www.iop.org/journals/physed

An introductory physics exercise using real extrasolar planet data

Michael C LoPresto and Rachel McKay

Henry Ford Community College, Dearborn, MI 48128, USA

E-mail: lopresto@hfcc.edu

Abstract

This paper describes an exercise that makes use of actual data available on the internet that were used in the discovery of extrasolar planets. The exercise involves deriving equations that are used to determine the planets' orbital radii and masses and to write equations that when plotted will match the waveforms of the fits to the actual data. The process necessitates learning about the methods of detection, the physics used in the calculations of the planets' properties and superposition of waves. The exercise is at the level of introductory physics and can be used as an independent or directed study.

Introduction

Data from a continuously growing list of over 100 'exoplanets' orbiting Sun-like stars are readily available on the internet [1]. What follows is an exercise focusing on the Doppler detection method [2, 3] that makes use of these data, thus providing the means to introduce this cutting-edge research to students of introductory physics, while also providing an educational experience beyond that which is generally available in the classroom, especially at teaching-oriented institutions.

Doppler detection method

The presence of a planet can be detected indirectly by observation of Doppler shifts in the spectrum of the star it orbits [4]. As the star and the planet orbit the centre of mass of the system, the period of oscillation between red and blue shifts in a star's spectrum will reveal the planet's orbital period, p (see figure 1).

Kepler's third law, approximated for a circular orbit and $M > m$, where M is the known mass of the star[1] and m is the mass of the planet, gives

$$\frac{p^2}{a^3} \approx \frac{4\pi^2}{GM}$$

which can be used to determine the orbital radius, a. The orbital speed of the star is $K = 2\pi r/p$, where r is the star's distance from the centre of mass of the system. Using the star as the origin, r can be approximated as $r \approx (m/M)a$ (see box 1). Substitution then gives $K = 2\pi ma/pM$. This expression can then be rearranged to calculate the lower limit of the planet's mass[2].

Eccentric orbits

A plot of the varying Doppler shifts will appear as a simple sine or cosine wave if the eccentricity

---

[1] Stellar masses derived from Hipparcos, metallicity and stellar evolution; see astro.estec.esa.nl/Hipparcos/ for details and access to the catalogue.

[2] The mass is considered a lower limit because we do not know whether we are viewing the systems edge-on, in which case our mass would be accurate, or if we are viewing the systems at an angle to the line between the plane of the orbits and Earth. In that case only a component of the star's velocity would be directed towards Earth, see obswww.unige.ch/~udry/planet/method.html for an excellent diagram showing this.

0031-9120/05/010046+05$30.00

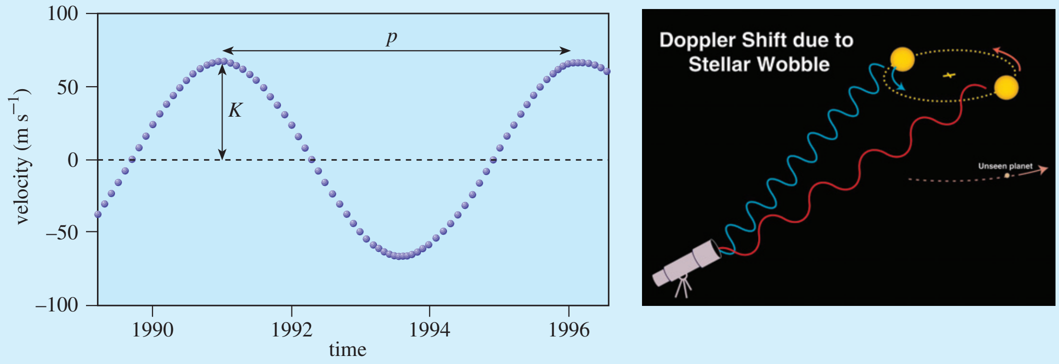
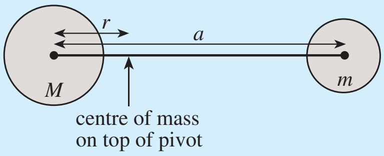


Figure 1. As a star and a planet orbit the centre of mass of their system, Doppler shifts in the star's spectrum will vary from a maximum blueshift, when the star is moving toward Earth, to a maximum redshift when it is moving away (from [2] with permission of webmaster).

Box 1.

Imagine that we could lift the star–planet system up and put it on a pivot:



If the pivot really were at the centre of mass then the whole thing should balance and the moments on each side should be the same, hence we can write

$$rM = m(a - r).$$

But if M is much bigger than m then r will be small compared with a so we can write

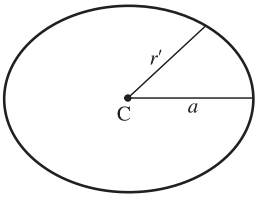
$$rM \simeq ma$$

which gives $r \approx (m/M)a$.

of the orbit is low (see figure 1). If the orbit is more elliptical, such as is the case for 70 Virginis (see figure 2) the plot will reflect that. The star's orbital velocity will change as the planet's distance from the centre of mass varies. The planet's orbital radius, r', is therefore given by the equation for an ellipse,

$$r' = a\frac{1 - e^2}{1 + e\cos(2\pi t/p)}$$

where a is now the semi-major axis of the elliptical orbit and e the eccentricity [5].



An equation for a waveform that should appear similar to the fit of the actual data for a planet in an eccentric orbit is

$$v = K\frac{1 - e^2}{1 + e\cos(2\pi t/p)}\sin\left(\frac{2\pi}{p}t + \phi\right). \quad (1)$$

See figure 2 for a comparison of a plot from this expression with $p = 116.6$ days, $K = 315$ m s^{-1} and $e = 0.4$ [6] and phase constant $\phi = \frac{3}{2}\pi$ with the fit of the data for 70 Virginis. Physically, the phase of the waveform represents the initial position of a planet in its orbit.

This can be done for any star with a planet in an eccentric orbit. Other planets were attempted because of their interesting waveforms, including 16 Cygni B [7] and HD 89744 [8].

Multiple planet systems

If a star has several planets in orbit, each planet will contribute to the variations in the Doppler

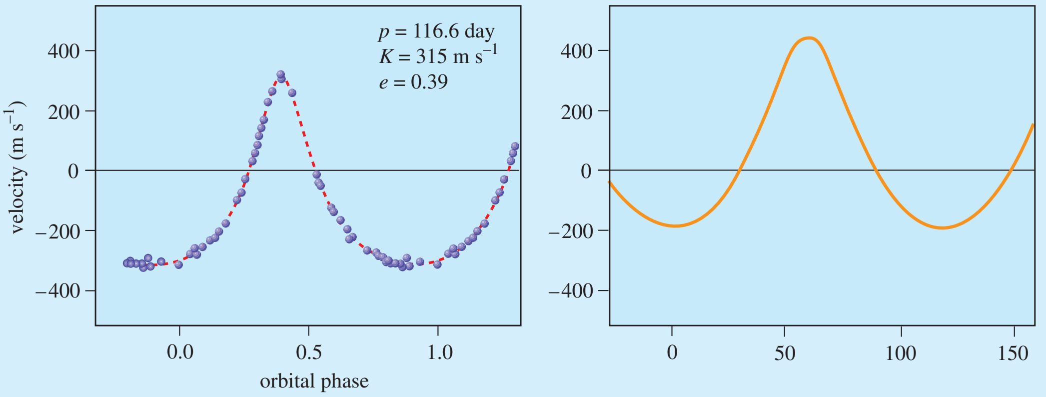


Figure 2. Fit to Doppler shift data for 70 Virginis (adapted from [6] with permission of webmaster) compared with a plot from equation (1).

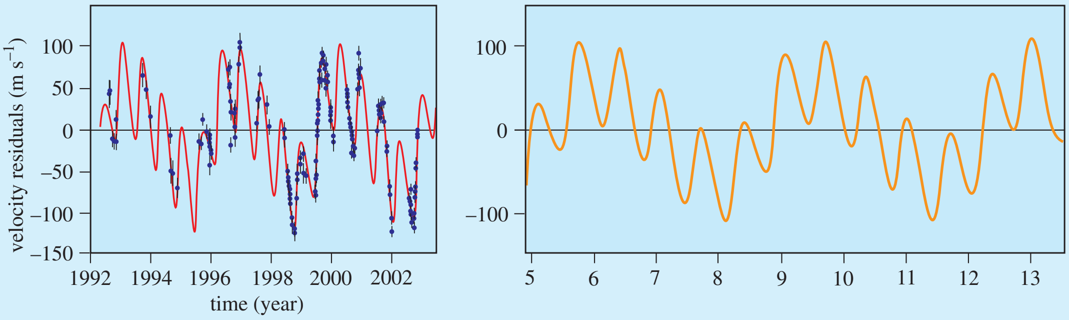


Figure 3. Fit to Doppler shift data for the Upsilon Andromedae system (adapted from [9] with permission of webmaster) compared with a plot from equation (2).

shifts, resulting in a complex waveform. Figure 3 shows the fit to the data for Upsilon Andromedae [9], the first multiple system discovered. In order to reproduce the waveform, the equations for the Doppler shifts of each individual planet must be combined. This amounts to an applied introduction to superposition of waves or even simple Fourier series.

An equation that will produce plots that match the fits of actual data must contain terms for each planet in the system,

$$v = \sum_{n=1}^{N} \frac{1 - e_n^2}{1 + e_n \cos(2\pi t/p_n)} K_n \sin\left(\frac{2\pi}{p_n} t + \phi_n\right). \tag{2}$$

A plot from this expression is compared with the actual fit to the Upsilon-Andromedae system data in figure 3. Choosing the phase constants that will result in a similar plot to the one from the actual data is simpler if the waveforms for each planet in the system are first matched individually[3].

Other multiple-planet systems for which the above was also done include HD 168443 [10] and 47 Ursa Majoris [11].

Discussion

This project had its origin in a directed study taken by the second author as a student with the first as the instructor. There were weekly meetings over the course of an academic year in which the student was concurrently taking the introductory calculus-based physics sequence (with a different instructor). Initially, the student

---

[3] The waveforms for individual planets with others in the systems removed are found in the references for the multiple systems.

was directed to exoplanets.org and instructed to become familiar with the Doppler detection method and the available data, then to describe the method to the instructor both verbally and in writing. After a review of the relevant physics, which had been recently studied in the introductory physics course, an attempt was made to derive the expressions (given above) used in the Doppler detection method and to verify the orbital radius and mass for several planets in low-eccentricity orbits. The values for the orbital periods, velocities and stellar masses given in the data [6–8] were used. The derivation did require some direction from the instructor, but ultimately the student was able to work it out independently.

At this point, equations (1) and (2) were developed and applied to selected systems. The plots were compared with the waveforms produced from the fits of the actual data. The focus was initially on single-planet systems with eccentric orbits [6–8] then afterward on multiple-planet systems [9–11]. Writing the equations and comparing the waveforms required guidance from the instructor at first, but by the end of the project the student was able to do this independently.

Another intriguing investigation that was also undertaken was to use the expression for K to calculate the orbital velocities of the Sun caused by each of the planets in our own solar system. An equation could then be written that produced a waveform that represented what an observer from afar might see when examining our solar system with the Doppler detection method [12].

The student eventually gave two presentations of a poster on the project—one at a seminar for Honor's program students and another as part of a campus-wide cultural activities program open to the public. The instructor presented the project at section [13] and national meetings of the American Association of Physics Teachers [14].

The availability of the actual data used in the discovery of exoplanets afforded the student a vastly different learning experience from what normally occurs in the traditional physics classroom at a (two-year or four-year) teaching-oriented institution in the United States. Students at institutions of this type often do not have much hands-on access to research, so this kind of experience is rare and therefore very valuable. This is especially true for a student who plans to continue his/her education at a research university.

Besides finding the project intriguing and challenging, the student commented that, in hindsight, just as she had hoped, the project turned out to be excellent preparation for the more advanced studies in Atmospheric and Space Science that she has since gone on to undertake.

The instructor found the experience of providing a student with examples of how the physics (and mathematics) being learned in the classroom can be applied to research very rewarding.

Acknowledgments

We thank HFCC Honor's Program Director Nabeel Abraham for allowing Rachel to continue in the program to finish this project. We also thank the UC-Berkeley Webmaster for permission to use figures 1–3.

Received 9 June 2004, in final form 6 September 2004
doi:10.1088/0031-9120/40/1/001

References

[1] California & Carnegie Planet Search, exoplanets.org

[2] exoplanets.org/doppframe.html

[3] Jones H and Newsam A 2003 Exoplanets *Phys. Educ.* **38** 211–7

[4] www.howstuffworks.com/planet-hunting2.htm

[5] Spiegel M R 1968 *Mathematical Handbook of Formulas and Tables* (New York: McGraw-Hill) equation 10.24, p 38 the equation for an ellipse in polar coordinates

[6] Data available from exoplanets.org/almanacframe.html. Source: Marcy G W and Butler R P 1996 A planetary companion to 70 Virginis *Astrophys. J. Lett.* **464** L147

[7] Data available from exoplanets.org/almanacframe.html. Source: Cochran W D, Hatzes A P, Butler R P and Marcy G W 1997 The discovery of a planetary companion to 16 Cygni B *Astrophys. J.* **483** 457

[8] Data available from exoplanets.org/almanacframe.html. Source: Korzennik S G, Brown T M, Fischer D A, Nisenson P and Noyes R W 2000 A high-eccentricity low-mass companion to HD 89744 *Astrophys. J. Lett.* **533** L147–50

[9] Values for K, p and e for each planet in the system are available from exoplanets.org/multframe.html. Source: Butler R P, Marcy G W, Fischer D A, Brown T M, Contos A R, Korzennik S G, Nisenson P and Noyes R W 1999 Evidence for multiple

companions to Upsilon Andromedae *Astrophys. J.* **526** 916

[10] Values for K, p and e for each planet in the system are available from exoplanets.org/multframe.html. Source: Marcy G W, Butler R P, Vogt S S, Fischer D and Liu M C 1999 Two new candidate planets in eccentric orbits *Astrophys. J.* **520** 239–47

[11] Values for K, p and e for each planet in the system are available from exoplanets.org/multframe.html. Source: Fischer D A, Marcy G W, Butler R P, Laughlin G and Vogt S S 2002 A second planet orbiting 47 Ursae Majoris *Astrophys. J.* **564** 1028–34

[12] LoPresto M and McKay R 2004 Detecting our own solar system from afar *Phys. Teacher* **42** 208–11

[13] American Association of Physics Teachers 2003 *Announcer* **33** (Summer) 141

[14] American Association of Physics Teachers 2003 *Announcer* **33** (Summer) 27–8

Michael C LoPresto is chair of the Physics Department at Henry Ford Community College in Dearborn, Michigan. He is the author of an astronomy laboratory manual and a textbook supplement *Cycles in the Sky*.

Rachel McKay is a former HFCC Honor's Program student who now attends the University of Michigan. She is on track to graduate in May 2005 with a degree from the College of Engineering's Department of Atmospheric, Oceanic and Space Sciences.