Flight without Bernoulli

Chris Waltham

The origin of this work is threefold. First, I have a lifelong interest in aviation. Second, I have been teaching first year-physics courses so long that I am beginning to share my students' mistrust of any explanation that cannot be encapsulated in a few well-chosen sentences and possibly a little algebra. Third, I was charged last year with creating the theoretical examination for the International Physics Olympiad, hosted by Canada, and my desire to include a question on flight at first seemed prohibited by the explicit exclusion of Bernoulli from the Olympiad syllabus. What follows is the result of my finding a way out of that impasse.

The science of flight has traditionally been the preserve of two camps. It can be understood in terms of some reasonably sophisticated mathematics, or it can be justifed using data obtained by experiment and read off tables and graphs. This dichotomy existed right from the start; the development of the first practical aircraft owed nothing to theoretical physics. Between these two extremes is the pedagogical physicist struggling to explain the wonders of flight in concise yet correct language.

From a teacher's point of view, the Bernoulli explanations of aerodynamic lift are both a blessing and a curse, and have a checkered pedagogical history. Since childhood I have read explanations of flight, or at least the action of the wing of a bird or aircraft, in the following terms: "The top of the wing is convex, the bottom of the wing is flat, so air going over the top has to accelerate to keep up with air going underneath, so the pressure on the top (thanks to Bernoulli) is reduced, etc. etc." I still see this version of events in many modern children's books. I also heard it earlier this year in an Air Canada in-flight video, which was presented by an engineering professor who cheerfully ignored the fact that his 250 viewers were then sitting on top of a supercritical wing that was flat on the top and convex on the bottom. It long puzzled me why the air going on one side of the wing should care what happened on the other, but it was many years before I discovered the truth: it doesn't. (For the standard textbook treatment, see for example, Acheson,[1] and for a systematic demolition of the common explanation, see Raskin[2]). Anyone who has thrown a balsa or paper glider with flat sheet wings can see the logical difficulty.

Later in life, while learning to fly, I ran across a version of events (also widespread) that there are two effects that generate lift: Bernoulli *and* air deflection.[3]

It seems reasonable that these two effects are one and the same, since Bernoulli's theorem can be derived from Newton's second law. Proving it is harder, because we cannot create a closed system in which we can consider in isolation the pressures and forces involved in aerodynamic lift. In recent years there have been significant developments in pedagogy of flight, and I warmly recommend a short and profound book by Henk Tennekes, *The Simple Science of Flight*.[4] Also, a straightforward resolution of Newton and Bernoulli has been presented by Klaus Weltner.[5] I am grateful to the editor for pointing out an even earlier paper in this journal that covers similar ground, although in a qualitative way.[6]

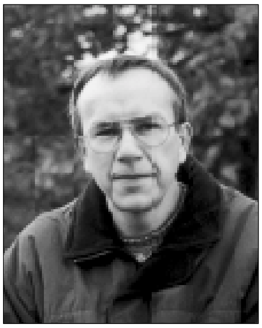

Photo by Susanna Waltham

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The Standard Treatment

Thin airfoil theory gives an approximate form for the lift force L in terms of a coefficient of lift C_L for a wing of area S and aspect ratio A (the ratio of the span l to the mean chord c), with an angle of incidence to the airstream α.[7] The air density is ρ.

$$L = \frac{1}{2} C_L \rho v^2 S; \;\; C_L \approx \frac{2\pi \sin \alpha}{1 + 2/A} \tag{1}$$

The only assumptions made to reach this result are that there is a viscous boundary layer and that there is a streamline fixed at the sharp trailing edge of the wing. (This is the so-called "Kutta" condition, which guarantees circulation of air around the wing and there-

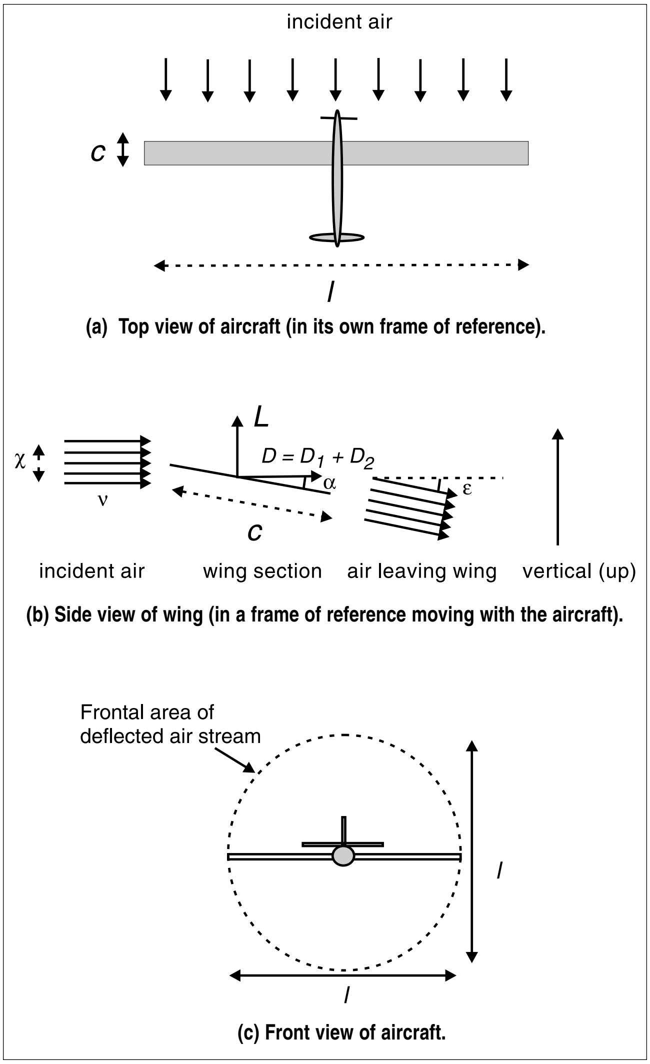

Fig. 1. Simple model of air flowing around a wing.

fore lift from a Bernoulli effect pressure difference.) The induced drag D_1 arises from the deflection of air downwards; since even a frictionless interaction with the wing cannot increase the speed of the air, then a deflection down must reduce the horizontal component of its velocity and thus cause a drag force. Its coefficient $C_{\mathrm{D,i}}$ always depends on the square of the lift coefficient:

$$D_1 = \frac{1}{2} C_{\mathrm{D,i}} \rho v^2 S; \quad C_{\mathrm{D,i}} \approx \frac{C_L^2}{\pi A} \tag{2}$$

This expression is strictly applicable only to the ideal case of a semi-elliptical lift distribution along the span of the wing; however, the correction for other typical distributions encountered is small.

The forms of Eqs. (1) and (2) can also be understood in terms of a force arising from the rate at which fluid mass is encountered (of order $\rho\, vS$) times the velocity imparted to that fluid (of order v).

A Simple Model

The germ of this idea came from Bradley Jones's *Elements of Practical Aerodynamics* of 1942,[8] although its origins are much earlier and the basic principles can be found in Rankine's 1858 classic text *A Manual of Applied Mechanics*.[9]

Birds and aircraft fly because they are constantly pushing air downwards:

$$L = \frac{\mathrm{d}p}{\mathrm{d}t} \tag{3}$$

Here L is the lift force and $\mathrm{d}p/\mathrm{d}t$ is the rate at which downward momentum is imparted to the airflow. The lift force can also be understood as the result of a pressure difference. Pushing air downwards naturally creates a pressure differential above and below the wing. A region of higher pressure beneath a wing will naturally slow down the air entering it, even as a region of low pressure above it will accelerate the incoming air.

Consider a long thin wing on an aircraft as shown in Fig. 1(a). The wing has a rectangular plan-form with span l, chord (width) c, and the aircraft is traveling at a velocity v with respect to the air mass. The wing intercepts the airflow with an angle α; see Fig. 1(b). We can consider a slice of air of height x and length l intercepting this wing and being deflected downward at a small angle ε with only a very small change in speed. The total mass of the aircraft is M. The projected wing area is $S = cl$ and the aspect ratio is $A = l / c$.

In reality, the influence of the wing will not extend merely to a well-defined slice of air like this, but to a much larger volume of air (in principle of infinite size), with the effect decreasing linearly with distance from the wing. In fluid dynamics this is usually understood in terms of the *circulation,* Γ, which is computed by integrating the fluid velocity around a circle enclosing the wing:

$$\Gamma = \oint v \cdot ds \tag{4}$$

There is no restriction on the size of the path used to evaluate the circulation; in principle you can calculate the lift of a balsa glider by integrating on a path around the Earth. The physical reason for this is that any body, no matter how small or slow, affects the entire body of air in which it is moving, i.e., the whole atmosphere. This feature contributes to the conceptual difficulty of flight since it prevents us from defining a usable closed system.

Considering the change in momentum of the air passing over the wing (with no change in speed), we can derive expressions for the vertical lift force L and the horizontal drag force D_1 on the wing in terms of l, v, x, ε, and the air density ρ. For now we assume the wings are so long that wing-tip effects can be ignored, as can the effects of the fuselage and propeller wash; the flow is completely two-dimensional. The force $\vec{F}$ required to change the velocity $\Delta\vec{v}$ of a fluid whose flow rate is $\frac{dm}{dt}$ is given by:

$$\vec{F} = \Delta\vec{v}\,\frac{dm}{dt} \tag{5}$$

In this case the flow rate is:

$$\frac{dm}{dt} = x\,l\rho v \tag{6}$$

The vertical component of $\Delta\vec{v}$ is:

$$\Delta v_V = v \sin\varepsilon \tag{7}$$

The horizontal component of $\Delta\vec{v}$ is:

$$\Delta v_H = v(1 - \cos\varepsilon) \tag{8}$$

We can now write expressions for the lift, L, and induced drag, D_1.

$$L = \rho v^2 x\,l \sin\varepsilon \tag{9}$$

$$D_1 = \rho v^2 x\,l(1 - \cos\varepsilon) \tag{10}$$

If we were to do this more correctly, we would box in the wing with a control volume[10] of infinite vertical thickness. The vertical dimension would be x and the deflection ε would be a function of x. Equation (9) would then look like:

$$L = \rho v^2\, l \int_{-\infty}^{\infty} \sin\varepsilon(x)\mathrm{d}x \tag{11}$$

This is still assuming two-dimensional flow and negligible changes in v. Equation (10) would take on a similar form.

Now we turn to the full dynamical analysis of fluids to understand the quantities x and ε.

Substituting Eqs. (9) and (10) into (1) and (2) gives expressions for the air deflection angle, ε, in terms of the angle of incidence, α, and also the height of the air stream involved, x, in terms of the wing dimensions.

$$\sin\varepsilon = \frac{4\sin\alpha}{2 + A} \tag{12}$$

$$x = \frac{\pi c A}{4} = \frac{\pi l}{4} \tag{13}$$

Here are two interesting results:

- The downward deflection angle of the air depends on the angle of incidence of the wing but goes to zero in the limit of a long thin wing, since the thickness of deflected air becomes infinite.
- The thickness of the deflected airstream is not dependent on the chord but on the length of the wing.

$$L = \frac{\pi}{4} l^2 \rho v^2 \sin\varepsilon \tag{14}$$

$$D_1 = \frac{\pi}{8} l^2 \rho v^2 (1 - \cos\varepsilon) \tag{15}$$

Hence for a given deflection angle, the relevant wing area is not S but $\frac{\pi}{4}l^2$, which is exactly the same area that would be involved if the aircraft were a helicopter and the wing was rotating. This is in contrast to writing these expressions in terms of the angle of incidence, in which case the area relevant to lift is S and that of induced drag is c^2. What happens to the lift if the chord is increased? Does it increase? Yes, but the reason is that the deflection angle increases, not the thickness of the deflected airstream.

In fact our result does not require that the deflected air be of uniform thickness along the wing, merely that the frontal area be $\frac{\pi}{4}l^2$ as shown in Fig. 1(c).

Within this result lies the reason why wings, against all engineering sense, tend to be long and thin and not short and broad. The more air deflected downwards, the less drag incurred for a given lift force. In fact, many attempts have been made to increase the depth of deflected air for a given wingspan; the splaying of an eagle's wingtip feathers is nature's response to the problem, and the upturned wingtips on a Boeing 747-400 is an engineering solution. We can also see the interference that occurs between a biplane's wings; the regions of influence of the two wings overlap unless they are spaced apart by more than the length of the span. It is not, however, practicable to space them by more than a small fraction of this distance.

Friction Drag

There is another drag force, D_2, caused by the friction of air flowing over the surface of the wing. The usual expression for friction drag is as follows:

$$D_2 = \frac{1}{2} C_{D,0} \rho\, v^2 S \tag{16}$$

The coefficient of friction $C_{D,0}$ is defined here using the area of the wing S as the relevant area. The zero subscript refers to the attitude (ε, α), which gives zero lift, and we will assume that the drag varies very little with small changes in angle of attack; this is a reasonable approximation for a good airfoil section. For a real wing D_2 would be a combination of friction and form drag (i.e., drag caused by air slowing in a direction parallel and perpendicular to the wing surface at any given point, respectively). In our case of a very thin wing, friction drag is dominant, and in any case both drags have the same form and are usually coalesced into one ("profile") drag coefficient. In our model we can express D_2 in terms of the rate of change of momentum of the air flowing past the wing due to friction:

$$D_2 = \nu_1 \frac{\mathrm{d}m_1}{\mathrm{d}t} - \nu^2 \frac{\mathrm{d}m_2}{\mathrm{d}t} \tag{17}$$

Here the subscripts 1 and 2 refer to the air mass before and after passing the wing, respectively. Since the wing is neither a source nor a sink, the mass flow of air toward the wing $\left(\frac{\mathrm{d}m_1}{\mathrm{d}t}\right)$ must be the same as the mass flow away from the wing $\left(\frac{\mathrm{d}m_2}{\mathrm{d}t}\right)$; therefore,

$$\frac{\mathrm{d}m_1}{\mathrm{d}t} = \frac{\mathrm{d}m_2}{\mathrm{d}t} = \frac{\mathrm{d}m}{\mathrm{d}t} = xl\rho\nu \tag{18}$$

Substituting $\nu_1 = \nu$ and $\nu_2 = \nu - \Delta\nu$:

$$D_2 = \nu xl\rho\nu - (\nu - \Delta\nu)xl\rho\nu = xl\rho\nu\Delta\nu$$

$$= \frac{\pi}{4} l^2 \rho\, \nu\Delta\nu \tag{19}$$

If we wish to recover the standard form for the friction drag, we can write:

$$\frac{\Delta\nu}{\nu} = \frac{2C_{D,0}}{\pi\,A} = \frac{2C_{D,0}}{\pi}\ \frac{c}{l} \tag{20}$$

This result makes intuitive sense in that the slowing of the air depends on the size of the chord and the inverse of the thickness of the airstream.

$$D_2 = \frac{\pi}{4}\ \frac{\Delta\nu}{\nu}\rho\nu^2\, l^2 \tag{21}$$

In comparing this with Eq. (16), we see that if we formulate the drag in terms of the fractional velocity change then the relevant area is l^2, whereas in terms of the conventional drag coefficient the relevant area is S. Typical values for $C_{D,0}$ for full-sized modern aircraft are around 0.02;[11] this means that the magnitude of $\Delta\nu/\nu$ is a few parts per thousand. Hence our assumption that $\Delta\nu/\nu$ is small is a good one.

[This drag is necessarily along the wing surface; when the wing is at an angle ε, the horizontal component is this value multiplied by $\cos\varepsilon$:

$$\frac{\pi}{4}\ \frac{\Delta\nu}{\nu}\rho\nu^2 l^2 \cos\varepsilon \approx \frac{\pi}{4}\ \frac{\Delta\nu}{\nu}\ \rho\nu^2 l^2\left(1 - \frac{\varepsilon^2}{2}\right)$$

$$\approx \frac{\pi}{4}\ \frac{\Delta\nu}{\nu}\rho\nu^2 l^2 + \mathrm{O}\left(\varepsilon^2\,\frac{\Delta\nu}{\nu}\right) \tag{22}$$

so to the order given, our simple answer is correct.]

Optimizing Flight Attitude

Total drag force $D = D_1 + D_2$ is dependent on deflection angle ε and drag coefficient f:

$$D = \frac{\pi}{4}\rho\nu^2 l^2\left[(1 - \cos\varepsilon) + \frac{\Delta\nu}{\nu}\right]$$

$$\approx \frac{\pi}{4}\rho\nu^2 l^2\left[\frac{1}{2}\sin^2\varepsilon + \frac{\Delta\nu}{\nu}\right] \tag{23}$$

In making this approximation, D can be expressed in terms of the mass, speed, and wing dimensions of the aircraft. Note that for level flight the lift has to be equal to the weight of the craft.

$$L = Mg = \frac{\pi}{4}\rho\nu^2 l^2 \sin\varepsilon;\quad \sin\varepsilon = \frac{4Mg}{\pi\rho\nu^2 l^2} \tag{24}$$

Minimum Power Condition

We can now find the minimum power required to keep this aircraft in the air. This occurs when the flight velocity is ν_0.

$$P = D\nu = \frac{\pi}{4}\rho\nu^3 l^2\left[\frac{\Delta\nu}{\nu} + \frac{1}{2}\,\frac{(4Mg)^2}{(\pi\rho\nu^2 l^2)^2}\right]$$

$$= \frac{\pi}{4}\rho\nu^3 l^2\,\frac{\Delta\nu}{\nu} + \frac{2(Mg)^2}{\pi\rho\nu l^2} \tag{25}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\nu} = \frac{3\pi}{4}\rho\nu^2 l^2\,\frac{\Delta\nu}{\nu} - \frac{2(Mg)^2}{\pi\rho\nu^2 l^2} = 0, \text{ when } \nu = \nu_0 \tag{26}$$

Hence the flight velocity for minimum power is given by:

$$\nu_0^{\,4} = \frac{8}{3}\left(\frac{\Delta\nu}{\nu}\right)^{-1}\left(\frac{Mg}{\pi\rho l^2}\right)^2 = \frac{4}{3C_{D,0}\pi A}\left(\frac{Mg}{\rho S}\right)^2 \tag{27}$$

Note that the velocity is proportional to the square root of the wing loading Mg/S and the inverse of the square root

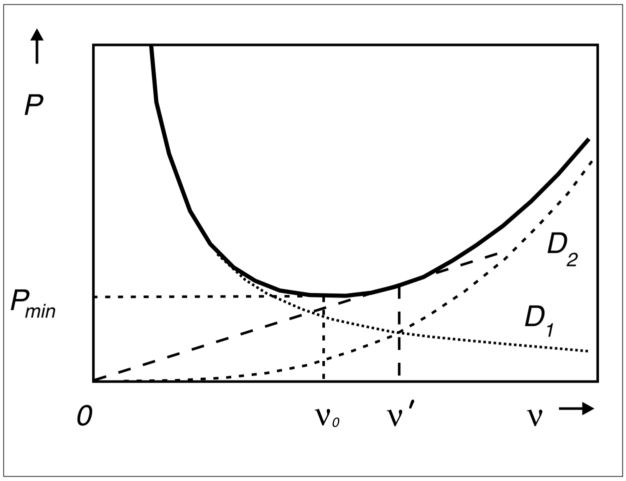

Fig. 2. Dependence of power on flight velocity for level flight without acceleration. The two velocities marked are those for minimum power (ν_0) and maximum range (ν'). The two types of drag responsible for power consumption are induced drag (D_1) and friction drag (D_2).

of the air density. These and other dependencies are the same as predicted by the full fluid dynamical theory. The graph of power versus velocity is shown in Fig. 2.

$$P_{\min} = \frac{\pi}{4}\rho\nu_0^3 l^2\left[\frac{\Delta\nu}{\nu} + \frac{1}{2}\frac{(4Mg)^2}{(\pi\rho\nu_0^2 l^2)^2}\right]$$

$$= \frac{\pi}{4}\rho\nu_0^3 l^2\left[\frac{\Delta\nu}{\nu} + \frac{(4Mg)^2}{2(\pi\rho l^2)^2}\frac{3\pi^2\rho^2 l^4}{8(Mg)^2}\frac{\Delta\nu}{\nu}\right]$$

$$= \pi\rho\nu_0^3 l^2\frac{\Delta\nu}{\nu} = 2C_{D,0}\rho\nu_0^3 S \qquad (28)$$

Substitute for ν_0:

$$P_{\min} = \left(\frac{8}{3}\right)^{3/4}\left(\frac{\Delta\nu}{\nu}\right)^{1/4}\frac{(Mg)^{3/2}}{(\pi\rho l^2)^{1/2}} = 2\left(\frac{4}{3A\pi}\right)^{3/4} C_{D,0}{}^{1/4}\frac{(Mg)^{3/2}}{(\rho S)^{1/2}} \qquad (29)$$

In this flight condition the induced drag (which depends on $1/\nu^2$ is three times the size of the friction drag (which depends on ν^2). The minimum power depends on the mass of the aircraft to the three-halves power. Once again, these and other dependencies are the same as predicted by the full theory.

Maximum Range Condition

A similar procedure allows us to maximize the range of an aircraft with a certain amount of energy available in the form of fuel, or height, in the case of a glider. Since work done is simply drag times distance, this represents the minimum drag condition. The necessary velocity can be found by drawing the line tangent to the power curve, and passing through the origin, which has the minimum slope. At the new velocity, ν', the two contributions to the drag can be shown to be equal and the value is given by:

$$\nu' = 3^{1/4}\nu_0 \approx 1.3\nu_0 \qquad (30)$$

The power for maximum range is given by:

$$P' = \frac{3^{3/4}}{2}P_{\min} \approx 1.14P_{\min} \qquad (31)$$

An Example

To illustrate the above analysis, let us estimate some quantities for a real aircraft; I take the ubiquitous Cessna 152,[12] a standard two-seat trainer.

Typical Mass, M = 700 kg
Span, l = 10.2 m; Wing area, S = 14.8 m^2; Aspect ratio, A = 7.0
Velocity for maximum range, ν' = 31 m/s; Lift-to-drag ratio at ν', $(L/D)_{\max}$ = 11
Power rating of engine, $P_{\max}$ = 82 kW

Now we can use the equations given above to derive quantities used in formal aerodynamics. Equation (1) for the maximum range condition gives C_L = 0.81, α = 9.5 deg. Equations (2) and (16) and the recognition that in the maximum range condition the two drag contributions are equal, gives $(L/D)_{\max} = [C_L/(C_{D,0} + C_{D,i})]_{\max} = 11$, hence $C_{D,0} = C_{D,i} = 0.037$. The power $P' = D\nu' = 19.3$ kW, well within the capability of the engine (maximum cruising speed is 57 m/s, considerably higher than ν').

We can also, using our simple model, estimate how much air is involved in flying the Cessna and what happens to it. The rate at which the aircraft encounters air is given by $dm/dt = \frac{\pi}{4}l^2\rho\nu' = 3000$ kg/s (3 tonnes per second!), and Eq. (12) gives the deflection angle, ε, to be 4.2 deg, about half the value of the angle of incidence, α. The downward velocity of the air (downwash) is $\nu'\sin\varepsilon = 2.3$ m/s.

Conclusions

We have used a very simple physical model relying only on Newton's second law to reproduce all the salient features of a rigorous fluid dynamical treatment of flight:

- Lift and drag have their origins in the change in momentum of the fluid flow.
- Induced drag is due to the deflection of the fluid.
- Frictional drag is due to the slowing of the fluid.
- In level, steady flight induced drag is proportional to $1/\nu^2$.
- In level, steady flight frictional drag is proportional to ν^2.
- Optimal flight conditions are obtained by balancing the two forms of drag.

The model has its limitations; we cannot calculate real performance with it. However, we have learned something about the depth of air flow involved in generating the lift, and how this affects induced drag. Above all, we can now explain in simple terms why birds and aircraft cannot fly by deflecting air with their bodies alone, and hence why they have to have appendages we call wings.

Acknowledgments

This treatment of flight grew out of a question written for the 28th International Physics Olympiad held in Sudbury, Ontario, Canada in July 1997. I would like to thank members of the marking team at this event for helping to debug the question and solution, in particular Michael Forbes, James Fraser, Andrzej Kotlicki, and Alan Poon.

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