

# FISICA DE LAS INTERACCIONES FUNDAMENTALES

1ER CUATRIMESTRE 2026

CLASE 10

RODOLFO SASSOT

# CLASE 10: Teoría lagrangiana de campos

Temas: formalismo lagrangiano, lagrangiano de KG, Dirac y Maxwell.

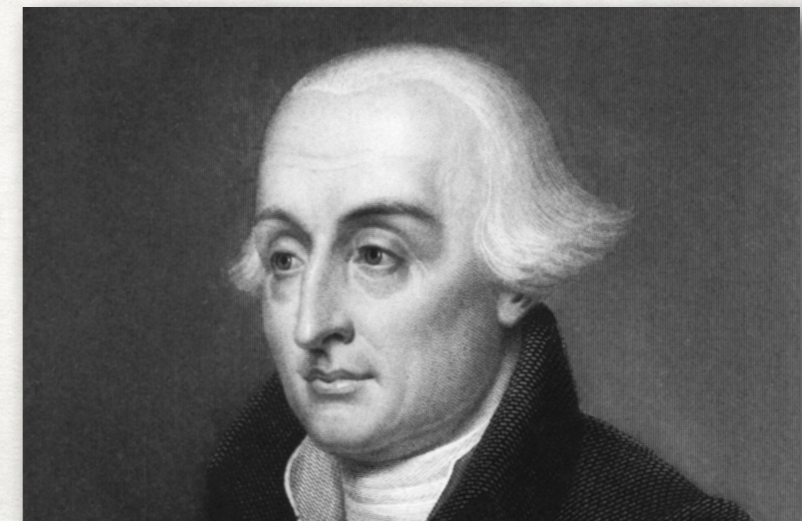
herramienta para analizar las interacciones: extiende Ec. Dirac para  $\infty$  grados de libertad  
pone en evidencia las simetrías (clave para las interacciones)  
su cuantización a) explica la relación entre spin-estadística  
b) imagen más satisfactoria de las antipartículas

porqué no simplemente Dirac? un sistema cuántico y relativista implica  $\infty$  grados de libertad

QFT  $\xrightarrow{\text{bajas energías}}$  Dirac  $\xrightarrow{\text{no relativista}}$  Schrödinger  $\xrightarrow{\text{clásico}}$  Newton



Giuseppe Ludovico Lagrangia (1736-1813)  
(alias Lagrange)



# CLASE 10: Teoría lagrangiana de campos

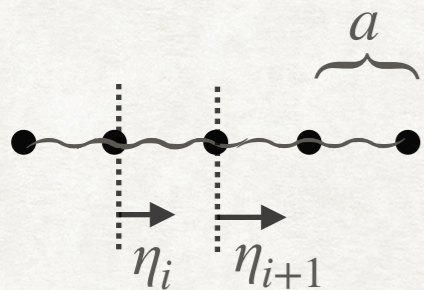
formalismo lagrangiano:

$$L(q, \dot{q}, t) = T - V$$

$$p \equiv \frac{\partial L}{\partial \dot{q}} \quad H = p\dot{q} - L \quad q, p \text{ variables generalizadas} \quad \{q, p\}_p = 1$$

$$\frac{dg}{dt} = \{g, H\}_p$$

$$q(t), \dot{q}(t) \quad \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0 \quad \delta \int dt L = 0$$



$$L = \sum_i^N \frac{1}{2} m_i \dot{\eta}_i^2 - \frac{1}{2} k (\eta_{i+1} - \eta_i)^2 = \sum_i^N a \frac{1}{2} \left[ \frac{m_i}{a} \dot{\eta}_i^2 - ka \left( \frac{\eta_{i+1} - \eta_i}{a} \right)^2 \right]$$

$$\mu \equiv \frac{m}{a} \quad Y \equiv ka$$

$$L \rightarrow \int dx \frac{1}{2} \left[ \mu \dot{\eta}^2(x) - Y \left( \frac{\partial \eta}{\partial x} \right)^2 \right] \equiv \int dx \mathcal{L}(\eta, \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial t})$$

$$\frac{\eta_{i+1} - \eta_i}{a} \rightarrow \frac{\partial \eta}{\partial x}$$

$$\eta, \frac{\partial L}{\partial \dot{\eta}} \quad \text{variables generalizadas}$$

$$\sum a \rightarrow \int dx$$

$$x, t \quad \text{parámetros}$$

# CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

$$L = \int dx \frac{1}{2} \left[ \mu \dot{\eta}^2(x) - Y \left( \frac{\partial \eta}{\partial x} \right)^2 \right] \equiv \int dx \mathcal{L}(\eta, \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial t})$$

$$\pi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\eta}} = \mu \dot{\eta} \quad p_i = \frac{\partial L}{\partial \dot{\eta}_i} \quad P = \sum_i p_i \dot{\eta}_i = \sum_i m \dot{\eta}_i^2 = \sum_i a \frac{m}{a} \dot{\eta}_i^2 \rightarrow \int dx \mu \dot{\eta}^2 = \int dx \pi^2$$

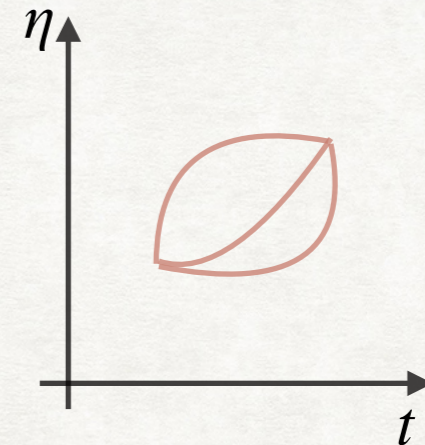
$$\mathcal{H} \equiv \pi \dot{\eta} - \mathcal{L} \quad H = \int dx \mathcal{H}$$

$$\{f, g\}_p \equiv \int dx \left[ \frac{\partial f}{\partial \eta} \frac{\partial g}{\partial \pi} - \frac{\partial f}{\partial \pi} \frac{\partial g}{\partial \eta} \right]$$

# CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

$$\delta \int dt L = \delta \int dt \int dx \mathcal{L} \quad \delta\eta(x, t) \begin{cases} \delta\eta(x, t_1) = 0 \\ \delta\eta(x, t_2) = 0 \end{cases}$$



$$\delta \int dt L = \int dt \int dx \left[ \frac{\partial \mathcal{L}}{\partial \eta} \delta\eta + \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \delta\left(\frac{\partial \eta}{\partial x}\right) + \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \delta\left(\frac{\partial \eta}{\partial t}\right) \right]$$

$$= \int dt \int dx \left[ \frac{\partial \mathcal{L}}{\partial \eta} \delta\eta - \frac{\partial}{\partial x} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) \delta\eta - \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \right) \delta\eta \right] = 0$$

$$\frac{\partial}{\partial x} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) + \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0$$

$$\frac{\partial}{\partial x} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \delta\eta \right) = \frac{\partial \delta\eta}{\partial x} \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} + \frac{\partial}{\partial x} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) \delta\eta$$

$$0 = \underbrace{\frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \delta\left(\frac{\partial \eta}{\partial x}\right)}_{\text{lo que tengo}} + \frac{\partial}{\partial x} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) \delta\eta$$

lo que tengo

como lo quiero

# CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

$$\frac{\partial}{\partial x} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x}} \right) + \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0 \quad \mathcal{L} = \frac{1}{2} \mu \dot{\eta}^2(x) - \frac{Y}{2} \left( \frac{\partial \eta}{\partial x} \right)^2$$

$$\frac{\partial}{\partial x} \left( -Y \frac{\partial \eta}{\partial x} \right) + \frac{\partial}{\partial t} (\mu \dot{\eta}) = 0 \quad -Y \frac{\partial^2 \eta}{\partial x^2} + \mu \frac{\partial^2 \eta}{\partial t^2} = 0$$

$$\sum_{k=1}^3 \frac{\partial}{\partial x_k} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x_k}} \right) + \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial t}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0 \quad \frac{\partial}{\partial x^\mu} \left( \frac{\partial \mathcal{L}}{\partial \frac{\partial \eta}{\partial x^\mu}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0$$

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \eta)} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0$$

# CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \quad \eta = \phi(x, t) \quad \mathcal{L} = \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - \mu^2 \phi^2 \right)$$

$$-\frac{\partial \mathcal{L}}{\partial \phi} = \mu^2 \phi \quad \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{\partial}{\partial(\partial_\mu \phi)} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) = \frac{1}{2} g^{\mu\nu} \left( \partial_\nu \phi + \partial_\mu \phi \delta_{\mu\nu} \right)$$

$$= \frac{1}{2} (\partial^\mu \phi + \partial^\mu \phi) = \partial^\mu \phi$$

$$\partial_\mu \partial^\mu \phi + \mu^2 \phi = 0 \quad \text{Klein-Gordon?}$$

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi = (\partial_\mu \phi_1 - i \partial_\mu \phi_2)(\partial^\mu \phi_1 + i \partial^\mu \phi_2) - \mu^2 (\phi_1 - i \phi_2)(\phi_1 + i \phi_2)$$

$$\begin{aligned} \phi &= \phi_1 + i \phi_2 \\ \phi^* &= \phi_1 - i \phi_2 \end{aligned} \quad \begin{aligned} &= \partial_\mu \phi_1 \partial^\mu \phi_1 - \mu^2 \phi_1^2 + \partial_\mu \phi_2 \partial^\mu \phi_2 - \mu^2 \phi_2^2 \\ &= \mathcal{L}_1 + \mathcal{L}_2 \end{aligned}$$

$$\left. \begin{aligned} \partial_\mu \partial^\mu \phi^* + \mu^2 \phi^* &= 0 \\ \partial_\mu \partial^\mu \phi + \mu^2 \phi &= 0 \end{aligned} \right\} \iff \begin{cases} \partial_\mu \partial^\mu \phi_1 + \mu^2 \phi_1 = 0 \\ \partial_\mu \partial^\mu \phi_2 + \mu^2 \phi_2 = 0 \end{cases} \quad \partial_\mu \partial^\mu \phi + \mu^2 \phi = 0$$

# CLASE 10: Teoría lagrangiana de campos

formalismo lagrangiano:

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0 \quad \eta = \psi(x, t) \quad \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} \right) - \frac{\partial \mathcal{L}}{\partial \bar{\psi}} = 0 \quad (i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \quad A_\mu(\vec{x}, t) \quad \text{campo cuadvectorial}$$

$$\square A_\mu - \partial^\mu(\partial_\nu A^\nu) = 0$$

Ec. Dirac

$\psi \sim$  función de onda

$x, p$

lagrangiano

$\psi \sim$  variable dinámica (clásica!)

$$\psi, \pi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\psi}}$$