$$\dot{x} = 4x - y$$

$$\dot{y} = 2x + y$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$y = 4x - \dot{x}$$

$$\Rightarrow \dot{y} = 4\dot{x} - \ddot{x}$$

$$4\dot{x} - \ddot{x} = 2x + 4x - \dot{x}$$

$$\boxed{\ddot{x} - 5\dot{x} + 6x = 0}$$

$$x = e^{\lambda t} \rightarrow \dot{x} = \lambda e^{\lambda t} \rightarrow \ddot{x} = \lambda^2 e^{\lambda t}$$

$$\ddot{x} - 5\dot{x} + 6x = 0$$

$$\lambda^2 \cancel{e^{\lambda t}} - 5\lambda \cancel{e^{\lambda t}} + 6\cancel{e^{\lambda t}} = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2} \begin{matrix} \nearrow 3 \\ \searrow 2 \end{matrix}$$

$$\dot{x} = 4x - y$$

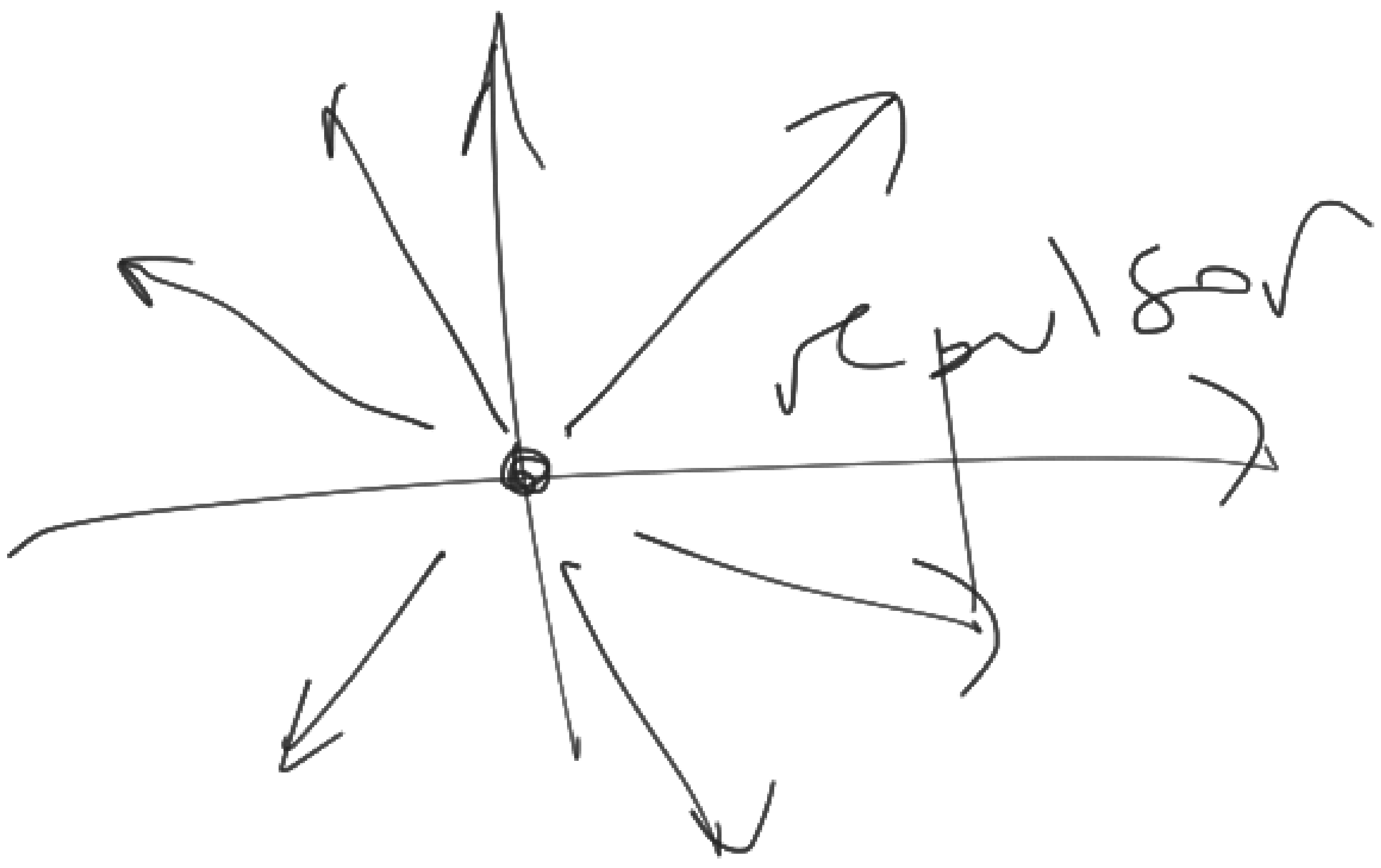
$$\dot{y} = 2x + y$$

Ptos fijos $\dot{x} = 0 = \dot{y}$

$$4x = y \quad ; \quad 2x = -y$$

$$x = y = 0$$

$$x = A e^{3t} + B e^{2t}$$


repulsor

$$\dot{x} = \mu x - y \Rightarrow y = \mu x - \dot{x}$$

$$\dot{y} = x + \mu y \qquad \dot{y} = \mu \dot{x} - \ddot{x}$$

$$\mu \dot{x} - \ddot{x} = x + \mu^2 x - \mu \dot{x}$$

$$\ddot{x} - 2\mu \dot{x} + (1+\mu^2) x = 0$$

$$\dot{x} = 0 = \dot{y} \Rightarrow \mu x = y \; ; \; x = -\mu y$$

$$-\mu^2 y = y$$

$x=0 \Rightarrow$ pto fijo

$$\ddot{x} - 2\mu\dot{x} + (1+\mu^2)x = 0$$

$$x = e^{\lambda t}$$

$$\lambda^2 - 2\mu\lambda + (1+\mu^2) = 0$$

$$\lambda = \frac{2\mu \pm \sqrt{4\mu^2 - 4(1+\mu^2)}}{2} = \mu \pm i$$

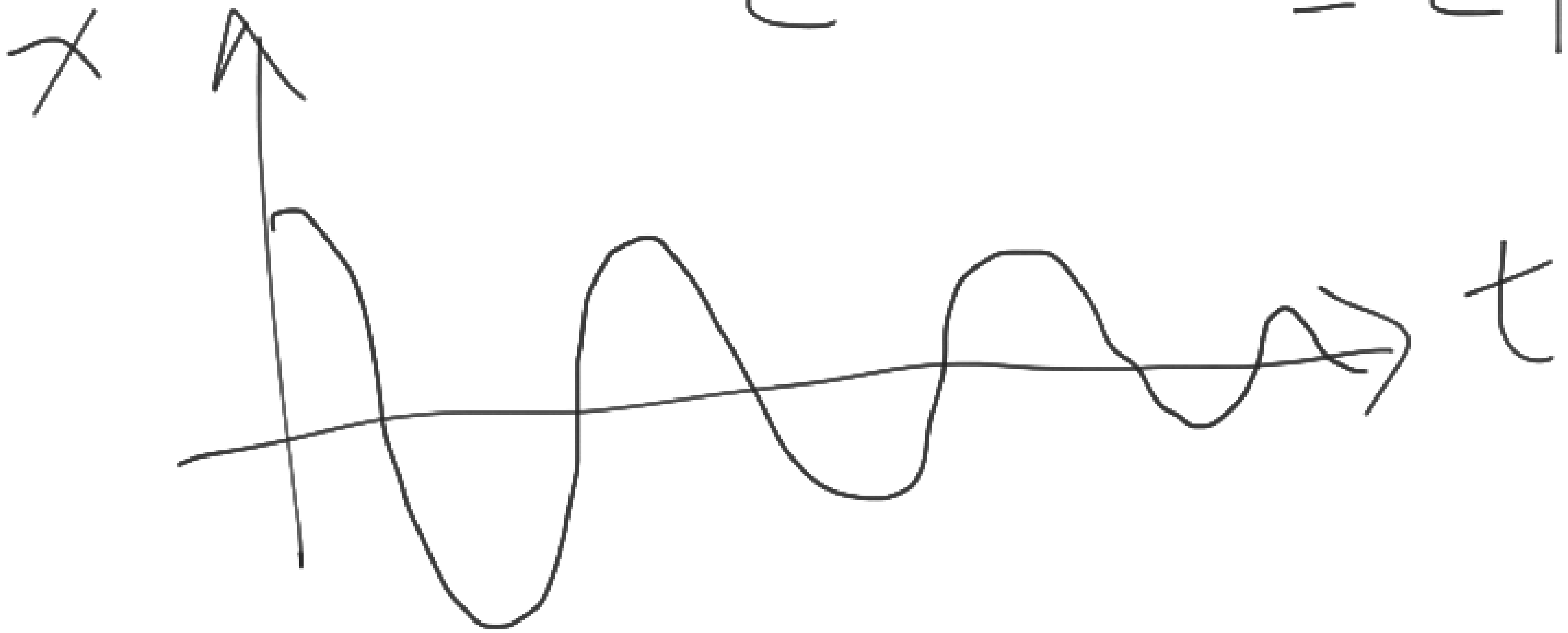
$$\lambda_\pm = \mu \pm i$$

$$e^{\lambda_+ t} = e^{\mu t} \, e^{it}$$

$$\boxed{\mu < 0}$$

$x = 0 \Rightarrow$ stabile

$$e^{\lambda_- t} = e^{\mu t} \, e^{-it}$$

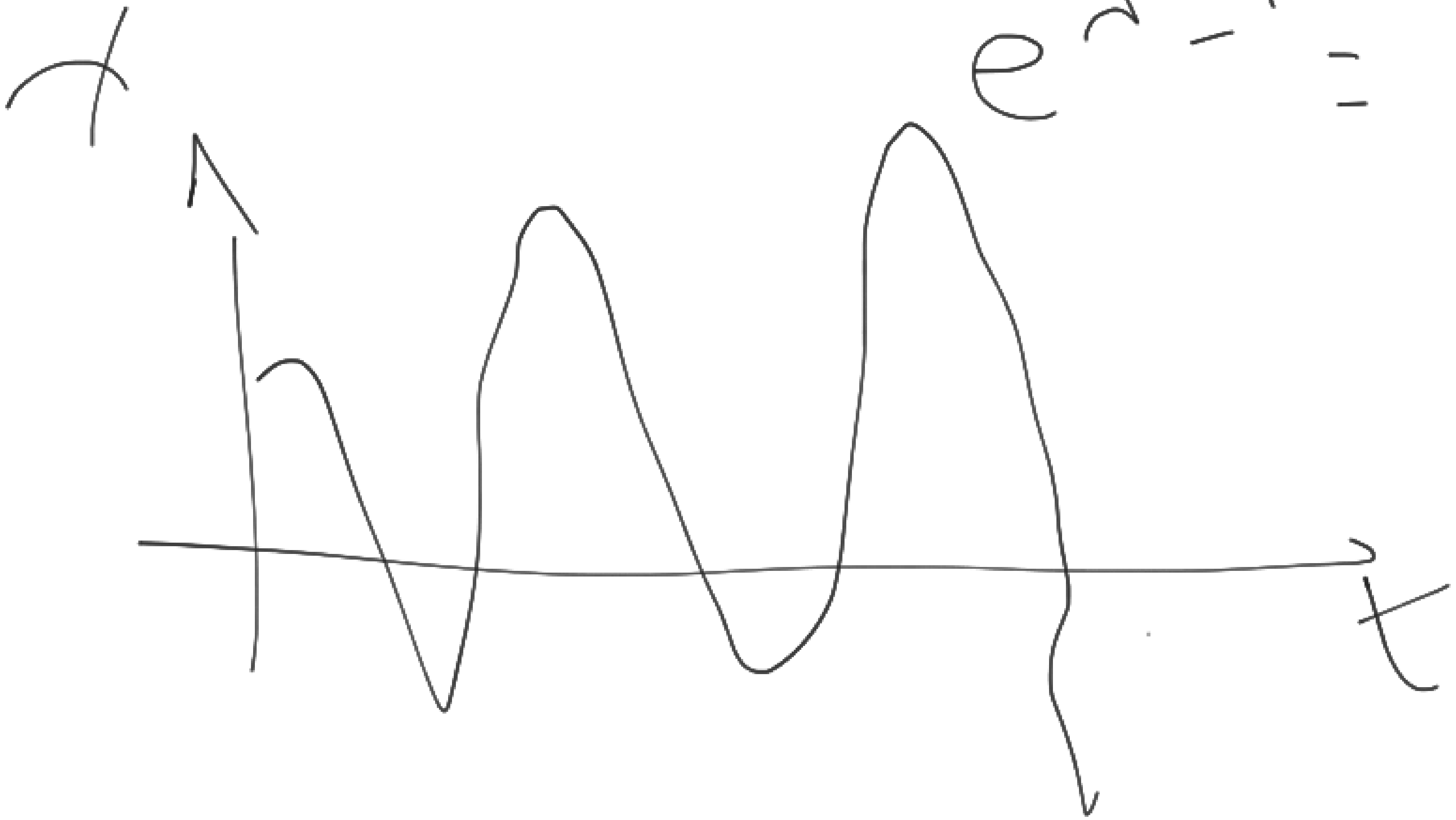


$$e^{i\hat{\omega} t}$$

$\mu > 0$

$$e^{\lambda_+ t} = e^{\mu t} e^{it}$$

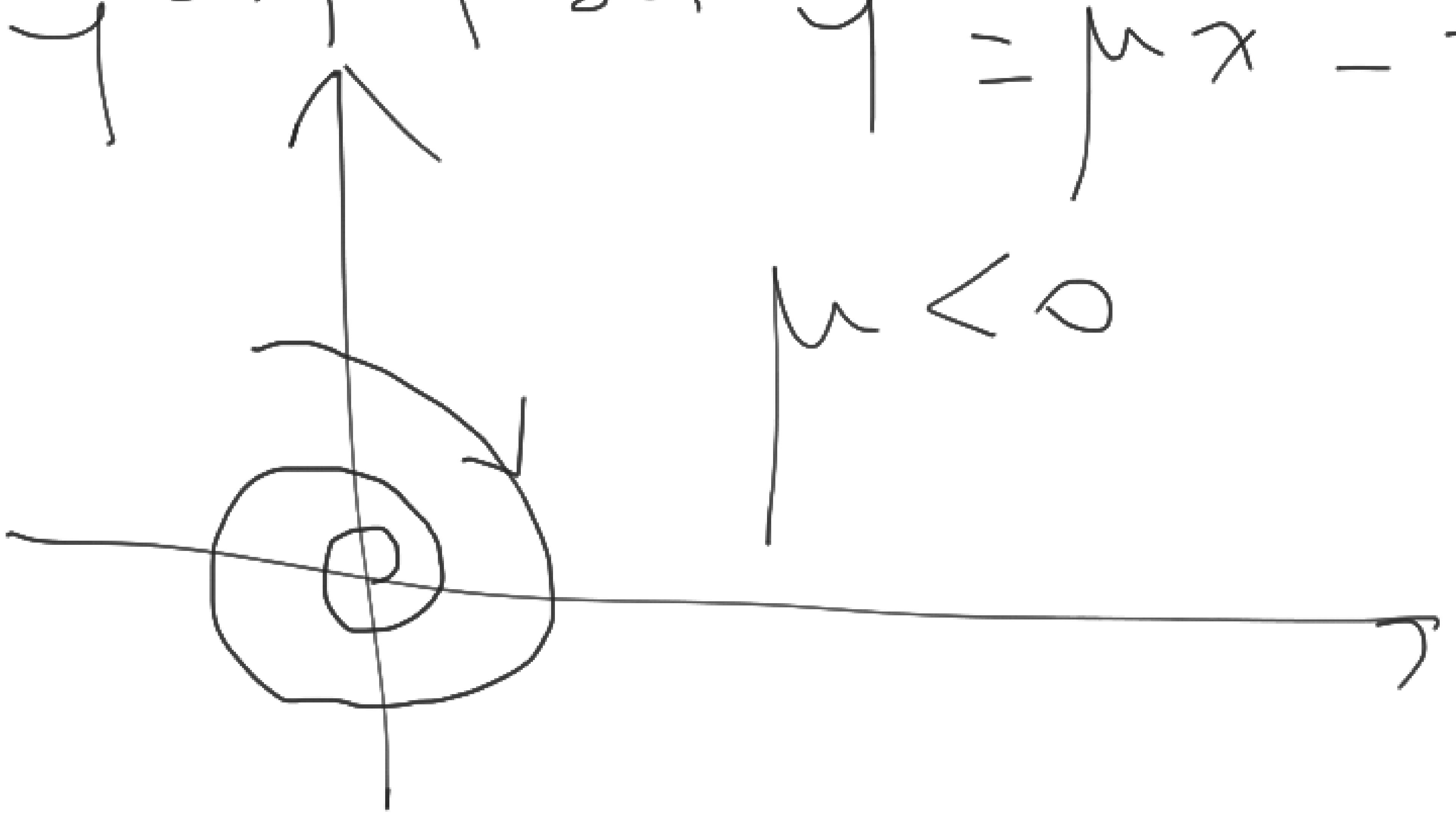
$$e^{\lambda_- t} = e^{\mu t} e^{-it}$$



Esp fases

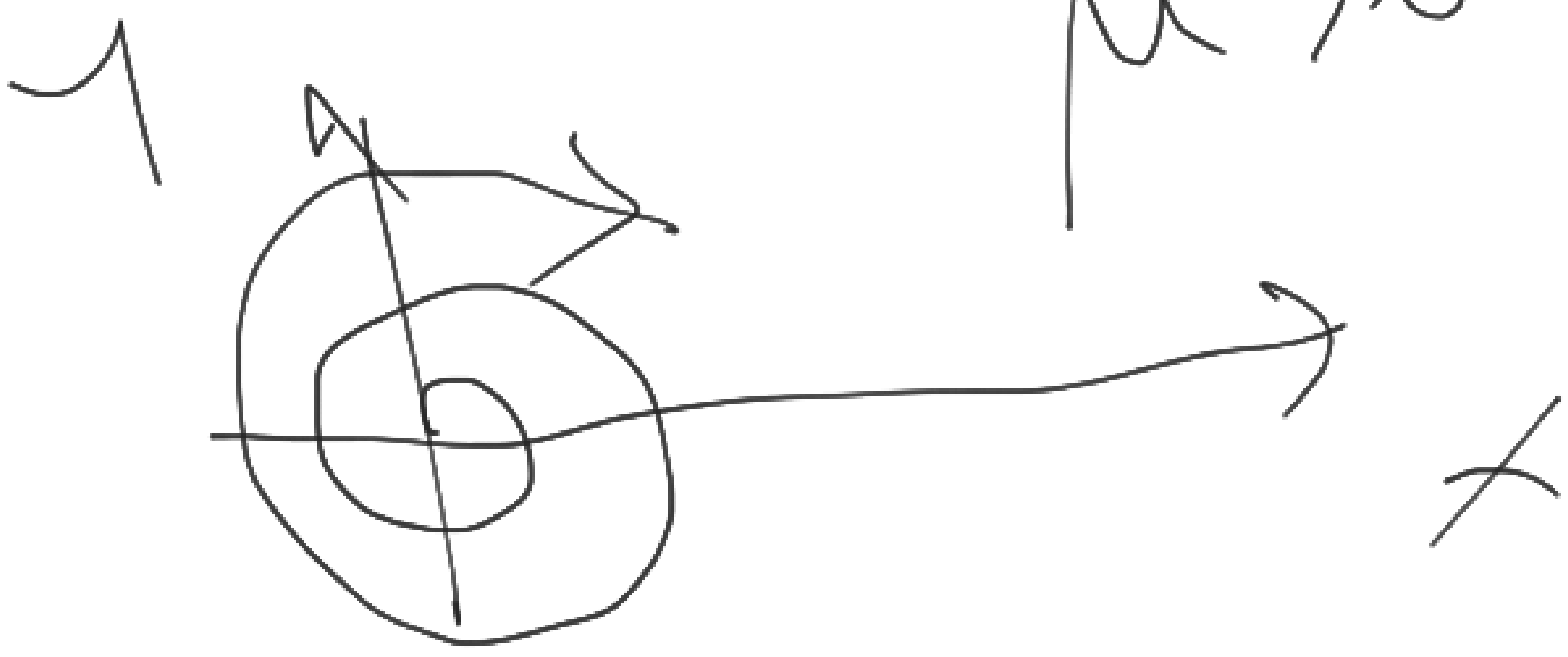
$$y = \mu x - \dot{x}$$

$\mu < 0$



Bifurcación

$\mu > 0$



$$\left.\begin{aligned}\dot{x} &= \mu x - y \\ \dot{y} &= x + \mu y\end{aligned}\right\}$$

x, y = variables

μ = parámetro

$\mu = 2$

Si al variar 1 parámetro hay 1 cambio cualitativo en el espacio de fases decimos que sucede 1 bifurcación

Las bifurcaciones suceden en cualquier sistema dinámico.
En muchas circunstancias interesa saber qué le pasa a los puntos fijos

- Busca puntos fijos
- Estudia su estabilidad como función de los parámetros.

→ Linealiza las ecuaciones alrededor del punto fijo y busca autovalores (λ)

Bif saddle-node

$$\dot{x} = \mu - x^2$$

Busco pto fijos

$$\dot{x} = 0 \Rightarrow \mu = x^2 \Rightarrow x = \pm\sqrt{\mu}$$

$$e^{i\omega t} + e^{-i\omega t} \qquad \mp(\cos\omega t)^2$$

$\mu < 0 \rightarrow$ No tengo puntos fijos

$\mu > 0 \rightarrow$ tengo 2 puntos fijos

$$\pm\sqrt{\mu} = x$$

$\mu = 0 \rightarrow$ 1 punto fijo; $x = 0$

$\downarrow$ Aniquilación de bifurcación saddle-node

$\mu < 0$

$\dot{x} = \mu - x^2 < 0$

$x(t) \to -\infty$

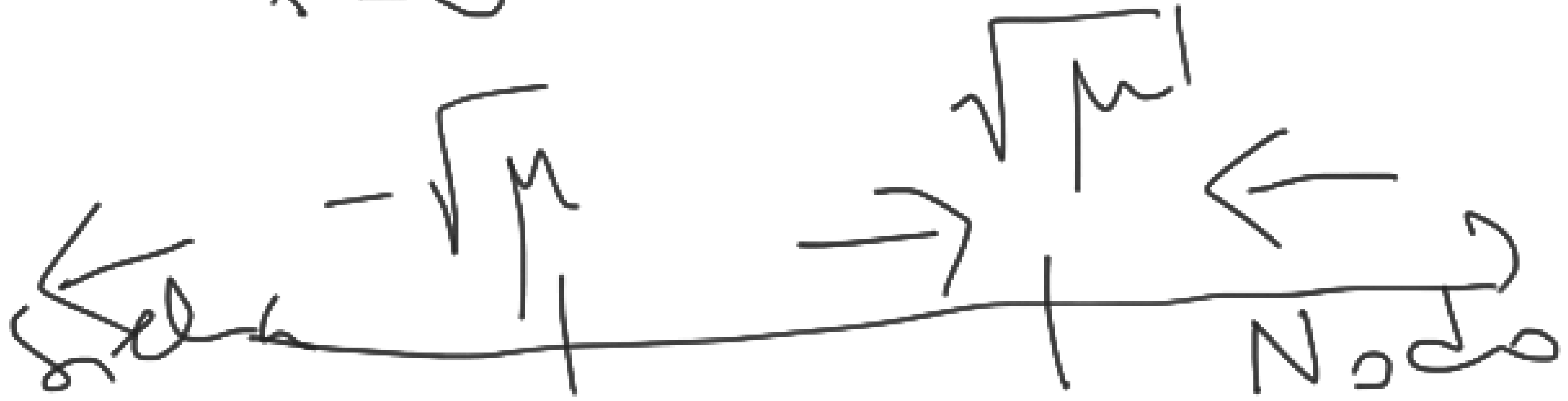
$\mu = 0$

$\dot{x} = -x^2$

$x = 0$

$\dot{x} < 0 \quad \forall x \neq 0$

$\mu > 0$



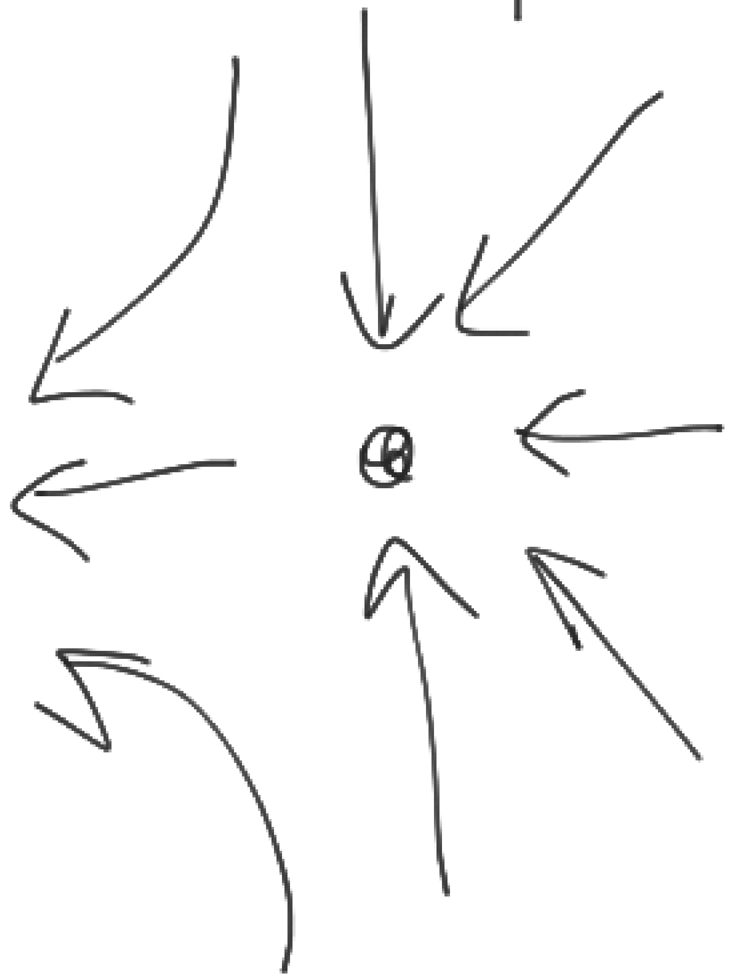
$\dot{x} = \mu - x^2$

Saddle node en más dimensiones

$\mu < 0$

No hay pto fijo

$\mu = 0$



$\mu > 0$

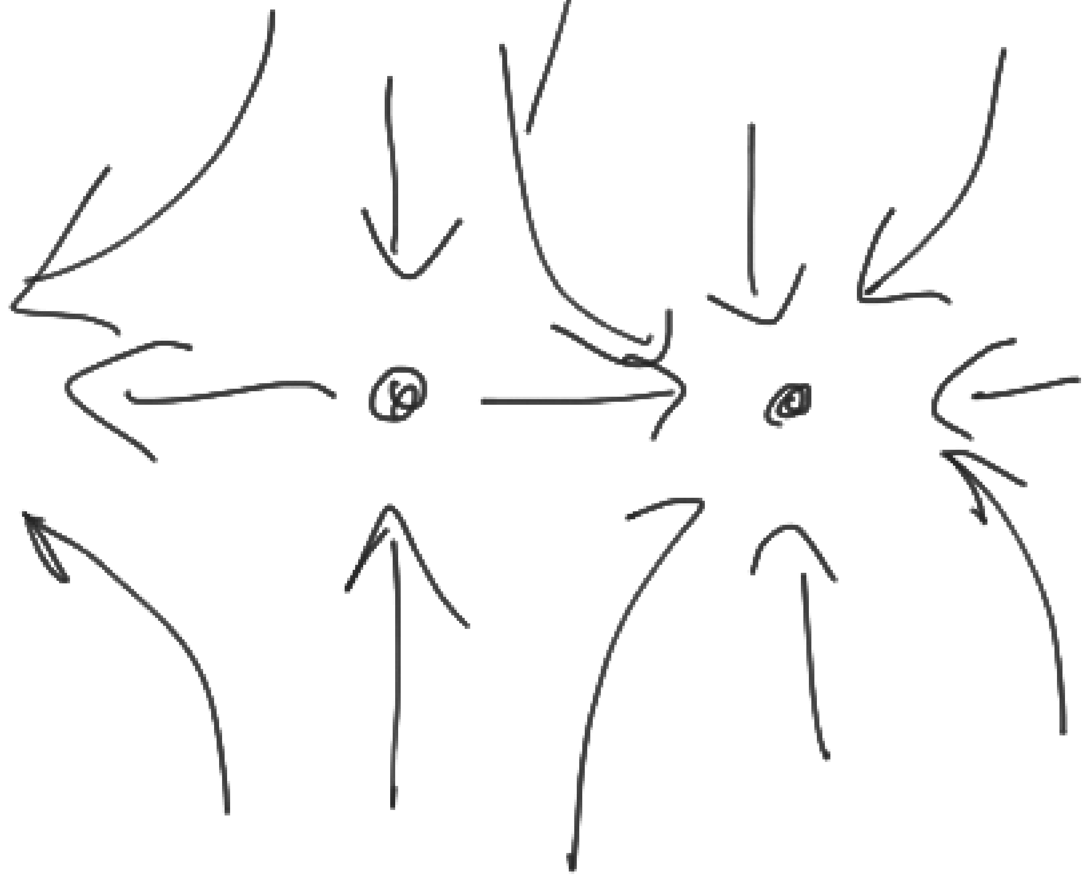
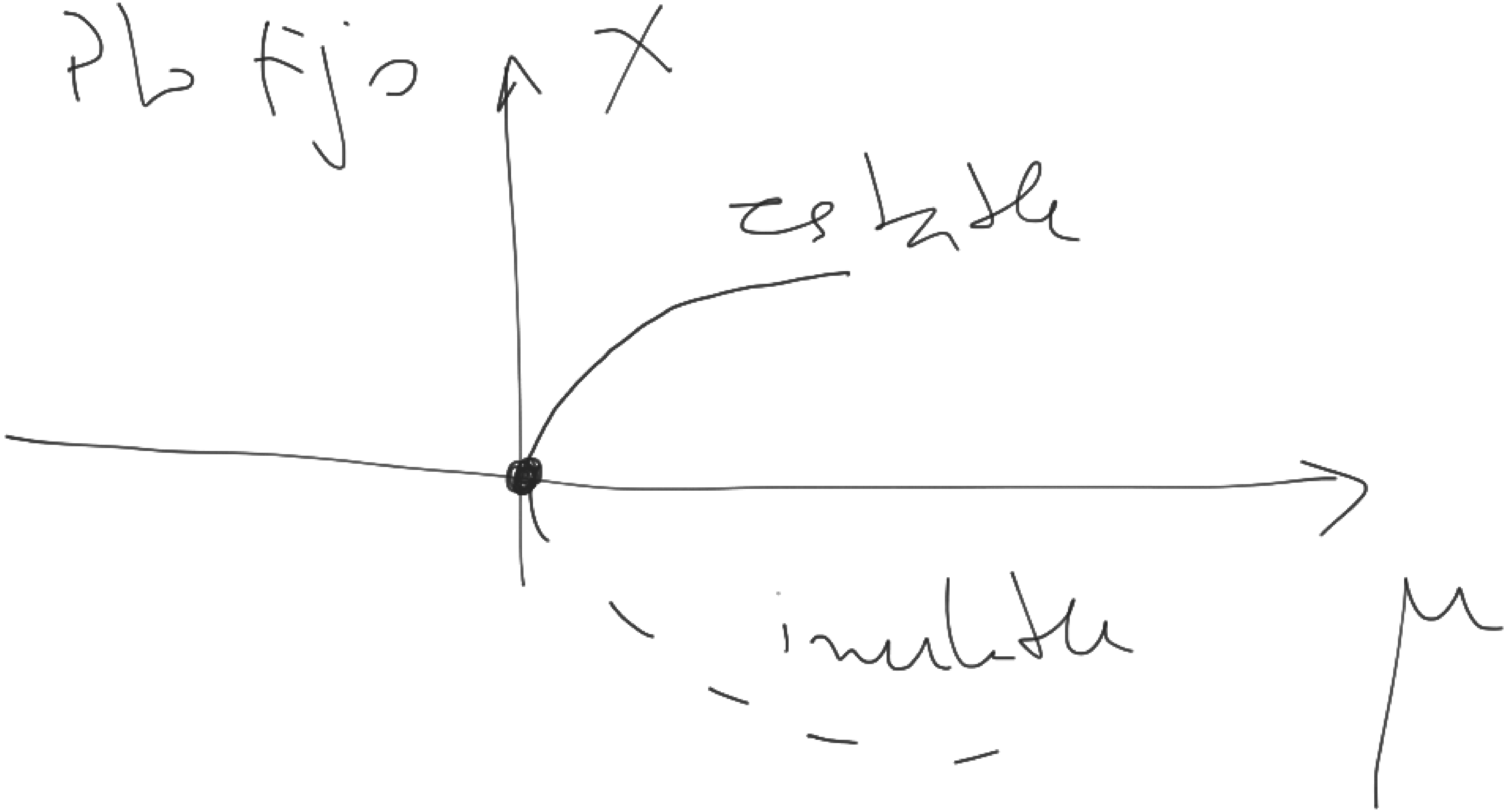


Diagrama de bifurcación



$$\cancel{m}\ddot{x} = -\frac{K}{m}x$$

$$\dot{x} = y$$

$$\ddot{x} = \dot{y}$$

$$\rightarrow$$

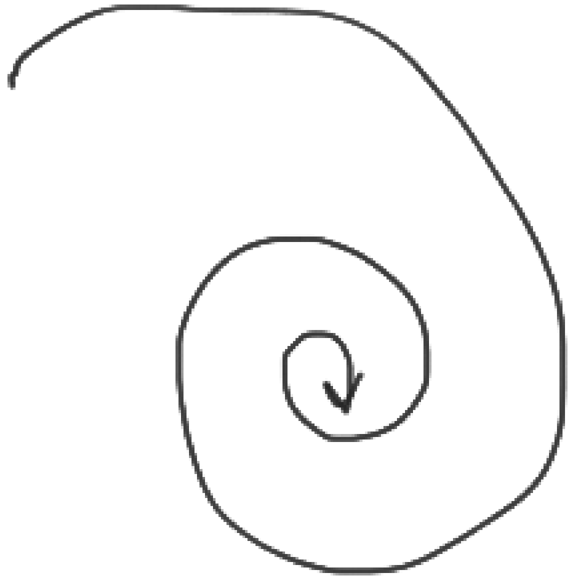
$$\dot{y} = -\frac{K}{m}x$$

$$\dot{x} = y$$

$$\dot{x} = y$$

$$\dot{y} = -\frac{K}{m}x$$

$\mu<0$ $\mu=0$ $\mu>0$



Hopf