Perturbaciones degeneradas, caso anomalo: Problema 5

```
H0 = DiagonalMatrix[{E0, E1, E1, E1}];
MatrixForm[H0]
V = a {{0, 1, 0, 1}, {1, 2, -1, -1}, {0, -1, 2, -1}, {1, -1, -1, 2}};
MatrixForm[V]
Vdeg = a {{2, -1, -1}, {-1, 2, -1}, {-1, -1, 2}};
MatrixForm[Vdeg]
```

$$\begin{pmatrix} E0 & 0 & 0 & 0 \\ 0 & E1 & 0 & 0 \\ 0 & 0 & E1 & 0 \\ 0 & 0 & 0 & E1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & a & 0 & a \\ a & 2\,a & -a & -a \\ 0 & -a & 2\,a & -a \\ a & -a & -a & 2\,a \end{pmatrix}$$

$$\begin{pmatrix} 2\,a & -a & -a \\ -a & 2\,a & -a \\ -a & -a & 2\,a \end{pmatrix}$$

Exacto

```
EnergiesExact = Sort[Eigenvalues[H0 + V] /. {E0 → 1, E1 → 1.5, a → 0.025}]
```

```
{0.99762, 1.50164, 1.575, 1.57574}
```

```
N[Eigensystem[H0 + V] /. {E0 → 1, E1 → 1.5, a → 0.025}]
```

```
{{1.575, 0.99762, 1.50164, 1.57574},
 {{0., -1., 0., 1.}, {-21.0047, 1., 0.0905173, 1.},
  {0.0996726, 1., 1.03397, 1.}, {0.0868451, 1., -1.94267, 1.}}}
```

```
N[Eigensystem[H0 + V] /. {E0 → 1, E1 → 1.5, a → 2.5 * 10^(-3)}, 15]
```

```
{{1.5075, 0.999975, 1.50002, 1.50751},
 {{0, -1.00000000000000, 0, 1.00000000000000},
  {-201., 1.00000000000000, 0.0099005, 1.00000000000000},
  {0.00999967, 1.00000000000000, 1.00334, 1.00000000000000},
  {0.00985206, 1.00000000000000, -1.99344, 1.00000000000000}}}
```

Primer Orden

Nivel no degenerado

```
(*Energia, No hay correccion , elemento de matriz nulo*)
delta1E0 = V[[1, 1]];
E0Orden1 = E0 + delta1E0
```

```
E0
```

```
(*Autoestado de orden cero, no hay correccion*)
V0Orden0 = {1, 0, 0, 0}
```

```
{1, 0, 0, 0}
```

Nivel degenerado

```
(*Energias, no se rompe totalmente la degeneracion*)
delta2E0 = Sort[Eigenvalues[Vdeg]]
E1Orden1 = Prepend[E1 + Sort[Eigenvalues[Vdeg]], E0]
EnegiasOrden1 =
 Prepend[E1 + Sort[Eigenvalues[Vdeg]], E0] /. {E0 → 1, E1 → 1.5, a → 0.025}
```

```
{0, 3 a, 3 a}
```

```
{E0, E1, 3 a + E1, 3 a + E1}
```

```
{1, 1.5, 1.575, 1.575}
```

```
(*Autoestado de orden cero, hay correccion*)
{delta1E1, eigenVector} = Sort[Eigensystem[Vdeg]]
```

```
{{0, 3 a, 3 a}, {{1, 1, 1}, {-1, 0, 1}, {-1, 1, 0}}}
```

```
V1Orden0 = Prepend[eigenVector[[1]], 0]
E1Orden1 = E1 + delta1E1[[1]] /. {E1 → 1.5}
```

```
{0, -1, 0, 1}
```

```
1.5
```

Persiste la degeneracion, quedan indeterminados dos autoestados a orden cero, con autoenergias E1+3a.
Debemos hacer el calculo de energias a segundo orden para obtener los autoestados a orden cero.

Segundo Orden

Nivel no degenerado

```
(*Energia*)
delta2E0 = Sum[V[[1, k]] (1 / (E0 - E1)) V[[k, 1]], {k, 2, 4}]
E0Orden2 = E0 + delta2E0
N[E0Orden2 /. {E0 → 1, E1 → 1.5, a → 0.025}]
```

$$\frac{2\,a^2}{E0 - E1}$$

$$E0 + \frac{2\,a^2}{E0 - E1}$$

```
0.9975
```

```
(*Autoestado*)
V0Orden1 = {1, 0, 0, 0} + Table[Sum[V[[k, i]] (1 / (E0 - E1)), {k, 2, 4}], {i, 1, 4}]
N[V0Orden1 /. {E0 → 1, E1 → 1.5, a → 0.025}]
```

$$\left\{1 + \frac{2\,a}{E0 - E1},\ 0,\ 0,\ 0\right\}$$

```
{0.9, 0., 0., 0.}
```

No cambia en primer orden el autoestado normalizado

Nivel degenerado

```
(*Energias*)
Mdeg2 =
 Table[Sum[V[[m, k]] (1 / (E1 - E0)) V[[k, n]], {k, 1, 1}], {m, 2, 4}, {n, 2, 4}]
Sort[Eigensystem[Vdeg + Mdeg2]] (*No sale analiticamente*)
{delta2E2, eigenVector} =
 Sort[Eigensystem[Vdeg + Mdeg2]] /. {E0 → 1, E1 → 1.5, a → 0.025}
Energias2 = E1 + delta2E2 /. {E1 → 1.5}
```

$$\left\{\left\{2.\,a^2,\ 0,\ 2.\,a^2\right\},\ \{0,\ 0,\ 0\},\ \left\{2.\,a^2,\ 0,\ 2.\,a^2\right\}\right\}$$

$$\left\{\left\{\{1.,\ 1.03408,\ 1.\},\ \left\{-1.,\ -1.21431 \times 10^{-12},\ 1.\right\},\ \{1.,\ -1.93408,\ 1.\}\right\},\ \{0.00164794,\ 0.075,\ 0.0758521\}\right\}$$

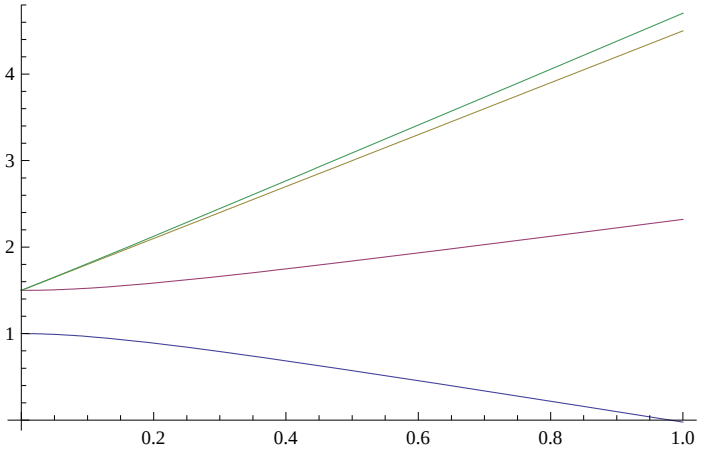
```
{{2.5, 2.53408, 2.5}, {0.5, 1.5, 2.5}, {2.5, -0.434082, 2.5}}
```

Se rompe la degeneracion, quedan determinados los autoestados a orden uno.

Plots

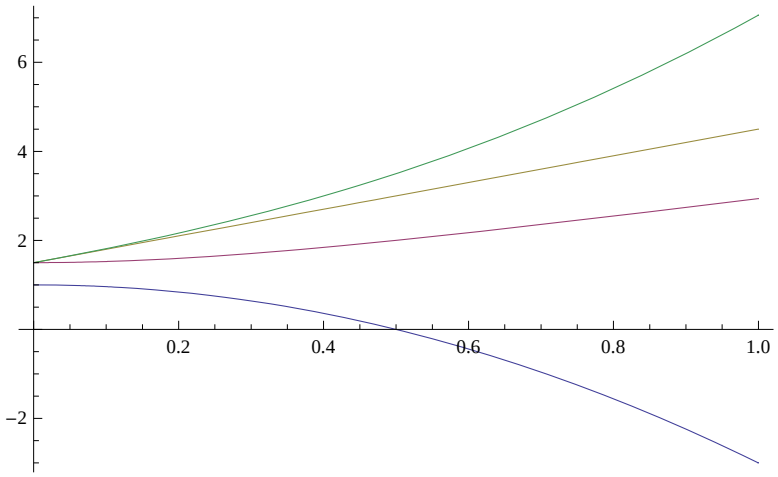
```
E0 = 1; E1 = 1.5;
EigenV = Table[{a, Sort[Eigenvalues[H0 + V]]}, {a, 0, 1, 0.01}];
EigenVP = Table[{a, Sort[Eigenvalues[Vdeg + Mdeg2]]}, {a, 0, 1, 0.01}];

Energia1 =
  Interpolation[Table[{EigenV[[k]][[1]], EigenV[[k]][[2, 1]]}, {k, 1, 100}]];
Energia2 = Interpolation[
   Table[{EigenV[[k]][[1]], EigenV[[k]][[2, 2]]}, {k, 1, 100}]];
Energia3 = Interpolation[Table[{EigenV[[k]][[1]], EigenV[[k]][[2, 3]]},
    {k, 1, 100}]];
Energia4 = Interpolation[Table[{EigenV[[k]][[1]], EigenV[[k]][[2, 4]]},
    {k, 1, 100}]];
Plot[{Energia1[a], Energia2[a], Energia3[a], Energia4[a]}, {a, 0, 1}]
```

Energias Exactas vs a



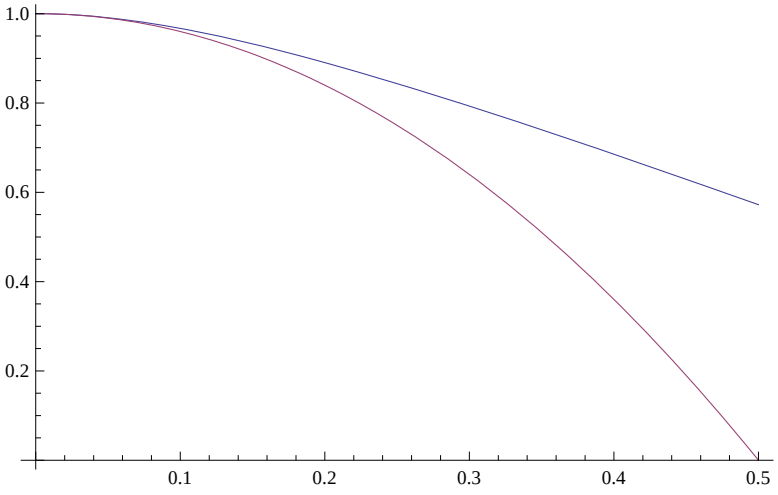
```
Energia1P[a_] := E0 + 2 a^2 / (E0 - E1)
Energia2P = Interpolation[
   Table[{EigenVP[[k]][[1]], E1 + EigenVP[[k]][[2, 1]]}, {k, 1, 101}]];
Energia3P = Interpolation[Table[{EigenVP[[k]][[1]],
     E1 + EigenVP[[k]][[2, 2]]}, {k, 1, 101}]];
Energia4P = Interpolation[Table[{EigenVP[[k]][[1]],
     E1 + EigenVP[[k]][[2, 3]]}, {k, 1, 101}]];
Plot[{Energia1P[a], Energia2P[a], Energia3P[a], Energia4P[a]}, {a, 0, 1}]
```

Energias en Perturbaciones vs a

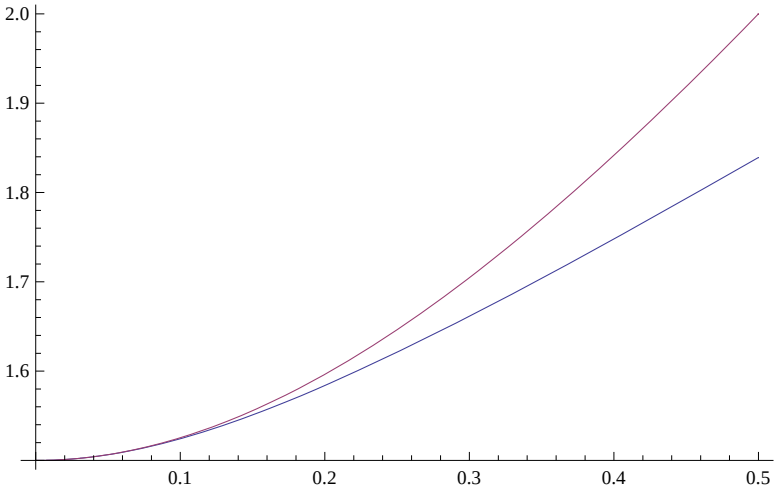


Exactas y Perturbativos por nivel vs a

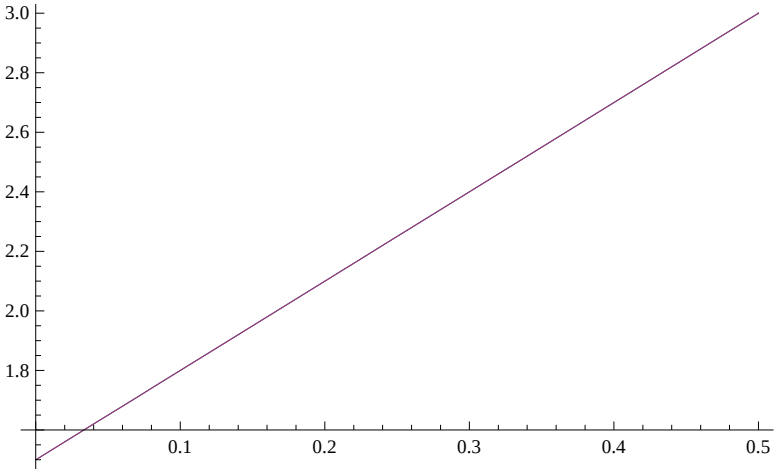
```
Plot[{Energia1[a], Energia1P[a]}, {a, 0, 0.5}]
```



```
Plot[{Energia2[a], Energia2P[a]}, {a, 0, 0.5}]
```




```
Plot[{Energia3[a], Energia3P[a]}, {a, 0, .5}]
```




```
Plot[{Energia4[a], Energia4P[a]}, {a, 0, 0.5}]
```