Coaxial cable analogs of multilayer dielectric optical coatings

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We study the properties of periodic coaxial cable structures (coaxial photonic crystals), where the periodicity consists of alternating cables with low and high impedance. We show that the electrical signal that propagates through these structures leads to similar phenomena as the light propagating through the corresponding multilayer dielectric optical coating structures. In this way, Bragg reflectors, optical thin-film filters, and Fabry–Perot resonators are reproduced in the megahertz frequency range. This represents an inexpensive way of experimentally investigating wave propagation in locally periodic media. © 2003 American Association of Physics Teachers. [DOI: 10.1119/1.1603271]

I. INTRODUCTION

Photonic crystals have become an important field in the last decade, both from the fundamental research point of view and because of their technological applications.1 A photonic crystal consists of a periodic arrangement of low-loss dielectric media having different refractive indices. Consequently, the reflection of light at the interfaces produces the same phenomena for photons as the atomic potential does for electrons in a solid state crystal. Thus, photonic crystals are the optical analogs of solid-state electron crystals. The main characteristic of a photonic crystal is the existence of a photonic band gap, i.e., a frequency region in which, for certain directions and/or polarizations, light cannot propagate through the crystal and is completely reflected. If this occurs for any direction and polarization, we talk about a complete photonic band gap.1

Just as in solid state crystals, if defects are introduced in the photonic structure, then allowed modes for light propagation appear in the gap.2 Thus by designing a proper defect pattern, we can control the propagation of light through the crystal. These exciting properties are the basis of novel devices, like optical fibers based on photonic crystals (the so-called “holey” fibers)3 or microstrip circuits that filter undesired frequencies using a photonic crystal as the substrate.4

In their one-dimensional version, photonic crystals are well known as optical multilayers of alternating dielectric materials.5,6 In these structures the photonic band gap arises from the constructive interference of the multiple waves reflected at the layer interfaces. This effect is the basis of many optical devices, such as dielectric mirrors (Bragg reflectors), Fabry–Perot filters, and distributed feedback lasers.7

Photonic crystals are scalable. This means that by changing the dimensions of the crystal the same properties are obtained, only in a different frequency range. For microwave control we need millimeter dimensions, while micron dimensions are required for infrared control. Despite the advances in material technology, the difficulties of fabricating very small-scale structures make this scaling property very useful. For instance, a quasi-one-dimensional photonic crystal based on coaxial BNC connectors was used to study impunity effects in the gigahertz frequency range.8 Also, a one-dimensional photonic crystal in the radio-frequency (rf) regime was made with coaxial cables to study the effect of linear9 and nonlinear impurities,10 and the propagation of superluminal pulses.11,12

In this work, several well-known multilayer dielectric optical structures are reproduced in the rf frequency range by means of a coaxial cable photonic crystal. We construct and characterize the coaxial analogs to Bragg reflectors, dielectric optical filters, and Fabry–Perot resonators. As we will show, a coaxial crystal with an impurity (doped coaxial crystal) can be regarded as a Fabry–Perot resonator. These are simple systems that require inexpensive equipment, that is available in most laboratories. Therefore, they are useful for designing teaching experiments for physics and engineering students to explore the effects of multilayer dielectric optical coatings, and wave interference phenomena in locally periodic systems in general.13

II. THE TRANSFER MATRIX METHOD ADAPTED TO COAXIAL CABLE SYSTEMS

When dealing with a system of multilayers it is convenient to use a transfer matrix method to describe the propagation of a wave through the structure.5,14 In this section we summarize how this formalism is adapted to a system of lossless coaxial cables.

A. The dynamic matrix

The layered system is described by a dynamic matrix given by

\[ M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \]

which relates the complex amplitudes of the electric field at both ends of the system in the following way:

\[ \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_s \\ b_s \end{pmatrix}. \]

Here \( a \) is the complex amplitude of the wave propagating to the right, \( b \) is the complex amplitude of the wave propagating to the left, and we refer to the incident and final media with subindices 0 and s, respectively, as shown in Fig. 1. The initial and final media are characterized by their respective impedances \( Z_0 \) and \( Z_s \).

The reflection and transmission coefficients are simple functions of the dynamic matrix elements given by:
coefficients at the impedance mismatch surface, respectively.

where when the wave arrives from medium 2 (~0), the elements A and C of $D_{12}$ are obtained, whereas when the wave arrives from medium 2 ($a_0=0$), the elements $B$ and $D$ are obtained, leading to:

$$D_{12} = \begin{pmatrix}
1 & r_{12} \\
\bar{r}_{12} & 1
\end{pmatrix}. \tag{7}
$$

Here $r_{12}$ and $\bar{r}_{12}$ are the transmission and reflection Fresnel coefficients at the impedance mismatch surface, respectively. These coefficients take the same form as in optics, if you substitute the index of refraction by the impedance. In the case of normal incidence they are:

$$r_{12} = \frac{Z_1 - Z_2}{Z_1 + Z_2}, \tag{8}
$$

$$t_{12} = \frac{2Z_1}{Z_1 + Z_2}. \tag{9}
$$

The matrix that describes the propagation through a single cable is analogous to the optical case:

$$P = \begin{pmatrix}
e^{i\phi} & 0 \\
0 & e^{-i\phi}
\end{pmatrix}. \tag{10}
$$

Here the phase $\phi$ acquired by the traveling wave is

$$\phi = \frac{2\pi f}{v} d, \tag{11}
$$

where $v$ is the phase velocity of the wave in the coaxial cable, $f$ its frequency, and $d$ the cable length.

Usually the lengths of the cables are designed such that $\phi = \pi/2$ at the design frequency. These quarter-wave layers are characterized by the letters $H$ and $L$ where the impedance is either high or low. The total dynamic matrix of any system of coaxial cables is obtained by the proper multiplication of these two kinds of elementary dynamic matrices. Thus, the evaluation of the transmission and reflection properties of the system is just a simple computational problem.

### III. EXPERIMENTAL SETUP

We have used coaxial cables of 50-$\Omega$ (RG-58/U) and 75-$\Omega$ (RG-59/U) characteristic impedance to build our one-dimensional coaxial crystals. The structures consist mainly of several quarter wave unit cells, where each unit cell has a 50-$\Omega$ cable (low impedance, $L$) and a 75-$\Omega$ cable (high impedance, $H$) connected in series. The cables are connected with 50-$\Omega$ barrel connectors. Three-feet (0.9-m) and six-feet (1.8-m) cables have been used for both types of cables $L$ and $H$. A 93-$\Omega$ (RG63/U) coaxial cable is used to introduce a defect in the structure. A sweep signal generator (Wavetek 1062) is used to produce the input sinusoidal waveform and also provides the internal detector circuit that converts the transmitted rf signal to a dc signal. A 500-MHz digital oscilloscope (Tektronix model TDS-3054) records the transmitted voltage as a function of frequency. A regular wave function generator and an analog oscilloscope can also be used. In this case, the frequency sweep must be performed manually.

### IV. RESULTS

#### A. Bragg reflectors

The first structures we have studied are the electric analogs of the $s(HL)^N a$ optical multilayer mirrors\textsuperscript{14} where $s$ is the glass substrate, $a$ is air, and $H$ and $L$ are the high and low index of refraction films, repeated $N$ times. The combination $HL$ is often called a unit cell and has a length equal to half of the design wavelength.

The generator and the oscilloscope are located at the ends of our coaxial crystal. Unlike optical structures, the initial medium $a$ and the substrate $s$ have the same impedance as the $L$ layers. Consequently to avoid reflections of the signal at the connections to the generator and to the oscilloscope (of 50-$\Omega$ output and input impedances, respectively), additional 50-$\Omega$ cables having an arbitrary length ($L'$) are placed in the extremes, thus eliminating the impedance mismatch. Therefore the cable structure would be written as $L'(HL)^N$. Because there is no impedance change between the last $L$ layer and the $L'$ layer, we introduce an additional $H$ layer and consider structures of the form $L'(HL)^N$. From now on, we will drop the $L'$ notation in order to simplify the expressions. Consequently our structures are written as...
\( (HL)^N H \). \hspace{1cm} (12)

Figure 2 shows the transmission of the \( (HL)^N H \) coaxial crystal structure for different numbers of unit cells of \( N = 3, 5, \) and 10. For these experiments, we used 3-ft cables, which correspond to a quarter wavelength for the design frequency of 55 MHz. The impedance mismatch at the interfaces between the \( L \) and the \( H \) cable causes the reflection of the wave, with a reflection coefficient given by Eq. (8) where \( Z_1 \) and \( Z_2 \) are the impedances of the first and second cables. A maximum in the reflectance, and consequently, a minimum in the transmittance, occurs when the multiple reflected waves in the backward direction interfere constructively. This occurs when the phase acquired by the wave in a round trip along the unit cell is a multiple of 2. This condition corresponds to the length of the unit cell being a multiple of half wavelengths; or for frequencies

\[
\hat{f} = \frac{m}{2d}, \hspace{1cm} (13)
\]

Here \( \hat{v} \) is the phase velocity of the signal and \( d \) is the unit cell length. The phase velocity of the wave in a 50-\( \Omega \) cable is \( 2c/3 \), where \( c \) is the speed of light in vacuum. As shown in Fig. 2, the central frequency of the transmission gap occurs at the expected value of 55 MHz. As we increase the number of unit cells we observe several consequences. The transmission minimum is centered at 55 MHz and becomes deeper and wider, eventually yielding a forbidden band in transmission that is 10 MHz wide [full width at half maximum (FWHM)], with a gap depth of almost 95% for \( N = 10 \). This behavior is similar to the results with dielectric stacks, where the optical transmission of the substrate is altered drastically by the multilayer structure. Therefore, Fig. 2 shows the electric analog of a dielectric mirror or Bragg reflector. Finally, the transmitted voltage decreases with increasing frequency and number of unit cells because of the attenuation of the signal along the cables. This attenuation increases as the structure length increases and as the frequency increases.

Next, we changed the length of the unit cell by using \( L \) cables having a length of 6 ft and \( H \) layers having a length of 3 ft. Now the central frequency of the gap decreases to 35 MHz as shown in Fig. 3. We also see the second-order gap at 70 MHz, in agreement with Eq. (13). This interesting result is not discussed in most initial treatments. The important factor in determining the central frequency of the gap is that the unit cell length must correspond to half a wavelength. This is independent of the individual lengths of the \( H \) and \( L \) layers that constitute the unit cell.

B. Apodized optical filters

One major issue in the design of optical filters is to remove the sidelobes that appear at both sides of the transmission gap. In the case of optical multilayers this is achieved by gradually decreasing the thickness of the layers on either side of the stack. This is known as apodizing. One of the simplest approaches consists of adding an eighth-wave high-index layer at both ends. Thus, the basic structure \( (HL)^N H \) becomes:

\[
(0.5H)L(0.5L)^N H (0.5H). \hspace{1cm} (14)
\]

Here 0.5\( H \) represents an \( H \) cable of half the length of the regular cable and corresponds to an eighth wavelength for the design frequency of 55 MHz. Figure 4(a) shows the results for \( N = 5 \) where no apodizing is present. The sidelobes at both high and low frequencies are quite high. By applying the apodizing structure mentioned above, the low-frequency sidelobes in Fig. 4(b) are now reduced while the high-frequency sidelobes have increased. Its counteranalog, the one reducing the high-frequency sidelobes, would be obtained by adding an eighth wave low-index layer at both ends. The corresponding coaxial cable structure would be \((0.5L)(HL)^N H (0.5L)\). But once again, since both eighth-wave segments have the same impedance as the cables connecting the equipment, this structure is just equivalent to \( (HL)^N H \), only with higher attenuation. Therefore we cannot build this structure.

More sophisticated apodized filters can be built by alternating the relative lengths of the \( H \) and \( L \) of each unit cell. As noted above, each unit cell length must correspond to half a wavelength but we can vary the individual lengths of each \( L \) and \( H \) segment. Some experimental results are also shown in Fig. 4(c), which corresponds to the structure:

\[
\{(\frac{1}{3}H)(\frac{2}{3}L)\} \{(\frac{1}{3}H)(\frac{1}{3}L)\} (HL)^N \{(\frac{1}{3}H)(\frac{2}{3}L)\} \{(\frac{1}{3}H)(\frac{2}{3}L)\}. \hspace{1cm} (15)
\]

The number that accompanies the impedance symbol is the length of the cable given as the fraction of the regular length (3 ft). The transmittance curves show that the low- and high-frequency sidelobes have been simultaneously removed.
C. Fabry–Perot interferometer and impurity layer

The simplest optical resonator, the Fabry–Perot interferometer, can also be reproduced with coaxial cables. In this case, the mirrors are coaxial mirrors of the kind discussed in Sec. IV A and the mirror spacing is another coaxial cable \( L' \) having a length \( d \). The structure we will consider is

\[
(HL)^N H - L' - (HL)^N H. \tag{16}
\]

Constructive interference of the multiple waves traveling along the spacing \( L' \) occurs when \( 2d \) is a multiple of the wavelength. The resonant frequencies can be written as

\[
f_m = m \frac{v}{2d}. \tag{17}
\]

These are the well-known resonant transmission frequencies of a Fabry–Perot interferometer.\(^5,6\) The multilayer mirrors for our Fabry–Perot have a transmission gap centered at about 55 MHz with a FWHM of about 10 MHz as shown in Fig. 2. Therefore the Fabry–Perot resonances of order \( m \) occur for distances \( d \) that are roughly integral multiples of 6 ft in order to coincide with the reflectivity maximum of the mirrors at 55 MHz.

The finesse \( (F) \) of a Fabry–Perot interferometer is given by the ratio of the free spectral range (the separation of adjacent resonances) and the FWHM of the resonances.\(^5,6\) It is a key parameter that measures the resolving power of the Fabry–Perot spectrum analyzer. We have studied the finesse...
of our coaxial Fabry–Perot varying both the length of the spacing $d$ and the reflectivity of the mirrors. The results are summarized in Figs. 5 and 6.

Figure 5 shows how the transmission changes when the length $d$ of the etalon is increased. Several effects are observed. First, the height of the resonance is not 100%, but is substantially lower. In addition, the height of the resonance decreases as the length of the separation $L''$ increases. This occurs because of losses within the Fabry–Perot cavity and these are usually ignored. Finally, more resonances appear in the gap as we increase the length $L''$, which is equivalent to a decrease in the free spectral range. This evolution occurs also in the optical Fabry–Perot interferometer.\textsuperscript{5,6}

Figure 6 shows changes in the transmission when we vary the number of unit cells and change the reflectivity of the coaxial mirrors. The reflectivity of the coaxial mirrors increases with the number of unit cells $N$, as we have previously discussed. For smaller reflectivities (smaller $N$) the resonance width increases, thereby lowering the finesse. In addition, the height of the resonance increases as we decrease the reflectivity of the mirrors. This is a consequence of the decreased absorption of the mirrors. Since we have coaxial mirrors, a change in $N$ not only changes the reflectivity but also the total absorption of the mirror structure. When we decrease the reflectivity of the mirrors by decreasing $N$, we are also decreasing the absorption and as a result, the resonance peak is higher.

Next we investigate the role of impurities in our mirror structure. Now we interrupt the periodicity in our coaxial crystal by introducing an impurity cable having a different impedance, for instance $93\, \Omega$, a length of 3 ft and represented by the letter $I$. Note that for this cable, the phase velocity $v$ is $0.85\, c$. The structure built is:

![Fig. 6. Transmitted voltage for a coaxial Fabry–Perot ($H_1L_1)^N H_1$] with cables $L_1=(50\, \Omega, 3\, \text{ft})$ and $H_1=(75\, \Omega, 3\, \text{ft})$ and mirror spacing $L''=6\, \text{ft}$ for (a) $N=2$, (b) $N=4$, (c) $N=5$.

![Fig. 7. Transmitted voltage for a Bragg reflector with an impurity $I$ = $(93\, \Omega, 3\, \text{ft})$ for different impurity locations. The structure is $(H_1L_1)^N I (H_1L_1)^N H_1$ with cables $L_1=(50\, \Omega, 3\, \text{ft})$ and $H_1=(75\, \Omega, 3\, \text{ft})$. (a) $N_1=N_2=5$, (b) $N_1=3$, $N_2=7$, (c) $N_1=7$, $N_2=3$.]


Here the indices $N_1$ and $N_2$ reflect the fact that the number of unit cells on either side of the impurity can be varied. The resulting transmittance curve shows a peak inside the gap (see Fig. 7). The same effect occurs when we dope a semiconductor and an impurity state appears in the forbidden band. Thus, the 93-Ω cable can be regarded as a defect or impurity of our coaxial crystal. Interestingly if we change $N_1$ and $N_2$ in order to move the defect along the crystal, the impurity peak also moves inside the gap. As the defect approaches one end of the coaxial crystal the peak moves to higher frequencies, whereas as it approaches the other end, it moves to lower frequencies. We believe that this is due to phase shifts introduced by the multilayer coaxial mirrors.

This subject is currently under investigation and will be the subject of future work. Also, there is a significant widening of the transmission gap when the impurity is introduced. These results agree with previous studies, and with the observed widening of the optical stop bands in real photonic crystals. Therefore, the universal behavior of waves in doped (locally) periodic structures can be used to study real crystals, and has in fact been used to mimic real photonic crystals.

The doped coaxial crystal of Eq. (18) can also be regarded as a Fabry–Perot interferometer with the impurity cable serving as the mirror spacing. Therefore, the resonance peak should appear when constructive interference of the multiple waves traveling along the mirror spacing occurs.

V. CONCLUSIONS

In this work we have explored the analogies between a system of coaxial cables with periodicity in the impedance, and a system of dielectric stacks with periodicity in the index of refraction. The latter is a photonic crystal with propagation control in the optical range, while the former can be regarded as a photonic crystal for rf control. We have reproduced electrical analogs for widely used thin-film optical devices, such as Bragg reflectors, dielectric filters, and Fabry–Perot optical resonators. These results show the similarities with multilayer dielectric optical coatings and allow students to perform experiments that would be extremely difficult in the optical region. We also stress that the underlying physics is common to the propagation of waves in many other locally periodic systems. Therefore, coaxial crystals can be of interest for pedagogical purposes. They are useful for physics and engineering students to experience the effects of light propagation in one-dimensional photonic crystals, and wave interference phenomena in general. These experiments are also relatively inexpensive and utilize equipment available in most science laboratories.

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