Caso 1: Ruptura espontánea de simetría. Simetría discreta, $\mathcal{L}(\phi) = \mathcal{L}(\phi'),\ \phi' = -\phi$.

$$\mathcal{L} = \tfrac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - \tfrac{1}{2}\,\mu^2\phi^2 - \tfrac{1}{4}\,\lambda\,\phi^4 \qquad \text{Mínimo } v = \sqrt{\tfrac{-\mu^2}{\lambda}} \quad \longrightarrow \quad \boxed{\phi(x) = v + h(x)}$$

$$\begin{aligned}
&= \tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \tfrac{1}{2}\,\mu^2(v+h)^2 - \tfrac{1}{4}\,\lambda(v+h)^4 \\
&= \tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \tfrac{1}{2}\,\mu^2 v^2 - \tfrac{1}{2}\,\mu^2 2vh - \tfrac{1}{2}\,\mu^2 h^2 - \tfrac{1}{4}\,\lambda v^4 - \tfrac{1}{4}\,\lambda 4v^3 h - \tfrac{1}{4}\,\lambda 6 v^2 h^2 - \tfrac{1}{4}\,\lambda 4 v h^3 - \tfrac{1}{4}\,\lambda h^4 \\
&= \tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \underbrace{\tfrac{1}{2}\,\mu^2 v^2 - \tfrac{1}{4}\,\lambda v^4}_{\text{constante}} - \underbrace{\mu^2 vh - \lambda v^3 h}_{\text{cero: } \mu^2 = -v^2\lambda} - \underbrace{\tfrac{1}{2}\,\mu^2 h^2 - \tfrac{2}{3}\,\lambda v^2 h^2}_{\lambda v^2 h^2} - \lambda v^3 h - \tfrac{1}{4}\,\lambda h^4
\end{aligned}$$

$$\mathcal{L} = \tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \lambda v^2 h^2 - \lambda v h^3 - \tfrac{1}{4}\,\lambda h^4 \qquad \text{Masa } m_h = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

Caso 2: Teorema de Goldstone. Simetría contínua, $\mathcal{L}(\phi) = \mathcal{L}(\phi'),\ \phi' = e^{i\alpha}\phi$.

$$\mathcal{L} = \tfrac{1}{2}\,(\partial_\mu\phi_1\,\partial^\mu\phi_1 + \partial_\mu\phi_2\,\partial^\mu\phi_2) - \tfrac{1}{2}\,\mu^2(\phi_1^2 + \phi_2^2) \qquad \text{Dos campos reales KG. Sea } \phi = \tfrac{1}{\sqrt{2}}\,(\phi_1 + i\phi_2)$$

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - \mu^2\phi^*\phi - \lambda\,(\phi^*\phi)^2 \qquad \text{Mínimo } \sqrt{\tfrac{-\mu^2}{2\lambda}} = \tfrac{1}{\sqrt{2}}\,v \quad \longrightarrow \quad \boxed{\phi(x) = \tfrac{1}{\sqrt{2}}\,e^{i\frac{\theta(x)}{v}}\,[v + h(x)]}$$

$$\mathcal{L} = \tfrac{1}{2}\,[\partial_\mu e^{-i\theta/v}(v+h)]\,[\partial^\mu e^{i\theta/v}(v+h)] - \tfrac{1}{2}\,\mu^2(v+h)^2 - \tfrac{1}{4}\,\lambda(v+h)^4$$

$$\mathcal{L} = \tfrac{1}{2}\,[-\tfrac{i}{v}\,\partial^\mu\theta\,e^{-i\theta/v}(v+h) + e^{-i\theta/v}\,\partial^\mu h]\,[\tfrac{i}{v}\,\partial_\mu\theta\,e^{i\theta/v}(v+h) + e^{i\theta/v}\,\partial_\mu h] \ + \cdots$$

$$\mathcal{L} = \underbrace{\tfrac{1}{2}\,\partial_\mu\theta\,\partial^\mu\theta}_{m_\theta^2 = 0} + \underbrace{\tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \lambda v^2 h^2}_{m_h^2 = -2\mu^2} - \lambda v h^3 - \tfrac{1}{4}\,\lambda h^4 + \frac{1}{v}\,\partial_\mu\theta\,\partial^\mu\theta\,h + \frac{1}{2v^2}\,\partial_\mu\theta\,\partial^\mu\theta\,h^2$$

Caso 3: Mecanismo de Higgs. Simetría continua local, $\mathcal{L}(\phi) = \mathcal{L}(\phi'),\ \phi' = e^{i\alpha(x)}\phi$.

Si remplazamos $\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$, $\mathcal{L}$ es invariante ante $\begin{cases} \phi(x) \to \phi'(x) = e^{i\alpha(x)}\,\phi(x) \\ A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \tfrac{1}{q}\,\partial_\mu\alpha(x) \end{cases}$

$$\mathcal{L} = (\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi - \mu^2\phi^*\phi - \lambda\,(\phi^*\phi)^2 - \tfrac{1}{4}\,F_{\mu\nu}F^{\mu\nu} \quad \longrightarrow \quad \boxed{\phi(x) = \tfrac{1}{\sqrt{2}}\,e^{i\frac{\theta(x)}{v}}\,[v + h(x)]}$$

$$\begin{aligned}
&= \tfrac{1}{2}\,(\partial_\mu - iqA_\mu)e^{-i\theta/v}(v+h)\,(\partial^\mu + iqA^\mu)e^{i\theta/v}(v+h) - \tfrac{1}{2}\,\mu^2(v+h)^2 - \tfrac{1}{4}\,\lambda(v+h)^4 - \tfrac{1}{4}\,F_{\mu\nu}F^{\mu\nu} \\
&= \tfrac{1}{2}\,[-\tfrac{i}{v}\,\partial_\mu\theta\,e^{-i\theta/v}(v+h) + e^{-i\theta/v}\,\partial_\mu h - iqA_\mu\,e^{-i\theta/v}(v+h)] \\
&\qquad [+\tfrac{i}{v}\,\partial^\mu\theta\,e^{+i\theta/v}(v+h) + e^{+i\theta/v}\,\partial^\mu h + iqA^\mu\,e^{+i\theta/v}(v+h)] \ + \cdots \\
&= \tfrac{1}{2}\,[\tfrac{1}{v^2}\,\partial_\mu\theta\,\partial^\mu\theta\,(v+h)^2 + \partial_\mu h\,\partial^\mu h + q^2 A_\mu A^\mu(v+h)^2 + 2\,\tfrac{q}{v}\,\partial^\mu\theta A_\mu(v+h)^2] \ + \cdots \\
&= \tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \lambda v^2 h^2 + \tfrac{1}{2}\,\partial_\mu\theta\,\partial^\mu\theta + \tfrac{1}{2}\,q^2 v^2\,A_\mu A^\mu + qvA_\mu\partial^\mu\theta - \tfrac{1}{4}\,F_{\mu\nu}F^{\mu\nu} \ + \text{términos de 3 y 4 campos} \\
&= \tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \lambda v^2 h^2 + \tfrac{1}{2}\,q^2 v^2 (A_\mu + \tfrac{1}{qv}\,\partial_\mu\theta)(A^\mu + \tfrac{1}{qv}\,\partial^\mu\theta) \ + \cdots
\end{aligned}$$

El lagrangiano electrodébil parte del lagrangiano libre quiral

$$\begin{aligned}
\mathcal{L}_0 &= \bar{e}\,(i\gamma^\mu\partial_\mu - \cancel{m_e})\,e + \bar{\nu}\,(i\gamma^\mu\partial_\mu - \cancel{m_\nu})\,\nu \\
&= \bar{e}_L\,(i\gamma^\mu\partial_\mu)\,e_L + \bar{e}_R\,(i\gamma^\mu\partial_\mu)\,e_R + \bar{\nu}_L\,(i\gamma^\mu\partial_\mu)\,\nu_L + \bar{\nu}_R\,(i\gamma^\mu\partial_\mu)\,\nu_R \\
&= \sum_{i=e_L,e_R,\nu_L,\nu_R} \bar{\psi}_i\,(i\gamma^\mu\partial_\mu)\,\psi_i
\end{aligned}$$

invariante ante transformaciones globales

$$\psi(x) \to \psi'(x) = e^{i\alpha_a T_a + i\beta Y}\,\psi(x)$$

	Q	T_3	Y
ν_L	0	$\frac{1}{2}$	-1
e_L	-1	-$\frac{1}{2}$	-1
ν_R	0	0	0
e_R	-1	0	-2

	Q	T_3	Y
u_L	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
d'_L	-$\frac{1}{3}$	-$\frac{1}{2}$	$\frac{1}{3}$
u_R	$\frac{2}{3}$	0	$\frac{4}{3}$
d'_R	-$\frac{1}{3}$	0	-$\frac{2}{3}$

Para que sea $\mathcal{L}(\psi) = \mathcal{L}(\psi')$ ante transformaciones locales $\psi(x) \to \psi'(x) = e^{i\alpha_a(x)T_a + i\beta(x)Y}\,\psi(x)$

$$\partial_\mu \to D_\mu = \partial_\mu + ig\,T^a W^a_\mu + ig'\tfrac{1}{2}\,Y B_\mu$$

$$\begin{aligned}
\mathcal{L}_{\text{EW}} &= \sum \bar{\psi}_i\,(i\gamma^\mu D_\mu)\,\psi_i \\
&= \bar{\psi}_i\,(i\gamma^\mu\partial_\mu)\,\psi_i - gW^a_\mu(\bar{\psi}_i\gamma^\mu T^a\psi_i) - g' B_\mu(\bar{\psi}_i\gamma^\mu\tfrac{1}{2}\,Y\psi_i) \\
&= \mathcal{L}_0 \underbrace{-\frac{g}{\sqrt{2}}\,W^\dagger_\mu(\bar{\psi}_i\gamma^\mu T^+\psi_i) - \frac{g}{\sqrt{2}}\,W_\mu(\bar{\psi}_i\gamma^\mu T^-\psi_i)}_{-\frac{g}{\sqrt{2}}\,W^\dagger_\mu(\bar{\nu}_L\gamma^\mu e_L) - \frac{g}{\sqrt{2}}\,W_\mu(\bar{e}_L\gamma^\mu\nu_L)} - g\,W^3_\mu(\bar{\psi}_i\gamma^\mu T^3\psi_i) - g'\,B_\mu(\bar{\psi}_i\gamma^\mu\tfrac{1}{2}\,Y\psi_i)
\end{aligned}$$

Con

$$\begin{cases} A_\mu = W^3_\mu \sin\theta_w + B_\mu\cos\theta_w \\ Z_\mu = W^3_\mu\cos\theta_w - B_\mu\sin\theta_w \end{cases}$$

y

$$g'\cos\theta_w = g\sin\theta_w = e$$

$$\boxed{\mathcal{L}_{\text{EW}} = \mathcal{L}_0 - \frac{g}{\sqrt{2}}\,W^\dagger_\mu(\bar{\psi}_i\gamma^\mu T^+\psi_i) - \frac{g}{\sqrt{2}}\,W_\mu(\bar{\psi}_i\gamma^\mu T^-\psi_i) - eA_\mu(\bar{\psi}_i\gamma^\mu Q\psi_i) - \frac{g}{\cos\theta_w}\,Z_\mu\big[\bar{\psi}_i\gamma^\mu(T^3 - Q\sin^2\theta_w)\psi_i\big]}$$

Ruptura de simetría: Agregamos a $\mathcal{L}_{\rm EW}$ un $\mathcal{L}_\phi$ escalar invariante local $U(1)_Y \otimes SU(2)_L$

$$\mathcal{L}_\phi = (D_\mu\phi)^\dagger (D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda\,(\phi^\dagger\phi)^2$$

$$\mathcal{L}_\phi = \left[(\partial_\mu + ig\tfrac{1}{2}\,\sigma^a W^a_\mu + ig'\tfrac{1}{2}\,YB_\mu)\,\phi\right]^\dagger \left[(\partial^\mu + ig\tfrac{1}{2}\,\sigma_a W_a^\mu + ig'\tfrac{1}{2}\,YB^\mu)\,\phi\right] - \mu^2\phi^\dagger\phi - \lambda\,(\phi^\dagger\phi)^2$$

Con $\mu^2 < 0$ hay un continuo de mínimos: $\frac{1}{2}\,(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2) = |\phi|^2 = \frac{1}{2}\,v^2$ con $v^2 = \frac{-\mu^2}{\lambda}$

$$\phi(x) = \tfrac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix} = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \qquad\qquad \phi(x) = \tfrac{1}{\sqrt{2}}\,e^{i\,\frac{\theta^a(x)\,\sigma^a}{2v}}\begin{pmatrix}0\\ v+h(x)\end{pmatrix}$$

Weinberg eligió un doblete con Y = -1 así el vacío elegido tiene T_3 = -$\frac{1}{2}$ y $Q = T_3 + \frac{1}{2}\,Y = 0$:

$$g\,W^a_\mu\sigma^a + g'B_\mu Y = g\,W^1_\mu\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + g\,W^2_\mu\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} + g\,W^3_\mu\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + g'B_\mu\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}gW^3_\mu + g'B_\mu & gW^1_\mu - igW^2_\mu\\ gW^1_\mu + igW^2_\mu & -gW^3_\mu + g'B_\mu\end{pmatrix}$$

$$\mathcal{L}_\phi = \left|(\partial_\mu + ig\tfrac{1}{2}\,\sigma^a W^a_\mu + ig'\tfrac{1}{2}\,YB_\mu)\,\phi\right|^2 - \mu^2\phi^\dagger\phi - \lambda\,(\phi^\dagger\phi)^2$$

$$= \left|\partial_\mu\tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix} + \frac{i}{2}\begin{pmatrix}gW^3_\mu + g'B_\mu & gW^1_\mu - igW^2_\mu\\ gW^1_\mu + igW^2_\mu & -gW^3_\mu + g'B_\mu\end{pmatrix}\tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix}\right|^2 + \tfrac{1}{2}\,(v+h)^2 + \tfrac{1}{4}\,(v+h)^4$$

$$= \frac{1}{2}\left|\begin{pmatrix}0\\ \partial_\mu h\end{pmatrix} + \frac{i}{2}\begin{pmatrix}gW^1_\mu - igW^2_\mu\\ -gW^3_\mu + g'B_\mu\end{pmatrix}(v+h)\right|^2 - \lambda v^2h^2 - \lambda v h^3 - \tfrac{1}{4}\,\lambda h^4$$

$$= \tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \lambda v^2 h^2 + \tfrac{1}{8}v^2g^2|W^1_\mu - iW^2_\mu|^2 + \tfrac{1}{8}\,v^2\,|g'B_\mu - gW^3_\mu|^2\ + \text{ términos de 3 y 4 campos}$$

Recordar $\qquad W_\mu = \frac{1}{\sqrt{2}}(W^1_\mu + iW^2_\mu) \qquad\qquad g'\cos\theta_w = e \qquad g\sin\theta_w = e$

$$g'B_\mu - gW^3_\mu = \frac{e}{\cos\theta_w}\,B_\mu - \frac{e}{\sin\theta_w}\,W^3_\mu = \frac{-e}{\sin\theta_w\cos\theta_w}\,\underbrace{(-\sin\theta_w B_\mu + \cos\theta_w W^3)}_{Z_\mu}$$

$$\mathcal{L}_\phi = \tfrac{1}{2}\,\partial_\mu h\,\partial^\mu h - \lambda v^2\,h^2 + (\tfrac{1}{2}\,vg)^2\,W^\dagger_\mu W^\mu + \tfrac{1}{2}\left(\tfrac{e\,v}{2\sin\theta_w\cos\theta_w}\right)^2 Z_\mu Z^\mu\ + \text{ términos de 3 y 4 campos}$$

$$\left.\begin{aligned} M_W &= \frac{1}{2}\,vg = \frac{v}{2}\,\frac{e}{\sin\theta_w}\\ M_Z &= \frac{v}{2}\,\frac{e}{\sin\theta_w\cos\theta_w}\end{aligned}\right\} \Rightarrow \boxed{\frac{M_W}{M_Z} = \cos\theta_w}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \Rightarrow M_W^2 = \frac{e^2}{\sin^2\theta_w}\,\frac{\sqrt{2}}{8\,G_F} \qquad \Rightarrow \qquad \boxed{\begin{aligned}M_W &= 80\,\text{GeV}\\ M_Z &= 91\,\text{GeV}\end{aligned}}$$

Masa de los fermiones

Se agregan acoplamientos entre fermiones y Higgs que preservan la invariancia local de gauge

$$\mathcal{L}_e = -G_e \left[\begin{pmatrix} \bar{\nu}_e & \bar{e} \end{pmatrix}_L \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} e_R + \bar{e}_R \begin{pmatrix} \bar{\phi}_+ & \bar{\phi}_0 \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right]$$

Remplazamos $\phi(x)$ por su desarrollo alrededor del mínimo, en el gauge $\theta_a(x) = 0$

$$\phi(x) = \tfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_e &= -\frac{G_e}{\sqrt{2}} \left[\begin{pmatrix} \bar{\nu}_e & \bar{e} \end{pmatrix}_L \begin{pmatrix} 0 \\ v + h \end{pmatrix} e_R + \bar{e}_R \begin{pmatrix} 0 & v + h \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right] \\ &= -\frac{G_e}{\sqrt{2}} (v + h) (\bar{e}_L e_R + \bar{e}_R e_L) \\ &= -\frac{G_e\, v}{\sqrt{2}}\ \bar{e}e - \frac{G_e}{\sqrt{2}}\ \bar{e}eh \end{aligned}$$

Llamando $$m_e = \frac{G_e\, v}{\sqrt{2}}$$

$$\begin{aligned} \mathcal{L}_0 + \mathcal{L}_e &= \bar{e}_L (i\gamma^\mu \partial_\mu) e_L + \bar{e}_R (i\gamma^\mu \partial_\mu) e_R - m_e\ \bar{e}e - \frac{m_e}{v}\ \bar{e}eh \\ &= \bar{e} (i\gamma^\mu \partial_\mu - m_e) e - \frac{m_e}{v}\ \bar{e}eh \end{aligned}$$