Sistemas dinámicos

→ varían en el tiempo

Variables que solo dependen de t → deterministas

Se describen típicamente con ec. diferenciales ordinarias

ODE

Supongamos partículas que se degradan

$N(t)$ = # de partículas al tiempo t

$$N(t+\Delta t) - N(t) = -\alpha \Delta t \, N(t)$$

$$\frac{N(t+\Delta t) - N(t)}{\Delta t} = -\alpha N$$

$$\frac{dN}{dt} = -\alpha N$$

$$\frac{1}{N}\frac{dN}{dt} = \frac{d\ln N}{dt} = -\alpha$$

?

$$\ln N = -\alpha t + \text{const}$$

$$N(t=0) = N_0$$

$$\rightarrow \ln N_0 = \text{const}$$

$$\ln N = -\alpha t + \ln N_0$$

$$\ln N - \ln N_0 = \ln\left(\frac{N}{N_0}\right) = -\alpha t$$

$$\boxed{N = N_0 e^{-\alpha t}}$$

$$N(t) = ?$$

$$\frac{dN}{dt} + \alpha N = 0$$

$$a\frac{d^3N}{dt^3} + b\frac{d^2N}{dt^2} + c\frac{dN}{dt} + dN = 0$$

Sol + General

$$N = e^{\lambda t} \rightarrow \frac{dN}{dt} = \lambda e^{\lambda t}$$

$$\frac{d^2N}{dt^2} = \lambda^2 e^{\lambda t}$$

$$\overbrace{a\lambda^3}^{\in \mathbb{R}} + b\lambda^2 + c\lambda + d = 0$$

Me interesan las raíces del polinomio

3 raíces de este polinomio

Sol. t general

$$A e^{\lambda_1 t} + B e^{\lambda_2 t} + C e^{\lambda_3 t}$$

condiciones

$$[\lambda] = \frac{1}{[\text{tiempo}]} \sim \frac{1}{\text{tiempos característicos}}$$

$$\sqrt{-2} = ?$$

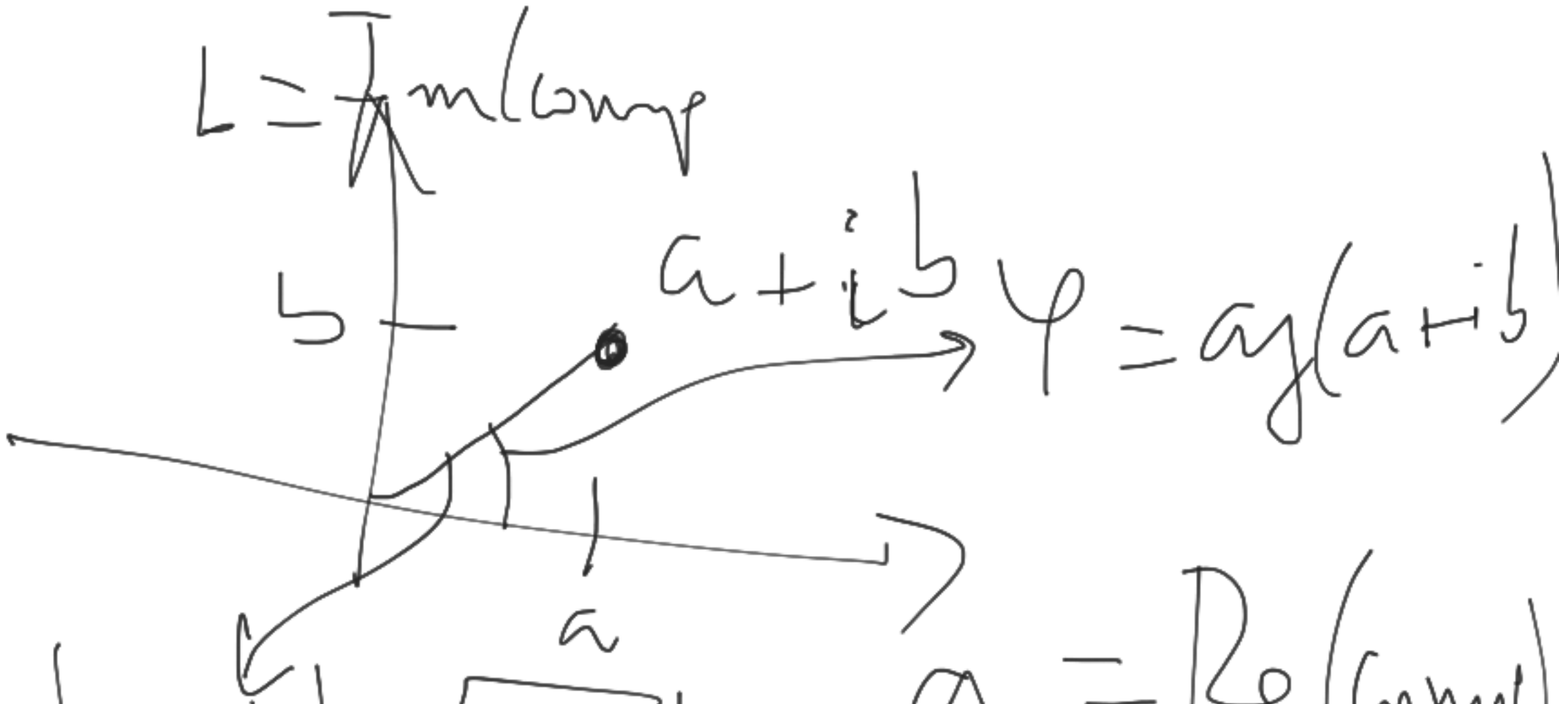
$$\sqrt{-1} = i$$

$$\underbrace{a + i\,b}_{\text{reales}} \longrightarrow \# \in \mathbb{C}$$

$$(a+ib)(a-ib) =$$

$$= a^2 - aib + iba + b^2$$

$$= a^2 + b^2$$



$$|a+ib| = \sqrt{a^2+b^2}$$

$$\lambda = a + ib \quad ?$$

$$e^{(\lambda t)} = e^{(a+ib)t} = e^{(at+ibt)}$$

$$= e^{at} \cdot e^{ibt}$$

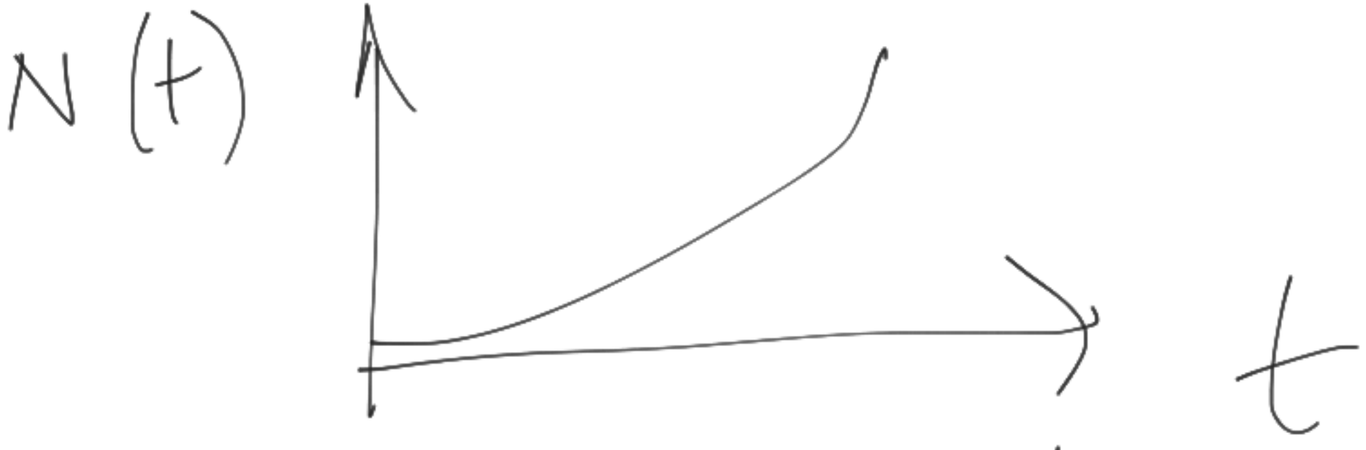
$$e^{ibt} = \cos(bt) + i\,\mathrm{sen}(bt)$$

$$b \propto \omega$$

$$a\frac{d^3N}{dt^3} + b\frac{d^2N}{dt^2} + c\frac{dN}{dt} + d = 0$$

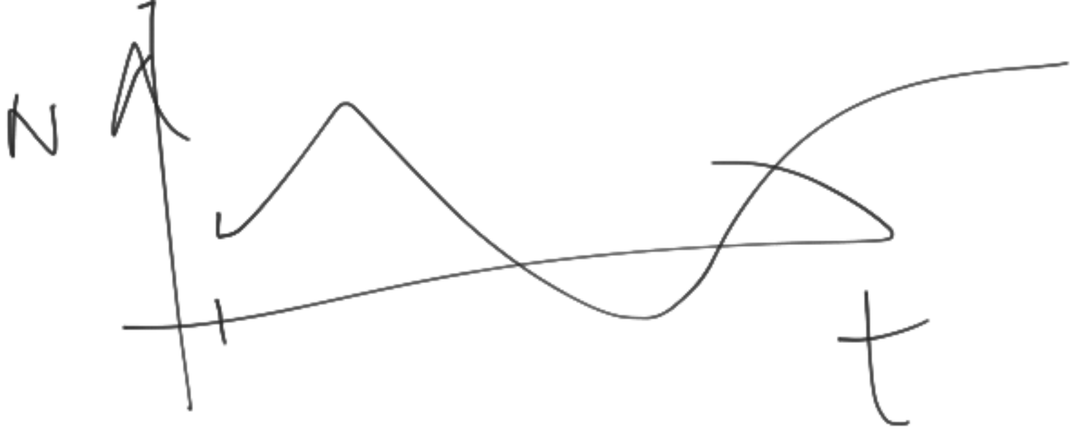
$\neq 0$

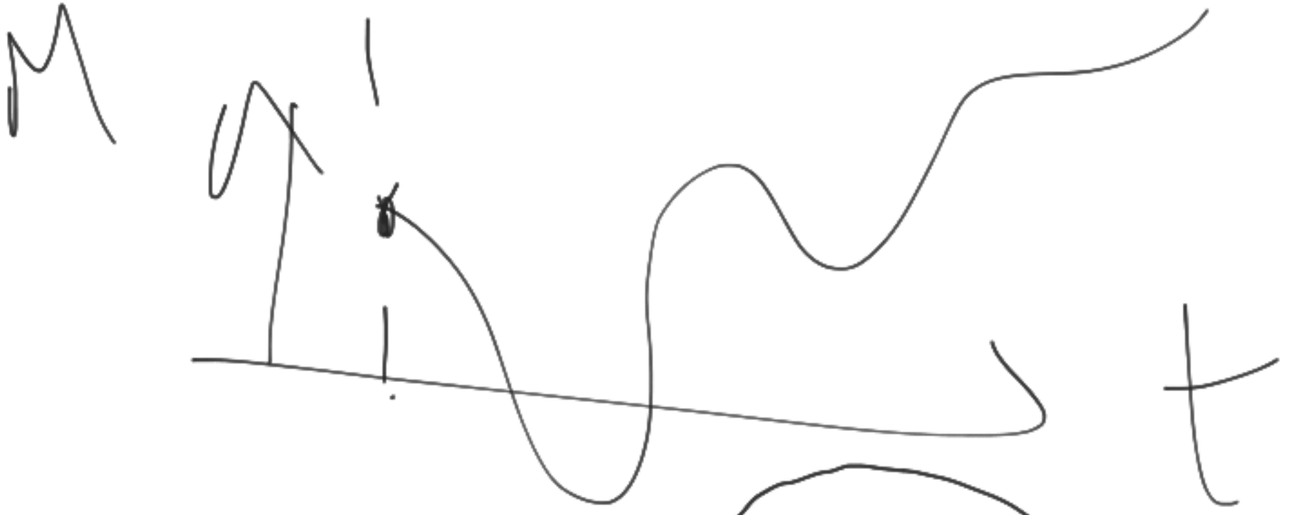
Dos formas de mirar a una solución.

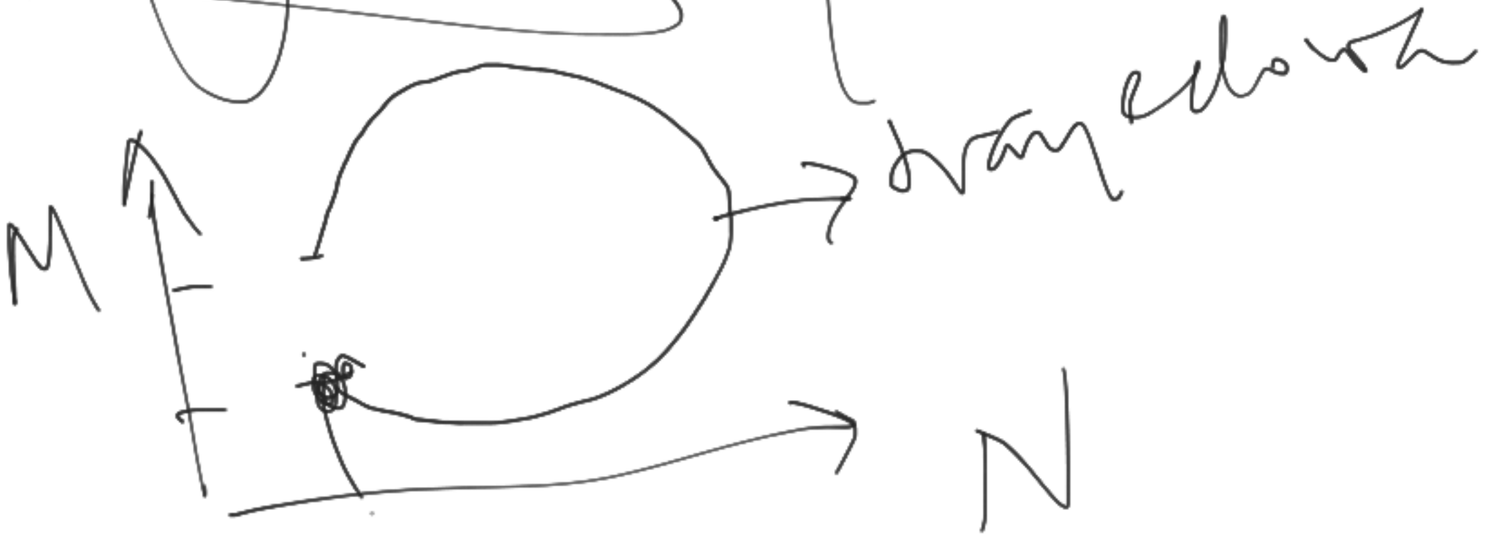


Problemas con más de 1 variable

N y M N(t) ; M(t)

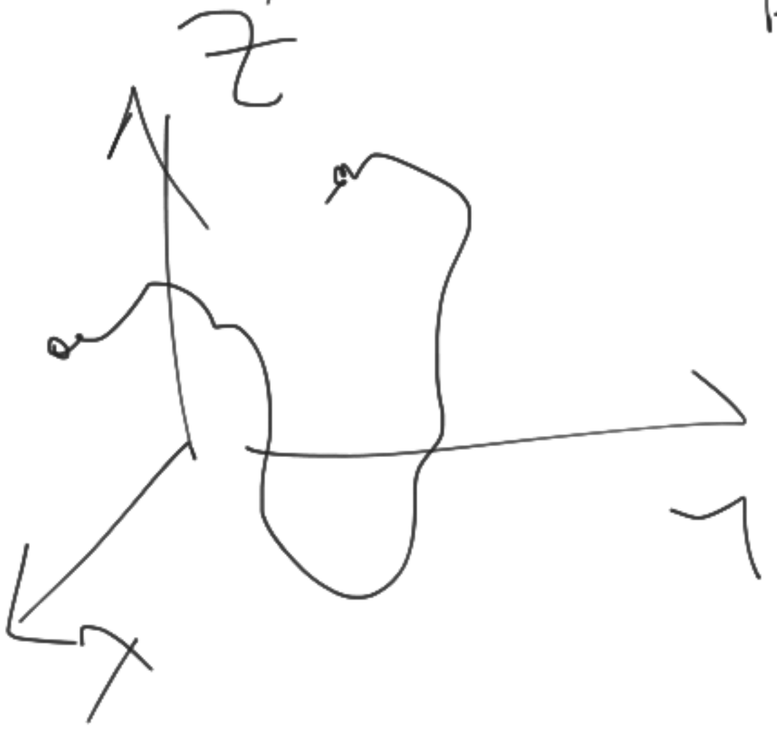

N
t

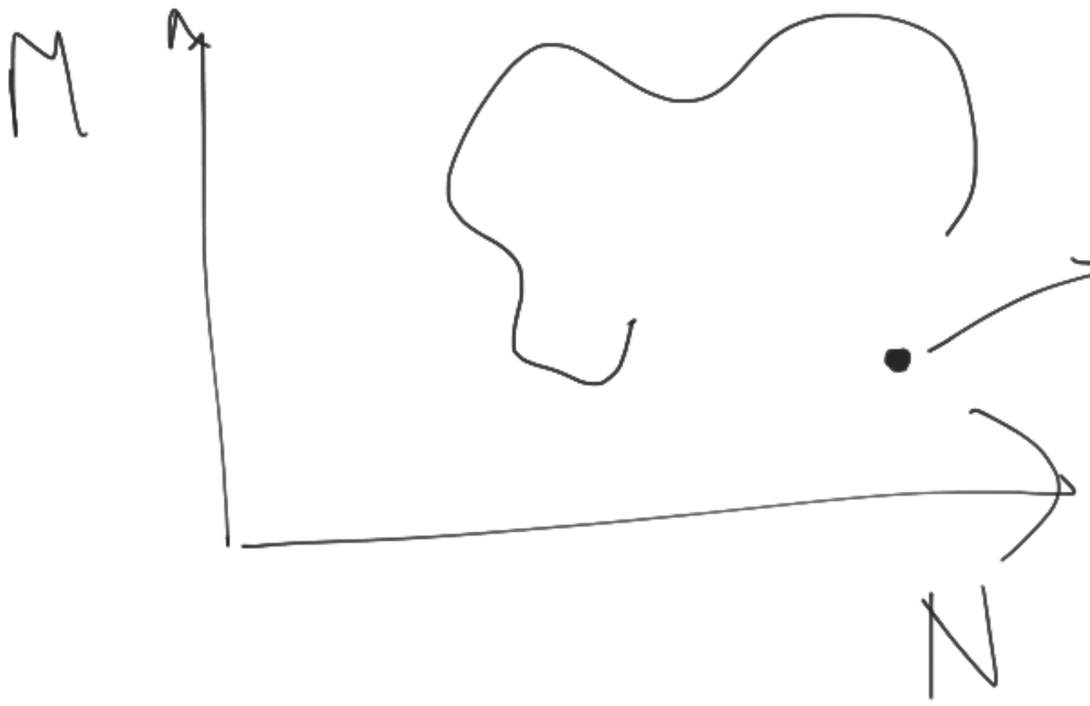

M
t


M
N

partícula que se mueve

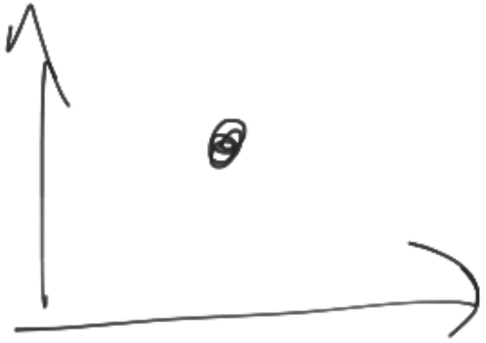
$\rightarrow \vec{r}(t)$





pto fijo
sobre
que no
varía
en el tiempo

Estabilidad
del pto fijo



N en el sto fijo en N^*

y M en M^*

$$\frac{dN}{dt} = f(N, M)$$

$$\frac{dM}{dt} = g(N, M)$$

$$0 = f(N^*, M^*)$$

$$0 = g(N^*, M^*)$$

$$\delta N = N - N^* \quad ; \quad \delta M = M - M^*$$

$$\frac{d(\delta N)}{dt} = \frac{dN}{dt} - \underbrace{\frac{dN^*}{dt}}_{=0} = \frac{dN}{dt}$$

$$= f(M, N) = \underbrace{f(M^*, N^*)}_{=0} + \frac{\partial f}{\partial M}(M - M^*)$$

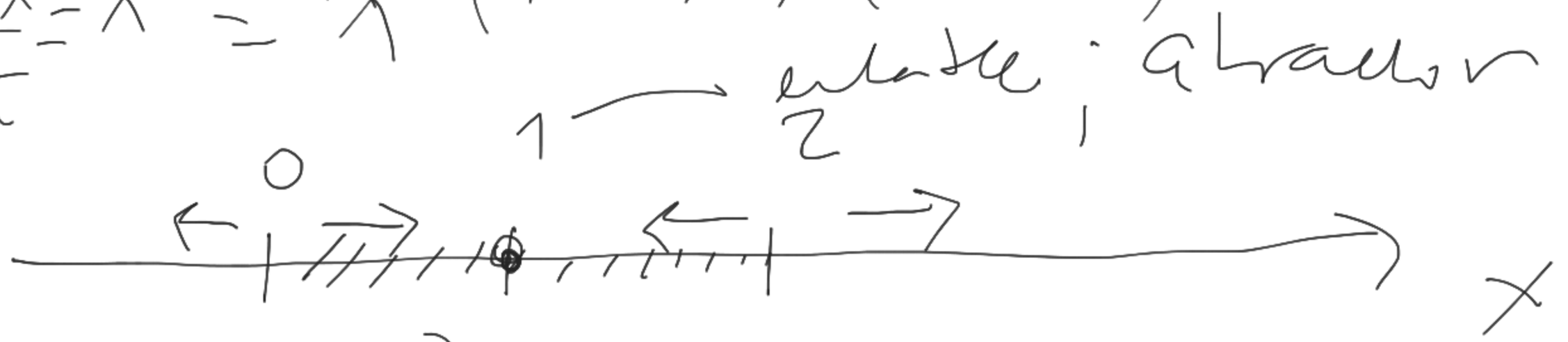
$$+ \frac{\partial f}{\partial N}(N - N^*) = \frac{\partial f}{\partial M} \cdot \delta M + \frac{\partial f}{\partial N} \delta N$$

$$\frac{d\delta N}{dt} = \frac{\partial f}{\partial M}\delta M + \frac{\partial f}{\partial N}\delta N$$

$$\frac{d\delta M}{dt} = \frac{\partial g}{\partial M}\delta M + \frac{\partial g}{\partial N}\delta N$$

Calculados en M^*, N^*

$$\frac{dx}{dt} = \dot{x} = x(1-x)(2-x)$$



Plus fijos?

$$\frac{dx}{dt} = 0 \quad \begin{cases} x = 0 \\ x = 1 \\ x = 2 \end{cases}$$