Changing speed of comets

Mike Follows

Sutton Coldfield Grammar School for Girls, Jockey Road, Sutton Coldfield, Birmingham B73 5PT, UK

E-mail: followsm@aol.com

Abstract
It is shown that highly elliptical orbits, such as those of comets, can be explained well in terms of energy rather than forces. The principle of conservation of energy allows a comet's velocity to be calculated at aphelion and periheilon. An example asks students to calculate whether they can run fast enough to escape from a small asteroid.

If only because energy is a scalar, it is easier to explain physical phenomena in terms of energy rather than forces. Despite this, teachers usually explain the orbit of comets in terms of forces. They will say that, when the comet is making its closest approach to the Sun, it will be travelling fastest to avoid crashing into the Sun's surface. This borrows from Isaac Newton's 'thought experiment' in which he imagined firing a cannon ball from a hill. The Earth's surface, being curved, falls away beneath the cannon ball and so the ball will travel further than if the Earth were flat. Fire it fast enough and the cannon ball will never reach the Earth. It will remain in orbit. However, in my experience, pupils are not always happy with this explanation. Indeed, I get the impression that some invent more comfortable explanations. Some believe that comets speed up on their approach to the Sun because, by the time they get there, much of the ice has 'evaporated'.

A more visually appealing explanation relies on the Sun's distortion of the fabric of space-time but, alas, this perhaps introduces more problems than it solves. Another way to tackle the problem is to explain that angular momentum is conserved. This has the advantage that it can be shown using video footage of an ice skater or demonstrated by spinning on a swivel chair. Outstretched limbs increase the moment of inertia and reduce the rate of spin. Pulling in one's arms increases the speed, directly analogous to the increasing speed of the comet as it approaches the Sun. Alas, linear momentum, let alone angular momentum, does not appear in 14–16 specifications. Besides, while some pupils will gain an intuitive understanding from these more visual approaches, they might well find that they are left unable to articulate this understanding in the context of answering examination questions.

I have found it much easier to resort to the principle of conservation of energy. When the comet is at its farthest point from the Sun (its aphelion), the comet is high above the Sun's surface where much of its energy is in the form of gravitational potential energy. As the comet approaches the Sun, its height falls and the difference in gravitational potential energy will be converted into kinetic energy. Hence a comet will be travelling fastest at its perihelion (i.e. when it is closest to the Sun). Of course, this is best explained with the help of sketches (figure 1). Strictly speaking, the 'height' of the comet is its distance from the centre of the Sun.

Indeed, a discussion of energy is perhaps also the best way to start the more sophisticated explanations suitable for A-level students and beyond. The total mechanical energy (ME) of a comet, or any orbiting body, is the sum of its kinetic energy (KE) and its gravitational potential energy (PE):

$$\mathrm{ME} = \mathrm{KE} + \mathrm{PE} = \text{constant} \qquad (1)$$

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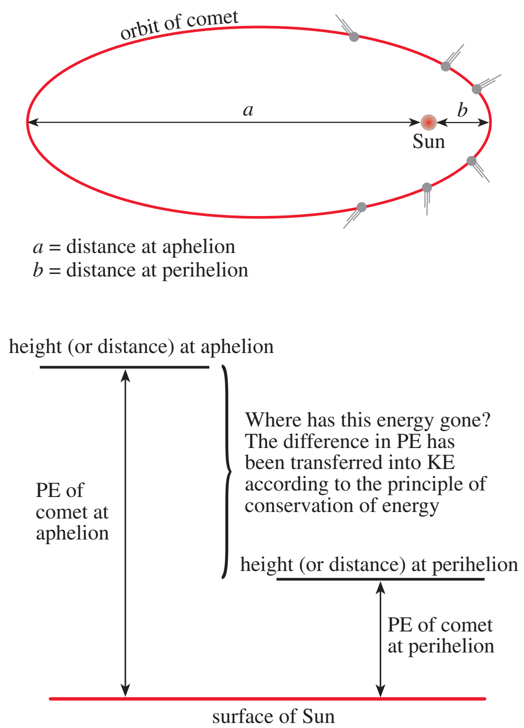

Figure 1. Variation of the height and the gravitational potential energy (PE) of a comet.

$$\mathrm{ME} = \tfrac{1}{2}mv^2 - m\frac{GM}{r} = \text{constant} \qquad (2)$$

where M is the mass of the Sun (1.99×10^{30} kg), m is the mass of the comet, r is its instantaneous distance from the Sun and G is the universal gravitational constant.

The relative magnitude of the kinetic and potential energies determines the shape of the orbit. If the velocity of the comet is too great then its KE $\geqslant$ PE. Consequently, the comet will escape the clutches of the Sun on a parabolic or hyperbolic orbit, never to return. Assuming that PE $>$ KE then the comet will have a bound elliptical orbit with its total mechanical energy given by

$$\mathrm{ME_{bound}} = -m\frac{GM}{2a}. \qquad (3)$$

Derivation of equation (3) is best left to undergraduates or very keen A-level students who are also confident mathematicians. It can be found at www.ac.wwu.edu/~vawter/PhysicsNet/Topics/TopicsMain.html. However, there is no reason why students should not be asked to substitute equation

Table 1. Various parameters for comet Halley.

Perihelion distance (AU)	0.587
Aphelion distance (AU)	35.11
Semi-major axis (AU)	17.84
Dimensions (km)	$16 \times 8 \times 8$
Density (kg m^{-3})	100
Mass (kg)	$\sim 1 \times 10^{14}$

Note that AU denotes astronomical units. 1 AU $= 1.49 \times 10^{11}$ m, the distance between the Sun and Earth. I encourage students to calculate this roughly for themselves by telling them that it takes approximately eight minutes for sunlight to reach us.

(3) into (2) to get

$$-m\frac{GM}{2a} = \tfrac{1}{2}mv^2 - m\frac{GM}{r} = \text{constant} \qquad (4)$$

in order to show that, as a function of its distance from the Sun, a comet has a velocity given by

$$v = \sqrt{\frac{GM}{a}\left(\frac{2a}{r} - 1\right)} \qquad (5)$$

where a is the mean radius of its orbit (sometimes referred to as the semi-major axis). Notice that, when $a = r$, equation (5) reduces to

$$v = \sqrt{\frac{GM}{a}} \qquad (6)$$

as would be expected for a circular orbit.

There is no reason why equation (5) cannot be given to advanced-level students in order that they can confirm that the velocity of a comet is less than its escape velocity. If students plug appropriate values from table 1 into equation (5) they will find that comet Halley has a velocity of 5.46×10^4 m s^{-1} when it is closest to the Sun (perihelion), reducing to approximately 900 m s^{-1} when it is at its most distant (aphelion). Students should be able to work out comet Halley's escape velocity by equating KE and PE:

$$\tfrac{1}{2}mv^2 = m\frac{GM}{r} \qquad (7)$$

and rearranging for v:

$$v \geqslant \sqrt{\frac{2GM}{r}}. \qquad (8)$$

At perihelion this is $v \geqslant 5.51 \times 10^4$ m s^{-1}, only just in excess of the actual speed of Halley calculated using equation (5).

Table 2. KE, PE and ME at the perihelion and aphelion of comet Halley's orbit.

	At perihelion	At aphelion
KE (J)	1.49×10^{23}	4.06×10^{19}
PE (J)	-1.52×10^{23}	-2.54×10^{21}
ME (J)	-2.47×10^{21}	-2.49×10^{21}

The velocity of comet Halley, already calculated using equation (5), can be substituted into the equation for KE (the left-hand term in equation (7)). PE can be worked out using the right-hand term of equation (7). The values of KE and PE at perihelion and aphelion can be substituted into equation (1) to get the ME. Table 2 shows that ME remains unchanged, within rounding errors, confirming that mechanical energy is conserved.

The mass of comet Halley is uncertain. Its uncertain density and volume imply that Halley has a mass of about 1×10^{14} kg. Its low density suggests that it is porous with much of its interstitial ice sublimated away. The uncertainty in its mass translates to an uncertainty in the values of KE, PE and ME for comet Halley shown in table 2. Despite this it should be clear that, at perihelion, KE is at a maximum while PE is at a minimum.

Figure 2. Comet Halley as seen in 1976.

The reverse is true at aphelion. Students might need to be reminded that gravitational potential energy is taken as zero at infinity so it is actually less negative and therefore bigger at aphelion, as expected.

Comet Halley loses about 3×10^{11} kg at each perihelion passage. While this reduces its ME, its dynamics does not change. Its (escape) velocity is independent of its mass, which might come as a surprise to those pupils who reasoned that it speeded up because it was losing material as it approached the Sun.

Some students may be tempted to use PE $= mgh$, the equation for a uniform field, mysteriously plugging in the gravitational field strength, g, for the Earth's surface. This is an excellent opportunity to remind students that the value of g falls off as the square of the distance from a massive body. It is also a chance to revise that $g = F/m$, where F is the force of attraction due to Newton's law of gravitation. This gives $g = GM/r^2$ which, when substituted into PE $= mgh$, reduces to the equation for PE for a radial field that appears on the right-hand side of equation (7). Pupils might need to be reminded that r can be substituted for h. After all, both symbols denote distance, h in a uniform field and r in a radial field.

I ask students to imagine that a particular comet is on a collision course with Earth. How much more kinetic energy would be needed in order to send the comet into an escape trajectory? How might this be achieved? Well, it should be possible to put a lander onto the surface of a comet. This is what is planned for the Rosetta spacecraft. A little research will show that the dramatic scenario portrayed in the film *Armageddon* would not be necessary, provided the Spaceguard Survey or other searches 'catch' Near Earth Objects in good time. As teachers we ought to be tapping into this popular interest. For example, why not ask A-level students to identify a comet or asteroid from which they could launch themselves into orbit simply by running. My experience suggests that, by making physics more 'personal' in this way,

Table 3. Data students need to answer the question on Toro.

Gravitational constant, G	6.673×10^{-11} N m^2 kg^{-2}
Mass of Earth, M	5.98×10^{24} kg
Radius of Earth, r	6.38×10^6 m

Table 4. Selected asteroids.

Asteroid number	Asteroid name	Radius (km)	Mass (10^{12} kg)	Gravitational field strength (N kg^{-1})	Escape velocity (m s^{-1})
1685	Toro	5.0	2880.0	7.68×10^{-3}	8.8
1566	Icarus	0.7	7.9	1.08×10^{-3}	1.2
1620	Geographus	1.0	23.0	1.53×10^{-3}	1.8
1862	Apollo	0.8	11.8	1.23×10^{-3}	1.4

physics can be perceived as less abstract. Here is an example of a question I use.

- The asteroid Toro (number 1685), discovered in 1964, has a radius of about 5 km. Suppose it has the same density as the Earth, find its mass and the gravitational field strength at its surface. Would you be able to run fast enough to launch yourself into orbit around Toro?

The data in table 3 are provided. Students may have forgotten the equation for the volume of a sphere so they may need to be reminded. In order to save time, the mean density of the Earth, approximately 5.5×10^3 kg m^{-3}, can be given to them. This gives a mass for Toro of 2.88×10^{15} kg. Students should know how to derive equation (8) and confirm that they would need to run at a speed of 8.77 m s^{-1}. The calculations are outlined as follows.

Finding the mass of Toro:

$$\begin{aligned} M_{\mathrm{Toro}} &= \rho V_{\mathrm{Toro}} = \rho \tfrac{4}{3}\pi R_{\mathrm{Toro}}^3 \\ &= 5500\ \mathrm{kg\,m^{-3}} \times \tfrac{4}{3}\pi (5 \times 10^3\ \mathrm{m})^3 \\ &= \mathbf{2.88 \times 10^{15}\ kg}. \end{aligned}$$

The gravitational field strength on the surface of Toro:

$$\begin{aligned} g &= \frac{G M_{\mathrm{Toro}}}{r_{\mathrm{Toro}}^2} \\ &= \frac{6.673 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}} \times 2.88 \times 10^{15}\ \mathrm{kg}}{(5 \times 10^3\ \mathrm{m})^2} \\ &= \mathbf{7.68 \times 10^{-3}\ N\,kg^{-1}}. \end{aligned}$$

The escape velocity:

$$\begin{aligned} v &= \sqrt{\frac{2 G M_{\mathrm{Toro}}}{r_{\mathrm{Toro}}}} \\ &= \sqrt{\frac{2 \times 6.673 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}} \times 2.88 \times 10^{15}\ \mathrm{kg}}{5 \times 10^3\ \mathrm{m}}} \\ &= \mathbf{8.77\ m\,s^{-1}}. \end{aligned}$$

Those who are not quite as nimble on their feet might want to choose one of the asteroids listed in table 4, which is taken from a catalogue of asteroids at nssdc.gsfc.nasa.gov/planetary/factsheet/asteroidfact.html. The final three columns of table 4 list the values that students should get.

An army on the march might expect to cover four miles in an hour. Get students to work out that this equates to a speed of a little under 2 m s^{-1} and comment on the hazards of simply walking on these asteroids.

No matter what the academic level, the behaviour of orbiting bodies is often more fruitfully dealt with in terms of energy, while the significance of comets goes beyond the fact that they are 'dirty snowballs'. Recent research suggests that, by colliding with Earth, comets might well have been responsible for watering our planet and seeding it with life. On the other hand they have also been implicated in mass extinctions. The demise of the dinosaurs 65 million years ago is generally accepted to coincide with the latest major cometary impact. Meteor showers are the funeral pyres of comets. The Perseids, the most spectacular, appear in early August and are all that remains of the comet Swift-Tuttle. Comets also have potential for cross-curricular links. For example, they appear in Shakespeare and history as portents of future misfortune—Halley's comet appeared just months before the Battle of Hastings in 1066. Perhaps alluding to the wider significance of physical phenomena in this way might go some way to stemming the exodus from physics.

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Mike Follows gained a PhD from Lancaster University for his research into dark matter. He now teaches at Sutton Coldfield Grammar School for Girls and is an Associate Lecturer with the Open University. He has been writing educational materials for major publishers and is currently on the Q&A expert panel of the science magazine *Focus*.