FISICA DE LAS INTERACCIONES FUNDAMENTALES

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CLASE 7

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CLASE 7: Covariancia y transformación de espinores

Covariancia: *todos tienen razón*

en S $x^\mu = (ct, \overrightarrow{x})$

en S′ $x'^\mu = (ct', \overrightarrow{x'})$

$$x'^\mu = \Lambda^\mu_\nu \, x^\nu$$

$$x^\nu \rightarrow \begin{pmatrix} ct \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \Lambda \rightarrow \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{boost\ en\ x} \quad \begin{array}{c} \gamma \equiv 1/\sqrt{1-\beta^2} \\ \beta \equiv v/c \end{array}$$

vector $V_i(x) \longrightarrow V'_i(x') = R_{ij}\, V_i(x)$ R_{ij} matriz de 3x3

cuadrivector $A^\mu(x) \longrightarrow A'^\mu(x') = \Lambda^\mu_\nu \, A^\nu(x)$

$\psi(x)?$ $\psi(x) \longrightarrow \psi'(x') = S(\Lambda)\, \psi(x)$

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Covariancia: *todos tienen razón*

en S $$\left(i\hbar\gamma^\mu \frac{\partial}{\partial x^\mu} - mc\right)\psi(x) = 0 \qquad (1)$$

en S′ $$\left(i\hbar\gamma^\mu \frac{\partial}{\partial x'^\mu} - mc\right)\psi'(x') = 0 \qquad (2)$$

$$x'^\nu = \Lambda^\nu_\mu \, x^\mu$$

$$\frac{\partial}{\partial x^\mu} = \frac{\partial x'^\nu}{\partial x^\mu}\frac{\partial}{\partial x'^\nu} = \Lambda^\nu_\mu \frac{\partial}{\partial x'^\nu}$$

$$\psi(x) = S^{-1}(\Lambda)\psi'(x')$$

en (1) $$\left(i\hbar\, \gamma^\mu \Lambda^\nu_\mu \frac{\partial}{\partial x'^\nu} - mc\right) S^{-1}(\Lambda)\, \psi'(x') = 0$$

$S \rightarrow$ $$\left(i\hbar\, S(\Lambda)\gamma^\mu\, \Lambda^\nu_\mu\, S^{-1}(\Lambda) \frac{\partial}{\partial x'^\nu} - mc\right)\psi'(x') = 0$$

$$S(\Lambda)\, \gamma^\mu\, \Lambda^\nu_\mu\, S^{-1}(\Lambda) = \gamma^\nu$$

$$S(\Lambda)\, \Lambda^\nu_\mu\, \gamma^\mu\, S^{-1}(\Lambda) = \gamma^\nu$$

$$\Lambda^\nu_\mu\, \gamma^\mu = S^{-1}\,(\Lambda)\,\gamma^\nu\, S(\Lambda)$$

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Ej. rotación en z:

para un espinor S=1/2 $\sim e^{-i\frac{\theta}{2}\vec{\sigma}\cdot\hat{n}}$

$$S_{rot_z}(\Lambda) = e^{-i\frac{\theta}{2}\Sigma_3} = \cos\frac{\theta}{2} - i\Sigma_3 \sin\frac{\theta}{2} = e^{\gamma_1\gamma_2\frac{\theta}{2}} \qquad \Sigma_3 \equiv \begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix} = i\gamma_1\gamma_2$$

$$\sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu, \gamma_\nu] = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) = i\gamma_\mu\gamma_\nu \qquad (\mu \neq \nu)$$

$$S_{rot_z}(\Lambda) = e^{-i\sigma_{12}\frac{\theta}{2}}$$

Ej. boost en x:

$$\begin{cases} x'^1 = x^1 cos\theta - x^2 sen\theta \\ x'^2 = x^2 cos\theta + x^1 sen\theta \end{cases} \qquad \begin{cases} x'^0 = \gamma x^0 - \gamma\beta x^1 = x^0 cosh\omega - x^1 senh\omega \\ x'^1 = \gamma x^1 - \gamma\beta x^0 = x^1 cosh\omega - x^0 senh\omega \end{cases}$$

$$S_{boost_x}(\Lambda) = e^{-i\sigma_{01}\frac{\omega}{2}} = e^{-\alpha_1\frac{\omega}{2}}$$

$$cosh\omega \equiv \gamma \qquad senh\omega \equiv \gamma\beta \qquad tgh\omega = \beta = v/c$$

$$-i\sigma_{01} = \gamma_0\gamma_1 = -\gamma^0\gamma^1 = -\beta\beta\alpha_1 = -\alpha_1$$

$$S_{boost_x}(\Lambda) = \cosh\frac{\omega}{2} - \alpha_1 \sinh\frac{\omega}{2} = \cosh\frac{\omega}{2} - \gamma^0\gamma^1 \sinh\frac{\omega}{2}$$

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inversa: $S^{-1}(\Lambda)$ $\quad \theta \to -\theta \quad \omega \to -\omega \qquad S^{-1}S = \mathbb{1}$

para boosts: $$S_{boost_x}(\Lambda) = \cosh\frac{\omega}{2} - \gamma^0\gamma^1 \sinh\frac{\omega}{2}$$

$$S^{-1}S = (\cosh\frac{\omega}{2} + \gamma^0\gamma^1 \sinh\frac{\omega}{2})\ (\cosh\frac{\omega}{2} - \gamma^0\gamma^1 \sinh\frac{\omega}{2})$$
$$= (\cosh^2\frac{\omega}{2} - \gamma^0\gamma^1\gamma^0\gamma^1 \sinh^2\frac{\omega}{2}) = (\cosh^2\frac{\omega}{2} + \underbrace{\gamma^0\gamma^0}_{(+1)}\underbrace{\gamma^1\gamma^1}_{(-1)} \sinh^2\frac{\omega}{2}) = \mathbb{1}$$

para rotaciones: $$S_{rot_z}(\Lambda) = \cos\frac{\theta}{2} + \gamma^1\gamma^2 \sin\frac{\theta}{2}$$

$$S^{-1}S = (\cos\frac{\theta}{2} - \gamma^1\gamma^2 \sin\frac{\theta}{2})\ (\cos\frac{\theta}{2} + \gamma^1\gamma^2 \sin\frac{\theta}{2})$$
$$= (\cos^2\frac{\theta}{2} - \gamma^1\gamma^2\gamma^1\gamma^2 \sin^2\frac{\theta}{2}) = (\cos^2\frac{\theta}{2} + \underbrace{\gamma^1\gamma^1}_{(-1)}\underbrace{\gamma^2\gamma^2}_{(-1)} \sin^2\frac{\theta}{2}) = \mathbb{1}$$

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$$S^{-1}(\Lambda)\,\gamma^{\mu}\,S(\Lambda) = \Lambda^{\mu}_{\nu}\,\gamma^{\nu}$$

para rotaciones:

$$(\cos\frac{\theta}{2} - \gamma^1\gamma^2\sin\frac{\theta}{2})\begin{pmatrix}\gamma^0\\ \gamma^1\\ \gamma^2\\ \gamma^3\end{pmatrix}(\cos\frac{\theta}{2} + \gamma^1\gamma^2\sin\frac{\theta}{2}) = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\gamma^0\\ \gamma^1\\ \gamma^2\\ \gamma^3\end{pmatrix}$$

1ra fila $\gamma^0 S^{-1} S = \gamma^0$

4ta fila $\gamma^3 S^{-1} S = \gamma^3$

2da fila $\gamma^1(\cos\frac{\theta}{2} + \gamma^1\gamma^2\sin\frac{\theta}{2})\,(\cos\frac{\theta}{2} + \gamma^1\gamma^2\sin\frac{\theta}{2}) = (\gamma^1\cos\theta - \gamma^2\sin\theta)$

$$\gamma^1(\cos^2\frac{\theta}{2} + \gamma^1\gamma^2\gamma^1\gamma^2\sin^2\frac{\theta}{2} + 2\gamma^1\gamma^2\sin\frac{\theta}{2}\cos\frac{\theta}{2})$$

$$\gamma^1(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} + 2\gamma^1\gamma^2\sin\frac{\theta}{2}\cos\frac{\theta}{2})$$

$$\gamma^1(\cos\theta + \gamma^1\gamma^2\sin\theta)$$

$$\gamma^1\cos\theta - \gamma^2\sin\theta$$

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$$S^\dagger\gamma^0 = \gamma^0 S^{-1} \qquad S^\dagger = \gamma^0 S^{-1}\gamma^0$$

para rotaciones:

$$S = (\cos\frac{\theta}{2} + \gamma^1\gamma^2 \sin\frac{\theta}{2})$$

$$S^\dagger = (\cos\frac{\theta}{2} + \gamma^{2\dagger}\gamma^{1\dagger}\sin\frac{\theta}{2}) = (\cos\frac{\theta}{2} - \gamma^1\gamma^2\sin\frac{\theta}{2}) = (\gamma^0\gamma^0\cos\frac{\theta}{2} - \gamma^0\gamma^0\gamma^1\gamma^2\sin\frac{\theta}{2})$$

$$= (\gamma^0\cos\frac{\theta}{2}\gamma^0 - \gamma^0\gamma^1\gamma^2\gamma^0\sin\frac{\theta}{2}) = \gamma^0(\cos\frac{\theta}{2} - \gamma^1\gamma^2\sin\frac{\theta}{2})\gamma^0$$

para boosts:

$$S = \cosh\frac{\omega}{2} - \gamma^0\gamma^1\sinh\frac{\omega}{2}$$

$$S^\dagger = \cosh\frac{\omega}{2} - \gamma^{1\dagger}\gamma^{0\dagger}\sinh\frac{\omega}{2} = \cosh\frac{\omega}{2} - \gamma^0\gamma^1\sinh\frac{\omega}{2} = \gamma^0\gamma^0\cosh\frac{\omega}{2} - \gamma^0\gamma^0\gamma^0\gamma^1\sinh\frac{\omega}{2}$$

$$= \gamma^0\cosh\frac{\omega}{2}\gamma^0 + \gamma^0\gamma^0\gamma^1\sinh\frac{\omega}{2}\gamma^0 = \gamma^0(\cosh\frac{\omega}{2} + \gamma^0\gamma^1\sinh\frac{\omega}{2})\gamma^0$$

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$$S^\dagger \gamma^0 = \gamma^0 S^{-1} \qquad S^\dagger = \gamma^0 S^{-1} \gamma^0$$

$$\overline{\psi} \equiv \psi^\dagger \gamma^0$$

$$\overline{\psi'} = ? \qquad \psi' = S\psi$$

$$\psi'^\dagger = (S\psi)^\dagger = \psi^\dagger S^\dagger$$

$$\overline{\psi'} = \psi'^\dagger \gamma^0 = \psi^\dagger S^\dagger \gamma^0 = \psi^\dagger \gamma^0 S^{-1} = \overline{\psi} S^{-1}$$

$$J'^\mu = \overline{\psi'} \gamma^\mu \psi' = \overline{\psi} S^{-1} \gamma^\mu S \psi = \Lambda^\mu_\nu \overline{\psi} \gamma^\nu \psi = \Lambda^\mu_\nu J^\nu \qquad \partial_\mu J^\mu = 0$$

$$\rho^0 = \overline{\psi} \gamma^0 \psi = \psi^\dagger \psi$$ *no es invariante*

$$\overline{\psi'} \psi' = \overline{\psi} S^{-1} S \psi = \overline{\psi} \psi = |\psi_1|^2 + |\psi_2|^2 - |\psi_3|^2 - |\psi_4|^2$$

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Paridad Intrínseca:

$$\vec{x} \longrightarrow -\vec{x} \qquad \Lambda = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$S_p = \,? \qquad S^{-1}(\Lambda)\,\gamma^\mu\, S(\Lambda) = \Lambda^\mu_\nu\, \gamma^\nu$$

$$S_p^{-1}\,\gamma^0\, S_p = \gamma^0 \qquad \gamma^0 S_p = S_p\,\gamma^0$$

$$S_p^{-1}\,\gamma^i\, S_p = -\,\gamma^i \qquad \gamma^i S_p = -\,S_p\,\gamma^i$$

$$S_p = \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

partículas y antipartículas tienen paridades opuestas