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Toward a global systematic analysis of sub-barrier fusion enhancement

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A global systematic analysis relating the asymptotic enhancement of sub-barrier fusion cross sections with the product of the coupling strength times the value of the Coulomb barrier is presented. It is found that all systems involving static deformations, inelastic excitations, and transfer degrees of freedom, follow the same systematic trend. In the analysis, the Coulomb barrier plays a central role in amplifying the relevance of the couplings associated with a particular degree of freedom. [S0556-2813(98)51106-5]

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Fusion cross sections between heavy ions, at bombarding energies near and below the barrier, show large enhancements relative to the predictions of a one-dimensional barrier penetration model. Early attempts to find a global interpretation of the fusion process have focused on extracting fusion barriers and testing several theoretical potentials [1–3]. Simple parametrizations of measured fusion excitation functions in terms of gross nuclear properties have also been published [4–6]. More recently considerable progress has been achieved in the understanding of these enhancements by including the internal structure of the participating nuclei in the dynamics of the reaction through coupled-channel calculations [7,8] and the interacting boson model [9]. In the present work, we propose a global systematic analysis relating the magnitude of the asymptotic enhancement of the sub-barrier cross sections with the product of the values of the Coulomb barrier and the values of the coupling strength necessary to account for the fusion cross section. In our analysis, the value of the Coulomb barrier, V_b, acts as an amplifier on the coupling strength of a system (associated with static deformations, inelastic excitations, or transfer reactions), i.e., for a nucleus with a quadrupole deformation β_2, the bigger the product $\beta_2 \cdot V_b$, the larger the sub-barrier fusion enhancement. All the systems studied, which include cases where static deformations, inelastic excitations, and transfer degrees of freedom are involved, fall nicely into the same systematic trend.

We have investigated an extensive set of available data on fusion cross section excitation functions and compared them to the *same* theoretical model. A simplified coupled-channels code, CCMOD [10], has been used to perform all the calculations. This code is a modified version of the code CCDEF [11] which can treat static deformations and couplings to inelastic excitations and transfer channels. The nuclear potential used in the code has a Wood-Saxon shape and together with the Coulomb potential determines a parabolic barrier characterized by three parameters: V_b (height), R_b (position), and $\hbar\omega$ (curvature). This model has been extensively used in describing (mostly successfully) a large number of fusion excitation functions [12]. In our coupled-channels calculations we have included quadrupole and hexadecapole deformations of the participating nuclei and the main inelastic and transfer reaction channels for each system (inelastic excitations with large coupling strengths and transfer channels with positive Q values). The values of the deformation parameters and the electromagnetic transition probabilities for the lowest excited states of the target and/or projectile nuclei for each system were taken from the literature [13–15]. We present an analysis of sixteen different systems: ^{4}He+^{154}Sm [16], ^{16}O+^{144}Sm [17,18], ^{16}O+^{148}Sm [18,19], ^{16}O+^{152}Sm [19], ^{16}O+^{154}Sm [18–20], ^{16}O+^{166}Er [21], ^{16}O+^{176}Yb [21], ^{16}O+^{186}W [18], ^{16}O+^{232}Th [22,23], ^{28}Si+^{142}Ce [24,25], ^{28}Si+^{154}Sm [26], ^{32}S+^{138}Ba [24,25], ^{32}S+^{154}Sm [27], ^{48}Ti+^{122}Sn [24,25], ^{40}Ar+^{154}Sm [28], and ^{40}Ca+^{192}Os [29].

An instructive way to visualize the effects of coupling in the enhancement of the fusion cross section with respect to the one-dimensional penetration model has been proposed by Vandenbosch [7]. It consists in taking the ratio of the experimental data, $\sigma_{\rm fus}$, to the corresponding values of the predictions of the one-dimensional calculation with no couplings, $\sigma_{\rm fus}^{\rm unc}$. Operationally, $\sigma_{\rm fus}^{\rm unc}$ is obtained in the following manner: (1) performing the calculations by first adjusting the barrier parameters (V_b, R_b, $\hbar\omega$) and including known values of the coupling strengths (permanent deformations, inelastic excitations, and transfer reaction channels) to reproduce the experimental fusion cross section; (2) turning off the coupling strengths to obtain $\sigma_{\rm fus}^{\rm unc}$, which is the fusion cross section predicted by the one-dimensional barrier penetration model without couplings.

Figure 1 displays a set of existing data on fusion cross sections for different systems normalized as explained above. We can see that the experimental fusion cross sections are increasingly enhanced as the bombarding energy $E_{\rm c.m.}$, decreases below the barrier V_b. Well below the barrier, these enhancements seem to remain constant. The lines in the figure represent the fusion cross section, $\sigma_{\rm fus}^{\rm cou}$, obtained by performing the coupled-channel model calculations [10,11] when the corresponding coupling strengths of the target and/or the projectile are included.

In order to quantify the fusion cross section enhancements we find it useful to define the concept of the asymptotic energy shift (AES), ΔE_t, as

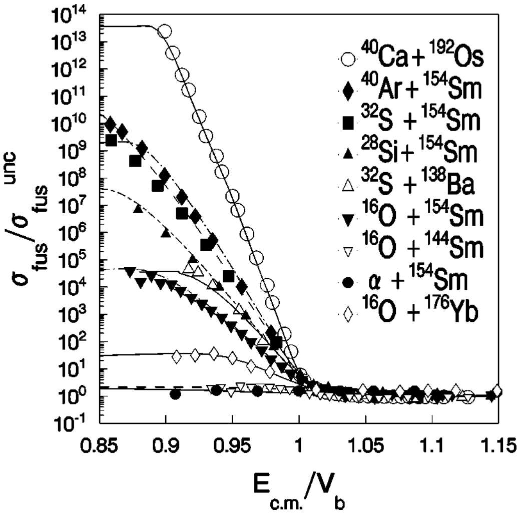

FIG. 1. The ratio of experimental fusion cross sections to the one-dimensional penetration model $\sigma_{\mathrm{fus}}^{\mathrm{unc}}$. The lines were obtained using a modified version of CCDEF, including permanent quadrupole and hexadecapole deformations, inelastic excitations, and transfer degrees of freedom of the target and/or the projectile.

$$\Delta E_t = \frac{\hbar\omega}{2\pi}\ln\left[\frac{\sigma_{\mathrm{fus}}^{\mathrm{cou}}(E_{\mathrm{c.m.}})}{\sigma_{\mathrm{fus}}^{\mathrm{unc}}(E_{\mathrm{c.m.}})}\right], \quad \text{for } E_{\mathrm{c.m.}} \ll V_b. \tag{1}$$

This definition is identical to the AES introduced by Aguiar *et al.* [6]. We can see in Fig. 1 that the extrapolated asymptotic enhancement varies from 1 to 10^{14}. We can also notice that for a given target, ^{154}Sm, this enhancement increases as the mass or Z of the projectile increases. The increase in the magnitude of the enhancement for more symmetric systems was noted in early studies of sub-barrier fusion [4]. However, this connection has not been clearly identified with the relevant physical processes or this trend has been associated with an onset of more exotic degrees of freedom such as neck formation [6].

In order to understand the trend observed in Fig. 1, let us consider the case of a system consisting of a spherical projectile and a permanently deformed target, characterized by a quadrupole deformation parameter β_2. The average variation in the interacting potential ΔV_b, produced by a deformation in the target, is, to a first order, given by

$$\frac{\Delta V_b}{V_b} \simeq \frac{\Delta R_{\mathrm{tgt}}}{R_b}. \tag{2}$$

The variation of the radius for a quadrupole deformed target is $\Delta R_{\mathrm{tgt}} \approx \beta_2 R_{\mathrm{tgt}}$; therefore

$$\Delta V_b \approx \beta_2 V_b \frac{R_{\mathrm{tgt}}}{R_b}. \tag{3}$$

Since, at sub-barrier energies, the fusion cross section is proportional to

$$\sigma_{\mathrm{fus}} \approx \exp\left[\frac{2\pi}{\hbar\omega}(V_b - E_{\mathrm{c.m.}})\right], \tag{4}$$

then, by combining Eqs. (1), (3), and (4), we obtain

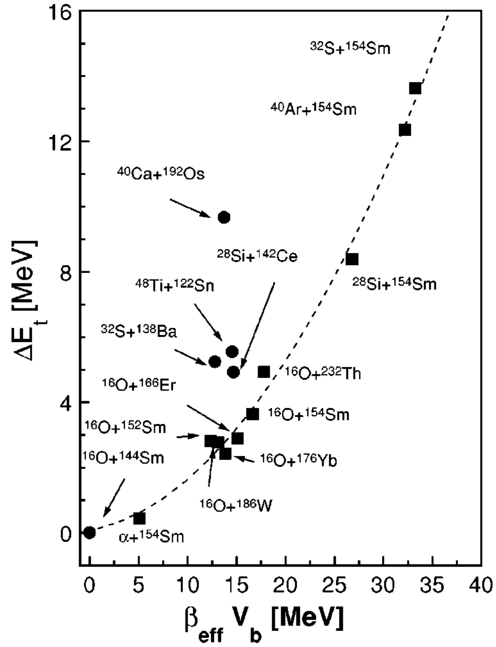

FIG. 2. An asymptotic energy shift, defined in Eq. (1) and obtained using coupled-channel calculations described in the text, as a function of the product $\beta_{\mathrm{eff}} V_b$. The dashed curve is drawn to guide the eye.

$$\Delta E_t \propto \beta_2 V_b \frac{R_{\mathrm{tgt}}}{R_b}. \tag{5}$$

Therefore, we see that following this heuristic argument, one would expect that for systems where the quadrupole deformation is the dominant degree of freedom, ΔE_t will be proportional to the product $\beta_2 V_b$. This physical picture can be generalized for the case in which both the projectile and the target have permanent quadrupole and hexadecapole deformations. Following Vandenbosch [30] we can define

$$\beta_{\mathrm{eff}} = \sqrt{\left[\left(\sum_\lambda \beta_\lambda^{\mathrm{proj}}\right)\frac{R_{\mathrm{proj}}}{R_b}\right]^2 + \left[\left(\sum_\lambda \beta_\lambda^{\mathrm{tgt}}\right)\frac{R_{\mathrm{tgt}}}{R_b}\right]^2}, \tag{6}$$

where R_{proj} and R_{tgt} are the radii of the projectile and target, respectively, and λ is the multipolarity of the deformation.

Figure 2 displays the values of ΔE_t versus $\beta_{\mathrm{eff}} V_b$. For each system, the values of the asymptotic energy shift, defined in Eq. (1), were obtained from the coupled-channel calculations described above, while the values of β_{eff} were calculated using Eq. (6) with values of the quadrupole and hexadecapole deformations of the respective targets and projectiles taken from Ref. [14]. We can observe that the systems represented by filled squares in Fig. 2 fall within the same trend (the dashed line through the data guides the eye), while the systems represented by solid circles fall outside this regular behavior. For the former systems (squares), the dominant effects responsible for the sub-barrier enhancements are the static deformations, whereas for the latter (circles) the more relevant degrees of freedom are the inelastic and transfer reactions channels. The fact that the systems indicated by solid circles fall outside the main trend can be explained because in the horizontal axis we have only included the parameters associated to permanent deformations

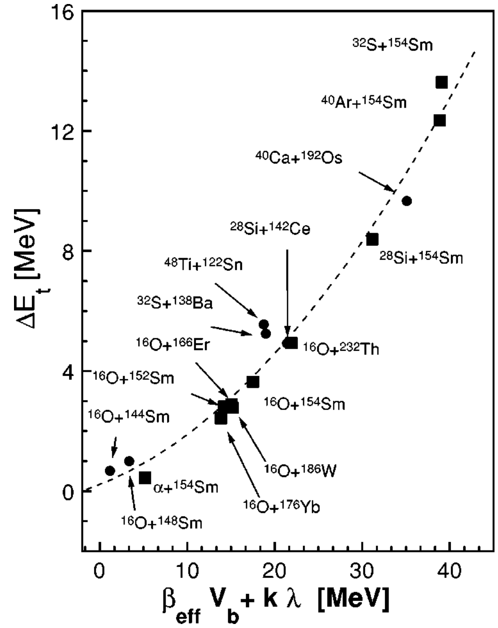

FIG. 3. An asymptotic energy shift, defined in Eq. (1) and obtained using coupled-channel calculations described in the text, as a function of the parameter $\beta_{\text{eff}}V_b+k\lambda$ calculated using Eq. (9) with $k=1.65$. The dashed curve is drawn to guide the eye.

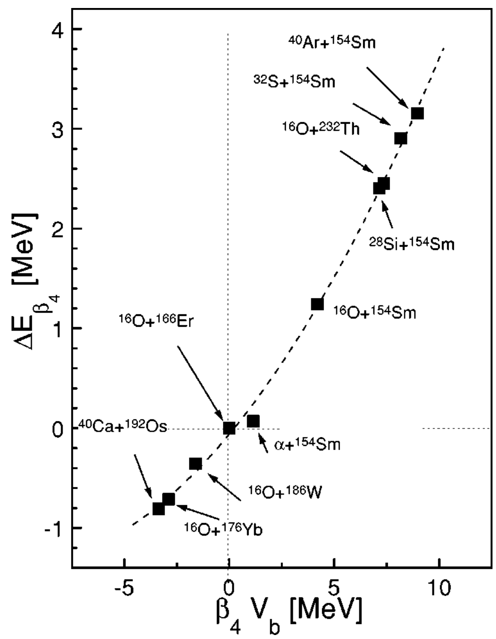

FIG. 4. An asymptotic energy shift associated to the hexadecapole deformation, ΔE_{β_4}, as a function of $\beta_4 V_b$. The dashed curve is drawn to guide the eye.

(which are not the dominant degrees of freedom for these systems). Moreover, according to the definition of β_{eff} given by Eq. (6), for a permanently deformed nucleus, with both quadrupole and hexadecapole deformations, these two deformations act coherently since their effects on the interaction potential are spatially correlated. The contributions of the target and the projectile are added incoherently, because they are, to a large extent, independent contributions. In general, the physics of the process dictates the mode for adding the different contributions. It is interesting to note, that the connection between ΔE_t and $\beta_{\text{eff}}V_b$ has been previously obtained by some authors [6], nevertheless, this connection has not been sufficiently exploited.

In an effort to include the rest of the degrees of freedom (inelastic excitations and transfer reaction channels) that affect the fusion cross section within a unified picture, we appeal to the physics involved in the coupled-channel approach. Following Dasso *et al.* [31], the effect of including n-couplings into the fusion process is to split the original barrier into $n+1$ barriers. In general, the coupling of the incident channels to other channel generates a lower and a higher barrier than the original one. At energies well below the barrier, the lower barriers are responsible for the fusion enhancement. It is a rather straightforward procedure to calculate the barrier shift λ associated with each coupled-channel [10,31]. They are obtained through a diagonalization method of the coupling matrix which depends on the coupling strength and the Q value of the relevant states. In the particular case of a single channel, with coupling strength f, and Q value Q_0, the lowering of the barrier is given by

$$\lambda^-=\frac{1}{2}\left(-Q_0^2-\sqrt{Q_0^2+4f^2}\right). \tag{7}$$

The values of λ^- have the analogous physical effect of the parameter $\beta_{\text{eff}}V_b$ associated with the permanent deformations, however, in general these parameters λ (associated with inelastic scattering and transfer channels) will affect the enhancement with different weighing factors. Therefore, the physics of the problem suggests defining the parameter,

$$\beta_{\text{eff}}V_b+k\lambda=\beta_{\text{eff}}V_b+k\sum_i |\lambda^-|_i\,, \tag{8}$$

where k is a scale factor (of the order of unity) that weights the effects on the fusion cross section of the deformation versus the inelastic and transfer reaction degrees of freedom.

In Fig. 3 we display the values of ΔE_t as a function of the parameter $\beta_{\text{eff}}\cdot V_b+k\cdot\lambda$ calculated using Eq. (8) with $k=1.65$. We can see that the scatter of the data observed in Fig. 2 has almost disappeared. The fact that all the systems fall nicely within a single trend indicates the soundness of this unified approach, which includes the effects of the quadrupole and hexadecapole permanent deformations, the inelastic excitations, and the transfer reaction channels to account for the enhancements of the fusion cross sections at energies below the barrier.

Another interesting aspect of this approach, is that it can also be applied to a single degree of freedom. For example the effect of the hexadecapole deformation component can be isolated and put as evidence if we introduce the following parameter:

$$\Delta E_{\beta_4}=\frac{\hbar\omega}{2\pi}\ln\left(\frac{\sigma_{\text{fus}}^{\text{cou}}(E_{\text{c.m.}},\beta_2,\beta_4,\alpha)}{\sigma_{\text{fus}}^{\text{cou}}(E_{\text{c.m.}},\beta_2,\beta_4=0,\alpha)}\right),\quad \text{for } E_{\text{c.m.}}\ll V_b\,. \tag{9}$$

The extent in which the quadrupole and the hexadecapole deformations contribute to the enhancement independently, ΔE_{β_4}, describes the contribution to the enhancement associated with β_4. In this expression α represents all the other coupling parameters associated with the inelastic excitations and transfer channels that are the same in the numerator and denominator of this expression.

In Fig. 4 we show the values of ΔE_{β_4} versus $\beta_4 V_b$. Once again we find a clear trend that nicely indicates the magnifying effect of the Coulomb barrier, in this case, on the isolated effect of the hexadecapole deformation. Moreover, this trend persists to *negative* values of β_4 where for energies below the Coulomb barrier the contribution of the hexadecapole deformation causes a well-known *decrease* in the fusion cross section [32]. The correlation shown in Fig. 4 strengthens the conclusion that a given coupling (in this case a permanent hexadecapole deformation) will become more relevant as the Coulomb interaction increases.

In conclusion, we have analyzed a set of sixteen fusion cross sections excitation functions and compared them to the same model. All the calculations were performed with the coupled-channel code, CCMOD [10], by taking into account the dominant degrees of freedom of the corresponding participating nuclei: quadrupole and hexadecapole deformations, inelastic excitations, and transfer reaction channels. We found that the asymptotic sub-barrier cross section enhancement characterized by the total asymptotic energy shift, ΔE_t, increases monotonously with the quantity $\beta_{\mathrm{eff}} \cdot V_b + k \cdot \lambda$. Similarly the asymptotic energy shift associated with the hexadecapole deformation, ΔE_{β_4}, increases monotonously with the product $\beta_4 \cdot V_b$. The correlations just mentioned are useful in predicting the effect on the enhancement of a given degree of freedom on the fusion cross sections. They also allow us to choose the most adequate reaction system for studying the effect on the fusion cross section of a given degree of freedom. In all cases, the magnitude of V_b can be thought of as a magnifying parameter of the corresponding coupling strength. In the case of permanent deformations this magnification effect is explicit; however, for the inelastic and transfer channels, this effect is indirect since the magnitude of the Coulomb barrier affects the value of the corresponding form factors f in Eq. (7) as described in Ref. [33]. Our physical interpretation of this general systematic overview emphasizes the central role of the Coulomb barrier in amplifying the relevance of the couplings associated with a particular degree of freedom.

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