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Thermally induced pulsatile motion of solids

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Solid objects lying on slopes for which gravity alone would be insufficient to overcome frictional resistance have long been known to experience downslope motions when subject to alternations in temperature. This paper will suggest that similar motions are possible in situations where gravity is either absent or even working against the prospective motions. It will be argued that the effects of differential material properties when the solid is subject to alternations of tension and compression, accompanying periodic cycles of high and low temperatures, can also produce motion. While most of the illustrative examples will be chosen from certain movements of either ice and ice-rich materials or asphalt pavements, it would appear that similar behaviour could be experienced by much wider classes of materials. It is suggested that this form of thermal ratchet process could be of importance far wider than is currently recognized.

Keywords: thermal loading; motion; expansion; contraction; ratchetting

1. Background

In a remarkable series of papers written by Moseley (1855, 1869*a*,*b*) in the middle decades of the nineteenth century, it was explained how a solid body when subject to alternations of temperature could move down a slope for which gravity alone would have been insufficient to cause the motion. This work had initially been inspired by the observation that lead sheets, providing the waterproofing on a south-facing roof of the Bristol Cathedral, gradually made their way down the slope to end up in the guttering whereas those on the north-facing roof did not. From a convincing experimental study, carried out in his back garden over the spring of 1855, Moseley confirmed that the downslope motions were most rapid on days when strong sunlight was regularly blocked out by the frequent passage of heavy cloud cover (Moseley 1855). There was, by contrast, virtually no movement on days when thick cloud cover prevented any fluctuations of insolation or during nights when temperatures varied little. At the time, this theory came to be known as the 'crawling theory'.

Surprisingly, Moseley's thermomechanical explanation for why solid bodies are able to move down gentle slopes has found very little recognition in an area where it would seem to be of considerable significance. Davison (1888) did pick up this model and used it to describe the process, sometimes referred to as insolation creep, which accounts for the downslope movements of surface

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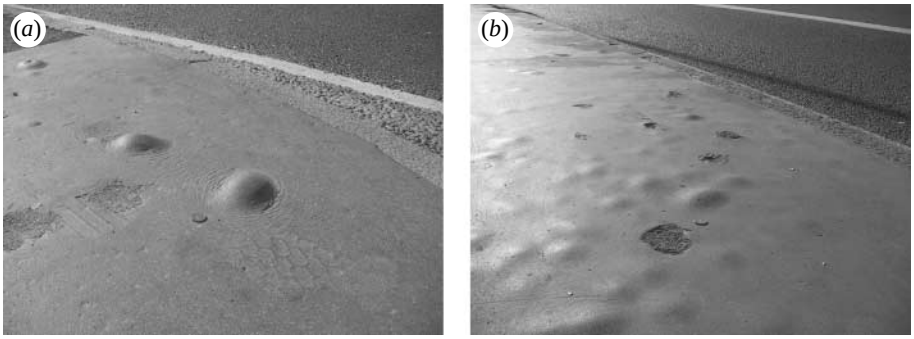

Figure 1. Asphalt pavement blisters showing (*a*) peripheral ring ridges and furrows and (*b*) the preference for axisymmetric forms. (Scale from 2 p coin diameter 20 mm.)

material by alternations in temperature alone. One of the few references to the work of Moseley and Davison in this area is given by Strathan (1977, p. 103), although it is also referred to in passing by Washburn (1979). Davison (1889) is himself better known for his contributions to the discussion of frost creep, a rather different form of frost-inspired ratchet process (Washburn 1979, p. 199) accounting for downslope motion.

One of the factors in history's seeming neglect of the work of Moseley (1869*a*,*b*) in this area may have been the result of his attempts to show that his crawling theory might also be of significance to the movement of glaciers. This work attracted considerable discussion (see Ball 1870; Mathews 1870; Wallace 1871) much of which was for its time of an unusually hostile nature. In a contemporary account of aspects of these exchanges, Benn (2002) quotes Wallace's (1871) claim that Moseley's crawling theory had been 'completely demolished' by Croll (1869, 1870). However, a reading of these contemporaneous exchanges does not support the view that Moseley's theory had been demolished, and certainly not by James Croll, who had been largely in support of Moseley's arguments. In his last contribution to the discussion, Moseley (1871) promised a follow-up paper outlining how he believed his model could be developed to account for certain forms of glacial motion. Regrettably his death in January 1872 meant that this intended work was never published.

Unaware of these earlier exchanges, the present author was encouraged in late 2003 to consider how a process of thermal upheaval buckling might be contributing to the development of 'pingos' within permafrost (Croll 2004, submitted). This followed earlier studies of how thermal buckling can explain the buckling of railway lines (Martinet 1934; Kerr 1974), subsea pipelines (Palmer & Maldry 1974; Hobbs 1984; Taylor & Gan 1986; Maltby & Calladine 1995; Croll 1997), rigid road pavements (Kerr & Shade 1984; Croll 2005) and heavy sheets (Hobbs 1989, 1990; Croll 2005), such as with the asphalt blisters shown in figure 1. In contrast to the dominant theories accounting for the growth of pingos, this new model (Croll 2004, submitted) relied upon a form of ratchet process powered by alternations in solar radiation that caused periodic variations of restrained horizontal expansion and contraction within the permafrost. Compression forces resulting from the restrained expansion during warming were shown to be sufficient to induce a form of incremental upward buckling growth of the pingo. Tensions arising when the subsequent cooling contractions are

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restrained would result in the formation of cracks. These cracks would eventually become locked when filled with moisture that would in turn freeze back to form ice. At the time, it seemed that a similar model could account for certain forms of glacial ice motion. It was during a search to see whether similar theories had been put forward in the past that Moseley's work came to light. But while it shared many common features with Moseley's crawling theory, the new ratchet model, accounting for certain forms of ice motion, would also be capable of explaining movements on horizontal surfaces or even those where movements occurred on uphill gradients.

Recent work on the upheaval buckling of heavy sheets had been inspired during a rather hot summer in 1999 and not more than 5 m from the writer's office, by the appearance of a host of relatively small blisters within the thin asphalt layers that had a short time previously been laid over the footpaths on Gower Street. Some of these blisters are illustrated in figure 1. The growth of these blisters also seemed to involve a form of thermal ratchet. Unlike ice, this thermal ratchet required a material whose viscoplastic creep rates at elevated temperatures would be higher than those exhibited at lower temperatures (Croll 2006, 2008*a*). It would appear that a number of other processes within asphalt (Croll in press *a*,*b*) rely upon forms of closely related thermal ratchet response. Indeed, it is possible that many other features of periglacial environments could also have, as part of their explanation, the working of similar forms of thermal ratchet (Croll & Jones 2006; Croll 2007, 2008*b*), involving some generically similar forms of behaviour (Croll 2006). First images being received from the Phoenix Mars Lander mission (New Scientist 2008) show periglacial morphologies that would suggest these processes are not confined to terrestrial environments.

This paper attempts to bring together some of the general ideas developed to account for all these seemingly disparate features of asphalt, glaciers and periglacial environments. It is particularly directed towards those features within the thermomechanical behaviours of either ice or asphalt that could be at work in their forward motion. For the most part, the phenomena described will be of an essentially qualitative nature, with more quantitative modelling beyond the present scope. However, some of the general forms of behaviour described could possibly be of such general significance as to justify this initial attempt to provide a more systematic qualitative framework for the forms of motion caused by alternations of temperature. With many of these motions depending upon fluctuations of the insolation levels reaching the Earth's surface, it will be suggested that a whole range of motions at different spatial scales may be the result of alternations of solar energy at various temporal periodicities. Some motions will be argued to depend upon circadian periodicities or even those measured in hours or minutes. At the other extreme, there appear to be motions that result from variations in average insolation occurring at periodicities measured in much longer time scales, such as the annual seasonal cycles up to those involving tens to hundreds of thousand years thought to be responsible for the ice ages. Whatever might be the spatial scales and associated temporal periodicities of thermal insolation levels, the following attempts to show that the development of closely related classes of thermomechanical motions could result.

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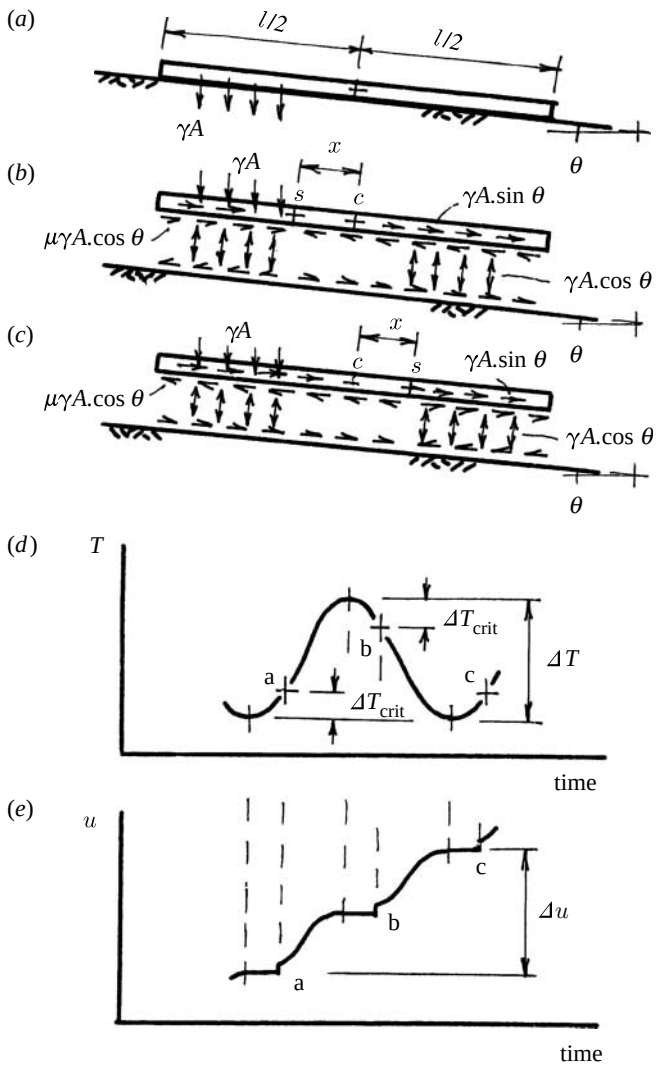

Figure 2. Simple mechanistic model illustrating pulsatile motion relying on gravity.

2. Pulsatile motions relying upon gravity

This is basically the form of ratchet motion described by Moseley (1869*a*,*b*). Apart from the movement of lead sheet on roofs, this form of thermomechanical motion is particularly troublesome in relation to the gradual downward movement of roof slates and tiles, often leading to leaks. It is an important source of the motions experienced by scree slopes, especially those in which the scree material has been formed from the break-up of sedimentary slates and shales (Davison 1888; Strathan 1977). Because it is closely related to the other forms of thermal ratchet motion to be developed below, a short summary, in a somewhat simpler form than that previously developed (Moseley 1869*a*,*b*), is provided in the present context.

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Consider a solid sheet shown in figure 2*a* of length l resting on a plane inclined at an angle, ϑ, that is lower than that required to overcome the shear stresses associated with a kinematic Coulomb friction coefficient, μ, i.e. $\tan\vartheta \leq \mu$. An increase in temperature, ΔT, reaching a level higher than the threshold required to fully mobilize the frictional resistance over the length of the sheet will result in the distribution of base shear stress shown in figure 2*b*. About a stagnation point, s, defined to be the point at which there is no downslope motion, the upper portion will move upwards and the lower portion downwards, with the distance between the stagnation point and the geometric centre of the sheet, c, being denoted, x. Equilibrium of the gravitational component and the base shear forces in the downslope direction requires that the distance x be determined from

$$2x\mu\gamma A\cos\vartheta = \gamma A l\sin\vartheta, \tag{2.1}$$

giving

$$x = \frac{l\tan\vartheta}{2\mu}, \tag{2.2}$$

in which γ represents the specific weight of solid having an assumed uniform cross-sectional area, A. Neglecting any effects of elastic shortening that would result from the compression stress parallel to the slope (and these will generally be orders of magnitude smaller), the geometric centre, c, of the solid sheet will, as a result of the expansion occurring, move down by a distance $\alpha\Delta Tx$ relative to the stagnation point, s, where α is the coefficient of linear expansion. When the temperature is lowered by the same amount, figure 2*c* shows that the stagnation point, s, will now be displaced by the same distance, x, but in the downslope direction relative to the geometric centre, c. As a result of the temperature drop, the geometric centre will again move down relative to the stagnation point a distance given as $\alpha\Delta Tx$. After completing a full cycle of heating and cooling, ΔT, the geometric centre of the solid will therefore have moved down a distance

$$\Delta u = 2\alpha\Delta Tx, \tag{2.3}$$

which on account of equation (2.2) will be given by

$$\Delta u = \alpha\Delta Tl\tan\vartheta/\mu. \tag{2.4}$$

Each successive cycle in which the temperature changes exceed the critical level

$$\Delta T > \Delta T_{\mathrm{crit}} = \frac{\mu\gamma\cos\vartheta\cdot l}{2\alpha E}\left(1-\left(\frac{\tan\vartheta}{\mu}\right)^2\right), \tag{2.5}$$

where E is the modulus of elasticity of the solid sheet, will cause the solid to move a little further down the slope. This critical lower bound for the instigation of downslope motion is determined by equating the axial force at the stagnation point, given as

$$F = 0.5\mu\gamma lA\cos\vartheta\left(1-\left(\frac{\tan\vartheta}{\mu}\right)^2\right), \tag{2.6}$$

to the force induced when the sheet is constrained but just at the point of fully mobilizing the kinematic friction motion at $\alpha\Delta T_{\mathrm{crit}}EA$. The rate of incremental deformation Δu, given by equation (2.4), will increase as $\tan\vartheta\rightarrow\mu$ and of course

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as α, ΔT, l increase or μ decreases. The temperature change ΔT_{crit} required to mobilize the incremental motion will decrease as $\tan\vartheta \rightarrow \mu$ and of course as α, E, θ increase or μ, γ decrease. Figure 2*e* summarizes in somewhat idealized form the nature of the cyclic ratcheting down the slope of the geometric centre of the solid during a typical thermal cycle.

For a sheet lying on a cohesionless solid having angle of internal friction, φ, undergoing plastic shear failure, equation (2.4) would become

$$\Delta u = 2\alpha\Delta Tx = \alpha\Delta Tl\tan\vartheta/\tan\varphi. \tag{2.7}$$

In the case of a plastically deforming solid, very similar reasoning would apply. Shear failure, assumed in the above simple model to be concentrated at the base of the solid, would for a plastically deforming solid be distributed throughout the body mass. But motion above a certain level would follow a similar form to that outlined above, with the motion rates dependent upon the thermal gradients through the thickness of the sheet. Motions caused by such a ratchet could be contributing to the slope deformation processes commonly referred to as solifluction and gelifluction (Washburn 1979), and, as was suggested at a very early stage in the awareness that glaciers moved, to the motion of glaciers (Moseley 1869*a*,*b*).

3. Pulsatile motions relying upon sign-dependent material strength

In this case, the alternations in temperature can produce motions as a result of materials possessing different failure properties when developing tension compared with those in compression. Ice, for example, suffers brittle tensile failure at low tensile stress. Under compression, ice can sustain very high stresses without failure occurring. Rock, concrete and even certain forms of embrittled asphalt, display similarly, sign-dependent, failure properties. When heated a body of such material is able to develop massive forces to overcome any resistance to its outward expansion. By contrast, when it cools, the tension stresses arising from the restrained contraction may be sufficient to cause cracking of the material before the forces become large enough to overcome the restraints to its inward contraction. Where some mechanism intervenes to fill the cracks, which have opened up after tension-related failures have occurred during restrained cooling, then not all the tensile crack strain will be recovered when the stress is again reversed under the effects of thermal heating. Within rocks it might be finely grained materials blown or washed into the cracks that offer a semi-rigid constraint when heating would otherwise cause closing of the cracks. Similar behaviour is exhibited when frozen ground cracks during cooling, with a cyclic process of accumulation of sand-filled cracks eventually forming what are known as sand wedges (Pewe 1959). In frozen ground or ice, any cracking during restrained cooling will in certain conditions attract moisture ingress. This might be in the form of surface-melt water or more likely haw frost or snow. Once this moisture or water turns to ice, or the snow is consolidated to form ice, any subsequent compression caused by warming of the ice will result in only partial closure of the crack. Over many such cycles an ice wedge will form (Leffingwell 1915). Both sand and ice wedges are commonly observed features of regions experiencing or that have in the past experienced permafrost. The wedge shapes

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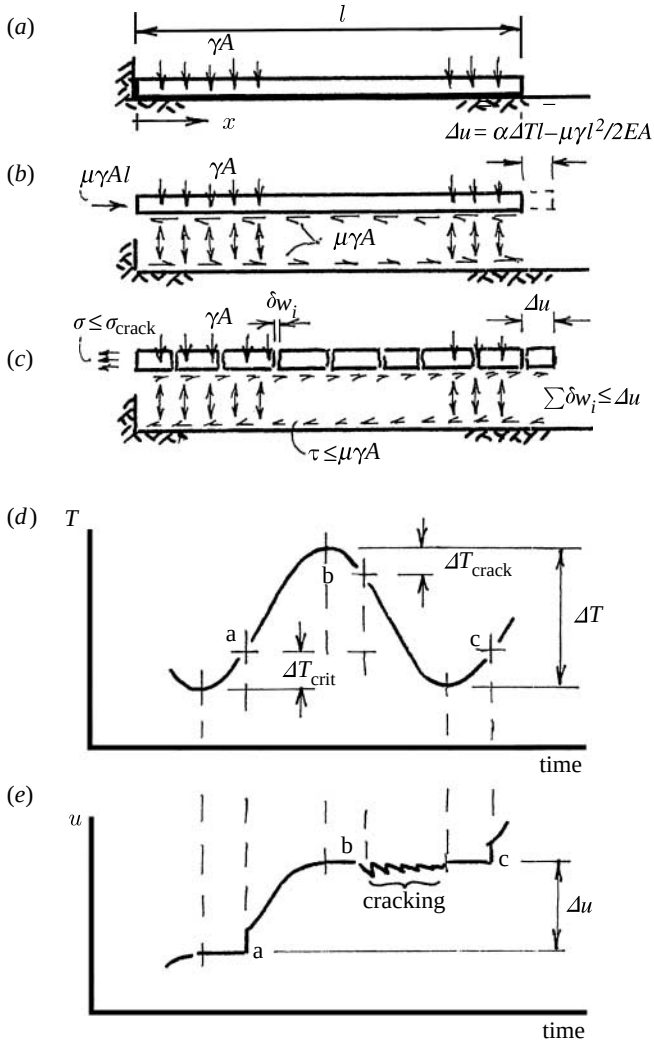

Figure 3. Simple mechanistic model illustrating pulsatile motion relying on sign-dependent material strength.

and depth of penetration reflect the attenuation of surface temperature cycles with depth. With the depth of the thermal wave being dependent upon the period of the thermal cycles, there exists for a given level of temperature fluctuation a close relationship between the spatial scales of the wedge geometries and the period of the thermal cycles.

For an idealized solid exhibiting the above sign-dependent material strength, the thermal ratchet that might cause motion is illustrated in figure 3. For the sake of illustration, it is assumed that one end of the uniform heavy sheet is restrained against movement, as depicted in figure 3*a*. This restraint could be real in the sense of a massive object preventing motion, or it could represent the geometric centre of a sheet expanding out equally in both directions. A temperature increase will initially have any expansion restrained by a bed

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shear strength that is in excess of the shears required to restrain the outward motion. Once the base shears exceed the bed shear strength, however, there will be a rapid loss of restraint as the system gradually mobilizes the kinematic friction resistance over the entire length of the solid. This stage is suggested to occur at point a (figure 3*e*) during the heating phase. Further heating will cause a fairly rapid increase in outward deformation, with the shear transfer being governed by the kinematic friction coefficient, μ. Upon cooling, little recovery of the outward motion would occur before the restrained tensile strains are sufficient to start to reverse the direction of the bed friction resistance. Initially, most of the bed frictional resistance will be concentrated at the outer end of the sheet. Depending upon the tensile failure properties of the sheet, this could eventually induce sufficient tension in the sheet to initiate tensile cracking. Once this occurs at point *b* (figure 3*e*), there will be little further recovery of the previous outward motion as a consequence of the cracks opening up and being propagated over the length of the once solid sheet. If some subsequent mechanism then leads to the cracks being filled with a rigid or semi-rigid material, the cycle would be repeated during the next thermal cycle. An idealized mechanism working in this way, would be the one in which bed friction is considered negligible in resisting restrained expansion, and the tensile capacity of the solid is effectively zero during restrained contraction, with the entire tensile strain developing cracks that are completely filled with an incompressible solid. Under these circumstances, the theoretical upper limit to the cyclic outward motion during a thermal cycle ΔT would, at a distance, x, from the constraint, be given as

$$\Delta u = \alpha \Delta T x. \tag{3.1}$$

While not the only cause, a situation where this form of material ratchet can occur is in the motion of lake or sea ice. It has been well documented that lake ice when heated may experience significant outward motion at its shoreline. Depending upon the nature of the shear resistance between the ice sheet and the shoreline, these outward motions can be considerable or alternatively cause considerable damage. The damage could either be to the ice sheet itself or whatever surrounds the edge of the lake. If the edge shear resistance is great, the ice sheet may experience a localized form of combined crushing and buckling failure. Alternatively, where the strength of the ice or its upheaval buckling strength is greater than the resistance offered by the lake shore, one of a number of failures can occur. Where the surrounding natural materials are relatively weak, the ice may, over repeated cycles of heating and cooling, shove it out to form ramparts (Gilbert 1890; Hobbs 1911) of loose debris, stone and rock (Scott 1926). Where the ice has a buckling resistance greater than the strength of any man-made structures impeding its outward deformation, then considerable damage can be inflicted on these structures (Frellsen 1963). If the shear resistance is low, each warming cycle will result in an outward motion. Whether this will be recovered during the subsequent cooling phase will be dependent upon the relative strengths of the lake shore shear capacity and the tensile strength of the ice sheet. Where the edge shear is small enough to allow outward overriding during warming but high enough to cause tensile cracking of the ice during cooling, then the mechanism described in figure 3 will operate. In the case of tensile cracking in lake ice, the underlying unfrozen lake provides an abundant

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source of moisture to enter the fissures and freeze before another warming cycle begins. Indeed, the reported behaviour of lake ice subject to alternations in temperature is an excellent example of the operation of this process. But there is evidence that a similar push–pull mechanism operates in other situations.

In the formation of ice wedges and ice-wedge polygons (see Leffingwell 1915; Lachenbruch 1962), there is, during the warming phase of the thermal cycle, considerable evidence to suggest there is a form of outward mass movement, dependent upon a thermomechanical process very similar to that described earlier. As the volume of ice within the ice wedges around the periphery of the emerging ice-wedge polygons grows, something must happen to the displaced mass of ground material. Much of the mass balance appears to be played out within the active layer overlying the permafrost. Over a period of more than 30 years, Mackay and his co-workers have been observing the motions within the seasonally frozen ground overlying networks of ice-wedge polygons (Mackay 1973, 1974, 2000; Mackay & Burn 2002). This work seems to make it clear that the conservation of mass within the ice-wedge polygons, at least those being studied, is associated with changes in the disposition of material within the active layer. Material is gradually moved from the central regions of the ice-wedge polygons out to the periphery. The mechanism that produces this outward motion of material would seem to be very closely related to the processes described recently to account for the development of the fascinating periglacial features, such as sorted and unsorted, stone circles, polygons and nets (Croll & Jones 2006; Croll 2008*b*), which in turn relate closely to the process believed to be responsible for the development of ramparts around the edges of certain lake shores (Hobbs 1911). Essentially this involves high levels of tension cracking near the surface, which would occur during any short-term, possibly circadian, cooling phase, not being recovered during the subsequent warming phase, as a result of the tensile cracks being filled with moisture that turns to relatively rigid ice. The result is an outward migration of active layer material with a rate that is directly related to the vertical profile of the cyclical changes in thermal gradient. That this form of motion is occurring seems to be clearly demonstrated by long-term and careful field measurements (Mackay 1973, 1974, 2000; Hallet & Prestrud 1986; Mackay & Burn 2002), and its existence has often led to the cause of these periglacial features being attributed to a form of density-induced convection within the active layer soils above the permafrost (Ray *et al.* 1983; Gleason *et al.* 1986; Hallet & Prestrud 1986), or attributed to generally only partially explained processes leading to what has become known as 'self-organization' (Kessler *et al.* 2001; Kessler & Werner 2003).

Continental and smaller land-based ice sheets appear to undergo similar forms of thermally induced ratchet motion. Figure 4*a* shows an area of the Greenland ice sheet that has been undergoing long-term motion around an outcrop of rock (a nunatak). It is noticeable that the surface topology of the ice exhibits distinctive wavy bands of ice build-up, similar to Forbes bands or ogives that often develop in alpine glaciers. While the present author has no direct empirical evidence in support of this view, it seems highly probable that these ridges are the result of the build-ups in compression and outward shoving of the ice arising from the seasonal thermal cycles; each band representing a seasonal contribution to the progressive motion. Some of the motion experienced in alpine glaciers could derive from a similar source. Figure 4*b*,*c* shows well-defined ogives being

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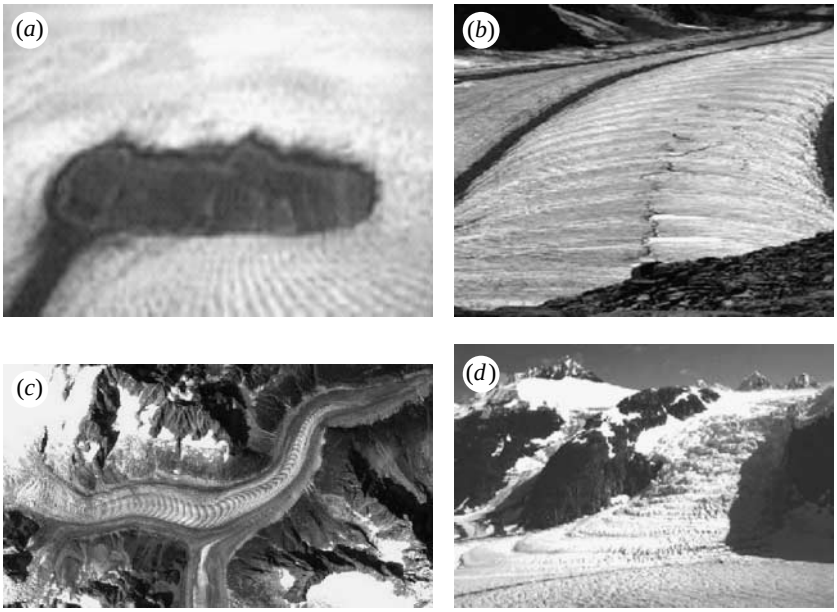

Figure 4. Possible evidence of pulsatile motions within glacial ice, showing seasonal bands of tension and compression action, in (*a*) an ice-sheet moving around a nunatak; (*b*,*c*) ogives formed in alpine glaciers; and (*d*) ogives forming at base of an icefall.

preserved as the glacial ice makes its way down the glacial valley. Each of these bands appears to be the result of the seasonal outflow from the accumulation zone at the head of the glacier, sometimes involving icefalls as shown in figure 4*d*. In this model, the build-up of compressive energy, due to the seasonal increase in temperature of the ice in the accumulation zone, is relieved by extruding ice into the head of the glacier. During the cooling period, this outfall zone will be subject to massive tensile stresses as a result of thermal shrinkage, leading to the development of fissured and creviced ice. Eventually, these fissures and crevices will be filled with accumulated snow and regelated surface-melt water. In this way, the ogives might well be the result of seasonal ice ramparts preserving the effects of seasonal expansion and contraction of the ice. Shorter term thermal cycles could also contribute to these motions. Especially during summer periods, high volumes of diurnal surface-melt water will capture in the form of latent heat energy much of the intense solar radiation that is so often experienced. Once this percolates into the body of the glacier, gives off its latent heat when refreezing, it provides another particularly effective form of heat transfer. Each of the many thermal cycles will interact and are easily able to account for much of the energy required to power glacial motion. Had he lived to prepare the paper promised (Moseley 1871), it is interesting to speculate whether Moseley might have developed a similar thermomechanical explanation of glacial ice motion.

Figure 5 shows a block pavement in an area that attracts regular cycles of high temperature. Over a period of years, the paving slabs have been undergoing a gradual outward motion. This is apparent at the outer boundary where a sheet of asphalt has experienced a shoving compression failure. It is even more apparent in the discrete cracking over the length of the pavement, normal to the direction

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Figure 5. Splitting of pavement blocks in motion relying upon sign-dependent material properties.

of motion. The cracks are as wide as 25 mm in places and have been filled with compressed detritus. The cracking generally shows a staggered form, as shown in figure 5, indicating that the weakest area tends to be within the joint. However, there are situations where the shear transfers between paving slabs are sufficient to induce tensile stresses within alternating slabs great enough to cause a tensile fracture of the slabs. In these cases, it is quite common for the cracking to follow a generally straight rather than staggered path.

4. Pulsatile motions relying upon temperature-dependent material properties

There appear to be other types of thermal ratchet-related motions that result from some form of differential material property (Croll 2006, 2008*a*,*b*, in press *b*). For example, many materials exhibit relatively high rates of viscoplastic creep flow at elevated temperatures compared with those at low temperatures. If significant differences occur over the typical temperature ranges experienced by a solid sheet, then again a gradual outward motion can occur. Figure 6 represents an idealized sheet of solid material having temperature-dependent viscoplastic properties. As for the phenomenological model of figure 3, the sheet is considered to be restrained at one end and resisted from outward motions by shear developed at its lower edge. The material is considered to have temperature-dependent viscosity and plasticity for which stiffness at elevated temperatures is considerably lower than that at lower temperatures. A rise in temperature from point *b* to point *c*, as shown in figure 6*d*, would, if there were no time-dependent relaxation, result in a build-up of compression stress as indicated by the dashed line in figure 6*e*. As a result of the high stress relaxation at elevated temperature, the actual stress would be more similar to that shown by the solid line. Associated with this, time-dependent stress relaxation would be a creep in the outward deformation experienced as indicated in figure 6*f*. Over the warming phase, *b* to *c*, the low elastic and viscoplastic stiffnesses would result in relatively large outward components of deformation, given, respectively, as $\delta u_{\mathrm{e}}^{+}$, $\delta u_{\mathrm{v}}^{+}$, where the positive superscript indicates that the elastic and viscoplastic

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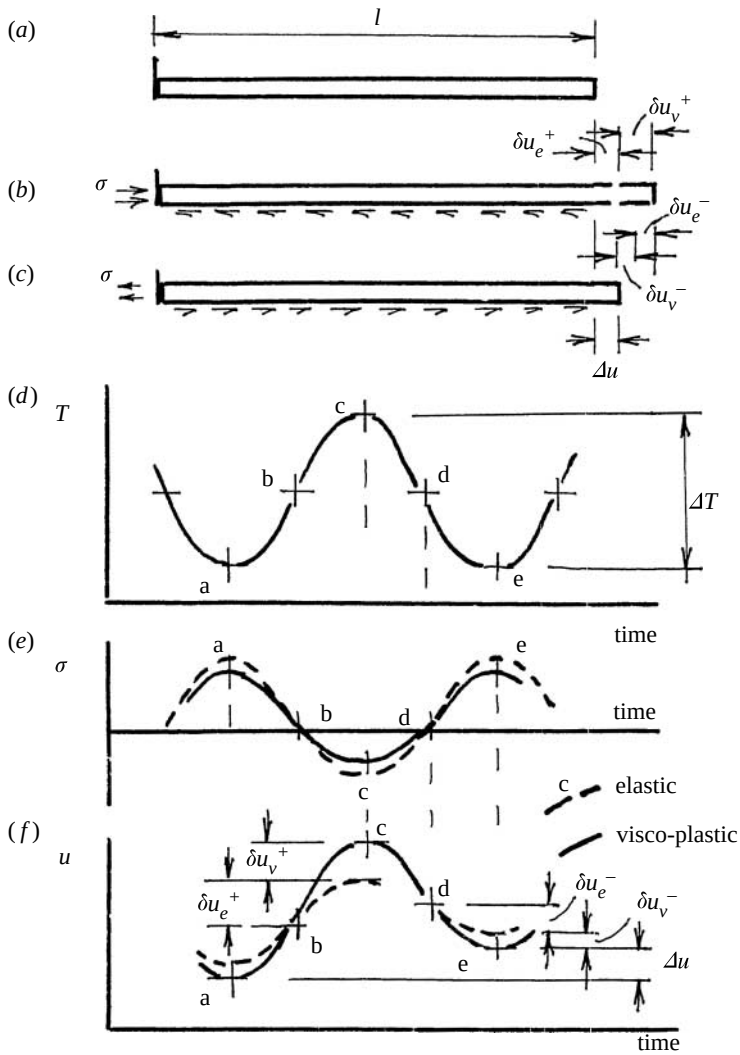

Figure 6. Simple mechanistic model illustrating pulsatile motion relying on temperature-dependent material properties.

components of the deformation are positive outwards during the compression phase associated with warming. When temperatures drop, the reverse will happen. But with the elastic and viscoplastic stiffnesses being so much higher at low temperatures, the deformation recovery $\delta u_{\mathrm{e}}^{-}$, $\delta u_{\mathrm{v}}^{-}$ will be considerably smaller. That the rates of creep relaxation are so much lower at low temperatures implies that there will not be a full recovery of the outward deformation at the end of the thermal cycle, leaving a small residual deformation Δu. Over many such cycles, there will be a progressive growth of the outward deformation occurring in the sheet.

Figure 7 shows evidence of such motions occurring in asphalt sheets covering essentially horizontal pavement at a location subject to fairly extreme circadian temperature cycles. That these motions occur is often best seen at the outer

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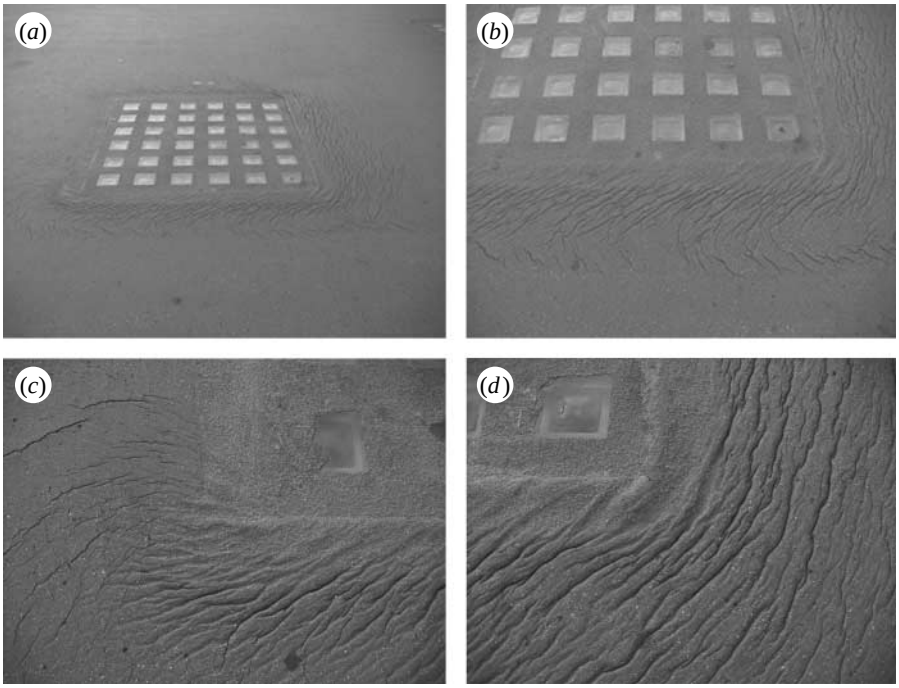

Figure 7. Pulsatile flow of asphalt sheet around rigid concrete skylight, showing (*a*) overall flow pattern, (*b*) details of upstream corner, (*c*) detail of downstream corner, and (*d*) closer detail of upstream corner.

edges of any rigid constraint that impedes motion; at these locations, there is a form of compression pile-up to form ridges. These are often observed to occur together with the adjacent tension-induced furrows. Sometimes the ridges take the form of an accumulating single large ridge and at other times there is a sequence of many ridge–furrow formations. Each ridge and furrow is generally the result of accumulations of failure occurring over many alternations in temperature. Figure 7*a* captures the clear effects of the gradual motions of the asphalt from right to left around a relatively rigid obstruction in the path of the outward motion. To describe this process as flow could be misinterpreted to imply some form of continuous motion, whereas what appears to take place is a discontinuous motion powered by compression and tension pulses originating from alternations in temperature. It is for this reason that the characterization 'thermal pulsatile motion' has been preferred.

Assuming that a given ridge line represents motions occurring over a particular period of time, it can be seen that the relatively unimpeded motions between the adjacent obstructions have moved a considerably greater distance forward than have those directly blocked at the upstream edge of the obstacle. On the upstream edge, shown by the corner detail of figure 7*b*, the accumulation of material in the form of multiple ridges is clearly visible. The ridges, or ramparts, result from the compression-related shoving action during warming, and may be viewed as forms of incremental, local, elastic–plastic–creep buckling. Between the ridges, the furrows are associated with the tensile action developed during cooling. In some of these furrows, there are cracks that at times show

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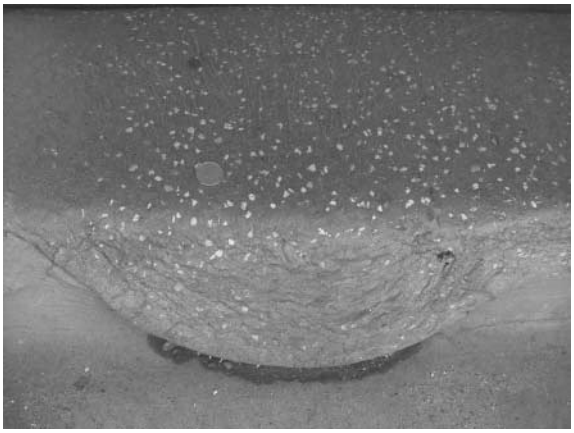

Figure 8. Pulsatile flow of asphalt sheet from a horizontal tread of a step.

signs of long-term build-up of detritus. At the upstream corners, the motion rates are clearly increasing with distance from the edge of the obstacle. Motion lines around the edge reflect the drag imposed upon the pulsatile motion. Downstream the motion-induced ridges and furrows could be viewed as relating to a form of flow separation that would occur in a fluid flow undergoing separation to form a downstream turbulent wake. This can be seen in the detail at a downstream, lower left, corner of the obstacle shown in figure 7*c*. It is almost possible to visualize a form of flow separation around this downstream corner and the formation of a stagnation region around the downstream face. As elsewhere, the direction of flow is normal to the lines of the ridges and furrows. The ridges and furrows on the upstream corner are shown in even greater detail in figure 7*d*.

A second clear example of thermal ratchet motion in asphalt is shown in figure 8. This depicts a step to a shop in North London, which had an asphalt sheet laid over its surface to prevent leaks into a basement area beneath. The top darker section is an essentially horizontal tread of the step and the bottom, lighter, section is the vertical riser. Over a period of years, a high proportion of the asphalt from the horizontal surface has been extruded over the lip to form a distinctive tongue having clear downslope convexity. That alternations of temperature have been powering this motion is again made clear by the bands of ridges and furrows. On the horizontal surface, the directions of the asphalt motions feeding this overflow are orthogonal to the bands of ridge–furrow formations. Comparing this with the ogives in glacial flow, it becomes possible to envisage a very similar form of thermally powered, pulsatile motion, driving each of these processes.

5. Discussion and conclusions

The above has attempted to generalize Moseley's crawling theory for the motions of solids to situations where there is a lack of gravity force to account for the motion. Conceptual models have been proposed as to how in two different ways the basic constitutive properties of materials might also allow motions powered by alternations in temperature. Materials displaying high compressive strengths

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but low levels of resistance to tensile fracture, or those that display lower elastic and viscoplastic stiffness when hot than when cold, have been postulated as allowing non-reversible deformations when subject to a complete heating and cooling cycle. Over many such cycles an accumulation of deformation can occur, in a process that has been referred to as pulsatile motion. Simple mechanistic models, as that adopted by Moseley to describe the pulsatile motion of solids subject to cyclic heating and cooling within a gravity field, have been used to illustrate the mechanics believed to be involved in each of these processes. Examples have been chosen from observed behaviour of sheets of ice and asphalt, which appear to be driven at least partially by these processes.

Moseley attracted considerable hostility (Croll 1869, 1870; Ball 1870; Mathews 1870; Wallace 1871) when he suggested that a thermal ratchet process explaining the motions of solids downslopes (Moseley 1855), for which gravity alone would not be sufficient, might be applicable to the movement of glacial ice (Moseley 1869*a*,*b*). Had he lived long enough to deliver the paper promised in his last contribution to the debate (Moseley 1871), it is interesting to speculate as to whether he may have described processes similar to those outlined above. Some of the evidence that suggests an extension of the thermal ratchet model he first outlined in 1855 might well be powering certain motions of ice, asphalt and indeed many other materials has been presented. This extension has been supported by reference to suitably simple mechanistic models of how the processes could be at work in heavy, thin sheets, overlying what are assumed to be rigid foundations providing a form of Coulomb frictional resistance to the motion. There is clearly a need for future theoretical work to extend these simple models into more realistic continuum situations. Any such theoretical developments will need to be supported by carefully planned experimental investigations.

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