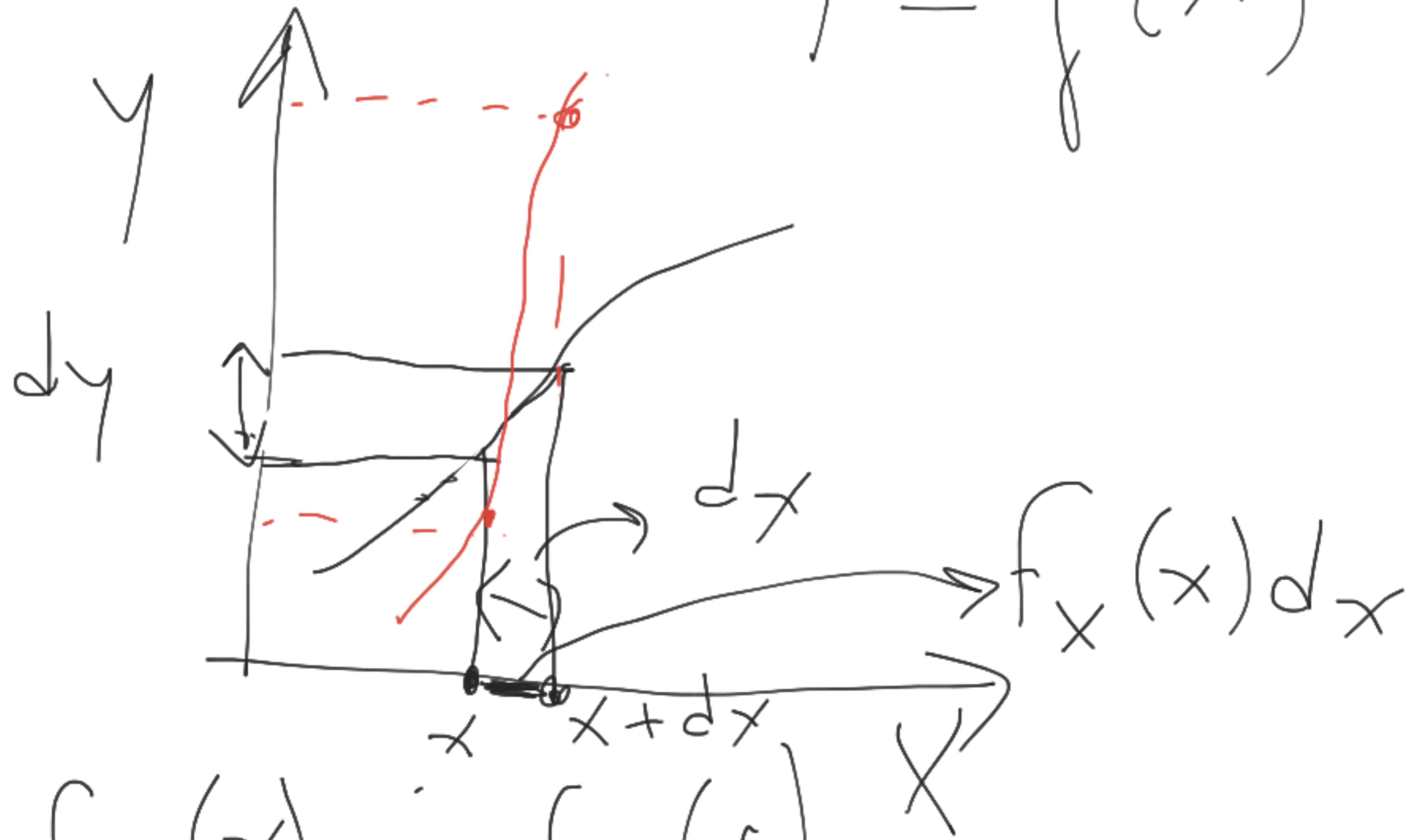
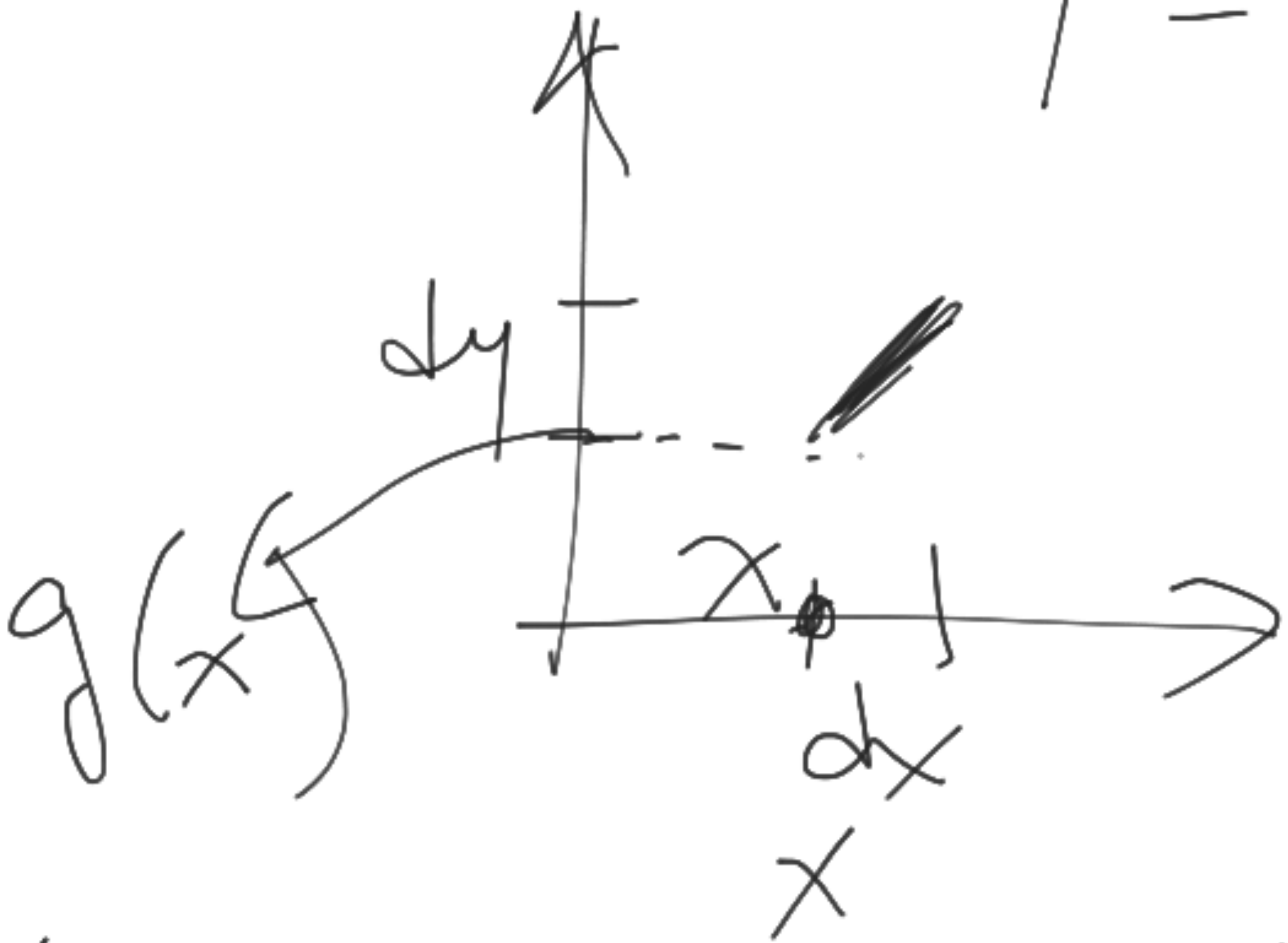
$$y = g(x)$$



$$f_X(x) \;;\; f_Y(y)$$

$$y = g(x)$$



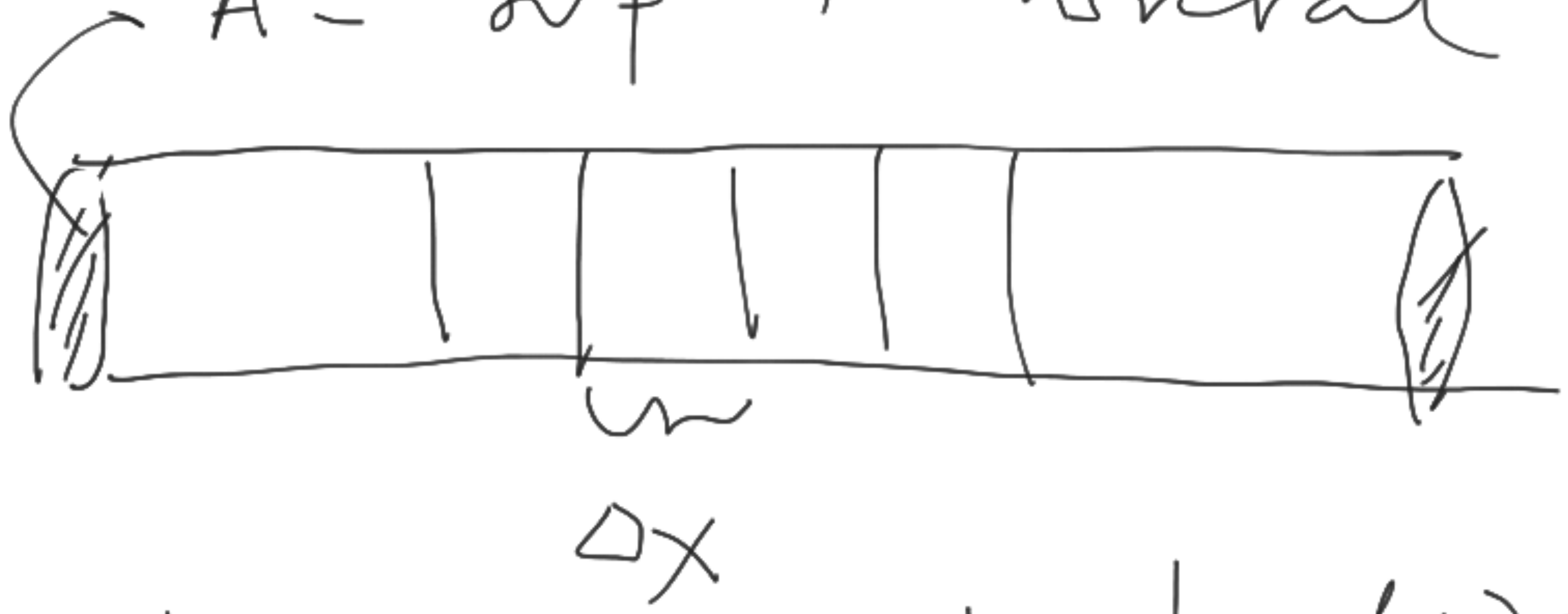
$$dy = g'(x)\,dx$$

$$dP(x) = f_x(x)\,dx$$

$$= dP(y) = f_y(y = g(x))\,\cdot$$

$$\cdot\, dy = f_y(y = g(x)) \cdot g'\,dx$$

$A =$ Sup transversal

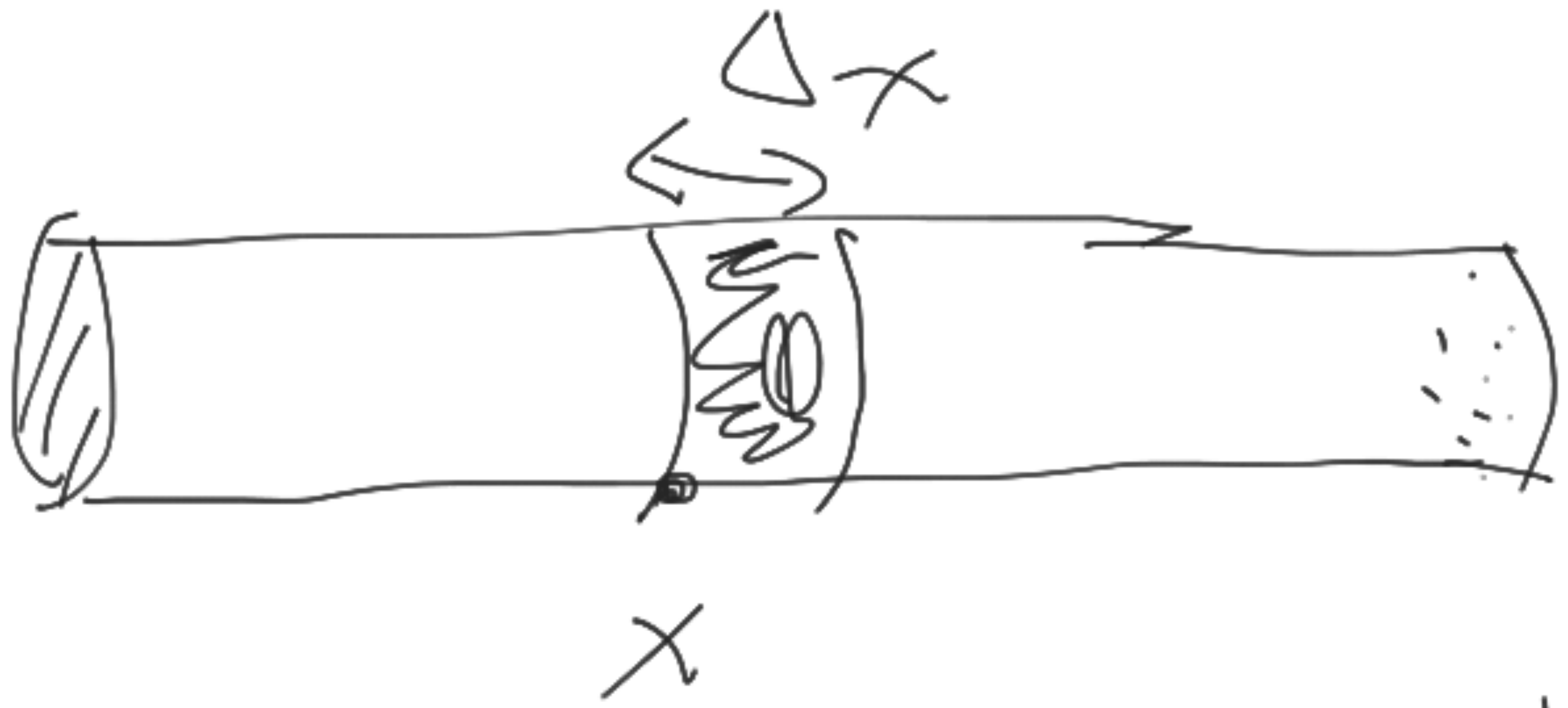


Partículas que se desplazan $\pm\lambda$ en 1 tiempo τ

$\Delta x = \lambda$; $\Delta t = \tau$

$t = 0, \tau, 2\tau, \ldots$

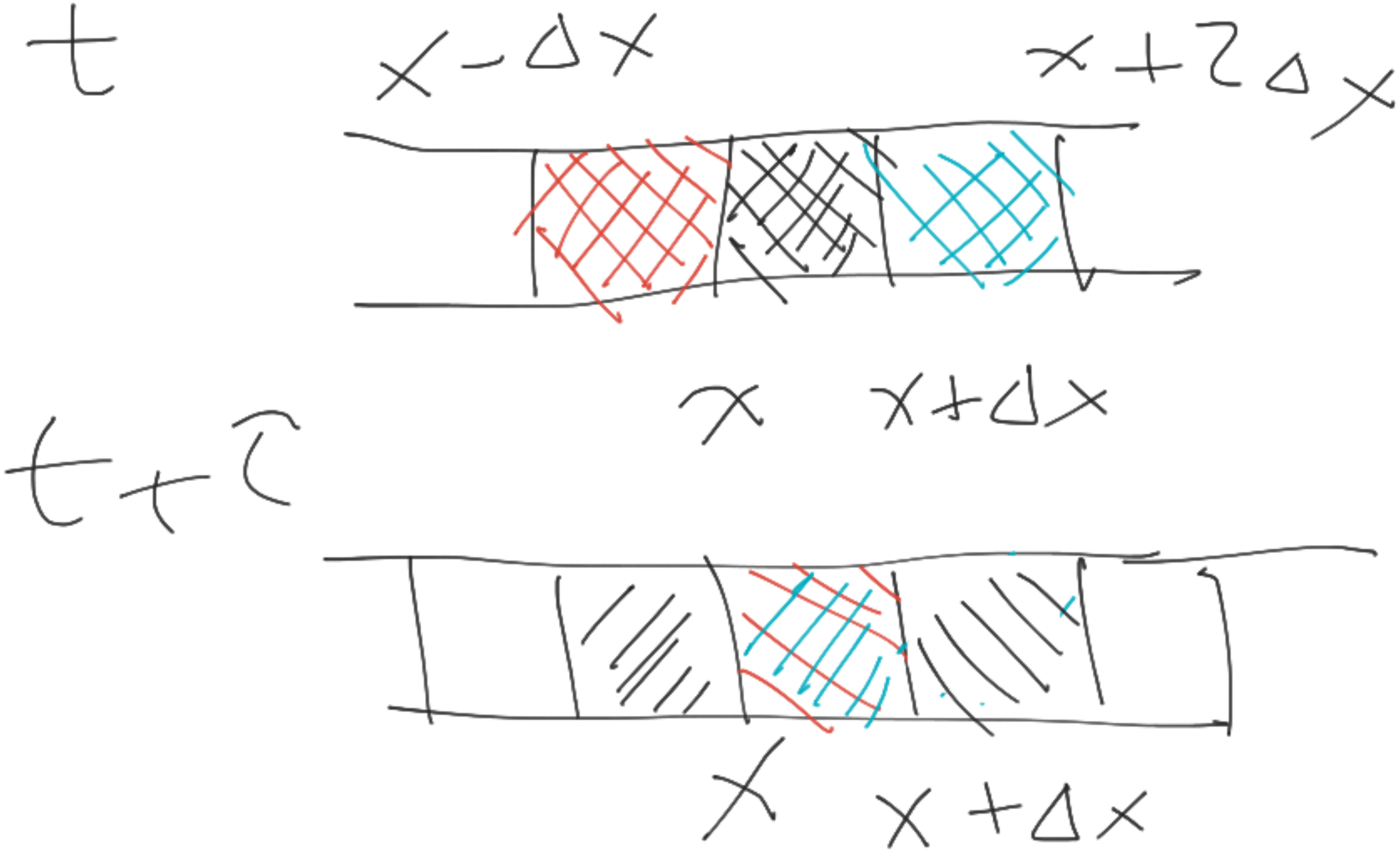
Quiero calcular cómo varía el número de paseadoras en la posición x (en el intervalo entre x y $x + dx$) entre el tiempo t y el tiempo $t + \tau$



\# paseadores "en x"
(entre x y $\cancel{x} + \Delta x$) al tiempo t

$$= n(x,t)\,\Delta x\,A$$

↳ \# paseadores por unid del volumen



$$n(x,t+\tau)\,A\Delta x =$$
$$= \frac{1}{2} n(x-\Delta x, t)\,A\Delta x + \frac{1}{2} n(x+\Delta x, t)$$

$$n(x,t+\Delta t)\cancel{A\Delta x} - n(x,t)\cancel{A\Delta x}$$

$$= \frac{1}{2} n(x-\Delta x,t)\cancel{A\Delta x} + \frac{1}{2} n(x+\Delta x,t)\cancel{A\Delta x}$$

$$- n(x,t)\cancel{A\Delta x}$$

$$n(x+\Delta x,t) \simeq n(x,t) + \frac{\partial n}{\partial x}\Delta x$$

$$+ \frac{1}{2}\frac{\partial^2 n}{\partial x^2}(\Delta x)^2$$

$$n(x-\Delta x,t) \simeq n(x,t) + \frac{\partial n}{\partial x}(-\Delta x) + \frac{1}{2}\frac{\partial^2 n}{\partial x^2}(-\Delta x)^2$$

$$n(x,t+\tau) - n(x,t) \simeq \frac{1}{2}\Big[\cancel{n(x,t)} + \cancel{\frac{\partial n}{\partial x}(-\Delta x)} + \frac{1}{2}\frac{\partial^2 n}{\partial x^2}(-\Delta x)^2 + \cancel{n(x,t)} + \cancel{\frac{\partial n}{\partial x}(\Delta x)} + \frac{1}{2}\frac{\partial^2 n}{\partial x^2}(\Delta x)^2\Big]$$

$$\cancel{-\,n(x,t)} = \frac{1}{2}\frac{\partial^2 n}{\partial x^2}(\Delta x)^2$$

$$n(x, t+\tau) - n(x,t) = \frac{1}{2} \frac{\partial^2 n}{\partial x^2} (\Delta x)^2$$

$$\frac{\partial n}{\partial t} = \frac{n(x, t+\tau) - n(x,t)}{\tau} = \frac{\Delta^2}{2\tau} \frac{\partial^2 n}{\partial x^2}$$

$$\frac{\Delta^2}{2\tau} = D$$

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}$$

$$\frac{\partial n}{\partial t} = D \underbrace{\left[\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + \frac{\partial^2 n}{\partial z^2} \right]}_{\nabla^2 n}$$

ecuación de difusión

$$m\vec{a} = \vec{g} \qquad \downarrow \hat{z} \qquad \vec{g}\downarrow$$

$$\vec{a} = a\hat{z} = \dot{\vec{v}} = \frac{dv}{dt}\hat{z} \qquad m\frac{dv}{dt} = g$$

Para encontrar $v(t)$ necesito dar condiciones iniciales.

$$\left(v(t=0) = v_0 \right)$$

$$m \frac{dv}{dt} \simeq m \frac{v(t+\Delta t) - v(t)}{\Delta t} = g$$

$$v(t+\Delta t) = \frac{g}{m} \Delta t + v(t)$$

Ec^s en derivadas parciales

$\frac{\partial}{\partial t} \rightarrow$ función en $t=0$

$n(x, t=0)$

$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$: equiv de las condiciones iniciales en el espacio

$\rightarrow$ condiciones de borde o de contorno

Para encontrar solución de la ec de difusión hay que dar condic iniciales y de borde

Ec difusión, en 1 dimensión

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$$

La función

$$n = \frac{N_o}{A}\,\frac{e^{-x^2/4Dt}}{\sqrt{4\pi D t}}$$

número sin unidades

$$[n] = \frac{1}{[\sqrt{Dt}]} = \frac{1}{\text{long}^3}$$

satisfacen la ec. de difusión

$$n = N_0 \frac{e^{-x^2/4Dt}}{\sqrt{4\pi D t}}$$

gaussiana

$$e^{-x^2/2\sigma^2}$$

con $\sigma^2 = 2Dt = \langle x^2(t) \rangle$

1 pateador

$t_0 < t$

$$\frac{1}{\sqrt{4\pi D t}}$$

$$\sigma = \sqrt{2Dt'}$$

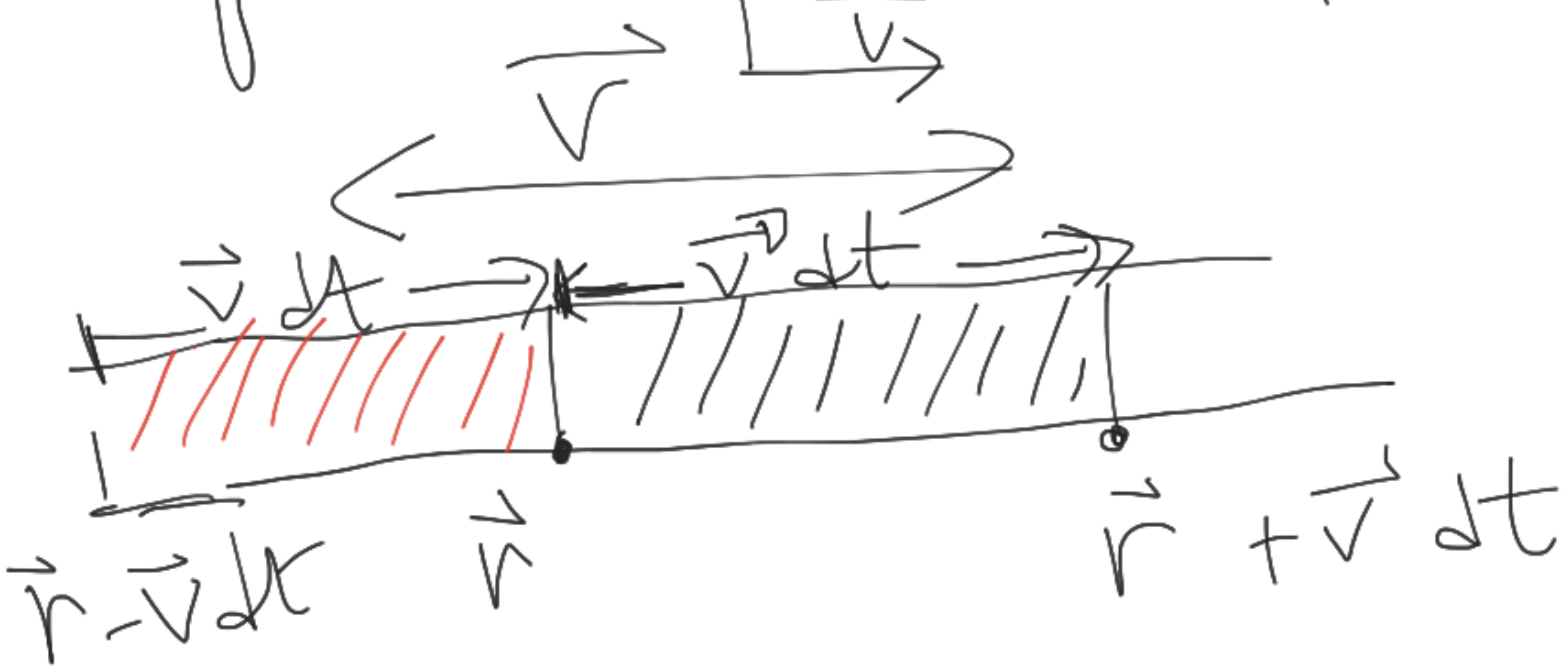
Para $t' > t$

$$\int_{-\infty}^{\infty} \frac{N_0 e^{-x^2/4Dt}}{\sqrt{4\pi D t}} = N_0$$

$$N_0 \frac{e^{-x^2/4Dt}}{\sqrt{\quad}} \xrightarrow[t \to 0]{} \delta(x)$$

Ec de difusión en 1 caso particular de l ec de transporte

Supongamos que tengo partículas que viajan con una veloc. $\vec{v}$



Quanto vale

$$\frac{n(\vec{r}, t+dt) - n(\vec{r}, t)}{dt} = \, ?$$

$$n(\vec{r}, t+dt) = n(\underbrace{\vec{r} - \vec{v}\,dt}_{=(x - v_x dt)\hat{x} + (y - v_y dt)\hat{y} + (z - v_z dt)\hat{z}}, t)$$

$$\cong n(\vec{r}, t) + \frac{\partial n}{\partial x}(-v_x dt) + \frac{\partial n}{\partial y}(-v_y\, dt) + \frac{\partial n}{\partial z}(-v_z dt)$$

$$n(\vec{r}, t+dt) - n(\vec{r}, t) =$$

$$\simeq \cancel{n(\vec{r}, t)} - \frac{\partial n}{\partial x} v_x \, dt - \frac{\partial n}{\partial y} v_y \, dt - \frac{\partial n}{\partial z} v_z \, dt$$

$$\cancel{+ n(\vec{r}, t)} \Rightarrow \frac{n(\vec{r}, t+dt) - n(\vec{r}, t)}{dt}$$

$$= -v_x \frac{\partial n}{\partial x} - v_y \frac{\partial n}{\partial y} - v_z \frac{\partial n}{\partial z}$$

$$\frac{n(\vec{r}, t+dt) - n(\vec{r}, t)}{dt} = -\frac{\partial}{\partial x}(n v_x)$$

$$-\frac{\partial}{\partial y}(n v_y) - \frac{\partial}{\partial z}(n v_z) =$$

$$= -\vec{\nabla} \cdot (n\vec{v})$$

$$n\vec{v} = n v_x \hat{x} + n v_y \hat{y} + n v_z \hat{z}$$

$$\vec{J} = n\vec{v}$$ flujo

$$\frac{\partial n}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$ ec continuidad

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$ conservación de la masa

Difusión 1D

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}$$

Ec continuidad 1D

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(j_x) = 0$$

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial x}(j_x) \longrightarrow -\frac{\partial}{\partial x}(j_x) = -\frac{\partial}{\partial x}\left(-D\frac{\partial n}{\partial x}\right)$$

$$j_x = -D \frac{\partial n}{\partial x}$$

$$\vec{J} = -D \frac{\partial n}{\partial x}\hat{x} - D \frac{\partial n}{\partial y}\hat{y} - D \frac{\partial n}{\partial z}\hat{z}$$

$$= -D \vec{\nabla} n = -D \left[\frac{\partial n}{\partial x}\hat{x} + \frac{\partial n}{\partial y}\hat{y} + \frac{\partial n}{\partial z}\hat{z} \right]$$

ley de Fick