Description of Numerical Model

Supporting Online Material for: Self-Organization of Sorted Patterned Ground

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1 Model Framework

In our numerical model for sorted patterned ground stones move in two horizontal dimensions representing an active layer in plan view (Fig. 1). Stone motion is not spatially discrete; however, a two-dimensional grid with cell size equal to stone size is used in searching for stones within an area, in calculations of stone concentration and to enforce a maximum depth of stones. Stone particles are moved by iterating through three transport algorithms representing lateral sorting, stone domain squeezing and hillslope diffusion. Stone transport arising from processes associated with soil domains is calculated from the current configuration of stones, which assumes that stone transport by soil domain processes is tightly coupled to the geometry of stone domains at the scale of sorted patterned ground.

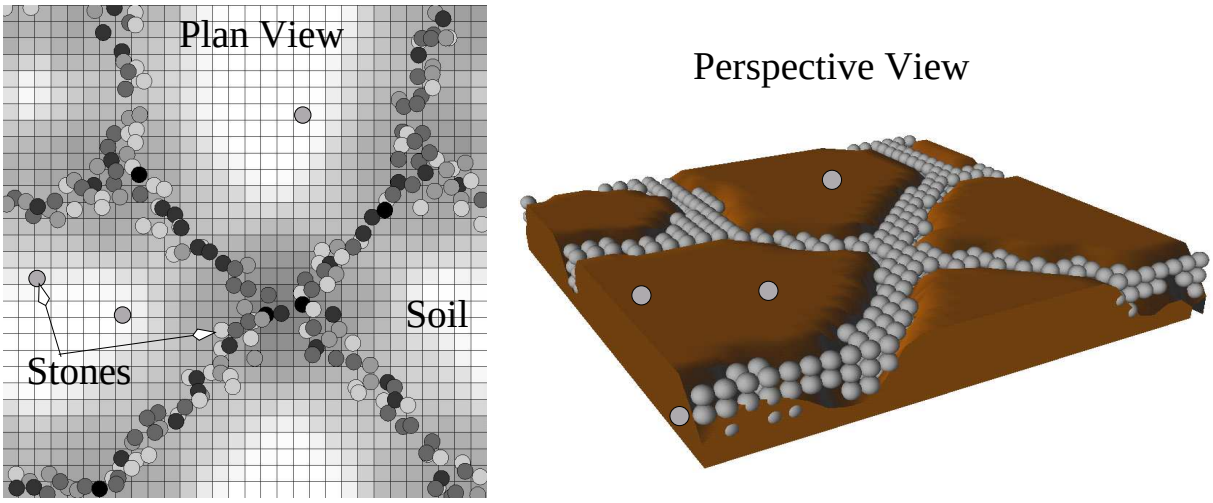

Figure 1: (A) Plan view of numerical model. Circles indicate the top stone in that cell. Stone color darkens with the number of stones in that cell from light gray for a single stone to black for the maximum number of stones, H_{max}. The surface used in the calculation of lateral sorting, S, is plotted as a gray-scale background; lighter shades indicate higher surfaces. (B) Perspective view of the three-dimensional geometry that corresponds to this two-dimensional simulation.

2 Lateral Sorting

The lateral sorting algorithm transports stones toward stone domains by calculating a surface (Fig. 1) whose gradient is the local strength of the lateral sorting processes. The height of this surface is proportional to the negative of the stone concentration calculated over a distance D_{ls} and weighted by inverse distance:

$$S = -\frac{\sum_{i=0,j=0}^{D_{ij}<D_{ls}} (N_{ij}/(1+D_{ij}))}{\sum H_{max}/(1+D_{ij})},$$

where D_{ij} is the distance to cell (i,j), N_{ij} is the number of stones in cell (i,j), and H_{max} is the maximum number of stones in a cell.

After calculating S, all cells are chosen in random order and the top stone in each cell is moved downslope (toward regions of high stone concentration) proportional to the local gradient in S: $\vec{x_{ls}} = K_{ls}\vec{\nabla}S$, where K_{ls} is a diffusion constant that relates stone transport (flux) to the local slope of S, unless this movement would cause the number of stones in any cell

to exceed H_{max}. Far from stone domains, S approximates the ground surface because lateral sorting results from downslope stone motion on the ground surface. Near stone domains lateral sorting also includes subsurface transport at inclined freezing fronts.

3 Stone Domain Squeezing and Confinement

Stone motion by lateral squeezing of stone domains is abstracted as diffusion of stones biased parallel to the axis of the stone domain: $\delta \vec{x_{sq}} = |K_{sq}\vec{\nabla}U|\hat{u}$, where U is the surface uplift owing to lateral squeezing of the stone domain and K_{sq} is a diffusion constant that relates stone transport to gradients in surface uplift. A first order equation for surface uplift U was derived by conserving the volume of a stone domain (rectangular cross section) being squeezed by an amount dw: $U = \frac{Tdw}{w}$, where T is the vertical thickness of the stone domain before squeezing and w is its width. In this model, we assume that wider stone domains are more easily compressed and that this effect is linear with stone domain width, making U proportional to T.

The stone domain squeezing algorithm operates on every cell in random order. After selecting a cell, the unit vector along the axis of the stone domain that includes the selected cell is calculated by a linear least squares fit to the positions of all the stones within a distance D_{sq} of the top stone in the selected cell. The direction of transport in this cell by stone domain squeezing, $\hat{u}$, is the average of the vector pointing along the axis of the stone domain weighted by a constant factor C_{sq} (representing confinement) and a randomly oriented unit vector weighted by the factor $1 - C_{sq}$. The gradient in vertical stone domain thickness, $\vec{\nabla}T$ along the vector $\hat{u}$ is calculated and the top stone in the selected cell is moved down gradient by an amount: $K_{sq}dw\vec{\nabla}T$ ($= K_{sq}\vec{\nabla}U$), unless this movement would cause the number of stones in any cell to exceed H_{max}.

4 Hillslope Transport

A hillslope gradient is prescribed within each cell. All cells are chosen in random order and the top stone in each cell is displaced downslope a distance proportional to the hillslope gradient, unless this transport displaces a stone into a cell with a greater number of stones. The downslope transport is given by: $\delta\vec{x} = K_h\vec{\nabla}H$, where K_h is a diffusion constant that relates stone transport (flux) to hillslope gradient. Because lateral sorting is due to downslope stone motion on the ground surface far from stone domains, K_h must equal K_{ls}.