



camino recorrido entre choques

$$v\, t_{\text{entre choques}} = \text{distancia entre choques}$$

$$\langle v^2 \rangle \sim \frac{kT}{m}$$

$$\langle t_{\text{entre choques}} \rangle = \tau = \text{tiempo entre colisiones}$$

$$\langle d_{\text{libre}} \rangle = \lambda = \text{camino libre medio}$$

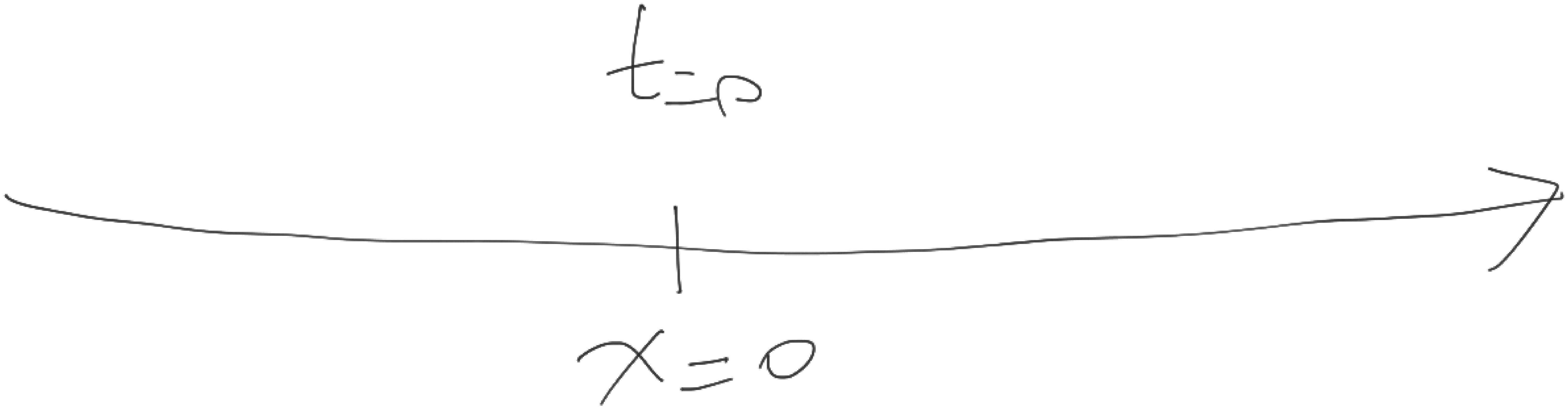
# Paso al azar

→ $|\text{desplazam entre choques}| = d$

$|\text{veloc}| = v_T$

tiempo entre choques $= \tau$

# Paso al azar 1 dim esp.




$$x(t=0) = 0$$

$$t = 0, \tau, 2\tau, \ldots, n\tau$$

$x(t_n)$ ; $x(t_0 = 0) = 0$

↓

variabile aleatoria

$\langle x(t_n) \rangle$ ; $Var(x(t_n))$

$= \langle (x - \langle x(t_n) \rangle)^2 \rangle$

$$x(t_n) = x(t_{n-1}) + \delta_{n-1}$$

$\forall n > 0,$

$$\delta_n = \begin{cases} +1 & \text{con prob } 1/2 \\ -1 & \text{" " } 1/2 \end{cases}$$

$$x(t_n) = x(t_{n-2}) + \delta_{n-2} + \delta_{n-1}$$

$$x(t_n) = x(t_{n-2}) + \delta_{n-2} + \delta_{n-1}$$

$$= \ldots\ldots = x(t_0) + \delta_0 + \delta_1 + \ldots + \delta_{n-1}$$

$$\langle x(t_n) \rangle = \underbrace{\langle x(t_0) \rangle}_{=0} + \langle \delta_0 \rangle + \ldots + \langle \delta_{n-1} \rangle = 0$$

$$\langle \delta_m \rangle = \lambda \frac{1}{2} + (-\lambda) \frac{1}{2} = 0$$

$$\langle (x(t_n) - \langle x(t_n) \rangle)^2 \rangle = \langle x^2(t_n) \rangle$$

$$x^2(t_n) = (x(t_{n-1}) + \delta_{n-1})^2$$

$$= x^2(t_{n-1}) + 2\delta_{n-1} x(t_{n-1}) + \delta_{n-1}^2$$

$$\langle x_n^2 \rangle = \langle x_{n-1}^2 + 2\delta_{n-1} x_{n-1} + \delta_{n-1}^2 \rangle$$

$$= \langle x_{n-1}^2 \rangle + 2\langle \delta_{n-1} x_{n-1} \rangle + \langle \delta_{n-1}^2 \rangle$$

$$\langle x_n^2 \rangle = \langle x_{n-1}^2 \rangle + 2\langle x_{n-1}\delta_{n-1} \rangle + \langle \delta_{n-1}^2 \rangle$$

$$\langle \delta_{n-1}^2 \rangle = \lambda^2$$

$$\langle x_{n-1}\delta_{n-1} \rangle = \underbrace{\langle x_{n-1} \rangle}_{=0} \underbrace{\langle \delta_{n-1} \rangle}_{=0}$$

$$\langle x_n^2 \rangle = \langle x_{n-1}^2 \rangle + \lambda^2$$

$$\langle x_n^2 \rangle = \langle x_{n-1}^2 \rangle + \lambda^2 = \langle x_{n-2}^2 \rangle + \lambda^2 + \lambda^2$$

$$= \ldots\ldots = \langle x_0^2 \rangle + \underbrace{\lambda^2 + \lambda^2 + \cdots + \lambda^2}_{n\lambda^2}$$

$$\langle \delta_n^2 \rangle = (+\lambda)^2 \frac{1}{2} + (-\lambda)^2 \frac{1}{2}$$

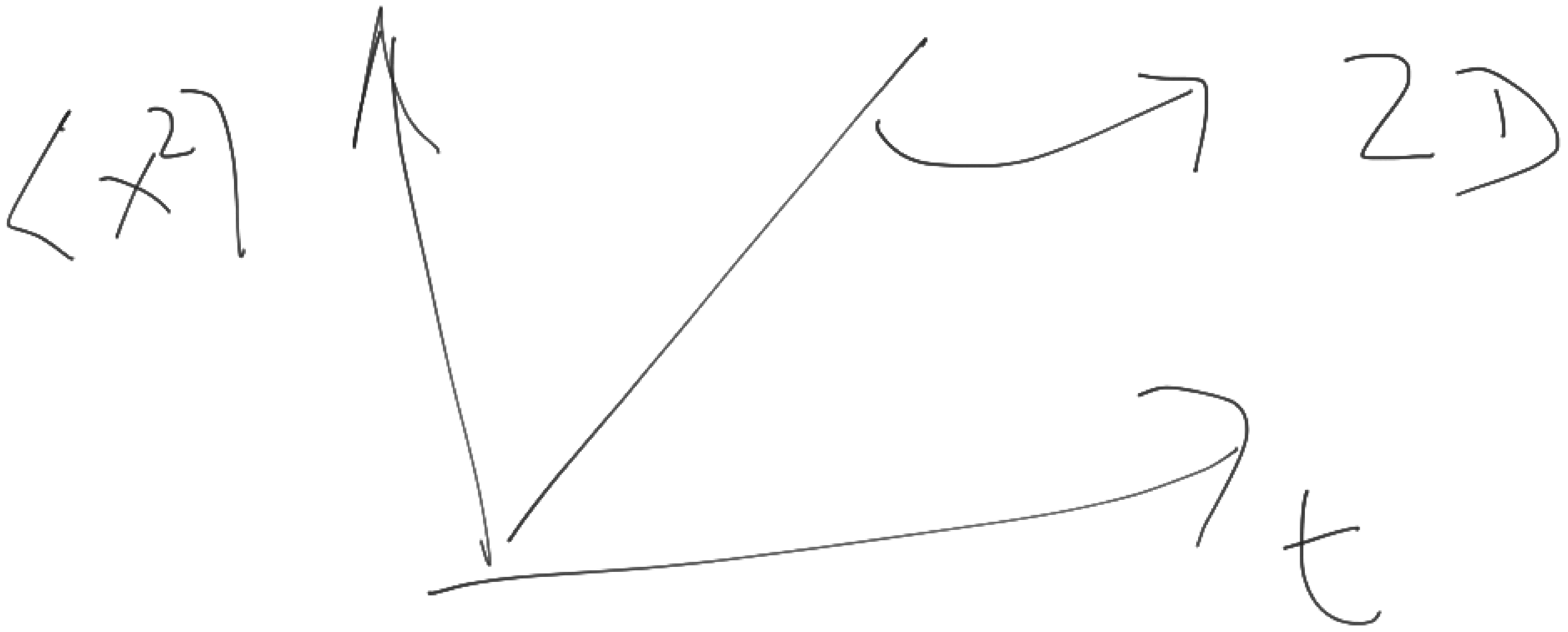
$$= \lambda^2$$

$$\langle x_n^2 \rangle = n \lambda^2$$

$$\langle x(t_n)^2 \rangle = \langle x(t = n\tau))^2 \rangle$$

$$= \underbrace{n\tau}_{t_n} \frac{\lambda^2}{\tau} = 2D\, t_n$$

↑ 1 dim espacial




$D$ = coef de difusión

$$2D = \frac{\lambda^2}{\tau}$$

$$[D] = \frac{\text{long}^2}{\text{tiempo}}$$

$$\langle x^2_{(t)} \rangle^{1/2} = \sqrt{2Dt}$$

En más dimensiones

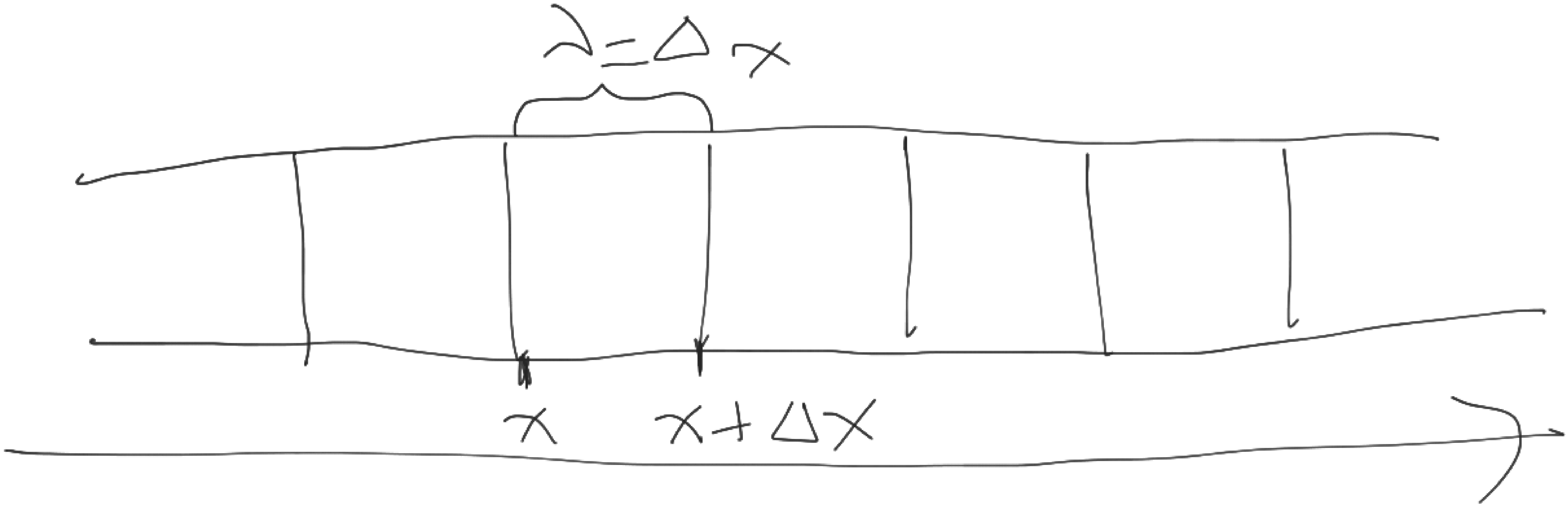
$\pm\lambda$ en $x$ ; $\pm\lambda$ en $y$ ; $\pm\lambda z$

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$

$$\langle r^2 \rangle = 2\,(\text{núm de dimensões do espaço})\,D\,t$$

↓ deslocamento quadrático médio

¿Qué pasa si pongo muchos pateadores a la vez en $x=0$? ¿Cómo va a ser la estadística, el histograma del número de pateadores que llegan a un lugar o a otro?




$n(x,t)$ = densidad del X

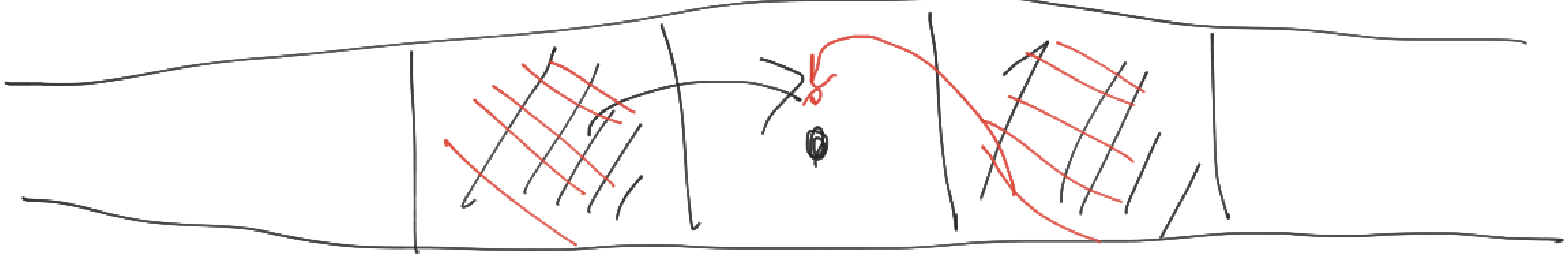
# de pasadores que están entre $x$ y $x + \Delta x$

$$N(x,t) = n(x,t)\,\Delta x$$

$$t = 0, \tau, \ldots, l\tau$$

$$n(x, t_m), \quad n(x, t_m + \tau)$$

$$n(x, t_m + \tau)\,\Delta x = \frac{1}{2} n(x - \Delta x, t_m)\,\Delta x + \frac{1}{2} n(x + \Delta x, t_m)\,\Delta x$$



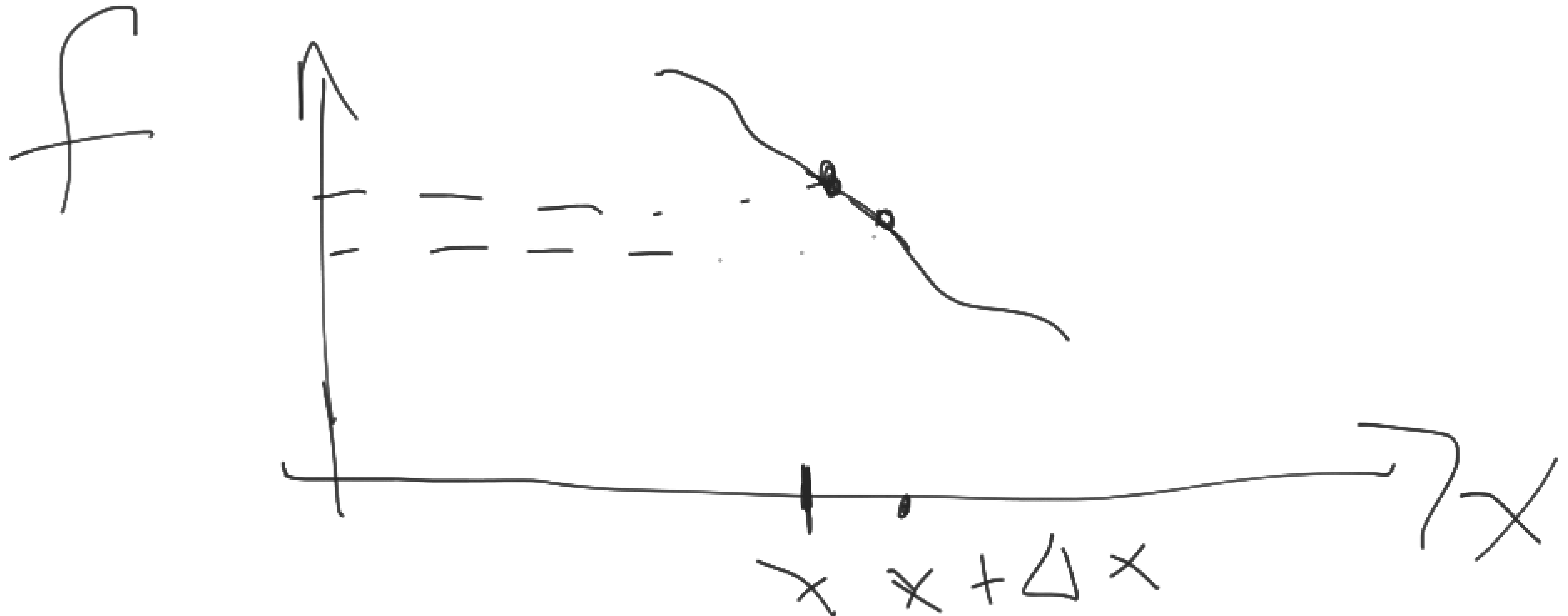
$x - \Delta x \qquad x \qquad x + \Delta x$

$$n(x, t+\tau)\Delta x = \frac{1}{2} n(x-\Delta x, t) + \frac{1}{2} n(x+\Delta x, t)$$

$$n(x, t+\tau)\Delta x - n(x, t) =$$

$$= \frac{1}{2} n(x-\Delta x, t) + \frac{1}{2} n(x+\Delta x, t) - n(x, t)$$

$$f(x+\Delta x, t) \cong f(x, t) + \frac{\partial f}{\partial x}(x, t)\cdot \Delta x + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(x, t)\cdot(\Delta x)^2$$



$$\frac{1}{2} n(x+\Delta x, t)\Delta x + \frac{1}{2} n(x-\Delta x, t)\Delta x$$

$$- n(x,t)\,\Delta x \cong$$

$$\cong \Delta x \frac{1}{2}\left[ n(x,t) + \frac{\partial n}{\partial x}\Delta x + \frac{1}{2}\frac{\partial^2 n}{\partial x^2}\Delta x^2 \right]$$

$$+ \frac{\Delta x}{2}\left( n(x,t) + \frac{\partial n}{\partial x}(-\Delta x) + \frac{1}{2}\frac{\partial^2 n}{\partial x^2}(-\Delta x)^2 \right]$$

$$- n(x,t)\Delta x$$