]{} i\, \frac{e^{i(E_n - E_i)t}}{E_i - E_n}$$

$$T_{fi} = -\, 2\pi i\, V_{fi}\, \delta(E_f - E_i) - 2\pi i \sum_{n \neq i} \frac{V_{fn} V_{ni}}{E_i - E_n}\, \delta(E_f - E_i)$$



$\sum_{n\neq i}$



$E_f = E_i$ sólo si $T \to \infty$!!

$\dfrac{1}{E_i - E_n}$ "propagador"

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formalismo relativista:

$$H_0 = \alpha\cdot p + \beta m \qquad \left(\gamma_\mu p^\mu - m\right)\psi = 0$$

$$H = \alpha\cdot p + \beta m + V \qquad \left(\gamma_\mu p^\mu - m - \gamma^0 V\right)\psi' = 0$$

$$-\gamma^0 V = e\gamma_\mu A^\mu \qquad V = -\,e\gamma^0\gamma_\mu A^\mu$$

$$p^\mu \to p^\mu + e\,A^\mu \qquad \left(\gamma_\mu p^\mu - m + e\gamma_\mu A^\mu\right)\psi' = 0$$

$$T_{fi} = -\,i\int d^4x\,\Phi_f^*(x^\mu)\,V\,\Phi_i(x^\mu) \quad\longrightarrow -\,i\int d^4x\,\psi_f^\dagger(x^\mu)\,V\,\psi_i(x^\mu) \;=-\,i\int d^4x\,\psi_f^\dagger(x^\mu)\,(-e\gamma^0\gamma_\mu A^\mu)\,\psi_i(x^\mu)$$

$$=-\,i\int d^4x\,(-e)\,\overline{\psi}_f(x^\mu)\,\gamma_\mu\,\psi_i(x^\mu)\,A^\mu$$

$$=-\,i\int d^4x\,J_\mu^{fi}\,A^\mu$$

$$J_\mu^{fi} \equiv -\,e\,\overline{\psi}_f\,\gamma_\mu\,\psi_i \;=-\,e\,\overline{u}_f\gamma_\mu\,u_i\,e^{i(p^f-p^i)_\mu x^\mu}$$

$$=-\,e\,(p_\mu^f - p_\mu^i)\;e^{i(p^f-p^i)_\mu x^\mu}$$

$$\Box^2 A^\mu = J^\mu \qquad J^\mu = -\,e\,(p_c^\mu - p_d^\mu)\;e^{i(p^d-p^c)_\mu x^\mu}$$

$$T_{fi} = -\,i\int d^4x\,J_\mu^{fi}\left(-\frac{1}{q^2}\right)J_{dc}^\mu$$

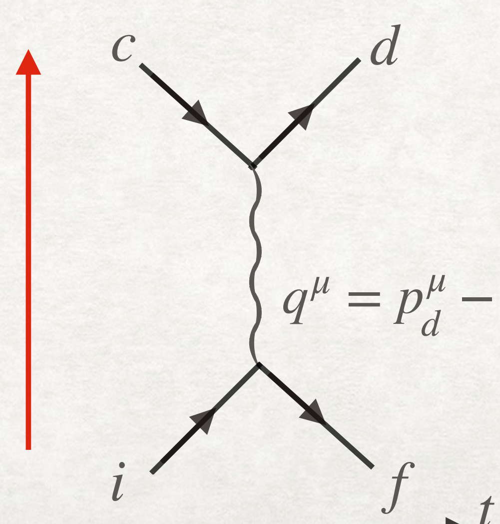
$$\Box^2\,e^{iq\cdot x} = -\,q^2 e^{iq\cdot x} \qquad (q^2 = q_\mu q^\mu)$$

$$A^\mu = -\frac{1}{q^2}J^\mu \qquad q^\mu = p_d^\mu - p_c^\mu$$

$\overline{u}_f \to \overline{v}_f$
$u_c \to v_c$

"aniquilación"

$i + \bar{f} \to \bar{c} + d$



$$q^\mu = p_d^\mu - p_c^\mu = p_f^\mu - p_i^\mu$$

"scattering" $\quad i + c \to f + d$

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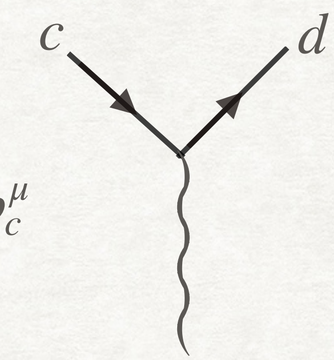
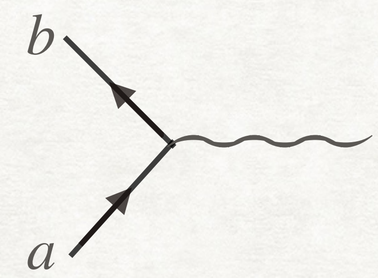
propagadores:

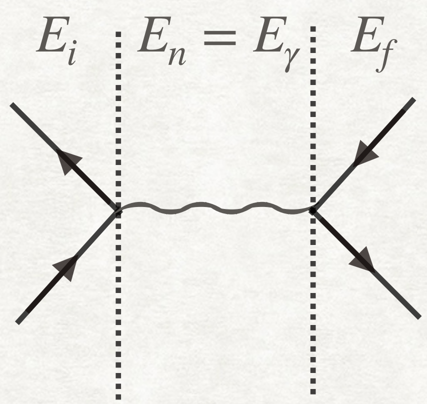
$$\frac{1}{E_i - E_f} \longrightarrow -\frac{1}{q^2}$$

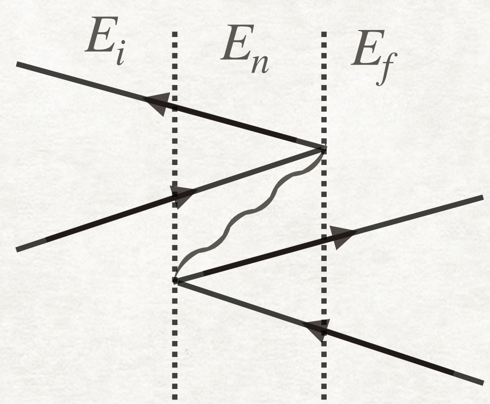
$$q^\mu = p_d^\mu - p_c^\mu$$



$$q^\mu = p_a^\mu + p_b^\mu$$







$$E_n = E_i + E_f + E_\gamma$$
$$= 2E_i + E_\gamma$$

$$T \sim V_{fn}\frac{1}{E_i - E_\gamma}V_{ni} + V_{fn}\frac{1}{E_i - 2E_i - E_\gamma}V_{ni} = V_{fn}\frac{-2E_i}{E_i^2 - E_\gamma^2}V_{ni}$$

$$-E_i - E_\gamma$$

en términos de p_a y p_b

$$p^2 = E^2 - \vec{p}^{\,2}$$

$$E_i^2 = (p_a + p_b)^2 - (\vec{p_a} + \vec{p_b})^2$$

$$E_\gamma^2 = m_\gamma^2 - \vec{p}_\gamma^{\,2}$$

$$\frac{1}{E_i^2 - E_\gamma^2} = \frac{1}{(p_a + p_b)^2 - m_\gamma^2} = \frac{1}{q^2 - m_\gamma^2}$$

$-\frac{1}{q^2}$ propagador de un mediador de m=0

(incluyendo todos los ordenamientos)

q^2 sería el cuadri-impulso al cuadrado del fotón si se conservara la energía en cada vértice

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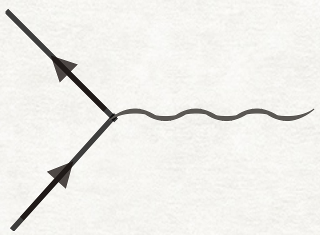
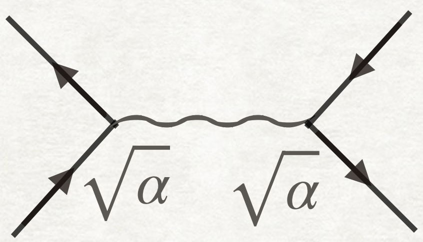
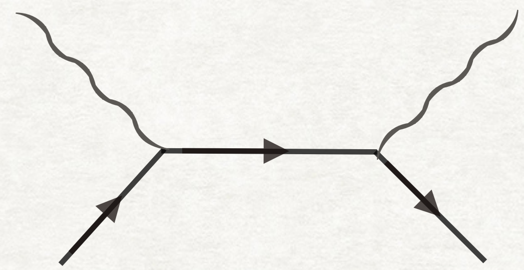
electrodinámica (QED):

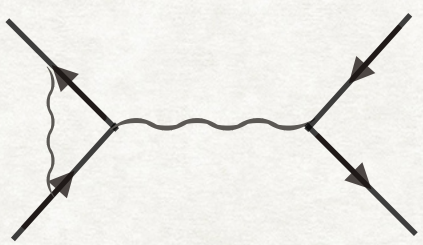
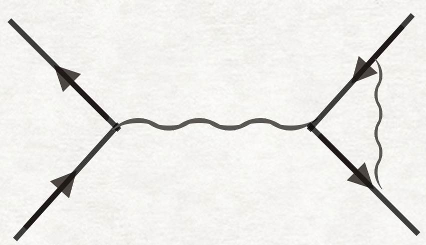
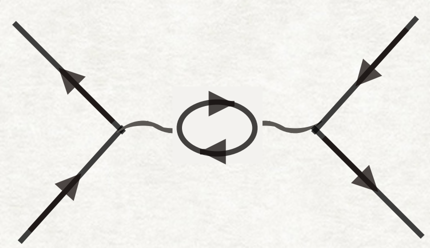
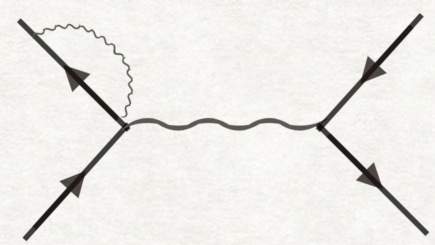


$\mathscr{A} \propto \sqrt{\alpha_{QED}}$ $\qquad \alpha_{QED} = \frac{e^2}{\hbar c} \simeq \frac{1}{137}$

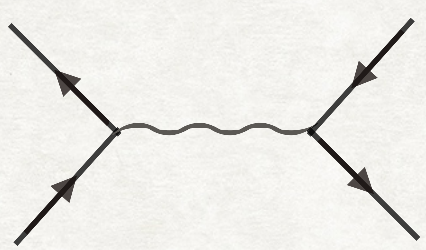
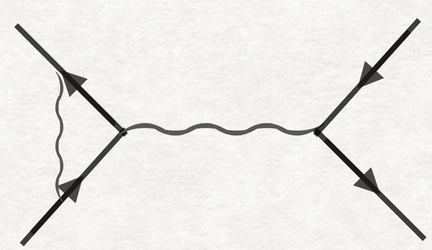


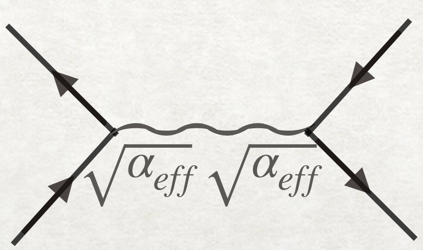
$\mathscr{A} \propto \alpha \qquad \sigma \propto \alpha^2$

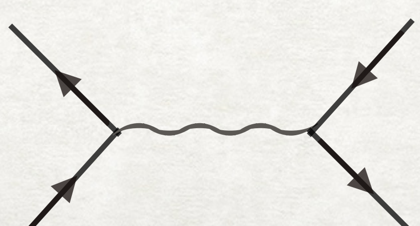
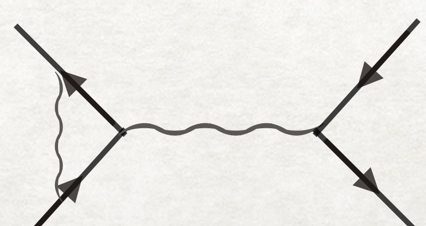


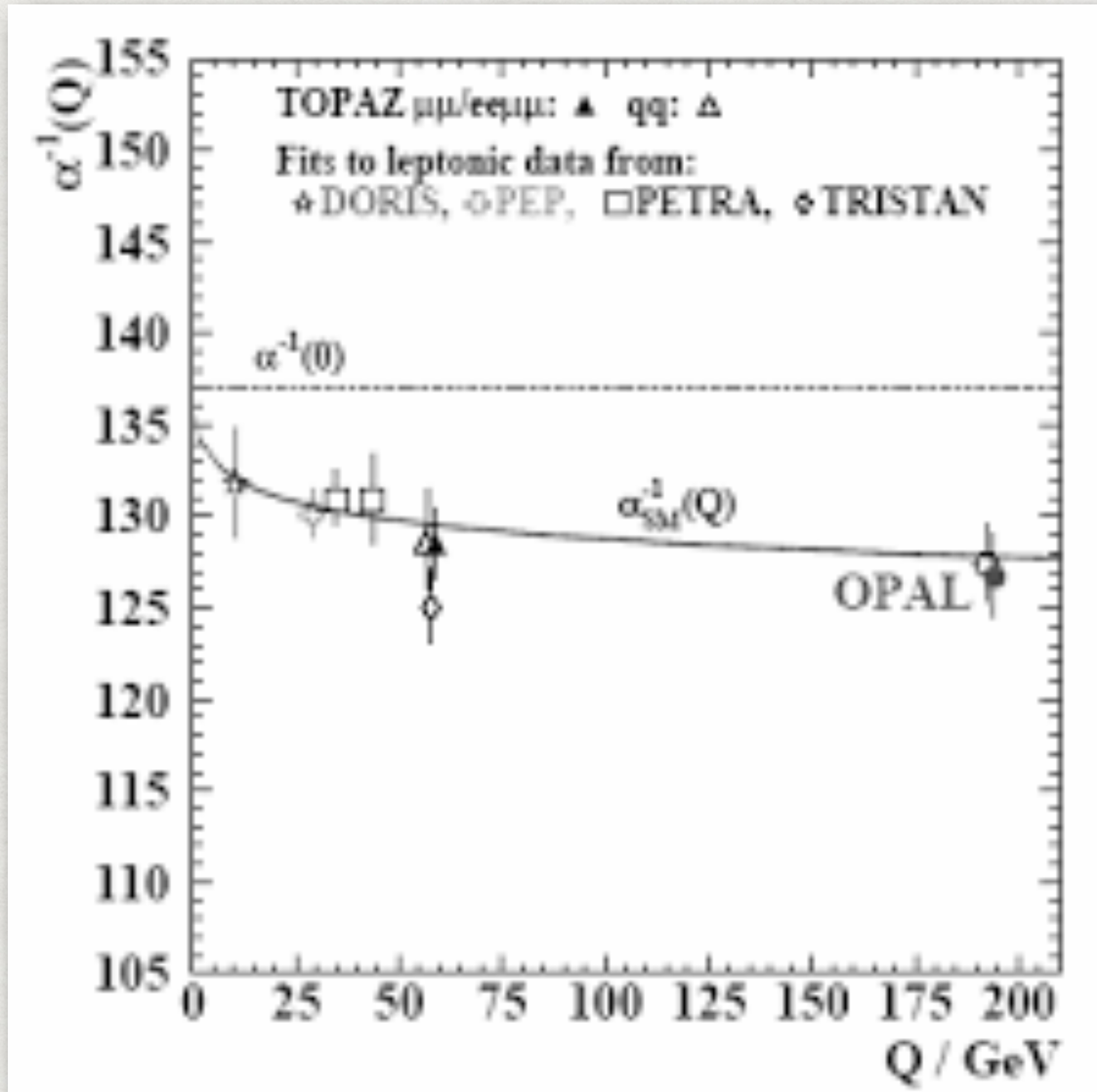
 +  +  +  $\cdots \qquad \sigma \propto \alpha^4$

$\sigma \sim \left| \sum \mathscr{A}_i \right|^2$

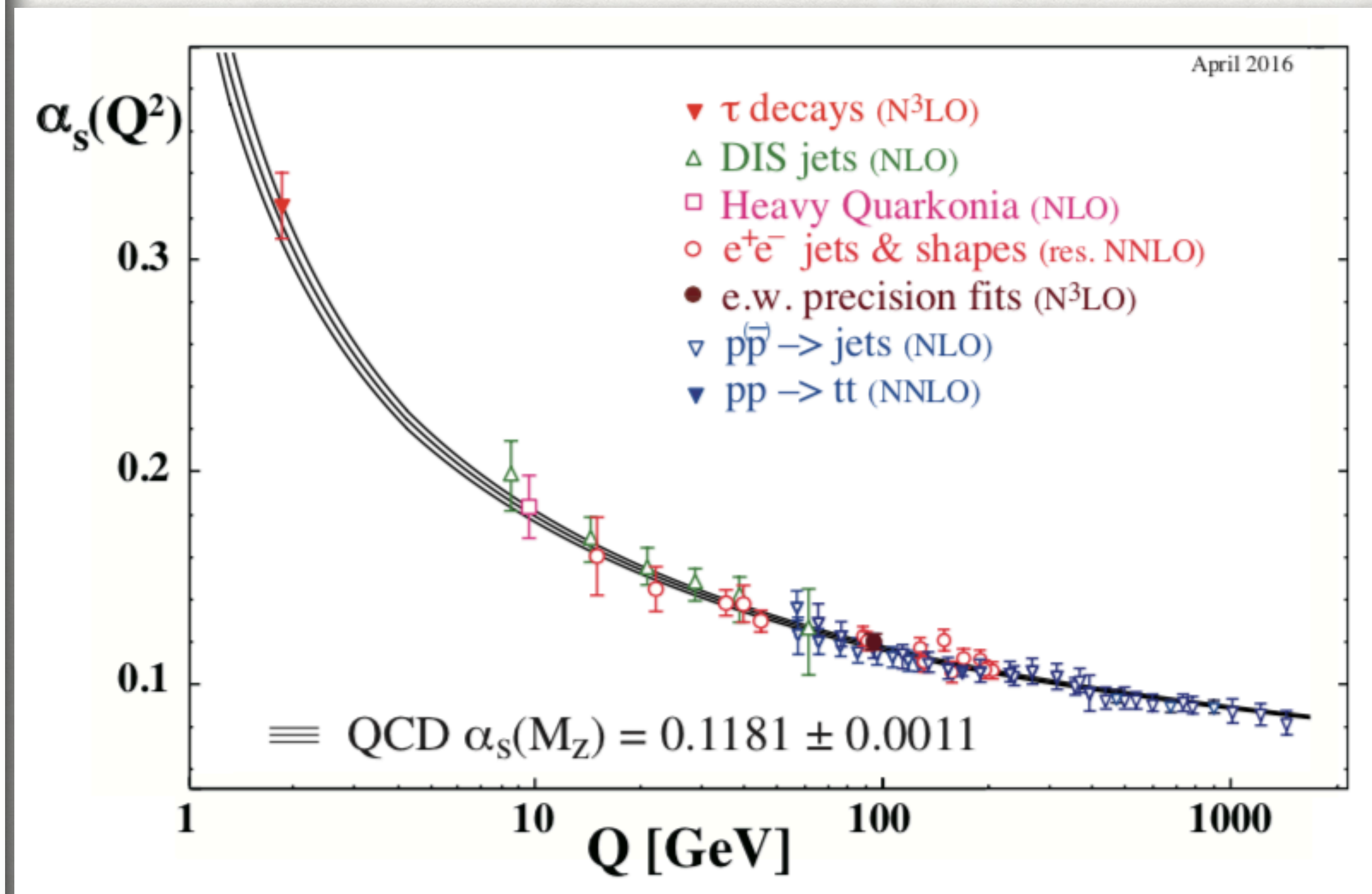
 $\times$  $\qquad \sigma \propto \alpha^3$



$\equiv$  +  $+ \cdots \qquad \alpha_{eff}(E) \qquad \frac{1}{137} \to \frac{1}{128}$



$\alpha^{-1}(Q)$
TOPAZ μμ/eeμμ: ▲ qq: △
Fits to leptonic data from:
DORIS, PEP, □PETRA, TRISTAN
$\alpha^{-1}(0)$
$\alpha^{-1}_{SM}(Q)$
OPAL
Q / GeV



April 2016
$\alpha_s(Q^2)$
τ decays (N³LO)
DIS jets (NLO)
Heavy Quarkonia (NLO)
e⁺e⁻ jets & shapes (res. NNLO)
e.w. precision fits (N³LO)
pp -> jets (NLO)
pp -> tt (NNLO)
QCD $\alpha_s(M_Z) = 0.1181 \pm 0.0011$
Q [GeV]